axiom™

The 30 Year Horizon

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Volume 2: Axiom Users Guide

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New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

0.1 Makefile

This book is actually a literate program[2] and can contain executable source code. In particular, the Makefile for this book is part of the source of the book and is included below. Axiom uses the “noweb” literate programming system by Norman Ramsey[6].

Chapter 1

Writing Spad Code

1.1 The Description: label and the)describe command

The describe command will print out the comments associated with Axiom source code elements. For the category, domain, and package sections the text is taken from the Description: keyword.

This information is stored in a database and can be queried with

```
  )lisp (getdatabase '|Integer| 'documentation)
```

for the Integer domain. However, this information has other uses in the system so it contains tags and control information. Most tags are removed by the describe function since the output is intended to be displayed in ASCII on the terminal.

The Description: keyword is in the comment block just after the abbreviation command. It is freeform and the paragraph will be reflowed automatically to allow for about 60 characters per line, adjusted for spaces. The Description: section should be written after the keyword in the "++" comments as in:

```
)abbrev package D03AGNT d03AgentsPackage
++ Description:
++ This package does some interesting stuff. We can write multiple
++ lines but they should all line up with the first character of
++ the Description keyword. Special \spad{terms} will be removed.
++
++ The above line will force a newline. So will ending a line with \br
++ \tab{5}This will allow primitive formatting\br
++ \tab{5}So you can align text\br
++ \tab{10}Start in column 11\tab{5}and skip 5 spaces\br
++ \tab{10}End in column 11\tab{7}and count out the needed spaces\br
++ \tab{5} note that the last line will not need the br command
```

As the comment says, the Description should all be aligned under the "D" in Description. You can indent using `\tab{n}` which will insert n spaces. You can force a newline in two ways. Either include a blank line (with the "++" comments) or use the `\br` keyword.

Due to lousy parsing algorithms for comments there are various ways this can all go wrong.

There should not be any macros between the Description: section and the beginning of the definition. This is wrong. It will cause the

```
)describe package d03AgentsPackage
```

to give the wrong output because it does not find the end of the description section properly.

```
)abbrev package D03AGNT d03AgentsPackage
++ Description:
++ This description does not work

LEDF  ==> List Expression DoubleFloat

d03AgentsPackage(): E == I where
```

In the Description: section the `\tab{nn}` function will be transformed into nn spaces. If you end each line with a `\br` you can control alignment.

```
++ Description:
++ This is an example of a table alignment\br
++ \tab{5}First Item\tab{5} This will line up with the following line\br
++ \tab{5}Second Item\tab{4} This will line up with the following line\br
++ \tab{5}Third Item\tab{5} This will line up with the following line
```

If the main body of the category, domain, or package begins with properties rather than functions the Description will be incorrectly recorded. This is a known bug finding the end of the Description section. For instance, this

```
++ Description:
++ The category of Lie Algebras.
++ It is used by the domains of non-commutative algebra,
++ LiePolynomial and XPBWPolynomial.

LieAlgebra(R: CommutativeRing): Category ==  Module(R) with
    NullSquare
      ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
    JacobiIdentity
      ++ \axiom{JacobiIdentity} means that
      ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
    construct:  ($,$) -> $
      ++ \axiom{construct(x,y)} returns the Lie bracket of \axiom{x}
      ++ and \axiom{y}.
```

will give the output

```
{JacobiIdentity} means that} [x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0 holds.
```

but reordering it to read:

```
++ Description:
++ The category of Lie Algebras.
++ It is used by the domains of non-commutative algebra,
++ LiePolynomial and XPBWPolynomial.

LieAlgebra(R: CommutativeRing): Category ==  Module(R) with
    construct:  ($,$) -> $
      ++ \axiom{construct(x,y)} returns the Lie bracket of \axiom{x}
      ++ and \axiom{y}.
    NullSquare
      ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
    JacobiIdentity
      ++ \axiom{JacobiIdentity} means that
      ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
```

will give the output

```
The category of Lie Algebras. It is used by the domains of
non-commutative algebra, LiePolynomial and XPBWPolynomial.
```

which is correct.

⟨*⟩≡

```
PROJECT=bookvol2
TANGLE=/usr/local/bin/NOTANGLE
WEAVE=/usr/local/bin/NOWEAVE
LATEX=/usr/bin/latex
MAKEINDEX=/usr/bin/makeindex

all:
	${WEAVE} -t8 -delay ${PROJECT}.pamphlet >${PROJECT}.tex
	${LATEX} ${PROJECT}.tex 2>/dev/null 1>/dev/null
	${MAKEINDEX} ${PROJECT}.idx
	${LATEX} ${PROJECT}.tex 2>/dev/null 1>/dev/null
```

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