axiom™

The 30 Year Horizon

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Volume 10: Axiom Algebra: Domains

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Fritz Schwarz	Nick Simicich	William Sit
Elena Smirnova	Jonathan Steinbach	Christine Sundaresan
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Stephen Wilson	Shmuel Winograd	Robert Wisbauer
Sandra Wityak	Waldemar Wiwianka	Knut Wolf
Clifford Yapp	David Yun	Richard Zippel
Evelyn Zoernack	Bruno Zuercher	Dan Zwillinger

Contents

New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

Chapter 1

Chapter Overview

This book contains the domains in Axiom, in alphabetical order.

Each domain has an associated 'dotpic' chunk which only lists the domains, categories, and packages that are in the layer immediately below in the build order. For the full list see the algebra Makefile where this information is maintained.

Each domain is preceded by a picture. The picture indicates several things. The colors indicate whether the name refers to a category, domain, or package. An ellipse means that the name refers to something in the bootstrap set. Thus,

Chapter 2

Chapter A

2.1 domain AFFPL AffinePlane

— AffinePlane.input —

```
)set break resume
)sys rm -f AffinePlane.output
)spool AffinePlane.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffinePlane
--R AffinePlane K: Field  is a domain constructor
--R Abbreviation for AffinePlane is AFFPL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AFFPL
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                affinePoint : List K -> %
--R coerce : List K -> %                  coerce : % -> List K
--R coerce : % -> OutputForm              conjugate : % -> %
--R definingField : % -> K                degree : % -> PositiveInteger
--R ?.? : (%,Integer) -> K                hash : % -> SingleInteger
--R latex : % -> String                   list : % -> List K
--R orbit : % -> List %                   origin : () -> %
--R pointValue : % -> List K              rational? : % -> Boolean
--R setelt : (%,Integer,K) -> K           ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
```

```
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)
```

———-

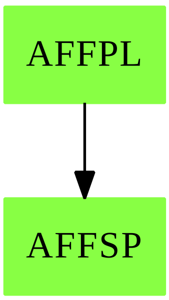
— AffinePlane.help —

```
====================================================================
AffinePlane examples
====================================================================

See Also:
o )show AffinePlane
```

———-

2.1.1 AffinePlane (AFFPL)



Exports:

— domain AFFPL AffinePlane —

```
)abbrev domain AFFPL AffinePlane
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
```

```
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffinePlane(K):Exports == Implementation where
  K:Field

  Exports ==> AffineSpaceCategory(K)

  Implementation ==> AffineSpace(2,K)
```

———-

— AFFPL.dotabb —

```
"AFFPL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFPL"];
"AFFSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFSP"];
"AFFPL" -> "AFFSP"
```

———-

2.2 domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField

— AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)spool AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffinePlaneOverPseudoAlgebraicClosureOfFiniteField
--R AffinePlaneOverPseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory  is a domain constructor
--R Abbreviation for AffinePlaneOverPseudoAlgebraicClosureOfFiniteField is AFFPLPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AFFPLPS
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R conjugate : % -> %                    degree : % -> PositiveInteger
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R orbit : % -> List %                    origin : () -> %
--R rational? : % -> Boolean               ?~=? : (%,%) -> Boolean
--R affinePoint : List PseudoAlgebraicClosureOfFiniteField K -> %
--R coerce : List PseudoAlgebraicClosureOfFiniteField K -> %
--R coerce : % -> List PseudoAlgebraicClosureOfFiniteField K
--R conjugate : (%,NonNegativeInteger) -> %
--R definingField : % -> PseudoAlgebraicClosureOfFiniteField K
--R ?.? : (%,Integer) -> PseudoAlgebraicClosureOfFiniteField K
--R list : % -> List PseudoAlgebraicClosureOfFiniteField K
--R orbit : (%,NonNegativeInteger) -> List %
--R pointValue : % -> List PseudoAlgebraicClosureOfFiniteField K
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R setelt : (%,Integer,PseudoAlgebraicClosureOfFiniteField K) -> PseudoAlgebraicClosureOfFi
--R
--E 1

)spool
)lisp (bye)
```

———

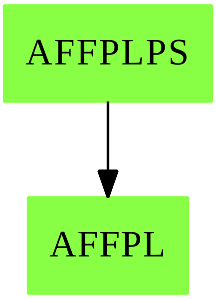
— AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.help —

```
oooo
====================================================================
AffinePlaneOverPseudoAlgebraicClosureOfFiniteField examples
====================================================================

See Also:
o )show AffinePlaneOverPseudoAlgebraicClosureOfFiniteField
```

———

2.2.1 AffinePlaneOverPseudoAlgebraicClosureOfFiniteField (AFFPLPS)



Exports:

?.?	?=?	?~=?	affinePoint	coerce
conjugate	definingField	degree	hash	latex
list	orbit	origin	pointValue	rational?
removeConjugate	setelt			

— **domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField** —

```
)abbrev domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffinePlaneOverPseudoAlgebraicClosureOfFiniteField(K):Exports == Impl where
  K:FiniteFieldCategory

  KK ==> PseudoAlgebraicClosureOfFiniteField(K)

  Exports ==> AffineSpaceCategory(KK)

  Impl ==> AffinePlane(KK)
```

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— **AFFPLPS.dotabb** —

```
"AFFPLPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFPLPS"];
"AFFPL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFPL"];
"AFFPLPS" -> "AFFPL"
```

———-

2.3 domain AFFSP AffineSpace

— AffineSpace.input —

```
)set break resume
)sys rm -f AffineSpace.output
)spool AffineSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffineSpace
--R AffineSpace(dim: NonNegativeInteger,K: Field)  is a domain constructor
--R Abbreviation for AffineSpace is AFFSP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AFFSP
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                affinePoint : List K -> %
--R coerce : List K -> %                  coerce : % -> List K
--R coerce : % -> OutputForm              conjugate : % -> %
--R definingField : % -> K                degree : % -> PositiveInteger
--R ?.? : (%,Integer) -> K                hash : % -> SingleInteger
--R latex : % -> String                   list : % -> List K
--R orbit : % -> List %                   origin : () -> %
--R pointValue : % -> List K              rational? : % -> Boolean
--R setelt : (%,Integer,K) -> K           ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)
```

———-

— AffineSpace.help —

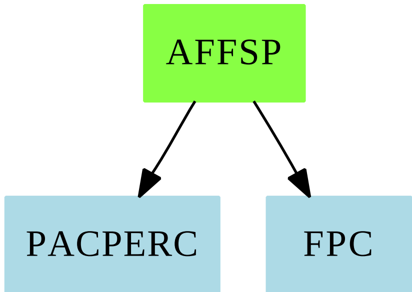
```
====================================================================
```

```
AffineSpace examples
====================================================================

See Also:
o )show AffineSpace
```

———-

2.3.1 AffineSpace (AFFSP)



Exports:

?.?	?=?	?~=?	affinePoint	coerce
conjugate	definingField	degree	hash	latex
list	orbit	origin	pointValue	rational?
removeConjugate	setelt			

— **domain AFFSP AffineSpace** —

```
)abbrev domain AFFSP AffineSpace
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffineSpace(dim,K):Exports == Implementation where
  dim:NonNegativeInteger
  K:Field

  LIST ==> List
  NNI  ==> NonNegativeInteger

  Exports ==> AffineSpaceCategory(K)
```

```
Implementation ==> List(K)  add

  Rep:= List(K)

  origin== new(dim,0$K)$List(K)

  coerce(pt:%):OutputForm ==
    dd:OutputForm:= ":" :: OutputForm
    llout:List(OutputForm):=[ hconcat(dd, a::OutputForm) for a in rest pt]
    lout:= cons( (first pt)::OutputForm , llout)
    out:= hconcat lout
    oo:=paren(out)
    ee:OutputForm:= degree(pt) :: OutputForm
    oo**ee

  definingField(pt)==
    K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
      maxTower(pt pretend Rep)
    1$K

  degree(pt)==
    K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
      extDegree definingField pt
    1

  coerce(pt:%):List(K) == pt pretend Rep

  affinePoint(pt:LIST(K))==
      pt :: %

  list(ptt)==
    ptt pretend Rep

  pointValue(ptt)==
    ptt pretend Rep

  conjugate(p,e)==
    lp:Rep:=p
    pc:List(K):=[c**e for c in lp]
    affinePoint(pc)

  rational?(p,n)== p=conjugate(p,n)

  rational?(p)==rational?(p,characteristic()$K)

  removeConjugate(l)==removeConjugate(l,characteristic()$K)

  removeConjugate(l:LIST(%),n:NNI):LIST(%)==
    if K has FiniteFieldCategory then
      allconj:LIST(%):=empty()
```

```
        conjrem:LIST(%):=empty()
        for p in l repeat
          if ^member?(p,allconj) then
            conjrem:=cons(p,conjrem)
            allconj:=concat(allconj,orbit(p,n))
        conjrem
      else
        error "The field is not finite"

    conjugate(p)==conjugate(p,characteristic()$K)

    orbit(p)==orbit(p,characteristic()$K)

    orbit(p,e)==
      if K has FiniteFieldCategory then
        l:LIST(%):=[p]
        np:%:=conjugate(p,e)
        flag:=^(np=p)::Boolean
        while flag repeat
          l:=concat(np,l)
          np:=conjugate(np,e)
          flag:=not (np=p)::Boolean
        l
      else
        error "Cannot compute the conjugate"

    aa:% = bb:% ==
      aa =$Rep bb

    coerce(pt:LIST(K))==
        ^(dim=#pt) => error "Le point n'a pas la bonne dimension"
        ptt:%:= pt
        ptt
```

———-

— AFFSP.dotabb —

```
"AFFSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFSP"];
"PACPERC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"FPC" [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"AFFSP" -> "FPC"
"AFFSP" -> "PACPERC"
```

———-

2.4 domain ALGSC AlgebraGivenByStructuralConstants

— AlgebraGivenByStructuralConstants.input —

```
)set break resume
)sys rm -f AlgebraGivenByStructuralConstants.output
)spool AlgebraGivenByStructuralConstants.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraGivenByStructuralConstants
--R AlgebraGivenByStructuralConstants(R: Field,n: PositiveInteger,ls: List Symbol,gamma: Vect
--R Abbreviation for AlgebraGivenByStructuralConstants is ALGSC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ALGSC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (SquareMatrix(n,R),%) -> %      ?*? : (R,%) -> %
--R ?*? : (%,R) -> %                      ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R alternative? : () -> Boolean          antiAssociative? : () -> Boolean
--R antiCommutative? : () -> Boolean      antiCommutator : (%,%) -> %
--R apply : (Matrix R,%) -> %             associative? : () -> Boolean
--R associator : (%,%,%) -> %             basis : () -> Vector %
--R coerce : Vector R -> %                coerce : % -> OutputForm
--R commutative? : () -> Boolean          commutator : (%,%) -> %
--R convert : Vector R -> %               convert : % -> Vector R
--R coordinates : % -> Vector R           ?.? : (%,Integer) -> R
--R flexible? : () -> Boolean             hash : % -> SingleInteger
--R jacobiIdentity? : () -> Boolean       jordanAdmissible? : () -> Boolean
--R jordanAlgebra? : () -> Boolean        latex : % -> String
--R leftAlternative? : () -> Boolean      leftDiscriminant : () -> R
--R leftDiscriminant : Vector % -> R      leftNorm : % -> R
--R leftTrace : % -> R                    leftTraceMatrix : () -> Matrix R
--R lieAdmissible? : () -> Boolean        lieAlgebra? : () -> Boolean
--R powerAssociative? : () -> Boolean     rank : () -> PositiveInteger
--R represents : Vector R -> %            rightAlternative? : () -> Boolean
--R rightDiscriminant : () -> R           rightDiscriminant : Vector % -> R
--R rightNorm : % -> R                    rightTrace : % -> R
--R rightTraceMatrix : () -> Matrix R     sample : () -> %
--R someBasis : () -> Vector %            zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
```

```
--R associatorDependence : () -> List Vector R if R has INTDOM
--R conditionsForIdempotents : () -> List Polynomial R
--R conditionsForIdempotents : Vector % -> List Polynomial R
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
--R leftRegularRepresentation : % -> Matrix R
--R leftRegularRepresentation : (%,Vector %) -> Matrix R
--R leftTraceMatrix : Vector % -> Matrix R
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if R has INTDOM
--R represents : (Vector R,Vector %) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : % -> Matrix R
--R rightRegularRepresentation : (%,Vector %) -> Matrix R
--R rightTraceMatrix : Vector % -> Matrix R
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R structuralConstants : () -> Vector Matrix R
--R structuralConstants : Vector % -> Vector Matrix R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— AlgebraGivenByStructuralConstants.help —

```
====================================================================
AlgebraGivenByStructuralConstants examples
====================================================================

See Also:
```

```
o )show AlgebraGivenByStructuralConstants
```

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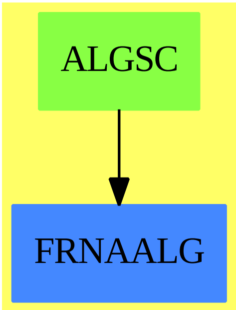
2.4.1 AlgebraGivenByStructuralConstants (ALGSC)



Exports:

0
alternative?
antiAssociative?
antiCommutative?
antiCommutator
apply
associative?
associator
associatorDependence
basis
coerce
commutative?
commutator
conditionsForIdempotents
convert
coordinates
flexible?
hash
jacobiIdentity?
jordanAdmissible?
jordanAlgebra?
latex
leftAlternative?
leftCharacteristicPolynomial
leftDiscriminant
leftMinimalPolynomial
leftNorm
leftPower
leftRankPolynomial
leftRecip
leftRegularRepresentation
leftTrace
leftTraceMatrix
leftUnit
leftUnits
lieAdmissible?
lieAlgebra?
noncommutativeJordanAlgebra?
plenaryPower
powerAssociative?
rank
recip
represents
rightAlternative?
rightCharacteristicPolynomial
rightDiscriminant
rightMinimalPolynomial
rightNorm
rightPower
rightRankPolynomial
rightRecip
rightRegularRepresentation
rightTrace
rightTraceMatrix
rightUnit
rightUnits
sample
someBasis
structuralConstants
subtractIfCan
unit
zero?
?*?
?**?
?+?
?-?
-?
?=?
?.?
?~=?
?*?

— domain ALGSC AlgebraGivenByStructuralConstants —

```
)abbrev domain ALGSC AlgebraGivenByStructuralConstants
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 22 January 1992
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords: algebra, structural constants
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++ AlgebraGivenByStructuralConstants implements finite rank algebras
++ over a commutative ring, given by the structural constants \spad{gamma}
++ with respect to a fixed  basis \spad{[a1,..,an]}, where
++ \spad{gamma} is an \spad{n}-vector of n by n matrices
++ \spad{[(gammaijk) for k in 1..rank()]} defined by
++ \spad{ai * aj = gammaij1 * a1 + ... + gammaijn * an}.
++ The symbols for the fixed basis
++ have to be given as a list of symbols.

AlgebraGivenByStructuralConstants(R:Field, n : PositiveInteger,_
  ls : List Symbol, gamma: Vector Matrix R ): public == private where

  V  ==> Vector
  M  ==> Matrix
  I  ==> Integer
  NNI  ==> NonNegativeInteger
  REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
  LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)

  --public ==> FramedNonAssociativeAlgebra(R) with
  public ==> Join(FramedNonAssociativeAlgebra(R), _
    LeftModule(SquareMatrix(n,R)) ) with

    coerce : Vector R -> %
      ++ coerce(v) converts a vector to a member of the algebra
      ++ by forming a linear combination with the basis element.
      ++ Note: the vector is assumed to have length equal to the
      ++ dimension of the algebra.

  private ==> DirectProduct(n,R) add

    Rep := DirectProduct(n,R)

    x,y : %
    dp : DirectProduct(n,R)
    v : V R


    recip(x) == recip(x)$FiniteRankNonAssociativeAlgebra_&(%,R)

    (m:SquareMatrix(n,R))*(x:%) == apply((m :: Matrix R),x)
    coerce v == directProduct(v) :: %

    structuralConstants() == gamma
```

```
    coordinates(x) == vector(entries(x :: Rep)$Rep)$Vector(R)

    coordinates(x,b) ==
      --not (maxIndex b = n) =>
      --  error("coordinates: your 'basis' has not the right length")
      m : NonNegativeInteger := (maxIndex b) :: NonNegativeInteger
      transitionMatrix   : Matrix R := new(n,m,0$R)$Matrix(R)
      for i in 1..m repeat
        setColumn_!(transitionMatrix,i,coordinates(b.i))
      res : REC := solve(transitionMatrix,coordinates(x))$LSMP
      if (not every?(zero?$R,first res.basis)) then
        error("coordinates: warning your 'basis' is linearly dependent")
      (res.particular  case "failed") =>
        error("coordinates: first argument is not in linear span of second argument")
      (res.particular) :: (Vector R)

    basis() == [unitVector(i::PositiveInteger)::% for i in 1..n]

    someBasis() == basis()$%

    rank() == n

    elt(x,i) == elt(x:Rep,i)$Rep

    coerce(x:%):OutputForm ==
      zero?(x::Rep)$Rep => (0$R) :: OutputForm
      le : List OutputForm := nil
      for i in 1..n repeat
        coef : R := elt(x::Rep,i)
        not zero?(coef)$R =>
--          one?(coef)$R =>
          ((coef) = 1)$R =>
            -- sy : OutputForm := elt(ls,i)$(List Symbol) :: OutputForm
            le := cons(elt(ls,i)$(List Symbol) :: OutputForm, le)
          le := cons(coef :: OutputForm *  elt(ls,i)$(List Symbol)_
             :: OutputForm, le)
      reduce("+",le)

    x * y ==
      v : Vector R :=  new(n,0)
      for k in 1..n repeat
        h : R := 0
        for i in 1..n repeat
          for j in 1..n repeat
            h := h  +$R elt(x,i) *$R elt(y,j) *$R elt(gamma.k,i,j )
        v.k := h
      directProduct v
```

```
alternative?() ==
  for i in 1..n repeat
    -- expression for check of left alternative is symmetric in i and j:
    -- expression for check of right alternative is symmetric in j and k:
    for j in 1..i-1 repeat
      for k in j..n repeat
        -- right check
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res - _
              (elt(gamma.l,j,k)+elt(gamma.l,k,j))*elt(gamma.r,i,l)+_
              (elt(gamma.l,i,j)*elt(gamma.r,l,k) + elt(gamma.l,i,k)*_
              elt(gamma.r,l,j) )
          not zero? res =>
            messagePrint("algebra is not right alternative")$OutputForm
            return false
    for j in i..n repeat
      for k in 1..j-1 repeat
        -- left check
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res + _
              (elt(gamma.l,i,j)+elt(gamma.l,j,i))*elt(gamma.r,l,k)-_
              (elt(gamma.l,j,k)*elt(gamma.r,i,l) + elt(gamma.l,i,k)*_
               elt(gamma.r,j,l) )
          not (zero? res) =>
            messagePrint("algebra is not left alternative")$OutputForm
            return false

      for k in j..n repeat
        -- left check
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res + _
              (elt(gamma.l,i,j)+elt(gamma.l,j,i))*elt(gamma.r,l,k)-_
              (elt(gamma.l,j,k)*elt(gamma.r,i,l) + elt(gamma.l,i,k)*_
               elt(gamma.r,j,l) )
          not (zero? res) =>
            messagePrint("algebra is not left alternative")$OutputForm
            return false
        -- right check
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res - _
              (elt(gamma.l,j,k)+elt(gamma.l,k,j))*elt(gamma.r,i,l)+_
              (elt(gamma.l,i,j)*elt(gamma.r,l,k) + elt(gamma.l,i,k)*_
```

```
              elt(gamma.r,l,j) )
            not (zero? res) =>
              messagePrint("algebra is not right alternative")$OutputForm
              return false

      messagePrint("algebra satisfies 2*associator(a,b,b) = 0 = 2*associator(a,a,b) = 0")$OutputForm
      true

    -- should be in the category, but is not exported
--    conditionsForIdempotents b  ==
--      n := rank()
--      --gamma : Vector Matrix R := structuralConstants b
--      listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
--      symbolsForCoef : Vector Symbol :=
--        [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
--      conditions : List Polynomial R := []
 --     for k in 1..n repeat
 --       xk := symbolsForCoef.k
 --       p : Polynomial R :=  monomial( - 1$Polynomial(R), [xk], [1] )
 --       for i in 1..n repeat
 --         for j in 1..n repeat
 --           xi := symbolsForCoef.i
 --           xj := symbolsForCoef.j
 --           p := p + monomial(_
 --             elt((gamma.k),i,j) :: Polynomial(R), [xi,xj], [1,1])
 --       conditions := cons(p,conditions)
 --     conditions

    associative?() ==
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
         for r in 1..n repeat
           res := 0$R
           for l in 1..n repeat
             res := res + elt(gamma.l,i,j)*elt(gamma.r,l,k)-_
                          elt(gamma.l,j,k)*elt(gamma.r,i,l)
           not (zero? res) =>
            messagePrint("algebra is not associative")$OutputForm
            return false
      messagePrint("algebra is associative")$OutputForm
      true


    antiAssociative?() ==
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
         for r in 1..n repeat
           res := 0$R
```

```
      for l in 1..n repeat
        res := res + elt(gamma.l,i,j)*elt(gamma.r,l,k)+_
                     elt(gamma.l,j,k)*elt(gamma.r,i,l)
      not (zero? res) =>
       messagePrint("algebra is not anti-associative")$OutputForm
       return false
  messagePrint("algebra is anti-associative")$OutputForm
  true

commutative?() ==
  for i in 1..n repeat
   for j in (i+1)..n repeat
    for k in 1..n repeat
      not ( elt(gamma.k,i,j)=elt(gamma.k,j,i) ) =>
       messagePrint("algebra is not commutative")$OutputForm
       return false
  messagePrint("algebra is commutative")$OutputForm
  true

antiCommutative?() ==
  for i in 1..n repeat
   for j in i..n repeat
    for k in 1..n repeat
     not zero? (i=j => elt(gamma.k,i,i); elt(gamma.k,i,j)+elt(gamma.k,j,i) ) =>
       messagePrint("algebra is not anti-commutative")$OutputForm
       return false
  messagePrint("algebra is anti-commutative")$OutputForm
  true

leftAlternative?() ==
  for i in 1..n repeat
   -- expression is symmetric in i and j:
   for j in i..n repeat
    for k in 1..n repeat
     for r in 1..n repeat
      res := 0$R
      for l in 1..n repeat
        res := res + (elt(gamma.l,i,j)+elt(gamma.l,j,i))*elt(gamma.r,l,k)-_
          (elt(gamma.l,j,k)*elt(gamma.r,i,l) + elt(gamma.l,i,k)*elt(gamma.r,j,l) )
      not (zero? res) =>
       messagePrint("algebra is not left alternative")$OutputForm
       return false
  messagePrint("algebra is left alternative")$OutputForm
  true


rightAlternative?() ==
  for i in 1..n repeat
   for j in 1..n repeat
   -- expression is symmetric in j and k:
```

```
    for k in j..n repeat
     for r in 1..n repeat
       res := 0$R
       for l in 1..n repeat
         res := res - (elt(gamma.l,j,k)+elt(gamma.l,k,j))*elt(gamma.r,i,l)+_
           (elt(gamma.l,i,j)*elt(gamma.r,l,k) + elt(gamma.l,i,k)*elt(gamma.r,l,j) )
       not (zero? res) =>
        messagePrint("algebra is not right alternative")$OutputForm
        return false
  messagePrint("algebra is right alternative")$OutputForm
  true


flexible?() ==
  for i in 1..n repeat
   for j in 1..n repeat
   -- expression is symmetric in i and k:
    for k in i..n repeat
     for r in 1..n repeat
       res := 0$R
       for l in 1..n repeat
         res := res + elt(gamma.l,i,j)*elt(gamma.r,l,k)-_
                      elt(gamma.l,j,k)*elt(gamma.r,i,l)+_
                      elt(gamma.l,k,j)*elt(gamma.r,l,i)-_
                      elt(gamma.l,j,i)*elt(gamma.r,k,l)
       not (zero? res) =>
        messagePrint("algebra is not flexible")$OutputForm
        return false
  messagePrint("algebra is flexible")$OutputForm
  true

lieAdmissible?() ==
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for r in 1..n repeat
       res := 0$R
       for l in 1..n repeat
         res := res_
          + (elt(gamma.l,i,j)-elt(gamma.l,j,i))*(elt(gamma.r,l,k)-elt(gamma.r,k,l)) _
          + (elt(gamma.l,j,k)-elt(gamma.l,k,j))*(elt(gamma.r,l,i)-elt(gamma.r,i,l)) _
          + (elt(gamma.l,k,i)-elt(gamma.l,i,k))*(elt(gamma.r,l,j)-elt(gamma.r,j,l))
       not (zero? res) =>
        messagePrint("algebra is not Lie admissible")$OutputForm
        return false
  messagePrint("algebra is Lie admissible")$OutputForm
  true

jordanAdmissible?()  ==
  recip(2 * 1$R) case "failed" =>
```

```
    messagePrint("this algebra is not Jordan admissible, as 2 is not invertible in the gr
    false
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for w in 1..n repeat
      for t in 1..n repeat
       res := 0$R
       for l in 1..n repeat
        for r in 1..n repeat
         res := res_
          + (elt(gamma.l,i,j)+elt(gamma.l,j,i))_
            * (elt(gamma.r,w,k)+elt(gamma.r,k,w))_
            * (elt(gamma.t,l,r)+elt(gamma.t,r,l))_
          - (elt(gamma.r,w,k)+elt(gamma.r,k,w))_
            * (elt(gamma.l,j,r)+elt(gamma.l,r,j))_
            * (elt(gamma.t,i,l)+elt(gamma.t,l,i))_
          + (elt(gamma.l,w,j)+elt(gamma.l,j,w))_
            * (elt(gamma.r,k,i)+elt(gamma.r,i,k))_
            * (elt(gamma.t,l,r)+elt(gamma.t,r,l))_
          - (elt(gamma.r,k,i)+elt(gamma.r,k,i))_
            * (elt(gamma.l,j,r)+elt(gamma.l,r,j))_
            * (elt(gamma.t,w,l)+elt(gamma.t,l,w))_
          + (elt(gamma.l,k,j)+elt(gamma.l,j,k))_
            * (elt(gamma.r,i,w)+elt(gamma.r,w,i))_
            * (elt(gamma.t,l,r)+elt(gamma.t,r,l))_
          - (elt(gamma.r,i,w)+elt(gamma.r,w,i))_
            * (elt(gamma.l,j,r)+elt(gamma.l,r,j))_
            * (elt(gamma.t,k,l)+elt(gamma.t,l,k))
       not (zero? res) =>
         messagePrint("algebra is not Jordan admissible")$OutputForm
         return false
  messagePrint("algebra is Jordan admissible")$OutputForm
  true

jordanAlgebra?()  ==
  recip(2 * 1$R) case "failed" =>
    messagePrint("this is not a Jordan algebra, as 2 is not invertible in the ground rin
    false
  not commutative?() =>
    messagePrint("this is not a Jordan algebra")$OutputForm
    false
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for l in 1..n repeat
       for t in 1..n repeat
         res := 0$R
         for r in 1..n repeat
           for s in 1..n repeat
```

```
                res := res +  _
                  elt(gamma.r,i,j)*elt(gamma.s,l,k)*elt(gamma.t,r,s) - _
                  elt(gamma.r,l,k)*elt(gamma.s,j,r)*elt(gamma.t,i,s) + _
                  elt(gamma.r,l,j)*elt(gamma.s,k,k)*elt(gamma.t,r,s) - _
                  elt(gamma.r,k,i)*elt(gamma.s,j,r)*elt(gamma.t,l,s) + _
                  elt(gamma.r,k,j)*elt(gamma.s,i,k)*elt(gamma.t,r,s) - _
                  elt(gamma.r,i,l)*elt(gamma.s,j,r)*elt(gamma.t,k,s)
                not zero? res =>
                  messagePrint("this is not a Jordan algebra")$OutputForm
                  return false
      messagePrint("this is a Jordan algebra")$OutputForm
      true


    jacobiIdentity?()  ==
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
         for r in 1..n repeat
           res := 0$R
           for s in 1..n repeat
                 res := res +  elt(gamma.r,i,j)*elt(gamma.s,j,k) +_
                               elt(gamma.r,j,k)*elt(gamma.s,k,i) +_
                               elt(gamma.r,k,i)*elt(gamma.s,i,j)
           not zero? res =>
                 messagePrint("Jacobi identity does not hold")$OutputForm
                 return false
      messagePrint("Jacobi identity holds")$OutputForm
      true
```

— ALGSC.dotabb —

```
"ALGSC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALGSC"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"ALGSC" -> "FRNAALG"
```

2.5 domain ALGFF AlgebraicFunctionField

— AlgebraicFunctionField.input —

```
)set break resume
```

```
)sys rm -f AlgebraicFunctionField.output
)spool AlgebraicFunctionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraicFunctionField
--R AlgebraicFunctionField(F: Field,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePolyno
--R Abbreviation for AlgebraicFunctionField is ALGFF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ALGFF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction UP,%) -> %            ?*? : (%,Fraction UP) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        basis : () -> Vector %
--R branchPoint? : UP -> Boolean          branchPoint? : F -> Boolean
--R coerce : Fraction UP -> %             coerce : Integer -> %
--R coerce : % -> OutputForm              convert : UPUP -> %
--R convert : % -> UPUP                   convert : Vector Fraction UP -> %
--R convert : % -> Vector Fraction UP     definingPolynomial : () -> UPUP
--R discriminant : () -> Fraction UP      elt : (%,F,F) -> F
--R generator : () -> %                   genus : () -> NonNegativeInteger
--R hash : % -> SingleInteger             integral? : (%,UP) -> Boolean
--R integral? : (%,F) -> Boolean          integral? : % -> Boolean
--R integralBasis : () -> Vector %        latex : % -> String
--R lift : % -> UPUP                      norm : % -> Fraction UP
--R one? : % -> Boolean                   primitivePart : % -> %
--R ramified? : UP -> Boolean             ramified? : F -> Boolean
--R rank : () -> PositiveInteger          rationalPoint? : (F,F) -> Boolean
--R recip : % -> Union(%,"failed")        reduce : UPUP -> %
--R represents : (Vector UP,UP) -> %      retract : % -> Fraction UP
--R sample : () -> %                      singular? : UP -> Boolean
--R singular? : F -> Boolean              trace : % -> Fraction UP
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Fraction UP has FIELD
--R ?*? : (Fraction Integer,%) -> % if Fraction UP has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if Fraction UP has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Fraction UP has FIELD
--R D : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIELI
--R D : (%,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FIELD or I
--R D : (%,Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
```

```
--R D : (%,Symbol,NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING
--R D : (%,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R D : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if Fraction UP has FIELD
--R ?^? : (%,Integer) -> % if Fraction UP has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R absolutelyIrreducible? : () -> Boolean
--R algSplitSimple : (%,(UP -> UP)) -> Record(num: %,den: UP,derivden: UP,gd: UP)
--R associates? : (%,%) -> Boolean if Fraction UP has FIELD
--R branchPointAtInfinity? : () -> Boolean
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UPUP
--R charthRoot : % -> Union(%,"failed") if Fraction UP has CHARNZ
--R charthRoot : % -> % if Fraction UP has FFIELDC
--R coerce : % -> % if Fraction UP has FIELD
--R coerce : Fraction Integer -> % if Fraction UP has FIELD or Fraction UP has RETRACT FRAC INT
--R complementaryBasis : Vector % -> Vector %
--R conditionP : Matrix % -> Union(Vector %,"failed") if Fraction UP has FFIELDC
--R coordinates : Vector % -> Matrix Fraction UP
--R coordinates : % -> Vector Fraction UP
--R coordinates : (Vector %,Vector %) -> Matrix Fraction UP
--R coordinates : (%,Vector %) -> Vector Fraction UP
--R createPrimitiveElement : () -> % if Fraction UP has FFIELDC
--R derivationCoordinates : (Vector %,(Fraction UP -> Fraction UP)) -> Matrix Fraction UP if Fraction UP
--R differentiate : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIEL
--R differentiate : (%,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FIELD or
--R differentiate : (%,Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRI
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction U
--R differentiate : (%,(UP -> UP)) -> %
--R differentiate : (%,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R differentiate : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if Fraction UP has FIELD
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction UP has FFIELDC
--R discreteLog : % -> NonNegativeInteger if Fraction UP has FFIELDC
--R discriminant : Vector % -> Fraction UP
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Fraction UP has FIELD
--R elliptic : () -> Union(UP,"failed")
--R euclideanSize : % -> NonNegativeInteger if Fraction UP has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Fraction UP has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Fraction UP has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Fraction UP has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Fraction UP has FIELD
--R factor : % -> Factored % if Fraction UP has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if Fraction UP has F
--R gcd : (%,%) -> % if Fraction UP has FIELD
--R gcd : List % -> % if Fraction UP has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R hyperelliptic : () -> Union(UP,"failed")
--R index : PositiveInteger -> % if Fraction UP has FINITE
```

```
--R init : () -> % if Fraction UP has FFIELDC
--R integralAtInfinity? : % -> Boolean
--R integralBasisAtInfinity : () -> Vector %
--R integralCoordinates : % -> Record(num: Vector UP,den: UP)
--R integralDerivationMatrix : (UP -> UP) -> Record(num: Matrix UP,den: UP)
--R integralMatrix : () -> Matrix Fraction UP
--R integralMatrixAtInfinity : () -> Matrix Fraction UP
--R integralRepresents : (Vector UP,UP) -> %
--R inv : % -> % if Fraction UP has FIELD
--R inverseIntegralMatrix : () -> Matrix Fraction UP
--R inverseIntegralMatrixAtInfinity : () -> Matrix Fraction UP
--R knownInfBasis : NonNegativeInteger -> Void
--R lcm : (%,%) -> % if Fraction UP has FIELD
--R lcm : List % -> % if Fraction UP has FIELD
--R lookup : % -> PositiveInteger if Fraction UP has FINITE
--R minimalPolynomial : % -> UPUP if Fraction UP has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Fraction UP has FIELD
--R nextItem : % -> Union(%,"failed") if Fraction UP has FFIELDC
--R nonSingularModel : Symbol -> List Polynomial F if F has FIELD
--R normalizeAtInfinity : Vector % -> Vector %
--R numberOfComponents : () -> NonNegativeInteger
--R order : % -> OnePointCompletion PositiveInteger if Fraction UP has FFIELDC
--R order : % -> PositiveInteger if Fraction UP has FFIELDC
--R prime? : % -> Boolean if Fraction UP has FIELD
--R primeFrobenius : % -> % if Fraction UP has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction UP has FFIELDC
--R primitive? : % -> Boolean if Fraction UP has FFIELDC
--R primitiveElement : () -> % if Fraction UP has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Fraction UP has FIELD
--R ?quo? : (%,%) -> % if Fraction UP has FIELD
--R ramifiedAtInfinity? : () -> Boolean
--R random : () -> % if Fraction UP has FINITE
--R rationalPoints : () -> List List F if F has FINITE
--R reduce : Fraction UPUP -> Union(%,"failed") if Fraction UP has FIELD
--R reduceBasisAtInfinity : Vector % -> Vector %
--R reducedSystem : Matrix % -> Matrix Fraction UP
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction UP,vec: Vector Fracti
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if Fraction UP has LINEXP INT
--R regularRepresentation : % -> Matrix Fraction UP
--R regularRepresentation : (%,Vector %) -> Matrix Fraction UP
--R ?rem? : (%,%) -> % if Fraction UP has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if Fraction UP has FF
--R represents : Vector Fraction UP -> %
--R represents : (Vector Fraction UP,Vector %) -> %
--R retract : % -> Fraction Integer if Fraction UP has RETRACT FRAC INT
--R retract : % -> Integer if Fraction UP has RETRACT INT
--R retractIfCan : % -> Union(Fraction UP,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if Fraction UP has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if Fraction UP has RETRACT INT
```

```
--R singularAtInfinity? : () -> Boolean
--R size : () -> NonNegativeInteger if Fraction UP has FINITE
--R sizeLess? : (%,%) -> Boolean if Fraction UP has FIELD
--R squareFree : % -> Factored % if Fraction UP has FIELD
--R squareFreePart : % -> % if Fraction UP has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Fraction UP has
--R traceMatrix : () -> Matrix Fraction UP
--R traceMatrix : Vector % -> Matrix Fraction UP
--R unit? : % -> Boolean if Fraction UP has FIELD
--R unitCanonical : % -> % if Fraction UP has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Fraction UP has FIELD
--R yCoordinates : % -> Record(num: Vector UP,den: UP)
--R
--E 1

)spool
)lisp (bye)
```

———-

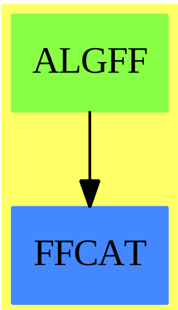
— **AlgebraicFunctionField.help** —

```
====================================================================
AlgebraicFunctionField examples
====================================================================

See Also:
o )show AlgebraicFunctionField
```

———-

2.5.1 AlgebraicFunctionField (ALGFF)



See

⇒ "RadicalFunctionField" (RADFF) 19.1.1 on page 2153

Exports:

1	0	absolutelyIrreducible?
algSplitSimple	associates?	basis
branchPoint?	branchPointAtInfinity?	characteristic
characteristicPolynomial	charthRoot	coerce
complementaryBasis	conditionP	convert
coordinates	createPrimitiveElement	D
definingPolynomial	derivationCoordinates	differentiate
discreteLog	discriminant	divide
elliptic	elt	euclideanSize
expressIdealMember	exquo	extendedEuclidean
factor	factorsOfCyclicGroupSize	gcd
gcdPolynomial	generator	genus
hash	hyperelliptic	index
init	integral?	integralAtInfinity?
integralBasis	integralBasisAtInfinity	integralCoordinates
integralDerivationMatrix	integralMatrix	integralMatrixAtInfinity
integralRepresents	inv	inverseIntegralMatrix
inverseIntegralMatrixAtInfinity	knownInfBasis	latex
lcm	lift	lookup
minimalPolynomial	multiEuclidean	nextItem
nonSingularModel	norm	normalizeAtInfinity
numberOfComponents	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	primitivePart	principalIdeal
ramified?	ramifiedAtInfinity?	random
rank	rationalPoint?	rationalPoints
recip	reduce	reduceBasisAtInfinity
reducedSystem	regularRepresentation	representationType
represents	retract	retractIfCan
sample	singular?	singularAtInfinity?
size	sizeLess?	squareFree
squareFreePart	subtractIfCan	tableForDiscreteLogarithm
trace	traceMatrix	unit?
unitCanonical	unitNormal	yCoordinates
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?quo?	?rem?

— domain ALGFF AlgebraicFunctionField —

```
)abbrev domain ALGFF AlgebraicFunctionField
++ Author: Manuel Bronstein
```

```
++ Date Created: 3 May 1988
++ Date Last Updated: 24 Jul 1990
++ Keywords: algebraic, curve, function, field.
++ Description:
++ Function field defined by f(x, y) = 0.

AlgebraicFunctionField(F, UP, UPUP, modulus): Exports == Impl where
  F      : Field
  UP     : UnivariatePolynomialCategory F
  UPUP   : UnivariatePolynomialCategory Fraction UP
  modulus: UPUP

  N   ==> NonNegativeInteger
  Z   ==> Integer
  RF  ==> Fraction UP
  QF  ==> Fraction UPUP
  UP2 ==> SparseUnivariatePolynomial UP
  SAE ==> SimpleAlgebraicExtension(RF, UPUP, modulus)
  INIT ==> if (deref brandNew?) then startUp false

  Exports ==> FunctionFieldCategory(F, UP, UPUP) with
    knownInfBasis: N -> Void
        ++ knownInfBasis(n) is not documented

  Impl ==> SAE add
    import ChangeOfVariable(F, UP, UPUP)
    import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
    import MatrixCommonDenominator(UP, RF)
    import UnivariatePolynomialCategoryFunctions2(RF, UPUP, UP, UP2)

    startUp    : Boolean -> Void
    vect       : Matrix RF -> Vector $
    getInfBasis: () -> Void

    brandNew?:Reference(Boolean) := ref true
    infBr?:Reference(Boolean) := ref true
    discPoly:Reference(RF) := ref 0
    n  := degree modulus
    n1 := (n - 1)::N
    ibasis:Matrix(RF)     := zero(n, n)
    invibasis:Matrix(RF)  := copy ibasis
    infbasis:Matrix(RF)   := copy ibasis
    invinfbasis:Matrix(RF):= copy ibasis

    branchPointAtInfinity?()   == (INIT; infBr?())
    discriminant()             == (INIT; discPoly())
    integralBasis()            == (INIT; vect ibasis)
    integralBasisAtInfinity()  == (INIT; vect infbasis)
    integralMatrix()           == (INIT; ibasis)
    inverseIntegralMatrix()    == (INIT; invibasis)
```

```
integralMatrixAtInfinity() == (INIT; infbasis)
branchPoint?(a:F)          == zero?((retract(discriminant())@UP) a)
definingPolynomial()       == modulus
inverseIntegralMatrixAtInfinity() == (INIT; invinfbasis)

vect m ==
  [represents row(m, i) for i in minRowIndex m .. maxRowIndex m]

integralCoordinates f ==
  splitDenominator(coordinates(f) * inverseIntegralMatrix())

knownInfBasis d ==
  if deref brandNew? then
    alpha := [monomial(1, d * i)$UP :: RF for i in 0..n1]$Vector(RF)
    ib := diagonalMatrix
      [inv qelt(alpha, i) for i in minIndex alpha .. maxIndex alpha]
    invib := diagonalMatrix alpha
    for i in minRowIndex ib .. maxRowIndex ib repeat
      for j in minColIndex ib .. maxColIndex ib repeat
        infbasis(i, j)    := qelt(ib, i, j)
        invinfbasis(i, j) := invib(i, j)
  void

getInfBasis() ==
  x           := inv(monomial(1, 1)$UP :: RF)
  invmod      := map(s +-> s(x), modulus)
  r           := mkIntegral invmod
  degree(r.poly) ^= n => error "Should not happen"
  ninvmod:UP2 := map(s +-> retract(s)@UP, r.poly)
  alpha       := [(r.coef ** i) x for i in 0..n1]$Vector(RF)
  invalpha := [inv qelt(alpha, i)
               for i in minIndex alpha .. maxIndex alpha]$Vector(RF)
  invib       := integralBasis()$FunctionFieldIntegralBasis(UP, UP2,
                         SimpleAlgebraicExtension(UP, UP2, ninvmod))
  for i in minRowIndex ibasis .. maxRowIndex ibasis repeat
    for j in minColIndex ibasis .. maxColIndex ibasis repeat
      infbasis(i, j)    := ((invib.basis)(i,j) / invib.basisDen) x
      invinfbasis(i, j) := ((invib.basisInv) (i, j)) x
  ib2    := infbasis * diagonalMatrix alpha
  invib2 := diagonalMatrix(invalpha) * invinfbasis
  for i in minRowIndex ib2 .. maxRowIndex ib2 repeat
    for j in minColIndex ibasis .. maxColIndex ibasis repeat
      infbasis(i, j)    := qelt(ib2, i, j)
      invinfbasis(i, j) := invib2(i, j)
  void

startUp b ==
  brandNew?() := b
  nmod:UP2    := map(retract, modulus)
  ib          := integralBasis()$FunctionFieldIntegralBasis(UP, UP2,
```

```
                                SimpleAlgebraicExtension(UP, UP2, nmod))
      for i in minRowIndex ibasis .. maxRowIndex ibasis repeat
        for j in minColIndex ibasis .. maxColIndex ibasis repeat
          qsetelt_!(ibasis, i, j, (ib.basis)(i, j) / ib.basisDen)
          invibasis(i, j) := ((ib.basisInv) (i, j))::RF
      if zero?(infbasis(minRowIndex infbasis, minColIndex infbasis))
        then getInfBasis()
      ib2    := coordinates normalizeAtInfinity vect ibasis
      invib2 := inverse(ib2)::Matrix(RF)
      for i in minRowIndex ib2 .. maxRowIndex ib2 repeat
        for j in minColIndex ib2 .. maxColIndex ib2 repeat
          ibasis(i, j)    := qelt(ib2, i, j)
          invibasis(i, j) := invib2(i, j)
      dsc  := resultant(modulus, differentiate modulus)
      dsc0 := dsc * determinant(infbasis) ** 2
      degree(numer dsc0) > degree(denom dsc0) =>error "Shouldn't happen"
      infBr?() := degree(numer dsc0) < degree(denom dsc0)
      dsc := dsc * determinant(ibasis) ** 2
      discPoly() := primitivePart(numer dsc) / denom(dsc)
      void

    integralDerivationMatrix d ==
      w := integralBasis()
      splitDenominator(coordinates([differentiate(w.i, d)
          for i in minIndex w .. maxIndex w]$Vector($))
               * inverseIntegralMatrix())

    integralRepresents(v, d) ==
      represents(coordinates(represents(v, d)) * integralMatrix())

    branchPoint?(p:UP) ==
      INIT
      (r:=retractIfCan(p)@Union(F,"failed")) case F =>branchPoint?(r::F)
      not ground? gcd(retract(discriminant())@UP, p)
```

— ALGFF.dotabb —

```
"ALGFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALGFF"]
"FFCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FFCAT"]
"ALGFF" -> "FFCAT"
```

2.6 domain AN AlgebraicNumber

— AlgebraicNumber.input —

```
)set break resume
)sys rm -f AlgebraicNumber.output
)spool AlgebraicNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraicNumber
--R AlgebraicNumber  is a domain constructor
--R Abbreviation for AlgebraicNumber is AN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AN
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?*? : (Integer,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (%,Fraction Integer) -> %
--R ?*? : (Fraction Integer,%) -> %      ?**? : (%,PositiveInteger) -> %
--R ?**? : (%,Integer) -> %              ?**? : (%,Fraction Integer) -> %
--R ?+? : (%,%) -> %                     -? : % -> %
--R ?-? : (%,%) -> %                     ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              D : % -> %
--R D : (%,NonNegativeInteger) -> %      1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R ?^? : (%,Integer) -> %               associates? : (%,%) -> Boolean
--R belong? : BasicOperator -> Boolean   box : List % -> %
--R box : % -> %                         coerce : Integer -> %
--R coerce : % -> %                      coerce : Fraction Integer -> %
--R coerce : Kernel % -> %               coerce : % -> OutputForm
--R convert : % -> Complex Float         convert : % -> DoubleFloat
--R convert : % -> Float                 differentiate : % -> %
--R distribute : (%,%) -> %              distribute : % -> %
--R elt : (BasicOperator,%,%) -> %       elt : (BasicOperator,%) -> %
--R eval : (%,List %,List %) -> %        eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %           eval : (%,List Equation %) -> %
--R eval : (%,Kernel %,%) -> %           factor : % -> Factored %
--R freeOf? : (%,Symbol) -> Boolean      freeOf? : (%,%) -> Boolean
--R gcd : (%,%) -> %                     gcd : List % -> %
--R hash : % -> SingleInteger            height : % -> NonNegativeInteger
--R inv : % -> %                         is? : (%,Symbol) -> Boolean
--R kernel : (BasicOperator,%) -> %      kernels : % -> List Kernel %
--R latex : % -> String                  lcm : (%,%) -> %
```

```
--R lcm : List % -> %                    map : ((% -> %),Kernel %) -> %
--R max : (%,%) -> %                     min : (%,%) -> %
--R norm : (%,List Kernel %) -> %        norm : (%,Kernel %) -> %
--R nthRoot : (%,Integer) -> %           one? : % -> Boolean
--R paren : List % -> %                  paren : % -> %
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       reduce : % -> %
--R ?rem? : (%,%) -> %                   retract : % -> Fraction Integer
--R retract : % -> Integer               retract : % -> Kernel %
--R rootOf : Polynomial % -> %           rootsOf : Polynomial % -> List %
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R sqrt : % -> %                        squareFree : % -> Factored %
--R squareFreePart : % -> %              subst : (%,Equation %) -> %
--R tower : % -> List Kernel %           unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R zeroOf : Polynomial % -> %           zerosOf : Polynomial % -> List %
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coerce : SparseMultivariatePolynomial(Integer,Kernel %) -> %
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(Integer,Kernel %)
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
--R eval : (%,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R is? : (%,BasicOperator) -> Boolean
--R kernel : (BasicOperator,List %) -> %
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
```

```
--R norm : (SparseUnivariatePolynomial %,List Kernel %) -> SparseUnivariatePolynomial %
--R norm : (SparseUnivariatePolynomial %,Kernel %) -> SparseUnivariatePolynomial %
--R numer : % -> SparseMultivariatePolynomial(Integer,Kernel %)
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reducedSystem : Matrix % -> Matrix Fraction Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction Integer,vec: Vector Fr
--R reducedSystem : Matrix % -> Matrix Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R rootOf : SparseUnivariatePolynomial % -> %
--R rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R rootsOf : SparseUnivariatePolynomial % -> List %
--R rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R zeroOf : SparseUnivariatePolynomial % -> %
--R zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R zerosOf : SparseUnivariatePolynomial % -> List %
--R zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R
--E 1

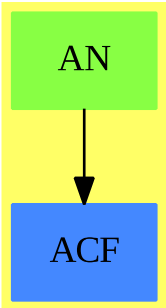
)spool
)lisp (bye)
```

— **AlgebraicNumber.help** —

```
====================================================================
AlgebraicNumber examples
====================================================================

See Also:
o )show AlgebraicNumber
```

2.6.1 AlgebraicNumber (AN)



See

⇒ "InnerAlgebraicNumber" (IAN) 10.20.1 on page 1240

Exports:

1	0	associates?	belong?
box	characteristic	coerce	convert
D	definingPolynomial	denom	differentiate
distribute	divide	elt	euclideanSize
eval	even?	expressIdealMember	exquo
extendedEuclidean	factor	freeOf?	gcd
gcdPolynomial	hash	height	inv
is?	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	norm
nthRoot	numer	odd?	one?
operator	operators	paren	prime?
principalIdeal	recip	reduce	reducedSystem
retract	retractIfCan	rootOf	rootsOf
sample	sizeLess?	sqrt	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	zero?
zeroOf	zerosOf	?*?	?**?
?+?	-?	?-?	?/?
?<?	?<=?	?=?	?>?
?>=?	?^?	?quo?	?rem?
?~=?			

— domain AN AlgebraicNumber —

```
)abbrev domain AN AlgebraicNumber
++ Author: James Davenport
++ Date Created: 9 October 1995
++ Date Last Updated: 10 October 1995 (JHD)
++ Keywords: algebraic, number.
```

```
++ Description:
++ Algebraic closure of the rational numbers, with mathematical =

AlgebraicNumber(): Exports == Implementation where
  Z   ==> Integer
  P   ==> SparseMultivariatePolynomial(Z, Kernel %)
  SUP ==>  SparseUnivariatePolynomial

  Exports ==> Join(ExpressionSpace, AlgebraicallyClosedField,
                   RetractableTo Z, RetractableTo Fraction Z,
                   LinearlyExplicitRingOver Z, RealConstant,
                   LinearlyExplicitRingOver Fraction Z,
                   CharacteristicZero,
                   ConvertibleTo Complex Float, DifferentialRing) with
    coerce : P -> %
      ++ coerce(p) returns p viewed as an algebraic number.
    numer  : % -> P
      ++ numer(f) returns the numerator of f viewed as a
      ++ polynomial in the kernels over Z.
    denom  : % -> P
      ++ denom(f) returns the denominator of f viewed as a
      ++ polynomial in the kernels over Z.
    reduce : % -> %
      ++ reduce(f) simplifies all the unreduced algebraic numbers
      ++ present in f by applying their defining relations.
    norm : (SUP(%),Kernel %) -> SUP(%)
      ++ norm(p,k) computes the norm of the polynomial p
      ++ with respect to the extension generated by kernel k
    norm : (SUP(%),List Kernel %) -> SUP(%)
      ++ norm(p,l) computes the norm of the polynomial p
      ++ with respect to the extension generated by kernels l
    norm : (%,Kernel %) -> %
      ++ norm(f,k) computes the norm of the algebraic number f
      ++ with respect to the extension generated by kernel k
    norm : (%,List Kernel %) -> %
      ++ norm(f,l) computes the norm of the algebraic number f
      ++ with respect to the extension generated by kernels l
  Implementation ==> InnerAlgebraicNumber add
    Rep:=InnerAlgebraicNumber
    a,b:%
    zero? a == trueEqual(a::Rep,0::Rep)
    one? a == trueEqual(a::Rep,1::Rep)
    a=b == trueEqual((a-b)::Rep,0::Rep)
```

———-

— AN.dotabb —

```
"AN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AN"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"AN" -> "ACF"
```

———-

2.7 domain ANON AnonymousFunction

— AnonymousFunction.input —

```
)set break resume
)sys rm -f AnonymousFunction.output
)spool AnonymousFunction.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AnonymousFunction
--R AnonymousFunction  is a domain constructor
--R Abbreviation for AnonymousFunction is ANON
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ANON
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

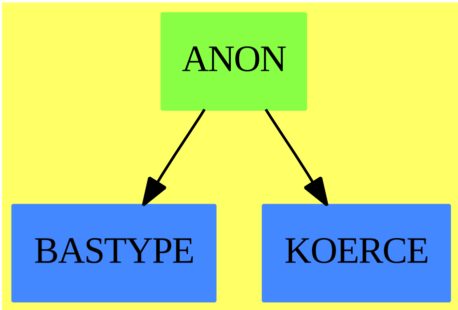
— AnonymousFunction.help —

```
====================================================================
AnonymousFunction examples
====================================================================

See Also:
o )show AnonymousFunction
```

———-

2.7.1 AnonymousFunction (ANON)



Exports:
coerce hash latex ?=? ?~=?

— domain ANON AnonymousFunction —

```
)abbrev domain ANON AnonymousFunction
++ Author: Mark Botch
++ Description:
++ This domain implements anonymous functions

AnonymousFunction():SetCategory == add
  coerce(x:%):OutputForm == x pretend OutputForm
```

———-

— ANON.dotabb —

```
"ANON" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ANON"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"ANON" -> "BASTYPE"
"ANON" -> "KOERCE"
```

———-

2.8 domain ANTISYM AntiSymm

— AntiSymm.input —

```
)set break resume
)sys rm -f AntiSymm.output
)spool AntiSymm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AntiSymm
--R AntiSymm(R: Ring,lVar: List Symbol)  is a domain constructor
--R Abbreviation for AntiSymm is ANTISYM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ANTISYM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coefficient : (%,%) -> R              coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger      exp : List Integer -> %
--R hash : % -> SingleInteger             homogeneous? : % -> Boolean
--R latex : % -> String                   leadingBasisTerm : % -> %
--R leadingCoefficient : % -> R           map : ((R -> R),%) -> %
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R reductum : % -> %                     retract : % -> R
--R retractable? : % -> Boolean           sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R generator : NonNegativeInteger -> %
--R retractIfCan : % -> Union(R,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

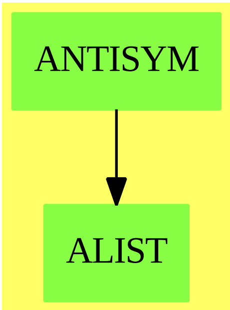
— AntiSymm.help —

```
====================================================================
AntiSymm examples
====================================================================

See Also:
o )show AntiSymm
```

———-

2.8.1 AntiSymm (ANTISYM)



See

⇒ “ExtAlgBasis” (EAB) 6.8.1 on page 711
⇒ “DeRhamComplex” (DERHAM) 5.6.1 on page 515

Exports:

1	0	coefficient	coerce	coerce
coerce	degree	exp	hash	homogeneous?
latex	leadingBasisTerm	leadingCoefficient	map	one?
recip	reductum	retract	retractable?	sample
zero?	characteristic	generator	retractIfCan	subtractIfCan
?*?	?**?	?+?	?-?	-?
?=?	?^?	?~=?		

— **domain ANTISYM AntiSymm** —

```
)abbrev domain ANTISYM AntiSymm
++ Author: Larry A. Lambe
++ Date     : 01/26/91.
++ Revised  : 30 Nov 94
++ Description:
++ The domain of antisymmetric polynomials.

AntiSymm(R:Ring, lVar:List Symbol): Export == Implement where
  LALG ==> LeftAlgebra
```

```
  FMR  ==> FM(R,EAB)
  FM   ==> FreeModule
  I    ==> Integer
  L    ==> List
  EAB  ==> ExtAlgBasis     -- these are exponents of basis elements in order
  NNI  ==> NonNegativeInteger
  O    ==> OutputForm
  base ==> k
  coef ==> c
  Term ==> Record(k:EAB,c:R)

  Export == Join(LALG(R), RetractableTo(R)) with
      leadingCoefficient : %           -> R
        ++ leadingCoefficient(p) returns the leading
        ++ coefficient of antisymmetric polynomial p.
--    leadingSupport       : %           -> EAB
      leadingBasisTerm     : %           -> %
        ++ leadingBasisTerm(p) returns the leading
        ++ basis term of antisymmetric polynomial p.
      reductum           : %           -> %
        ++ reductum(p), where p is an antisymmetric polynomial,
        ++ returns p minus the leading
        ++ term of p if p has at least two terms, and 0 otherwise.
      coefficient        : (%,%)     -> R
        ++ coefficient(p,u) returns the coefficient of
        ++ the term in p containing the basis term u if such
        ++ a term exists, and 0 otherwise.
        ++ Error: if the second argument u is not a basis element.
      generator          : NNI         -> %
        ++ generator(n) returns the nth multiplicative generator,
        ++ a basis term.
      exp                : L I         -> %
        ++  exp([i1,...in]) returns \spad{u_1\^{i_1} ... u_n\^{i_n}}
      homogeneous?       : %           -> Boolean
        ++  homogeneous?(p) tests if all of the terms of
        ++  p have the same degree.
      retractable?       : %           -> Boolean
        ++  retractable?(p) tests if p is a 0-form,
        ++  i.e., if degree(p) = 0.
      degree             : %           -> NNI
        ++  degree(p) returns the homogeneous degree of p.
      map                : (R -> R, %) -> %
        ++  map(f,p) changes each coefficient of p by the
        ++  application of f.


--    1 corresponds to the empty monomial Nul = [0,...,0]
--    from EAB.  In terms of the exterior algebra on X,
--    it corresponds to the identity element which lives
--    in homogeneous degree 0.
```

```
Implement == FMR add
    Rep := L Term
    x,y :  EAB
    a,b :  %
    r   :  R
    m   :  I

    dim := #lVar

    1 == [[ Nul(dim)$EAB, 1$R ]]

    coefficient(a,u) ==
      not null u.rest => error "2nd argument must be a basis element"
      x := u.first.base
      for t in a repeat
        if t.base = x then return t.coef
        if t.base < x then return 0
      0

    retractable?(a) ==
      null a or (a.first.k  =  Nul(dim))

    retractIfCan(a):Union(R,"failed") ==
      null a               => 0$R
      a.first.k = Nul(dim) => leadingCoefficient a
      "failed"

    retract(a):R ==
      null a => 0$R
      leadingCoefficient a

    homogeneous? a ==
      null a => true
      siz := _+/exponents(a.first.base)
      for ta in reductum a repeat
        _+/exponents(ta.base) ^= siz => return false
      true

    degree a ==
      null a => 0$NNI
      homogeneous? a => (_+/exponents(a.first.base)) :: NNI
      error "not a homogeneous element"

    zo : (I,I) -> L I
    zo(p,q) ==
      p = 0 => [1,q]
      q = 0 => [1,1]
      [0,0]
```

```
getsgn : (EAB,EAB) -> I
getsgn(x,y) ==
  sgn:I  := 0
  xx:L I := exponents x
  yy:L I := exponents y
  for i in 1 .. (dim-1) repeat
    xx  := rest xx
    sgn := sgn + (_+/xx)*yy.i
  sgn rem 2 = 0 => 1
  -1

Nalpha: (EAB,EAB) -> L I
Nalpha(x,y) ==
  i:I := 1
  dum2:L I := [0 for i in 1..dim]
  for j in 1..dim repeat
    dum:=zo((exponents x).j,(exponents y).j)
    (i:= i*dum.1) = 0 => leave
    dum2.j := dum.2
  i = 0 => cons(i, dum2)
  cons(getsgn(x,y), dum2)

a * b ==
  null a => 0
  null b => 0
  ((null a.rest) and (a.first.k = Nul(dim))) => a.first.c * b
  ((null b.rest) and (b.first.k = Nul(dim))) => b.first.c * a
  z:% := 0
  for tb in b repeat
    for ta in a repeat
      stuff:=Nalpha(ta.base,tb.base)
      r:=first(stuff)*ta.coef*tb.coef
      if r ^= 0 then z := z + [[rest(stuff)::EAB, r]]
  z

coerce(r):% ==
  r = 0 => 0
  [ [Nul(dim), r] ]

coerce(m):% ==
  m = 0 => 0
  [ [Nul(dim), m::R] ]

characteristic() == characteristic()$R

generator(j) ==
  -- j < 1 or j > dim => error "your subscript is out of range"
  -- error will be generated by dum.j if out of range
  dum:L I := [0 for i in 1..dim]
  dum.j:=1
```

```
        [[dum::EAB, 1::R]]

      exp(li:(L I)) ==  [[li::EAB, 1]]

      leadingBasisTerm a ==
        [[a.first.k, 1]]

      displayList:EAB -> O
      displayList(x):O ==
        le: L I := exponents(x)$EAB
--      reduce(_*,[(lVar.i)::O for i in 1..dim | le.i = 1])$L(O)
--        reduce(_*,[(lVar.i)::O for i in 1..dim | one?(le.i)])$L(O)
        reduce(_*,[(lVar.i)::O for i in 1..dim | ((le.i) = 1)])$L(O)

      makeTerm:(R,EAB) -> O
      makeTerm(r,x) ==
      -- we know that r ^= 0
        x = Nul(dim)$EAB  => r::O
--        one? r => displayList(x)
        (r = 1) => displayList(x)
--      r = 1 => displayList(x)
--      r = 0 => 0$I::O
--      x = Nul(dim)$EAB  => r::O
        r::O * displayList(x)

      coerce(a):O ==
        zero? a     => 0$I::O
        null rest(a @ Rep) =>
                 t := first(a @ Rep)
                 makeTerm(t.coef,t.base)
        reduce(_+,[makeTerm(t.coef,t.base) for t in (a @ Rep)])$L(O)
```

———-

— **ANTISYM.dotabb** —

```
"ANTISYM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ANTISYM"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ANTISYM" -> "ALIST"
```

———-

2.9 domain ANY Any

— **Any.input** —

```
)set break resume
)sys rm -f Any.output
)spool Any.output
)set message test on
)set message auto off
)clear all

--S 1 of 18
a:Any := [1,2]
--R
--R
--R   (1)  [1,2]
--R                                                  Type: List PositiveInteger
--E 1

--S 2 of 18
b:Any := [1,2]
--R
--R
--R   (2)  [1,2]
--R                                                  Type: List PositiveInteger
--E 2

--S 3 of 18
(a = b)@Boolean
--R
--R
--R   (3)  true
--R                                                              Type: Boolean
--E 3

--S 4 of 18
c := [1,2]
--R
--R
--R   (4)  [1,2]
--R                                                  Type: List PositiveInteger
--E 4

--S 5 of 18
typeOf a
--R
--R
--R   (5)  Any
--R                                                               Type: Domain
--E 5

--S 6 of 18
typeOf c
--R
```

```
--R
--R   (6)  List PositiveInteger
--R                                                          Type: Domain
--E 6

--S 7 of 18
(a = c)@Boolean
--R
--R
--R   (7)  true
--R                                                         Type: Boolean
--E 7

--S 8 of 18
b := [1,3]
--R
--R
--R   (8)  [1,3]
--R                                           Type: List PositiveInteger
--E 8

--S 9 of 18
(a = b)@Boolean
--R
--R
--R   (9)  false
--R                                                         Type: Boolean
--E 9

--S 10 of 18
a := "A"
--R
--R
--R   (10)  "A"
--R                                                          Type: String
--E 10

--S 11 of 18
(a = b)@Boolean
--R
--R
--R   (11)  false
--R                                                         Type: Boolean
--E 11

--S 12 of 18
b := "A"
--R
--R
--R   (12)  "A"
```

```
--R                                                            Type: String
--E 12

--S 13 of 18
(a = b)@Boolean
--R
--R
--R   (13)  true
--R                                                           Type: Boolean
--E 13

--S 14 of 18
Sae := SAE(FRAC INT, UP(x, FRAC INT), x^2-3)
--R
--R
--R   (14)
--R  SimpleAlgebraicExtension(Fraction Integer,UnivariatePolynomial(x,Fraction Int
--R  eger),x*x-3)
--R                                                            Type: Domain
--E 14

--S 15 of 18
a := generator()$Sae
--R
--R
--R   (15)  x
--RType: SimpleAlgebraicExtension(Fraction Integer,UnivariatePolynomial(x,Fraction Integer),x*x-3)
--E 15

--S 16 of 18
b := generator()$Sae
--R
--R
--R   (16)  x
--RType: SimpleAlgebraicExtension(Fraction Integer,UnivariatePolynomial(x,Fraction Integer),x*x-3)
--E 16

--S 17 of 18
(a = b)@Boolean
--R
--R
--R   (17)  true
--R                                                           Type: Boolean
--E 17

--S 18 of 18
)show Any
--R
--R Any  is a domain constructor
--R Abbreviation for Any is ANY
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ANY
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                 any : (SExpression,None) -> %
--R coerce : % -> OutputForm               dom : % -> SExpression
--R domainOf : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                    obj : % -> None
--R objectOf : % -> OutputForm             ?~=? : (%,%) -> Boolean
--R showTypeInOutput : Boolean -> String
--R
--E 18

)spool
)lisp (bye)
```

———

— Any.help —

```
====================================================================
Any examples
====================================================================

Any implements a type that packages up objects and their types in
objects of Any. Roughly speaking that means that if s : S then when
converted to Any, the new object will include both the original object
and its type. This is a way of converting arbitrary objects into a
single type without losing any of the original information. Any object
can be converted to one of Any.

So we can convert a list to type Any

a:Any := [1,2]
   [1,2]

and another list to type Any

b:Any := [1,2]
   [1,2]

Equality works

(a = b)@Boolean
   true

We can compare the Any type with other types:

c := [1,2]
```

```
typeOf a

typeOf c

(a = c)@Boolean


If the values are differennt than we see the difference:

b := [1,3]
   [1,3]

(a = b)@Boolean
    false

The Any type works with many types:

a := "A"
    "A"

(a = b)@Boolean
    false

b := "A"
    "A"

(a = b)@Boolean
   true

This is true for more complex types:

Sae := SAE(FRAC INT, UP(x, FRAC INT), x^2-3)

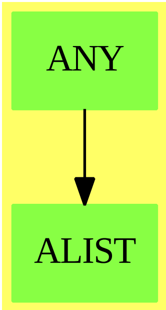
a := generator()$Sae
   x

b := generator()$Sae
   x

(a = b)@Boolean
   true

See Also:
o )show Any
```

2.9.1 Any (ANY)



See

⇒ "None" (NONE) 15.4.1 on page 1700

Exports:

any	coerce	dom	domainOf	hash
latex	obj	objectOf	showTypeInOutput	?=?
?~=?				

— domain ANY Any —

```
)abbrev domain ANY Any
++ Author: Robert S. Sutor
++ Date Created:
++ Change History:
++ Basic Functions: any, domainOf, objectOf, dom, obj, showTypeInOutput
++ Related Constructors: AnyFunctions1
++ Also See: None
++ AMS Classification:
++ Keywords:
++ Description:
++ \spadtype{Any} implements a type that packages up objects and their
++ types in objects of \spadtype{Any}. Roughly speaking that means
++ that if \spad{s : S} then when converted to \spadtype{Any}, the new
++ object will include both the original object and its type. This is
++ a way of converting arbitrary objects into a single type without
++ losing any of the original information. Any object can be converted
++ to one of \spadtype{Any}.

Any(): SetCategory with
        any             : (SExpression, None) -> %
          ++ any(type,object) is a technical function for creating
          ++ an object of \spadtype{Any}. Arugment \spad{type} is a
          ++ \spadgloss{LISP} form for the type of \spad{object}.
        domainOf        : % -> OutputForm
          ++ domainOf(a) returns a printable form of the type of the
          ++ original object that was converted to \spadtype{Any}.
```

```
      objectOf        : % -> OutputForm
        ++ objectOf(a) returns a printable form of the
        ++ original object that was converted to \spadtype{Any}.
      dom             : % -> SExpression
        ++ dom(a) returns a \spadgloss{LISP} form of the type of the
        ++ original object that was converted to \spadtype{Any}.
      obj             : % -> None
        ++ obj(a) essentially returns the original object that was
        ++ converted to \spadtype{Any} except that the type is forced
        ++ to be \spadtype{None}.
      showTypeInOutput: Boolean -> String
        ++ showTypeInOutput(bool) affects the way objects of
        ++ \spadtype{Any} are displayed. If \spad{bool} is true
        ++ then the type of the original object that was converted
        ++ to \spadtype{Any} will be printed. If \spad{bool} is
        ++ false, it will not be printed.

 == add
    Rep := Record(dm: SExpression, ob: None)

    printTypeInOutputP:Reference(Boolean) := ref false

    obj x      == x.ob
    dom x      == x.dm
    domainOf x == x.dm pretend OutputForm
    x = y      == (x.dm = y.dm) and EQ(x.ob, y.ob)$Lisp
    x = y      ==
     (x.dm = y.dm) and EQUAL(x.ob, y.ob)$Lisp

    objectOf(x : %) : OutputForm ==
      spad2BootCoerce(x.ob, x.dm,
         list("OutputForm"::Symbol)$List(Symbol))$Lisp

    showTypeInOutput(b : Boolean) : String ==
     printTypeInOutputP := ref b
     b=> "Type of object will be displayed in output of a member of Any"
     "Type of object will not be displayed in output of a member of Any"

    coerce(x):OutputForm ==
      obj1 : OutputForm := objectOf x
      not deref printTypeInOutputP => obj1
      dom1 :=
        p:Symbol := prefix2String(devaluate(x.dm)$Lisp)$Lisp
        atom?(p pretend SExpression) => list(p)$List(Symbol)
        list(p)$Symbol
      hconcat cons(obj1,
        cons(":"::OutputForm, [a::OutputForm for a in dom1]))

    any(domain, object) ==
      (isValidType(domain)$Lisp)@Boolean => [domain, object]
```

```
      domain := devaluate(domain)$Lisp
      (isValidType(domain)$Lisp)@Boolean => [domain, object]
      error "function any must have a domain as first argument"
```

———-

— ANY.dotabb —

```
"ANY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ANY"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ANY" -> "ALIST"
```

———-

2.10 domain ASTACK ArrayStack

— ArrayStack.input —

```
)set break resume
)sys rm -f ArrayStack.output
)spool ArrayStack.output
)set message test on
)set message auto off
)clear all

--S 1 of 44
a:ArrayStack INT:= arrayStack [1,2,3,4,5]
--R
--R
--R   (1)  [1,2,3,4,5]
--R                                                        Type: ArrayStack Integer
--E 1

--S 2 of 44
pop! a
--R
--R
--R   (2)  1
--R                                                           Type: PositiveInteger
--E 2

--S 3 of 44
a
--R
```

```
--R
--R   (3)  [2,3,4,5]
--R                                                    Type: ArrayStack Integer
--E 3

--S 4 of 44
extract! a
--R
--R
--R   (4)  2
--R                                                       Type: PositiveInteger
--E 4

--S 5 of 44
a
--R
--R
--R   (5)  [3,4,5]
--R                                                    Type: ArrayStack Integer
--E 5

--S 6 of 44
push!(9,a)
--R
--R
--R   (6)  9
--R                                                       Type: PositiveInteger
--E 6

--S 7 of 44
a
--R
--R
--R   (7)  [9,3,4,5]
--R                                                    Type: ArrayStack Integer
--E 7

--S 8 of 44
insert!(8,a)
--R
--R
--R   (8)  [8,9,3,4,5]
--R                                                    Type: ArrayStack Integer
--E 8

--S 9 of 44
a
--R
--R
--R   (9)  [8,9,3,4,5]
```

```
--R                                                Type: ArrayStack Integer
--E 9

--S 10 of 44
inspect a
--R
--R
--R   (10)  8
--R                                                   Type: PositiveInteger
--E 10

--S 11 of 44
empty? a
--R
--R
--R   (11)  false
--R                                                           Type: Boolean
--E 11

--S 12 of 44
top a
--R
--R
--R   (12)  8
--R                                                   Type: PositiveInteger
--E 12

--S 13 of 44
depth a
--R
--R
--R   (13)  5
--R                                                   Type: PositiveInteger
--E 13

--S 14 of 44
#a
--R
--R
--R   (14)  5
--R                                                   Type: PositiveInteger
--E 14

--S 15 of 44
less?(a,9)
--R
--R
--R   (15)  true
--R                                                           Type: Boolean
--E 15
```

```
--S 16 of 44
more?(a,9)
--R
--R
--R   (16)  false
--R                                                            Type: Boolean
--E 16

--S 17 of 44
size?(a,#a)
--R
--R
--R   (17)  true
--R                                                            Type: Boolean
--E 17

--S 18 of 44
size?(a,9)
--R
--R
--R   (18)  false
--R                                                            Type: Boolean
--E 18

--S 19 of 44
parts a
--R
--R
--R   (19)  [8,9,3,4,5]
--R                                                       Type: List Integer
--E 19

--S 20 of 44
bag([1,2,3,4,5])$ArrayStack(INT)
--R
--R
--R   (20)  [5,4,3,2,1]
--R                                                 Type: ArrayStack Integer
--E 20

--S 21 of 44
b:=empty()$(ArrayStack INT)
--R
--R
--R   (21)  []
--R                                                 Type: ArrayStack Integer
--E 21

--S 22 of 44
```

```
empty? b
--R
--R
--R   (22)  true
--R                                                            Type: Boolean
--E 22

--S 23 of 44
sample()$ArrayStack(INT)
--R
--R
--R   (23)  []
--R                                                  Type: ArrayStack Integer
--E 23

--S 24 of 44
c:=copy a
--R
--R
--R   (24)  [8,9,3,4,5]
--R                                                  Type: ArrayStack Integer
--E 24

--S 25 of 44
eq?(a,c)
--R
--R
--R   (25)  false
--R                                                            Type: Boolean
--E 25

--S 26 of 44
eq?(a,a)
--R
--R
--R   (26)  true
--R                                                            Type: Boolean
--E 26

--S 27 of 44
(a=c)@Boolean
--R
--R
--R   (27)  true
--R                                                            Type: Boolean
--E 27

--S 28 of 44
(a=a)@Boolean
--R
```

```
--R
--R   (28)  true
--R                                                              Type: Boolean
--E 28

--S 29 of 44
a~=c
--R
--R
--R   (29)  false
--R                                                              Type: Boolean
--E 29

--S 30 of 44
any?(x+->(x=4),a)
--R
--R
--R   (30)  true
--R                                                              Type: Boolean
--E 30

--S 31 of 44
any?(x+->(x=11),a)
--R
--R
--R   (31)  false
--R                                                              Type: Boolean
--E 31

--S 32 of 44
every?(x+->(x=11),a)
--R
--R
--R   (32)  false
--R                                                              Type: Boolean
--E 32

--S 33 of 44
count(4,a)
--R
--R
--R   (33)  1
--R                                                      Type: PositiveInteger
--E 33

--S 34 of 44
count(x+->(x>2),a)
--R
--R
--R   (34)  5
```

```
--R                                                    Type: PositiveInteger
--E 34

--S 35 of 44
map(x+->x+10,a)
--R
--R
--R   (35)  [18,19,13,14,15]
--R                                                 Type: ArrayStack Integer
--E 35

--S 36 of 44
a
--R
--R
--R   (36)  [8,9,3,4,5]
--R                                                 Type: ArrayStack Integer
--E 36

--S 37 of 44
map!(x+->x+10,a)
--R
--R
--R   (37)  [18,19,13,14,15]
--R                                                 Type: ArrayStack Integer
--E 37

--S 38 of 44
a
--R
--R
--R   (38)  [18,19,13,14,15]
--R                                                 Type: ArrayStack Integer
--E 38

--S 39 of 44
members a
--R
--R
--R   (39)  [18,19,13,14,15]
--R                                                       Type: List Integer
--E 39

--S 40 of 44
member?(14,a)
--R
--R
--R   (40)  true
--R                                                            Type: Boolean
--E 40
```

```
--S 41 of 44
coerce a
--R
--R
--R   (41)  [18,19,13,14,15]
--R                                                          Type: OutputForm
--E 41

--S 42 of 44
hash a
--R
--R
--I   (42)  36310821
--R                                                        Type: SingleInteger
--E 42

--S 43 of 44
latex a
--R
--R
--R   (43)  "\mbox{\bf Unimplemented}"
--R                                                             Type: String
--E 43

--S 44 of 44
)show ArrayStack
--R
--R ArrayStack S: SetCategory  is a domain constructor
--R Abbreviation for ArrayStack is ASTACK
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASTACK
--R
--R------------------------------- Operations --------------------------------
--R arrayStack : List S -> %            bag : List S -> %
--R copy : % -> %                       depth : % -> NonNegativeInteger
--R empty : () -> %                     empty? : % -> Boolean
--R eq? : (%,%) -> Boolean              extract! : % -> S
--R insert! : (S,%) -> %                inspect : % -> S
--R map : ((S -> S),%) -> %             pop! : % -> S
--R push! : (S,%) -> S                  sample : () -> %
--R top : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
```

```
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 44

)spool
)lisp (bye)
```

———-

— ArrayStack.help —

```
====================================================================
ArrayStack examples
====================================================================

An ArrayStack object is represented as a list ordered by last-in, first-out.
It operates like a pile of books, where the "next" book is the one
on the top of the pile.

Here we create an array stack of integers from a list. Notice that the
order in the list is the order in the stack.

   a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      [1,2,3,4,5]

We can remove the top of the stack using pop!:

   pop! a
      1

Notice that the use of pop! is destructive (destructive operations
in Axiom usually end with ! to indicate that the underlying data
structure is changed).

   a
      [2,3,4,5]
```

The extract! operation is another name for the pop! operation and has the same effect. This operation treats the stack as a BagAggregate:

```
extract! a
   2
```

and you can see that it also has destructively modified the stack:

```
a
   [3,4,5]
```

Next we push a new element on top of the stack:

```
push!(9,a)
   9
```

Again, the push! operation is destructive so the stack is changed:

```
a
   [9,3,4,5]
```

Another name for push! is insert!, which treats the stack as a BagAggregate:

```
insert!(8,a)
   [8,9,3,4,5]
```

and it modifies the stack:

```
a
   [8,9,3,4,5]
```

The inspect function returns the top of the stack without modification, viewed as a BagAggregate:

```
inspect a
   8
```

The empty? operation returns true only if there are no element on the stack, otherwise it returns false:

```
empty? a
   false
```

The top operation returns the top of stack without modification, viewed as a Stack:

```
top a
   8
```

The depth operation returns the number of elements on the stack:

```
depth a
   5
```

which is the same as the # (length) operation:

```
#a
    5
```

The less? predicate will compare the stack length to an integer:

```
less?(a,9)
     true
```

The more? predicate will compare the stack length to an integer:

```
more?(a,9)
     false
```

The size? operation will compare the stack length to an integer:

```
size?(a,#a)
     true
```

and since the last computation must alwasy be true we try:

```
size?(a,9)
     false
```

The parts function will return the stack as a list of its elements:

```
parts a
     [8,9,3,4,5]
```

If we have a BagAggregate of elements we can use it to construct a stack.
Notice that the elements are pushed in reverse order:

```
bag([1,2,3,4,5])$ArrayStack(INT)
     [5,4,3,2,1]
```

The empty function will construct an empty stack of a given type:

```
b:=empty()$(ArrayStack INT)
     []
```

and the empty? predicate allows us to find out if a stack is empty:

```
empty? b
     true
```

The sample function returns a sample, empty stack:

```
   sample()$ArrayStack(INT)
        []
```

We can copy a stack and it does not share storage so subsequent
modifications of the original stack will not affect the copy:

```
   c:=copy a
        [8,9,3,4,5]
```

The eq? function is only true if the lists are the same reference,
so even though c is a copy of a, they are not the same:

```
   eq?(a,c)
        false
```

However, a clearly shares a reference with itself:

```
   eq?(a,a)
        true
```

But we can compare a and c for equality:

```
   (a=c)@Boolean
        true
```

and clearly a is equal to itself:

```
   (a=a)@Boolean
        true
```

and since a and c are equal, they are clearly NOT not-equal:

```
   a~=c
        false
```

We can use the any? function to see if a predicate is true for any element:

```
   any?(x+->(x=4),a)
        true
```

or false for every element:

```
   any?(x+->(x=11),a)
        false
```

We can use the every? function to check every element satisfies a predicate:

```
   every?(x+->(x=11),a)
        false
```

We can count the elements that are equal to an argument of this type:

```
   count(4,a)
        1
```

or we can count against a boolean function:

```
   count(x+->(x>2),a)
        5
```

You can also map a function over every element, returning a new stack:

```
   map(x+->x+10,a)
        [18,19,13,14,15]
```

Notice that the orignal stack is unchanged:

```
   a
        [8,9,3,4,5]
```

You can use map! to map a function over every element and change
the original stack since map! is destructive:

```
   map!(x+->x+10,a)
       [18,19,13,14,15]
```

Notice that the orignal stack has been changed:

```
   a
       [18,19,13,14,15]
```

The member function can also get the element of the stack as a list:

```
   members a
       [18,19,13,14,15]
```

and using member? we can test if the stack holds a given element:

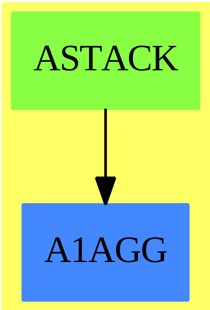
```
   member?(14,a)
       true
```

See Also:
o)show Stack
o)show ArrayStack
o)show Queue
o)show Dequeue
o)show Heap

```
o )show BagAggregate
```

———-

2.10.1 ArrayStack (ASTACK)



See

⇒ "Stack" (STACK) 20.28.1 on page 2521
⇒ "Queue" (QUEUE) 18.5.1 on page 2143
⇒ "Dequeue" (DEQUEUE) 5.5.1 on page 497
⇒ "Heap" (HEAP) 9.2.1 on page 1100

Exports:

any?	arrayStack	bag	coerce	copy
count	depth	empty	empty?	eq?
eval	every?	extract!	hash	insert!
inspect	latex	less?	map	map!
member?	members	more?	parts	pop!
push!	sample	size?	top	#?
?~=?	?=?			

— **domain ASTACK ArrayStack** —

```
)abbrev domain ASTACK ArrayStack
++ Author: Michael Monagan, Stephen Watt, Timothy Daly
++ Date Created:June 86 and July 87
++ Date Last Updated:Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
```

```
++ Description:
++ A stack represented as a flexible array.
--% Dequeue and Heap data types

ArrayStack(S:SetCategory): StackAggregate(S) with
    arrayStack: List S -> %
      ++ arrayStack([x,y,...,z]) creates an array stack with first (top)
      ++ element x, second element y,...,and last element z.
      ++
      ++E c:ArrayStack INT:= arrayStack [1,2,3,4,5]

 -- Inherited Signatures repeated for examples documentation

    pop_! : % -> S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X pop! a
      ++X a
    extract_! : % -> S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X extract! a
      ++X a
    push_! : (S,%) -> S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X push!(9,a)
      ++X a
    insert_! : (S,%) -> %
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X insert!(8,a)
      ++X a
    inspect : % -> S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X inspect a
    top : % -> S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X top a
    depth : % -> NonNegativeInteger
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X depth a
    less? : (%,NonNegativeInteger) -> Boolean
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X less?(a,9)
    more? : (%,NonNegativeInteger) -> Boolean
```

```
    ++
    ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
    ++X more?(a,9)
  size? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
    ++X size?(a,5)
  bag : List S -> %
    ++
    ++X bag([1,2,3,4,5])$ArrayStack(INT)
  empty? : % -> Boolean
    ++
    ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
    ++X empty? a
  empty : () -> %
    ++
    ++X b:=empty()$(ArrayStack INT)
  sample : () -> %
    ++
    ++X sample()$ArrayStack(INT)
  copy : % -> %
    ++
    ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
    ++X copy a
  eq? : (%,%) -> Boolean
    ++
    ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
    ++X b:=copy a
    ++X eq?(a,b)
  map :  ((S -> S),%) -> %
    ++
    ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
    ++X map(x+->x+10,a)
    ++X a
  if $ has shallowlyMutable then
    map! :  ((S -> S),%) -> %
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X map!(x+->x+10,a)
      ++X a
  if S has SetCategory then
    latex : % -> String
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X latex a
    hash : % -> SingleInteger
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X hash a
    coerce : % -> OutputForm
```

```
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X coerce a
    "=": (%,%) -> Boolean
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X b:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X (a=b)@Boolean
    "~=" : (%,%) -> Boolean
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X b:=copy a
      ++X (a~=b)
  if % has finiteAggregate then
    every? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X every?(x+->(x=4),a)
    any? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X any?(x+->(x=4),a)
    count :  ((S -> Boolean),%) -> NonNegativeInteger
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X count(x+->(x>2),a)
    _# : % -> NonNegativeInteger
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X #a
    parts : % -> List S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X parts a
    members : % -> List S
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X members a
  if % has finiteAggregate and S has SetCategory then
    member? : (S,%) -> Boolean
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X member?(3,a)
    count : (S,%) -> NonNegativeInteger
      ++
      ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
      ++X count(4,a)

== add
  Rep := IndexedFlexibleArray(S,0)
```

```
    -- system operations
    # s == _#(s)$Rep
    s = t == s =$Rep t
    copy s == copy(s)$Rep
    coerce(d):OutputForm ==
        empty? d => empty()$(List S) ::OutputForm
        [(d.i::OutputForm) for i in 0..#d-1] ::OutputForm

    -- stack operations
    depth s == # s
    empty? s == empty?(s)$Rep
    extract_! s == pop_! s
    insert_!(e,s) == (push_!(e,s);s)
    push_!(e,s) == (concat(e,s); e)
    pop_! s ==
        if empty? s then error "empty stack"
        r := s.0
        delete_!(s,0)
        r
    top s == if empty? s then error "empty stack" else s.0
    arrayStack l == construct(l)$Rep
    empty() == new(0,0 pretend S)
    parts s == [s.i for i in 0..#s-1]::List(S)
    map(f,s) == construct [f(s.i) for i in 0..#s-1]
    map!(f,s) == ( for i in 0..#s-1 repeat qsetelt!(s,i,f(s.i)) ; s )
    inspect(s) ==
        if empty? s then error "empty stack"
        qelt(s,0)
```

— **ASTACK.dotabb** —

```
"ASTACK" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASTACK"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ASTACK" -> "A1AGG"
```

2.11 domain ASP1 Asp1

— **Asp1.input** —

```
)set break resume
```

```
)sys rm -f Asp1.output
)spool Asp1.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp1
--R Asp1 name: Symbol  is a domain constructor
--R Abbreviation for Asp1 is ASP1
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP1
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %     retract : Polynomial Float -> %
--R retract : Expression Integer -> %     retract : Expression Float -> %
--R coerce : FortranExpression([construct,QUOTEX],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Polynomial Float -> Union(%,"failed")
--R retractIfCan : Expression Integer -> Union(%,"failed")
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

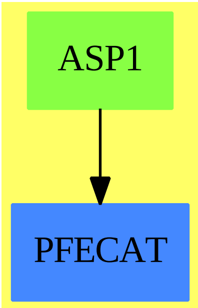
— Asp1.help —

```
====================================================================
Asp1 examples
====================================================================

See Also:
o )show Asp1
```

———-

2.11.1 Asp1 (ASP1)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP1 Asp1 —

```
)abbrev domain ASP1 Asp1
++ Author: Mike Dewar, Grant Keady, Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994, 6 October 1994
++ Related Constructors: FortranFunctionCategory, FortranProgramCategory.
++ Description:
++ \spadtype{Asp1} produces Fortran for Type 1 ASPs, needed for various
++ NAG routines. Type 1 ASPs take a univariate expression (in the symbol x)
++ and turn it into a Fortran Function like the following:
++
++ \tab{5}DOUBLE PRECISION FUNCTION F(X)\br
++ \tab{5}DOUBLE PRECISION X\br
++ \tab{5}F=DSIN(X)\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp1(name): Exports == Implementation where
  name : Symbol

  FEXPR  ==> FortranExpression
  FST    ==> FortranScalarType
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
```

```
Exports ==> FortranFunctionCategory with
  coerce : FEXPR(['X],[],MachineFloat) -> $
    ++coerce(f) takes an object from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns it into an ASP.

Implementation ==> add

  -- Build Symbol Table for Rep
  syms : SYMTAB := empty()$SYMTAB
  declare!(X,fortranReal()$FT,syms)$SYMTAB
  real : FST := "real"::FST

  Rep := FortranProgram(name,[real]$Union(fst:FST,void:"void"),[X],syms)

  retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR(['X],[],MachineFloat))::$
  retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
    foo : Union(FEXPR(['X],[],MachineFloat),"failed")
    foo := retractIfCan(u)$FEXPR(['X],[],MachineFloat)
    foo case "failed" => "failed"
    foo::FEXPR(['X],[],MachineFloat)::$

  retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR(['X],[],MachineFloat))::$
  retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
    foo : Union(FEXPR(['X],[],MachineFloat),"failed")
    foo := retractIfCan(u)$FEXPR(['X],[],MachineFloat)
    foo case "failed" => "failed"
    foo::FEXPR(['X],[],MachineFloat)::$

  retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR(['X],[],MachineFloat))::$
  retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
    foo : Union(FEXPR(['X],[],MachineFloat),"failed")
    foo := retractIfCan(u)$FEXPR(['X],[],MachineFloat)
    foo case "failed" => "failed"
    foo::FEXPR(['X],[],MachineFloat)::$

  retract(u:EXPR INT):$ == (retract(u)@FEXPR(['X],[],MachineFloat))::$
  retractIfCan(u:EXPR INT):Union($,"failed") ==
    foo : Union(FEXPR(['X],[],MachineFloat),"failed")
    foo := retractIfCan(u)$FEXPR(['X],[],MachineFloat)
    foo case "failed" => "failed"
    foo::FEXPR(['X],[],MachineFloat)::$

  retract(u:POLY FLOAT):$ == (retract(u)@FEXPR(['X],[],MachineFloat))::$
  retractIfCan(u:POLY FLOAT):Union($,"failed") ==
    foo : Union(FEXPR(['X],[],MachineFloat),"failed")
    foo := retractIfCan(u)$FEXPR(['X],[],MachineFloat)
    foo case "failed" => "failed"
    foo::FEXPR(['X],[],MachineFloat)::$

  retract(u:POLY INT):$ == (retract(u)@FEXPR(['X],[],MachineFloat))::$
```

```
    retractIfCan(u:POLY INT):Union($,"failed") ==
      foo : Union(FEXPR(['X],[],MachineFloat),"failed")
      foo := retractIfCan(u)$FEXPR(['X],[],MachineFloat)
      foo case "failed" => "failed"
      foo::FEXPR(['X],[],MachineFloat)::$

    coerce(u:FEXPR(['X],[],MachineFloat)):$ ==
      coerce((u::Expression(MachineFloat))$FEXPR(['X],[],MachineFloat))$Rep

    coerce(c:List FortranCode):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FortranCode):$ == coerce(c)$Rep

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

———

— ASP1.dotabb —

```
"ASP1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP1"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP1" -> "PFECAT"
```

———

2.12 domain ASP10 Asp10

— Asp10.input —

```
)set break resume
)sys rm -f Asp10.output
)spool Asp10.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show Asp10
--R Asp10 name: Symbol  is a domain constructor
--R Abbreviation for Asp10 is ASP10
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP10
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEJINT,QUOTEX,QUOTEELAM],[construct],Mac
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

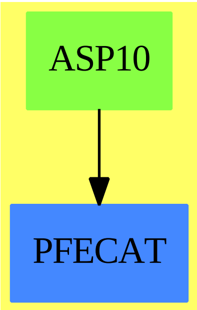
— Asp10.help —

```
====================================================================
Asp10 examples
====================================================================

See Also:
o )show Asp10
```

———

2.12.1 Asp10 (ASP10)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP10 Asp10 —

```
)abbrev domain ASP10 Asp10
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                    6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{ASP10} produces Fortran for Type 10 ASPs, needed for NAG routine
++ d02kef. This ASP computes the values of a set of functions, for example:
++
++ \tab{5}SUBROUTINE COEFFN(P,Q,DQDL,X,ELAM,JINT)\br
++ \tab{5}DOUBLE PRECISION ELAM,P,Q,X,DQDL\br
++ \tab{5}INTEGER JINT\br
++ \tab{5}P=1.0D0\br
++ \tab{5}Q=((-1.0D0*X**3)+ELAM*X*X-2.0D0)/(X*X)\br
++ \tab{5}DQDL=1.0D0\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp10(name): Exports == Implementation where
  name : Symbol

  FST  ==> FortranScalarType
  FT   ==> FortranType
  SYMTAB ==> SymbolTable
  EXF ==> Expression Float
  RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FEXPR ==> FortranExpression(['JINT,'X,'ELAM],[],MFLOAT)
  MFLOAT ==> MachineFloat
  FRAC   ==> Fraction
```

```
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector
VF2    ==> VectorFunctions2

Exports ==> FortranVectorFunctionCategory with
  coerce : Vector FEXPR -> %
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : FST := "real"::FST
  syms : SYMTAB := empty()$SYMTAB
  declare!(P,fortranReal()$FT,syms)$SYMTAB
  declare!(Q,fortranReal()$FT,syms)$SYMTAB
  declare!(DQDL,fortranReal()$FT,syms)$SYMTAB
  declare!(X,fortranReal()$FT,syms)$SYMTAB
  declare!(ELAM,fortranReal()$FT,syms)$SYMTAB
  declare!(JINT,fortranInteger()$FT,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$Union(fst:FST,void:"void"),
                        [P,Q,DQDL,X,ELAM,JINT],syms)

  retract(u:VEC FRAC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC FRAC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
    v case "failed" => "failed"
```

```
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  coerce(c:FortranCode):% == coerce(c)$Rep

  coerce(r:RSFC):% == coerce(r)$Rep

  coerce(c:List FortranCode):% == coerce(c)$Rep

  -- To help the poor old compiler!
  localAssign(s:Symbol,u:Expression MFLOAT):FortranCode ==
    assign(s,u)$FortranCode

  coerce(u:Vector FEXPR):% ==
    import Vector FEXPR
    not (#u = 3) => error "Incorrect Dimension For Vector"
    ([localAssign(P,elt(u,1)::Expression MFLOAT),_
      localAssign(Q,elt(u,2)::Expression MFLOAT),_
      localAssign(DQDL,elt(u,3)::Expression MFLOAT),_
      returns()$FortranCode ]$List(FortranCode))::Rep

  coerce(u:%):OutputForm == coerce(u)$Rep

  outputAsFortran(u):Void ==
```

```
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage
```

———-

— **ASP10.dotabb** —

```
"ASP10" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP10"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP10" -> "PFECAT"
```

———-

2.13 domain ASP12 Asp12

— **Asp12.input** —

```
)set break resume
)sys rm -f Asp12.output
)spool Asp12.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp12
--R Asp12 name: Symbol  is a domain constructor
--R Abbreviation for Asp12 is ASP12
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP12
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              outputAsFortran : () -> Void
--R outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)
```

———-

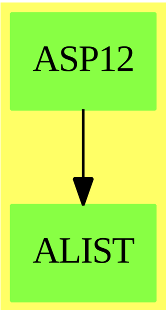
— **Asp12.help** —

```
====================================================================
Asp12 examples
====================================================================

See Also:
o )show Asp12
```

———-

2.13.1 Asp12 (ASP12)



Exports:
coerce outputAsFortran

— domain ASP12 Asp12 —

```
)abbrev domain ASP12 Asp12
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Oct 1993
++ Date Last Updated: 18 March 1994
++                    21 June 1994 Changed print to printStatement
++ Related Constructors:
++ Description:
++ \spadtype{Asp12} produces Fortran for Type 12 ASPs, needed for NAG routine
++ d02kef etc., for example:
++
++ \tab{5}SUBROUTINE MONIT (MAXIT,IFLAG,ELAM,FINFO)\br
++ \tab{5}DOUBLE PRECISION ELAM,FINFO(15)\br
++ \tab{5}INTEGER MAXIT,IFLAG\br
++ \tab{5}IF(MAXIT.EQ.-1)THEN\br
++ \tab{7}PRINT*,"Output from Monit"\br
++ \tab{5}ENDIF\br
++ \tab{5}PRINT*,MAXIT,IFLAG,ELAM,(FINFO(I),I=1,4)\br
++ \tab{5}RETURN\br
```

```
++ \tab{5}END\

Asp12(name): Exports == Implementation where
  name : Symbol

  O      ==> OutputForm
  S      ==> Symbol
  FST    ==> FortranScalarType
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  EXI    ==> Expression Integer
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  U      ==> Union(I: Expression Integer,F: Expression Float,_
                 CF: Expression Complex Float,switch:Switch)
  UFST   ==> Union(fst:FST,void:"void")

  Exports ==> FortranProgramCategory with
     outputAsFortran:() -> Void
      ++outputAsFortran() generates the default code for \spadtype{ASP12}.

  Implementation ==> add

    import FC
    import Switch

    real : FST := "real"::FST
    syms : SYMTAB := empty()$SYMTAB
    declare!(MAXIT,fortranInteger()$FT,syms)$SYMTAB
    declare!(IFLAG,fortranInteger()$FT,syms)$SYMTAB
    declare!(ELAM,fortranReal()$FT,syms)$SYMTAB
    fType : FT := construct([real]$UFST,["15"::Symbol],false)$FT
    declare!(FINFO,fType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST,[MAXIT,IFLAG,ELAM,FINFO],syms)

    -- eqn : O := (I::O)=(1@Integer::EXI::O)
    code:=([cond(EQ([MAXIT@S::EXI]$U,[-1::EXI]$U),
                 printStatement(["_"Output from Monit_""::O])),
            printStatement([MAXIT::O,IFLAG::O,ELAM::O,subscript("(FINFO"::S,[I::O])::O,"I=1"
            returns()]$List(FortranCode))::Rep

    coerce(u:%):OutputForm == coerce(u)$Rep

    outputAsFortran(u:%):Void == outputAsFortran(u)$Rep
    outputAsFortran():Void == outputAsFortran(code)$Rep
```

— ASP12.dotabb —

```
"ASP12" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP12"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP12" -> "ALIST"
```

2.14 domain ASP19 Asp19

— Asp19.input —

```
)set break resume
)sys rm -f Asp19.output
)spool Asp19.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp19
--R Asp19 name: Symbol  is a domain constructor
--R Abbreviation for Asp19 is ASP19
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP19
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEXC],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

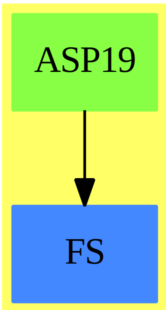
— Asp19.help —

```
====================================================================
Asp19 examples
====================================================================

See Also:
o )show Asp19
```

———-

2.14.1 Asp19 (ASP19)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP19 Asp19 —

```
)abbrev domain ASP19 Asp19
++ Author: Mike Dewar, Godfrey Nolan, Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                    6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp19} produces Fortran for Type 19 ASPs, evaluating a set of
++functions and their jacobian at a given point, for example:
++
```

```
++\tab{5}SUBROUTINE LSFUN2(M,N,XC,FVECC,FJACC,LJC)\br
++\tab{5}DOUBLE PRECISION FVECC(M),FJACC(LJC,N),XC(N)\br
++\tab{5}INTEGER M,N,LJC\br
++\tab{5}INTEGER I,J\br
++\tab{5}DO 25003 I=1,LJC\br
++\tab{7}DO 25004 J=1,N\br
++\tab{9}FJACC(I,J)=0.0D0\br
++25004   CONTINUE\br
++25003 CONTINUE\br
++\tab{5}FVECC(1)=((XC(1)-0.14D0)*XC(3)+(15.0D0*XC(1)-2.1D0)*XC(2)+1.0D0)/(\br
++\tab{4}&XC(3)+15.0D0*XC(2))\br
++\tab{5}FVECC(2)=((XC(1)-0.18D0)*XC(3)+(7.0D0*XC(1)-1.26D0)*XC(2)+1.0D0)/(\br
++\tab{4}&XC(3)+7.0D0*XC(2))\br
++\tab{5}FVECC(3)=((XC(1)-0.22D0)*XC(3)+(4.333333333333333D0*XC(1)-0.953333\br
++\tab{4}&3333333333D0)*XC(2)+1.0D0)/(XC(3)+4.333333333333333D0*XC(2))\br
++\tab{5}FVECC(4)=((XC(1)-0.25D0)*XC(3)+(3.0D0*XC(1)-0.75D0)*XC(2)+1.0D0)/(\br
++\tab{4}&XC(3)+3.0D0*XC(2))\br
++\tab{5}FVECC(5)=((XC(1)-0.29D0)*XC(3)+(2.2D0*XC(1)-0.6379999999999999D0)*\br
++\tab{4}&XC(2)+1.0D0)/(XC(3)+2.2D0*XC(2))\br
++\tab{5}FVECC(6)=((XC(1)-0.32D0)*XC(3)+(1.666666666666667D0*XC(1)-0.533333\br
++\tab{4}&3333333333D0)*XC(2)+1.0D0)/(XC(3)+1.666666666666667D0*XC(2))\br
++\tab{5}FVECC(7)=((XC(1)-0.35D0)*XC(3)+(1.285714285714286D0*XC(1)-0.45D0)*\br
++\tab{4}&XC(2)+1.0D0)/(XC(3)+1.285714285714286D0*XC(2))\br
++\tab{5}FVECC(8)=((XC(1)-0.39D0)*XC(3)+(XC(1)-0.39D0)*XC(2)+1.0D0)/(XC(3)+\br
++\tab{4}&XC(2))\br
++\tab{5}FVECC(9)=((XC(1)-0.37D0)*XC(3)+(XC(1)-0.37D0)*XC(2)+1.285714285714\br
++\tab{4}&286D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(10)=((XC(1)-0.58D0)*XC(3)+(XC(1)-0.58D0)*XC(2)+1.66666666666\br
++\tab{4}&6667D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(11)=((XC(1)-0.73D0)*XC(3)+(XC(1)-0.73D0)*XC(2)+2.2D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FVECC(12)=((XC(1)-0.96D0)*XC(3)+(XC(1)-0.96D0)*XC(2)+3.0D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FVECC(13)=((XC(1)-1.34D0)*XC(3)+(XC(1)-1.34D0)*XC(2)+4.33333333333\br
++\tab{4}&3333D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(14)=((XC(1)-2.1D0)*XC(3)+(XC(1)-2.1D0)*XC(2)+7.0D0)/(XC(3)+X\br
++\tab{4}&C(2))\br
++\tab{5}FVECC(15)=((XC(1)-4.39D0)*XC(3)+(XC(1)-4.39D0)*XC(2)+15.0D0)/(XC(3\br
++\tab{4}&)+XC(2))\br
++\tab{5}FJACC(1,1)=1.0D0\br
++\tab{5}FJACC(1,2)=-15.0D0/(XC(3)**2+30.0D0*XC(2)*XC(3)+225.0D0*XC(2)**2)\br
++\tab{5}FJACC(1,3)=-1.0D0/(XC(3)**2+30.0D0*XC(2)*XC(3)+225.0D0*XC(2)**2)\br
++\tab{5}FJACC(2,1)=1.0D0\br
++\tab{5}FJACC(2,2)=-7.0D0/(XC(3)**2+14.0D0*XC(2)*XC(3)+49.0D0*XC(2)**2)\br
++\tab{5}FJACC(2,3)=-1.0D0/(XC(3)**2+14.0D0*XC(2)*XC(3)+49.0D0*XC(2)**2)\br
++\tab{5}FJACC(3,1)=1.0D0\br
++\tab{5}FJACC(3,2)=((-0.1110223024625157D-15*XC(3))-4.333333333333333D0)/(\br
++\tab{4}&XC(3)**2+8.666666666666666D0*XC(2)*XC(3)+18.77777777777778D0*XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(3,3)=(0.1110223024625157D-15*XC(2)-1.0D0)/(XC(3)**2+8.666666\br
```

```
++\tab{4}&666666666D0*XC(2)*XC(3)+18.77777777777778D0*XC(2)**2)\br
++\tab{5}FJACC(4,1)=1.0D0\br
++\tab{5}FJACC(4,2)=-3.0D0/(XC(3)**2+6.0D0*XC(2)*XC(3)+9.0D0*XC(2)**2)\br
++\tab{5}FJACC(4,3)=-1.0D0/(XC(3)**2+6.0D0*XC(2)*XC(3)+9.0D0*XC(2)**2)\br
++\tab{5}FJACC(5,1)=1.0D0\br
++\tab{5}FJACC(5,2)=((-0.1110223024625157D-15*XC(3))-2.2D0)/(XC(3)**2+4.399\br
++\tab{4}&999999999999D0*XC(2)*XC(3)+4.839999999999998D0*XC(2)**2)\br
++\tab{5}FJACC(5,3)=(0.1110223024625157D-15*XC(2)-1.0D0)/(XC(3)**2+4.399999\br
++\tab{4}&999999999D0*XC(2)*XC(3)+4.839999999999998D0*XC(2)**2)\br
++\tab{5}FJACC(6,1)=1.0D0\br
++\tab{5}FJACC(6,2)=((-0.2220446049250313D-15*XC(3))-1.666666666666667D0)/(\br
++\tab{4}&XC(3)**2+3.333333333333333D0*XC(2)*XC(3)+2.777777777777777D0*XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(6,3)=(0.2220446049250313D-15*XC(2)-1.0D0)/(XC(3)**2+3.333333\br
++\tab{4}&333333333D0*XC(2)*XC(3)+2.777777777777777D0*XC(2)**2)\br
++\tab{5}FJACC(7,1)=1.0D0\br
++\tab{5}FJACC(7,2)=((-0.5551115123125783D-16*XC(3))-1.285714285714286D0)/(\br
++\tab{4}&XC(3)**2+2.571428571428571D0*XC(2)*XC(3)+1.653061224489796D0*XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(7,3)=(0.5551115123125783D-16*XC(2)-1.0D0)/(XC(3)**2+2.571428\br
++\tab{4}&571428571D0*XC(2)*XC(3)+1.653061224489796D0*XC(2)**2)\br
++\tab{5}FJACC(8,1)=1.0D0\br
++\tab{5}FJACC(8,2)=-1.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(8,3)=-1.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(9,1)=1.0D0\br
++\tab{5}FJACC(9,2)=-1.285714285714286D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)*\br
++\tab{4}&*2)\br
++\tab{5}FJACC(9,3)=-1.285714285714286D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)*\br
++\tab{4}&*2)\br
++\tab{5}FJACC(10,1)=1.0D0\br
++\tab{5}FJACC(10,2)=-1.666666666666667D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(10,3)=-1.666666666666667D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(11,1)=1.0D0\br
++\tab{5}FJACC(11,2)=-2.2D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(11,3)=-2.2D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(12,1)=1.0D0\br
++\tab{5}FJACC(12,2)=-3.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(12,3)=-3.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(13,1)=1.0D0\br
++\tab{5}FJACC(13,2)=-4.333333333333333D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(13,3)=-4.333333333333333D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(14,1)=1.0D0\br
++\tab{5}FJACC(14,2)=-7.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(14,3)=-7.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(15,1)=1.0D0\br
++\tab{5}FJACC(15,2)=-15.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
```

```
++\tab{5}FJACC(15,3)=-15.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}RETURN\br
++\tab{5}END

Asp19(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
  FSTU   ==> Union(fst:FST,void:"void")
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  MFLOAT ==> MachineFloat
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,EXPR MFLOAT,VEC EXPR MFLOAT
  FEXPR  ==> FortranExpression([],['XC],MFLOAT)
  S      ==> Symbol

  Exports ==> FortranVectorFunctionCategory with
    coerce : VEC FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add

    real : FSTU := ["real"::FST]$FSTU
    syms : SYMTAB := empty()$SYMTAB
    declare!(M,fortranInteger()$FT,syms)$SYMTAB
    declare!(N,fortranInteger()$FT,syms)$SYMTAB
    declare!(LJC,fortranInteger()$FT,syms)$SYMTAB
    xcType : FT := construct(real,[N],false)$FT
    declare!(XC,xcType,syms)$SYMTAB
    fveccType : FT := construct(real,[M],false)$FT
    declare!(FVECC,fveccType,syms)$SYMTAB
    fjaccType : FT := construct(real,[LJC,N],false)$FT
    declare!(FJACC,fjaccType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$FSTU,[M,N,XC,FVECC,FJACC,LJC],syms)

    coerce(c:List FC):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FC):$ == coerce(c)$Rep
```

```
-- Take a symbol, pull of the script and turn it into an integer!!
o2int(u:S):Integer ==
  o : OutputForm := first elt(scripts(u)$S,sub)
  o pretend Integer

-- To help the poor old compiler!
fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:S,j:Matrix FEXPR):FC ==
  j' : Matrix EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

coerce(u:VEC FEXPR):$ ==
  -- First zero the Jacobian matrix in case we miss some derivatives which
  -- are zero.
  import POLY INT
  seg1 : Segment (POLY INT) := segment(1::(POLY INT),LJC@S::(POLY INT))
  seg2 : Segment (POLY INT) := segment(1::(POLY INT),N@S::(POLY INT))
  s1 : SegmentBinding POLY INT := equation(I@S,seg1)
  s2 : SegmentBinding POLY INT := equation(J@S,seg2)
  as : FC := assign(FJACC,[I@S::(POLY INT),J@S::(POLY INT)],0.0::EXPR FLOAT)
  clear : FC := forLoop(s1,forLoop(s2,as))
  j:Integer
  x:S := XC::S
  pu:List(S) := []
  -- Work out which variables appear in the expressions
  for e in entries(u) repeat
    pu := setUnion(pu,variables(e)$FEXPR)
  scriptList : List Integer := map(o2int,pu)$ListFunctions2(S,Integer)
  -- This should be the maximum XC_n which occurs (there may be others
  -- which don't):
  n:Integer := reduce(max,scriptList)$List(Integer)
  p:List(S) := []
  for j in 1..n repeat p:= cons(subscript(x,[j::OutputForm])$S,p)
  p:= reverse(p)
  jac:Matrix(FEXPR) := _
  jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  c1:FC := localAssign2(FVECC,u)
  c2:FC := localAssign1(FJACC,jac)
  [clear,c1,c2,returns()]$List(FC)::$

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
```

```
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage


  retract(u:VEC FRAC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC FRAC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC POLY FLOAT):$ ==
```

```
    v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$
```

— **ASP19.dotabb** —

```
"ASP19" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP19"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP19" -> "FS"
```

2.15 domain ASP20 Asp20

— **Asp20.input** —

```
)set break resume
)sys rm -f Asp20.output
)spool Asp20.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp20
--R Asp20 name: Symbol  is a domain constructor
--R Abbreviation for Asp20 is ASP20
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP20
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct],[construct,QUOTEX,QUOTEHESS],MachineFloat)
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
```

```
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Expression Integer -> Union(%,"failed")
--R retractIfCan : Matrix Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

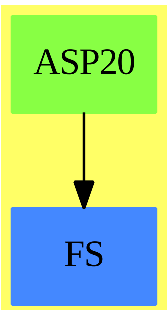
— **Asp20.help** —

```
====================================================================
Asp20 examples
====================================================================

See Also:
o )show Asp20
```

———-

2.15.1 Asp20 (ASP20)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP20 Asp20** —

```
)abbrev domain ASP20 Asp20
++ Author: Mike Dewar and Godfrey Nolan and Grant Keady
++ Date Created: Dec 1993
++ Date Last Updated: 21 March 1994
++                     6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp20} produces Fortran for Type 20 ASPs, for example:
++
++ \tab{5}SUBROUTINE QPHESS(N,NROWH,NCOLH,JTHCOL,HESS,X,HX)\br
++ \tab{5}DOUBLE PRECISION HX(N),X(N),HESS(NROWH,NCOLH)\br
++ \tab{5}INTEGER JTHCOL,N,NROWH,NCOLH\br
++ \tab{5}HX(1)=2.0D0*X(1)\br
++ \tab{5}HX(2)=2.0D0*X(2)\br
++ \tab{5}HX(3)=2.0D0*X(4)+2.0D0*X(3)\br
++ \tab{5}HX(4)=2.0D0*X(4)+2.0D0*X(3)\br
++ \tab{5}HX(5)=2.0D0*X(5)\br
++ \tab{5}HX(6)=(-2.0D0*X(7))+(-2.0D0*X(6))\br
++ \tab{5}HX(7)=(-2.0D0*X(7))+(-2.0D0*X(6))\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp20(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  PI     ==> PositiveInteger
  UFST   ==> Union(fst:FST,void:"void")
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  MAT    ==> Matrix
  VF2    ==> VectorFunctions2
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression([],['X,'HESS],MFLOAT)
  O      ==> OutputForm
  M2     ==> MatrixCategoryFunctions2
  MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
               MAT FRAC POLY INT,FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
               MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
               FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
```

```
              MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2e   ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
              FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2f   ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
              MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)


Exports ==> FortranMatrixFunctionCategory with
  coerce: MAT FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : UFST := ["real"::FST]$UFST
  syms : SYMTAB := empty()
  declare!(N,fortranInteger(),syms)$SYMTAB
  declare!(NROWH,fortranInteger(),syms)$SYMTAB
  declare!(NCOLH,fortranInteger(),syms)$SYMTAB
  declare!(JTHCOL,fortranInteger(),syms)$SYMTAB
  hessType : FT := construct(real,[NROWH,NCOLH],false)$FT
  declare!(HESS,hessType,syms)$SYMTAB
  xType : FT := construct(real,[N],false)$FT
  declare!(X,xType,syms)$SYMTAB
  declare!(HX,xType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$UFST,
                        [N,NROWH,NCOLH,JTHCOL,HESS,X,HX],syms)

  coerce(c:List FortranCode):$ == coerce(c)$Rep

  coerce(r:RSFC):$ == coerce(r)$Rep

  coerce(c:FortranCode):$ == coerce(c)$Rep

  -- To help the poor old compiler!
  fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

  localAssign(s:Symbol,j:VEC FEXPR):FortranCode ==
    j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
    assign(s,j')$FortranCode

  coerce(u:MAT FEXPR):$ ==
    j:Integer
    x:Symbol := X::Symbol
    n := nrows(u)::PI
    p:VEC FEXPR := [retract(subscript(x,[j::O])$Symbol)@FEXPR for j in 1..n]
    prod:VEC FEXPR := u*p
    ([localAssign(HX,prod),returns()$FortranCode]$List(FortranCode))::$

  retract(u:MAT FRAC POLY INT):$ ==
```

```
    v : MAT FEXPR := map(retract,u)$MF2a
    v::$

  retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT FRAC POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2b
    v::$

  retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT EXPR INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2e
    v::$

  retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT EXPR FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2f
    v::$

  retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT POLY INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2c
    v::$

  retractIfCan(u:MAT POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2d
    v::$

  retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
```

```
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  coerce(u:$):O == coerce(u)$Rep

  outputAsFortran(u):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage
```

———-

— ASP20.dotabb —

```
"ASP20" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP20"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP20" -> "FS"
```

———-

2.16 domain ASP24 Asp24

— Asp24.input —

```
)set break resume
)sys rm -f Asp24.output
)spool Asp24.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp24
--R Asp24 name: Symbol  is a domain constructor
--R Abbreviation for Asp24 is ASP24
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP24
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %     retract : Polynomial Float -> %
--R retract : Expression Integer -> %     retract : Expression Float -> %
--R coerce : FortranExpression([construct],[construct,QUOTEXC],MachineFloat) -> %
```

```
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Polynomial Float -> Union(%,"failed")
--R retractIfCan : Expression Integer -> Union(%,"failed")
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

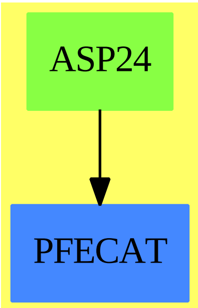
— Asp24.help —

```
====================================================================
Asp24 examples
====================================================================

See Also:
o )show Asp24
```

———-

2.16.1 Asp24 (ASP24)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP24 Asp24 —

```
)abbrev domain ASP24 Asp24
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 21 March 1994
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp24} produces Fortran for Type 24 ASPs which evaluate a
++multivariate function at a point (needed for NAG routine e04jaf),
++for example:
++
++\tab{5}SUBROUTINE FUNCT1(N,XC,FC)\br
++\tab{5}DOUBLE PRECISION FC,XC(N)\br
++\tab{5}INTEGER N\br
++\tab{5}FC=10.0D0*XC(4)**4+(-40.0D0*XC(1)*XC(4)**3)+(60.0D0*XC(1)**2+5\br
++\tab{4}&.0D0)*XC(4)**2+((-10.0D0*XC(3))+(-40.0D0*XC(1)**3))*XC(4)+16.0D0*X\br
++\tab{4}&C(3)**4+(-32.0D0*XC(2)*XC(3)**3)+(24.0D0*XC(2)**2+5.0D0)*XC(3)**2+\br
++\tab{4}&(-8.0D0*XC(2)**3*XC(3))+XC(2)**4+100.0D0*XC(2)**2+20.0D0*XC(1)*XC(\br
++\tab{4}&2)+10.0D0*XC(1)**4+XC(1)**2\br
++\tab{5}RETURN\br
++\tab{5}END\br

Asp24(name): Exports == Implementation where
  name : Symbol

  FST  ==> FortranScalarType
  FT   ==> FortranType
  SYMTAB ==> SymbolTable
  RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FSTU ==> Union(fst:FST,void:"void")
  FEXPR ==> FortranExpression([],['XC],MachineFloat)
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float

  Exports ==> FortranFunctionCategory with
      coerce : FEXPR -> $
        ++ coerce(f) takes an object from the appropriate instantiation of
              ++ \spadtype{FortranExpression} and turns it into an ASP.


  Implementation ==> add


    real : FSTU := ["real"::FST]$FSTU
    syms : SYMTAB := empty()
    declare!(N,fortranInteger(),syms)$SYMTAB
    xcType : FT := construct(real,[N::Symbol],false)$FT
```

```
declare!(XC,xcType,syms)$SYMTAB
declare!(FC,fortranReal(),syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,[N,XC,FC],syms)

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:FEXPR):$ ==
  coerce(assign(FC,u::Expression(MachineFloat))$FortranCode)$Rep

retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==
```

```
      foo : Union(FEXPR,"failed")
      foo := retractIfCan(u)$FEXPR
      foo case "failed" => "failed"
      (foo::FEXPR)::$

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

——

— ASP24.dotabb —

```
"ASP24" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP24"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP24" -> "PFECAT"
```

——

2.17 domain ASP27 Asp27

— Asp27.input —

```
)set break resume
)sys rm -f Asp27.output
)spool Asp27.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp27
--R Asp27 name: Symbol  is a domain constructor
--R Abbreviation for Asp27 is ASP27
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP27
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
```

```
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

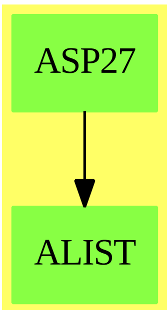
— Asp27.help —

```
====================================================================
Asp27 examples
====================================================================

See Also:
o )show Asp27
```

———-

2.17.1 Asp27 (ASP27)



Exports:
coerce outputAsFortran

— domain ASP27 Asp27 —

```
)abbrev domain ASP27 Asp27
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 27 April 1994
++                     6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
```

```
++ Description:
++\spadtype{Asp27} produces Fortran for Type 27 ASPs, needed for NAG routine
++f02fjf ,for example:
++
++\tab{5}FUNCTION DOT(IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION W(N),Z(N),RWORK(LRWORK)\br
++\tab{5}INTEGER N,LIWORK,IFLAG,LRWORK,IWORK(LIWORK)\br
++\tab{5}DOT=(W(16)+(-0.5D0*W(15)))*Z(16)+((-0.5D0*W(16))+W(15)+(-0.5D0*W(1\br
++\tab{4}&4)))*Z(15)+((-0.5D0*W(15))+W(14)+(-0.5D0*W(13)))*Z(14)+((-0.5D0*W(\br
++\tab{4}&14))+W(13)+(-0.5D0*W(12)))*Z(13)+((-0.5D0*W(13))+W(12)+(-0.5D0*W(1\br
++\tab{4}&1)))*Z(12)+((-0.5D0*W(12))+W(11)+(-0.5D0*W(10)))*Z(11)+((-0.5D0*W(\br
++\tab{4}&11))+W(10)+(-0.5D0*W(9)))*Z(10)+((-0.5D0*W(10))+W(9)+(-0.5D0*W(8))\br
++\tab{4}&)*Z(9)+((-0.5D0*W(9))+W(8)+(-0.5D0*W(7)))*Z(8)+((-0.5D0*W(8))+W(7)\br
++\tab{4}&+(-0.5D0*W(6)))*Z(7)+((-0.5D0*W(7))+W(6)+(-0.5D0*W(5)))*Z(6)+((-0.\br
++\tab{4}&5D0*W(6))+W(5)+(-0.5D0*W(4)))*Z(5)+((-0.5D0*W(5))+W(4)+(-0.5D0*W(3\br
++\tab{4}&)))*Z(4)+((-0.5D0*W(4))+W(3)+(-0.5D0*W(2)))*Z(3)+((-0.5D0*W(3))+W(\br
++\tab{4}&2)+(-0.5D0*W(1)))*Z(2)+((-0.5D0*W(2))+W(1))*Z(1)\br
++\tab{5}RETURN\br
++\tab{5}END

Asp27(name): Exports == Implementation where
  name : Symbol

  O      ==> OutputForm
  FST    ==> FortranScalarType
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  UFST   ==> Union(fst:FST,void:"void")
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  EXPR   ==> Expression
  MAT    ==> Matrix
  MFLOAT ==> MachineFloat



  Exports == FortranMatrixCategory

  Implementation == add


    real : UFST := ["real"::FST]$UFST
    integer : UFST := ["integer"::FST]$UFST
    syms : SYMTAB := empty()$SYMTAB
    declare!(IFLAG,fortranInteger(),syms)$SYMTAB
    declare!(N,fortranInteger(),syms)$SYMTAB
    declare!(LRWORK,fortranInteger(),syms)$SYMTAB
    declare!(LIWORK,fortranInteger(),syms)$SYMTAB
    zType : FT := construct(real,[N],false)$FT
```

```
    declare!(Z,zType,syms)$SYMTAB
    declare!(W,zType,syms)$SYMTAB
    rType : FT := construct(real,[LRWORK],false)$FT
    declare!(RWORK,rType,syms)$SYMTAB
    iType : FT := construct(integer,[LIWORK],false)$FT
    declare!(IWORK,iType,syms)$SYMTAB
    Rep := FortranProgram(name,real,
                          [IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK],syms)

    -- To help the poor old compiler!
    localCoerce(u:Symbol):EXPR(MFLOAT) == coerce(u)$EXPR(MFLOAT)

    coerce (u:MAT MFLOAT):$ ==
      Ws: Symbol := W
      Zs: Symbol := Z
      code : List FC
      l:EXPR MFLOAT := "+"/ _
          [("+"/[localCoerce(elt(Ws,[j::O])$Symbol) * u(j,i)_
                                           for j in 1..nrows(u)::PI])_
           *localCoerce(elt(Zs,[i::O])$Symbol) for i in 1..ncols(u)::PI]
      c := assign(name,l)$FC
      code := [c,returns()]$List(FC)
      code::$

    coerce(c:List FortranCode):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FortranCode):$ == coerce(c)$Rep

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

— **ASP27.dotabb** —

```
"ASP27" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP27"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP27" -> "ALIST"
```

2.18 domain ASP28 Asp28

— **Asp28.input** —

```
)set break resume
)sys rm -f Asp28.output
)spool Asp28.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp28
--R Asp28 name: Symbol  is a domain constructor
--R Abbreviation for Asp28 is ASP28
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP28
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

———

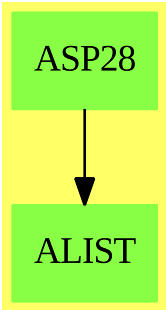
— **Asp28.help** —

```
====================================================================
Asp28 examples
====================================================================

See Also:
o )show Asp28
```

———

2.18.1 Asp28 (ASP28)



Exports:
coerce outputAsFortran

— domain ASP28 Asp28 —

```
)abbrev domain ASP28 Asp28
++ Author: Mike Dewar
++ Date Created: 21 March 1994
++ Date Last Updated: 28 April 1994
++                   6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp28} produces Fortran for Type 28 ASPs, used in NAG routine
++f02fjf, for example:
++
++\tab{5}SUBROUTINE IMAGE(IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION Z(N),W(N),IWORK(LRWORK),RWORK(LRWORK)\br
++\tab{5}INTEGER N,LIWORK,IFLAG,LRWORK\br
++\tab{5}W(1)=0.01707454969713436D0*Z(16)+0.001747395874954051D0*Z(15)+0.00\br
++\tab{4}&2106973900813502D0*Z(14)+0.002957434991769087D0*Z(13)+(-0.00700554\br
++\tab{4}&0882865317D0*Z(12))+(-0.01219194009813166D0*Z(11))+0.0037230647365\br
++\tab{4}&3087D0*Z(10)+0.04932374658377151D0*Z(9)+(-0.03586220812223305D0*Z(\br
++\tab{4}&8))+(-0.04723268012114625D0*Z(7))+(-0.02434652144032987D0*Z(6))+0.\br
++\tab{4}&2264766947290192D0*Z(5)+(-0.1385343580686922D0*Z(4))+(-0.116530050\br
++\tab{4}&8238904D0*Z(3))+(-0.2803531651057233D0*Z(2))+1.019463911841327D0*Z\br
++\tab{4}&(1)\br
++\tab{5}W(2)=0.0227345011107737D0*Z(16)+0.008812321197398072D0*Z(15)+0.010\br
++\tab{4}&94012210519586D0*Z(14)+(-0.01764072463999744D0*Z(13))+(-0.01357136\br
++\tab{4}&72105995D0*Z(12))+0.00157466157362272D0*Z(11)+0.05258889186338282D\br
++\tab{4}&0*Z(10)+(-0.01981532388243379D0*Z(9))+(-0.06095390688679697D0*Z(8)\br
++\tab{4}&)+(-0.04153119955569051D0*Z(7))+0.2176561076571465D0*Z(6)+(-0.0532\br
++\tab{4}&5555586632358D0*Z(5))+(-0.1688977368984641D0*Z(4))+(-0.32440166056\br
++\tab{4}&67343D0*Z(3))+0.9128222941872173D0*Z(2)+(-0.2419652703415429D0*Z(1\br
++\tab{4}&))\br
++\tab{5}W(3)=0.03371198197190302D0*Z(16)+0.02021603150122265D0*Z(15)+(-0.0\br
```

```
++\tab{4}&06607305534689702D0*Z(14))+(-0.03032392238968179D0*Z(13))+0.002033\br
++\tab{4}&305231024948D0*Z(12)+0.05375944956767728D0*Z(11)+(-0.0163213312502\br
++\tab{4}&9967D0*Z(10))+(-0.05483186562035512D0*Z(9))+(-0.0490142882257987 2D\br
++\tab{4}&0*Z(8))+0.2091097927887612D0*Z(7)+(-0.05760560341383113D0*Z(6))+(-\br
++\tab{4}&0.1236679206156403D0*Z(5))+(-0.3523683853026259D0*Z(4))+0.88929961\br
++\tab{4}&32269974D0*Z(3)+(-0.2995429545781457D0*Z(2))+(-0.02986582812574917\br
++\tab{4}&D0*Z(1))\br
++\tab{5}W(4)=0.05141563713660119D0*Z(16)+0.005239165960779299D0*Z(15)+(-0.\br
++\tab{4}&01623427735779699D0*Z(14))+(-0.01965809746040371D0*Z(13))+0.054688\br
++\tab{4}&97337339577D0*Z(12)+(-0.014224695935687D0*Z(11))+(-0.0505181779315\br
++\tab{4}&6355D0*Z(10))+(-0.04353074206076491D0*Z(9))+0.2012230497530726D0*Z\br
++\tab{4}&(8)+(-0.06630874514535952D0*Z(7))+(-0.1280829963720053D0*Z(6))+(-0\br
++\tab{4}&.305169742604165D0*Z(5))+0.8600427128450191D0*Z(4)+(-0.32415033802\br
++\tab{4}&68184D0*Z(3))+(-0.09033531980693314D0*Z(2))+0.09089205517109111D0*\br
++\tab{4}&Z(1)\br
++\tab{5}W(5)=0.04556369767776375D0*Z(16)+(-0.001822737697581869D0*Z(15))+(\br
++\tab{4}&-0.002512226501941856D0*Z(14))+0.02947046460707379D0*Z(13)+(-0.014\br
++\tab{4}&45079632086177D0*Z(12))+(-0.05034242196614937D0*Z(11))+(-0.0376966\br
++\tab{4}&3291725935D0*Z(10))+0.2171103102175198D0*Z(9)+(-0.0824949256021352\br
++\tab{4}&4D0*Z(8))+(-0.1473995209288945D0*Z(7))+(-0.315042193418466D0*Z(6))\br
++\tab{4}&+0.9591623347824002D0*Z(5)+(-0.3852396953763045D0*Z(4))+(-0.141718\br
++\tab{4}&5427288274D0*Z(3))+(-0.03423495461011043D0*Z(2))+0.319820917706851\br
++\tab{4}&6D0*Z(1)\br
++\tab{5}W(6)=0.04015147277405744D0*Z(16)+0.01328585741341559D0*Z(15)+0.048\br
++\tab{4}&26082005465965D0*Z(14)+(-0.04319641116207706D0*Z(13))+(-0.04931323\br
++\tab{4}&319055762D0*Z(12))+(-0.03526886317505474D0*Z(11))+0.22295383396730\br
++\tab{4}&01D0*Z(10)+(-0.07375317649315155D0*Z(9))+(-0.1589391311991561D0*Z(\br
++\tab{4}&8))+(-0.328001910890377D0*Z(7))+0.952576555482747D0*Z(6)+(-0.31583\br
++\tab{4}&09975786731D0*Z(5))+(-0.1846882042225383D0*Z(4))+(-0.0703762046700\br
++\tab{4}&4427D0*Z(3))+0.2311852964327382D0*Z(2)+0.04254083491825025D0*Z(1)\br
++\tab{5}W(7)=0.06069778964023718D0*Z(16)+0.06681263884671322D0*Z(15)+(-0.0\br
++\tab{4}&2113506688615768D0*Z(14))+(-0.083996867458326D0*Z(13))+(-0.0329843\br
++\tab{4}&8523869648D0*Z(12))+0.2276878326327734D0*Z(11)+(-0.067356038933017\br
++\tab{4}&95D0*Z(10))+(-0.1559813965382218D0*Z(9))+(-0.3363262957694705D0*Z(\br
++\tab{4}&8))+0.9442791158560948D0*Z(7)+(-0.3199955249404657D0*Z(6))+(-0.136\br
++\tab{4}&2463839920727D0*Z(5))+(-0.1006185171570586D0*Z(4))+0.2057504515015\br
++\tab{4}&423D0*Z(3)+(-0.02065879269286707D0*Z(2))+0.03160990266745513D0*Z(1\br
++\tab{4}&)\br
++\tab{5}W(8)=0.126386868896738D0*Z(16)+0.002563370039476418D0*Z(15)+(-0.05\br
++\tab{4}&581757739455641D0*Z(14))+(-0.07777893205900685D0*Z(13))+0.23117338\br
++\tab{4}&45834199D0*Z(12)+(-0.06031581134427592D0*Z(11))+(-0.14805474755869\br
++\tab{4}&52D0*Z(10))+(-0.3364014128402243D0*Z(9))+0.9364014128402244D0*Z(8)\br
++\tab{4}&+(-0.3269452524413048D0*Z(7))+(-0.1396841886557241D0*Z(6))+(-0.056\br
++\tab{4}&1733845834199D0*Z(5))+0.1777789320590069D0*Z(4)+(-0.04418242260544\br
++\tab{4}&359D0*Z(3))+(-0.02756337003947642D0*Z(2))+0.07361313110326199D0*Z(\br
++\tab{4}&1)\br
++\tab{5}W(9)=0.07361313110326199D0*Z(16)+(-0.02756337003947642D0*Z(15))+(-\br
++\tab{4}&0.04418242260544359D0*Z(14))+0.1777789320590069D0*Z(13)+(-0.056173\br
++\tab{4}&3845834199D0*Z(12))+(-0.1396841886557241D0*Z(11))+(-0.326945252441\br
++\tab{4}&3048D0*Z(10))+0.9364014128402244D0*Z(9)+(-0.3364014128402243D0*Z(8\br
```

```
++\tab{4}&))+(-0.1480547475586952D0*Z(7))+(-0.06031581134427592D0*Z(6))+0.23\br
++\tab{4}&11733845834199D0*Z(5)+(-0.07777893205900685D0*Z(4))+(-0.0558175773\br
++\tab{4}&9455641D0*Z(3))+0.002563370039476418D0*Z(2)+0.126386868896738D0*Z(\br
++\tab{4}&1)\br
++\tab{5}W(10)=0.0316099026674551 3D0*Z(16)+(-0.02065879269286707D0*Z(15))+0\br
++\tab{4}&.2057504515015423D0*Z(14)+(-0.1006185171570586D0*Z(13))+(-0.136246\br
++\tab{4}&3839920727D0*Z(12))+(-0.3199955249404657D0*Z(11))+0.94427911585609\br
++\tab{4}&48D0*Z(10)+(-0.3363262957694705D0*Z(9))+(-0.1559813965382218D0*Z(8\br
++\tab{4}&))+(-0.06735603893301795D0*Z(7))+0.227687832632773 4D0*Z(6)+(-0.032\br
++\tab{4}&98438523869648D0*Z(5))+(-0.083996867458326D0*Z(4))+(-0.02113506688\br
++\tab{4}&615768D0*Z(3))+0.06681263884671322D0*Z(2)+0.06069778964023718D0*Z(\br
++\tab{4}&1)\br
++\tab{5}W(11)=0.04254083491825025D0*Z(16)+0.2311852964327382D0*Z(15)+(-0.0\br
++\tab{4}&7037620467004427D0*Z(14))+(-0.1846882042225383D0*Z(13))+(-0.315830\br
++\tab{4}&9975786731D0*Z(12))+0.952576555482747D0*Z(11)+(-0.328001910890377D\br
++\tab{4}&0*Z(10))+(-0.1589391311991561D0*Z(9))+(-0.07375317649315155D0*Z(8)\br
++\tab{4}&)+0.2229538339673001D0*Z(7)+(-0.03526886317505474D0*Z(6))+(-0.0493\br
++\tab{4}&1323319055762D0*Z(5))+(-0.04319641116207706D0*Z(4))+0.048260820054\br
++\tab{4}&65965D0*Z(3)+0.01328585741341559D0*Z(2)+0.04015147277405744D0*Z(1)\br
++\tab{5}W(12)=0.3198209177068516D0*Z(16)+(-0.03423495461011043D0*Z(15))+(-\br
++\tab{4}&0.1417185427288274D0*Z(14))+(-0.3852396953763045D0*Z(13))+0.959162\br
++\tab{4}&3347824002D0*Z(12)+(-0.315042193418466D0*Z(11))+(-0.14739952092889\br
++\tab{4}&45D0*Z(10))+(-0.08249492560213524D0*Z(9))+0.2171103102175198D0*Z(8\br
++\tab{4}&)+(-0.03769663291725935D0*Z(7))+(-0.05034242196614937D0*Z(6))+(-0.\br
++\tab{4}&01445079632086177D0*Z(5))+0.02947046460707379D0*Z(4)+(-0.002512226\br
++\tab{4}&501941856D0*Z(3))+(-0.001822737697581869D0*Z(2))+0.04556369767776 3\br
++\tab{4}&75D0*Z(1)\br
++\tab{5}W(13)=0.09089205517109111D0*Z(16)+(-0.09033531980693314D0*Z(15))+(\br
++\tab{4}&-0.3241503380268184D0*Z(14))+0.8600427128450191D0*Z(13)+(-0.305169\br
++\tab{4}&742604165D0*Z(12))+(-0.1280829963720053D0*Z(11))+(-0.066308745145 3\br
++\tab{4}&5952D0*Z(10))+0.2012230497530726D0*Z(9)+(-0.04353074206076491D0*Z(\br
++\tab{4}&8))+(-0.05051817793156355D0*Z(7))+(-0.014224695935687D0*Z(6))+0.05\br
++\tab{4}&468897337339577D0*Z(5)+(-0.01965809746040371D0*Z(4))+(-0.016234277\br
++\tab{4}&35779699D0*Z(3))+0.005239165960779299D0*Z(2)+0.05141563713660119D0\br
++\tab{4}&*Z(1)\br
++\tab{5}W(14)=(-0.02986582812574917D0*Z(16))+(-0.2995429545781457D0*Z(15))\br
++\tab{4}&+0.8892996132269974D0*Z(14)+(-0.3523683853026259D0*Z(13))+(-0.1236\br
++\tab{4}&679206156403D0*Z(12))+(-0.05760560341383113D0*Z(11))+0.20910979278\br
++\tab{4}&87612D0*Z(10)+(-0.04901428822579872D0*Z(9))+(-0.05483186562035512D\br
++\tab{4}&0*Z(8))+(-0.01632133125029967D0*Z(7))+0.05375944956767728D0*Z(6)+0\br
++\tab{4}&.002033305231024948D0*Z(5)+(-0.03032392238968179D0*Z(4))+(-0.00660\br
++\tab{4}&7305534689702D0*Z(3))+0.02021603150122265D0*Z(2)+0.033711981971903\br
++\tab{4}&02D0*Z(1)\br
++\tab{5}W(15)=(-0.2419652703415429D0*Z(16))+0.9128222941872173D0*Z(15)+(-0\br
++\tab{4}&.3244016605667343D0*Z(14))+(-0.1688977368984641D0*Z(13))+(-0.05325\br
++\tab{4}&555586632358D0*Z(12))+0.2176561076571465D0*Z(11)+(-0.0415311995556\br
++\tab{4}&9051D0*Z(10))+(-0.06095390688679697D0*Z(9))+(-0.01981532388243379D\br
++\tab{4}&0*Z(8))+0.05258889186338282D0*Z(7)+0.001574661573622 72D0*Z(6)+(-0.\br
++\tab{4}&0135713672105995D0*Z(5))+(-0.01764072463999744D0*Z(4))+0.010940122\br
++\tab{4}&10519586D0*Z(3)+0.008812321197398072D0*Z(2)+0.02273450111077 37D0*Z\br
```

```
++\tab{4}&(1)\br
++\tab{5}W(16)=1.019463911841327D0*Z(16)+(-0.2803531651057233D0*Z(15))+(-0.\br
++\tab{4}&1165300508238904D0*Z(14))+(-0.1385343580686922D0*Z(13))+0.22647669\br
++\tab{4}&47290192D0*Z(12)+(-0.02434652144032987D0*Z(11))+(-0.04723268012114\br
++\tab{4}&625D0*Z(10))+(-0.03586220812223305D0*Z(9))+0.04932374658377151D0*Z\br
++\tab{4}&(8)+0.00372306473653087D0*Z(7)+(-0.01219194009813166D0*Z(6))+(-0.0\br
++\tab{4}&07005540882865317D0*Z(5))+0.002957434991769087D0*Z(4)+0.0021069739\br
++\tab{4}&00813502D0*Z(3)+0.001747395874954051D0*Z(2)+0.01707454969713436D0*\br
++\tab{4}&Z(1)\br
++\tab{5}RETURN\br
++\tab{5}END\br

Asp28(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  EXPR   ==> Expression
  MFLOAT ==> MachineFloat
  VEC    ==> Vector
  UFST   ==> Union(fst:FST,void:"void")
  MAT    ==> Matrix

  Exports == FortranMatrixCategory

  Implementation == add


    real : UFST := ["real"::FST]$UFST
    syms : SYMTAB := empty()
    declare!(IFLAG,fortranInteger(),syms)$SYMTAB
    declare!(N,fortranInteger(),syms)$SYMTAB
    declare!(LRWORK,fortranInteger(),syms)$SYMTAB
    declare!(LIWORK,fortranInteger(),syms)$SYMTAB
    xType : FT := construct(real,[N],false)$FT
    declare!(Z,xType,syms)$SYMTAB
    declare!(W,xType,syms)$SYMTAB
    rType : FT := construct(real,[LRWORK],false)$FT
    declare!(RWORK,rType,syms)$SYMTAB
    iType : FT := construct(real,[LIWORK],false)$FT
    declare!(IWORK,rType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST,
                          [IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK],syms)

    -- To help the poor old compiler!
    localCoerce(u:Symbol):EXPR(MFLOAT) == coerce(u)$EXPR(MFLOAT)
```

```
    coerce (u:MAT MFLOAT):$ ==
      Zs: Symbol := Z
      code : List FC
      r: List EXPR MFLOAT
      r := ["+"/[u(j,i)*localCoerce(elt(Zs,[i::OutputForm])$Symbol)_
                         for i in 1..ncols(u)$MAT(MFLOAT)::PI]_
                         for j in 1..nrows(u)$MAT(MFLOAT)::PI]
      code := [assign(W@Symbol,vector(r)$VEC(EXPR MFLOAT)),returns()]$List(FC)
      code::$

    coerce(c:FortranCode):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:List FortranCode):$ == coerce(c)$Rep

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

———-

— **ASP28.dotabb** —

```
"ASP28" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP28"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP28" -> "ALIST"
```

———-

2.19 domain ASP29 Asp29

— **Asp29.input** —

```
)set break resume
)sys rm -f Asp29.output
)spool Asp29.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show Asp29
--R Asp29 name: Symbol  is a domain constructor
--R Abbreviation for Asp29 is ASP29
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP29
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              outputAsFortran : () -> Void
--R outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)
```

———-

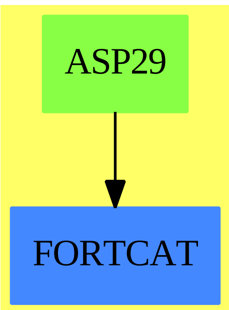
— Asp29.help —

```
====================================================================
Asp29 examples
====================================================================

See Also:
o )show Asp29
```

———-

2.19.1 Asp29 (ASP29)



Exports:
coerce outputAsFortran

— domain ASP29 Asp29 —

```
)abbrev domain ASP29 Asp29
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp29} produces Fortran for Type 29 ASPs, needed for NAG routine
++f02fjf, for example:
++
++\tab{5}SUBROUTINE MONIT(ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D)\br
++\tab{5}DOUBLE PRECISION D(K),F(K)\br
++\tab{5}INTEGER K,NEXTIT,NEVALS,NVECS,ISTATE\br
++\tab{5}CALL F02FJZ(ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D)\br
++\tab{5}RETURN\br
++\tab{5}END\br

Asp29(name): Exports == Implementation where
  name : Symbol

  FST  ==> FortranScalarType
  FT   ==> FortranType
  FSTU   ==> Union(fst:FST,void:"void")
  SYMTAB ==> SymbolTable
  FC   ==> FortranCode
  PI   ==> PositiveInteger
  EXF   ==> Expression Float
  EXI   ==> Expression Integer
  VEF  ==> Vector Expression Float
  VEI  ==> Vector Expression Integer
  MEI  ==> Matrix Expression Integer
  MEF  ==> Matrix Expression Float
  UEXPR ==> Union(I: Expression Integer,F: Expression Float,_
                CF: Expression Complex Float)
  RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))

  Exports == FortranProgramCategory  with
     outputAsFortran:() -> Void
       ++outputAsFortran() generates the default code for \spadtype{ASP29}.


  Implementation == add

    import FST
    import FT
    import FC
    import SYMTAB
```

```
    real : FSTU := ["real"::FST]$FSTU
    integer : FSTU := ["integer"::FST]$FSTU
    syms : SYMTAB := empty()
    declare!(ISTATE,fortranInteger(),syms)
    declare!(NEXTIT,fortranInteger(),syms)
    declare!(NEVALS,fortranInteger(),syms)
    declare!(NVECS,fortranInteger(),syms)
    declare!(K,fortranInteger(),syms)
    kType : FT := construct(real,[K],false)$FT
    declare!(F,kType,syms)
    declare!(D,kType,syms)
    Rep := FortranProgram(name,["void"]$FSTU,
                          [ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D],syms)


    outputAsFortran():Void ==
      callOne := call("F02FJZ(ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D)")
      code : List FC := [callOne,returns()]$List(FC)
      outputAsFortran(coerce(code)@Rep)$Rep
```

———-

— **ASP29.dotabb** —

```
"ASP29" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP29"]
"FORTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FORTCAT"]
"ASP29" -> "FORTCAT"
```

———-

2.20 domain ASP30 Asp30

— **Asp30.input** —

```
)set break resume
)sys rm -f Asp30.output
)spool Asp30.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp30
--R Asp30 name: Symbol  is a domain constructor
```

```
--R Abbreviation for Asp30 is ASP30
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP30
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

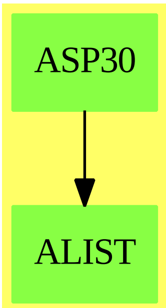
— **Asp30.help** —

```
====================================================================
Asp30 examples
====================================================================

See Also:
o )show Asp30
```

———-

2.20.1 Asp30 (ASP30)



Exports:
coerce outputAsFortran

— **domain ASP30 Asp30** —

```
)abbrev domain ASP30 Asp30
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 28 March 1994
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp30} produces Fortran for Type 30 ASPs, needed for NAG routine
++f04qaf, for example:
++
++\tab{5}SUBROUTINE APROD(MODE,M,N,X,Y,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION X(N),Y(M),RWORK(LRWORK)\br
++\tab{5}INTEGER M,N,LIWORK,IFAIL,LRWORK,IWORK(LIWORK),MODE\br
++\tab{5}DOUBLE PRECISION A(5,5)\br
++\tab{5}EXTERNAL F06PAF\br
++\tab{5}A(1,1)=1.0D0\br
++\tab{5}A(1,2)=0.0D0\br
++\tab{5}A(1,3)=0.0D0\br
++\tab{5}A(1,4)=-1.0D0\br
++\tab{5}A(1,5)=0.0D0\br
++\tab{5}A(2,1)=0.0D0\br
++\tab{5}A(2,2)=1.0D0\br
++\tab{5}A(2,3)=0.0D0\br
++\tab{5}A(2,4)=0.0D0\br
++\tab{5}A(2,5)=-1.0D0\br
++\tab{5}A(3,1)=0.0D0\br
++\tab{5}A(3,2)=0.0D0\br
++\tab{5}A(3,3)=1.0D0\br
++\tab{5}A(3,4)=-1.0D0\br
++\tab{5}A(3,5)=0.0D0\br
++\tab{5}A(4,1)=-1.0D0\br
++\tab{5}A(4,2)=0.0D0\br
++\tab{5}A(4,3)=-1.0D0\br
++\tab{5}A(4,4)=4.0D0\br
++\tab{5}A(4,5)=-1.0D0\br
++\tab{5}A(5,1)=0.0D0\br
++\tab{5}A(5,2)=-1.0D0\br
++\tab{5}A(5,3)=0.0D0\br
++\tab{5}A(5,4)=-1.0D0\br
++\tab{5}A(5,5)=4.0D0\br
++\tab{5}IF(MODE.EQ.1)THEN\br
++\tab{7}CALL F06PAF('N',M,N,1.0D0,A,M,X,1,1.0D0,Y,1)\br
++\tab{5}ELSEIF(MODE.EQ.2)THEN\br
++\tab{7}CALL F06PAF('T',M,N,1.0D0,A,M,Y,1,1.0D0,X,1)\br
++\tab{5}ENDIF\br
++\tab{5}RETURN\br
++\tab{5}END

Asp30(name): Exports == Implementation where
  name : Symbol
```

```
FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
FC     ==> FortranCode
PI     ==> PositiveInteger
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
UFST   ==> Union(fst:FST,void:"void")
MAT    ==> Matrix
MFLOAT ==> MachineFloat
EXI    ==> Expression Integer
UEXPR  ==> Union(I:Expression Integer,F:Expression Float,_
                 CF:Expression Complex Float,switch:Switch)
S      ==> Symbol

Exports == FortranMatrixCategory

Implementation == add

  import FC
  import FT
  import Switch

  real : UFST := ["real"::FST]$UFST
  integer : UFST := ["integer"::FST]$UFST
  syms : SYMTAB := empty()$SYMTAB
  declare!(MODE,fortranInteger()$FT,syms)$SYMTAB
  declare!(M,fortranInteger()$FT,syms)$SYMTAB
  declare!(N,fortranInteger()$FT,syms)$SYMTAB
  declare!(LRWORK,fortranInteger()$FT,syms)$SYMTAB
  declare!(LIWORK,fortranInteger()$FT,syms)$SYMTAB
  xType : FT := construct(real,[N],false)$FT
  declare!(X,xType,syms)$SYMTAB
  yType : FT := construct(real,[M],false)$FT
  declare!(Y,yType,syms)$SYMTAB
  rType : FT := construct(real,[LRWORK],false)$FT
  declare!(RWORK,rType,syms)$SYMTAB
  iType : FT := construct(integer,[LIWORK],false)$FT
  declare!(IWORK,iType,syms)$SYMTAB
  declare!(IFAIL,fortranInteger()$FT,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$UFST,
                        [MODE,M,N,X,Y,RWORK,LRWORK,IWORK,LIWORK],syms)

  coerce(a:MAT MFLOAT):$ ==
    locals : SYMTAB := empty()
    numRows := nrows(a) :: Polynomial Integer
    numCols := ncols(a) :: Polynomial Integer
    declare!(A,[real,[numRows,numCols],false]$FT,locals)
    declare!(F06PAF@S,construct(["void"]$UFST,[]@List(S),true)$FT,locals)
    ptA:UEXPR := [("MODE"::S)::EXI]
```

```
      ptB:UEXPR := [1::EXI]
      ptC:UEXPR := [2::EXI]
      sw1 : Switch := EQ(ptA,ptB)$Switch
      sw2 : Switch := EQ(ptA,ptC)$Switch
      callOne := call("F06PAF('N',M,N,1.0D0,A,M,X,1,1.0D0,Y,1)")
      callTwo := call("F06PAF('T',M,N,1.0D0,A,M,Y,1,1.0D0,X,1)")
      c : FC := cond(sw1,callOne,cond(sw2,callTwo))
      code : List FC := [assign(A,a),c,returns()]
      ([locals,code]$RSFC)::$

    coerce(c:List FortranCode):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FortranCode):$ == coerce(c)$Rep

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

———-

— ASP30.dotabb —

```
"ASP30" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP30"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP30" -> "ALIST"
```

———-

2.21 domain ASP31 Asp31

— Asp31.input —

```
)set break resume
)sys rm -f Asp31.output
)spool Asp31.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show Asp31
--R Asp31 name: Symbol  is a domain constructor
--R Abbreviation for Asp31 is ASP31
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP31
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX],[construct,QUOTEY],MachineFloat) ->
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

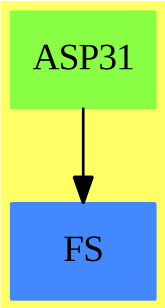
)spool
)lisp (bye)
```

— Asp31.help —

```
====================================================================
Asp31 examples
====================================================================

See Also:
o )show Asp31
```

2.21.1 Asp31 (ASP31)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP31 Asp31** —

```
)abbrev domain ASP31 Asp31
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 22 March 1994
++                    6 October 1994
++ Related Constructors: FortranMatrixFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp31} produces Fortran for Type 31 ASPs, needed for NAG routine
++d02ejf, for example:
++
++\tab{5}SUBROUTINE PEDERV(X,Y,PW)\br
++\tab{5}DOUBLE PRECISION X,Y(*)\br
++\tab{5}DOUBLE PRECISION PW(3,3)\br
++\tab{5}PW(1,1)=-0.03999999999999999D0\br
++\tab{5}PW(1,2)=10000.0D0*Y(3)\br
++\tab{5}PW(1,3)=10000.0D0*Y(2)\br
++\tab{5}PW(2,1)=0.03999999999999999D0\br
++\tab{5}PW(2,2)=(-10000.0D0*Y(3))+(-60000000.0D0*Y(2))\br
++\tab{5}PW(2,3)=-10000.0D0*Y(2)\br
++\tab{5}PW(3,1)=0.0D0\br
++\tab{5}PW(3,2)=60000000.0D0*Y(2)\br
++\tab{5}PW(3,3)=0.0D0\br
++\tab{5}RETURN\br
++\tab{5}END

Asp31(name): Exports == Implementation where
  name : Symbol

  O       ==> OutputForm
  FST     ==> FortranScalarType
```

```
UFST   ==> Union(fst:FST,void:"void")
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['X],['Y],MFLOAT)
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector
MAT    ==> Matrix
VF2    ==> VectorFunctions2
MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR,
                 EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,MAT EXPR MFLOAT)



Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add


  real : UFST := ["real"::FST]$UFST
  syms : SYMTAB := empty()
  declare!(X,fortranReal(),syms)$SYMTAB
  yType : FT := construct(real,["*"::Symbol],false)$FT
  declare!(Y,yType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$UFST,[X,Y,PW],syms)

  -- To help the poor old compiler!
  fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

  localAssign(s:Symbol,j:MAT FEXPR):FC ==
    j' : MAT EXPR MFLOAT := map(fexpr2expr,j)$MF2
    assign(s,j')$FC

  makeXList(n:Integer):List(Symbol) ==
    j:Integer
    y:Symbol := Y::Symbol
    p:List(Symbol) := []
    for j in 1 .. n repeat p:= cons(subscript(y,[j::OutputForm])$Symbol,p)
    p:= reverse(p)

  coerce(u:VEC FEXPR):$ ==
```

```
    dimension := #u::Polynomial Integer
    locals : SYMTAB := empty()
    declare!(PW,[real,[dimension,dimension],false]$FT,locals)$SYMTAB
    n:Integer := maxIndex(u)$VEC(FEXPR)
    p:List(Symbol) := makeXList(n)
    jac: MAT FEXPR := jacobian(u,p)$MultiVariableCalculusFunctions(_
                                 Symbol,FEXPR ,VEC FEXPR,List(Symbol))
    code : List FC := [localAssign(PW,jac),returns()$FC]$List(FC)
    ([locals,code]$RSFC)::$

  retract(u:VEC FRAC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC FRAC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
    v::$
```

```
  retractIfCan(u:VEC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  coerce(c:List FC):$ == coerce(c)$Rep

  coerce(r:RSFC):$ == coerce(r)$Rep

  coerce(c:FC):$ == coerce(c)$Rep

  coerce(u:$):O == coerce(u)$Rep

  outputAsFortran(u):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage
```

— ASP31.dotabb —

```
"ASP31" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP31"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP31" -> "FS"
```

2.22 domain ASP33 Asp33

— Asp33.input —

```
)set break resume
)sys rm -f Asp33.output
)spool Asp33.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp33
--R Asp33 name: Symbol  is a domain constructor
--R Abbreviation for Asp33 is ASP33
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP33
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              outputAsFortran : () -> Void
--R outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)
```

———-

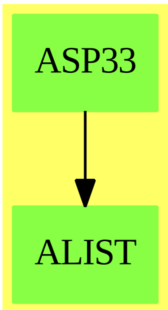
— **Asp33.help** —

```
====================================================================
Asp33 examples
====================================================================

See Also:
o )show Asp33
```

———-

2.22.1 Asp33 (ASP33)



Exports:
coerce outputAsFortran

— domain ASP33 Asp33 —

```
)abbrev domain ASP33 Asp33
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 30 March 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory.
++ Description:
++\spadtype{Asp33} produces Fortran for Type 33 ASPs, needed for NAG routine
++d02kef.  The code is a dummy ASP:
++
++\tab{5}SUBROUTINE REPORT(X,V,JINT)\br
++\tab{5}DOUBLE PRECISION V(3),X\br
++\tab{5}INTEGER JINT\br
++\tab{5}RETURN\br
++\tab{5}END

Asp33(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  UFST   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))

  Exports ==> FortranProgramCategory with
     outputAsFortran:() -> Void
      ++outputAsFortran() generates the default code for \spadtype{ASP33}.


  Implementation ==> add

    real : UFST := ["real"::FST]$UFST
    syms : SYMTAB := empty()
    declare!(JINT,fortranInteger(),syms)$SYMTAB
    declare!(X,fortranReal(),syms)$SYMTAB
    vType : FT := construct(real,["3"::Symbol],false)$FT
    declare!(V,vType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST,[X,V,JINT],syms)

    outputAsFortran():Void ==
      outputAsFortran( (returns()$FortranCode)::Rep )$Rep

    outputAsFortran(u):Void == outputAsFortran(u)$Rep
```

```
    coerce(u:$):OutputForm == coerce(u)$Rep
```

— ASP33.dotabb —

```
"ASP33" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP33"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP33" -> "ALIST"
```

2.23 domain ASP34 Asp34

— Asp34.input —

```
)set break resume
)sys rm -f Asp34.output
)spool Asp34.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp34
--R Asp34 name: Symbol  is a domain constructor
--R Abbreviation for Asp34 is ASP34
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP34
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

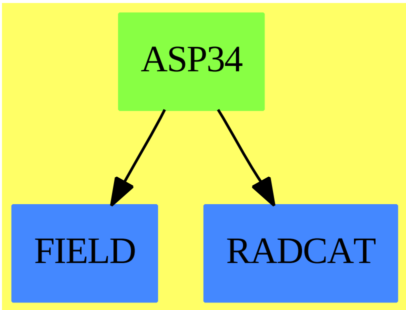
— Asp34.help —

```
====================================================================
Asp34 examples
====================================================================

See Also:
o )show Asp34
```

———-

2.23.1 Asp34 (ASP34)



Exports:
coerce outputAsFortran

— domain ASP34 Asp34 —

```
)abbrev domain ASP34 Asp34
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 14 June 1994 (Themos Tsikas)
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp34} produces Fortran for Type 34 ASPs, needed for NAG routine
++f04mbf, for example:
++
++\tab{5}SUBROUTINE MSOLVE(IFLAG,N,X,Y,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION RWORK(LRWORK),X(N),Y(N)\br
++\tab{5}INTEGER I,J,N,LIWORK,IFLAG,LRWORK,IWORK(LIWORK)\br
++\tab{5}DOUBLE PRECISION W1(3),W2(3),MS(3,3)\br
++\tab{5}IFLAG=-1\br
```

```
++\tab{5}MS(1,1)=2.0D0\br
++\tab{5}MS(1,2)=1.0D0\br
++\tab{5}MS(1,3)=0.0D0\br
++\tab{5}MS(2,1)=1.0D0\br
++\tab{5}MS(2,2)=2.0D0\br
++\tab{5}MS(2,3)=1.0D0\br
++\tab{5}MS(3,1)=0.0D0\br
++\tab{5}MS(3,2)=1.0D0\br
++\tab{5}MS(3,3)=2.0D0\br
++\tab{5}CALL F04ASF(MS,N,X,N,Y,W1,W2,IFLAG)\br
++\tab{5}IFLAG=-IFLAG\br
++\tab{5}RETURN\br
++\tab{5}END

Asp34(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  UFST   ==> Union(fst:FST,void:"void")
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  EXI    ==> Expression Integer
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))

  Exports == FortranMatrixCategory

  Implementation == add

    real : UFST := ["real"::FST]$UFST
    integer : UFST := ["integer"::FST]$UFST
    syms : SYMTAB := empty()$SYMTAB
    declare!(IFLAG,fortranInteger(),syms)$SYMTAB
    declare!(N,fortranInteger(),syms)$SYMTAB
    xType : FT := construct(real,[N],false)$FT
    declare!(X,xType,syms)$SYMTAB
    declare!(Y,xType,syms)$SYMTAB
    declare!(LRWORK,fortranInteger(),syms)$SYMTAB
    declare!(LIWORK,fortranInteger(),syms)$SYMTAB
    rType : FT := construct(real,[LRWORK],false)$FT
    declare!(RWORK,rType,syms)$SYMTAB
    iType : FT := construct(integer,[LIWORK],false)$FT
    declare!(IWORK,iType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST,
                      [IFLAG,N,X,Y,RWORK,LRWORK,IWORK,LIWORK],syms)

    -- To help the poor old compiler
    localAssign(s:Symbol,u:EXI):FC == assign(s,u)$FC
```

```
  coerce(u:Matrix MachineFloat):$ ==
    dimension := nrows(u) ::Polynomial Integer
    locals : SYMTAB := empty()$SYMTAB
    declare!(I,fortranInteger(),syms)$SYMTAB
    declare!(J,fortranInteger(),syms)$SYMTAB
    declare!(W1,[real,[dimension],false]$FT,locals)$SYMTAB
    declare!(W2,[real,[dimension],false]$FT,locals)$SYMTAB
    declare!(MS,[real,[dimension,dimension],false]$FT,locals)$SYMTAB
    assign1 : FC := localAssign(IFLAG@Symbol,(-1)@EXI)
    call : FC := call("F04ASF(MS,N,X,N,Y,W1,W2,IFLAG)")$FC
    assign2 : FC := localAssign(IFLAG::Symbol,-(IFLAG@Symbol::EXI))
    assign3 : FC := assign(MS,u)$FC
    code : List FC := [assign1,assign3,call,assign2,returns()]$List(FC)
    ([locals,code]$RSFC)::$

  coerce(c:List FortranCode):$ == coerce(c)$Rep

  coerce(r:RSFC):$ == coerce(r)$Rep

  coerce(c:FortranCode):$ == coerce(c)$Rep

  coerce(u:$):OutputForm == coerce(u)$Rep

  outputAsFortran(u):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage
```

———-

— **ASP34.dotabb** —

```
"ASP34" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP34"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"ASP34" -> "FIELD"
"ASP34" -> "RADCAT"
```

———-

2.24 domain ASP35 Asp35

— **Asp35.input** —

```
)set break resume
```

```
)sys rm -f Asp35.output
)spool Asp35.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp35
--R Asp35 name: Symbol  is a domain constructor
--R Abbreviation for Asp35 is ASP35
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP35
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

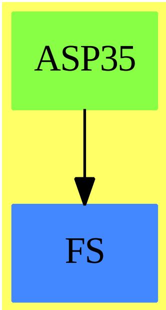
— Asp35.help —

```
====================================================================
Asp35 examples
====================================================================

See Also:
o )show Asp35
```

———

2.24.1 Asp35 (ASP35)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP35 Asp35 —

```
)abbrev domain ASP35 Asp35
++ Author: Mike Dewar, Godfrey Nolan, Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 22 March 1994
++                    6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp35} produces Fortran for Type 35 ASPs, needed for NAG routines
++c05pbf, c05pcf, for example:
++
++\tab{5}SUBROUTINE FCN(N,X,FVEC,FJAC,LDFJAC,IFLAG)\br
++\tab{5}DOUBLE PRECISION X(N),FVEC(N),FJAC(LDFJAC,N)\br
++\tab{5}INTEGER LDFJAC,N,IFLAG\br
++\tab{5}IF(IFLAG.EQ.1)THEN\br
++\tab{7}FVEC(1)=(-1.0D0*X(2))+X(1)\br
++\tab{7}FVEC(2)=(-1.0D0*X(3))+2.0D0*X(2)\br
++\tab{7}FVEC(3)=3.0D0*X(3)\br
++\tab{5}ELSEIF(IFLAG.EQ.2)THEN\br
++\tab{7}FJAC(1,1)=1.0D0\br
++\tab{7}FJAC(1,2)=-1.0D0\br
++\tab{7}FJAC(1,3)=0.0D0\br
++\tab{7}FJAC(2,1)=0.0D0\br
++\tab{7}FJAC(2,2)=2.0D0\br
++\tab{7}FJAC(2,3)=-1.0D0\br
++\tab{7}FJAC(3,1)=0.0D0\br
++\tab{7}FJAC(3,2)=0.0D0\br
++\tab{7}FJAC(3,3)=3.0D0\br
++\tab{5}ENDIF\br
++\tab{5}END
```

```
Asp35(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  UFST   ==> Union(fst:FST,void:"void")
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  MAT    ==> Matrix
  VF2    ==> VectorFunctions2
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression([],['X],MFLOAT)
  MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR,
                    EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,MAT EXPR MFLOAT)
  SWU    ==> Union(I:Expression Integer,F:Expression Float,
                  CF:Expression Complex Float,switch:Switch)

  Exports ==> FortranVectorFunctionCategory with
    coerce : VEC FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add

    real : UFST := ["real"::FST]$UFST
    syms : SYMTAB := empty()$SYMTAB
    declare!(N,fortranInteger(),syms)$SYMTAB
    xType : FT := construct(real,[N],false)$FT
    declare!(X,xType,syms)$SYMTAB
    declare!(FVEC,xType,syms)$SYMTAB
    declare!(LDFJAC,fortranInteger(),syms)$SYMTAB
    jType : FT := construct(real,[LDFJAC,N],false)$FT
    declare!(FJAC,jType,syms)$SYMTAB
    declare!(IFLAG,fortranInteger(),syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST,[N,X,FVEC,FJAC,LDFJAC,IFLAG],syms)

    coerce(u:$):OutputForm == coerce(u)$Rep

    makeXList(n:Integer):List(Symbol) ==
      x:Symbol := X::Symbol
      [subscript(x,[j::OutputForm])$Symbol for j in 1..n]
```

```
fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:Symbol,j:MAT FEXPR):FC ==
  j' : MAT EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:Symbol,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

coerce(u:VEC FEXPR):$ ==
  n:Integer := maxIndex(u)
  p:List(Symbol) := makeXList(n)
  jac: MAT FEXPR := jacobian(u,p)$MultiVariableCalculusFunctions(_
                                   Symbol,FEXPR,VEC FEXPR,List(Symbol))
  assf:FC := localAssign2(FVEC,u)
  assj:FC := localAssign1(FJAC,jac)
  iflag:SWU := [IFLAG@Symbol::EXPR(INT)]$SWU
  sw1:Switch := EQ(iflag,[1::EXPR(INT)]$SWU)
  sw2:Switch := EQ(iflag,[2::EXPR(INT)]$SWU)
  cond(sw1,assf,cond(sw2,assj)$FC)$FC::$

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$
```

```
    retract(u:VEC EXPR INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC EXPR FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$
```

—— **ASP35.dotabb** ——

```
"ASP35" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP35"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP35" -> "FS"
```

2.25 domain ASP4 Asp4

— **Asp4.input** —

```
)set break resume
)sys rm -f Asp4.output
)spool Asp4.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp4
--R Asp4 name: Symbol  is a domain constructor
--R Abbreviation for Asp4 is ASP4
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP4
--R
--R------------------------------ Operations -------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %     retract : Polynomial Float -> %
--R retract : Expression Integer -> %     retract : Expression Float -> %
--R coerce : FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Polynomial Float -> Union(%,"failed")
--R retractIfCan : Expression Integer -> Union(%,"failed")
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

— **Asp4.help** —

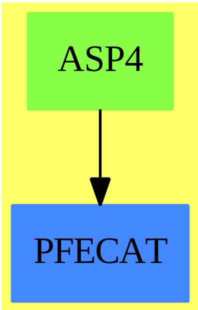
```
====================================================================
Asp4 examples
====================================================================

See Also:
```

```
o )show Asp4
```

———-

2.25.1 Asp4 (ASP4)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP4 Asp4** —

```
)abbrev domain ASP4 Asp4
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp4} produces Fortran for Type 4 ASPs, which take an expression
++in X(1) .. X(NDIM) and produce a real function of the form:
++
++\tab{5}DOUBLE PRECISION FUNCTION FUNCTN(NDIM,X)\br
++\tab{5}DOUBLE PRECISION X(NDIM)\br
++\tab{5}INTEGER NDIM\br
++\tab{5}FUNCTN=(4.0D0*X(1)*X(3)**2*DEXP(2.0D0*X(1)*X(3)))/(X(4)**2+(2.0D0*\br
++\tab{4}&X(2)+2.0D0)*X(4)+X(2)**2+2.0D0*X(2)+1.0D0)\br
++\tab{5}RETURN\br
++\tab{5}END

Asp4(name): Exports == Implementation where
  name : Symbol

  FEXPR   ==> FortranExpression([],['X],MachineFloat)
  FST     ==> FortranScalarType
```

```
FT     ==> FortranType
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FSTU   ==> Union(fst:FST,void:"void")
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float

Exports ==> FortranFunctionCategory with
  coerce : FEXPR -> $
    ++coerce(f) takes an object from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns it into an ASP.

Implementation ==> add

  real : FSTU := ["real"::FST]$FSTU
  syms : SYMTAB := empty()$SYMTAB
  declare!(NDIM,fortranInteger(),syms)$SYMTAB
  xType : FT := construct(real,[NDIM],false)$FT
  declare!(X,xType,syms)$SYMTAB
  Rep := FortranProgram(name,real,[NDIM,X],syms)

  retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
  retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
    foo : Union(FEXPR,"failed")
    foo := retractIfCan(u)$FEXPR
    foo case "failed" => "failed"
    foo::FEXPR::$

  retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
  retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
    foo : Union(FEXPR,"failed")
    foo := retractIfCan(u)$FEXPR
    foo case "failed" => "failed"
    foo::FEXPR::$

  retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
  retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
    foo : Union(FEXPR,"failed")
    foo := retractIfCan(u)$FEXPR
    foo case "failed" => "failed"
    foo::FEXPR::$

  retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
  retractIfCan(u:EXPR INT):Union($,"failed") ==
    foo : Union(FEXPR,"failed")
    foo := retractIfCan(u)$FEXPR
    foo case "failed" => "failed"
```

```
      foo::FEXPR::$

    retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
    retractIfCan(u:POLY FLOAT):Union($,"failed") ==
      foo : Union(FEXPR,"failed")
      foo := retractIfCan(u)$FEXPR
      foo case "failed" => "failed"
      foo::FEXPR::$

    retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
    retractIfCan(u:POLY INT):Union($,"failed") ==
      foo : Union(FEXPR,"failed")
      foo := retractIfCan(u)$FEXPR
      foo case "failed" => "failed"
      foo::FEXPR::$

    coerce(u:FEXPR):$ ==
      coerce((u::Expression(MachineFloat))$FEXPR)$Rep

    coerce(c:List FortranCode):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FortranCode):$ == coerce(c)$Rep

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

— ASP4.dotabb —

```
"ASP4" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP4"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP4" -> "PFECAT"
```

2.26 domain ASP41 Asp41

— Asp41.input —

```
)set break resume
)sys rm -f Asp41.output
)spool Asp41.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp41
--R Asp41(nameOne: Symbol,nameTwo: Symbol,nameThree: Symbol)  is a domain constructor
--R Abbreviation for Asp41 is ASP41
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP41
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX,QUOTEEPS],[construct,QUOTEY],Machine
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

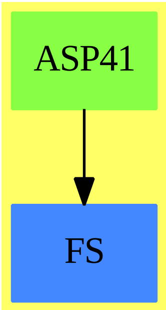
— **Asp41.help** —

```
====================================================================
Asp41 examples
====================================================================

See Also:
o )show Asp41
```

———

2.26.1 Asp41 (ASP41)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP41 Asp41 —

```
)abbrev domain ASP41 Asp41
++ Author: Mike Dewar, Godfrey Nolan
++ Date Created:
++ Date Last Updated: 29 March 1994
++                     6 October 1994
++ Related Constructors: FortranFunctionCategory, FortranProgramCategory.
++ Description:
++\spadtype{Asp41} produces Fortran for Type 41 ASPs, needed for NAG
++routines d02raf and d02saf in particular.  These ASPs are in fact
++three Fortran routines which return a vector of functions, and their
++derivatives wrt Y(i) and also a continuation parameter EPS, for example:
++
++\tab{5}SUBROUTINE FCN(X,EPS,Y,F,N)\br
++\tab{5}DOUBLE PRECISION EPS,F(N),X,Y(N)\br
++\tab{5}INTEGER N\br
++\tab{5}F(1)=Y(2)\br
++\tab{5}F(2)=Y(3)\br
++\tab{5}F(3)=(-1.0D0*Y(1)*Y(3))+2.0D0*EPS*Y(2)**2+(-2.0D0*EPS)\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACOBF(X,EPS,Y,F,N)\br
++\tab{5}DOUBLE PRECISION EPS,F(N,N),X,Y(N)\br
++\tab{5}INTEGER N\br
++\tab{5}F(1,1)=0.0D0\br
++\tab{5}F(1,2)=1.0D0\br
++\tab{5}F(1,3)=0.0D0\br
++\tab{5}F(2,1)=0.0D0\br
++\tab{5}F(2,2)=0.0D0\br
++\tab{5}F(2,3)=1.0D0\br
++\tab{5}F(3,1)=-1.0D0*Y(3)\br
```

```
++\tab{5}F(3,2)=4.0D0*EPS*Y(2)\br
++\tab{5}F(3,3)=-1.0D0*Y(1)\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACEPS(X,EPS,Y,F,N)\br
++\tab{5}DOUBLE PRECISION EPS,F(N),X,Y(N)\br
++\tab{5}INTEGER N\br
++\tab{5}F(1)=0.0D0\br
++\tab{5}F(2)=0.0D0\br
++\tab{5}F(3)=2.0D0*Y(2)**2-2.0D0\br
++\tab{5}RETURN\br
++\tab{5}END

Asp41(nameOne,nameTwo,nameThree): Exports == Implementation where
  nameOne : Symbol
  nameTwo : Symbol
  nameThree : Symbol

  D      ==> differentiate
  FST    ==> FortranScalarType
  UFST   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X,'EPS],['Y],MFLOAT)
  S      ==> Symbol
  MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,
                 EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,Matrix EXPR MFLOAT)

  Exports ==> FortranVectorFunctionCategory with
    coerce : VEC FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add
    real : UFST := ["real"::FST]$UFST

    symOne : SYMTAB := empty()$SYMTAB
    declare!(N,fortranInteger(),symOne)$SYMTAB
    declare!(X,fortranReal(),symOne)$SYMTAB
    declare!(EPS,fortranReal(),symOne)$SYMTAB
```

```
yType : FT := construct(real,[N],false)$FT
declare!(Y,yType,symOne)$SYMTAB
declare!(F,yType,symOne)$SYMTAB

symTwo : SYMTAB := empty()$SYMTAB
declare!(N,fortranInteger(),symTwo)$SYMTAB
declare!(X,fortranReal(),symTwo)$SYMTAB
declare!(EPS,fortranReal(),symTwo)$SYMTAB
declare!(Y,yType,symTwo)$SYMTAB
fType : FT := construct(real,[N,N],false)$FT
declare!(F,fType,symTwo)$SYMTAB

symThree : SYMTAB := empty()$SYMTAB
declare!(N,fortranInteger(),symThree)$SYMTAB
declare!(X,fortranReal(),symThree)$SYMTAB
declare!(EPS,fortranReal(),symThree)$SYMTAB
declare!(Y,yType,symThree)$SYMTAB
declare!(F,yType,symThree)$SYMTAB

R1:=FortranProgram(nameOne,["void"]$UFST,[X,EPS,Y,F,N],symOne)
R2:=FortranProgram(nameTwo,["void"]$UFST,[X,EPS,Y,F,N],symTwo)
R3:=FortranProgram(nameThree,["void"]$UFST,[X,EPS,Y,F,N],symThree)
Rep := Record(f:R1,fJacob:R2,eJacob:R3)
Fsym:Symbol:=coerce "F"

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:S,j:Matrix FEXPR):FC ==
  j' : Matrix EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

makeCodeOne(u:VEC FEXPR):FortranCode ==
  -- simple assign
  localAssign2(Fsym,u)

makeCodeThree(u:VEC FEXPR):FortranCode ==
  -- compute jacobian wrt to eps
  jacEps:VEC FEXPR := [D(v,EPS) for v in entries(u)]$VEC(FEXPR)
  makeCodeOne(jacEps)

makeYList(n:Integer):List(Symbol) ==
  j:Integer
  y:Symbol := Y::Symbol
  p:List(Symbol) := []
  [subscript(y,[j::OutputForm])$Symbol for j in 1..n]
```

```
makeCodeTwo(u:VEC FEXPR):FortranCode ==
  -- compute jacobian wrt to f
  n:Integer := maxIndex(u)$VEC(FEXPR)
  p:List(Symbol) := makeYList(n)
  jac:Matrix(FEXPR) := _
  jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  localAssign1(Fsym,jac)

coerce(u:VEC FEXPR):$ ==
  aF:FortranCode := makeCodeOne(u)
  bF:FortranCode := makeCodeTwo(u)
  cF:FortranCode := makeCodeThree(u)
  -- add returns() to complete subroutines
  aLF:List(FortranCode) := [aF,returns()$FortranCode]$List(FortranCode)
  bLF:List(FortranCode) := [bF,returns()$FortranCode]$List(FortranCode)
  cLF:List(FortranCode) := [cF,returns()$FortranCode]$List(FortranCode)
  [coerce(aLF)$R1,coerce(bLF)$R2,coerce(cLF)$R3]

coerce(u:$):OutputForm ==
  bracket commaSeparate
    [nameOne::OutputForm,nameTwo::OutputForm,nameThree::OutputForm]

outputAsFortran(u:$):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran elt(u,f)$Rep
  outputAsFortran elt(u,fJacob)$Rep
  outputAsFortran elt(u,eJacob)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$
```

```
retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$
```

———

— **ASP41.dotabb** —

```
"ASP41" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP41"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP41" -> "FS"
```

———

2.27 domain ASP42 Asp42

— Asp42.input —

```
)set break resume
)sys rm -f Asp42.output
)spool Asp42.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp42
--R Asp42(nameOne: Symbol,nameTwo: Symbol,nameThree: Symbol)  is a domain constructor
--R Abbreviation for Asp42 is ASP42
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP42
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEEPS],[construct,QUOTEYA,QUOTEYB],Machi
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

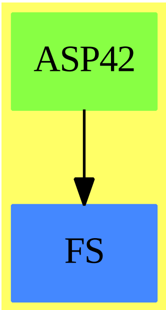
— Asp42.help —

```
====================================================================
Asp42 examples
====================================================================

See Also:
o )show Asp42
```

———-

2.27.1 Asp42 (ASP42)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP42 Asp42 —

```
)abbrev domain ASP42 Asp42
++ Author: Mike Dewar, Godfrey Nolan
++ Date Created:
++ Date Last Updated: 29 March 1994
++                    6 October 1994
++ Related Constructors: FortranFunctionCategory, FortranProgramCategory.
++ Description:
++\spadtype{Asp42} produces Fortran for Type 42 ASPs, needed for NAG
++routines d02raf and d02saf
++in particular.  These ASPs are in fact
++three Fortran routines which return a vector of functions, and their
++derivatives wrt Y(i) and also a continuation parameter EPS, for example:
++
++\tab{5}SUBROUTINE G(EPS,YA,YB,BC,N)\br
++\tab{5}DOUBLE PRECISION EPS,YA(N),YB(N),BC(N)\br
++\tab{5}INTEGER N\br
++\tab{5}BC(1)=YA(1)\br
++\tab{5}BC(2)=YA(2)\br
++\tab{5}BC(3)=YB(2)-1.0D0\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACOBG(EPS,YA,YB,AJ,BJ,N)\br
++\tab{5}DOUBLE PRECISION EPS,YA(N),AJ(N,N),BJ(N,N),YB(N)\br
++\tab{5}INTEGER N\br
```

```
++\tab{5}AJ(1,1)=1.0D0\br
++\tab{5}AJ(1,2)=0.0D0\br
++\tab{5}AJ(1,3)=0.0D0\br
++\tab{5}AJ(2,1)=0.0D0\br
++\tab{5}AJ(2,2)=1.0D0\br
++\tab{5}AJ(2,3)=0.0D0\br
++\tab{5}AJ(3,1)=0.0D0\br
++\tab{5}AJ(3,2)=0.0D0\br
++\tab{5}AJ(3,3)=0.0D0\br
++\tab{5}BJ(1,1)=0.0D0\br
++\tab{5}BJ(1,2)=0.0D0\br
++\tab{5}BJ(1,3)=0.0D0\br
++\tab{5}BJ(2,1)=0.0D0\br
++\tab{5}BJ(2,2)=0.0D0\br
++\tab{5}BJ(2,3)=0.0D0\br
++\tab{5}BJ(3,1)=0.0D0\br
++\tab{5}BJ(3,2)=1.0D0\br
++\tab{5}BJ(3,3)=0.0D0\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACGEP(EPS,YA,YB,BCEP,N)\br
++\tab{5}DOUBLE PRECISION EPS,YA(N),YB(N),BCEP(N)\br
++\tab{5}INTEGER N\br
++\tab{5}BCEP(1)=0.0D0\br
++\tab{5}BCEP(2)=0.0D0\br
++\tab{5}BCEP(3)=0.0D0\br
++\tab{5}RETURN\br
++\tab{5}END

Asp42(nameOne,nameTwo,nameThree): Exports == Implementation where
  nameOne : Symbol
  nameTwo : Symbol
  nameThree : Symbol

  D      ==> differentiate
  FST    ==> FortranScalarType
  FT     ==> FortranType
  FP     ==> FortranProgram
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  NNI    ==> NonNegativeInteger
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  UFST   ==> Union(fst:FST,void:"void")
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
```

```
VF2    ==> VectorFunctions2
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['EPS],['YA,'YB],MFLOAT)
S      ==> Symbol
MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,
               EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,Matrix EXPR MFLOAT)

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add
  real : UFST := ["real"::FST]$UFST

  symOne : SYMTAB := empty()$SYMTAB
  declare!(EPS,fortranReal(),symOne)$SYMTAB
  declare!(N,fortranInteger(),symOne)$SYMTAB
  yType : FT := construct(real,[N],false)$FT
  declare!(YA,yType,symOne)$SYMTAB
  declare!(YB,yType,symOne)$SYMTAB
  declare!(BC,yType,symOne)$SYMTAB

  symTwo : SYMTAB := empty()$SYMTAB
  declare!(EPS,fortranReal(),symTwo)$SYMTAB
  declare!(N,fortranInteger(),symTwo)$SYMTAB
  declare!(YA,yType,symTwo)$SYMTAB
  declare!(YB,yType,symTwo)$SYMTAB
  ajType : FT := construct(real,[N,N],false)$FT
  declare!(AJ,ajType,symTwo)$SYMTAB
  declare!(BJ,ajType,symTwo)$SYMTAB

  symThree : SYMTAB := empty()$SYMTAB
  declare!(EPS,fortranReal(),symThree)$SYMTAB
  declare!(N,fortranInteger(),symThree)$SYMTAB
  declare!(YA,yType,symThree)$SYMTAB
  declare!(YB,yType,symThree)$SYMTAB
  declare!(BCEP,yType,symThree)$SYMTAB

  rt := ["void"]$UFST
  R1:=FortranProgram(nameOne,rt,[EPS,YA,YB,BC,N],symOne)
  R2:=FortranProgram(nameTwo,rt,[EPS,YA,YB,AJ,BJ,N],symTwo)
  R3:=FortranProgram(nameThree,rt,[EPS,YA,YB,BCEP,N],symThree)
  Rep := Record(g:R1,gJacob:R2,geJacob:R3)
  BCsym:Symbol:=coerce "BC"
  AJsym:Symbol:=coerce "AJ"
  BJsym:Symbol:=coerce "BJ"
  BCEPsym:Symbol:=coerce "BCEP"

  makeList(n:Integer,s:Symbol):List(Symbol) ==
```

```
  j:Integer
  p:List(Symbol) := []
  for j in 1 .. n repeat p:= cons(subscript(s,[j::OutputForm])$Symbol,p)
  reverse(p)

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:S,j:Matrix FEXPR):FC ==
  j' : Matrix EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

makeCodeOne(u:VEC FEXPR):FortranCode ==
  -- simple assign
  localAssign2(BCsym,u)

makeCodeTwo(u:VEC FEXPR):List(FortranCode) ==
  -- compute jacobian wrt to ya
  n:Integer := maxIndex(u)
  p:List(Symbol) := makeList(n,YA::Symbol)
  jacYA:Matrix(FEXPR) := _
    jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  -- compute jacobian wrt to yb
  p:List(Symbol) := makeList(n,YB::Symbol)
  jacYB: Matrix(FEXPR) := _
    jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  -- assign jacobians to AJ & BJ
  [localAssign1(AJsym,jacYA),localAssign1(BJsym,jacYB),returns()$FC]$List(FC)

makeCodeThree(u:VEC FEXPR):FortranCode ==
  -- compute jacobian wrt to eps
  jacEps:VEC FEXPR := [D(v,EPS) for v in entries u]$VEC(FEXPR)
  localAssign2(BCEPsym,jacEps)

coerce(u:VEC FEXPR):$ ==
  aF:FortranCode := makeCodeOne(u)
  bF:List(FortranCode) := makeCodeTwo(u)
  cF:FortranCode := makeCodeThree(u)
  -- add returns() to complete subroutines
  aLF:List(FortranCode) := [aF,returns()$FC]$List(FortranCode)
  cLF:List(FortranCode) := [cF,returns()$FC]$List(FortranCode)
  [coerce(aLF)$R1,coerce(bF)$R2,coerce(cLF)$R3]

coerce(u:$) : OutputForm ==
  bracket commaSeparate
    [nameOne::OutputForm,nameTwo::OutputForm,nameThree::OutputForm]
```

```
outputAsFortran(u:$):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran elt(u,g)$Rep
  outputAsFortran elt(u,gJacob)$Rep
  outputAsFortran elt(u,geJacob)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
```

```
    (v::VEC FEXPR)::$

  retract(u:VEC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$
```

———-

— ASP42.dotabb —

```
"ASP42" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP42"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP42" -> "FS"
```

———-

2.28 domain ASP49 Asp49

— Asp49.input —

```
)set break resume
)sys rm -f Asp49.output
)spool Asp49.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp49
--R Asp49 name: Symbol  is a domain constructor
--R Abbreviation for Asp49 is ASP49
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP49
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %            coerce : List FortranCode -> %
--R coerce : % -> OutputForm             outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %    retract : Polynomial Float -> %
--R retract : Expression Integer -> %    retract : Expression Float -> %
```

```
--R coerce : FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Polynomial Float -> Union(%,"failed")
--R retractIfCan : Expression Integer -> Union(%,"failed")
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1

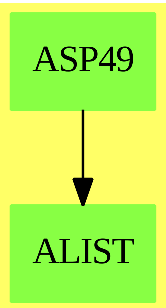
)spool
)lisp (bye)
```

— **Asp49.help** —

```
====================================================================
Asp49 examples
====================================================================

See Also:
o )show Asp49
```

2.28.1 Asp49 (ASP49)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP49 Asp49 —

```
)abbrev domain ASP49 Asp49
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 23 March 1994
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp49} produces Fortran for Type 49 ASPs, needed for NAG routines
++e04dgf, e04ucf, for example:
++
++\tab{5}SUBROUTINE OBJFUN(MODE,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER)\br
++\tab{5}DOUBLE PRECISION X(N),OBJF,OBJGRD(N),USER(*)\br
++\tab{5}INTEGER N,IUSER(*),MODE,NSTATE\br
++\tab{5}OBJF=X(4)*X(9)+((-1.0D0*X(5))+X(3))*X(8)+((-1.0D0*X(3))+X(1))*X(7)\br
++\tab{4}&+(-1.0D0*X(2)*X(6))\br
++\tab{5}OBJGRD(1)=X(7)\br
++\tab{5}OBJGRD(2)=-1.0D0*X(6)\br
++\tab{5}OBJGRD(3)=X(8)+(-1.0D0*X(7))\br
++\tab{5}OBJGRD(4)=X(9)\br
++\tab{5}OBJGRD(5)=-1.0D0*X(8)\br
++\tab{5}OBJGRD(6)=-1.0D0*X(2)\br
++\tab{5}OBJGRD(7)=(-1.0D0*X(3))+X(1)\br
++\tab{5}OBJGRD(8)=(-1.0D0*X(5))+X(3)\br
++\tab{5}OBJGRD(9)=X(4)\br
++\tab{5}RETURN\br
++\tab{5}END

Asp49(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  UFST   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression([],['X],MFLOAT)
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  S      ==> Symbol
```

```
  Exports ==> FortranFunctionCategory with
    coerce : FEXPR -> $
      ++coerce(f) takes an object from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns it into an ASP.

  Implementation ==> add

    real : UFST := ["real"::FST]$UFST
    integer : UFST := ["integer"::FST]$UFST
    syms : SYMTAB := empty()$SYMTAB
    declare!(MODE,fortranInteger(),syms)$SYMTAB
    declare!(N,fortranInteger(),syms)$SYMTAB
    xType : FT := construct(real,[N::S],false)$FT
    declare!(X,xType,syms)$SYMTAB
    declare!(OBJF,fortranReal(),syms)$SYMTAB
    declare!(OBJGRD,xType,syms)$SYMTAB
    declare!(NSTATE,fortranInteger(),syms)$SYMTAB
    iuType : FT := construct(integer,["*"::S],false)$FT
    declare!(IUSER,iuType,syms)$SYMTAB
    uType : FT := construct(real,["*"::S],false)$FT
    declare!(USER,uType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST,
                          [MODE,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER],syms)

    fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

    localAssign(s:S,j:VEC FEXPR):FC ==
      j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
      assign(s,j')$FC

    coerce(u:FEXPR):$ ==
      vars:List(S) := variables(u)
      grd:VEC FEXPR := gradient(u,vars)$MultiVariableCalculusFunctions(_
                                         S,FEXPR,VEC FEXPR,List(S))
      code : List(FC) := [assign(OBJF@S,fexpr2expr u)$FC,_
                          localAssign(OBJGRD@S,grd),_
                          returns()$FC]
      code::$

    coerce(u:$):OutputForm == coerce(u)$Rep

    coerce(c:List FC):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FC):$ == coerce(c)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
```

```
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$
```

— ASP49.dotabb —

```
"ASP49" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP49"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP49" -> "ALIST"
```

2.29 domain ASP50 Asp50

— Asp50.input —

```
)set break resume
)sys rm -f Asp50.output
)spool Asp50.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp50
--R Asp50 name: Symbol  is a domain constructor
--R Abbreviation for Asp50 is ASP50
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP50
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEXC],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

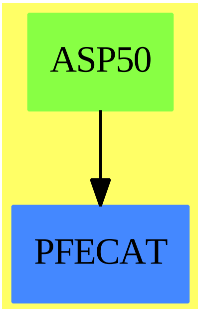
)spool
)lisp (bye)
```

—— Asp50.help ——

```
====================================================================
Asp50 examples
====================================================================

See Also:
o )show Asp50
```

2.29.1 Asp50 (ASP50)



Exports:
coerce outputAsFortran retract retractIfCan

—— domain ASP50 Asp50 ——

```
)abbrev domain ASP50 Asp50
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 23 March 1994
++                     6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp50} produces Fortran for Type 50 ASPs, needed for NAG routine
++e04fdf, for example:
++
++\tab{5}SUBROUTINE LSFUN1(M,N,XC,FVECC)\br
++\tab{5}DOUBLE PRECISION FVECC(M),XC(N)\br
++\tab{5}INTEGER I,M,N\br
++\tab{5}FVECC(1)=((XC(1)-2.4D0)*XC(3)+(15.0D0*XC(1)-36.0D0)*XC(2)+1.0D0)/(\br
```

```
++\tab{4}&XC(3)+15.0D0*XC(2))\br
++\tab{5}FVECC(2)=((XC(1)-2.8D0)*XC(3)+(7.0D0*XC(1)-19.6D0)*XC(2)+1.0D0)/(X\br
++\tab{4}&C(3)+7.0D0*XC(2))\br
++\tab{5}FVECC(3)=((XC(1)-3.2D0)*XC(3)+(4.333333333333333D0*XC(1)-13.866666\br
++\tab{4}&66666667D0)*XC(2)+1.0D0)/(XC(3)+4.333333333333333D0*XC(2))\br
++\tab{5}FVECC(4)=((XC(1)-3.5D0)*XC(3)+(3.0D0*XC(1)-10.5D0)*XC(2)+1.0D0)/(X\br
++\tab{4}&C(3)+3.0D0*XC(2))\br
++\tab{5}FVECC(5)=((XC(1)-3.9D0)*XC(3)+(2.2D0*XC(1)-8.579999999999998D0)*XC\br
++\tab{4}&(2)+1.0D0)/(XC(3)+2.2D0*XC(2))\br
++\tab{5}FVECC(6)=((XC(1)-4.199999999999999D0)*XC(3)+(1.666666666666667D0*X\br
++\tab{4}&C(1)-7.0D0)*XC(2)+1.0D0)/(XC(3)+1.666666666666667D0*XC(2))\br
++\tab{5}FVECC(7)=((XC(1)-4.5D0)*XC(3)+(1.285714285714286D0*XC(1)-5.7857142\br
++\tab{4}&85714286D0)*XC(2)+1.0D0)/(XC(3)+1.285714285714286D0*XC(2))\br
++\tab{5}FVECC(8)=((XC(1)-4.899999999999999D0)*XC(3)+(XC(1)-4.8999999999999\br
++\tab{4}&99D0)*XC(2)+1.0D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(9)=((XC(1)-4.699999999999999D0)*XC(3)+(XC(1)-4.6999999999999\br
++\tab{4}&99D0)*XC(2)+1.285714285714286D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(10)=((XC(1)-6.8D0)*XC(3)+(XC(1)-6.8D0)*XC(2)+1.6666666666666\br
++\tab{4}&67D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(11)=((XC(1)-8.299999999999999D0)*XC(3)+(XC(1)-8.299999999999\br
++\tab{4}&999D0)*XC(2)+2.2D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(12)=((XC(1)-10.6D0)*XC(3)+(XC(1)-10.6D0)*XC(2)+3.0D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FVECC(13)=((XC(1)-1.34D0)*XC(3)+(XC(1)-1.34D0)*XC(2)+4.33333333333\br
++\tab{4}&3333D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(14)=((XC(1)-2.1D0)*XC(3)+(XC(1)-2.1D0)*XC(2)+7.0D0)/(XC(3)+X\br
++\tab{4}&C(2))\br
++\tab{5}FVECC(15)=((XC(1)-4.39D0)*XC(3)+(XC(1)-4.39D0)*XC(2)+15.0D0)/(XC(3\br
++\tab{4}&)+XC(2))\br
++\tab{5}END

Asp50(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  UFST   ==> Union(fst:FST,void:"void")
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  FEXPR  ==> FortranExpression([],['XC],MFLOAT)
  MFLOAT ==> MachineFloat

  Exports ==> FortranVectorFunctionCategory with
```

```
    coerce : VEC FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add

    real : UFST := ["real"::FST]$UFST
    syms : SYMTAB := empty()$SYMTAB
    declare!(M,fortranInteger(),syms)$SYMTAB
    declare!(N,fortranInteger(),syms)$SYMTAB
    xcType : FT := construct(real,[N],false)$FT
    declare!(XC,xcType,syms)$SYMTAB
    fveccType : FT := construct(real,[M],false)$FT
    declare!(FVECC,fveccType,syms)$SYMTAB
    declare!(I,fortranInteger(),syms)$SYMTAB
    tType : FT := construct(real,[M,N],false)$FT
--    declare!(TC,tType,syms)$SYMTAB
--    declare!(Y,fveccType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$UFST, [M,N,XC,FVECC],syms)

    fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

    coerce(u:VEC FEXPR):$ ==
      u' : VEC EXPR MFLOAT := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
      assign(FVECC,u')$FortranCode::$

    retract(u:VEC FRAC POLY INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
      v::$

    retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC FRAC POLY FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC EXPR INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
```

```
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC EXPR FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    coerce(c:List FortranCode):$ == coerce(c)$Rep

    coerce(r:RSFC):$ == coerce(r)$Rep

    coerce(c:FortranCode):$ == coerce(c)$Rep

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

— **ASP50.dotabb** —

```
"ASP50" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP50"]
```

```
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP50" -> "PFECAT"
```

———-

2.30 domain ASP55 Asp55

— Asp55.input —

```
)set break resume
)sys rm -f Asp55.output
)spool Asp55.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp55
--R Asp55 name: Symbol  is a domain constructor
--R Abbreviation for Asp55 is ASP55
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP55
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

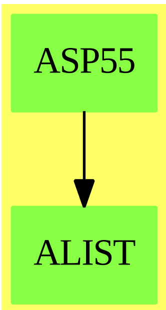
— **Asp55.help** —

```
====================================================================
Asp55 examples
====================================================================

See Also:
o )show Asp55
```

———-

2.30.1 Asp55 (ASP55)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP55 Asp55** —

```
)abbrev domain ASP55 Asp55
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: June 1993
++ Date Last Updated: 23 March 1994
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp55} produces Fortran for Type 55 ASPs, needed for NAG routines
++e04dgf and e04ucf, for example:
++
++\tab{5}SUBROUTINE CONFUN(MODE,NCNLN,N,NROWJ,NEEDC,X,C,CJAC,NSTATE,IUSER\br
++\tab{4}&,USER)\br
++\tab{5}DOUBLE PRECISION C(NCNLN),X(N),CJAC(NROWJ,N),USER(*)\br
++\tab{5}INTEGER N,IUSER(*),NEEDC(NCNLN),NROWJ,MODE,NCNLN,NSTATE\br
```

```
++\tab{5}IF(NEEDC(1).GT.0)THEN\br
++\tab{7}C(1)=X(6)**2+X(1)**2\br
++\tab{7}CJAC(1,1)=2.0D0*X(1)\br
++\tab{7}CJAC(1,2)=0.0D0\br
++\tab{7}CJAC(1,3)=0.0D0\br
++\tab{7}CJAC(1,4)=0.0D0\br
++\tab{7}CJAC(1,5)=0.0D0\br
++\tab{7}CJAC(1,6)=2.0D0*X(6)\br
++\tab{5}ENDIF\br
++\tab{5}IF(NEEDC(2).GT.0)THEN\br
++\tab{7}C(2)=X(2)**2+(-2.0D0*X(1)*X(2))+X(1)**2\br
++\tab{7}CJAC(2,1)=(-2.0D0*X(2))+2.0D0*X(1)\br
++\tab{7}CJAC(2,2)=2.0D0*X(2)+(-2.0D0*X(1))\br
++\tab{7}CJAC(2,3)=0.0D0\br
++\tab{7}CJAC(2,4)=0.0D0\br
++\tab{7}CJAC(2,5)=0.0D0\br
++\tab{7}CJAC(2,6)=0.0D0\br
++\tab{5}ENDIF\br
++\tab{5}IF(NEEDC(3).GT.0)THEN\br
++\tab{7}C(3)=X(3)**2+(-2.0D0*X(1)*X(3))+X(2)**2+X(1)**2\br
++\tab{7}CJAC(3,1)=(-2.0D0*X(3))+2.0D0*X(1)\br
++\tab{7}CJAC(3,2)=2.0D0*X(2)\br
++\tab{7}CJAC(3,3)=2.0D0*X(3)+(-2.0D0*X(1))\br
++\tab{7}CJAC(3,4)=0.0D0\br
++\tab{7}CJAC(3,5)=0.0D0\br
++\tab{7}CJAC(3,6)=0.0D0\br
++\tab{5}ENDIF\br
++\tab{5}RETURN\br
++\tab{5}END

Asp55(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  FSTU   ==> Union(fst:FST,void:"void")
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  S      ==> Symbol
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  MAT    ==> Matrix
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression([],['X],MFLOAT)
```

```
MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR,
                 EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,MAT EXPR MFLOAT)
SWU    ==> Union(I:Expression Integer,F:Expression Float,
                CF:Expression Complex Float,switch:Switch)

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : FSTU := ["real"::FST]$FSTU
  integer : FSTU := ["integer"::FST]$FSTU
  syms : SYMTAB := empty()$SYMTAB
  declare!(MODE,fortranInteger(),syms)$SYMTAB
  declare!(NCNLN,fortranInteger(),syms)$SYMTAB
  declare!(N,fortranInteger(),syms)$SYMTAB
  declare!(NROWJ,fortranInteger(),syms)$SYMTAB
  needcType : FT := construct(integer,[NCNLN::Symbol],false)$FT
  declare!(NEEDC,needcType,syms)$SYMTAB
  xType : FT := construct(real,[N::Symbol],false)$FT
  declare!(X,xType,syms)$SYMTAB
  cType : FT := construct(real,[NCNLN::Symbol],false)$FT
  declare!(C,cType,syms)$SYMTAB
  cjacType : FT := construct(real,[NROWJ::Symbol,N::Symbol],false)$FT
  declare!(CJAC,cjacType,syms)$SYMTAB
  declare!(NSTATE,fortranInteger(),syms)$SYMTAB
  iuType : FT := construct(integer,["*"::Symbol],false)$FT
  declare!(IUSER,iuType,syms)$SYMTAB
  uType : FT := construct(real,["*"::Symbol],false)$FT
  declare!(USER,uType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$FSTU,
                  [MODE,NCNLN,N,NROWJ,NEEDC,X,C,CJAC,NSTATE,IUSER,USER],syms)

  -- Take a symbol, pull of the script and turn it into an integer!!
  o2int(u:S):Integer ==
    o : OutputForm := first elt(scripts(u)$S,sub)
    o pretend Integer

  localAssign(s:Symbol,dim:List POLY INT,u:FEXPR):FC ==
    assign(s,dim,(u::EXPR MFLOAT)$FEXPR)$FC

  makeCond(index:INT,fun:FEXPR,jac:VEC FEXPR):FC ==
    needc : EXPR INT := (subscript(NEEDC,[index::OutputForm])$S)::EXPR(INT)
    sw : Switch := GT([needc]$SWU,[0::EXPR(INT)]$SWU)$Switch
    ass : List FC := [localAssign(CJAC,[index::POLY INT,i::POLY INT],jac.i)_
                                              for i in 1..maxIndex(jac)]
    cond(sw,block([localAssign(C,[index::POLY INT],fun),:ass])$FC)$FC
```

```
coerce(u:VEC FEXPR):$ ==
  ncnln:Integer := maxIndex(u)
  x:S := X::S
  pu:List(S) := []
  -- Work out which variables appear in the expressions
  for e in entries(u) repeat
    pu := setUnion(pu,variables(e)$FEXPR)
  scriptList : List Integer := map(o2int,pu)$ListFunctions2(S,Integer)
  -- This should be the maximum X_n which occurs (there may be others
  -- which don't):
  n:Integer := reduce(max,scriptList)$List(Integer)
  p:List(S) := []
  for j in 1..n repeat p:= cons(subscript(x,[j::OutputForm])$S,p)
  p:= reverse(p)
  jac:MAT FEXPR := _
     jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  code : List FC := [makeCond(j,u.j,row(jac,j)) for j in 1..ncnln]
  [:code,returns()$FC]::$

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$
```

```
    retract(u:VEC EXPR INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC EXPR FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$
```

— **ASP55.dotabb** —

```
"ASP55" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP55"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP55" -> "ALIST"
```

2.31 domain ASP6 Asp6

— Asp6.input —

```
)set break resume
)sys rm -f Asp6.output
)spool Asp6.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp6
--R Asp6 name: Symbol  is a domain constructor
--R Abbreviation for Asp6 is ASP6
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP6
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

————

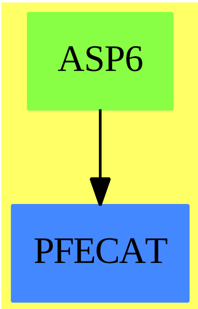
— Asp6.help —

```
====================================================================
Asp6 examples
====================================================================
```

```
See Also:
o )show Asp6
```

———–

2.31.1 Asp6 (ASP6)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP6 Asp6** —

```
)abbrev domain ASP6 Asp6
++ Author: Mike Dewar and Godfrey Nolan and Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                     6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp6} produces Fortran for Type 6 ASPs, needed for NAG routines
++ c05nbf, c05ncf. These represent vectors of functions of X(i) and look like:
++
++ \tab{5}SUBROUTINE FCN(N,X,FVEC,IFLAG)
++ \tab{5}DOUBLE PRECISION X(N),FVEC(N)
++ \tab{5}INTEGER N,IFLAG
++ \tab{5}FVEC(1)=(-2.0D0*X(2))+(-2.0D0*X(1)**2)+3.0D0*X(1)+1.0D0
++ \tab{5}FVEC(2)=(-2.0D0*X(3))+(-2.0D0*X(2)**2)+3.0D0*X(2)+(-1.0D0*X(1))+1.
++ \tab{4}&0D0
++ \tab{5}FVEC(3)=(-2.0D0*X(4))+(-2.0D0*X(3)**2)+3.0D0*X(3)+(-1.0D0*X(2))+1.
++ \tab{4}&0D0
++ \tab{5}FVEC(4)=(-2.0D0*X(5))+(-2.0D0*X(4)**2)+3.0D0*X(4)+(-1.0D0*X(3))+1.
++ \tab{4}&0D0
++ \tab{5}FVEC(5)=(-2.0D0*X(6))+(-2.0D0*X(5)**2)+3.0D0*X(5)+(-1.0D0*X(4))+1.
```

```
++ \tab{4}&0D0
++ \tab{5}FVEC(6)=(-2.0D0*X(7))+(-2.0D0*X(6)**2)+3.0D0*X(6)+(-1.0D0*X(5))+1.
++ \tab{4}&0D0
++ \tab{5}FVEC(7)=(-2.0D0*X(8))+(-2.0D0*X(7)**2)+3.0D0*X(7)+(-1.0D0*X(6))+1.
++ \tab{4}&0D0
++ \tab{5}FVEC(8)=(-2.0D0*X(9))+(-2.0D0*X(8)**2)+3.0D0*X(8)+(-1.0D0*X(7))+1.
++ \tab{4}&0D0
++ \tab{5}FVEC(9)=(-2.0D0*X(9)**2)+3.0D0*X(9)+(-1.0D0*X(8))+1.0D0
++ \tab{5}RETURN
++ \tab{5}END

Asp6(name): Exports == Implementation where
  name : Symbol

  FEXPR  ==> FortranExpression([],['X],MFLOAT)
  MFLOAT ==> MachineFloat
  FST    ==> FortranScalarType
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  UFST   ==> Union(fst:FST,void:"void")
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2

  Exports == FortranVectorFunctionCategory with
    coerce: Vector FEXPR -> %
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation == add

    real : UFST := ["real"::FST]$UFST
    syms : SYMTAB := empty()$SYMTAB
    declare!(N,fortranInteger()$FT,syms)$SYMTAB
    xType : FT := construct(real,[N],false)$FT
    declare!(X,xType,syms)$SYMTAB
    declare!(FVEC,xType,syms)$SYMTAB
    declare!(IFLAG,fortranInteger()$FT,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$Union(fst:FST,void:"void"),
                          [N,X,FVEC,IFLAG],syms)

    retract(u:VEC FRAC POLY INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
      v::$
```

```
retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=_
    map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$
```

```
    fexpr2expr(u:FEXPR):EXPR MFLOAT ==
      (u::EXPR MFLOAT)$FEXPR

    coerce(u:VEC FEXPR):% ==
      v : VEC EXPR MFLOAT
      v := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
      ([assign(FVEC,v)$FortranCode,returns()$FortranCode]$List(FortranCode))::$

    coerce(c:List FortranCode):% == coerce(c)$Rep

    coerce(r:RSFC):% == coerce(r)$Rep

    coerce(c:FortranCode):% == coerce(c)$Rep

    coerce(u:%):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

———-

— ASP6.dotabb —

```
"ASP6" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP6"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP6" -> "PFECAT"
```

———-

2.32 domain ASP7 Asp7

— Asp7.input —

```
)set break resume
)sys rm -f Asp7.output
)spool Asp7.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show Asp7
--R Asp7 name: Symbol  is a domain constructor
--R Abbreviation for Asp7 is ASP7
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP7
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX],[construct,QUOTEY],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

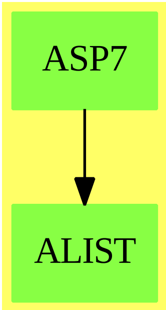
)spool
)lisp (bye)
```

— **Asp7.help** —

```
====================================================================
Asp7 examples
====================================================================

See Also:
o )show Asp7
```

2.32.1 Asp7 (ASP7)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP7 Asp7 —

```
)abbrev domain ASP7 Asp7
++ Author: Mike Dewar and Godfrey Nolan and Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                    6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp7} produces Fortran for Type 7 ASPs, needed for NAG routines
++ d02bbf, d02gaf. These represent a vector of functions of the scalar X and
++ the array Z, and look like:
++
++ \tab{5}SUBROUTINE FCN(X,Z,F)\br
++ \tab{5}DOUBLE PRECISION F(*),X,Z(*)\br
++ \tab{5}F(1)=DTAN(Z(3))\br
++ \tab{5}F(2)=((-0.031999999999999999D0*DCOS(Z(3))*DTAN(Z(3)))+(-0.02D0*Z(2)\br
++ \tab{4}&**2))/(Z(2)*DCOS(Z(3)))\br
++ \tab{5}F(3)=-0.031999999999999999D0/(X*Z(2)**2)\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp7(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X],['Y],MFLOAT)
  UFST   ==> Union(fst:FST,void:"void")
```

```
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector
VF2    ==> VectorFunctions2

Exports ==> FortranVectorFunctionCategory with
  coerce : Vector FEXPR -> %
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : UFST := ["real"::FST]$UFST
  syms : SYMTAB := empty()$SYMTAB
  declare!(X,fortranReal(),syms)$SYMTAB
  yType : FT := construct(real,["*"::Symbol],false)$FT
  declare!(Y,yType,syms)$SYMTAB
  declare!(F,yType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$UFST,[X,Y,F],syms)

  retract(u:VEC FRAC POLY INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC FRAC POLY FLOAT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$

  retract(u:VEC EXPR INT):$ ==
    v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
    v::$

  retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$
```

```
retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

fexpr2expr(u:FEXPR):EXPR MFLOAT ==
  (u::EXPR MFLOAT)$FEXPR

coerce(u:Vector FEXPR ):% ==
  v : Vector EXPR MFLOAT
  v:=map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
  ([assign(F,v)$FortranCode,returns()$FortranCode]$List(FortranCode))::%

coerce(c:List FortranCode):% == coerce(c)$Rep

coerce(r:RSFC):% == coerce(r)$Rep

coerce(c:FortranCode):% == coerce(c)$Rep

coerce(u:%):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage
```

———

— ASP7.dotabb —

```
"ASP7" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP7"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP7" -> "ALIST"
```

———-

2.33 domain ASP73 Asp73

— Asp73.input —

```
)set break resume
)sys rm -f Asp73.output
)spool Asp73.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp73
--R Asp73 name: Symbol  is a domain constructor
--R Abbreviation for Asp73 is ASP73
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP73
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX,QUOTEY],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

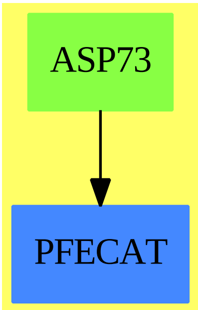
— **Asp73.help** —

```
====================================================================
Asp73 examples
====================================================================

See Also:
o )show Asp73
```

———-

2.33.1 Asp73 (ASP73)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP73 Asp73** —

```
)abbrev domain ASP73 Asp73
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 30 March 1994, 6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp73} produces Fortran for Type 73 ASPs, needed for NAG routine
++ d03eef, for example:
++
```

```
++ \tab{5}SUBROUTINE PDEF(X,Y,ALPHA,BETA,GAMMA,DELTA,EPSOLN,PHI,PSI)\br
++ \tab{5}DOUBLE PRECISION ALPHA,EPSOLN,PHI,X,Y,BETA,DELTA,GAMMA,PSI\br
++ \tab{5}ALPHA=DSIN(X)\br
++ \tab{5}BETA=Y\br
++ \tab{5}GAMMA=X*Y\br
++ \tab{5}DELTA=DCOS(X)*DSIN(Y)\br
++ \tab{5}EPSOLN=Y+X\br
++ \tab{5}PHI=X\br
++ \tab{5}PSI=Y\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp73(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FEXPR  ==> FortranExpression(['X,'Y],[],MachineFloat)
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2

  Exports ==> FortranVectorFunctionCategory with
    coerce : VEC FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add

    syms : SYMTAB := empty()$SYMTAB
    declare!(X,fortranReal(),syms) $SYMTAB
    declare!(Y,fortranReal(),syms) $SYMTAB
    declare!(ALPHA,fortranReal(),syms)$SYMTAB
    declare!(BETA,fortranReal(),syms) $SYMTAB
    declare!(GAMMA,fortranReal(),syms) $SYMTAB
    declare!(DELTA,fortranReal(),syms) $SYMTAB
    declare!(EPSOLN,fortranReal(),syms) $SYMTAB
    declare!(PHI,fortranReal(),syms) $SYMTAB
    declare!(PSI,fortranReal(),syms) $SYMTAB
    Rep := FortranProgram(name,["void"]$FSTU,
                      [X,Y,ALPHA,BETA,GAMMA,DELTA,EPSOLN,PHI,PSI],syms)

    -- To help the poor compiler!
```

```
localAssign(u:Symbol,v:FEXPR):FortranCode ==
  assign(u,(v::EXPR MachineFloat)$FEXPR)$FortranCode

coerce(u:VEC FEXPR):$ ==
  maxIndex(u) ^= 7 => error "Vector is not of dimension 7"
  [localAssign(ALPHA@Symbol,elt(u,1)),_
   localAssign(BETA@Symbol,elt(u,2)),_
   localAssign(GAMMA@Symbol,elt(u,3)),_
   localAssign(DELTA@Symbol,elt(u,4)),_
   localAssign(EPSOLN@Symbol,elt(u,5)),_
   localAssign(PHI@Symbol,elt(u,6)),_
   localAssign(PSI@Symbol,elt(u,7)),_
   returns()$FortranCode]$List(FortranCode)::$

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
```

```
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC EXPR FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY INT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY INT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$

    retract(u:VEC POLY FLOAT):$ ==
      v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
      v::$

    retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
      v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
      v case "failed" => "failed"
      (v::VEC FEXPR)::$
```

— **ASP73.dotabb** —

```
"ASP73" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP73"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP73" -> "PFECAT"
```

2.34 domain ASP74 Asp74

— **Asp74.input** —

```
)set break resume
)sys rm -f Asp74.output
)spool Asp74.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp74
--R Asp74 name: Symbol  is a domain constructor
--R Abbreviation for Asp74 is ASP74
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP74
--R
--R------------------------------ Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct,QUOTEX,QUOTEY],[construct],MachineFloat) ->
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Expression Integer -> Union(%,"failed")
--R retractIfCan : Matrix Expression Float -> Union(%,"failed")
--R
--E 1

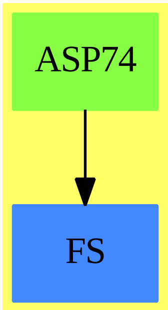
)spool
)lisp (bye)
```

— Asp74.help —

```
====================================================================
Asp74 examples
====================================================================

See Also:
o )show Asp74
```

2.34.1 Asp74 (ASP74)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP74 Asp74 —

```
)abbrev domain ASP74 Asp74
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Oct 1993
++ Date Last Updated: 30 March 1994
++                    6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory.
++ Description:
++ \spadtype{Asp74} produces Fortran for Type 74 ASPs, needed for NAG routine
++ d03eef, for example:
++
++ \tab{5} SUBROUTINE BNDY(X,Y,A,B,C,IBND)\br
++ \tab{5} DOUBLE PRECISION A,B,C,X,Y\br
++ \tab{5} INTEGER IBND\br
++ \tab{5} IF(IBND.EQ.0)THEN\br
++ \tab{7} A=0.0D0\br
++ \tab{7} B=1.0D0\br
++ \tab{7} C=-1.0D0*DSIN(X)\br
++ \tab{5} ELSEIF(IBND.EQ.1)THEN\br
++ \tab{7} A=1.0D0\br
++ \tab{7} B=0.0D0\br
++ \tab{7} C=DSIN(X)*DSIN(Y)\br
++ \tab{5} ELSEIF(IBND.EQ.2)THEN\br
++ \tab{7} A=1.0D0\br
++ \tab{7} B=0.0D0\br
++ \tab{7} C=DSIN(X)*DSIN(Y)\br
++ \tab{5} ELSEIF(IBND.EQ.3)THEN\br
++ \tab{7} A=0.0D0\br
++ \tab{7} B=1.0D0\br
++ \tab{7} C=-1.0D0*DSIN(Y)\br
++ \tab{5} ENDIF\br
```

```
++ \tab{5} END

Asp74(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X,'Y],[],MFLOAT)
  U      ==> Union(I: Expression Integer,F: Expression Float,_
                 CF: Expression Complex Float,switch:Switch)
  VEC    ==> Vector
  MAT    ==> Matrix
  M2     ==> MatrixCategoryFunctions2
  MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
              MAT FRAC POLY INT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
              MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
              FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
              MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2e   ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
              FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2f   ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
              MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)

  Exports ==> FortranMatrixFunctionCategory with
    coerce : MAT FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add

    syms : SYMTAB := empty()$SYMTAB
    declare!(X,fortranReal(),syms)$SYMTAB
    declare!(Y,fortranReal(),syms)$SYMTAB
    declare!(A,fortranReal(),syms)$SYMTAB
    declare!(B,fortranReal(),syms)$SYMTAB
    declare!(C,fortranReal(),syms)$SYMTAB
```

```
declare!(IBND,fortranInteger(),syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,[X,Y,A,B,C,IBND],syms)

-- To help the poor compiler!
localAssign(u:Symbol,v:FEXPR):FC == assign(u,(v::EXPR MFLOAT)$FEXPR)$FC

coerce(u:MAT FEXPR):$ ==
  (nrows(u) ^= 4 or ncols(u) ^= 3) => error "Not a 4X3 matrix"
  flag:U := [IBND@Symbol::EXPR INT]$U
  pt0:U  := [0::EXPR INT]$U
  pt1:U  := [1::EXPR INT]$U
  pt2:U  := [2::EXPR INT]$U
  pt3:U  := [3::EXPR INT]$U
  sw1: Switch := EQ(flag,pt0)$Switch
  sw2: Switch := EQ(flag,pt1)$Switch
  sw3: Switch := EQ(flag,pt2)$Switch
  sw4: Switch := EQ(flag,pt3)$Switch
  a11 : FC := localAssign(A,u(1,1))
  a12 : FC := localAssign(B,u(1,2))
  a13 : FC := localAssign(C,u(1,3))
  a21 : FC := localAssign(A,u(2,1))
  a22 : FC := localAssign(B,u(2,2))
  a23 : FC := localAssign(C,u(2,3))
  a31 : FC := localAssign(A,u(3,1))
  a32 : FC := localAssign(B,u(3,2))
  a33 : FC := localAssign(C,u(3,3))
  a41 : FC := localAssign(A,u(4,1))
  a42 : FC := localAssign(B,u(4,2))
  a43 : FC := localAssign(C,u(4,3))
  c : FC := cond(sw1,block([a11,a12,a13])$FC,
               cond(sw2,block([a21,a22,a23])$FC,
                    cond(sw3,block([a31,a32,a33])$FC,
                         cond(sw4,block([a41,a42,a43])$FC)$FC)$FC)$FC)$FC
  c::$

coerce(u:$):OutputForm == coerce(u)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:MAT FRAC POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2a
```

```
    v::$

  retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT FRAC POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2b
    v::$

  retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT EXPR INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2e
    v::$

  retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT EXPR FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2f
    v::$

  retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT POLY INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2c
    v::$

  retractIfCan(u:MAT POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2d
    v::$

  retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
    v case "failed" => "failed"
```

```
(v::MAT FEXPR)::$
```

———-

— ASP74.dotabb —

```
"ASP74" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP74"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP74" -> "FS"
```

———-

2.35 domain ASP77 Asp77

— Asp77.input —

```
)set break resume
)sys rm -f Asp77.output
)spool Asp77.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp77
--R Asp77 name: Symbol  is a domain constructor
--R Abbreviation for Asp77 is ASP77
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP77
--R
--R------------------------------ Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct,QUOTEX],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
```

```
--R retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Expression Integer -> Union(%,"failed")
--R retractIfCan : Matrix Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

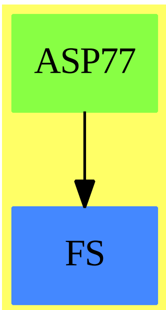
— **Asp77.help** —

```
====================================================================
Asp77 examples
====================================================================

See Also:
o )show Asp77
```

———-

2.35.1 Asp77 (ASP77)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP77 Asp77** —

```
)abbrev domain ASP77 Asp77
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 30 March 1994
```

```
++                    6 October 1994
++ Related Constructors: FortranMatrixFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp77} produces Fortran for Type 77 ASPs, needed for NAG routine
++ d02gbf, for example:
++
++ \tab{5}SUBROUTINE FCNF(X,F)\br
++ \tab{5}DOUBLE PRECISION X\br
++ \tab{5}DOUBLE PRECISION F(2,2)\br
++ \tab{5}F(1,1)=0.0D0\br
++ \tab{5}F(1,2)=1.0D0\br
++ \tab{5}F(2,1)=0.0D0\br
++ \tab{5}F(2,2)=-10.0D0\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp77(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X],[],MFLOAT)
  VEC    ==> Vector
  MAT    ==> Matrix
  M2     ==> MatrixCategoryFunctions2
  MF2    ==> M2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,EXPR MFLOAT,
                VEC EXPR MFLOAT,VEC EXPR MFLOAT,Matrix EXPR MFLOAT)
  MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
                MAT FRAC POLY INT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
                MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
                FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
                MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2e   ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
                FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2f   ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
                MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
```

```
Exports ==> FortranMatrixFunctionCategory with
  coerce : MAT FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : FSTU := ["real"::FST]$FSTU
  syms : SYMTAB := empty()$SYMTAB
  declare!(X,fortranReal(),syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$FSTU,[X,F],syms)

  fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

  localAssign(s:Symbol,j:MAT FEXPR):FortranCode ==
    j' : MAT EXPR MFLOAT := map(fexpr2expr,j)$MF2
    assign(s,j')$FortranCode

  coerce(u:MAT FEXPR):$ ==
    dimension := nrows(u)::POLY(INT)
    locals : SYMTAB := empty()
    declare!(F,[real,[dimension,dimension]$List(POLY(INT)),false]$FT,locals)
    code : List FC := [localAssign(F,u),returns()$FC]
    ([locals,code]$RSFC)::$

  coerce(c:List FC):$ == coerce(c)$Rep

  coerce(r:RSFC):$ == coerce(r)$Rep

  coerce(c:FC):$ == coerce(c)$Rep

  coerce(u:$):OutputForm == coerce(u)$Rep

  outputAsFortran(u):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage

  retract(u:MAT FRAC POLY INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2a
    v::$

  retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT FRAC POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2b
```

```
      v::$

    retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
      v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
      v case "failed" => "failed"
      (v::MAT FEXPR)::$

    retract(u:MAT EXPR INT):$ ==
      v : MAT FEXPR := map(retract,u)$MF2e
      v::$

    retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
      v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
      v case "failed" => "failed"
      (v::MAT FEXPR)::$

    retract(u:MAT EXPR FLOAT):$ ==
      v : MAT FEXPR := map(retract,u)$MF2f
      v::$

    retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
      v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
      v case "failed" => "failed"
      (v::MAT FEXPR)::$

    retract(u:MAT POLY INT):$ ==
      v : MAT FEXPR := map(retract,u)$MF2c
      v::$

    retractIfCan(u:MAT POLY INT):Union($,"failed") ==
      v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
      v case "failed" => "failed"
      (v::MAT FEXPR)::$

    retract(u:MAT POLY FLOAT):$ ==
      v : MAT FEXPR := map(retract,u)$MF2d
      v::$

    retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
      v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
      v case "failed" => "failed"
      (v::MAT FEXPR)::$
```

— **ASP77.dotabb** —

```
"ASP77" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP77"]
```

```
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP77" -> "FS"
```

———-

2.36 domain ASP78 Asp78

— Asp78.input —

```
)set break resume
)sys rm -f Asp78.output
)spool Asp78.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp78
--R Asp78 name: Symbol  is a domain constructor
--R Abbreviation for Asp78 is ASP78
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP78
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

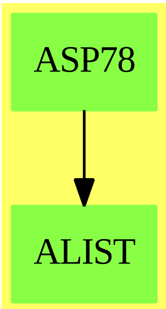
— **Asp78.help** —

```
====================================================================
Asp78 examples
====================================================================

See Also:
o )show Asp78
```

———-

2.36.1 Asp78 (ASP78)



Exports:
coerce outputAsFortran retract retractIfCan

— **domain ASP78 Asp78** —

```
)abbrev domain ASP78 Asp78
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 30 March 1994
++                    6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp78} produces Fortran for Type 78 ASPs, needed for NAG routine
++ d02gbf, for example:
++
++ \tab{5}SUBROUTINE FCNG(X,G)\br
++ \tab{5}DOUBLE PRECISION G(*),X\br
++ \tab{5}G(1)=0.0D0\br
++ \tab{5}G(2)=0.0D0\br
```

```
++ \tab{5}END

Asp78(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X],[],MFLOAT)

  Exports ==> FortranVectorFunctionCategory with
    coerce : VEC FEXPR -> $
      ++coerce(f) takes objects from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns them into an ASP.

  Implementation ==> add

    real : FSTU := ["real"::FST]$FSTU
    syms : SYMTAB := empty()$SYMTAB
    declare!(X,fortranReal(),syms)$SYMTAB
    gType : FT := construct(real,["*"::Symbol],false)$FT
    declare!(G,gType,syms)$SYMTAB
    Rep := FortranProgram(name,["void"]$FSTU,[X,G],syms)

    fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

    coerce(u:VEC FEXPR):$ ==
      u' : VEC EXPR MFLOAT := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
      (assign(G,u')$FC)::$

    coerce(u:$):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage

    coerce(c:List FC):$ == coerce(c)$Rep
```

```
coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
```

```
    v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
    v::$

  retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
    v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
    v case "failed" => "failed"
    (v::VEC FEXPR)::$
```

— ASP78.dotabb —

```
"ASP78" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP78"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP78" -> "ALIST"
```

2.37 domain ASP8 Asp8

— Asp8.input —

```
)set break resume
)sys rm -f Asp8.output
)spool Asp8.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp8
--R Asp8 name: Symbol  is a domain constructor
--R Abbreviation for Asp8 is ASP8
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP8
--R
--R------------------------------ Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : Vector MachineFloat -> %     coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

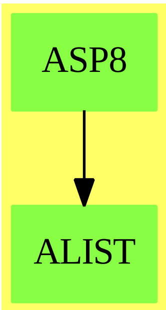
— Asp8.help —

```
====================================================================
Asp8 examples
====================================================================

See Also:
o )show Asp8
```

———-

2.37.1 Asp8 (ASP8)



Exports:
coerce outputAsFortran

— domain ASP8 Asp8 —

```
)abbrev domain ASP8 Asp8
++ Author: Godfrey Nolan and Mike Dewar
++ Date Created: 11 February 1994
++ Date Last Updated: 18 March 1994
++                    31 May 1994 to use alternative interface. MCD
++                    30 June 1994 to handle the end condition correctly. MCD
++                     6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp8} produces Fortran for Type 8 ASPs, needed for NAG routine
```

```
++ d02bbf.  This ASP prints intermediate values of the computed solution of
++ an ODE and might look like:
++
++ \tab{5}SUBROUTINE OUTPUT(XSOL,Y,COUNT,M,N,RESULT,FORWRD)\br
++ \tab{5}DOUBLE PRECISION Y(N),RESULT(M,N),XSOL\br
++ \tab{5}INTEGER M,N,COUNT\br
++ \tab{5}LOGICAL FORWRD\br
++ \tab{5}DOUBLE PRECISION X02ALF,POINTS(8)\br
++ \tab{5}EXTERNAL X02ALF\br
++ \tab{5}INTEGER I\br
++ \tab{5}POINTS(1)=1.0D0\br
++ \tab{5}POINTS(2)=2.0D0\br
++ \tab{5}POINTS(3)=3.0D0\br
++ \tab{5}POINTS(4)=4.0D0\br
++ \tab{5}POINTS(5)=5.0D0\br
++ \tab{5}POINTS(6)=6.0D0\br
++ \tab{5}POINTS(7)=7.0D0\br
++ \tab{5}POINTS(8)=8.0D0\br
++ \tab{5}COUNT=COUNT+1\br
++ \tab{5}DO 25001 I=1,N\br
++ \tab{7} RESULT(COUNT,I)=Y(I)\br
++ 25001 CONTINUE\br
++ \tab{5}IF(COUNT.EQ.M)THEN\br
++ \tab{7}IF(FORWRD)THEN\br
++ \tab{9}XSOL=X02ALF()\br
++ \tab{7}ELSE\br
++ \tab{9}XSOL=-X02ALF()\br
++ \tab{7}ENDIF\br
++ \tab{5}ELSE\br
++ \tab{7} XSOL=POINTS(COUNT)\br
++ \tab{5}ENDIF\br
++ \tab{5}END

Asp8(name): Exports == Implementation where
  name : Symbol

  O      ==> OutputForm
  S      ==> Symbol
  FST    ==> FortranScalarType
  UFST   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  EX     ==> Expression Integer
  MFLOAT ==> MachineFloat
  EXPR   ==> Expression
  PI     ==> Polynomial Integer
  EXU    ==> Union(I: EXPR Integer,F: EXPR Float,CF: EXPR Complex Float,
                 switch: Switch)
```

```
Exports ==> FortranVectorCategory

Implementation ==> add

  real : UFST := ["real"::FST]$UFST
  syms : SYMTAB := empty()$SYMTAB
  declare!([COUNT,M,N],fortranInteger(),syms)$SYMTAB
  declare!(XSOL,fortranReal(),syms)$SYMTAB
  yType : FT := construct(real,[N],false)$FT
  declare!(Y,yType,syms)$SYMTAB
  declare!(FORWRD,fortranLogical(),syms)$SYMTAB
  declare!(RESULT,construct(real,[M,N],false)$FT,syms)$SYMTAB
  Rep := _
   FortranProgram(name,["void"]$UFST,[XSOL,Y,COUNT,M,N,RESULT,FORWRD],syms)

  coerce(c:List FC):% == coerce(c)$Rep

  coerce(r:RSFC):% == coerce(r)$Rep

  coerce(c:FC):% == coerce(c)$Rep

  coerce(u:%):O == coerce(u)$Rep

  outputAsFortran(u:%):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage


  f2ex(u:MFLOAT):EXPR MFLOAT == (u::EXPR MFLOAT)$EXPR(MFLOAT)

  coerce(points:Vector MFLOAT):% ==
    import PI
    import EXPR Integer
    -- Create some extra declarations
    locals : SYMTAB := empty()$SYMTAB
    nPol : PI := "N"::S::PI
    iPol : PI := "I"::S::PI
    countPol : PI := "COUNT"::S::PI
    pointsDim : PI := max(#points,1)::PI
    declare!(POINTS,[real,[pointsDim],false]$FT,locals)$SYMTAB
    declare!(X02ALF,[real,[],true]$FT,locals)$SYMTAB
    -- Now build up the code fragments
    index : SegmentBinding PI := equation(I@S,1::PI..nPol)$SegmentBinding(PI)
    ySym : EX := (subscript("Y"::S,[I::O])$S)::EX
    loop := forLoop(index,assign(RESULT,[countPol,iPol],ySym)$FC)$FC
    v:Vector EXPR MFLOAT
    v := map(f2ex,points)$VectorFunctions2(MFLOAT,EXPR MFLOAT)
    assign1 : FC := assign(POINTS,v)$FC
```

```
    countExp: EX := COUNT@S::EX
    newValue: EX := 1 + countExp
    assign2 : FC := assign(COUNT,newValue)$FC
    newSymbol : S := subscript(POINTS,[COUNT]@List(0))$S
    assign3 : FC := assign(XSOL, newSymbol::EX )$FC
    fphuge : EX := kernel(operator X02ALF,empty()$List(EX))
    assign4 : FC := assign(XSOL, fphuge)$FC
    assign5 : FC := assign(XSOL, -fphuge)$FC
    innerCond : FC := cond("FORWRD"::Symbol::Switch,assign4,assign5)
    mExp : EX := M@S::EX
    endCase : FC := cond(EQ([countExp]$EXU,[mExp]$EXU)$Switch,innerCond,assign3)
    code := [assign1, assign2, loop, endCase]$List(FC)
    ([locals,code]$RSFC)::%
```

——

— ASP8.dotabb —

```
"ASP8" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP8"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP8" -> "ALIST"
```

——

2.38 domain ASP80 Asp80

— Asp80.input —

```
)set break resume
)sys rm -f Asp80.output
)spool Asp80.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp80
--R Asp80 name: Symbol  is a domain constructor
--R Abbreviation for Asp80 is ASP80
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP80
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
```

```
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct,QUOTEXL,QUOTEXR,QUOTEELAM],[construct],MachineFloat) ->
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Expression Integer -> Union(%,"failed")
--R retractIfCan : Matrix Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

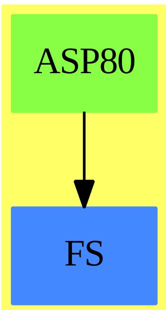
— Asp80.help —

```
====================================================================
Asp80 examples
====================================================================

See Also:
o )show Asp80
```

———-

2.38.1 Asp80 (ASP80)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP80 Asp80 —

```
)abbrev domain ASP80 Asp80
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Oct 1993
++ Date Last Updated: 30 March 1994
++                    6 October 1994
++ Related Constructors: FortranMatrixFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp80} produces Fortran for Type 80 ASPs, needed for NAG routine
++ d02kef, for example:
++
++ \tab{5}SUBROUTINE BDYVAL(XL,XR,ELAM,YL,YR)\br
++ \tab{5}DOUBLE PRECISION ELAM,XL,YL(3),XR,YR(3)\br
++ \tab{5}YL(1)=XL\br
++ \tab{5}YL(2)=2.0D0\br
++ \tab{5}YR(1)=1.0D0\br
++ \tab{5}YR(2)=-1.0D0*DSQRT(XR+(-1.0D0*ELAM))\br
++ \tab{5}RETURN\br
++ \tab{5}END

Asp80(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['XL,'XR,'ELAM],[],MFLOAT)
  VEC    ==> Vector
  MAT    ==> Matrix
  VF2    ==> VectorFunctions2
  M2     ==> MatrixCategoryFunctions2
  MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
               MAT FRAC POLY INT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
               MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
```

```
MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
              FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
              MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2e   ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
              FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2f   ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
              MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)

Exports ==> FortranMatrixFunctionCategory with
  coerce : MAT FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : FSTU := ["real"::FST]$FSTU
  syms : SYMTAB := empty()$SYMTAB
  declare!(XL,fortranReal(),syms)$SYMTAB
  declare!(XR,fortranReal(),syms)$SYMTAB
  declare!(ELAM,fortranReal(),syms)$SYMTAB
  yType : FT := construct(real,["3"::Symbol],false)$FT
  declare!(YL,yType,syms)$SYMTAB
  declare!(YR,yType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$FSTU, [XL,XR,ELAM,YL,YR],syms)

  fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

  vecAssign(s:Symbol,u:VEC FEXPR):FC ==
    u' : VEC EXPR MFLOAT := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
    assign(s,u')$FC

  coerce(u:MAT FEXPR):$ ==
    [vecAssign(YL,row(u,1)),vecAssign(YR,row(u,2)),returns()$FC]$List(FC)::$

  coerce(c:List FortranCode):$ == coerce(c)$Rep

  coerce(r:RSFC):$ == coerce(r)$Rep

  coerce(c:FortranCode):$ == coerce(c)$Rep

  coerce(u:$):OutputForm == coerce(u)$Rep

  outputAsFortran(u):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage

  retract(u:MAT FRAC POLY INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2a
```

```
    v::$

  retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT FRAC POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2b
    v::$

  retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT EXPR INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2e
    v::$

  retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT EXPR FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2f
    v::$

  retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT POLY INT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2c
    v::$

  retractIfCan(u:MAT POLY INT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
    v case "failed" => "failed"
    (v::MAT FEXPR)::$

  retract(u:MAT POLY FLOAT):$ ==
    v : MAT FEXPR := map(retract,u)$MF2d
    v::$

  retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
    v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
    v case "failed" => "failed"
```

```
(v::MAT FEXPR)::$
```

———-

— **ASP80.dotabb** —

```
"ASP80" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP80"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP80" -> "FS"
```

———-

2.39 domain ASP9 Asp9

— **Asp9.input** —

```
)set break resume
)sys rm -f Asp9.output
)spool Asp9.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp9
--R Asp9 name: Symbol  is a domain constructor
--R Abbreviation for Asp9 is ASP9
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP9
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %     retract : Polynomial Float -> %
--R retract : Expression Integer -> %     retract : Expression Float -> %
--R coerce : FortranExpression([construct,QUOTEX],[construct,QUOTEY],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Polynomial Float -> Union(%,"failed")
--R retractIfCan : Expression Integer -> Union(%,"failed")
```

```
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

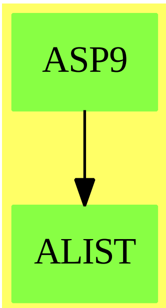
— Asp9.help —

```
====================================================================
Asp9 examples
====================================================================

See Also:
o )show Asp9
```

———-

2.39.1 Asp9 (ASP9)



Exports:
coerce outputAsFortran retract retractIfCan

— domain ASP9 Asp9 —

```
)abbrev domain ASP9 Asp9
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                    12 July 1994 added COMMON blocks for d02cjf, d02ejf
++                     6 October 1994
```

```
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp9} produces Fortran for Type 9 ASPs, needed for NAG routines
++ d02bhf, d02cjf, d02ejf.
++ These ASPs represent a function of a scalar X and a vector Y, for example:
++
++ \tab{5}DOUBLE PRECISION FUNCTION G(X,Y)\br
++ \tab{5}DOUBLE PRECISION X,Y(*)\br
++ \tab{5}G=X+Y(1)\br
++ \tab{5}RETURN\br
++ \tab{5}END
++
++ If the user provides a constant value for G, then extra information is added
++ via COMMON blocks used by certain routines.  This specifies that the value
++ returned by G in this case is to be ignored.

Asp9(name): Exports == Implementation where
  name : Symbol

  FEXPR   ==> FortranExpression(['X],['Y],MFLOAT)
  MFLOAT  ==> MachineFloat
  FC      ==> FortranCode
  FST     ==> FortranScalarType
  FT      ==> FortranType
  SYMTAB  ==> SymbolTable
  RSFC    ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  UFST    ==> Union(fst:FST,void:"void")
  FRAC    ==> Fraction
  POLY    ==> Polynomial
  EXPR    ==> Expression
  INT     ==> Integer
  FLOAT   ==> Float

  Exports ==> FortranFunctionCategory with
    coerce : FEXPR -> %
      ++coerce(f) takes an object from the appropriate instantiation of
      ++\spadtype{FortranExpression} and turns it into an ASP.

  Implementation ==> add

    real : FST := "real"::FST
    syms : SYMTAB := empty()$SYMTAB
    declare!(X,fortranReal()$FT,syms)$SYMTAB
    yType : FT := construct([real]$UFST,["*"::Symbol],false)$FT
    declare!(Y,yType,syms)$SYMTAB
    Rep := FortranProgram(name,[real]$UFST,[X,Y],syms)

    retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
    retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
      foo : Union(FEXPR,"failed")
```

```
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

coerce(u:FEXPR):% ==
  expr : Expression MachineFloat := (u::Expression(MachineFloat))$FEXPR
  (retractIfCan(u)@Union(MFLOAT,"failed"))$FEXPR case "failed" =>
      coerce(expr)$Rep
  locals : SYMTAB := empty()
  charType : FT := construct(["character"::FST]$UFST,[6::POLY(INT)],false)$FT
  declare!([CHDUM1,CHDUM2,GOPT1,CHDUM,GOPT2],charType,locals)$SYMTAB
  common1 := common(CD02EJ,[CHDUM1,CHDUM2,GOPT1] )$FC
  common2 := common(AD02CJ,[CHDUM,GOPT2] )$FC
  assign1 := assign(GOPT1,"NOGOPT")$FC
  assign2 := assign(GOPT2,"NOGOPT")$FC
```

```
      result  := assign(name,expr)$FC
      code : List FC := [common1,common2,assign1,assign2,result]
      ([locals,code]$RSFC)::Rep

    coerce(c:List FortranCode):% == coerce(c)$Rep

    coerce(r:RSFC):% == coerce(r)$Rep

    coerce(c:FortranCode):% == coerce(c)$Rep

    coerce(u:%):OutputForm == coerce(u)$Rep

    outputAsFortran(u):Void ==
      p := checkPrecision()$NAGLinkSupportPackage
      outputAsFortran(u)$Rep
      p => restorePrecision()$NAGLinkSupportPackage
```

———-

— **ASP9.dotabb** —

```
"ASP9" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP9"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP9" -> "ALIST"
```

———-

2.40 domain JORDAN AssociatedJordanAlgebra

— **AssociatedJordanAlgebra.input** —

```
)set break resume
)sys rm -f AssociatedJordanAlgebra.output
)spool AssociatedJordanAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AssociatedJordanAlgebra
--R AssociatedJordanAlgebra(R: CommutativeRing,A: NonAssociativeAlgebra R)  is a domain constructor
--R Abbreviation for AssociatedJordanAlgebra is JORDAN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for JORDAN
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antiCommutator : (%,%) -> %
--R associator : (%,%,%) -> %             coerce : A -> %
--R coerce : % -> A                       coerce : % -> OutputForm
--R commutator : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R alternative? : () -> Boolean if A has FINAALG R
--R antiAssociative? : () -> Boolean if A has FINAALG R
--R antiCommutative? : () -> Boolean if A has FINAALG R
--R apply : (Matrix R,%) -> % if A has FRNAALG R
--R associative? : () -> Boolean if A has FINAALG R
--R associatorDependence : () -> List Vector R if A has FINAALG R and R has INTDOM or A has F
--R basis : () -> Vector % if A has FRNAALG R
--R commutative? : () -> Boolean if A has FINAALG R
--R conditionsForIdempotents : Vector % -> List Polynomial R if A has FINAALG R
--R conditionsForIdempotents : () -> List Polynomial R if A has FRNAALG R
--R convert : % -> Vector R if A has FRNAALG R
--R convert : Vector R -> % if A has FRNAALG R
--R coordinates : (%,Vector %) -> Vector R if A has FINAALG R
--R coordinates : (Vector %,Vector %) -> Matrix R if A has FINAALG R
--R coordinates : % -> Vector R if A has FRNAALG R
--R coordinates : Vector % -> Matrix R if A has FRNAALG R
--R ?.? : (%,Integer) -> R if A has FRNAALG R
--R flexible? : () -> Boolean if A has FINAALG R
--R jacobiIdentity? : () -> Boolean if A has FINAALG R
--R jordanAdmissible? : () -> Boolean if A has FINAALG R
--R jordanAlgebra? : () -> Boolean if A has FINAALG R
--R leftAlternative? : () -> Boolean if A has FINAALG R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R leftDiscriminant : Vector % -> R if A has FINAALG R
--R leftDiscriminant : () -> R if A has FRNAALG R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has IN
--R leftNorm : % -> R if A has FINAALG R
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and
--R leftRecip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R
--R leftRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R
--R leftRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R leftTrace : % -> R if A has FINAALG R
--R leftTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R leftTraceMatrix : () -> Matrix R if A has FRNAALG R
--R leftUnit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R
```

```
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if A has FINAALG R and R has I
--R lieAdmissible? : () -> Boolean if A has FINAALG R
--R lieAlgebra? : () -> Boolean if A has FINAALG R
--R noncommutativeJordanAlgebra? : () -> Boolean if A has FINAALG R
--R plenaryPower : (%,PositiveInteger) -> %
--R powerAssociative? : () -> Boolean if A has FINAALG R
--R rank : () -> PositiveInteger if A has FINAALG R
--R recip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTD
--R represents : (Vector R,Vector %) -> % if A has FINAALG R
--R represents : Vector R -> % if A has FRNAALG R
--R rightAlternative? : () -> Boolean if A has FINAALG R
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R rightDiscriminant : Vector % -> R if A has FINAALG R
--R rightDiscriminant : () -> R if A has FRNAALG R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has INTDOM or A
--R rightNorm : % -> R if A has FINAALG R
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and R has FIE
--R rightRecip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has
--R rightRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R
--R rightRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R rightTrace : % -> R if A has FINAALG R
--R rightTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R rightTraceMatrix : () -> Matrix R if A has FRNAALG R
--R rightUnit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if A has FINAALG R and R has
--R someBasis : () -> Vector % if A has FINAALG R
--R structuralConstants : Vector % -> Vector Matrix R if A has FINAALG R
--R structuralConstants : () -> Vector Matrix R if A has FRNAALG R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTD
--R
--E 1

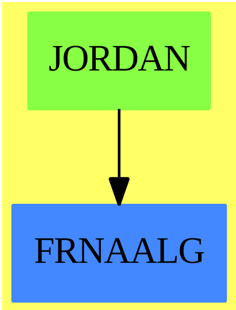
)spool
)lisp (bye)
```

— **AssociatedJordanAlgebra.help** —

```
====================================================================
AssociatedJordanAlgebra examples
====================================================================

See Also:
o )show AssociatedJordanAlgebra
```

2.40.1 AssociatedJordanAlgebra (JORDAN)



See

⇒ "AssociatedLieAlgebra" (LIE) 2.41.1 on page 211
⇒ "LieSquareMatrix" (LSQM) 13.5.1 on page 1419

Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	apply
associative?	associator
associatorDependence	basis
coerce	commutative?
commutator	conditionsForIdempotents
convert	coordinates
flexible?	hash
jacobiIdentity?	jordanAdmissible?
jordanAlgebra?	latex
leftAlternative?	leftCharacteristicPolynomial
leftDiscriminant	leftMinimalPolynomial
leftNorm	leftPower
leftRankPolynomial	leftRecip
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftUnit
leftUnits	lieAdmissible?
lieAlgebra?	noncommutativeJordanAlgebra?
plenaryPower	powerAssociative?
rank	recip
represents	rightAlternative?
rightCharacteristicPolynomial	rightDiscriminant
rightMinimalPolynomial	rightNorm
rightPower	rightRankPolynomial
rightRecip	rightRegularRepresentation
rightTrace	rightTraceMatrix
rightUnit	rightUnits
sample	someBasis
structuralConstants	subtractIfCan
unit	zero?
?*?	?**?
?+?	?-?
-?	?=?
?~=?	?.?

— domain JORDAN AssociatedJordanAlgebra —

```
)abbrev domain JORDAN AssociatedJordanAlgebra
++ Author: J. Grabmeier
++ Date Created: 14 June 1991
++ Date Last Updated: 14 June 1991
++ Basic Operations: *,**,+,-
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: associated Jordan algebra
```

```
++ References:
++ Description:
++ AssociatedJordanAlgebra takes an algebra \spad{A} and uses \spadfun{*$A}
++ to define the new multiplications \spad{a*b := (a *$A b + b *$A a)/2}
++ (anticommutator).
++ The usual notation \spad{{a,b}_+} cannot be used due to
++ restrictions in the current language.
++ This domain only gives a Jordan algebra if the
++ Jordan-identity \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0} holds
++ for all \spad{a},\spad{b},\spad{c} in \spad{A}.
++ This relation can be checked by
++ \spadfun{jordanAdmissible?()$A}.
++
++ If the underlying algebra is of type
++ \spadtype{FramedNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free R-module of finite
++ rank, together with a fixed R-module basis), then the same
++ is true for the associated Jordan algebra.
++ Moreover, if the underlying algebra is of type
++ \spadtype{FiniteRankNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free R-module of finite
++ rank), then the same true for the associated Jordan algebra.

AssociatedJordanAlgebra(R:CommutativeRing,A:NonAssociativeAlgebra R):
    public == private where
  public ==> Join (NonAssociativeAlgebra R, CoercibleTo A)  with
    coerce : A -> %
      ++ coerce(a) coerces the element \spad{a} of the algebra \spad{A}
      ++ to an element of the Jordan algebra
      ++ \spadtype{AssociatedJordanAlgebra}(R,A).
    if A has FramedNonAssociativeAlgebra(R) then _
      FramedNonAssociativeAlgebra(R)
    if A has FiniteRankNonAssociativeAlgebra(R) then _
      FiniteRankNonAssociativeAlgebra(R)

  private ==> A add
    Rep := A
    two  : R := (1$R + 1$R)
    oneHalf : R := (recip two) :: R
    (a:%) * (b:%) ==
      zero? two => error
        "constructor must no be called with Ring of characteristic 2"
      ((a::Rep) * $Rep (b::Rep) +$Rep (b::Rep) * $Rep (a::Rep)) * oneHalf
      -- (a::Rep) * $Rep (b::Rep) +$Rep (b::Rep) * $Rep (a::Rep)
    coerce(a:%):A == a :: Rep
    coerce(a:A):% == a :: %
    (a:%) ** (n:PositiveInteger) == a
```

———

— JORDAN.dotabb —

```
"JORDAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=JORDAN"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"JORDAN" -> "FRNAALG"
```

2.41 domain LIE AssociatedLieAlgebra

— AssociatedLieAlgebra.input —

```
)set break resume
)sys rm -f AssociatedLieAlgebra.output
)spool AssociatedLieAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AssociatedLieAlgebra
--R AssociatedLieAlgebra(R: CommutativeRing,A: NonAssociativeAlgebra R)  is a domain constructor
--R Abbreviation for AssociatedLieAlgebra is LIE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LIE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antiCommutator : (%,%) -> %
--R associator : (%,%,%) -> %             coerce : A -> %
--R coerce : % -> A                       coerce : % -> OutputForm
--R commutator : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R alternative? : () -> Boolean if A has FINAALG R
--R antiAssociative? : () -> Boolean if A has FINAALG R
--R antiCommutative? : () -> Boolean if A has FINAALG R
--R apply : (Matrix R,%) -> % if A has FRNAALG R
--R associative? : () -> Boolean if A has FINAALG R
--R associatorDependence : () -> List Vector R if A has FINAALG R and R has INTDOM or A has FRNAALG R an
```

```
--R basis : () -> Vector % if A has FRNAALG R
--R commutative? : () -> Boolean if A has FINAALG R
--R conditionsForIdempotents : Vector % -> List Polynomial R if A has FINAALG R
--R conditionsForIdempotents : () -> List Polynomial R if A has FRNAALG R
--R convert : % -> Vector R if A has FRNAALG R
--R convert : Vector R -> % if A has FRNAALG R
--R coordinates : (%,Vector %) -> Vector R if A has FINAALG R
--R coordinates : (Vector %,Vector %) -> Matrix R if A has FINAALG R
--R coordinates : % -> Vector R if A has FRNAALG R
--R coordinates : Vector % -> Matrix R if A has FRNAALG R
--R ?.? : (%,Integer) -> R if A has FRNAALG R
--R flexible? : () -> Boolean if A has FINAALG R
--R jacobiIdentity? : () -> Boolean if A has FINAALG R
--R jordanAdmissible? : () -> Boolean if A has FINAALG R
--R jordanAlgebra? : () -> Boolean if A has FINAALG R
--R leftAlternative? : () -> Boolean if A has FINAALG R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R leftDiscriminant : Vector % -> R if A has FINAALG R
--R leftDiscriminant : () -> R if A has FRNAALG R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has IN
--R leftNorm : % -> R if A has FINAALG R
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and
--R leftRecip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R
--R leftRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R
--R leftRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R leftTrace : % -> R if A has FINAALG R
--R leftTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R leftTraceMatrix : () -> Matrix R if A has FRNAALG R
--R leftUnit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if A has FINAALG R
--R lieAdmissible? : () -> Boolean if A has FINAALG R
--R lieAlgebra? : () -> Boolean if A has FINAALG R
--R noncommutativeJordanAlgebra? : () -> Boolean if A has FINAALG R
--R plenaryPower : (%,PositiveInteger) -> %
--R powerAssociative? : () -> Boolean if A has FINAALG R
--R rank : () -> PositiveInteger if A has FINAALG R
--R recip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and
--R represents : (Vector R,Vector %) -> % if A has FINAALG R
--R represents : Vector R -> % if A has FRNAALG R
--R rightAlternative? : () -> Boolean if A has FINAALG R
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R rightDiscriminant : Vector % -> R if A has FINAALG R
--R rightDiscriminant : () -> R if A has FRNAALG R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has I
--R rightNorm : % -> R if A has FINAALG R
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R an
--R rightRecip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG
--R rightRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R
```

```
--R rightRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R rightTrace : % -> R if A has FINAALG R
--R rightTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R rightTraceMatrix : () -> Matrix R if A has FRNAALG R
--R rightUnit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if A has FINAALG R and R has
--R someBasis : () -> Vector % if A has FINAALG R
--R structuralConstants : Vector % -> Vector Matrix R if A has FINAALG R
--R structuralConstants : () -> Vector Matrix R if A has FRNAALG R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTD
--R
--E 1

)spool
)lisp (bye)
```

———-

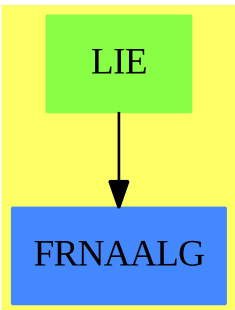
— **AssociatedLieAlgebra.help** —

```
====================================================================
AssociatedLieAlgebra examples
====================================================================

See Also:
o )show AssociatedLieAlgebra
```

———-

2.41.1 AssociatedLieAlgebra (LIE)



See

⇒ "AssociatedJordanAlgebra" (JORDAN) 2.40.1 on page 206
⇒ "LieSquareMatrix" (LSQM) 13.5.1 on page 1419

Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	apply
associative?	associator
associatorDependence	basis
coerce	commutative?
commutator	conditionsForIdempotents
convert	coordinates
flexible?	hash
jacobiIdentity?	jordanAdmissible?
jordanAlgebra?	latex
leftAlternative?	leftCharacteristicPolynomial
leftDiscriminant	leftMinimalPolynomial
leftNorm	leftPower
leftRankPolynomial	leftRecip
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftUnit
leftUnits	lieAdmissible?
lieAlgebra?	noncommutativeJordanAlgebra?
plenaryPower	powerAssociative?
rank	recip
represents	represents
rightAlternative?	rightCharacteristicPolynomial
rightDiscriminant	rightMinimalPolynomial
rightNorm	rightPower
rightRankPolynomial	rightRecip
rightRegularRepresentation	rightTrace
rightTraceMatrix	rightUnit
rightUnits	sample
someBasis	structuralConstants
structuralConstants	subtractIfCan
unit	zero?
?*?	?**?
?+?	?-?
-?	?=?
?~=?	?.?

— domain LIE AssociatedLieAlgebra —

```
)abbrev domain LIE AssociatedLieAlgebra
++ Author: J. Grabmeier
++ Date Created: 07 March 1991
++ Date Last Updated: 14 June 1991
++ Basic Operations: *,**,+,-
++ Related Constructors:
++ Also See:
```

```
++ AMS Classifications:
++ Keywords: associated Liealgebra
++ References:
++ Description:
++ AssociatedLieAlgebra takes an algebra \spad{A}
++ and uses \spadfun{*$A} to define the
++ Lie bracket \spad{a*b := (a *$A b - b *$A a)} (commutator). Note that
++ the notation \spad{[a,b]} cannot be used due to
++ restrictions of the current compiler.
++ This domain only gives a Lie algebra if the
++ Jacobi-identity \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0} holds
++ for all \spad{a},\spad{b},\spad{c} in \spad{A}.
++ This relation can be checked by
++ \spad{lieAdmissible?()$A}.
++
++ If the underlying algebra is of type
++ \spadtype{FramedNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free \spad{R}-module of finite
++ rank, together with a fixed \spad{R}-module basis), then the same
++ is true for the associated Lie algebra.
++ Also, if the underlying algebra is of type
++ \spadtype{FiniteRankNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free R-module of finite
++ rank), then the same is true for the associated Lie algebra.

AssociatedLieAlgebra(R:CommutativeRing,A:NonAssociativeAlgebra R):
    public == private where
  public ==> Join (NonAssociativeAlgebra R, CoercibleTo A)  with
    coerce : A -> %
      ++ coerce(a) coerces the element \spad{a} of the algebra \spad{A}
      ++ to an element of the Lie
      ++ algebra \spadtype{AssociatedLieAlgebra}(R,A).
    if A has FramedNonAssociativeAlgebra(R) then
      FramedNonAssociativeAlgebra(R)
    if A has FiniteRankNonAssociativeAlgebra(R) then
      FiniteRankNonAssociativeAlgebra(R)

  private ==> A add
    Rep := A
    (a:%) * (b:%) == (a::Rep) * $Rep (b::Rep) -$Rep (b::Rep) * $Rep (a::Rep)
    coerce(a:%):A == a :: Rep
    coerce(a:A):% == a :: %
    (a:%) ** (n:PositiveInteger) ==
      n = 1 => a
      0
```

———

— LIE.dotabb —

```
"LIE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LIE"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"LIE" -> "FRNAALG"
```

2.42 domain ALIST AssociationList

— AssociationList.input —

```
)set break resume
)sys rm -f AssociationList.output
)spool AssociationList.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
Data := Record(monthsOld : Integer, gender : String)
--R
--R
--R   (1)  Record(monthsOld: Integer,gender: String)
--R                                                                 Type: Domain
--E 1

--S 2 of 10
al : AssociationList(String,Data)
--R
--R                                                                   Type: Void
--E 2

--S 3 of 10
al := table()
--R
--R
--R   (3)  table()
--R      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
--E 3

--S 4 of 10
al."bob" := [407,"male"]$Data
--R
--R
--R   (4)  [monthsOld= 407,gender= "male"]
--R                                 Type: Record(monthsOld: Integer,gender: String)
```

```
--E 4

--S 5 of 10
al."judith" := [366,"female"]$Data
--R
--R
--R   (5)  [monthsOld= 366,gender= "female"]
--R                              Type: Record(monthsOld: Integer,gender: String)
--E 5

--S 6 of 10
al."katie" := [24,"female"]$Data
--R
--R
--R   (6)  [monthsOld= 24,gender= "female"]
--R                              Type: Record(monthsOld: Integer,gender: String)
--E 6

--S 7 of 10
al."smokie" := [200,"female"]$Data
--R
--R
--R   (7)  [monthsOld= 200,gender= "female"]
--R                              Type: Record(monthsOld: Integer,gender: String)
--E 7

--S 8 of 10
al
--R
--R
--R   (8)
--R   table
--R      "smokie"= [monthsOld= 200,gender= "female"]
--R  ,
--R      "katie"= [monthsOld= 24,gender= "female"]
--R  ,
--R      "judith"= [monthsOld= 366,gender= "female"]
--R  ,
--R      "bob"= [monthsOld= 407,gender= "male"]
--R      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
--E 8

--S 9 of 10
al."katie" := [23,"female"]$Data
--R
--R
--R   (9)  [monthsOld= 23,gender= "female"]
--R                              Type: Record(monthsOld: Integer,gender: String)
--E 9
```

```
--S 10 of 10
delete!(al,1)
--R
--R
--R   (10)
--R   table
--R      "katie"= [monthsOld= 23,gender= "female"]
--R   ,
--R      "judith"= [monthsOld= 366,gender= "female"]
--R   ,
--R      "bob"= [monthsOld= 407,gender= "male"]
--R      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
--E 10
)spool
)lisp (bye)
```

— **AssociationList.help** —

```
====================================================================
AssociationList examples
====================================================================

The AssociationList constructor provides a general structure for
associative storage.  This type provides association lists in which
data objects can be saved according to keys of any type.  For a given
association list, specific types must be chosen for the keys and
entries.  You can think of the representation of an association list
as a list of records with key and entry fields.

Association lists are a form of table and so most of the operations
available for Table are also available for AssociationList.  They can
also be viewed as lists and can be manipulated accordingly.

This is a Record type with age and gender fields.

  Data := Record(monthsOld : Integer, gender : String)
    Record(monthsOld: Integer,gender: String)
                      Type: Domain

In this expression, al is declared to be an association
list whose keys are strings and whose entries are the above records.

  al : AssociationList(String,Data)
                      Type: Void

The table operation is used to create an empty association list.
```

```
  al := table()
   table()
    Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
```

You can use assignment syntax to add things to the association list.

```
  al."bob" := [407,"male"]$Data
   [monthsOld=407, gender= "male"]
                      Type: Record(monthsOld: Integer,gender: String)

  al."judith" := [366,"female"]$Data
   [monthsOld=366, gender= "female"]
                      Type: Record(monthsOld: Integer,gender: String)

  al."katie" := [24,"female"]$Data
   [monthsOld=24, gender= "female"]
                      Type: Record(monthsOld: Integer,gender: String)
```

Perhaps we should have included a species field.

```
  al."smokie" := [200,"female"]$Data
   [monthsOld=200, gender= "female"]
                      Type: Record(monthsOld: Integer,gender: String)
```

Now look at what is in the association list. Note that the last-added (key, entry) pair is at the beginning of the list.

```
  al
   table("smokie" = [monthsOld=200, gender= "female"],
         "katie" = [monthsOld=24, gender= "female"],
         "judith" = [monthsOld=366, gender= "female"],
         "bob" = [monthsOld=407, gender= "male"])
      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
```

You can reset the entry for an existing key.

```
  al."katie" := [23,"female"]$Data
   [monthsOld=23, gender= "female"]
                      Type: Record(monthsOld: Integer,gender: String)
```

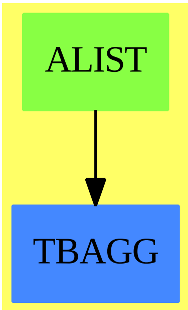
Use delete! to destructively remove an element of the association list. Use delete to return a copy of the association list with the element deleted. The second argument is the index of the element to delete.

```
  delete!(al,1)
   table("katie" = [monthsOld=23, gender= "female"],
         "judith" = [monthsOld=366, gender= "female"],
         "bob" = [monthsOld=407, gender= "male"])
    Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
```

```
See Also:
o )help Table
o )help List
o )show AssociationList
```

———-

2.42.1 AssociationList (ALIST)



See

⇒ "IndexedList" (ILIST) 10.11.1 on page 1196
⇒ "List" (LIST) 13.9.1 on page 1468

Exports:

any?	assoc	bag	child?	children
coerce	concat	concat!	construct	convert
copy	copyInto!	count	cycleEntry	cycleLength
cycleSplit!	cycleTail	cyclic?	delete	delete!
dictionary	distance	elt	empty	empty?
entries	entry?	eq?	eval	every?
explicitlyFinite?	extract!	fill!	find	first
hash	index?	indices	insert	insert!
inspect	key?	keys	last	latex
leaf?	leaves	less?	list	map
map!	max	maxIndex	member?	members
merge	merge!	min	minIndex	more?
new	node?	nodes	parts	position
possiblyInfinite?	qelt	qsetelt!	reduce	remove
remove!	removeDuplicates	removeDuplicates!	rest	reverse
reverse!	sample	search	second	select
select!	setchildren!	setelt	setfirst!	setlast!
setrest!	setvalue!	size?	sort	sort!
sorted?	split!	swap!	table	tail
third	value	#?	?<?	?<=?
?=?	?>?	?>=?	?~=?	?.rest
?.value	?.first	?.last	?.?	

— domain ALIST AssociationList —

```
)abbrev domain ALIST AssociationList
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations: empty, empty?, keys, \#, concat, first, rest,
++   setrest!, search, setelt, remove!
++ Related Constructors:
++ Also See: List
++ AMS Classification:
++ Keywords: list, association list
++ Description:
++ \spadtype{AssociationList} implements association lists. These
++ may be viewed as lists of pairs where the first part is a key
++ and the second is the stored value. For example, the key might
++ be a string with a persons employee identification number and
++ the value might be a record with personnel data.

AssociationList(Key:SetCategory, Entry:SetCategory):
 AssociationListAggregate(Key, Entry) == add
        Pair ==> Record(key:Key, entry:Entry)
        Rep := Reference List Pair

        dictionary()            == ref empty()
```

```
        empty()                  == dictionary()
        empty? t                 == empty? deref t
        entries(t:%):List(Pair) == deref t
        parts(t:%):List(Pair)   == deref t
        keys t                   == [k.key for k in deref t]
        # t                      == # deref t
        first(t:%):Pair         == first deref t
        rest t                   == ref rest deref t
        concat(p:Pair, t:%)     == ref concat(p, deref t)
        setrest_!(a:%, b:%)     == ref setrest_!(deref a, deref b)
        setfirst_!(a:%, p:Pair) == setfirst_!(deref a,p)
        minIndex(a:%):Integer   == minIndex(deref a)
        maxIndex(a:%):Integer   == maxIndex(deref a)

        search(k, t) ==
          for r in deref t repeat
            k = r.key => return(r.entry)
          "failed"

        latex(a : %) : String ==
          l : List Pair := entries a
          s : String := "\left["
          while not empty?(l) repeat
            r : Pair := first l
            l        := rest l
            s := concat(s, concat(latex r.key, concat(" = ", latex r.entry)$String)$String)$S
            if not empty?(l) then s := concat(s, ", ")$String
          concat(s, " \right]")$String

--      assoc(k, l) ==
--        (r := find(#1.key=k, l)) case "failed" => "failed"
--        r

        assoc(k, t) ==
          for r in deref t repeat
            k = r.key => return r
          "failed"

        setelt(t:%, k:Key, e:Entry) ==
          (r := assoc(k, t)) case Pair => (r::Pair).entry := e
          setref(t, concat([k, e], deref t))
          e

        remove_!(k:Key, t:%) ==
          empty?(l := deref t) => "failed"
          k = first(l).key =>
            setref(t, rest l)
            first(l).entry
          prev := l
          curr := rest l
```

```
          while not empty? curr and first(curr).key ^= k repeat
            prev := curr
            curr := rest curr
          empty? curr => "failed"
          setrest_!(prev, rest curr)
          first(curr).entry
```

———-

— ALIST.dotabb —

```
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"ALIST" -> "TBAGG"
```

———-

2.43 domain ATTRBUT AttributeButtons

— AttributeButtons.input —

```
)set break resume
)sys rm -f AttributeButtons.output
)spool AttributeButtons.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AttributeButtons
--R AttributeButtons  is a domain constructor
--R Abbreviation for AttributeButtons is ATTRBUT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ATTRBUT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R decrease : String -> Float            hash : % -> SingleInteger
--R increase : String -> Float            latex : % -> String
--R resetAttributeButtons : () -> Void    ?~=? : (%,%) -> Boolean
--R decrease : (String,String) -> Float
--R getButtonValue : (String,String) -> Float
--R increase : (String,String) -> Float
--R setAttributeButtonStep : Float -> Float
```

```
--R setButtonValue : (String,String,Float) -> Float
--R setButtonValue : (String,Float) -> Float
--R
--E 1

)spool
)lisp (bye)
```

———–

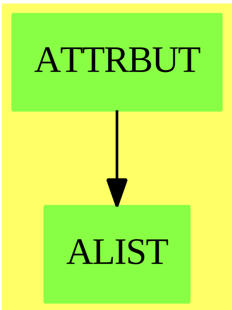
— **AttributeButtons.help** —

```
====================================================================
AttributeButtons examples
====================================================================

See Also:
o )show AttributeButtons
```

———–

2.43.1 AttributeButtons (ATTRBUT)



Exports:

coerce	decrease	getButtonValue	hash
increase	latex	resetAttributeButtons	setAttributeButtonStep
setButtonValue	?~=?	?=?	

— **domain ATTRBUT AttributeButtons** —

```
)abbrev domain ATTRBUT AttributeButtons
++ Author: Brian Dupee
++ Date Created: April 1996
++ Date Last Updated: December 1997
```

```
++ Basic Operations: increase, decrease, getButtonValue, setButtonValue
++ Related Constructors: Table(String,Float)
++ Description:
++ \axiomType{AttributeButtons} implements a database and associated
++ adjustment mechanisms for a set of attributes.
++
++ For ODEs these attributes are "stiffness", "stability" (i.e. how much
++ affect the cosine or sine component of the solution has on the stability of
++ the result), "accuracy" and "expense" (i.e. how expensive is the evaluation
++ of the ODE).  All these have bearing on the cost of calculating the
++ solution given that reducing the step-length to achieve greater accuracy
++ requires considerable number of evaluations and calculations.
++
++ The effect of each of these attributes can be altered by increasing or
++ decreasing the button value.
++
++ For Integration there is a button for increasing and decreasing the preset
++ number of function evaluations for each method.  This is automatically used
++ by ANNA when a method fails due to insufficient workspace or where the
++ limit of function evaluations has been reached before the required
++ accuracy is achieved.

AttributeButtons(): E == I where
  F     ==> Float
  ST    ==> String
  LST   ==> List String
  Rec   ==> Record(key:Symbol,entry:Any)
  RList ==> List(Record(key:Symbol,entry:Any))
  IFL   ==> List(Record(ifail:Integer,instruction:ST))
  Entry ==> Record(chapter:ST, type:ST, domainName: ST,
                    defaultMin:F, measure:F, failList:IFL, explList:LST)


  E ==> SetCategory with

    increase:(ST,ST) -> F
      ++ \axiom{increase(routineName,attributeName)} increases the value
      ++ for the effect of the attribute \axiom{attributeName} with routine
      ++ \axiom{routineName}.
      ++
      ++ \axiom{attributeName} should be one of the values
      ++ "stiffness", "stability", "accuracy", "expense" or
      ++ "functionEvaluations".
    increase:(ST) -> F
      ++ \axiom{increase(attributeName)} increases the value for the
      ++ effect of the attribute \axiom{attributeName} with all routines.
      ++
      ++ \axiom{attributeName} should be one of the values
      ++ "stiffness", "stability", "accuracy", "expense" or
      ++ "functionEvaluations".
```

```
  decrease:(ST,ST) -> F
    ++ \axiom{decrease(routineName,attributeName)} decreases the value
    ++ for the effect of the attribute \axiom{attributeName} with routine
    ++ \axiom{routineName}.
    ++
    ++ \axiom{attributeName} should be one of the values
    ++ "stiffness", "stability", "accuracy", "expense" or
    ++ "functionEvaluations".
  decrease:(ST) -> F
    ++ \axiom{decrease(attributeName)} decreases the value for the
    ++ effect of the attribute \axiom{attributeName} with all routines.
    ++
    ++ \axiom{attributeName} should be one of the values
    ++ "stiffness", "stability", "accuracy", "expense" or
    ++ "functionEvaluations".
  getButtonValue:(ST,ST) -> F
    ++ \axiom{getButtonValue(routineName,attributeName)} returns the
    ++ current value for the effect of the attribute \axiom{attributeName}
    ++ with routine \axiom{routineName}.
    ++
    ++ \axiom{attributeName} should be one of the values
    ++ "stiffness", "stability", "accuracy", "expense" or
    ++ "functionEvaluations".
  resetAttributeButtons:() -> Void
    ++ \axiom{resetAttributeButtons()} resets the Attribute buttons to a
    ++ neutral level.
  setAttributeButtonStep:(F) -> F
    ++ \axiom{setAttributeButtonStep(n)} sets the value of the steps for
    ++ increasing and decreasing the button values. \axiom{n} must be
    ++ greater than 0 and less than 1.  The preset value is 0.5.
  setButtonValue:(ST,F) -> F
    ++ \axiom{setButtonValue(attributeName,n)} sets the
    ++ value of all buttons of attribute \spad{attributeName}
    ++ to \spad{n}. \spad{n} must be in the range [0..1].
    ++
    ++ \axiom{attributeName} should be one of the values
    ++ "stiffness", "stability", "accuracy", "expense" or
    ++ "functionEvaluations".
  setButtonValue:(ST,ST,F) -> F
    ++ \axiom{setButtonValue(attributeName,routineName,n)} sets the
    ++ value of the button of attribute \spad{attributeName} to routine
    ++ \spad{routineName} to \spad{n}. \spad{n} must be in the range [0..1].
    ++
    ++ \axiom{attributeName} should be one of the values
    ++ "stiffness", "stability", "accuracy", "expense" or
    ++ "functionEvaluations".
  finiteAggregate

I ==> add
```

```
Rep := StringTable(F)
import Rep

buttons:() -> $

buttons():$ ==
  eList := empty()$List(Record(key:ST,entry:F))
  l1:List String := ["stability","stiffness","accuracy","expense"]
  l2:List String := ["functionEvaluations"]
  ro1 := selectODEIVPRoutines(r := routines()$RoutinesTable)$RoutinesTable
  ro2 := selectIntegrationRoutines(r)$RoutinesTable
  k1:List String := [string(i)$Symbol for i in keys(ro1)$RoutinesTable]
  k2:List String := [string(i)$Symbol for i in keys(ro2)$RoutinesTable]
  for i in k1 repeat
    for j in l1 repeat
      e:Record(key:ST,entry:F) := [i j,0.5]
      eList := cons(e,eList)$List(Record(key:ST,entry:F))
  for i in k2 repeat
    for j in l2 repeat
      e:Record(key:ST,entry:F) := [i j,0.5]
      eList := cons(e,eList)$List(Record(key:ST,entry:F))
  construct(eList)$Rep

attributeButtons:$ := buttons()

attributeStep:F := 0.5

setAttributeButtonStep(n:F):F ==
  positive?(n)$F and (n<1$F) => attributeStep:F := n
  error("setAttributeButtonStep","New value must be in (0..1)")$ErrorFunctions

resetAttributeButtons():Void ==
  attributeButtons := buttons()
  void()$Void

setButtonValue(routineName:ST,attributeName:ST,n:F):F ==
  f := search(routineName attributeName,attributeButtons)$Rep
  f case Float =>
    n>=0$F and n<=1$F =>
      setelt(attributeButtons,routineName attributeName,n)$Rep
    error("setAttributeButtonStep","New value must be in [0..1]")$ErrorFunctions
  error("setButtonValue","attribute name " attributeName
         " not found for routine " routineName)$ErrorFunctions

setButtonValue(attributeName:ST,n:F):F ==
  ro1 := selectODEIVPRoutines(r := routines()$RoutinesTable)$RoutinesTable
  ro2 := selectIntegrationRoutines(r)$RoutinesTable
  l1:List String := ["stability","stiffness","accuracy","expense"]
  l2:List String := ["functionEvaluations"]
  if attributeName="functionEvaluations" then
```

```
    for i in keys(ro2)$RoutinesTable repeat
      setButtonValue(string(i)$Symbol,attributeName,n)
  else
    for i in keys(ro1)$RoutinesTable repeat
      setButtonValue(string(i)$Symbol,attributeName,n)
  n

increase(routineName:ST,attributeName:ST):F ==
  f := search(routineName attributeName,attributeButtons)$Rep
  f case Float =>
    newValue:F := (1$F-attributeStep)*f+attributeStep
    setButtonValue(routineName,attributeName,newValue)
  error("increase","attribute name " attributeName
         " not found for routine " routineName)$ErrorFunctions

increase(attributeName:ST):F ==
  ro1 := selectODEIVPRoutines(r := routines()$RoutinesTable)$RoutinesTable
  ro2 := selectIntegrationRoutines(r)$RoutinesTable
  l1:List String := ["stability","stiffness","accuracy","expense"]
  l2:List String := ["functionEvaluations"]
  if attributeName="functionEvaluations" then
    for i in keys(ro2)$RoutinesTable repeat
      increase(string(i)$Symbol,attributeName)
  else
    for i in keys(ro1)$RoutinesTable repeat
      increase(string(i)$Symbol,attributeName)
  getButtonValue(string(i)$Symbol,attributeName)

decrease(routineName:ST,attributeName:ST):F ==
  f := search(routineName attributeName,attributeButtons)$Rep
  f case Float =>
    newValue:F := (1$F-attributeStep)*f
    setButtonValue(routineName,attributeName,newValue)
  error("increase","attribute name " attributeName
         " not found for routine " routineName)$ErrorFunctions

decrease(attributeName:ST):F ==
  ro1 := selectODEIVPRoutines(r := routines()$RoutinesTable)$RoutinesTable
  ro2 := selectIntegrationRoutines(r)$RoutinesTable
  l1:List String := ["stability","stiffness","accuracy","expense"]
  l2:List String := ["functionEvaluations"]
  if attributeName="functionEvaluations" then
    for i in keys(ro2)$RoutinesTable repeat
      decrease(string(i)$Symbol,attributeName)
  else
    for i in keys(ro1)$RoutinesTable repeat
      decrease(string(i)$Symbol,attributeName)
  getButtonValue(string(i)$Symbol,attributeName)
```

```
  getButtonValue(routineName:ST,attributeName:ST):F ==
    f := search(routineName attributeName,attributeButtons)$Rep
    f case Float => f
    error("getButtonValue","attribute name " attributeName
            " not found for routine " routineName)$ErrorFunctions
```

———-

— ATTRBUT.dotabb —

```
"ATTRBUT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ATTRBUT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ATTRBUT" -> "ALIST"
```

———-

2.44 domain AUTOMOR Automorphism

— Automorphism.input —

```
)set break resume
)sys rm -f Automorphism.output
)spool Automorphism.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Automorphism
--R Automorphism R: Ring  is a domain constructor
--R Abbreviation for Automorphism is AUTOMOR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AUTOMOR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %        ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                 1 : () -> %
--R ?^? : (%,Integer) -> %                 ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm               commutator : (%,%) -> %
--R conjugate : (%,%) -> %                 ?.? : (%,R) -> R
--R hash : % -> SingleInteger              inv : % -> %
--R latex : % -> String                    morphism : (R -> R) -> %
--R one? : % -> Boolean                    recip : % -> Union(%,"failed")
```

```
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R morphism : ((R,Integer) -> R) -> %
--R morphism : ((R -> R),(R -> R)) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

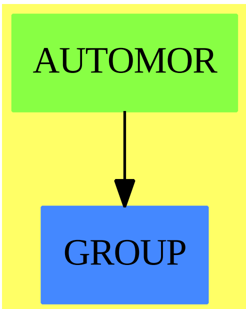
— **Automorphism.help** —

```
====================================================================
Automorphism examples
====================================================================

See Also:
o )show Automorphism
```

———-

2.44.1 Automorphism (AUTOMOR)



See

⇒ "SparseUnivariateSkewPolynomial" (ORESUP) 20.21.1 on page 2450
⇒ "UnivariateSkewPolynomial" (OREUP) 22.8.1 on page 2829

Exports:

1	coerce	commutator	conjugate	hash
inv	latex	morphism	one?	recip
sample	?~=?	?**?	?^?	?.?
?*?	?/?	?=?		

— domain AUTOMOR Automorphism —

```
)abbrev domain AUTOMOR Automorphism
++ Author: Manuel Bronstein
++ Date Created: 31 January 1994
++ Date Last Updated: 31 January 1994
++ References:
++ Description:
++ Automorphism R is the multiplicative group of automorphisms of R.
-- In fact, non-invertible endomorphism are allowed as partial functions.
-- This domain is noncanonical in that f*f^{-1} will be the identity
-- function but won't be equal to 1.

Automorphism(R:Ring): Join(Group, Eltable(R, R)) with
      morphism: (R -> R) -> %
        ++ morphism(f) returns the non-invertible morphism given by f.
      morphism: (R -> R, R -> R) -> %
        ++ morphism(f, g) returns the invertible morphism given by f, where
        ++ g is the inverse of f..
      morphism: ((R, Integer) -> R) -> %
        ++ morphism(f) returns the morphism given by \spad{f^n(x) = f(x,n)}.
   == add
      err:   R -> R
      ident: (R, Integer) -> R
      iter:  (R -> R, NonNegativeInteger, R) -> R
      iterat: (R -> R, R -> R, Integer, R) -> R
      apply: (%, R, Integer) -> R

      Rep := ((R, Integer) -> R)

      1                               == ident
      err r                           == error "Morphism is not invertible"
      ident(r, n)                     == r
      f = g                           == EQ(f, g)$Lisp
      elt(f, r)                       == apply(f, r, 1)
      inv f  == (r1:R, i2:Integer):R +-> apply(f, r1, - i2)
      f ** n == (r1:R, i2:Integer):R +-> apply(f, r1, n * i2)
      coerce(f:%):OutputForm          == message("R -> R")
      morphism(f:(R, Integer) -> R):% == f
      morphism(f:R -> R):%            == morphism(f, err)
      morphism(f, g) == (r1:R, i2:Integer):R +-> iterat(f, g, i2, r1)
      apply(f, r, n) == (g := f pretend ((R, Integer) -> R); g(r, n))

      iterat(f, g, n, r) ==
          n < 0 => iter(g, (-n)::NonNegativeInteger, r)
          iter(f, n::NonNegativeInteger, r)

      iter(f, n, r) ==
          for i in 1..n repeat r := f r
```

```
          r

      f * g ==
        f = g => f**2
        (r1:R, i2:Integer):R +->
          iterat((u1:R):R +-> f g u1,
                 (v1:R):R +-> (inv g)(inv f) v1,
                 i2, r1)
```

— AUTOMOR.dotabb —

```
"AUTOMOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AUTOMOR"]
"GROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=GROUP"]
"AUTOMOR" -> "GROUP"
```

Chapter 3

Chapter B

3.1 domain BBTREE BalancedBinaryTree

— BalancedBinaryTree.input —

```
)set break resume
)sys rm -f BalancedBinaryTree.output
)spool BalancedBinaryTree.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
lm := [3,5,7,11]
--E 1

--S 2 of 7
t := balancedBinaryTree(#lm, 0)
--E 2

--S 3 of 7
setleaves!(t,lm)
--E 3

--S 4 of 7
mapUp!(t,_*)
--E 4

--S 5 of 7
t
--E 5

--S 6 of 7
```

```
mapDown!(t,12,_rem)
--E 6

--S 7 of 7
leaves %
--E 7

)spool
)lisp (bye)
```

— BalancedBinaryTree.help —

```
====================================================================
BalancedBinaryTree examples
====================================================================

BalancedBinaryTrees(S) is the domain of balanced binary trees with
elements of type S at the nodes.  A binary tree is either empty or
else consists of a node having a value and two branches, each branch a
binary tree.  A balanced binary tree is one that is balanced with
respect its leaves.  One with 2^k leaves is perfectly "balanced": the
tree has minimum depth, and the left and right branch of every
interior node is identical in shape.

Balanced binary trees are useful in algebraic computation for
so-called "divide-and-conquer" algorithms.  Conceptually, the data
for a problem is initially placed at the root of the tree.  The
original data is then split into two subproblems, one for each
subtree.  And so on.  Eventually, the problem is solved at the leaves
of the tree.  A solution to the original problem is obtained by some
mechanism that can reassemble the pieces.  In fact, an implementation
of the Chinese Remainder Algorithm using balanced binary trees was
first proposed by David Y. Y.  Yun at the IBM T. J.  Watson Research
Center in Yorktown Heights, New York, in 1978.  It served as the
prototype for polymorphic algorithms in Axiom.

In what follows, rather than perform a series of computations with a
single expression, the expression is reduced modulo a number of
integer primes, a computation is done with modular arithmetic for each
prime, and the Chinese Remainder Algorithm is used to obtain the
answer to the original problem.  We illustrate this principle with the
computation of 12^2 = 144.

A list of moduli:

  lm := [3,5,7,11]
   [3,5,7,11]
```

```
                      Type: PositiveInteger
```

The expression modTree(n, lm) creates a balanced binary tree with leaf values n mod m for each modulus m in lm.

```
  modTree(12,lm)
   [0, 2, 5, 1]
                      Type: List Integer
```

Operation modTree does this using operations on balanced binary trees. We trace its steps. Create a balanced binary tree t of zeros with four leaves.

```
  t := balancedBinaryTree(#lm, 0)
   [[0, 0, 0], 0, [0, 0, 0]]
                      Type: BalancedBinaryTree NonNegativeInteger
```

The leaves of the tree are set to the individual moduli.

```
  setleaves!(t,lm)
   [[3, 0, 5], 0, [7, 0, 11]]
                      Type: BalancedBinaryTree NonNegativeInteger
```

mapUp! to do a bottom-up traversal of t, setting each interior node to the product of the values at the nodes of its children.

```
  mapUp!(t,_*)
   1155
                      Type: PositiveInteger
```

The value at the node of every subtree is the product of the moduli of the leaves of the subtree.

```
  t
   [[3, 15, 5], 1155, [7, 77, 11]]
                      Type: BalancedBinaryTree NonNegativeInteger
```

Operation mapDown!(t,a,fn) replaces the value v at each node of t by fn(a,v).

```
  mapDown!(t,12,_rem)
   [[0, 12, 2], 12, [5, 12, 1]]
                      Type: BalancedBinaryTree NonNegativeInteger
```

The operation leaves returns the leaves of the resulting tree. In this case, it returns the list of 12 mod m for each modulus m.

```
  leaves %
   [0, 2, 5, 1]
                      Type: List NonNegativeInteger
```

```
Compute the square of the images of 12 modulo each m.

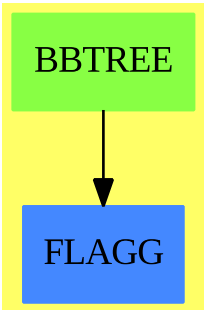
  squares := [x**2 rem m for x in % for m in lm]
   [0, 4, 4, 1]
                         Type: List NonNegativeInteger

Call the Chinese Remainder Algorithm to get the answer for 12^2.

  chineseRemainder(%,lm)
   144
                         Type: PositiveInteger

See Also:
o )show BalancedBinaryTree
```

3.1.1 BalancedBinaryTree (BBTREE)



See

⇒ "Tree" (TREE) 21.10.1 on page 2699
⇒ "BinaryTree" (BTREE) 3.11.1 on page 292
⇒ "BinarySearchTree" (BSTREE) 3.9.1 on page 285
⇒ "BinaryTournament" (BTOURN) 3.10.1 on page 289
⇒ "PendantTree" (PENDTREE) 17.13.1 on page 1904

Exports:

any?	balancedBinaryTree	child?	children	coerce
copy	count	cyclic?	distance	empty
empty?	eq?	eval	every?	hash
latex	leaf?	leaves	left	less?
map	map!	mapDown!	mapUp!	member?
members	more?	node?	node	nodes
parts	right	sample	setchildren!	setelt
setleaves!	setleft!	setright!	setvalue!	size?
value	#?	?=?	?~=?	?.right
?.left	?.value			

— domain BBTREE BalancedBinaryTree —

```
)abbrev domain BBTREE BalancedBinaryTree
++ Description:
++ \spadtype{BalancedBinaryTree(S)} is the domain of balanced
++ binary trees (bbtree). A balanced binary tree of \spad{2**k} leaves,
++ for some \spad{k > 0}, is symmetric, that is, the left and right
++ subtree of each interior node have identical shape.
++ In general, the left and right subtree of a given node can differ
++ by at most leaf node.

BalancedBinaryTree(S: SetCategory): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
    finiteAggregate
    shallowlyMutable
--  BUG: applies wrong fnct for balancedBinaryTree(0,[1,2,3,4])
--    balancedBinaryTree: (S, List S) -> %
--      ++ balancedBinaryTree(s, ls) creates a balanced binary tree with
--      ++ s at the interior nodes and elements of ls at the
--      ++ leaves.
    balancedBinaryTree: (NonNegativeInteger, S) -> %
      ++ balancedBinaryTree(n, s) creates a balanced binary tree with
      ++ n nodes each with value s.
      ++
      ++X balancedBinaryTree(4, 0)

    setleaves_!: (%, List S) -> %
      ++ setleaves!(t, ls) sets the leaves of t in left-to-right order
      ++ to the elements of ls.
      ++
      ++X t1:=balancedBinaryTree(4, 0)
      ++X setleaves!(t1,[1,2,3,4])

    mapUp_!: (%, (S,S) -> S) -> S
      ++ mapUp!(t,f) traverses balanced binary tree t in an "endorder"
      ++ (left then right then node) fashion returning t with the value
      ++ at each successive interior node of t replaced by
      ++ f(l,r) where l and r are the values at the immediate
```

```
  ++ left and right nodes.
  ++
  ++X T1:=BalancedBinaryTree Integer
  ++X t2:=balancedBinaryTree(4, 0)$T1
  ++X setleaves!(t2,[1,2,3,4]::List(Integer))
  ++X adder(a:Integer,b:Integer):Integer == a+b
  ++X mapUp!(t2,adder)
  ++X t2

mapUp_!: (%, %, (S,S,S,S) -> S) -> %
  ++ mapUp!(t,t1,f) traverses balanced binary tree t in an "endorder"
  ++ (left then right then node) fashion returning t with the value
  ++ at each successive interior node of t replaced by
  ++ f(l,r,l1,r1) where l and r are the values at the immediate
  ++ left and right nodes. Values l1 and r1 are values at the
  ++ corresponding nodes of a balanced binary tree t1, of identical
  ++ shape at t.
  ++
  ++X T1:=BalancedBinaryTree Integer
  ++X t2:=balancedBinaryTree(4, 0)$T1
  ++X setleaves!(t2,[1,2,3,4]::List(Integer))
  ++X adder4(i:INT,j:INT,k:INT,l:INT):INT == i+j+k+l
  ++X mapUp!(t2,t2,adder4)
  ++X t2

mapDown_!: (%,S,(S,S) -> S) -> %
  ++ mapDown!(t,p,f) returns t after traversing t in "preorder"
  ++ (node then left then right) fashion replacing the successive
  ++ interior nodes as follows. The root value x is
  ++ replaced by q := f(p,x). The mapDown!(l,q,f) and
  ++ mapDown!(r,q,f) are evaluated for the left and right subtrees
  ++ l and r of t.
  ++
  ++X T1:=BalancedBinaryTree Integer
  ++X t2:=balancedBinaryTree(4, 0)$T1
  ++X setleaves!(t2,[1,2,3,4]::List(Integer))
  ++X adder(i:Integer,j:Integer):Integer == i+j
  ++X mapDown!(t2,4::INT,adder)
  ++X t2

mapDown_!: (%,S, (S,S,S) -> List S) -> %
  ++ mapDown!(t,p,f) returns t after traversing t in "preorder"
  ++ (node then left then right) fashion replacing the successive
  ++ interior nodes as follows. Let l and r denote the left and
  ++ right subtrees of t. The root value x of t is replaced by p.
  ++ Then f(value l, value r, p), where l and r denote the left
  ++ and right subtrees of t, is evaluated producing two values
  ++ pl and pr. Then \spad{mapDown!(l,pl,f)} and \spad{mapDown!(l,pr,f)}
  ++ are evaluated.
  ++
```

```
      ++X T1:=BalancedBinaryTree Integer
      ++X t2:=balancedBinaryTree(4, 0)$T1
      ++X setleaves!(t2,[1,2,3,4]::List(Integer))
      ++X adder3(i:Integer,j:Integer,k:Integer):List Integer == [i+j,j+k]
      ++X mapDown!(t2,4::INT,adder3)
      ++X t2

  Implementation == BinaryTree(S) add
    Rep := BinaryTree(S)
    leaf? x ==
      empty? x => false
      empty? left x and empty? right x
--    balancedBinaryTree(x: S, u: List S) ==
--      n := #u
--      n = 0 => empty()
--      setleaves_!(balancedBinaryTree(n, x), u)
    setleaves_!(t, u) ==
      n := #u
      n = 0 =>
        empty? t => t
        error "the tree and list must have the same number of elements"
      n = 1 =>
        setvalue_!(t,first u)
        t
      m := n quo 2
      acc := empty()$(List S)
      for i in 1..m repeat
        acc := [first u,:acc]
        u := rest u
      setleaves_!(left t, reverse_! acc)
      setleaves_!(right t, u)
      t
    balancedBinaryTree(n: NonNegativeInteger, val: S) ==
      n = 0 => empty()
      n = 1 => node(empty(),val,empty())
      m := n quo 2
      node(balancedBinaryTree(m, val), val,
           balancedBinaryTree((n - m) pretend NonNegativeInteger, val))
    mapUp_!(x,fn) ==
      empty? x => error "mapUp! called on a null tree"
      leaf? x  => x.value
      x.value := fn(mapUp_!(x.left,fn),mapUp_!(x.right,fn))
    mapUp_!(x,y,fn) ==
      empty? x  => error "mapUp! is called on a null tree"
      leaf? x  =>
        leaf? y => x
        error "balanced binary trees are incompatible"
      leaf? y  =>  error "balanced binary trees are incompatible"
      mapUp_!(x.left,y.left,fn)
      mapUp_!(x.right,y.right,fn)
```

```
      x.value := fn(x.left.value,x.right.value,y.left.value,y.right.value)
      x
    mapDown_!(x: %, p: S, fn: (S,S) -> S ) ==
      empty? x => x
      x.value := fn(p, x.value)
      mapDown_!(x.left, x.value, fn)
      mapDown_!(x.right, x.value, fn)
      x
    mapDown_!(x: %, p: S, fn: (S,S,S) -> List S) ==
      empty? x => x
      x.value := p
      leaf? x => x
      u := fn(x.left.value, x.right.value, p)
      mapDown_!(x.left, u.1, fn)
      mapDown_!(x.right, u.2, fn)
      x
```

— BBTREE.dotabb —

```
"BBTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BBTREE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"BBTREE" -> "FLAGG"
```

3.2 domain BPADIC BalancedPAdicInteger

— BalancedPAdicInteger.input —

```
)set break resume
)sys rm -f BalancedPAdicInteger.output
)spool BalancedPAdicInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BalancedPAdicInteger
--R BalancedPAdicInteger p: Integer  is a domain constructor
--R Abbreviation for BalancedPAdicInteger is BPADIC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BPADIC
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R digits : % -> Stream Integer          extend : (%,Integer) -> %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R moduloP : % -> Integer                modulus : () -> Integer
--R one? : % -> Boolean                   order : % -> NonNegativeInteger
--R ?quo? : (%,%) -> %                    quotientByP : % -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R sqrt : (%,Integer) -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R root : (SparseUnivariatePolynomial Integer,Integer) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

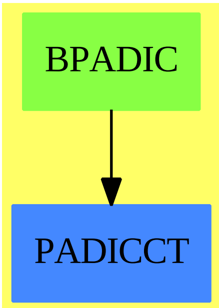
— **BalancedPAdicInteger.help** —

```
====================================================================
BalancedPAdicInteger examples
====================================================================

See Also:
o )show BalancedPAdicInteger
```

———-

3.2.1 BalancedPAdicInteger (BPADIC)



See

⇒ “InnerPAdicInteger” (IPADIC) 10.24.1 on page 1258
⇒ “PAdicInteger” (PADIC) 17.1.1 on page 1841
⇒ “PAdicRationalConstructor” (PADICRC) 17.3.1 on page 1850
⇒ “PAdicRational” (PADICRAT) 17.2.1 on page 1845
⇒ “BalancedPAdicRational” (BPADICRT) 3.3.1 on page 244

Exports:

0	1	approximate	associates?
characteristic	coerce	complete	digits
divide	euclideanSize	expressIdealMember	exquo
extend	extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm	moduloP
modulus	multiEuclidean	one?	order
principalIdeal	quotientByP	recip	root
sample	sizeLess?	sqrt	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?
?quo?	?rem?		

— domain BPADIC BalancedPAdicInteger —

```
)abbrev domain BPADIC BalancedPAdicInteger
```

```
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: p-adic, complementation
++ Examples:
++ References:
++ Description:
++ Stream-based implementation of Zp: p-adic numbers are represented as
++ sum(i = 0.., a[i] * p^i), where the a[i] lie in -(p - 1)/2,...,(p - 1)/2.

BalancedPAdicInteger(p:Integer) == InnerPAdicInteger(p,false$Boolean)
```

———-

— BPADIC.dotabb —

```
"BPADIC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BPADIC"]
"PADICCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PADICCT"]
"BPADIC" -> "PADICCT"
```

———-

3.3 domain BPADICRT BalancedPAdicRational

— BalancedPAdicRational.input —

```
)set break resume
)sys rm -f BalancedPAdicRational.output
)spool BalancedPAdicRational.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BalancedPAdicRational
--R BalancedPAdicRational p: Integer  is a domain constructor
--R Abbreviation for BalancedPAdicRational is BPADICRT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BPADICRT
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean       coerce : Fraction Integer -> %
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             denominator : % -> %
--R factor : % -> Factored %             gcd : List % -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R numerator : % -> %                   one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R removeZeroes : (Integer,%) -> %      removeZeroes : % -> %
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,BalancedPAdicInteger p) -> %
--R ?*? : (BalancedPAdicInteger p,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (BalancedPAdicInteger p,BalancedPAdicInteger p) -> %
--R ?<? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R ?<=? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R ?>? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R ?>=? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R D : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p)) -> %
--R D : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYI
--R D : (%,Symbol,NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,List Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if BalancedPAdicInteger p has DIFRING
--R D : % -> % if BalancedPAdicInteger p has DIFRING
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if BalancedPAdicInteger p has OINTDOM
--R approximate : (%,Integer) -> Fraction Integer
--R ceiling : % -> BalancedPAdicInteger p if BalancedPAdicInteger p has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and BalancedPAdicInteger p has PFECA
--R coerce : Symbol -> % if BalancedPAdicInteger p has RETRACT SYMBOL
--R coerce : BalancedPAdicInteger p -> %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and BalancedPAdicInteg
```

```
--R continuedFraction : % -> ContinuedFraction Fraction Integer
--R convert : % -> DoubleFloat if BalancedPAdicInteger p has REAL
--R convert : % -> Float if BalancedPAdicInteger p has REAL
--R convert : % -> InputForm if BalancedPAdicInteger p has KONVERT INFORM
--R convert : % -> Pattern Float if BalancedPAdicInteger p has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if BalancedPAdicInteger p has KONVERT PATTERN INT
--R denom : % -> BalancedPAdicInteger p
--R differentiate : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p)) -> %
--R differentiate : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SY
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if BalancedPAdicInteger p has DIFRING
--R differentiate : % -> % if BalancedPAdicInteger p has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has ELTAB(BPADIC p,BPADIC p)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has IEVALAB(SYMBOL,BPADIC p)
--R eval : (%,List Symbol,List BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has IEVALAB(SYMBOL
--R eval : (%,List Equation BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has EVALAB BPADIC p
--R eval : (%,Equation BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has EVALAB BPADIC p
--R eval : (%,BalancedPAdicInteger p,BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has EVALAB B
--R eval : (%,List BalancedPAdicInteger p,List BalancedPAdicInteger p) -> % if BalancedPAdicInteger p ha
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Balanced
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R floor : % -> BalancedPAdicInteger p if BalancedPAdicInteger p has INS
--R fractionPart : % -> % if BalancedPAdicInteger p has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R init : () -> % if BalancedPAdicInteger p has STEP
--R map : ((BalancedPAdicInteger p -> BalancedPAdicInteger p),%) -> %
--R max : (%,%) -> % if BalancedPAdicInteger p has ORDSET
--R min : (%,%) -> % if BalancedPAdicInteger p has ORDSET
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if BalancedPAdicInteger p has OINTDOM
--R nextItem : % -> Union(%,"failed") if BalancedPAdicInteger p has STEP
--R numer : % -> BalancedPAdicInteger p
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Balan
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R positive? : % -> Boolean if BalancedPAdicInteger p has OINTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if BalancedPAdicInteger p has INS
--R reducedSystem : Matrix % -> Matrix BalancedPAdicInteger p
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix BalancedPAdicInteger p,vec: Vector Balance
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if BalancedPA
--R reducedSystem : Matrix % -> Matrix Integer if BalancedPAdicInteger p has LINEXP INT
```

```
--R retract : % -> Integer if BalancedPAdicInteger p has RETRACT INT
--R retract : % -> Fraction Integer if BalancedPAdicInteger p has RETRACT INT
--R retract : % -> Symbol if BalancedPAdicInteger p has RETRACT SYMBOL
--R retract : % -> BalancedPAdicInteger p
--R retractIfCan : % -> Union(Integer,"failed") if BalancedPAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if BalancedPAdicInteger p has RETRAC
--R retractIfCan : % -> Union(Symbol,"failed") if BalancedPAdicInteger p has RETRACT SYMBOL
--R retractIfCan : % -> Union(BalancedPAdicInteger p,"failed")
--R sign : % -> Integer if BalancedPAdicInteger p has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R wholePart : % -> BalancedPAdicInteger p if BalancedPAdicInteger p has EUCDOM
--R
--E 1

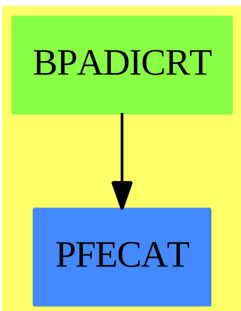
)spool
)lisp (bye)
```

— **BalancedPAdicRational.help** —

```
====================================================================
BalancedPAdicRational examples
====================================================================

See Also:
o )show BalancedPAdicRational
```

3.3.1 BalancedPAdicRational (BPADICRT)



See

⇒ "InnerPAdicInteger" (IPADIC) 10.24.1 on page 1258
⇒ "PAdicInteger" (PADIC) 17.1.1 on page 1841
⇒ "BalancedPAdicInteger" (BPADIC) 3.2.1 on page 240
⇒ "PAdicRationalConstructor" (PADICRC) 17.3.1 on page 1850
⇒ "PAdicRational" (PADICRAT) 17.2.1 on page 1845

Exports:

0	1	abs
approximate	associates?	ceiling
characteristic	charthRoot	coerce
conditionP	continuedFraction	convert
D	denom	denominator
differentiate	divide	?.?
euclideanSize	eval	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	floor
fractionPart	gcd	gcdPolynomial
hash	init	inv
latex	lcm	map
max	min	multiEuclidean
negative?	nextItem	numer
numerator	one?	patternMatch
positive?	prime?	principalIdeal
random	recip	reducedSystem
removeZeroes	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
zero?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?quo?
?rem?		

— domain BPADICRT BalancedPAdicRational —

```
)abbrev domain BPADICRT BalancedPAdicRational
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Keywords: p-adic, complementation
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ Stream-based implementation of Qp: numbers are represented as
++ sum(i = k.., a[i] * p^i), where the a[i] lie in -(p - 1)/2,...,(p - 1)/2.

BalancedPAdicRational(p:Integer) ==
  PAdicRationalConstructor(p,BalancedPAdicInteger p)
```

———-

— BPADICRT.dotabb —

```
"BPADICRT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BPADICRT"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"BPADICRT" -> "PFECAT"
```

———-

3.4 domain BFUNCT BasicFunctions

— BasicFunctions.input —

```
)set break resume
)sys rm -f BasicFunctions.output
)spool BasicFunctions.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BasicFunctions
--R BasicFunctions  is a domain constructor
--R Abbreviation for BasicFunctions is BFUNCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BFUNCT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                bfKeys : () -> List Symbol
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R bfEntry : Symbol -> Record(zeros: Stream DoubleFloat,ones: Stream DoubleFloat,singularit
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

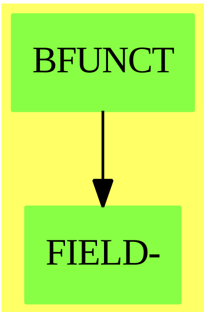
— BasicFunctions.help —

```
====================================================================
BasicFunctions examples
====================================================================

See Also:
o )show BasicFunctions
```

———-

3.4.1 BasicFunctions (BFUNCT)



Exports:

bfEntry	bfKeys	coerce	hash	latex
?~=?	?=?			

— domain BFUNCT BasicFunctions —

```
)abbrev domain BFUNCT BasicFunctions
++ Author: Brian Dupee
++ Date Created: August 1994
++ Date Last Updated: April 1996
++ Basic Operations: bfKeys, bfEntry
++ Description:
++ A Domain which implements a table containing details of
++ points at which particular functions have evaluation problems.
```

```
BasicFunctions():  E == I where
  DF  ==> DoubleFloat
  SDF ==> Stream DoubleFloat
  RS  ==> Record(zeros: SDF, ones: SDF, singularities: SDF)

  E ==> SetCategory with
    bfKeys:() -> List Symbol
      ++ bfKeys() returns the names of each function in the
      ++ \axiomType{BasicFunctions} table
    bfEntry:Symbol -> RS
      ++ bfEntry(k) returns the entry in the \axiomType{BasicFunctions} table
      ++ corresponding to \spad{k}
    finiteAggregate

  I ==> add

    Rep := Table(Symbol,RS)
    import Rep, SDF

    f(x:DF):DF ==
      positive?(x) => -x
      -x+1

    bf():$ ==
      import RS
      dpi := pi()$DF
      ndpi:SDF := map(x1+->x1*dpi,(z := generate(f,0))) -- [n pi for n in Z]
      n1dpi:SDF := map(x1+->-(2*(x1)-1)*dpi/2,z) -- [(n+1) pi /2]
      n2dpi:SDF := map(x1+->2*x1*dpi,z) -- [2 n pi for n in Z]
      n3dpi:SDF := map(x1+->-(4*(x1)-1)*dpi/4,z)
      n4dpi:SDF := map(x1+->-(4*(x1)-1)*dpi/2,z)
      sinEntry:RS := [ndpi, n4dpi, empty()$SDF]
      cosEntry:RS := [n1dpi, n2dpi, esdf := empty()$SDF]
      tanEntry:RS := [ndpi, n3dpi, n1dpi]
      asinEntry:RS := [construct([0$DF])$SDF,
                        construct([float(8414709848078965,-16,10)$DF]), esdf]
      acosEntry:RS := [construct([1$DF])$SDF,
                        construct([float(54030230586813977,-17,10)$DF]), esdf]
      atanEntry:RS := [construct([0$DF])$SDF,
                        construct([float(15574077246549023,-16,10)$DF]), esdf]
      secEntry:RS := [esdf, n2dpi, n1dpi]
      cscEntry:RS := [esdf, n4dpi, ndpi]
      cotEntry:RS := [n1dpi, n3dpi, ndpi]
      logEntry:RS := [construct([1$DF])$SDF,esdf, construct([0$DF])$SDF]
      entryList:List(Record(key:Symbol,entry:RS)) :=
         [[sin@Symbol, sinEntry], [cos@Symbol, cosEntry],
           [tan@Symbol, tanEntry], [sec@Symbol, secEntry],
            [csc@Symbol, cscEntry], [cot@Symbol, cotEntry],
             [asin@Symbol, asinEntry], [acos@Symbol, acosEntry],
```

```
            [atan@Symbol, atanEntry], [log@Symbol, logEntry]]
      construct(entryList)$Rep

    bfKeys():List Symbol == keys(bf())$Rep

    bfEntry(k:Symbol):RS == qelt(bf(),k)$Rep
```

— BFUNCT.dotabb —

```
"BFUNCT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BFUNCT"]
"FIELD-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FIELD"]
"BFUNCT" -> "FIELD-"
```

3.5 domain BOP BasicOperator

— BasicOperator.input —

```
)set break resume
)sys rm -f BasicOperator.output
)spool BasicOperator.output
)set message test on
)set message auto off
)clear all
--S 1 of 18
y := operator 'y
--R
--R
--R   (1)  y
--R                                                    Type: BasicOperator
--E 1

--S 2 of 18
deq := D(y x, x, 2) + D(y x, x) + y x = 0
--R
--R
--R          ,,       ,
--R   (2)  y  (x) + y (x) + y(x)= 0
--R
--R                                          Type: Equation Expression Integer
--E 2
```

```
--S 3 of 18
solve(deq, y, x)
--R
--R
--R                                              x      x
--R                                       +-+   - -    - -      +-+
--R                                     x\|3     2      2    x\|3
--R   (3)  [particular= 0,basis= [cos(-----)%e   ,%e   sin(-----)]]
--R                                       2                    2
--RType: Union(Record(particular: Expression Integer,basis: List Expression Integer),...)
--E 3

--S 4 of 18
nary? y
--R
--R
--R   (4)  true
--R                                                                  Type: Boolean
--E 4

--S 5 of 18
unary? y
--R
--R
--R   (5)  false
--R                                                                  Type: Boolean
--E 5

--S 6 of 18
opOne := operator('opOne, 1)
--R
--R
--R   (6)  opOne
--R                                                            Type: BasicOperator
--E 6

--S 7 of 18
nary? opOne
--R
--R
--R   (7)  false
--R                                                                  Type: Boolean
--E 7

--S 8 of 18
unary? opOne
--R
--R
--R   (8)  true
```

```
--R                                                          Type: Boolean
--E 8

--S 9 of 18
arity opOne
--R
--R
--R   (9)  1
--R                                         Type: Union(NonNegativeInteger,...)
--E 9

--S 10 of 18
name opOne
--R
--R
--R   (10)  opOne
--R                                                           Type: Symbol
--E 10

--S 11 of 18
is?(opOne, 'z2)
--R
--R
--R   (11)  false
--R                                                          Type: Boolean
--E 11

--S 12 of 18
is?(opOne, "opOne")
--R
--R
--R   (12)  true
--R                                                          Type: Boolean
--E 12

--S 13 of 18
properties y
--R
--R
--R   (13)  table()
--R                                         Type: AssociationList(String,None)
--E 13

--S 14 of 18
setProperty(y, "use", "unknown function" :: None )
--R
--R
--R   (14)  y
--R                                                    Type: BasicOperator
--E 14
```

```
--S 15 of 18
properties y
--R
--R
--R   (15)  table("use"= NONE)
--R                                          Type: AssociationList(String,None)
--E 15

--S 16 of 18
property(y, "use") :: None pretend String
--R
--R
--R   (16)  "unknown function"
--R                                                                Type: String
--E 16

--S 17 of 18
deleteProperty!(y, "use")
--R
--R
--R   (17)  y
--R                                                         Type: BasicOperator
--E 17

--S 18 of 18
properties y
--R
--R
--R   (18)  table()
--R                                          Type: AssociationList(String,None)
--E 18
)spool
)lisp (bye)
```

— BasicOperator.help —

```
====================================================================
BasicOperator examples
====================================================================

A basic operator is an object that can be symbolically applied to a
list of arguments from a set, the result being a kernel over that set
or an expression.

You create an object of type BasicOperator by using the operator
operation.  This first form of this operation has one argument and it
```

must be a symbol. The symbol should be quoted in case the name has been used as an identifier to which a value has been assigned.

A frequent application of BasicOperator is the creation of an operator to represent the unknown function when solving a differential equation.

Let y be the unknown function in terms of x.

```
  y := operator 'y
   y
                     Type: BasicOperator
```

This is how you enter the equation y'' + y' + y = 0.

```
  deq := D(y x, x, 2) + D(y x, x) + y x = 0
    ,,        ,
   y  (x) + y (x) + y(x) = 0
                     Type: Equation Expression Integer
```

To solve the above equation, enter this.

```
  solve(deq, y, x)
                                        x     x
                              +-+    - -   - -      +-+
                            x\|3      2     2    x\|3
   [particular= 0,basis= [cos(-----)%e   ,%e   sin(-----)]]
                              2                    2
                     Type: Union(Record(particular: Expression Integer,
                                 basis: List Expression Integer),...)
```

Use the single argument form of BasicOperator (as above) when you intend to use the operator to create functional expressions with an arbitrary number of arguments

Nary means an arbitrary number of arguments can be used in the functional expressions.

```
  nary? y
   true
                     Type: Boolean

  unary? y
   false
                     Type: Boolean
```

Use the two-argument form when you want to restrict the number of arguments in the functional expressions created with the operator.

This operator can only be used to create functional expressions with one argument.

```
  opOne := operator('opOne, 1)
   opOne
                     Type: BasicOperator

  nary? opOne
   false
                     Type: Boolean

  unary? opOne
   true
                     Type: Boolean
```

Use arity to learn the number of arguments that can be used. It returns "false" if the operator is nary.

```
  arity opOne
   1
                     Type: Union(NonNegativeInteger,...)
```

Use name to learn the name of an operator.

```
  name opOne
   opOne
                     Type: Symbol
```

Use is? to learn if an operator has a particular name.

```
  is?(opOne, 'z2)
   false
                     Type: Boolean
```

You can also use a string as the name to be tested against.

```
  is?(opOne, "opOne")
   true
                     Type: Boolean
```

You can attached named properties to an operator. These are rarely used at the top-level of the Axiom interactive environment but are used with Axiom library source code.

By default, an operator has no properties.

```
  properties y
   table()
                     Type: AssociationList(String,None)
```

The interface for setting and getting properties is somewhat awkward because the property values are stored as values of type None.

Attach a property by using setProperty.

```
  setProperty(y, "use", "unknown function" :: None )
   y
                    Type: BasicOperator

  properties y
   table("use"=NONE)
                    Type: AssociationList(String,None)
```

We know the property value has type String.

```
  property(y, "use") :: None pretend String
   "unknown function"
                    Type: String
```

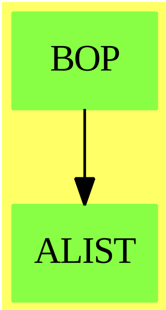
Use deleteProperty! to destructively remove a property.

```
 deleteProperty!(y, "use")
  y
                    Type: BasicOperator

  properties y
   table()
                    Type: AssociationList(String,None)
```

See Also
o)help Expression
o)help Kernel
o)show BasicOperator

3.5.1 BasicOperator (BOP)



Exports:

arity	assert	coerce	comparison	copy
deleteProperty!	display	display	display	equality
has?	hash	input	input	is?
latex	max	min	name	nary?
nullary?	operator	operator	properties	property
setProperties	setProperty	unary?	weight	weight
?~=?	?<?	?<=?	?=?	?>?
?>=?				

— domain BOP BasicOperator —

```
)abbrev domain BOP BasicOperator
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 11 October 1993
++ Keywords: operator, kernel.
++ Description:
++ Basic system operators.
++ A basic operator is an object that can be applied to a list of
++ arguments from a set, the result being a kernel over that set.

BasicOperator(): Exports == Implementation where
  O   ==> OutputForm
  P   ==> AssociationList(String, None)
  L   ==> List Record(key:String, entry:None)
  SEX ==> InputForm
-- some internal properties
  LESS?   ==> "%less?"
  EQUAL?  ==> "%equal?"
  WEIGHT  ==> "%weight"
  DISPLAY ==> "%display"
  SEXPR   ==> "%input"

  Exports ==> OrderedSet with
```

```
name      : $ -> Symbol
  ++ name(op) returns the name of op.
properties: $ -> P
  ++ properties(op) returns the list of all the properties
  ++ currently attached to op.
copy      : $ -> $
  ++ copy(op) returns a copy of op.
operator  : Symbol -> $
  ++ operator(f) makes f into an operator with arbitrary arity.
operator  : (Symbol, NonNegativeInteger) -> $
  ++ operator(f, n) makes f into an n-ary operator.
arity     : $ -> Union(NonNegativeInteger, "failed")
  ++ arity(op) returns n if op is n-ary, and
  ++ "failed" if op has arbitrary arity.
nullary?  : $ -> Boolean
  ++ nullary?(op) tests if op is nullary.
unary?    : $ -> Boolean
  ++ unary?(op) tests if op is unary.
nary?     : $ -> Boolean
  ++ nary?(op) tests if op has arbitrary arity.
weight    : $ -> NonNegativeInteger
  ++ weight(op) returns the weight attached to op.
weight    : ($, NonNegativeInteger) -> $
  ++ weight(op, n) attaches the weight n to op.
equality   : ($, ($, $) -> Boolean) -> $
  ++ equality(op, foo?) attaches foo? as the "%equal?" property
  ++ to op. If op1 and op2 have the same name, and one of them
  ++ has an "%equal?" property f, then \spad{f(op1, op2)} is called to
  ++ decide whether op1 and op2 should be considered equal.
comparison : ($, ($, $) -> Boolean) -> $
  ++ comparison(op, foo?) attaches foo? as the "%less?" property
  ++ to op. If op1 and op2 have the same name, and one of them
  ++ has a "%less?" property f, then \spad{f(op1, op2)} is called to
  ++ decide whether \spad{op1 < op2}.
display    : $ -> Union(List O -> O, "failed")
  ++ display(op) returns the "%display" property of op if
  ++ it has one attached, and "failed" otherwise.
display    : ($, List O -> O)      -> $
  ++ display(op, foo) attaches foo as the "%display" property
  ++ of op. If op has a "%display" property f, then \spad{op(a1,...,an)}
  ++ gets converted to OutputForm as \spad{f(a1,...,an)}.
display    : ($, O -> O)           -> $
  ++ display(op, foo) attaches foo as the "%display" property
  ++ of op. If op has a "%display" property f, then \spad{op(a)}
  ++ gets converted to OutputForm as \spad{f(a)}.
  ++ Argument op must be unary.
input      : ($, List SEX -> SEX)  -> $
  ++ input(op, foo) attaches foo as the "%input" property
  ++ of op. If op has a "%input" property f, then \spad{op(a1,...,an)}
  ++ gets converted to InputForm as \spad{f(a1,...,an)}.
```

```
    input      : $ -> Union(List SEX -> SEX, "failed")
      ++ input(op) returns the "%input" property of op if
      ++ it has one attached, "failed" otherwise.
    is?        : ($, Symbol) -> Boolean
      ++ is?(op, s) tests if the name of op is s.
    has?       : ($, String) -> Boolean
      ++ has?(op, s) tests if property s is attached to op.
    assert     : ($, String) -> $
      ++ assert(op, s) attaches property s to op.
      ++ Argument op is modified "in place", i.e. no copy is made.
    deleteProperty_!: ($, String) -> $
      ++ deleteProperty!(op, s) unattaches property s from op.
      ++ Argument op is modified "in place", i.e. no copy is made.
    property      : ($, String) -> Union(None, "failed")
      ++ property(op, s) returns the value of property s if
      ++ it is attached to op, and "failed" otherwise.
    setProperty   : ($, String, None) -> $
      ++ setProperty(op, s, v) attaches property s to op,
      ++ and sets its value to v.
      ++ Argument op is modified "in place", i.e. no copy is made.
    setProperties : ($, P) -> $
      ++ setProperties(op, l) sets the property list of op to l.
      ++ Argument op is modified "in place", i.e. no copy is made.

  Implementation ==> add
    -- if narg < 0 then the operator has variable arity.
    Rep := Record(opname:Symbol, narg:SingleInteger, props:P)

    oper: (Symbol, SingleInteger, P) -> $

    is?(op, s)           == name(op) = s
    name op              == op.opname
    properties op        == op.props
    setProperties(op, l) == (op.props := l; op)
    operator s           == oper(s, -1::SingleInteger, table())
    operator(s, n)       == oper(s, n::Integer::SingleInteger, table())
    property(op, name)   == search(name, op.props)
    assert(op, s)        == setProperty(op, s, NIL$Lisp)
    has?(op, name)       == key?(name, op.props)
    oper(se, n, prop)    == [se, n, prop]
    weight(op, n)        == setProperty(op, WEIGHT, n pretend None)
    nullary? op          == zero?(op.narg)
--    unary? op            == one?(op.narg)
    unary? op            == ((op.narg) = 1)
    nary? op             == negative?(op.narg)
    equality(op, func)   == setProperty(op, EQUAL?, func pretend None)
    comparison(op, func) == setProperty(op, LESS?, func pretend None)
    display(op:$, f:O -> O)        == display(op,(x1:List(O)):O +-> f first x1)
    deleteProperty_!(op, name)     == (remove_!(name, properties op); op)
    setProperty(op, name, valu)    == (op.props.name := valu; op)
```

```
    coerce(op:$):OutputForm        == name(op)::OutputForm
    input(op:$, f:List SEX -> SEX) == setProperty(op, SEXPR, f pretend None)
    display(op:$, f:List O -> O)   == setProperty(op, DISPLAY, f pretend None)

    display op ==
      (u := property(op, DISPLAY)) case "failed" => "failed"
      (u::None) pretend (List O -> O)

    input op ==
      (u := property(op, SEXPR)) case "failed" => "failed"
      (u::None) pretend (List SEX -> SEX)

    arity op ==
      negative?(n := op.narg) => "failed"
      convert(n)@Integer :: NonNegativeInteger

    copy op ==
      oper(name op, op.narg,
          table([[r.key, r.entry] for r in entries(properties op)@L]$L))

-- property EQUAL? contains a function f: (BOP, BOP) -> Boolean
-- such that f(o1, o2) is true iff o1 = o2
    op1 = op2 ==
      (EQ$Lisp)(op1, op2) => true
      name(op1) ^= name(op2) => false
      op1.narg ^= op2.narg => false
      brace(keys properties op1)^=$Set(String) _
                     brace(keys properties op2) => false
      (func := property(op1, EQUAL?)) case None =>
                   ((func::None) pretend (($, $) -> Boolean)) (op1, op2)
      true

-- property WEIGHT allows one to change the ordering around
-- by default, every operator has weigth 1
    weight op ==
      (w := property(op, WEIGHT)) case "failed" => 1
      (w::None) pretend NonNegativeInteger

-- property LESS? contains a function f: (BOP, BOP) -> Boolean
-- such that f(o1, o2) is true iff o1 < o2
    op1 < op2 ==
      (w1 := weight op1) ^= (w2 := weight op2) => w1 < w2
      op1.narg ^= op2.narg => op1.narg < op2.narg
      name(op1) ^= name(op2) => name(op1) < name(op2)
      n1 := #(k1 := brace(keys(properties op1))$Set(String))
      n2 := #(k2 := brace(keys(properties op2))$Set(String))
      n1 ^= n2 => n1 < n2
      not zero?(n1 := #(d1 := difference(k1, k2))) =>
        n1 ^= (n2 := #(d2 := difference(k2, k1))) => n1 < n2
        inspect(d1) < inspect(d2)
```

```
      (func := property(op1, LESS?)) case None =>
                   ((func::None) pretend (($, $) -> Boolean)) (op1, op2)
      (func := property(op1, EQUAL?)) case None =>
              not(((func::None) pretend (($, $) -> Boolean)) (op1, op2))
      false
```

———-

— BOP.dotabb —

```
"BOP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BOP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"BOP" -> "ALIST"
```

———-

3.6 domain BSD BasicStochasticDifferential

— BasicStochasticDifferential.input —

```
)set break resume
)sys rm -f BasicStochasticDifferential.output
)spool BasicStochasticDifferential.output
)set message test on
)set message auto off
)clear all

--S 1 of 30
print copyBSD()
--R
--R   []
--R                                                                   Type: Void
--E 1

--S 2 of 30
print copyIto()
--R
--R   table()
--R                                                                   Type: Void
--E 2

--S 3 of 30
dX:=introduce!(X,dX) -- dX is a new stochastic differential
--R
```

```
--R
--R   (3)  dX
--R                              Type: Union(BasicStochasticDifferential,...)
--E 3

--S 4 of 30
print copyBSD()
--R
--R   [dX]
--R                                                                  Type: Void
--E 4

--S 5 of 30
print copyIto()
--R
--R   table(X= dX)
--R                                                                  Type: Void
--E 5

--S 6 of 30
getSmgl(dX)
--R
--R
--R   (6)  X
--R                                                    Type: Union(Symbol,...)
--E 6

--S 7 of 30
print dX
--R
--R   dX
--R                                                                  Type: Void
--E 7

--S 8 of 30
print domainOf(dX)
--R
--R   BasicStochasticDifferential()
--R                                                                  Type: Void
--E 8

--S 9 of 30
print d X
--R
--R   dX
--R                                                                  Type: Void
--E 9

--S 10 of 30
print ((d$BSD) X)
```

```
--R
--R   dX
--R                                                              Type: Void
--E 10

--S 11 of 30
print domainOf((d$BSD) X)
--R
--R   BasicStochasticDifferential()
--R                                                              Type: Void
--E 11

--S 12 of 30
introduce!(t,dt) -- dt is a new stochastic differential
--R
--R
--R   (12)  dt
--R                                Type: Union(BasicStochasticDifferential,...)
--E 12

--S 13 of 30
print copyBSD()
--R
--R   [dX,dt]
--R                                                              Type: Void
--E 13

--S 14 of 30
print copyIto()
--R
--R   table(t= dt,X= dX)
--R                                                              Type: Void
--E 14

--S 15 of 30
introduce!(t,dt) -- fails (can't repeat an introduction!)
--R
--R
--R   (15)  "failed"
--R                                                Type: Union("failed",...)
--E 15

--S 16 of 30
print copyBSD()
--R
--R   [dX,dt]
--R                                                              Type: Void
--E 16

--S 17 of 30
```

```
print copyIto()
--R
--R   table(t= dt,X= dX)
--R                                                                  Type: Void
--E 17

--S 18 of 30
'dZ::BSD           -- can fail
--R
--R   dZ is not a stochastic differential
--R
--RDaly Bug
--R   >> Error detected within library code:
--R   above causes failure in convert$BSD
--R
--R   Continuing to read the file...
--R
--E 18

--S 19 of 30
dZ::BSD            -- fails
--R
--R   dZ is not a stochastic differential
--R
--RDaly Bug
--R   >> Error detected within library code:
--R   above causes failure in convert$BSD
--R
--R   Continuing to read the file...
--R
--E 19

--S 20 of 30
string(dY)::BSD    -- fails
--R
--R
--RDaly Bug
--R   Cannot convert from type String to BasicStochasticDifferential for
--R      value
--R   "dY"
--R
--E 20

--S 21 of 30
print domainOf(dt)
--R
--R   Variable(dt)
--R                                                                  Type: Void
--E 21
```

```
--S 22 of 30
dt::BSD           -- succeeds!
--R
--R
--R   (19)  dt
--R                                        Type: BasicStochasticDifferential
--E 22

--S 23 of 30
getSmgl(dt::BSD)
--R
--R
--R   (20)  t
--R                                                  Type: Union(Symbol,...)
--E 23

--S 24 of 30
dX::Symbol        -- succeeds
--R
--R
--R   (21)  dX
--R                                                             Type: Symbol
--E 24

--S 25 of 30
string(dX)        -- fails
--R
--R   There are 3 exposed and 1 unexposed library operations named string
--R      having 1 argument(s) but none was determined to be applicable.
--R      Use HyperDoc Browse, or issue
--R                              )display op string
--R      to learn more about the available operations. Perhaps
--R      package-calling the operation or using coercions on the arguments
--R      will allow you to apply the operation.
--R
--RDaly Bug
--R   Cannot find a definition or applicable library operation named
--R      string with argument type(s)
--R                          BasicStochasticDifferential
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--E 25

--S 26 of 30
string(dX::Symbol) -- succeeds
--R
--R
--R   (22)  "dX"
--R                                                             Type: String
```

```
--E 26

--S 27 of 30
[introduce!(A[i],dA[i]) for i in 1..2]
--R
--R
--R   (23)  [dA ,dA ]
--R           1   2
--R                       Type: List Union(BasicStochasticDifferential,"failed")
--E 27

--S 28 of 30
print copyBSD()
--R
--R   [dA ,dA ,dX,dt]
--R      1   2
--R                                                                  Type: Void
--E 28

--S 29 of 30
print copyIto()
--R
--R   table(A = dA ,A = dA ,t= dt,X= dX)
--R          2    2  1    1
--R                                                                  Type: Void
--E 29

--S 30 of 30
[d A[i] for i in 1..2]
--R
--R
--R   (26)  [dA ,dA ]
--R           1   2
--R                        Type: List Union(BasicStochasticDifferential,Integer)
--E 30


)spool
)lisp (bye)
```

———-

— BasicStochasticDifferential.help —

```
====================================================================
BasicStochasticDifferential examples
====================================================================

Based on Symbol: a domain of symbols representing basic stochastic
```

differentials, used in StochasticDifferential(R) in the underlying sparse multivariate polynomial representation.

We create new BSD only by coercion from Symbol using a special function introduce! first of all to add to a private set SDset. We allow a separate function convertIfCan which will check whether the argument has previously been declared as a BSD.

The copyBSD() returns setBSD as a list of BSD.
Initially the BSD table of symbols is empty:

```
   print copyBSD()
      []
```

as is the Ito table. The copyIto() returns the table relating semimartingales to basic stochastic differentials.

```
   print copyIto()
      table()
```

We introduce a new stochastic differential:

```
   dX:=introduce!(X,dX) -- dX is a new stochastic differential
      dX
```

and now it is in the BSD table of symbols:

```
   print copyBSD()
      [dX]
```

and the Ito table:

```
   print copyIto()
      table(X= dX)
```

The getSmgl(bsd) returns the semimartingale S related to the basic stochastic differential bsd by introduce!:

```
   getSmgl(dX)
      X
```

Note that the dX symbol is of type BasicStochasticDifferential:

```
   print domainOf(dX)
     BasicStochasticDifferential()
```

The d function, d(X) returns dX if tableIto(X)=dX otherwise 0:

```
   print d X
      dX
```

and we can specify the domain:

```
   print ((d$BSD) X)
      dX
```

which clearly is of domain BasicStochasticDifferential

```
   print domainOf((d$BSD) X)
```

```
      BasicStochasticDifferential()

The introduce!(X,dX) returns dX as BSD if it isn't already in BSD
   introduce!(t,dt) -- dt is a new stochastic differential
       dt

Now we can see that dt is in the BSD table:
   print copyBSD()
       [dX,dt]

and the Ito table:
   print copyIto()
       table(t= dt,X= dX)

You cannot repeat an introduce! for an existing entry:
   introduce!(t,dt)
       "failed"

Regular symbols can be added, especially with subscripts:
   [introduce!(A[i],dA[i]) for i in 1..2]
            [dA ,dA ]
               1   2

And we can see them in the BSD table:
   print copyBSD()
      [dA ,dA ,dX,dt]
         1   2

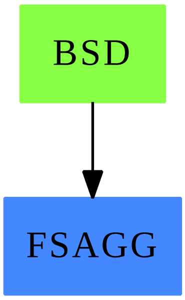
and the Ito table:
   print copyIto()
      table(A = dA ,A = dA ,t= dt,X= dX)
             2    2  1    1

The d function can extract these symbols:
   [d A[i] for i in 1..2]
       [dA ,dA ]
          1   2

See Also:
o )show BasicStochasticDifferential
```

———-

3.6.1 BasicStochasticDifferential (BSD)



See

⇒ "StochasticDifferential" (SD) 20.29.1 on page 2530

Exports:

coerce	convert	convertIfCan	copyBSD
copyIto	d	getSmgl	hash
introduce!	latex	max	min
?~=?	?<?	?<=?	?=?
?>?	?>=?		

— domain BSD BasicStochasticDifferential —

```
)abbrev domain BSD BasicStochasticDifferential
++ Author: Wilfrid S. Kendall
++ Date Last Updated: July 26, 1999
++ Related Domains: StochasticDifferential(R)
++ Keywords: stochastic differential, semimartingale.
++ References: Ito (1975), Kendall (1991a,b; 1993a,b; 1999a,b).
++ Description:
++ Based on Symbol: a domain of symbols representing basic stochastic
++ differentials, used in StochasticDifferential(R) in the underlying
++ sparse multivariate polynomial representation.
++
++ We create new BSD only by coercion from Symbol using a special
++ function introduce! first of all to add to a private set SDset.
++ We allow a separate function convertIfCan which will check whether the
++ argument has previously been declared as a BSD.

BasicStochasticDifferential(): Category == Implementation where
 INT ==> Integer
 OF  ==> OutputForm
 Category ==> OrderedSet with
  ConvertibleTo(Symbol)

  convertIfCan: Symbol -> Union(%, "failed")
   ++ convertIfCan(ds) transforms \axiom{dX} into a \axiom{BSD}
```

```
    ++ if possible (if \axiom{introduce(X,dX)} has
    ++ been invoked previously).

   convert: Symbol -> %
    ++ convert(dX) transforms \axiom{dX} into a \axiom{BSD}
    ++ if possible and otherwise produces an error.

   introduce!: (Symbol,Symbol) -> Union(%, "failed")
    ++ introduce!(X,dX) returns \axiom{dX} as \axiom{BSD} if it
    ++ isn't already in \axiom{BSD}
    ++
    ++X introduce!(t,dt) -- dt is a new stochastic differential
    ++X copyBSD()

   d: Symbol -> Union(%,INT)
    ++ d(X) returns \axiom{dX} if \axiom{tableIto(X)=dX}
    ++ and otherwise returns \axiom{0}

   copyBSD:() -> List %
    ++ copyBSD() returns \axiom{setBSD} as a list of \axiom{BSD}.
    ++
    ++X introduce!(t,dt) -- dt is a new stochastic differential
    ++X copyBSD()

   copyIto:() -> Table(Symbol,%)
    ++ copyIto() returns the table relating semimartingales
    ++ to basic stochastic differentials.
    ++
    ++X introduce!(t,dt) -- dt is a new stochastic differential
    ++X copyIto()

   getSmgl: % -> Union(Symbol,"failed")
    ++ getSmgl(bsd) returns the semimartingale \axiom{S} related
    ++ to the basic stochastic differential \axiom{bsd} by
    ++ \axiom{introduce!}
    ++
    ++X introduce!(t,dt) -- dt is a new stochastic differential
    ++X getSmgl(dt::BSD)

  Implementation ==> Symbol add

   Rep := Symbol

   setBSD := empty()$Set(Symbol)
   tableIto:Table(Symbol,%) := table()
   tableBSD:Table(%,Symbol) := table()

   convertIfCan(ds:Symbol):Union(%,"failed") ==
    not(member?(ds,setBSD)) => "failed"
    ds::%
```

```
  convert(ds:Symbol):% ==
   (du:=convertIfCan(ds))
    case "failed" =>
     print(hconcat(ds::Symbol::OF,
       message(" is not a stochastic differential")$OF))
     error "above causes failure in convert$BSD"
   du

  introduce!(X,dX) ==
   member?(dX,setBSD) => "failed"
   insert!(dX,setBSD)
   tableBSD(dX::%) := X
   tableIto(X) := dX::%

  d(X) ==
   search(X,tableIto) case "failed" => 0::INT
   tableIto(X)

  copyBSD() == [ds::% for ds in members(setBSD)]
  copyIto() == tableIto
  getSmgl(ds:%):Union(Symbol,"failed") == tableBSD(ds)
```

— **BSD.dotabb** —

```
"BSD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BSD"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"BSD" -> "FSAGG"
```

3.7 domain BINARY BinaryExpansion

— **BinaryExpansion.input** —

```
)set break resume
)sys rm -f BinaryExpansion.output
)spool BinaryExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
```

```
r := binary(22/7)
--R
--R
--R           ---
--R   (1)  11.001
--R                                                        Type: BinaryExpansion
--E 1

--S 2 of 7
r + binary(6/7)
--R
--R
--R   (2)  100
--R                                                        Type: BinaryExpansion
--E 2

--S 3 of 7
[binary(1/i) for i in 102..106]
--R
--R
--R   (3)
--R       --------    ---------------------------------------------------
--R   [0.000000101, 0.000000100111110001000101100101111001110010010101001,
--R         ------------    ------------
--R    0.000000100111011, 0.000000100111,
--R       ----------------------------------------------------
--R    0.000000100110101001000011100111110110010101101111100011]
--R                                                   Type: List BinaryExpansion
--E 3

--S 4 of 7
binary(1/1007)
--R
--R
--R   (4)
--R   0.
--R     OVERBAR
--R        000000000100000100010100100101111000001111110000101111110010110001111100
--R          1000100111001001100110001100100101010111101101001100000000110000110001
--R          1110111000110100010111101001000111101100001010111011100111010101110001
--R          1001010010111000000001110001111001000000100100100110111001010100111010
--R          0011011101101011100010010000011001011011000000101100101111100010100000
--R          0101010101101011000001101101110100101011111110101110101001100100000101
--R          00110110001001100010001000010000110001110100111100001
--R                                                        Type: BinaryExpansion
--E 4

--S 5 of 7
p := binary(1/4)*x**2 + binary(2/3)*x + binary(4/9)
--R
```

```
--R
--R              2     --      ------
--R   (5)  0.01x  + 0.10x + 0.011100
--R                                              Type: Polynomial BinaryExpansion
--E 5

--S 6 of 7
q := D(p, x)
--R
--R
--R                     --
--R   (6)  0.1x + 0.10
--R                                              Type: Polynomial BinaryExpansion
--E 6

--S 7 of 7
g := gcd(p, q)
--R
--R
--R                  --
--R   (7)  x + 1.01
--R                                              Type: Polynomial BinaryExpansion
--E 7
)spool
)lisp (bye)
```

— **BinaryExpansion.help** —

```
====================================================================
BinaryExpansion examples
====================================================================
All rational numbers have repeating binary expansions.  Operations to
access the individual bits of a binary expansion can be obtained by
converting the value to RadixExpansion(2).  More examples of
expansions are available with

The expansion (of type BinaryExpansion) of a rational number is
returned by the binary operation.

  r := binary(22/7)
      ---
   11.001
                      Type: BinaryExpansion

Arithmetic is exact.

  r + binary(6/7)
```

```
   100
                     Type: BinaryExpansion
```

The period of the expansion can be short or long.

```
  [binary(1/i) for i in 102..106]
```

```
      --------
   [0.00000101,
      ---------------------------------------------------
    0.000000100111111000100010110010111100111001001010100 1,
```

```
       ------------    ------------
    0.000000100111011, 0.000000100111,
       ----------------------------------------------------
    0.00000010011010100100001110011111011001010110111100011]
                     Type: List BinaryExpansion
```

or very long.

```
  binary(1/1007)
```

```
     ------------------------------------------------------------------------
   0.000000000100000100010100100101111000001111110000101111110010110001111101
     ------------------------------------------------------------------------
     000100111001001100110001100100101010111101101001100000000110000110011110
     ------------------------------------------------------------------------
     111000110100010111101001000111101100001010111011100111010101110011001010
     ------------------------------------------------------------------------
     010111000000011100011110010000000100100100110111001010100111010001101110 1
     ------------------------------------------------------------------------
     101011100010010000011001011011000000101100101111100010100000101010101101
     ------------------------------------------------------------------------
     011000001101101110100101011111110101110101001100100001010011011000100110
     ------------------------------------
     001000100001000011000111010011110001
                     Type: BinaryExpansion
```

These numbers are bona fide algebraic objects.

```
  p := binary(1/4)*x**2 + binary(2/3)*x + binary(4/9)
```

```
             --       ------
   0.01 x^2 +0.10 x + 0.011100
                     Type: Polynomial BinaryExpansion
```

```
  q := D(p, x)
```

```
            --
   0.1 x + 0.10
                     Type: Polynomial BinaryExpansion
```

```
  g := gcd(p, q)
```

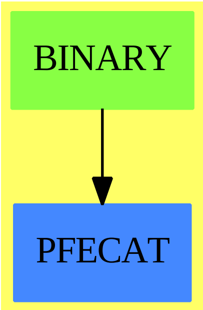
```
       --
   x+1.01
```

```
                      Type: Polynomial BinaryExpansion

See Also:
o )help DecimalExpansion
o )help HexadecimalExpansion
o )show BinaryExpansion
```

———-

3.7.1 BinaryExpansion (BINARY)



See

⇒ "RadixExpansion" (RADIX) 19.2.1 on page 2165
⇒ "DecimalExpansion" (DECIMAL) 5.3.1 on page 451
⇒ "HexadecimalExpansion" (HEXADEC) 9.3.1 on page 1108

Exports:

0	1	abs
associates?	binary	ceiling
characteristic	charthRoot	coerce
conditionP	convert	D
denom	denominator	differentiate
divide	euclideanSize	eval
expressIdealMember	exquo	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractionPart	gcd
gcdPolynomial	hash	init
inv	latex	lcm
map	max	min
multiEuclidean	negative?	nextItem
numer	numerator	one?
patternMatch	positive?	prime?
principalIdeal	random	recip
reducedSystem	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
zero?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?.?
?quo?	?rem?	

— domain BINARY BinaryExpansion —

```
)abbrev domain BINARY BinaryExpansion
++ Author: Clifton J. Williamson
++ Date Created: April 26, 1990
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains: RadixExpansion
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, binary
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ binary expansions.

BinaryExpansion(): Exports == Implementation where
  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
```

```
      ++ coerce(b) converts a binary expansion to a rational number.
    coerce: % -> RadixExpansion(2)
      ++ coerce(b) converts a binary expansion to a radix expansion with base 2.
    fractionPart: % -> Fraction Integer
      ++ fractionPart(b) returns the fractional part of a binary expansion.
    binary: Fraction Integer -> %
      ++ binary(r) converts a rational number to a binary expansion.
      ++
      ++X binary(22/7)

  Implementation ==> RadixExpansion(2) add
    binary r == r :: %
    coerce(x:%): RadixExpansion(2) == x pretend RadixExpansion(2)
```

— BINARY.dotabb —

```
"BINARY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BINARY"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"BINARY" -> "PFECAT"
```

3.8 domain BINFILE BinaryFile

— BinaryFile.input —

```
)set break resume
)sys rm -f BinaryFile.output
)spool BinaryFile.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryFile
--R BinaryFile  is a domain constructor
--R Abbreviation for BinaryFile is BINFILE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BINFILE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                close! : % -> %
```

```
--R coerce : % -> OutputForm              flush : % -> Void
--R hash : % -> SingleInteger             iomode : % -> String
--R latex : % -> String                   name : % -> FileName
--R open : (FileName,String) -> %         open : FileName -> %
--R position : % -> SingleInteger         read! : % -> SingleInteger
--R reopen! : (%,String) -> %             ?~=? : (%,%) -> Boolean
--R position! : (%,SingleInteger) -> SingleInteger
--R readIfCan! : % -> Union(SingleInteger,"failed")
--R write! : (%,SingleInteger) -> SingleInteger
--R
--E 1

)spool
)lisp (bye)
```

———–

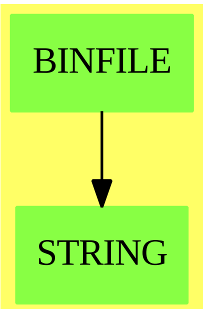
— **BinaryFile.help** —

```
====================================================================
BinaryFile examples
====================================================================

See Also:
o )show BinaryFile
```

———–

3.8.1 BinaryFile (BINFILE)



See

⇒ "File" (FILE) 7.2.1 on page 770
⇒ "TextFile" (TEXTFILE) 21.5.1 on page 2651
⇒ "KeyedAccessFile" (KAFILE) 12.2.1 on page 1377
⇒ "Library" (LIB) 13.2.1 on page 1392

Exports:

close!	coerce	hash	iomode	latex
name	open	position	position!	read!
readIfCan!	reopen!	write!	?=?	?~=?

— domain BINFILE BinaryFile —

```
)abbrev domain BINFILE BinaryFile
++ Author: Barry M. Trager
++ Date Created: 1993
++ Date Last Updated:
++ Basic Operations: writeByte! readByte! readByteIfCan!
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain provides an implementation of binary files. Data is
++ accessed one byte at a time as a small integer.

BinaryFile: Cat == Def where

    Cat == FileCategory(FileName, SingleInteger) with
        readIfCan_!: % -> Union(SingleInteger, "failed")
            ++ readIfCan!(f) returns a value from the file f, if possible.
            ++ If f is not open for reading, or if f is at the end of file
            ++ then \spad{"failed"} is the result.

--      "#": % -> SingleInteger
--          ++ #(f) returns the length of the file f in bytes.

        position: % -> SingleInteger
            ++ position(f) returns the current byte-position in the file f.

        position_!: (%, SingleInteger) -> SingleInteger
            ++ position!(f, i) sets the current byte-position to i.

    Def == File(SingleInteger) add
        FileState ==> SExpression

        Rep := Record(fileName:   FileName,    _
                      fileState:  FileState,   _
                      fileIOmode: String)

--      direc : Symbol := INTERN("DIRECTION","KEYWORD")$Lisp
--      input : Symbol := INTERN("INPUT","KEYWORD")$Lisp
--      output : Symbol := INTERN("OUTPUT","KEYWORD")$Lisp
--      eltype : Symbol := INTERN("ELEMENT-TYPE","KEYWORD")$Lisp
--      bytesize : SExpression := LIST(QUOTE(UNSIGNED$Lisp)$Lisp,8)$Lisp
```

```
        defstream(fn: FileName, mode: String): FileState ==
            mode = "input"  =>
              not readable? fn => error ["File is not readable", fn]
              BINARY__OPEN__INPUT(fn::String)$Lisp
--            OPEN(fn::String, direc, input, eltype, bytesize)$Lisp
            mode = "output" =>
              not writable? fn => error ["File is not writable", fn]
              BINARY__OPEN__OUTPUT(fn::String)$Lisp
--            OPEN(fn::String, direc, output, eltype, bytesize)$Lisp
            error ["IO mode must be input or output", mode]

        open(fname, mode) ==
            fstream := defstream(fname, mode)
            [fname, fstream, mode]

        reopen_!(f, mode) ==
            fname := f.fileName
            f.fileState := defstream(fname, mode)
            f.fileIOmode:= mode
            f

        close_! f ==
            f.fileIOmode = "output" =>
                 BINARY__CLOSE__OUTPUT()$Lisp
                 f
            f.fileIOmode = "input" =>
                  BINARY__CLOSE__INPUT()$Lisp
                  f
            error "file must be in read or write state"

        read! f ==
            f.fileIOmode ^= "input"  => error "File not in read state"
            BINARY__SELECT__INPUT(f.fileState)$Lisp
            BINARY__READBYTE()$Lisp
--          READ_-BYTE(f.fileState)$Lisp
        readIfCan_! f ==
            f.fileIOmode ^= "input"  => error "File not in read state"
            BINARY__SELECT__INPUT(f.fileState)$Lisp
            n:SingleInteger:=BINARY__READBYTE()$Lisp
            n = -1 => "failed"
            n::Union(SingleInteger,"failed")
--          READ_-BYTE(f.fileState,NIL$Lisp,
--                   "failed"::Union(SingleInteger,"failed"))$Lisp
        write_!(f, x) ==
            f.fileIOmode ^= "output" => error "File not in write state"
            x < 0 or x>255 => error "integer cannot be represented as a byte"
            BINARY__PRINBYTE(x)$Lisp
--          WRITE_-BYTE(x, f.fileState)$Lisp
```

```
            x

--      # f == FILE_-LENGTH(f.fileState)$Lisp
        position f ==
           f.fileIOmode ^= "input"  => error "file must be in read state"
           FILE_-POSITION(f.fileState)$Lisp
        position_!(f,i) ==
           f.fileIOmode ^= "input"  => error "file must be in read state"
           (FILE_-POSITION(f.fileState,i)$Lisp ; i)
```

— **BINFILE.dotabb** —

```
"BINFILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BINFILE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"BINFILE" -> "STRING"
```

3.9 domain BSTREE BinarySearchTree

— **BinarySearchTree.input** —

```
)set break resume
)sys rm -f BinarySearchTree.output
)spool BinarySearchTree.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
lv := [8,3,5,4,6,2,1,5,7]
--R
--R
--R   (1)  [8,3,5,4,6,2,1,5,7]
--R                                                   Type: List PositiveInteger
--E 1

--S 2 of 12
t := binarySearchTree lv
--R
--R
--R   (2)  [[[1,2,.],3,[4,5,[5,6,7]]],8,.]
--R                                             Type: BinarySearchTree PositiveInteger
```

```
--E 2

--S 3 of 12
emptybst := empty()$BSTREE(INT)
--R
--R
--R   (3)  []
--R                                              Type: BinarySearchTree Integer
--E 3

--S 4 of 12
t1 := insert!(8,emptybst)
--R
--R
--R   (4)  8
--R                                              Type: BinarySearchTree Integer
--E 4

--S 5 of 12
insert!(3,t1)
--R
--R
--R   (5)  [3,8,.]
--R                                              Type: BinarySearchTree Integer
--E 5

--S 6 of 12
leaves t
--R
--R
--R   (6)  [1,4,5,7]
--R                                                  Type: List PositiveInteger
--E 6

--S 7 of 12
split(3,t)
--R
--R
--R   (7)  [less= [1,2,.],greater= [[.,3,[4,5,[5,6,7]]],8,.]]
--RType: Record(less: BinarySearchTree PositiveInteger,greater: BinarySearchTree PositiveInteger)
--E 7

--S 8 of 12
insertRoot: (INT,BSTREE INT) -> BSTREE INT
--R
--R                                                                   Type: Void
--E 8

--S 9 of 12
insertRoot(x, t) ==
```

```
    a := split(x, t)
    node(a.less, x, a.greater)
--R
--R                                                                   Type: Void
--E 9

--S 10 of 12
buildFromRoot ls == reduce(insertRoot,ls,emptybst)
--R
--R                                                                   Type: Void
--E 10

--S 11 of 12
rt := buildFromRoot reverse lv
--R
--R   Compiling function buildFromRoot with type List PositiveInteger ->
--R      BinarySearchTree Integer
--R   Compiling function insertRoot with type (Integer,BinarySearchTree
--R      Integer) -> BinarySearchTree Integer
--R
--R   (11)  [[[1,2,.],3,[4,5,[5,6,7]]],8,.]
--R                                            Type: BinarySearchTree Integer
--E 11

--S 12 of 12
(t = rt)@Boolean
--R
--R
--R   (12)  true
--R                                                                Type: Boolean
--E 12
)spool
)lisp (bye)
```

— **BinarySearchTree.help** —

```
====================================================================
BinarySearchTree examples
====================================================================

BinarySearchTree(R) is the domain of binary trees with elements of
type R, ordered across the nodes of the tree.  A non-empty binary
search tree has a value of type R, and right and left binary search
subtrees.  If a subtree is empty, it is displayed as a period (".").

Define a list of values to be placed across the tree.  The resulting
tree has 8 at the root; all other elements are in the left subtree.
```

```
  lv := [8,3,5,4,6,2,1,5,7]
   [8, 3, 5, 4, 6, 2, 1, 5, 7]
                      Type: List PositiveInteger
```

A convenient way to create a binary search tree is to apply the operation binarySearchTree to a list of elements.

```
  t := binarySearchTree lv
   [[[1, 2, .], 3, [4, 5, [5, 6, 7]]], 8, .]
                      Type: BinarySearchTree PositiveInteger
```

Another approach is to first create an empty binary search tree of integers.

```
  emptybst := empty()$BSTREE(INT)
   []
                      Type: BinarySearchTree Integer
```

Insert the value 8. This establishes 8 as the root of the binary search tree. Values inserted later that are less than 8 get stored in the left subtree, others in the right subtree.

```
  t1 := insert!(8,emptybst)
   8
                      Type: BinarySearchTree Integer
```

Insert the value 3. This number becomes the root of the left subtree of t1. For optimal retrieval, it is thus important to insert the middle elements first.

```
  insert!(3,t1)
   [3, 8, .]
                      Type: BinarySearchTree Integer
```

We go back to the original tree t. The leaves of the binary search tree are those which have empty left and right subtrees.

```
  leaves t
   [1, 4, 5, 7]
                      Type: List PositiveInteger
```

The operation split(k,t) returns a record containing the two subtrees: one with all elements "less" than k, another with elements "greater" than k.

```
  split(3,t)
   [less=[1, 2, .], greater=[[., 3, [4, 5, [5, 6, 7]]], 8, .]]
              Type: Record(less: BinarySearchTree PositiveInteger,greater:
                           BinarySearchTree PositiveInteger)
```

```
Define insertRoot to insert new elements by creating a new node.

  insertRoot: (INT,BSTREE INT) -> BSTREE INT
                    Type: Void

The new node puts the inserted value between its "less" tree and
"greater" tree.


  insertRoot(x, t) ==
    a := split(x, t)
    node(a.less, x, a.greater)
                    Type: Void


Function buildFromRoot builds a binary search tree from a list
of elements ls and the empty tree emptybst.

  buildFromRoot ls == reduce(insertRoot,ls,emptybst)
                    Type: Void

Apply this to the reverse of the list lv.

  rt := buildFromRoot reverse lv
   [[[1, 2, . ], 3, [4, 5, [5, 6, 7]]], 8, .]
                    Type: BinarySearchTree Integer

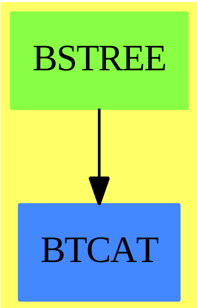
Have Axiom check that these are equal.

  (t = rt)@Boolean
   true
                    Type: Boolean

See Also:
o )show BinarySearchTree
```

————

3.9.1 BinarySearchTree (BSTREE)



See

⇒ “Tree” (TREE) 21.10.1 on page 2699
⇒ “BinaryTree” (BTREE) 3.11.1 on page 292
⇒ “BinaryTournament” (BTOURN) 3.10.1 on page 289
⇒ “BalancedBinaryTree” (BBTREE) 3.1.1 on page 234
⇒ “PendantTree” (PENDTREE) 17.13.1 on page 1904

Exports:

any?	binarySearchTree	child?	children	coerce
copy	count	count	cyclic?	distance
empty	empty?	eq?	eval	eval
eval	eval	every?	hash	insert!
insertRoot!	latex	leaf?	leaves	left
less?	map	map!	member?	members
more?	node	node?	nodes	parts
right	sample	setchildren!	setelt	setelt
setelt	setleft!	setright!	setvalue!	size?
split	value	#?	?=?	?~=?
?.right	?.left	?.value		

— domain BSTREE BinarySearchTree —

```
)abbrev domain BSTREE BinarySearchTree
++ Author: Mark Botch
++ Description:
++ BinarySearchTree(S) is the domain of
++ a binary trees where elements are ordered across the tree.
++ A binary search tree is either empty or has
++ a value which is an S, and a
++ right and left which are both BinaryTree(S)
++ Elements are ordered across the tree.

BinarySearchTree(S: OrderedSet): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
    shallowlyMutable
```

```
    finiteAggregate
    binarySearchTree: List S -> %
     ++ binarySearchTree(l) is not documented
     ++
     ++X binarySearchTree [1,2,3,4]

    insert_!: (S,%) -> %
     ++ insert!(x,b) inserts element x as leaves into binary search tree b.
     ++
     ++X t1:=binarySearchTree [1,2,3,4]
     ++X insert!(5,t1)

    insertRoot_!: (S,%) -> %
     ++ insertRoot!(x,b) inserts element x as a root of binary search tree b.
     ++
     ++X t1:=binarySearchTree [1,2,3,4]
     ++X insertRoot!(5,t1)

    split:      (S,%) -> Record(less: %, greater: %)
     ++ split(x,b) splits binary tree b into two trees, one with elements
     ++ greater than x, the other with elements less than x.
     ++
     ++X t1:=binarySearchTree [1,2,3,4]
     ++X split(3,t1)

  Implementation == BinaryTree(S) add
    Rep := BinaryTree(S)
    binarySearchTree(u:List S) ==
      null u => empty()
      tree := binaryTree(first u)
      for x in rest u repeat insert_!(x,tree)
      tree
    insert_!(x,t) ==
      empty? t => binaryTree(x)
      x >= value t =>
        setright_!(t,insert_!(x,right t))
        t
      setleft_!(t,insert_!(x,left t))
      t
    split(x,t) ==
      empty? t => [empty(),empty()]
      x > value t =>
        a := split(x,right t)
        [node(left t, value t, a.less), a.greater]
      a := split(x,left t)
      [a.less, node(a.greater, value t, right t)]
    insertRoot_!(x,t) ==
      a := split(x,t)
      node(a.less, x, a.greater)
```

———-

— **BSTREE.dotabb** —

```
"BSTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BSTREE"]
"BTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BTCAT"]
"BSTREE" -> "BTCAT"
```

———-

3.10 domain BTOURN BinaryTournament

A BinaryTournament(S) is the domain of binary trees where elements are ordered down the tree. A binary search tree is either empty or is a node containing a value of type S, and a right and a left which are both BinaryTree(S)

— **BinaryTournament.input** —

```
)set break resume
)sys rm -f BinaryTournament.output
)spool BinaryTournament.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryTournament
--R BinaryTournament S: OrderedSet  is a domain constructor
--R Abbreviation for BinaryTournament is BTOURN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BTOURN
--R
--R------------------------------- Operations --------------------------------
--R binaryTournament : List S -> %       children : % -> List %
--R copy : % -> %                        cyclic? : % -> Boolean
--R distance : (%,%) -> Integer          ?.right : (%,right) -> %
--R ?.left : (%,left) -> %               ?.value : (%,value) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               insert! : (S,%) -> %
--R leaf? : % -> Boolean                 leaves : % -> List S
--R left : % -> %                        map : ((S -> S),%) -> %
--R node : (%,S,%) -> %                  nodes : % -> List %
--R right : % -> %                       sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
```

```
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

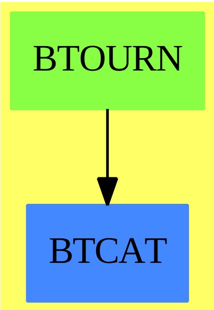
— **BinaryTournament.help** —

```
====================================================================
BinaryTournament examples
====================================================================

See Also:
o )show BinaryTournament
```

———-

3.10.1 BinaryTournament (BTOURN)



See

⇒ "Tree" (TREE) 21.10.1 on page 2699
⇒ "BinaryTree" (BTREE) 3.11.1 on page 292
⇒ "BinarySearchTree" (BSTREE) 3.9.1 on page 285
⇒ "BalancedBinaryTree" (BBTREE) 3.1.1 on page 234
⇒ "PendantTree" (PENDTREE) 17.13.1 on page 1904

Exports:

any?	binaryTournament	child?	children	coerce
copy	count	cyclic?	distance	empty
empty?	eq?	eval	every?	hash
insert!	latex	leaf?	leaves	left
less?	map	map!	member?	members
more?	node	node?	nodes	parts
right	sample	setchildren!	setelt	setleft!
setright!	setvalue!	size?	value	#?
?=?	?~=?	?.right	?.left	?.value

— domain BTOURN BinaryTournament —

```
)abbrev domain BTOURN BinaryTournament
++ Author: Mark Botch
++ Description:
++ BinaryTournament creates a binary tournament with the
++ elements of ls as values at the nodes.

BinaryTournament(S: OrderedSet): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
    shallowlyMutable
    binaryTournament: List S -> %
      ++ binaryTournament(ls) creates a binary tournament with the
      ++ elements of ls as values at the nodes.
      ++
      ++X binaryTournament [1,2,3,4]
```

```
    insert_!: (S,%) -> %
      ++ insert!(x,b) inserts element x as leaves into binary tournament b.
      ++
      ++X t1:=binaryTournament [1,2,3,4]
      ++X insert!(5,t1)
      ++X t1

  Implementation == BinaryTree(S) add
    Rep := BinaryTree(S)
    binaryTournament(u:List S) ==
      null u => empty()
      tree := binaryTree(first u)
      for x in rest u repeat insert_!(x,tree)
      tree
    insert_!(x,t) ==
      empty? t => binaryTree(x)
      x > value t =>
        setleft_!(t,copy t)
        setvalue_!(t,x)
        setright_!(t,empty())
      setright_!(t,insert_!(x,right t))
      t
```

———-

— BTOURN.dotabb —

```
"BTOURN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BTOURN"]
"BTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BTCAT"]
"BTOURN" -> "BTCAT"
```

———-

3.11 domain BTREE BinaryTree

— BinaryTree.input —

```
)set break resume
)sys rm -f BinaryTree.output
)spool BinaryTree.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show BinaryTree
--R BinaryTree S: SetCategory  is a domain constructor
--R Abbreviation for BinaryTree is BTREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BTREE
--R
--R------------------------------- Operations --------------------------------
--R binaryTree : (%,S,%) -> %            binaryTree : S -> %
--R children : % -> List %               copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.right : (%,right) -> %             ?.left : (%,left) -> %
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                 leaves : % -> List S
--R left : % -> %                        map : ((S -> S),%) -> %
--R node : (%,S,%) -> %                  nodes : % -> List %
--R right : % -> %                       sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

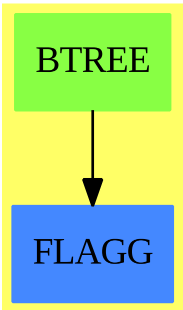
— **BinaryTree.help** —

```
====================================================================
BinaryTree examples
====================================================================

See Also:
o )show BinaryTree
```

———-

3.11.1 BinaryTree (BTREE)



See

⇒ "Tree" (TREE) 21.10.1 on page 2699
⇒ "BinarySearchTree" (BSTREE) 3.9.1 on page 285
⇒ "BinaryTournament" (BTOURN) 3.10.1 on page 289
⇒ "BalancedBinaryTree" (BBTREE) 3.1.1 on page 234
⇒ "PendantTree" (PENDTREE) 17.13.1 on page 1904

Exports:

any?	binaryTree	child?	children	coerce
copy	count	cyclic?	distance	empty
empty?	eq?	eval	every?	hash
latex	leaf?	leaves	left	less?
map	map!	member?	members	more?
node	node?	nodes	parts	right
sample	setchildren!	setelt	setleft!	setright!
setvalue!	size?	value	#?	?=?
?~=?	?.right	?.left	?.value	

— domain BTREE BinaryTree —

```
)abbrev domain BTREE BinaryTree
++ Author: Mark Botch
++ Description:
++ \spadtype{BinaryTree(S)} is the domain of all
++ binary trees. A binary tree over \spad{S} is either empty or has
++ a \spadfun{value} which is an S and a \spadfun{right}
++ and \spadfun{left} which are both binary trees.

BinaryTree(S: SetCategory): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
   binaryTree: S -> %
    ++ binaryTree(v) is an non-empty binary tree
    ++ with value v, and left and right empty.
    ++
    ++X t1:=binaryTree([1,2,3])

   binaryTree: (%,S,%) -> %
    ++ binaryTree(l,v,r) creates a binary tree with
    ++ value v with left subtree l and right subtree r.
    ++
    ++X t1:=binaryTree([1,2,3])
    ++X t2:=binaryTree([4,5,6])
    ++X binaryTree(t1,[7,8,9],t2)

  Implementation == add
     Rep := List Tree S
     t1 = t2 == (t1::Rep) =$Rep (t2::Rep)
     empty()== [] pretend %
     empty()== [] pretend %
     node(l,v,r) == cons(tree(v,l:Rep),r:Rep)
     binaryTree(l,v,r) == node(l,v,r)
     binaryTree(v:S) == node(empty(),v,empty())
     empty? t == empty?(t)$Rep
     leaf? t  == empty? t or empty? left t and empty? right t
     right t ==
       empty? t => error "binaryTree:no right"
```

```
      rest t
    left t ==
      empty? t => error "binaryTree:no left"
      children first t
    value t==
      empty? t => error "binaryTree:no value"
      value first t
    setvalue_! (t,nd)==
      empty? t => error "binaryTree:no value to set"
      setvalue_!(first(t:Rep),nd)
      nd
    setleft_!(t1,t2) ==
      empty? t1 => error "binaryTree:no left to set"
      setchildren_!(first(t1:Rep),t2:Rep)
      t1
    setright_!(t1,t2) ==
      empty? t1 => error "binaryTree:no right to set"
      setrest_!(t1:List Tree S,t2)
```

— **BTREE.dotabb** —

```
"BTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BTREE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"BTREE" -> "FLAGG"
```

3.12 domain BITS Bits

— **Bits.input** —

```
)set break resume
)sys rm -f Bits.output
)spool Bits.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Bits
--R Bits  is a domain constructor
--R Abbreviation for Bits is BITS
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BITS
--R
--R------------------------------- Operations --------------------------------
--R ?/\? : (%,%) -> %                    ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean              ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean               ?>=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                    ^? : % -> %
--R ?and? : (%,%) -> %                   coerce : % -> OutputForm
--R concat : (%,Boolean) -> %            concat : (Boolean,%) -> %
--R concat : (%,%) -> %                  concat : List % -> %
--R construct : List Boolean -> %        copy : % -> %
--R delete : (%,Integer) -> %            ?.? : (%,Integer) -> Boolean
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List Boolean          eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger            index? : (Integer,%) -> Boolean
--R indices : % -> List Integer          insert : (%,%,Integer) -> %
--R latex : % -> String                  max : (%,%) -> %
--R min : (%,%) -> %                     nand : (%,%) -> %
--R nor : (%,%) -> %                     not? : % -> %
--R ?or? : (%,%) -> %                    qelt : (%,Integer) -> Boolean
--R reverse : % -> %                     sample : () -> %
--R xor : (%,%) -> %                     ~? : % -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bits : (NonNegativeInteger,Boolean) -> %
--R convert : % -> InputForm if Boolean has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : ((Boolean -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Boolean,%) -> NonNegativeInteger if $ has finiteAggregate and Boolean has SETCAT
--R delete : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Boolean) -> Boolean
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (Boolean,%) -> Boolean if $ has finiteAggregate and Boolean has SETCAT
--R eval : (%,List Equation Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,Equation Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,Boolean,Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,List Boolean,List Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R every? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Boolean) -> % if $ has shallowlyMutable
--R find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
--R first : % -> Boolean if Integer has ORDSET
--R insert : (Boolean,%,Integer) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Boolean -> Boolean),%) -> %
--R map : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R map! : ((Boolean -> Boolean),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Boolean,%) -> Boolean if $ has finiteAggregate and Boolean has SETCAT
```

```
--R members : % -> List Boolean if $ has finiteAggregate
--R merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if Boolean has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Boolean) -> %
--R parts : % -> List Boolean if $ has finiteAggregate
--R position : ((Boolean -> Boolean),%) -> Integer
--R position : (Boolean,%) -> Integer if Boolean has SETCAT
--R position : (Boolean,%,Integer) -> Integer if Boolean has SETCAT
--R qsetelt! : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean if $ has finiteAgg
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean if $ has finiteAggregate
--R reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R remove : (Boolean,%) -> % if $ has finiteAggregate and Boolean has SETCAT
--R remove : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Boolean has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((Boolean,Boolean) -> Boolean),%) -> %
--R sort : % -> % if Boolean has ORDSET
--R sort! : (((Boolean,Boolean) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if $ has shallowlyMutable and Boolean has ORDSET
--R sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if Boolean has ORDSET
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R
--E 1

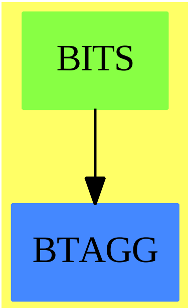
)spool
)lisp (bye)
```

— **Bits.help** —

```
====================================================================
Bits examples
====================================================================

See Also:
o )show Bits
```

3.12.1 Bits (BITS)



See

⇒ “Reference” (REF) 19.5.1 on page 2209
⇒ “Boolean” (BOOLEAN) 3.15.1 on page 304
⇒ “IndexedBits” (IBITS) 10.3.1 on page 1165

Exports:

any?	bits	coerce	concat	construct
convert	copy	copyInto!	count	delete
elt	empty	empty?	entries	entry?
eq?	eval	every?	fill!	find
first	hash	index?	indices	insert
latex	less?	map	map!	max
maxIndex	member?	members	merge	min
minIndex	more?	nand	new	nor
not?	parts	position	qelt	qsetelt!
reduce	remove	removeDuplicates	reverse	reverse!
sample	setelt	size?	sort	sort!
sorted?	swap!	xor	#?	?.?
?/\?	?<?	?<=?	?=?	?>?
?>=?	?\/?	^?	?and?	?or?
?.?	~?	?~=?		

— domain BITS Bits —

```
)abbrev domain BITS Bits
++ Author: Stephen M. Watt
++ Date Created:
++ Change History:
++ Basic Operations: And, Not, Or
++ Related Constructors:
++ Keywords: bits
++ Description:
++ \spadtype{Bits} provides logical functions for Indexed Bits.

Bits(): Exports == Implementation where
```

```
  Exports == BitAggregate() with
    bits: (NonNegativeInteger, Boolean) -> %
      ++ bits(n,b) creates bits with n values of b
  Implementation == IndexedBits(1) add
    bits(n,b)    == new(n,b)
```

———-

— BITS.dotabb —

```
"BITS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BITS"]
"BTAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BTAGG"]
"BITS" -> "BTAGG"
```

———-

3.13 domain BLHN BlowUpWithHamburgerNoether

— BlowUpWithHamburgerNoether.input —

```
)set break resume
)sys rm -f BlowUpWithHamburgerNoether.output
)spool BlowUpWithHamburgerNoether.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BlowUpWithHamburgerNoether
--R BlowUpWithHamburgerNoether  is a domain constructor
--R Abbreviation for BlowUpWithHamburgerNoether is BLHN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BLHN
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                chartCoord : % -> Integer
--R coerce : List Integer -> %            coerce : % -> OutputForm
--R excepCoord : % -> Integer             hash : % -> SingleInteger
--R infClsPt? : % -> Boolean              latex : % -> String
--R quotValuation : % -> Integer          ramifMult : % -> Integer
--R transCoord : % -> Integer             ?~=? : (%,%) -> Boolean
--R createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,Union(left,center,right,verti
--R type : % -> Union(left,center,right,vertical,horizontal)
--R
```

```
--E 1

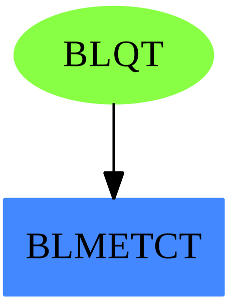
)spool
)lisp (bye)
```

— BlowUpWithHamburgerNoether.help —

```
====================================================================
BlowUpWithHamburgerNoether examples
====================================================================

See Also:
o )show BlowUpWithHamburgerNoether
```

3.13.1 BlowUpWithHamburgerNoether (BLHN)



Exports:

?=?	?~=?	chartCoord	coerce	createHN
excepCoord	hash	infClsPt?	latex	quotValuation
ramifMult	transCoord	type		

— domain BLHN BlowUpWithHamburgerNoether —

```
)abbrev domain BLHN BlowUpWithHamburgerNoether
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
```

```
++ This domain is part of the PAFF package
BlowUpWithHamburgerNoether: Exports == Implementation where
  MetRec ==> Record(_
     ex:Integer, tr:Integer, ch:Integer , quotVal:Integer, _
     ramif:Integer, infClsPt:Boolean, _
     type:Union("left","center","right","vertical","horizontal") )

  Exports ==> BlowUpMethodCategory with HamburgerNoether

  Implementation ==  add
    Rep := MetRec

    infClsPt_? a == a.infClsPt

    createHN( a,b,c,d,e,f,g)==[a,b,c,d,e,f,g]$Rep

    excepCoord a == a.ex

    chartCoord a == a.ch

    transCoord a == a.tr

    ramifMult a == a.ramif

    quotValuation a == a.quotVal

    type a == a.type

    coerce(c:%):OutputForm== ( (c :: Rep ) :: MetRec) :: OutputForm
```

— BLHN.dotabb —

```
"BLHN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BLHN",
           shape=ellipse]
"BLMETCT" [color="#4488FF",href="bookvol10.3.pdf#nameddest=BLMETCT"]
"BLHN" -> "BLMETCT"
```

3.14 domain BLQT BlowUpWithQuadTrans

— BlowUpWithQuadTrans.input —

```
)set break resume
```

```
)sys rm -f BlowUpWithQuadTrans.output
)spool BlowUpWithQuadTrans.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BlowUpWithQuadTrans
--R BlowUpWithQuadTrans  is a domain constructor
--R Abbreviation for BlowUpWithQuadTrans is BLQT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BLQT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                chartCoord : % -> Integer
--R coerce : List Integer -> %            coerce : % -> OutputForm
--R excepCoord : % -> Integer             hash : % -> SingleInteger
--R infClsPt? : % -> Boolean              latex : % -> String
--R quotValuation : % -> Integer          ramifMult : % -> Integer
--R transCoord : % -> Integer             ?~=? : (%,%) -> Boolean
--R createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,Union(left,center,right,vertical,horizon
--R type : % -> Union(left,center,right,vertical,horizontal)
--R
--E 1

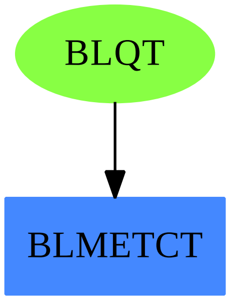
)spool
)lisp (bye)
```

— BlowUpWithQuadTrans.help —

```
====================================================================
BlowUpWithQuadTrans examples
====================================================================

See Also:
o )show BlowUpWithQuadTrans
```

3.14.1 BlowUpWithQuadTrans (BLQT)



Exports:

?=?	?˜=?	chartCoord	coerce	createHN
excepCoord	hash	infClsPt?	latex	quotValuation
ramifMult	transCoord	type		

— domain BLQT BlowUpWithQuadTrans —

```
)abbrev domain BLQT BlowUpWithQuadTrans
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
BlowUpWithQuadTrans: Exports == Implementation where

  MetRec ==> Record( ex:Integer, tr: Integer, ch: Integer , ramif: Integer )
  outRec ==> Record( exCoord:Integer, affNeigh: Integer )
  Exports ==> BlowUpMethodCategory with

    QuadraticTransform

  Implementation ==  add
    Rep := MetRec

    coerce(la:List(Integer)):% == [la.1, la.2,la.3,  1 ]$Rep

    ramifMult a == One$Integer

    excepCoord a == a.ex

    chartCoord a == a.ch

    transCoord a == a.tr

    ramifMult a == a.ramif
```

```
    quotValuation a == One$Integer

    coerce(c:%):OutputForm==
      oo: outRec := [ excepCoord(c) , chartCoord(c) ]$outRec
      oo :: OutputForm
```

— BLQT.dotabb —

```
"BLQT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BLQT",
           shape=ellipse]
"BLMETCT" [color="#4488FF",href="bookvol10.3.pdf#nameddest=BLMETCT"]
"BLQT" -> "BLMETCT"
```

3.15 domain BOOLEAN Boolean

— Boolean.input —

```
)set break resume
)sys rm -f Boolean.output
)spool Boolean.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Boolean
--R Boolean  is a domain constructor
--R Abbreviation for Boolean is BOOLEAN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BOOLEAN
--R
--R------------------------------- Operations --------------------------------
--R ?/\? : (%,%) -> %                    ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean              ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean               ?>=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                    ^? : % -> %
--R ?and? : (%,%) -> %                   coerce : % -> OutputForm
--R convert : % -> InputForm             false : () -> %
--R hash : % -> SingleInteger            implies : (%,%) -> %
```

```
--R index : PositiveInteger -> %          latex : % -> String
--R lookup : % -> PositiveInteger         max : (%,%) -> %
--R min : (%,%) -> %                      nand : (%,%) -> %
--R nor : (%,%) -> %                      not? : % -> %
--R ?or? : (%,%) -> %                     random : () -> %
--R size : () -> NonNegativeInteger       test : % -> Boolean
--R true : () -> %                        xor : (%,%) -> %
--R ~? : % -> %                           ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

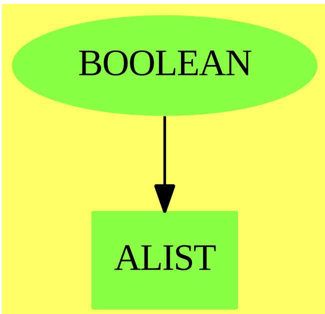
— Boolean.help —

```
====================================================================
Boolean examples
====================================================================

See Also:
o )show Boolean
```

———-

3.15.1 Boolean (BOOLEAN)



See

⇒ "Reference" (REF) 19.5.1 on page 2209
⇒ "IndexedBits" (IBITS) 10.3.1 on page 1165
⇒ "Bits" (BITS) 3.12.1 on page 297

Exports:

coerce	convert	false	hash	implies
index	latex	lookup	max	min
nand	nor	not?	random	size
test	true	xor	˜?	?˜=?
?/\?	?<?	?<=?	?=?	?>?
?>=?	?\/?	ˆ?	?and?	?or?

— domain BOOLEAN Boolean —

```
)abbrev domain BOOLEAN Boolean
++ Author: Stephen M. Watt
++ Date Created:
++ Change History:
++ Basic Operations: true, false, not, and, or, xor, nand, nor, implies, ^
++ Related Constructors:
++ Keywords: boolean
++ Description:
++ \spadtype{Boolean} is the elementary logic with 2 values:
++ true and false

Boolean(): Join(OrderedSet, Finite, Logic, ConvertibleTo InputForm) with
    true   : constant -> %
      ++ true is a logical constant.
    false  : constant -> %
      ++ false is a logical constant.
    _^    : % -> %
      ++ ^ n returns the negation of n.
    _not : % -> %
      ++ not n returns the negation of n.
    _and  : (%, %) -> %
      ++ a and b  returns the logical and of Boolean \spad{a} and b.
    _or  : (%, %) -> %
      ++ a or b returns the logical inclusive or
      ++ of Boolean \spad{a} and b.
    xor    : (%, %) -> %
      ++ xor(a,b) returns the logical exclusive or
      ++ of Boolean \spad{a} and b.
    nand   : (%, %) -> %
      ++ nand(a,b) returns the logical negation of \spad{a} and b.
    nor    : (%, %) -> %
      ++ nor(a,b) returns the logical negation of \spad{a} or b.
    implies: (%, %) -> %
      ++ implies(a,b) returns the logical implication
      ++ of Boolean \spad{a} and b.
    test: % -> Boolean
      ++ test(b) returns b and is provided for compatibility with the
      ++ new compiler.
  == add
```

```
    nt: % -> %

    test a        == a pretend Boolean

    nt b          == (b pretend Boolean => false; true)
    true          == EQ(2,2)$Lisp   --well, 1 is rather special
    false         == NIL$Lisp
    sample()      == true
    not b         == (test b => false; true)
    _^ b          == (test b => false; true)
    _~ b          == (test b => false; true)
    _and(a, b)    == (test a => b; false)
    _/_\(a, b)    == (test a => b; false)
    _or(a, b)     == (test a => true; b)
    _\_/(a, b)     == (test a => true; b)
    xor(a, b)     == (test a => nt b; b)
    nor(a, b)     == (test a => false; nt b)
    nand(a, b)    == (test a => nt b; true)
    a = b         == BooleanEquality(a, b)$Lisp
    implies(a, b) == (test a => b; true)
    a < b         == (test b => not(test a);false)

    size()        == 2
    index i       ==
      even?(i::Integer) => false
      true
    lookup a      ==
      a pretend Boolean => 1
      2
    random()      ==
      even?(random()$Integer) => false
      true

    convert(x:%):InputForm ==
      x pretend Boolean => convert("true"::Symbol)
      convert("false"::Symbol)

    coerce(x:%):OutputForm ==
      x pretend Boolean => message "true"
      message "false"
```

———

— BOOLEAN.dotabb —

```
"BOOLEAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BOOLEAN",
           shape=ellipse]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
```

```
"BOOLEAN" -> "ALIST"
```

Chapter 4

Chapter C

4.1 domain CARD CardinalNumber

— CardinalNumber.input —

```
)set break resume
)sys rm -f CardinalNumber.output
)spool CardinalNumber.output
)set message test on
)set message auto off
)clear all
--S 1 of 20
c0 := 0 :: CardinalNumber
--R
--R
--R   (1)  0
--R                                                           Type: CardinalNumber
--E 1

--S 2 of 20
c1 := 1 :: CardinalNumber
--R
--R
--R   (2)  1
--R                                                           Type: CardinalNumber
--E 2

--S 3 of 20
c2 := 2 :: CardinalNumber
--R
--R
--R   (3)  2
```

```
--R                                                    Type: CardinalNumber
--E 3

--S 4 of 20
c3 := 3 :: CardinalNumber
--R
--R
--R   (4)  3
--R                                                    Type: CardinalNumber
--E 4

--S 5 of 20
A0 := Aleph 0
--R
--R
--R   (5)  Aleph(0)
--R                                                    Type: CardinalNumber
--E 5

--S 6 of 20
A1 := Aleph 1
--R
--R
--R   (6)  Aleph(1)
--R                                                    Type: CardinalNumber
--E 6

--S 7 of 20
finite? c2
--R
--R
--R   (7)  true
--R                                                           Type: Boolean
--E 7

--S 8 of 20
finite? A0
--R
--R
--R   (8)  false
--R                                                           Type: Boolean
--E 8

--S 9 of 20
countable? c2
--R
--R
--R   (9)  true
--R                                                           Type: Boolean
--E 9
```

```
--S 10 of 20
countable? A0
--R
--R
--R   (10)  true
--R                                                            Type: Boolean
--E 10

--S 11 of 20
countable? A1
--R
--R
--R   (11)  false
--R                                                            Type: Boolean
--E 11

--S 12 of 20
[c2 + c2, c2 + A1]
--R
--R
--R   (12)  [4,Aleph(1)]
--R                                                Type: List CardinalNumber
--E 12

--S 13 of 20
[c0*c2, c1*c2, c2*c2, c0*A1, c1*A1, c2*A1, A0*A1]
--R
--R
--R   (13)  [0,2,4,0,Aleph(1),Aleph(1),Aleph(1)]
--R                                                Type: List CardinalNumber
--E 13

--S 14 of 20
[c2**c0, c2**c1, c2**c2, A1**c0, A1**c1, A1**c2]
--R
--R
--R   (14)  [1,2,4,1,Aleph(1),Aleph(1)]
--R                                                Type: List CardinalNumber
--E 14

--S 15 of 20
[c2-c1, c2-c2, c2-c3, A1-c2, A1-A0, A1-A1]
--R
--R
--R   (15)  [1,0,"failed",Aleph(1),Aleph(1),"failed"]
--R                                 Type: List Union(CardinalNumber,"failed")
--E 15

--S 16 of 20
```

```
generalizedContinuumHypothesisAssumed true
--R
--R
--R   (16)  true
--R                                                                  Type: Boolean
--E 16

--S 17 of 20
[c0**A0, c1**A0, c2**A0, A0**A0, A0**A1, A1**A0, A1**A1]
--R
--R
--R   (17)  [0,1,Aleph(1),Aleph(1),Aleph(2),Aleph(1),Aleph(2)]
--R                                                      Type: List CardinalNumber
--E 17

--S 18 of 20
a := Aleph 0
--R
--R
--R   (18)  Aleph(0)
--R                                                           Type: CardinalNumber
--E 18

--S 19 of 20
c := 2**a
--R
--R
--R   (19)  Aleph(1)
--R                                                           Type: CardinalNumber
--E 19

--S 20 of 20
f := 2**c
--R
--R
--R   (20)  Aleph(2)
--R                                                           Type: CardinalNumber
--E 20
)spool
)lisp (bye)
```

— **CardinalNumber.help** —

```
====================================================================
CardinalNumber examples
====================================================================
```

```
The CardinalNumber domain can be used for values indicating the
cardinality of sets, both finite and infinite.  For example, the
dimension operation in the category VectorSpace returns a cardinal
number.

The non-negative integers have a natural construction as cardinals

  0 = #{ }, 1 = {0}, 2 = {0, 1}, ..., n = {i | 0 <= i < n}.

The fact that 0 acts as a zero for the multiplication of cardinals is
equivalent to the axiom of choice.

Cardinal numbers can be created by conversion from non-negative integers.

  c0 := 0 :: CardinalNumber
   0
                      Type: CardinalNumber

  c1 := 1 :: CardinalNumber
   1
                      Type: CardinalNumber

  c2 := 2 :: CardinalNumber
   2
                      Type: CardinalNumber

  c3 := 3 :: CardinalNumber
   3
                      Type: CardinalNumber

They can also be obtained as the named cardinal Aleph(n).

  A0 := Aleph 0
   Aleph(0)
                      Type: CardinalNumber

  A1 := Aleph 1
   Aleph(1)
                      Type: CardinalNumber

The finite? operation tests whether a value is a finite cardinal, that
is, a non-negative integer.

  finite? c2
   true
                      Type: Boolean

  finite? A0
   false
                      Type: Boolean
```

Similarly, the countable? operation determines whether a value is a countable cardinal, that is, finite or Aleph(0).

```
  countable? c2
   true
                     Type: Boolean

  countable? A0
   true
                     Type: Boolean

  countable? A1
   false
                     Type: Boolean
```

Arithmetic operations are defined on cardinal numbers as follows:
If x = #X and y = #Y then

```
  x+y  = #(X+Y) cardinality of the disjoint union
  x-y  = #(X-Y) cardinality of the relative complement
  x*y  = #(X*Y) cardinality of the Cartesian product
  x**y = #(X**Y) cardinality of the set of maps from Y to X
```

Here are some arithmetic examples.

```
  [c2 + c2, c2 + A1]
   [4, Aleph(1)]
                     Type: List CardinalNumber

  [c0*c2, c1*c2, c2*c2, c0*A1, c1*A1, c2*A1, A0*A1]
   [0, 2, 4, 0, Aleph(1), Aleph(1), Aleph(1)]
                     Type: List CardinalNumber

  [c2**c0, c2**c1, c2**c2, A1**c0, A1**c1, A1**c2]
   [1, 2, 4, 1, Aleph(1), Aleph(1)]
                     Type: List CardinalNumber
```

Subtraction is a partial operation: it is not defined when subtracting a larger cardinal from a smaller one, nor when subtracting two equal infinite cardinals.

```
  [c2-c1, c2-c2, c2-c3, A1-c2, A1-A0, A1-A1]
   [1, 0, "failed", Aleph(1), Aleph(1), "failed"]
                     Type: List Union(CardinalNumber,"failed")
```

The generalized continuum hypothesis asserts that

```
  2**Aleph i = Aleph(i+1)
```

```
and is independent of the axioms of set theory.

(reference: Goedel, The consistency of the continuum hypothesis,
Ann. Math. Studies, Princeton Univ. Press, 1940.)

The CardinalNumber domain provides an operation to assert whether the
hypothesis is to be assumed.

  generalizedContinuumHypothesisAssumed true
   true
                      Type: Boolean

When the generalized continuum hypothesis is assumed, exponentiation
to a transfinite power is allowed.

  [c0**A0, c1**A0, c2**A0, A0**A0, A0**A1, A1**A0, A1**A1]
   [0, 1, Aleph(1), Aleph(1), Aleph(2), Aleph(1), Aleph(2)]
                      Type: List CardinalNumber

Three commonly encountered cardinal numbers are

  a = #Z countable infinity
  c = #R the continuum
  f = #{g| g: [0,1] -> R}

In this domain, these values are obtained under the generalized
continuum hypothesis in this way.

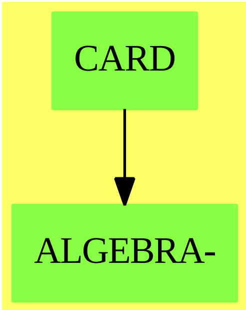
  a := Aleph 0
   Aleph(0)
                      Type: CardinalNumber

  c := 2**a
   Aleph(1)
                      Type: CardinalNumber

  f := 2**c
   Aleph(2)
                      Type: CardinalNumber

See Also:
o )show CardinalNumber
```

4.1.1 CardinalNumber (CARD)



Exports:

0	1
Aleph	coerce
countable?	finite?
generalizedContinuumHypothesisAssumed	hash
generalizedContinuumHypothesisAssumed?	latex
max	min
one?	recip
retract	retractIfCan
sample	zero?
?^?	?~=?
?*?	?**?
?-?	?+?
?<?	?<=?
?=?	?>?
?>=?	

— domain CARD CardinalNumber —

```
)abbrev domain CARD CardinalNumber
++ Author: S.M. Watt
++ Date Created: June 1986
++ Date Last Updated: May 1990
++ Basic Operations: Aleph, +, -, *, **
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: cardinal number, transfinite arithmetic
++ Examples:
++ References:
++   Goedel, "The consistency of the continuum hypothesis",
++   Ann. Math. Studies, Princeton Univ. Press, 1940
++ Description:
++ Members of the domain CardinalNumber are values indicating the
```

```
++ cardinality of sets, both finite and infinite.  Arithmetic operations
++ are defined on cardinal numbers as follows.
++
++ If \spad{x = #X}  and  \spad{y = #Y} then\br
++ \tab{5}\spad{x+y  = #(X+Y)} \tab{5}disjoint union\br
++ \tab{5}\spad{x-y  = #(X-Y)} \tab{5}relative complement\br
++ \tab{5}\spad{x*y  = #(X*Y)} \tab{5}cartesian product\br
++ \tab{5}\spad{x**y = #(X**Y)} \tab{4}\spad{X**Y = g| g:Y->X}
++
++ The non-negative integers have a natural construction as cardinals\br
++ \spad{0 = #\{\}}, \spad{1 = \{0\}},
++ \spad{2 = \{0, 1\}}, ..., \spad{n = \{i| 0 <= i < n\}}.
++
++ That \spad{0} acts as a zero for the multiplication of cardinals is
++ equivalent to the axiom of choice.
++
++ The generalized continuum hypothesis asserts \br
++ \spad{2**Aleph i = Aleph(i+1)}
++ and is independent of the axioms of set theory [Goedel 1940].
++
++ Three commonly encountered cardinal numbers are\br
++ \tab{5}\spad{a = #Z} \tab{5}countable infinity\br
++ \tab{5}\spad{c = #R} \tab{5}the continuum\br
++ \tab{5}\spad{f = # g | g:[0,1]->R\}
++
++ In this domain, these values are obtained using\br
++ \tab{5}\spad{a := Aleph 0}, \spad{c := 2**a}, \spad{f := 2**c}.

CardinalNumber: Join(OrderedSet, AbelianMonoid, Monoid,
                        RetractableTo NonNegativeInteger) with
        commutative "*"
            ++ a domain D has \spad{commutative("*")} if it has an operation
            ++ \spad{"*": (D,D) -> D} which is commutative.

        "-": (%,%) -> Union(%,"failed")
            ++ \spad{x - y} returns an element z such that
            ++ \spad{z+y=x} or "failed" if no such element exists.
            ++
            ++X c2:=2::CardinalNumber
            ++X c2-c2
            ++X A1:=Aleph 1
            ++X A1-c2

        "**": (%, %) -> %
            ++ \spad{x**y} returns \spad{#(X**Y)} where \spad{X**Y} is defined
            ++  as \spad{\{g| g:Y->X\}}.
            ++
            ++X c2:=2::CardinalNumber
            ++X c2**c2
            ++X A1:=Aleph 1
```

```
        ++X A1**c2
        ++X generalizedContinuumHypothesisAssumed true
        ++X A1**A1

    Aleph: NonNegativeInteger -> %
        ++ Aleph(n) provides the named (infinite) cardinal number.
        ++
        ++X A0:=Aleph 0

    finite?: % -> Boolean
        ++ finite?(\spad{a}) determines whether
        ++ \spad{a} is a finite cardinal, i.e. an integer.
        ++
        ++X c2:=2::CardinalNumber
        ++X finite? c2
        ++X A0:=Aleph 0
        ++X finite? A0

    countable?: % -> Boolean
        ++ countable?(\spad{a}) determines
        ++ whether \spad{a} is a countable cardinal,
        ++ i.e. an integer or \spad{Aleph 0}.
        ++
        ++X c2:=2::CardinalNumber
        ++X countable? c2
        ++X A0:=Aleph 0
        ++X countable? A0
        ++X A1:=Aleph 1
        ++X countable? A1

    generalizedContinuumHypothesisAssumed?: () -> Boolean
        ++ generalizedContinuumHypothesisAssumed?()
        ++ tests if the hypothesis is currently assumed.
        ++
        ++X generalizedContinuumHypothesisAssumed?

    generalizedContinuumHypothesisAssumed:  Boolean -> Boolean
        ++ generalizedContinuumHypothesisAssumed(bool)
        ++ is used to dictate whether the hypothesis is to be assumed.
        ++
        ++X generalizedContinuumHypothesisAssumed true
        ++X a:=Aleph 0
        ++X c:=2**a
        ++X f:=2**c
== add
    NNI ==> NonNegativeInteger
    FINord   ==> -1
    DUMMYval ==> -1

    Rep := Record(order: Integer, ival: Integer)
```

```
GCHypothesis: Reference(Boolean) := ref false

-- Creation
0           == [FINord, 0]
1           == [FINord, 1]
coerce(n:NonNegativeInteger):% == [FINord, n]
Aleph n     == [n, DUMMYval]

-- Output
ALEPHexpr := "Aleph"::OutputForm

coerce(x: %): OutputForm ==
    x.order = FINord => (x.ival)::OutputForm
    prefix(ALEPHexpr, [(x.order)::OutputForm])

-- Manipulation
x = y ==
    x.order ^= y.order => false
    finite? x          => x.ival = y.ival
    true     -- equal transfinites
x < y ==
    x.order < y.order => true
    x.order > y.order => false
    finite? x         => x.ival < y.ival
    false    -- equal transfinites
x:% + y:% ==
    finite? x and finite? y => [FINord, x.ival+y.ival]
    max(x, y)
x - y ==
    x < y     => "failed"
    finite? x => [FINord, x.ival-y.ival]
    x > y     => x
    "failed" -- equal transfinites
x:% * y:% ==
    finite? x and finite? y => [FINord, x.ival*y.ival]
    x = 0 or y = 0          => 0
    max(x, y)
n:NonNegativeInteger * x:% ==
    finite? x => [FINord, n*x.ival]
    n = 0     => 0
    x
x**y ==
    y = 0 =>
        x ^= 0 => 1
        error "0**0 not defined for cardinal numbers."
    finite? y =>
        not finite? x => x
        [FINord,x.ival**(y.ival):NNI]
    x = 0 => 0
```

```
            x = 1 => 1
            GCHypothesis() => [max(x.order-1, y.order) + 1, DUMMYval]
            error "Transfinite exponentiation only implemented under GCH"

        finite? x    == x.order = FINord
        countable? x == x.order < 1

        retract(x:%):NonNegativeInteger ==
          finite? x => (x.ival)::NNI
          error "Not finite"

        retractIfCan(x:%):Union(NonNegativeInteger, "failed") ==
          finite? x => (x.ival)::NNI
          "failed"

        -- State manipulation
        generalizedContinuumHypothesisAssumed?() == GCHypothesis()
        generalizedContinuumHypothesisAssumed b == (GCHypothesis() := b)
```

— CARD.dotabb —

```
"CARD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CARD"]
"ALGEBRA-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALGEBRA"]
"CARD" -> "ALGEBRA-"
```

4.2 domain CARTEN CartesianTensor

— CartesianTensor.input —

```
)set break resume
)sys rm -f CartesianTensor.output
)spool CartesianTensor.output
)set message test on
)set message auto off
)clear all
--S 1 of 48
CT := CARTEN(i0 := 1, 2, Integer)
--R
--R
--R   (1)  CartesianTensor(1,2,Integer)
```

```
--R                                                             Type: Domain
--E 1

--S 2 of 48
t0: CT := 8
--R
--R
--R   (2)  8
--R                                       Type: CartesianTensor(1,2,Integer)
--E 2

--S 3 of 48
rank t0
--R
--R
--R   (3)  0
--R                                                 Type: NonNegativeInteger
--E 3

--S 4 of 48
v: DirectProduct(2, Integer) := directProduct [3,4]
--R
--R
--R   (4)  [3,4]
--R                                         Type: DirectProduct(2,Integer)
--E 4

--S 5 of 48
Tv: CT := v
--R
--R
--R   (5)  [3,4]
--R                                       Type: CartesianTensor(1,2,Integer)
--E 5

--S 6 of 48
m: SquareMatrix(2, Integer) := matrix [ [1,2],[4,5] ]
--R
--R
--R        +1  2+
--R   (6)  |    |
--R        +4  5+
--R                                          Type: SquareMatrix(2,Integer)
--E 6

--S 7 of 48
Tm: CT := m
--R
--R
--R        +1  2+
```

```
--R   (7)  |    |
--R        +4  5+
--R                                          Type: CartesianTensor(1,2,Integer)
--E 7

--S 8 of 48
n: SquareMatrix(2, Integer) := matrix [ [2,3],[0,1] ]
--R
--R
--R        +2  3+
--R   (8)  |    |
--R        +0  1+
--R                                               Type: SquareMatrix(2,Integer)
--E 8

--S 9 of 48
Tn: CT := n
--R
--R
--R        +2  3+
--R   (9)  |    |
--R        +0  1+
--R                                          Type: CartesianTensor(1,2,Integer)
--E 9

--S 10 of 48
t1: CT := [2, 3]
--R
--R
--R   (10)  [2,3]
--R                                          Type: CartesianTensor(1,2,Integer)
--E 10

--S 11 of 48
rank t1
--R
--R
--R   (11)  1
--R                                                       Type: PositiveInteger
--E 11

--S 12 of 48
t2: CT := [t1, t1]
--R
--R
--R         +2  3+
--R   (12)  |    |
--R         +2  3+
--R                                          Type: CartesianTensor(1,2,Integer)
--E 12
```

```
--S 13 of 48
t3: CT := [t2, t2]
--R
--R
--R          +2  3+ +2  3+
--R   (13)  [|    |,|    |]
--R          +2  3+ +2  3+
--R                                          Type: CartesianTensor(1,2,Integer)
--E 13

--S 14 of 48
tt: CT := [t3, t3]; tt := [tt, tt]
--R
--R
--R          ++2  3+  +2  3++ ++2  3+  +2  3++
--R          ||    |  |    || ||    |  |    ||
--R          |+2  3+  +2  3+| |+2  3+  +2  3+|
--R   (14)  [|              |,|              |]
--R          |+2  3+  +2  3+| |+2  3+  +2  3+|
--R          ||    |  |    || ||    |  |    ||
--R          ++2  3+  +2  3++ ++2  3+  +2  3++
--R                                          Type: CartesianTensor(1,2,Integer)
--E 14

--S 15 of 48
rank tt
--R
--R
--R   (15)  5
--R                                                       Type: PositiveInteger
--E 15

--S 16 of 48
Tmn := product(Tm, Tn)
--R
--R
--R         ++2  3+    +4  6+ +
--R         ||    |    |    | |
--R         |+0  1+    +0  2+ |
--R   (16)  |                 |
--R         |+8  12+  +10  15+|
--R         ||     |  |      ||
--R         ++0  4 +  +0   5 ++
--R                                          Type: CartesianTensor(1,2,Integer)
--E 16

--S 17 of 48
Tmv := contract(Tm,2,Tv,1)
--R
```

```
--R
--R   (17)  [11,32]
--R                                          Type: CartesianTensor(1,2,Integer)
--E 17

--S 18 of 48
Tm*Tv
--R
--R
--R   (18)  [11,32]
--R                                          Type: CartesianTensor(1,2,Integer)
--E 18

--S 19 of 48
Tmv = m * v
--R
--R
--R   (19)  [11,32]= [11,32]
--R                                 Type: Equation CartesianTensor(1,2,Integer)
--E 19

--S 20 of 48
t0()
--R
--R
--R   (20)  8
--R                                                        Type: PositiveInteger
--E 20

--S 21 of 48
t1(1+1)
--R
--R
--R   (21)  3
--R                                                        Type: PositiveInteger
--E 21

--S 22 of 48
t2(2,1)
--R
--R
--R   (22)  2
--R                                                        Type: PositiveInteger
--E 22

--S 23 of 48
t3(2,1,2)
--R
--R
--R   (23)  3
```

```
--R                                                    Type: PositiveInteger
--E 23

--S 24 of 48
Tmn(2,1,2,1)
--R
--R
--R   (24)  0
--R                                                 Type: NonNegativeInteger
--E 24

--S 25 of 48
t0[]
--R
--R
--R   (25)  8
--R                                                    Type: PositiveInteger
--E 25

--S 26 of 48
t1[2]
--R
--R
--R   (26)  3
--R                                                    Type: PositiveInteger
--E 26

--S 27 of 48
t2[2,1]
--R
--R
--R   (27)  2
--R                                                    Type: PositiveInteger
--E 27

--S 28 of 48
t3[2,1,2]
--R
--R
--R   (28)  3
--R                                                    Type: PositiveInteger
--E 28

--S 29 of 48
Tmn[2,1,2,1]
--R
--R
--R   (29)  0
--R                                                 Type: NonNegativeInteger
--E 29
```

```
--S 30 of 48
cTmn := contract(Tmn,1,2)
--R
--R
--R         +12  18+
--R   (30)  |      |
--R         +0   6 +
--R                                      Type: CartesianTensor(1,2,Integer)
--E 30

--S 31 of 48
trace(m) * n
--R
--R
--R         +12  18+
--R   (31)  |      |
--R         +0   6 +
--R                                           Type: SquareMatrix(2,Integer)
--E 31

--S 32 of 48
contract(Tmn,1,2) = trace(m) * n
--R
--R
--R         +12  18+  +12  18+
--R   (32)  |      |= |      |
--R         +0   6 +  +0   6 +
--R                             Type: Equation CartesianTensor(1,2,Integer)
--E 32

--S 33 of 48
contract(Tmn,1,3) = transpose(m) * n
--R
--R
--R         +2  7 +  +2  7 +
--R   (33)  |     |= |     |
--R         +4  11+  +4  11+
--R                             Type: Equation CartesianTensor(1,2,Integer)
--E 33

--S 34 of 48
contract(Tmn,1,4) = transpose(m) * transpose(n)
--R
--R
--R         +14  4+  +14  4+
--R   (34)  |     |= |     |
--R         +19  5+  +19  5+
--R                             Type: Equation CartesianTensor(1,2,Integer)
--E 34
```

```
--S 35 of 48
contract(Tmn,2,3) = m * n
--R
--R
--R         +2  5 +  +2  5 +
--R   (35)  |     |= |     |
--R         +8  17+  +8  17+
--R                                Type: Equation CartesianTensor(1,2,Integer)
--E 35

--S 36 of 48
contract(Tmn,2,4) = m * transpose(n)
--R
--R
--R         +8   2+  +8   2+
--R   (36)  |     |= |     |
--R         +23  5+  +23  5+
--R                                Type: Equation CartesianTensor(1,2,Integer)
--E 36

--S 37 of 48
contract(Tmn,3,4) = trace(n) * m
--R
--R
--R         +3   6 +  +3   6 +
--R   (37)  |      |= |      |
--R         +12  15+  +12  15+
--R                                Type: Equation CartesianTensor(1,2,Integer)
--E 37

--S 38 of 48
tTmn := transpose(Tmn,1,3)
--R
--R
--R         ++2  3 +  +4   6 ++
--R         ||     |  |      ||
--R         |+8  12+  +10  15+|
--R   (38)  |                 |
--R         |+0  1+    +0  2+ |
--R         ||    |    |    | |
--R         ++0  4+    +0  5+ +
--R                                         Type: CartesianTensor(1,2,Integer)
--E 38

--S 39 of 48
transpose Tmn
--R
--R
--R         ++2  8+   +4  10++
```

```
--R         ||     |   |      ||
--R         |+0  0+   +0  0 +|
--R   (39)  |                |
--R         |+3  12+  +6  15+|
--R         ||      |  |     ||
--R         ++1  4 +  +2  5 ++
--R                                              Type: CartesianTensor(1,2,Integer)
--E 39

--S 40 of 48
transpose Tm = transpose m
--R
--R
--R         +1  4+  +1  4+
--R   (40)  |     |= |     |
--R         +2  5+  +2  5+
--R                                     Type: Equation CartesianTensor(1,2,Integer)
--E 40

--S 41 of 48
rTmn := reindex(Tmn, [1,4,2,3])
--R
--R
--R         ++2  0+   +3  1+ +
--R         ||     |   |    | |
--R         |+4  0+   +6  2+ |
--R   (41)  |                |
--R         |+8   0+  +12  4+|
--R         ||      |  |     ||
--R         ++10  0+  +15  5++
--R                                              Type: CartesianTensor(1,2,Integer)
--E 41

--S 42 of 48
tt := transpose(Tm)*Tn - Tn*transpose(Tm)
--R
--R
--R         +- 6  - 16+
--R   (42)  |         |
--R         + 2    6  +
--R                                              Type: CartesianTensor(1,2,Integer)
--E 42

--S 43 of 48
Tv*(tt+Tn)
--R
--R
--R   (43)  [- 4,- 11]
--R                                              Type: CartesianTensor(1,2,Integer)
--E 43
```

```
--S 44 of 48
reindex(product(Tn,Tn),[4,3,2,1])+3*Tn*product(Tm,Tm)
--R
--R
--R         ++46   84 +  +57   114++
--R         ||        |  |        ||
--R         |+174  212+  +228  285+|
--R   (44)  |                      |
--R         | +18  24+    +17  30+ |
--R         | |      |    |      | |
--R         + +57  63+    +63  76+ +
--R                                  Type: CartesianTensor(1,2,Integer)
--E 44

--S 45 of 48
delta:  CT := kroneckerDelta()
--R
--R
--R         +1  0+
--R   (45)  |    |
--R         +0  1+
--R                                  Type: CartesianTensor(1,2,Integer)
--E 45

--S 46 of 48
contract(Tmn, 2, delta, 1) = reindex(Tmn, [1,3,4,2])
--R
--R
--R         + +2  4+   +0  0++  + +2  4+   +0  0++
--R         | |    |   |    ||  | |    |   |    ||
--R         | +3  6+   +1  2+|  | +3  6+   +1  2+|
--R   (46)  |                |= |                |
--R         |+8   10+  +0  0+|  |+8   10+  +0  0+|
--R         ||      |  |    ||  ||      |  |    ||
--R         ++12  15+  +4  5++  ++12  15+  +4  5++
--R                         Type: Equation CartesianTensor(1,2,Integer)
--E 46

--S 47 of 48
epsilon:CT := leviCivitaSymbol()
--R
--R
--R         + 0   1+
--R   (47)  |      |
--R         +- 1  0+
--R                                  Type: CartesianTensor(1,2,Integer)
--E 47

--S 48 of 48
```

```
contract(epsilon*Tm*epsilon, 1,2) = 2 * determinant m
--R
--R
--R   (48)  - 6= - 6
--R                                 Type: Equation CartesianTensor(1,2,Integer)
--E 48
)spool
)lisp (bye)
```

— CartesianTensor.help —

```
====================================================================
CartesianTensor examples
====================================================================

CartesianTensor(i0,dim,R) provides Cartesian tensors with components
belonging to a commutative ring R.  Tensors can be described as a
generalization of vectors and matrices.  This gives a concise tensor
algebra for multilinear objects supported by the CartesianTensor
domain.  You can form the inner or outer product of any two tensors
and you can add or subtract tensors with the same number of components.
Additionally, various forms of traces and transpositions are useful.

The CartesianTensor constructor allows you to specify the minimum
index for subscripting.  In what follows we discuss in detail how to
manipulate tensors.

Here we construct the domain of Cartesian tensors of dimension 2 over the
integers, with indices starting at 1.

  CT := CARTEN(i0 := 1, 2, Integer)
   CartesianTensor(1,2,Integer)
                      Type: Domain

====================================================================
Forming tensors
====================================================================

Scalars can be converted to tensors of rank zero.

  t0: CT := 8
   8
                      Type: CartesianTensor(1,2,Integer)

  rank t0
   0
                      Type: NonNegativeInteger
```

Vectors (mathematical direct products, rather than one dimensional array structures) can be converted to tensors of rank one.

```
  v: DirectProduct(2, Integer) := directProduct [3,4]
   [3, 4]
                     Type: DirectProduct(2,Integer)

  Tv: CT := v
   [3, 4]
                     Type: CartesianTensor(1,2,Integer)
```

Matrices can be converted to tensors of rank two.

```
  m: SquareMatrix(2, Integer) := matrix [ [1,2],[4,5] ]
   +1  2+
   |    |
   +4  5+
                     Type: SquareMatrix(2,Integer)

  Tm: CT := m
   +1  2+
   |    |
   +4  5+
                     Type: CartesianTensor(1,2,Integer)

  n: SquareMatrix(2, Integer) := matrix [ [2,3],[0,1] ]
   +2  3+
   |    |
   +0  1+
                     Type: SquareMatrix(2,Integer)

  Tn: CT := n
   +2  3+
   |    |
   +0  1+
                     Type: CartesianTensor(1,2,Integer)
```

In general, a tensor of rank k can be formed by making a list of rank k-1 tensors or, alternatively, a k-deep nested list of lists.

```
  t1: CT := [2, 3]
   [2, 3]
                     Type: CartesianTensor(1,2,Integer)

  rank t1
   1
                     Type: PositiveInteger

  t2: CT := [t1, t1]
```

```
   +2  3+
   |    |
   +2  3+
                    Type: CartesianTensor(1,2,Integer)

 t3: CT := [t2, t2]

    +2  3+ +2  3+
   [|    |,|    |]
    +2  3+ +2  3+
                    Type: CartesianTensor(1,2,Integer)

 tt: CT := [t3, t3]; tt := [tt, tt]
    ++2  3+  +2  3++ ++2  3+  +2  3++
    ||    |  |    || ||    |  |    ||
    |+2  3+  +2  3+| |+2  3+  +2  3+|
   [|              |,|              |]
    |+2  3+  +2  3+| |+2  3+  +2  3+|
    ||    |  |    || ||    |  |    ||
    ++2  3+  +2  3++ ++2  3+  +2  3++
                    Type: CartesianTensor(1,2,Integer)

 rank tt
  5
                    Type: PositiveInteger

====================================================================
Multiplication
====================================================================

Given two tensors of rank k1 and k2, the outer product forms a new
tensor of rank k1+k2. Here

 Tmn(i,j,k,l) = Tm(i,j)Tn(k,l)

 Tmn := product(Tm, Tn)
    ++2  3+    +4  6+ +
    ||    |    |    | |
    |+0  1+    +0  2+ |
    |                 |
    |+8  12+  +10  15+|
    ||     |  |      ||
    ++0  4 +  +0   5 ++
                    Type: CartesianTensor(1,2,Integer)

The inner product (contract) forms a tensor of rank k1+k2-2.  This
product generalizes the vector dot product and matrix-vector product
by summing component products along two indices.

Here we sum along the second index of Tm and the first index of Tv. Here
```

```
 Tmv = sum {j=1..dim} Tm(i,j) Tv(j)

 Tmv := contract(Tm,2,Tv,1)
   [11,32]
                     Type: CartesianTensor(1,2,Integer)

The multiplication operator * is scalar multiplication or an inner
product depending on the ranks of the arguments.

If either argument is rank zero it is treated as scalar multiplication.
Otherwise, a*b is the inner product summing the last index of a with the
first index of b.

  Tm*Tv
   [11,32]
                    Type: CartesianTensor(1,2,Integer)

This definition is consistent with the inner product on matrices
and vectors.

  Tmv = m * v
   [11,32] = [11,32]
                    Type: Equation CartesianTensor(1,2,Integer)

====================================================================
Selecting Components
====================================================================

For tensors of low rank (that is, four or less), components can be selected
by applying the tensor to its indices.

  t0()
   8
                    Type: PositiveInteger

  t1(1+1)
   3
                    Type: PositiveInteger

  t2(2,1)
   2
                    Type: PositiveInteger

  t3(2,1,2)
    3
                    Type: PositiveInteger

  Tmn(2,1,2,1)
    0
```

```
                     Type: NonNegativeInteger
```

A general indexing mechanism is provided for a list of indices.

```
  t0[]
    8
                     Type: PositiveInteger

  t1[2]
    3
                     Type: PositiveInteger

  t2[2,1]
    2
                     Type: PositiveInteger
```

The general mechanism works for tensors of arbitrary rank, but is somewhat less efficient since the intermediate index list must be created.

```
  t3[2,1,2]
    3
                     Type: PositiveInteger

  Tmn[2,1,2,1]
    0
                     Type: NonNegativeInteger
```

==
Contraction
==

A "contraction" between two tensors is an inner product, as we have seen above. You can also contract a pair of indices of a single tensor. This corresponds to a "trace" in linear algebra. The expression contract(t,k1,k2) forms a new tensor by summing the diagonal given by indices in position k1 and k2.

This is the tensor given by

```
  xTmn = sum{k=1..dim} Tmn(k,k,i,j)

  cTmn := contract(Tmn,1,2)
    +12  18+
    |      |
    +0   6 +
                         Type: CartesianTensor(1,2,Integer)
```

Since Tmn is the outer product of matrix m and matrix n, the above is equivalent to this.

```
  trace(m) * n
```

```
    +12  18+
    |      |
    +0   6 +
                         Type: SquareMatrix(2,Integer)
```

In this and the next few examples, we show all possible contractions of Tmn and their matrix algebra equivalents.

```
  contract(Tmn,1,2) = trace(m) * n
   +12  18+  +12  18+
   |      |= |      |
   +0   6 +  +0   6 +
                         Type: Equation CartesianTensor(1,2,Integer)

  contract(Tmn,1,3) = transpose(m) * n
   +2  7 +  +2  7 +
   |     |= |     |
   +4  11+  +4  11+
                         Type: Equation CartesianTensor(1,2,Integer)

  contract(Tmn,1,4) = transpose(m) * transpose(n)
   +14  4+  +14  4+
   |     |= |     |
   +19  5+  +19  5+
                         Type: Equation CartesianTensor(1,2,Integer)

  contract(Tmn,2,3) = m * n
   +2  5 +  +2  5 +
   |     |= |     |
   +8  17+  +8  17+
                         Type: Equation CartesianTensor(1,2,Integer)

  contract(Tmn,2,4) = m * transpose(n)
   +8   2+  +8   2+
   |     |= |     |
   +23  5+  +23  5+
                         Type: Equation CartesianTensor(1,2,Integer)

  contract(Tmn,3,4) = trace(n) * m
   +3   6 +  +3   6 +
   |      |= |      |
   +12  15+  +12  15+
                         Type: Equation CartesianTensor(1,2,Integer)
```

==
Transpositions
==

You can exchange any desired pair of indices using the transpose operation.

```
Here the indices in positions one and three are exchanged, that is,
  tTmn(i,j,k,l) = Tmn(k,j,i,l)

  tTmn := transpose(Tmn,1,3)
   ++2  3 +  +4   6 ++
   ||     |  |      ||
   |+8  12+  +10  15+|
   |                 |
   |+0  1+    +0  2+ |
   ||    |    |    | |
   ++0  4+    +0  5+ +
                             Type: CartesianTensor(1,2,Integer)

If no indices are specified, the first and last index are exchanged.

  transpose Tmn
   ++2  8+   +4  10++
   ||    |   |     ||
   |+0  0+   +0  0 +|
   |                |
   |+3  12+  +6  15+|
   ||     |  |     ||
   ++1  4 +  +2  5 ++
                             Type: CartesianTensor(1,2,Integer)

This is consistent with the matrix transpose.

  transpose Tm = transpose m
   +1  4+  +1  4+
   |    |= |    |
   +2  5+  +2  5+
                             Type: Equation CartesianTensor(1,2,Integer)


If a more complicated reordering of the indices is required, then the
reindex operation can be used.  This operation allows the indices to
be arbitrarily permuted.

  rTmn(i,j,k,l) = Tmn(i,l,j,k)

  rTmn := reindex(Tmn, [1,4,2,3])
   ++2  0+   +3  1+ +
   ||    |   |    | |
   |+4  0+   +6  2+ |
   |                |
   |+8   0+  +12  4+|
   ||     |  |     ||
   ++10  0+  +15  5++
                              Type: CartesianTensor(1,2,Integer)
```

```
====================================================================
Arithmetic
====================================================================

Tensors of equal rank can be added or subtracted so arithmetic
expressions can be used to produce new tensors.

  tt := transpose(Tm)*Tn - Tn*transpose(Tm)
   +- 6  - 16+
   |         |
   + 2    6  +
                              Type: CartesianTensor(1,2,Integer)


  Tv*(tt+Tn)
   [- 4,- 11]
                              Type: CartesianTensor(1,2,Integer)

  reindex(product(Tn,Tn),[4,3,2,1])+3*Tn*product(Tm,Tm)
   ++46   84 +  +57   114++
   ||         |  |        ||
   |+174  212+  +228  285+|
   |                      |
   | +18  24+    +17  30+ |
   | |      |    |      | |
   + +57  63+    +63  76+ +
                              Type: CartesianTensor(1,2,Integer)

====================================================================
Specific Tensors
====================================================================

Two specific tensors have properties which depend only on the dimension.

The Kronecker delta satisfies

             +-              -+
             |   1  if i  = j |
delta(i,j) = |                |
             |   0  if i ^= j |
             +-              -+


  delta:  CT := kroneckerDelta()
   +1  0+
   |    |
   +0  1+
                              Type: CartesianTensor(1,2,Integer)
```

```
This can be used to reindex via contraction.

  contract(Tmn, 2, delta, 1) = reindex(Tmn, [1,3,4,2])
   + +2  4+   +0  0++  + +2  4+   +0  0++
   | |    |   |    ||  | |    |   |    ||
   | +3  6+   +1  2+|  | +3  6+   +1  2+|
   |                |= |                |
   |+8   10+  +0  0+|  |+8   10+  +0  0+|
   ||      |  |    ||  ||      |  |    ||
   ++12  15+  +4  5++  ++12  15+  +4  5++
                             Type: Equation CartesianTensor(1,2,Integer)

The Levi Civita symbol determines the sign of a permutation of indices.

  epsilon:CT := leviCivitaSymbol()
   + 0   1+
   |      |
   +- 1  0+
                              Type: CartesianTensor(1,2,Integer)

Here we have:

  epsilon(i1,...,idim)
     = +1  if i1,...,idim is an even permutation of i0,...,i0+dim-1
     = -1  if i1,...,idim is an  odd permutation of i0,...,i0+dim-1
     =  0  if i1,...,idim is not   a permutation of i0,...,i0+dim-1

This property can be used to form determinants.

  contract(epsilon*Tm*epsilon, 1,2) = 2 * determinant m
   - 6= - 6
                            Type: Equation CartesianTensor(1,2,Integer)


====================================================================
Properties of the CartesianTensor domain
====================================================================

GradedModule(R,E) denotes "E-graded R-module", that is, a collection
of R-modules indexed by an abelian monoid E. An element g of G[s] for
some specific s in E is said to be an element of G with degree s.
Sums are defined in each module G[s] so two elements of G can be added
if they have the same degree.  Morphisms can be defined and composed
by degree to give the mathematical category of graded modules.

GradedAlgebra(R,E) denotes "E-graded R-algebra".  A graded algebra is
a graded module together with a degree preserving R-bilinear map,
called the product.

  degree(product(a,b))    = degree(a) + degree(b)
```

```
  product(r*a,b)          = product(a,r*b) = r*product(a,b)
  product(a1+a2,b)        = product(a1,b) + product(a2,b)
  product(a,b1+b2)        = product(a,b1) + product(a,b2)
  product(a,product(b,c)) = product(product(a,b),c)
```

The domain CartesianTensor(i0, dim, R) belongs to the category GradedAlgebra(R, NonNegativeInteger). The non-negative integer degree is the tensor rank and the graded algebra product is the tensor outer product. The graded module addition captures the notion that only tensors of equal rank can be added.

If V is a vector space of dimension dim over R, then the tensor module T[k](V) is defined as

```
 T[0](V) = R
  T[k](V) = T[k-1](V) * V
```

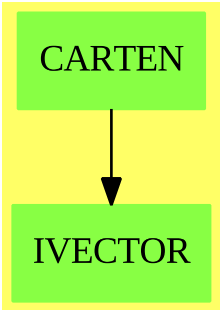
where * denotes the R-module tensor product. CartesianTensor(i0,dim,R) is the graded algebra in which the degree k module is T[k](V).

==
Tensor Calculus
==

It should be noted here that often tensors are used in the context of tensor-valued manifold maps. This leads to the notion of covariant and contravariant bases with tensor component functions transforming in specific ways under a change of coordinates on the manifold. This is no more directly supported by the CartesianTensor domain than it is by the Vector domain. However, it is possible to have the components implicitly represent component maps by choosing a polynomial or expression type for the components. In this case, it is up to the user to satisfy any constraints which arise on the basis of this interpretation.

See Also
o)show CartesianTensor

4.2.1 CartesianTensor (CARTEN)



Exports:

0	1	coerce	contract	degree
elt	hash	kroneckerDelta	latex	leviCivitaSymbol
product	rank	ravel	reindex	retract
retractIfCan	sample	transpose	unravel	?~=?
?.?	?*?	?+?	?-?	-?
?=?				

— domain CARTEN CartesianTensor —

```
)abbrev domain CARTEN CartesianTensor
++ Author: Stephen M. Watt
++ Date Created: December 1986
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: tensor, graded algebra
++ Examples:
++ References:
++ Description:
++ CartesianTensor(minix,dim,R) provides Cartesian tensors with
++ components belonging to a commutative ring R.  These tensors
++ can have any number of indices.  Each index takes values from
++ \spad{minix} to \spad{minix + dim - 1}.

CartesianTensor(minix, dim, R): Exports == Implementation where
    NNI ==> NonNegativeInteger
    I   ==> Integer
    DP  ==> DirectProduct
    SM  ==> SquareMatrix

    minix: Integer
    dim: NNI
```

```
R: CommutativeRing

Exports ==> Join(GradedAlgebra(R, NNI), GradedModule(I, NNI)) with

    coerce: DP(dim, R) -> %
        ++ coerce(v) views a vector as a rank 1 tensor.
        ++
        ++X v:DirectProduct(2,Integer):=directProduct [3,4]
        ++X tv:CartesianTensor(1,2,Integer):=v

    coerce: SM(dim, R)  -> %
        ++ coerce(m) views a matrix as a rank 2 tensor.
        ++
        ++X v:SquareMatrix(2,Integer):=[[1,2],[3,4]]
        ++X tv:CartesianTensor(1,2,Integer):=v

    coerce: List R -> %
        ++ coerce([r_1,...,r_dim]) allows tensors to be constructed
        ++ using lists.
        ++
        ++X v:=[2,3]
        ++X tv:CartesianTensor(1,2,Integer):=v

    coerce: List % -> %
        ++ coerce([t_1,...,t_dim]) allows tensors to be constructed
        ++ using lists.
        ++
        ++X v:=[2,3]
        ++X tv:CartesianTensor(1,2,Integer):=v
        ++X tm:CartesianTensor(1,2,Integer):=[tv,tv]

    rank: % -> NNI
        ++ rank(t) returns the tensorial rank of t (that is, the
        ++ number of indices).  This is the same as the graded module
        ++ degree.
        ++
        ++X CT:=CARTEN(1,2,Integer)
        ++X t0:CT:=8
        ++X rank t0

    elt: (%) -> R
        ++ elt(t) gives the component of a rank 0 tensor.
        ++
        ++X tv:CartesianTensor(1,2,Integer):=8
        ++X elt(tv)
        ++X tv[]

    elt: (%, I) -> R
        ++ elt(t,i) gives a component of a rank 1 tensor.
        ++
```

```
    ++X v:=[2,3]
    ++X tv:CartesianTensor(1,2,Integer):=v
    ++X elt(tv,2)
    ++X tv[2]

elt: (%, I, I) -> R
    ++ elt(t,i,j) gives a component of a rank 2 tensor.
    ++
    ++X v:=[2,3]
    ++X tv:CartesianTensor(1,2,Integer):=v
    ++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
    ++X elt(tm,2,2)
    ++X tm[2,2]

elt: (%, I, I, I) -> R
    ++ elt(t,i,j,k) gives a component of a rank 3 tensor.
    ++
    ++X v:=[2,3]
    ++X tv:CartesianTensor(1,2,Integer):=v
    ++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
    ++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
    ++X elt(tn,2,2,2)
    ++X tn[2,2,2]

elt: (%, I, I, I, I) -> R
    ++ elt(t,i,j,k,l) gives a component of a rank 4 tensor.
    ++
    ++X v:=[2,3]
    ++X tv:CartesianTensor(1,2,Integer):=v
    ++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
    ++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
    ++X tp:CartesianTensor(1,2,Integer):=[tn,tn]
    ++X elt(tp,2,2,2,2)
    ++X tp[2,2,2,2]

elt: (%, List I) -> R
    ++ elt(t,[i1,...,iN]) gives a component of a rank \spad{N} tensor.
    ++
    ++X v:=[2,3]
    ++X tv:CartesianTensor(1,2,Integer):=v
    ++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
    ++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
    ++X tp:CartesianTensor(1,2,Integer):=[tn,tn]
    ++X tq:CartesianTensor(1,2,Integer):=[tp,tp]
    ++X elt(tq,[2,2,2,2,2])

-- This specializes the documentation from GradedAlgebra.
product: (%,%) -> %
    ++ product(s,t) is the outer product of the tensors s and t.
    ++ For example, if \spad{r = product(s,t)} for rank 2 tensors
```

```
    ++ s and t, then \spad{r} is a rank 4 tensor given by
    ++     \spad{r(i,j,k,l) = s(i,j)*t(k,l)}.
    ++
    ++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
    ++X Tm:CartesianTensor(1,2,Integer):=m
    ++X n:SquareMatrix(2,Integer):=matrix [[2,3],[0,1]]
    ++X Tn:CartesianTensor(1,2,Integer):=n
    ++X Tmn:=product(Tm,Tn)

"*": (%, %) -> %
    ++ s*t is the inner product of the tensors s and t which contracts
    ++ the last index of s with the first index of t, that is,
    ++     \spad{t*s = contract(t,rank t, s, 1)}
    ++     \spad{t*s = sum(k=1..N, t[i1,..,iN,k]*s[k,j1,..,jM])}
    ++ This is compatible with the use of \spad{M*v} to denote
    ++ the matrix-vector inner product.
    ++
    ++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
    ++X Tm:CartesianTensor(1,2,Integer):=m
    ++X v:DirectProduct(2,Integer):=directProduct [3,4]
    ++X Tv:CartesianTensor(1,2,Integer):=v
    ++X Tm*Tv

contract:  (%, Integer, %, Integer) -> %
    ++ contract(t,i,s,j) is the inner product of tenors s and t
    ++ which sums along the \spad{k1}-th index of
    ++ t and the \spad{k2}-th index of s.
    ++ For example, if \spad{r = contract(s,2,t,1)} for rank 3 tensors
    ++ rank 3 tensors \spad{s} and \spad{t}, then \spad{r} is
    ++ the rank 4 \spad{(= 3 + 3 - 2)} tensor  given by
    ++     \spad{r(i,j,k,l) = sum(h=1..dim,s(i,h,j)*t(h,k,l))}.
    ++
    ++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
    ++X Tm:CartesianTensor(1,2,Integer):=m
    ++X v:DirectProduct(2,Integer):=directProduct [3,4]
    ++X Tv:CartesianTensor(1,2,Integer):=v
    ++X Tmv:=contract(Tm,2,Tv,1)

contract:  (%, Integer, Integer)    -> %
    ++ contract(t,i,j) is the contraction of tensor t which
    ++ sums along the \spad{i}-th and \spad{j}-th indices.
    ++ For example,  if
    ++ \spad{r = contract(t,1,3)} for a rank 4 tensor t, then
    ++ \spad{r} is the rank 2 \spad{(= 4 - 2)} tensor given by
    ++     \spad{r(i,j) = sum(h=1..dim,t(h,i,h,j))}.
    ++
    ++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
    ++X Tm:CartesianTensor(1,2,Integer):=m
    ++X v:DirectProduct(2,Integer):=directProduct [3,4]
    ++X Tv:CartesianTensor(1,2,Integer):=v
```

```
    ++X Tmv:=contract(Tm,2,1)

transpose: % -> %
    ++ transpose(t) exchanges the first and last indices of t.
    ++ For example, if \spad{r = transpose(t)} for a rank 4
    ++ tensor t, then \spad{r} is the rank 4 tensor given by
    ++     \spad{r(i,j,k,l) = t(l,j,k,i)}.
    ++
    ++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
    ++X Tm:CartesianTensor(1,2,Integer):=m
    ++X transpose(Tm)

transpose: (%, Integer, Integer) -> %
    ++ transpose(t,i,j) exchanges the \spad{i}-th and \spad{j}-th
    ++ indices of t. For example, if \spad{r = transpose(t,2,3)}
    ++ for a rank 4 tensor t, then \spad{r} is the rank 4 tensor
    ++ given by
    ++     \spad{r(i,j,k,l) = t(i,k,j,l)}.
    ++
    ++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
    ++X tm:CartesianTensor(1,2,Integer):=m
    ++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
    ++X transpose(tn,1,2)

reindex: (%, List Integer) -> %
    ++ reindex(t,[i1,...,idim]) permutes the indices of t.
    ++ For example, if \spad{r = reindex(t, [4,1,2,3])}
    ++ for a rank 4 tensor t,
    ++ then \spad{r} is the rank for tensor given by
    ++     \spad{r(i,j,k,l) = t(l,i,j,k)}.
    ++
    ++X n:SquareMatrix(2,Integer):=matrix [[2,3],[0,1]]
    ++X tn:CartesianTensor(1,2,Integer):=n
    ++X p:=product(tn,tn)
    ++X reindex(p,[4,3,2,1])

kroneckerDelta:  () -> %
    ++ kroneckerDelta() is the rank 2 tensor defined by
    ++    \spad{kroneckerDelta()(i,j)}
    ++       \spad{= 1  if i = j}
    ++       \spad{= 0 if  i \^= j}
    ++
    ++X delta:CartesianTensor(1,2,Integer):=kroneckerDelta()

leviCivitaSymbol: () -> %
    ++ leviCivitaSymbol() is the rank \spad{dim} tensor defined by
    ++ \spad{leviCivitaSymbol()(i1,...idim) = +1/0/-1}
    ++ if \spad{i1,...,idim} is an even/is nota /is an odd permutation
    ++ of \spad{minix,...,minix+dim-1}.
    ++
```

```
        ++X lcs:CartesianTensor(1,2,Integer):=leviCivitaSymbol()

    ravel:     % -> List R
        ++ ravel(t) produces a list of components from a tensor such that
        ++   \spad{unravel(ravel(t)) = t}.
        ++
        ++X n:SquareMatrix(2,Integer):=matrix [[2,3],[0,1]]
        ++X tn:CartesianTensor(1,2,Integer):=n
        ++X ravel tn

    unravel:   List R -> %
        ++ unravel(t) produces a tensor from a list of
        ++ components such that
        ++   \spad{unravel(ravel(t)) = t}.

    sample:    () -> %
        ++ sample() returns an object of type %.

Implementation ==> add

    PERM  ==> Vector Integer  -- 1-based entries from 1..n
    INDEX ==> Vector Integer  -- 1-based entries from minix..minix+dim-1


    get   ==> elt$Rep
    set_! ==> setelt$Rep

    -- Use row-major order:
    --   x[h,i,j] <-> x[(h-minix)*dim**2+(i-minix)*dim+(j-minix)]

    Rep := IndexedVector(R,0)

    n:     Integer
    r,s:   R
    x,y,z: %

    ---- Local stuff
    dim2: NNI := dim**2
    dim3: NNI := dim**3
    dim4: NNI := dim**4

    sample()==kroneckerDelta()$%
    int2index(n: Integer, indv: INDEX): INDEX ==
        n < 0 => error "Index error (too small)"
        rnk := #indv
        for i in 1..rnk repeat
            qr := divide(n, dim)
            n  := qr.quotient
            indv.((rnk-i+1) pretend NNI) := qr.remainder + minix
        n ^= 0 => error "Index error (too big)"
```

```
    indv

index2int(indv: INDEX): Integer ==
    n: I := 0
    for i in 1..#indv repeat
        ix := indv.i - minix
        ix<0 or ix>dim-1 => error "Index error (out of range)"
        n := dim*n + ix
    n

lengthRankOrElse(v: Integer): NNI ==
    v = 1    => 0
    v = dim  => 1
    v = dim2 => 2
    v = dim3 => 3
    v = dim4 => 4
    rx := 0
    while v ^= 0 repeat
        qr := divide(v, dim)
        v  := qr.quotient
        if v ^= 0 then
            qr.remainder ^= 0 => error "Rank is not a whole number"
            rx := rx + 1
    rx

-- l must be a list of the numbers 1..#l
mkPerm(n: NNI, l: List Integer): PERM ==
    #l ^= n =>
        error "The list is not a permutation."
    p:    PERM           := new(n, 0)
    seen: Vector Boolean := new(n, false)
    for i in 1..n for e in l repeat
        e < 1 or e > n => error "The list is not a permutation."
        p.i    := e
        seen.e := true
    for e in 1..n repeat
        not seen.e => error "The list is not a permutation."
    p

-- permute s according to p into result t.
permute_!(t: INDEX, s: INDEX, p: PERM): INDEX ==
    for i in 1..#p repeat t.i := s.(p.i)
    t

-- permsign!(v) = 1, 0, or -1  according as
-- v is an even, is not, or is an odd permutation of minix..minix+#v-1.
permsign_!(v: INDEX): Integer ==
    -- sum minix..minix+#v-1.
    maxix := minix+#v-1
    psum  := (((maxix+1)*maxix - minix*(minix-1)) exquo 2)::Integer
```

```
    -- +/v ^= psum => 0
    n := 0
    for i in 1..#v repeat n := n + v.i
    n ^= psum => 0
    -- Bubble sort!  This is pretty grotesque.
    totTrans: Integer := 0
    nTrans:   Integer := 1
    while nTrans ^= 0 repeat
        nTrans := 0
        for i in 1..#v-1 for j in 2..#v repeat
            if v.i > v.j then
                nTrans := nTrans + 1
                e := v.i; v.i := v.j; v.j := e
        totTrans := totTrans + nTrans
    for i in 1..dim repeat
        if v.i ^= minix+i-1 then return 0
    odd? totTrans => -1
    1


---- Exported functions
ravel x ==
    [get(x,i) for i in 0..#x-1]

unravel l ==
    -- lengthRankOrElse #l gives sytnax error
    nz: NNI := # l
    lengthRankOrElse nz
    z := new(nz, 0)
    for i in 0..nz-1 for r in l repeat set_!(z, i, r)
    z

kroneckerDelta() ==
    z := new(dim2, 0)
    for i in 1..dim for zi in 0.. by (dim+1) repeat set_!(z, zi, 1)
    z
leviCivitaSymbol() ==
    nz := dim**dim
    z  := new(nz, 0)
    indv: INDEX := new(dim, 0)
    for i in 0..nz-1 repeat
        set_!(z, i, permsign_!(int2index(i, indv))::R)
    z

-- from GradedModule
degree x ==
    rank x

rank x ==
    n := #x
```

```
        lengthRankOrElse n

    elt(x) ==
        #x ^= 1    => error "Index error (the rank is not 0)"
        get(x,0)
    elt(x, i: I) ==
        #x ^= dim  => error "Index error (the rank is not 1)"
        get(x,(i-minix))
    elt(x, i: I, j: I) ==
        #x ^= dim2 => error "Index error (the rank is not 2)"
        get(x,(dim*(i-minix) + (j-minix)))
    elt(x, i: I, j: I, k: I) ==
        #x ^= dim3 => error "Index error (the rank is not 3)"
        get(x,(dim2*(i-minix) + dim*(j-minix) + (k-minix)))
    elt(x, i: I, j: I, k: I, l: I) ==
        #x ^= dim4 => error "Index error (the rank is not 4)"
        get(x,(dim3*(i-minix)+dim2*(j-minix)+dim*(k-minix)+(l-minix)))

    elt(x, i: List I) ==
        #i ^= rank x => error "Index error (wrong rank)"
        n: I := 0
        for ii in i repeat
            ix := ii - minix
            ix<0 or ix>dim-1 => error "Index error (out of range)"
            n := dim*n + ix
        get(x,n)

    coerce(lr: List R): % ==
        #lr ^= dim => error "Incorrect number of components"
        z := new(dim, 0)
        for r in lr for i in 0..dim-1 repeat set_!(z, i, r)
        z
    coerce(lx: List %): % ==
        #lx ^= dim => error "Incorrect number of slices"
        rx := rank first lx
        for x in lx repeat
            rank x ^= rx => error "Inhomogeneous slice ranks"
        nx := # first lx
        z  := new(dim * nx, 0)
        for x in lx for offz in 0.. by nx repeat
            for i in 0..nx-1 repeat set_!(z, offz + i, get(x,i))
        z

    retractIfCan(x:%):Union(R,"failed") ==
        zero? rank(x) => x()
        "failed"
    Outf ==> OutputForm

    mkOutf(x:%, i0:I, rnk:NNI): Outf ==
        odd? rnk =>
```

```
            rnk1  := (rnk-1) pretend NNI
            nskip := dim**rnk1
            [mkOutf(x, i0+nskip*i, rnk1) for i in 0..dim-1]::Outf
        rnk = 0 =>
            get(x,i0)::Outf
        rnk1  := (rnk-2) pretend NNI
        nskip := dim**rnk1
        matrix [[mkOutf(x, i0+nskip*(dim*i + j), rnk1)
                        for j in 0..dim-1] for i in 0..dim-1]
    coerce(x): Outf ==
        mkOutf(x, 0, rank x)

    0 == 0$R::Rep
    1 == 1$R::Rep

    --coerce(n: I): % == new(1, n::R)
    coerce(r: R): % == new(1,r)

    coerce(v: DP(dim,R)): % ==
        z := new(dim, 0)
        for i in 0..dim-1 for j in minIndex v .. maxIndex v repeat
            set_!(z, i, v.j)
        z
    coerce(m: SM(dim,R)): % ==
        z := new(dim**2, 0)
        offz := 0
        for i in 0..dim-1 repeat
            for j in 0..dim-1 repeat
                set_!(z, offz + j, m(i+1,j+1))
            offz := offz + dim
        z

    x = y ==
        #x ^= #y => false
        for i in 0..#x-1 repeat
           if get(x,i) ^= get(y,i) then return false
        true
    x + y ==
        #x ^= #y => error "Rank mismatch"
        -- z := [xi + yi for xi in x for yi in y]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, get(x,i) + get(y,i))
        z
    x - y ==
        #x ^= #y => error "Rank mismatch"
        -- [xi - yi for xi in x for yi in y]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, get(x,i) - get(y,i))
        z
    - x ==
```

```
        -- [-xi for xi in x]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, -get(x,i))
        z
    n * x ==
        -- [n * xi for xi in x]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, n * get(x,i))
        z
    x * n ==
        -- [n * xi for xi in x]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, n* get(x,i))  -- Commutative!!
        z
    r * x ==
        -- [r * xi for xi in x]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, r * get(x,i))
        z
    x * r ==
        -- [xi*r for xi in x]
        z := new(#x, 0)
        for i in 0..#x-1 repeat set_!(z, i, r* get(x,i))  -- Commutative!!
        z
    product(x, y) ==
        nx := #x; ny := #y
        z  := new(nx * ny, 0)
        for i in 0..nx-1 for ioff in 0.. by ny repeat
            for j in 0..ny-1 repeat
                set_!(z, ioff + j, get(x,i) * get(y,j))
        z
    x * y ==
        rx := rank x
        ry := rank y
        rx = 0 => get(x,0) * y
        ry = 0 => x * get(y,0)
        contract(x, rx, y, 1)

    contract(x, i, j) ==
        rx := rank x
        i < 1 or i > rx or j < 1 or j > rx or i = j =>
            error "Improper index for contraction"
        if i > j then (i,j) := (j,i)

        rl:= (rx- j) pretend NNI; nl:= dim**rl; zol:= 1;     xol:= zol
        rm:= (j-i-1) pretend NNI; nm:= dim**rm; zom:= nl;    xom:= zom*dim
        rh:= (i - 1) pretend NNI; nh:= dim**rh; zoh:= nl*nm
        xoh:= zoh*dim**2
        xok := nl*(1 + nm*dim)
        z   := new(nl*nm*nh, 0)
```

```
        for h in 1..nh _
        for xh in 0.. by xoh for zh in 0.. by zoh repeat
            for m in 1..nm _
            for xm in xh.. by xom for zm in zh.. by zom repeat
                for l in 1..nl _
                for xl in xm.. by xol for zl in zm.. by zol repeat
                    set_!(z, zl, 0)
                    for k in 1..dim for xk in xl.. by xok repeat
                        set_!(z, zl, get(z,zl) + get(x,xk))
        z

    contract(x, i, y, j) ==
        rx := rank x
        ry := rank y

        i < 1 or i > rx or j < 1 or j > ry =>
            error "Improper index for contraction"

        rly:= (ry-j) pretend NNI;  nly:= dim**rly;  oly:= 1;    zoly:= 1
        rhy:= (j -1) pretend NNI; nhy:= dim**rhy
        ohy:= nly*dim; zohy:= zoly*nly
        rlx:= (rx-i) pretend NNI;  nlx:= dim**rlx
        olx:= 1;        zolx:= zohy*nhy
        rhx:= (i -1) pretend NNI;  nhx:= dim**rhx
        ohx:= nlx*dim;  zohx:= zolx*nlx

        z := new(nlx*nhx*nly*nhy, 0)

        for dxh in 1..nhx _
        for xh in 0.. by ohx for zhx in 0.. by zohx repeat
            for dxl in 1..nlx _
            for xl in xh.. by olx for zlx in zhx.. by zolx repeat
                for dyh in 1..nhy _
                for yh in 0.. by ohy for zhy in zlx.. by zohy repeat
                    for dyl in 1..nly _
                    for yl in yh.. by oly for zly in zhy.. by zoly repeat
                        set_!(z, zly, 0)
                        for k in 1..dim _
                        for xk in xl.. by nlx for yk in yl.. by nly repeat
                            set_!(z, zly, get(z,zly)+get(x,xk)*get(y,yk))
        z

    transpose x ==
        transpose(x, 1, rank x)
    transpose(x, i, j) ==
        rx := rank x
        i < 1 or i > rx or j < 1 or j > rx or i = j =>
            error "Improper indicies for transposition"
        if i > j then (i,j) := (j,i)
```

```
            rl:= (rx- j) pretend NNI; nl:= dim**rl; zol:= 1;      zoi := zol*nl
            rm:= (j-i-1) pretend NNI; nm:= dim**rm; zom:= nl*dim; zoj := zom*nm
            rh:= (i - 1) pretend NNI; nh:= dim**rh; zoh:= nl*nm*dim**2
            z   := new(#x, 0)
            for h in 1..nh for zh in 0..  by zoh repeat _
            for m in 1..nm for zm in zh.. by zom repeat _
            for l in 1..nl for zl in zm.. by zol repeat _
                for p in 1..dim _
                for zp in zl.. by zoi for xp in zl.. by zoj repeat
                    for q in 1..dim _
                    for zq in zp.. by zoj for xq in xp.. by zoi repeat
                        set_!(z, zq, get(x,xq))
            z

        reindex(x, l) ==
            nx := #x
            z: % := new(nx, 0)

            rx := rank x
            p  := mkPerm(rx, l)
            xiv: INDEX := new(rx, 0)
            ziv: INDEX := new(rx, 0)

            -- Use permutation
            for i in 0..#x-1 repeat
                pi := index2int(permute_!(ziv, int2index(i,xiv),p))
                set_!(z, pi, get(x,i))
            z
```

— **CARTEN.dotabb** —

```
"CARTEN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CARTEN"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"CARTEN" -> "IVECTOR"
```

4.3 domain CHAR Character

— **Character.input** —

```
)set break resume
```

```
)sys rm -f Character.output
)spool Character.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
chars := [char "a", char "A", char "X", char "8", char "+"]
--R
--R
--R   (1)  [a,A,X,8,+]
--R                                                    Type: List Character
--E 1

--S 2 of 13
space()
--R
--R
--R   (2)
--R                                                         Type: Character
--E 2

--S 3 of 13
quote()
--R
--R
--R   (3)  "
--R                                                         Type: Character
--E 3

--S 4 of 13
escape()
--R
--R
--R   (4)  _
--R                                                         Type: Character
--E 4

--S 5 of 13
[ord c for c in chars]
--R
--R
--R   (5)  [97,65,88,56,43]
--R                                                      Type: List Integer
--E 5

--S 6 of 13
[upperCase c for c in chars]
--R
--R
--R   (6)  [A,A,X,8,+]
```

```
--R                                                        Type: List Character
--E 6

--S 7 of 13
[lowerCase c for c in chars]
--R
--R
--R   (7)  [a,a,x,8,+]
--R                                                        Type: List Character
--E 7

--S 8 of 13
[alphabetic? c for c in chars]
--R
--R
--R   (8)  [true,true,true,false,false]
--R                                                          Type: List Boolean
--E 8

--S 9 of 13
[upperCase? c for c in chars]
--R
--R
--R   (9)  [false,true,true,false,false]
--R                                                          Type: List Boolean
--E 9

--S 10 of 13
[lowerCase? c for c in chars]
--R
--R
--R   (10)  [true,false,false,false,false]
--R                                                          Type: List Boolean
--E 10

--S 11 of 13
[digit? c for c in chars]
--R
--R
--R   (11)  [false,false,false,true,false]
--R                                                          Type: List Boolean
--E 11

--S 12 of 13
[hexDigit? c for c in chars]
--R
--R
--R   (12)  [true,true,false,true,false]
--R                                                          Type: List Boolean
--E 12
```

```
--S 13 of 13
[alphanumeric? c for c in chars]
--R
--R
--R   (13)  [true,true,true,true,false]
--R                                                    Type: List Boolean
--E 13
)spool
)lisp (bye)
```

———-

— Character.help —

```
====================================================================
Character examples
====================================================================

The members of the domain Character are values representing letters,
numerals and other text elements.

Characters can be obtained using String notation.

  chars := [char "a", char "A", char "X", char "8", char "+"]
   [a,A,X,8,+]
                      Type: List Character

Certain characters are available by name. This is the blank character.

  space()

                      Type: Character

This is the quote that is used in strings.

  quote()
   "
                      Type: Character

This is the escape character that allows quotes and other characters
within strings.

  escape()
   _
                      Type: Character

Characters are represented as integers in a machine-dependent way.
The integer value can be obtained using the ord operation.  It is
```

always true that char(ord c) = c and ord(char i) = i, provided that i is in the range 0..size()$Character-1.

```
  [ord c for c in chars]
   [97,65,88,56,43]
                      Type: List Integer
```

The lowerCase operation converts an upper case letter to the corresponding lower case letter. If the argument is not an upper case letter, then it is returned unchanged.

```
  [upperCase c for c in chars]
    [A,A,X,8,+]
                      Type: List Character
```

The upperCase operation converts lower case letters to upper case.

```
  [lowerCase c for c in chars]
   [a,a,x,8,+]
                      Type: List Character
```

A number of tests are available to determine whether characters belong to certain families.

```
  [alphabetic? c for c in chars]
   [true,true,true,false,false]
                      Type: List Boolean

  [upperCase? c for c in chars]
   [false,true,true,false,false]
                      Type: List Boolean

  [lowerCase? c for c in chars]
    [true,false,false,false,false]
                      Type: List Boolean

  [digit? c for c in chars]
   [false,false,false,true,false]
                      Type: List Boolean

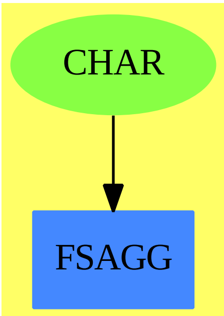
  [hexDigit? c for c in chars]
   [true,true,false,true,false]
                      Type: List Boolean

  [alphanumeric? c for c in chars]
   [true,true,true,true,false]
                      Type: List Boolean
```

See Also:
o)help CharacterClass

```
o )help String
o )show Character
```

4.3.1 Character (CHAR)



See

⇒ "CharacterClass" (CCLASS) 4.4.1 on page 365
⇒ "IndexedString" (ISTRING) 10.14.1 on page 1214
⇒ "String" (STRING) 20.31.1 on page 2565

Exports:

alphabetic?	alphanumeric?	char	coerce	digit?
escape	hash	hexDigit?	index	latex
lookup	lowerCase	lowerCase?	max	min
ord	quote	random	size	space
upperCase	upperCase?	?~=?	?<?	?<=?
?=?	?>?	?>=?		

— domain CHAR Character —

```
)abbrev domain CHAR Character
++ Author: Stephen M. Watt
++ Date Created: July 1986
++ Date Last Updated: June 20, 1991
++ Basic Operations: char
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: character, string
++ Examples:
++ References:
++ Description:
```

```
++ This domain provides the basic character data type.

Character: OrderedFinite() with
  ord: % -> Integer
    ++ ord(c) provides an integral code corresponding to the
    ++ character c.  It is always true that \spad{char ord c = c}.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [ord c for c in chars]
  char: Integer  -> %
    ++ char(i) provides a character corresponding to the integer
    ++ code i. It is always true that \spad{ord char i = i}.
    ++
    ++X [char c for c in [97,65,88,56,43]]
  char: String   -> %
    ++ char(s) provides a character from a string s of length one.
    ++
    ++X [char c for c in ["a","A","X","8","+"]]
  space: () -> %
    ++ space() provides the blank character.
    ++
    ++X space()
  quote: () -> %
    ++ quote() provides the string quote character, \spad{"}.
    ++
    ++X quote()
  escape: () -> %
    ++ escape() provides the escape character, \spad{_}, which
    ++ is used to allow quotes and other characters within
    ++ strings.
    ++
    ++X escape()
  upperCase: % -> %
    ++ upperCase(c) converts a lower case letter to the corresponding
    ++ upper case letter.  If c is not a lower case letter, then
    ++ it is returned unchanged.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [upperCase c for c in chars]
  lowerCase: % -> %
    ++ lowerCase(c) converts an upper case letter to the corresponding
    ++ lower case letter.  If c is not an upper case letter, then
    ++ it is returned unchanged.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [lowerCase c for c in chars]
  digit?: % -> Boolean
    ++ digit?(c) tests if c is a digit character,
    ++ i.e. one of 0..9.
    ++
```

```
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [digit? c for c in chars]
  hexDigit?: % -> Boolean
    ++ hexDigit?(c) tests if c is a hexadecimal numeral,
    ++ i.e. one of 0..9, a..f or A..F.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [hexDigit? c for c in chars]
  alphabetic?: % -> Boolean
    ++ alphabetic?(c) tests if c is a letter,
    ++ i.e. one of a..z or A..Z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [alphabetic? c for c in chars]
  upperCase?: % -> Boolean
    ++ upperCase?(c) tests if c is an upper case letter,
    ++ i.e. one of A..Z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [upperCase? c for c in chars]
  lowerCase?: % -> Boolean
    ++ lowerCase?(c) tests if c is an lower case letter,
    ++ i.e. one of a..z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [lowerCase? c for c in chars]
  alphanumeric?: % -> Boolean
    ++ alphanumeric?(c) tests if c is either a letter or number,
    ++ i.e. one of 0..9, a..z or A..Z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [alphanumeric? c for c in chars]

 == add

  Rep := SingleInteger  -- 0..255

  CC ==> CharacterClass()
  import CC

  OutChars:PrimitiveArray(OutputForm) :=
   construct [CODE_-CHAR(i)$Lisp for i in 0..255]

  minChar := minIndex OutChars

  a = b                  == a =$Rep b
  a < b                  == a <$Rep b
  size()                 == 256
  index n                == char((n - 1)::Integer)
  lookup c               == (1 + ord c)::PositiveInteger
```

```
  char(n:Integer)        == n::%
  ord c                  == convert(c)$Rep
  random()               == char(random()$Integer rem size())
  space                  == QENUM("   ", 0$Lisp)$Lisp
  quote                  == QENUM("_" ", 0$Lisp)$Lisp
  escape                 == QENUM("__ ", 0$Lisp)$Lisp
  coerce(c:%):OutputForm == OutChars(minChar + ord c)
  digit? c               == member?(c pretend Character, digit())
  hexDigit? c            == member?(c pretend Character, hexDigit())
  upperCase? c           == member?(c pretend Character, upperCase())
  lowerCase? c           == member?(c pretend Character, lowerCase())
  alphabetic? c          == member?(c pretend Character, alphabetic())
  alphanumeric? c        == member?(c pretend Character, alphanumeric())

  latex c ==
    concat("\mbox{`", concat(new(1,c pretend Character)$String, "'}")_
       $String)$String

  char(s:String) ==
   (#s) = 1 => s(minIndex s) pretend %
   error "String is not a single character"

  upperCase c ==
    QENUM(PNAME(UPCASE(CODE_-CHAR(ord c)$Lisp)$Lisp)$Lisp,0$Lisp)$Lisp

  lowerCase c ==
    QENUM(PNAME(DOWNCASE(CODE_-CHAR(ord c)$Lisp)$Lisp)$Lisp,0$Lisp)$Lisp
```

———-

— CHAR.dotabb —

```
"CHAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CHAR",shape=ellipse]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"CHAR" -> "FSAGG"
```

———-

4.4 domain CCLASS CharacterClass

— CharacterClass.input —

```
)set break resume
)sys rm -f CharacterClass.output
```

```
)spool CharacterClass.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
cl1:=charClass[char "a",char "e",char "i",char "o",char "u",char "y"]
--R
--R
--R   (1)  "aeiouy"
--R                                                    Type: CharacterClass
--E 1

--S 2 of 16
cl2 := charClass "bcdfghjklmnpqrstvwxyz"
--R
--R
--R   (2)  "bcdfghjklmnpqrstvwxyz"
--R                                                    Type: CharacterClass
--E 2

--S 3 of 16
digit()
--R
--R
--R   (3)  "0123456789"
--R                                                    Type: CharacterClass
--E 3

--S 4 of 16
hexDigit()
--R
--R
--R   (4)  "0123456789ABCDEFabcdef"
--R                                                    Type: CharacterClass
--E 4

--S 5 of 16
upperCase()
--R
--R
--R   (5)  "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
--R                                                    Type: CharacterClass
--E 5

--S 6 of 16
lowerCase()
--R
--R
--R   (6)  "abcdefghijklmnopqrstuvwxyz"
--R                                                    Type: CharacterClass
```

```
--E 6

--S 7 of 16
alphabetic()
--R
--R
--R   (7)  "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz"
--R                                                     Type: CharacterClass
--E 7

--S 8 of 16
alphanumeric()
--R
--R
--R   (8)  "0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz"
--R                                                     Type: CharacterClass
--E 8

--S 9 of 16
member?(char "a", cl1)
--R
--R
--R   (9)  true
--R                                                            Type: Boolean
--E 9

--S 10 of 16
member?(char "a", cl2)
--R
--R
--R   (10)  false
--R                                                            Type: Boolean
--E 10

--S 11 of 16
intersect(cl1, cl2)
--R
--R
--R   (11)  "y"
--R                                                     Type: CharacterClass
--E 11

--S 12 of 16
union(cl1,cl2)
--R
--R
--R   (12)  "abcdefghijklmnopqrstuvwxyz"
--R                                                     Type: CharacterClass
--E 12
```

```
--S 13 of 16
difference(cl1,cl2)
--R
--R
--R   (13)  "aeiou"
--R                                                           Type: CharacterClass
--E 13

--S 14 of 16
intersect(complement(cl1),cl2)
--R
--R
--R   (14)  "bcdfghjklmnpqrstvwxz"
--R                                                           Type: CharacterClass
--E 14

--S 15 of 16
insert!(char "a", cl2)
--R
--R
--R   (15)  "abcdfghjklmnpqrstvwxyz"
--R                                                           Type: CharacterClass
--E 15

--S 16 of 16
remove!(char "b", cl2)
--R
--R
--R   (16)  "acdfghjklmnpqrstvwxyz"
--R                                                           Type: CharacterClass
--E 16
)spool
)lisp (bye)
```

— **CharacterClass.help** —

```
====================================================================
CharacterClass examples
====================================================================

The CharacterClass domain allows classes of characters to be defined
and manipulated efficiently.

Character classes can be created by giving either a string or a list
of characters.

  cl1:=charClass[char "a",char "e",char "i",char "o",char "u",char "y"]
```

```
 "aeiouy"
                    Type: CharacterClass

cl2 := charClass "bcdfghjklmnpqrstvwxyz"
 "bcdfghjklmnpqrstvwxyz"
                    Type: CharacterClass
```

A number of character classes are predefined for convenience.

```
digit()
 "0123456789"
                    Type: CharacterClass

hexDigit()
 "0123456789ABCDEFabcdef"
                    Type: CharacterClass

upperCase()
 "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
                    Type: CharacterClass

lowerCase()
 "abcdefghijklmnopqrstuvwxyz"
                    Type: CharacterClass

alphabetic()
 "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz"
                    Type: CharacterClass

alphanumeric()
 "0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz"
                    Type: CharacterClass
```

You can quickly test whether a character belongs to a class.

```
member?(char "a", cl1)
 true
                    Type: Boolean

member?(char "a", cl2)
 false
                    Type: Boolean
```

Classes have the usual set operations because the CharacterClass domain belongs to the category FiniteSetAggregate(Character).

```
intersect(cl1, cl2)
 "y"
                    Type: CharacterClass
```

```
  union(cl1,cl2)
   "abcdefghijklmnopqrstuvwxyz"
                       Type: CharacterClass

  difference(cl1,cl2)
   "aeiou"
                       Type: CharacterClass

  intersect(complement(cl1),cl2)
   "bcdfghjklmnpqrstvwxz"
                       Type: CharacterClass

You can modify character classes by adding or removing characters.

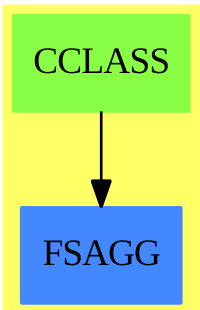
  insert!(char "a", cl2)
   "abcdfghjklmnpqrstvwxyz"
                       Type: CharacterClass

  remove!(char "b", cl2)
   "acdfghjklmnpqrstvwxyz"
                       Type: CharacterClass

See Also:
o )help Character
o )help String
o )show CharacterClass
```

——————

4.4.1 CharacterClass (CCLASS)



See

⇒ "Character" (CHAR) 4.3.1 on page 357
⇒ "IndexedString" (ISTRING) 10.14.1 on page 1214

⇒ "String" (STRING) 20.31.1 on page 2565

Exports:

any?	alphabetic	alphanumeric	bag	brace
brace	cardinality	charClass	coerce	complement
construct	convert	copy	count	count
dictionary	difference	digit	empty	empty?
eq?	eval	eval	eval	eval
every?	extract!	find	hash	hexDigit
index	insert!	inspect	intersect	latex
less?	lookup	lowerCase	map	map!
max	member?	members	min	more?
parts	random	reduce	reduce	reduce
remove	remove	remove!	remove!	removeDuplicates
sample	select	select!	set	size
size?	subset?	symmetricDifference	union	universe
upperCase	#?	?<?	?=?	?~=?

— domain CCLASS CharacterClass —

```
)abbrev domain CCLASS CharacterClass
++ Author: Stephen M. Watt
++ Date Created: July 1986
++ Date Last Updated: June 20, 1991
++ Basic Operations: charClass
++ Related Domains: Character, Bits
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain allows classes of characters to be defined and manipulated
++ efficiently.

CharacterClass: Join(SetCategory, ConvertibleTo String,
  FiniteSetAggregate Character, ConvertibleTo List Character) with
        charClass: String -> %
          ++ charClass(s) creates a character class which contains
          ++ exactly the characters given in the string s.
        charClass: List Character -> %
          ++ charClass(l) creates a character class which contains
          ++ exactly the characters given in the list l.
        digit: constant -> %
          ++ digit() returns the class of all characters
          ++ for which digit? is true.
        hexDigit: constant -> %
          ++ hexDigit() returns the class of all characters for which
          ++ hexDigit? is true.
```

```
    upperCase: constant -> %
      ++ upperCase() returns the class of all characters for which
      ++ upperCase? is true.
    lowerCase:  constant -> %
      ++ lowerCase() returns the class of all characters for which
      ++ lowerCase? is true.
    alphabetic  :  constant -> %
      ++ alphabetic() returns the class of all characters for which
      ++ alphabetic? is true.
    alphanumeric:  constant -> %
      ++ alphanumeric() returns the class of all characters for which
      ++ alphanumeric? is true.

== add
    Rep := IndexedBits(0)
    N   := size()$Character

    a, b: %

    digit()         == charClass "0123456789"
    hexDigit()      == charClass "0123456789abcdefABCDEF"
    upperCase()     == charClass "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
    lowerCase()     == charClass "abcdefghijklmnopqrstuvwxyz"
    alphabetic()    == union(upperCase(), lowerCase())
    alphanumeric()  == union(alphabetic(), digit())

    a = b           == a =$Rep b

    member?(c, a)   == a(ord c)
    union(a,b)      == Or(a, b)
    intersect (a,b) == And(a, b)
    difference(a,b) == And(a, Not b)
    complement a    == Not a

    convert(cl):String ==
      construct(convert(cl)@List(Character))
    convert(cl:%):List(Character) ==
      [char(i) for i in 0..N-1 | cl.i]

    charClass(s: String) ==
      cl := new(N, false)
      for i in minIndex(s)..maxIndex(s) repeat cl(ord s.i) := true
      cl

    charClass(l: List Character) ==
      cl := new(N, false)
      for c in l repeat cl(ord c) := true
      cl

    coerce(cl):OutputForm == (convert(cl)@String)::OutputForm
```

```
        -- Stuff to make a legal SetAggregate view
        # a == (n := 0; for i in 0..N-1 | a.i repeat n := n+1; n)
        empty():% == charClass []
        brace():% == charClass []

        insert_!(c, a) == (a(ord c) := true; a)
        remove_!(c, a) == (a(ord c) := false; a)

        inspect(a) ==
            for i in 0..N-1 | a.i repeat
                 return char i
            error "Cannot take a character from an empty class."
        extract_!(a) ==
            for i in 0..N-1 | a.i repeat
                 a.i := false
                 return char i
            error "Cannot take a character from an empty class."

        map(f, a) ==
            b := new(N, false)
            for i in 0..N-1 | a.i repeat b(ord f char i) := true
            b

        temp: % := new(N, false)$Rep
        map_!(f, a) ==
            fill_!(temp, false)
            for i in 0..N-1 | a.i repeat temp(ord f char i) := true
            copyInto_!(a, temp, 0)

        parts a ==
            [char i for i in 0..N-1 | a.i]
```

— CCLASS.dotabb —

```
"CCLASS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CCLASS"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"CCLASS" -> "FSAGG"
```

4.5 domain CLIF CliffordAlgebra[?, ?]

4.5.1 Vector (linear) spaces

This information is originally from Paul Leopardi's presentation on the *Introduction to Clifford Algebras* and is included here as an outline with his permission. Further details are based on the book by Doran and Lasenby called *Geometric Algebra for Physicists*.

Consider the various kinds of products that can occur between vectors. There are scalar and vector products from 3D geometry. There are the complex and quaterion products. There is also the *outer* or *exterior* product.

Vector addition commutes:

$$a + b = b + a$$

Vector addtion is associative:

$$a + (b + c) = (a + b) + c$$

The identity vector exists:

$$a + 0 = a$$

Every vector has an inverse:

$$a + (-a) = 0$$

If we consider vectors to be directed line segments, thus establishing a geometric meaning for a vector, then each of these properties has a geometric meaning.

A multiplication operator exists between scalars and vectors with the properties:

$$\lambda(a + b) = \lambda a + \lambda b$$

$$(\lambda + \mu)a = \lambda a + \mu a$$

$$(\lambda\mu)a = \lambda(\mu a)$$

$$\text{If } 1\lambda = \lambda \text{ for all scalars } \lambda \text{ then } 1a = a \text{ for all vectors } a$$

These properties completely define a vector (linear) space. The + operation for scalar arithmetic is not the same as the + operation for vectors.

Definition: Isomorphic The vector space A is isomorphic to the vector space B if their exists a one-to-one correspondence between their elements which preserves sums and there is a one-to-one correspondence between the scalars which preserves sums and products.

Definition: Subspace Vector space B is a subspace of vector space A if all of the elements of B are contained in A and they share the same scalars.

Definition: Linear Combination Given vectors $a_1, \ldots, a_n$ the vector b is a linear combination of the vectors if we can find scalars λ_i such that

$$b = \lambda_1 a_1 + \ldots + \lambda_n a_n = \sum_{k=1}^{n} \lambda_i a_i$$

Definition: Linearly Independent If there exists scalars λ_i such that

$$\lambda_1 a_1 + \ldots + \lambda_n a_n = 0$$

and at least one of the λ_i is not zero then the vectors $a_1, \ldots, a_n$ are linearly dependent. If no such scalars exist then the vectors are linearly independent.

Definition: Span If every vector can be written as a linear combination of a fixed set of vectors $a_1, \ldots, a_n$ then this set of vectors is said to span the vector space.

Definition: Basis If a set of vectors $a_1, \ldots, a_n$ is linearly independent and spans a vector space A then the vectors form a basis for A.

Definition: Dimension The dimension of a vector space is the number of basis elements, which is unique since all bases of a vector space have the same number of elements.

4.5.2 Quadratic Forms[?]

For vector space $\mathbb{V}$ over field $\mathbb{F}$, characteristic $\neq 2$:

Map $f : \mathbb{V} \to \mathbb{F}$, with

$$f(\lambda x) = \lambda^2 f(x), \forall \lambda \in \mathbb{F}, x \in \mathbb{V}$$

$f(x) = b(x, x)$, where

$$b : \mathbb{V} \text{ x } \mathbb{V} \to \mathbb{F} \text{ ,given by}$$

$$b(x, y) := \frac{1}{2}(f(x + y) - f(x) = f(y))$$

is a symmetric bilinear form

4.5.3 Quadratic spaces, Clifford Maps[?, ?]

A quadratic space is the pair($\mathbb{V}$,f), where f is a quadratic form on $\mathbb{V}$

A Clifford map is a vector space homomorphism

$$\rho : \mathbb{V} \to \mathbb{A}$$

where $\mathbb{A}$ is an associated algebra, and

$$(\rho v)^2 = f(v), \quad \forall v \in \mathbb{V}$$

4.5.4 Universal Clifford algebras[?]

The *universal Clifford algebra* $Cl(f)$ for the quadratic space $(\mathbb{V}, f)$ is the algebra generated by the image of the Clifford map ϕ_f such that $Cl(f)$ is the universal initial object such that $\forall$ suitable algebra $\mathbb{A}$ with Clifford map $\phi_{\mathbb{A}} \exists$ a homomorphism

$$P_{\mathbb{A}} : Cl(f) \to \mathbb{A}$$

$$\rho_{\mathbb{A}} = P_{\mathbb{A}} \circ \rho_f$$

4.5.5 Real Clifford algebras $\mathbb{R}_{p,q}$[?]

The real quadratic space $\mathbb{R}^{p,q}$ is $\mathbb{R}^{p+q}$ with

$$\phi(x) := -\sum_{k:=-q}^{-1} x_k^2 + \sum_{k=1}^{p} x_k^2$$

For each $p, q \in \mathbb{N}$, the real universal Clifford algebra for $\mathbb{R}^{p,q}$ is called $\mathbb{R}_{p,q}$

$\mathbb{R}_{p,q}$ is isomorphic to some matrix algebra over one of: $\mathbb{R}$,$\mathbb{R} \oplus \mathbb{R}$,$\mathbb{C}$, $\mathbb{H}$,$\mathbb{H} \oplus \mathbb{H}$

For example, $\mathbb{R}_{1,1} \cong \mathbb{R}(2)$

4.5.6 Notation for integer sets

For $S \subseteq \mathbb{Z}$, define

$$\sum_{k \in S} f_k := \sum_{k=\min\ S, k \in S}^{\max\ S} f_k$$

$$\prod_{k \in S} f_k := \prod_{k=\min\ S, k \in S}^{\max\ S} f_k$$

$$\mathbb{P}(S) := \text{ the } \textit{power set} \text{ of } S$$

For $m \leq n \in \mathbb{Z}$, define

$$\zeta(m, n) := \{m, m+1, \ldots, n-1, n\} \backslash \{0\}$$

4.5.7 Frames for Clifford algebras[?, ?, ?]

A *frame* is an ordered basis $(\gamma_{-q}, \ldots, \gamma_p)$ for $\mathbb{R}^{p,q}$ which puts a quadratic form into the canonical form ϕ

For $p, q \in \mathbb{N}$, embed the frame for $\mathbb{R}^{p,q}$ into $\mathbb{R}_{p,q}$ via the maps

$$\gamma : \zeta(-q, p) \to \mathbb{R}^{p,q}$$

$$\rho : \mathbb{R}^{p,q} \to \mathbb{R}_{p,q}$$

$$(\rho\gamma k)^2 = \phi\gamma k = \text{ sgn } k$$

4.5.8 Real frame groups[?, ?]

For $p, q \in \mathbb{N}$, define the real *frame group* $\mathbb{G}_{p,q}$ via the map

$$g : \zeta(-q, p) \to \mathbb{G}_{p,q}$$

with generators and relations

$$\langle \mu, g_k | \mu g_k = g_k \mu, \quad \mu^2 = 1,$$

$$(g_k)^2 = \begin{cases} \mu, & \text{if } k < 0 \\ 1 & \text{if } k > 0 \end{cases}$$

$$g_k g_m = \mu g_m g_k \quad \forall k \neq m \rangle$$

4.5.9 Canonical products[?, ?, ?]

The real frame group $\mathbb{G}_{p,q}$ has order 2^{p+q+1}

Each member w can be expressed as the canonically ordered product

$$w = \mu^a \prod_{k \in T} g_k$$

$$= \mu^a \prod_{k=-q, k\neq 0}^{p} g_k^{b_k}$$

where $T \subseteq \zeta(-q,p), a, b_k \in \{0,1\}$

4.5.10 Clifford algebra of frame group[?, ?, ?, ?]

For $p, q \in \mathbb{N}$ embed $\mathbb{G}_{p,q}$ into $\mathbb{R}_{p,q}$ via the map

$$\alpha \mathbb{G}_{p,q} \to \mathbb{R}_{p,q}$$

$$\alpha 1 := 1, \qquad \alpha\mu := -1$$

$$\alpha g_k := \rho\gamma_k, \qquad \alpha(gh) := (\alpha g)(\alpha h)$$

Define *basis elements* via the map

$$e : \mathbb{P}\zeta(-q,p) \to \mathbb{R}_{p,q}, \qquad e_T := \alpha \prod_{k \in T} g_k$$

Each $a \in \mathbb{R}_{p,q}$ can be expressed as

$$a = \sum_{T \subseteq \zeta(-q,p)} a_T e_T$$

4.5.11 Neutral matrix representations[?, ?, ?]

The *representation map* P_m and *representation matrix* R_m make the following diagram commute:

$$\begin{tikzcd} \mathbb{R}_{m,m} \arrow[r, "\text{coord}"] \arrow[d, "P_m"] & \mathbb{R}^{4^m} \arrow[d, "R_m"] \\ \mathbb{R}(2^m) \arrow[r, "\text{reshape}"'] & \mathbb{R}^{4^m} \end{tikzcd}$$

— CliffordAlgebra.input —

```
)set break resume
)sys rm -f CliffordAlgebra.output
)spool CliffordAlgebra.output
)set message test on
)set message auto off
)clear all
--S 1 of 36
K := Fraction Polynomial Integer
--R
--R
--R   (1)  Fraction Polynomial Integer
--R                                                            Type: Domain
--E 1

--S 2 of 36
m := matrix [ [-1] ]
--R
--R
--R   (2)  [- 1]
--R                                                    Type: Matrix Integer
--E 2

--S 3 of 36
C := CliffordAlgebra(1, K, quadraticForm m)
--R
--R
--R   (3)  CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--R                                                            Type: Domain
--E 3

--S 4 of 36
i: C := e(1)
--R
--R
```

```
--R   (4)  e
--R         1
--R                  Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 4

--S 5 of 36
x := a + b * i
--R
--R
--R   (5)  a + b e
--R               1
--R                  Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 5

--S 6 of 36
y := c + d * i
--R
--R
--R   (6)  c + d e
--R               1
--R                  Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 6

--S 7 of 36
x * y
--R
--R
--R   (7)  - b d + a c + (a d + b c)e
--R                                  1
--R                  Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 7
)clear all

--S 8 of 36
K := Fraction Polynomial Integer
--R
--R
--R   (1)  Fraction Polynomial Integer
--R                                                                  Type: Domain
--E 8

--S 9 of 36
m := matrix [ [-1,0],[0,-1] ]
--R
--R
--R        +- 1   0 +
--R   (2)  |        |
--R        + 0   - 1+
--R                                                          Type: Matrix Integer
--E 9
```

```
--S 10 of 36
H  := CliffordAlgebra(2, K, quadraticForm m)
--R
--R
--R   (3)  CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--R                                                                 Type: Domain
--E 10

--S 11 of 36
i: H  := e(1)
--R
--R
--R   (4)  e
--R         1
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 11

--S 12 of 36
j: H  := e(2)
--R
--R
--R   (5)  e
--R         2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 12

--S 13 of 36
k: H  := i * j
--R
--R
--R   (6)  e e
--R         1 2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 13

--S 14 of 36
x := a + b * i + c * j + d * k
--R
--R
--R   (7)  a + b e  + c e  + d e e
--R               1      2      1 2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 14

--S 15 of 36
y := e + f * i + g * j + h * k
--R
--R
--R   (8)  e + f e  + g e  + h e e
```

```
--R               1      2      1 2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 15

--S 16 of 36
x + y
--R
--R
--R   (9)  e + a + (f + b)e  + (g + c)e  + (h + d)e e
--R                        1          2          1 2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 16

--S 17 of 36
x * y
--R
--R
--R   (10)
--R     - d h - c g - b f + a e + (c h - d g + a f + b e)e
--R                                                      1
--R   +
--R     (- b h + a g + d f + c e)e  + (a h + b g - c f + d e)e e
--R                               2                          1 2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 17

--S 18 of 36
y * x
--R
--R
--R   (11)
--R     - d h - c g - b f + a e + (- c h + d g + a f + b e)e
--R                                                        1
--R   +
--R     (b h + a g - d f + c e)e  + (a h - b g + c f + d e)e e
--R                             2                          1 2
--R                  Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 18
)clear all

--S 19 of 36
K := Fraction Polynomial Integer
--R
--R
--R   (1)  Fraction Polynomial Integer
--R                                                                 Type: Domain
--E 19

--S 20 of 36
Ext := CliffordAlgebra(3, K, quadraticForm 0)
```

```
--R
--R
--R   (2)  CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--R                                                               Type: Domain
--E 20

--S 21 of 36
i: Ext := e(1)
--R
--R
--R   (3)  e
--R         1
--R                     Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 21

--S 22 of 36
j: Ext := e(2)
--R
--R
--R   (4)  e
--R         2
--R                     Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 22

--S 23 of 36
k: Ext := e(3)
--R
--R
--R   (5)  e
--R         3
--R                     Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 23

--S 24 of 36
x := x1*i + x2*j + x3*k
--R
--R
--R   (6)  x1 e  + x2 e  + x3 e
--R            1       2       3
--R                     Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 24

--S 25 of 36
y := y1*i + y2*j + y3*k
--R
--R
--R   (7)  y1 e  + y2 e  + y3 e
--R            1       2       3
--R                     Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 25
```

```
--S 26 of 36
x + y
--R
--R
--R   (8)  (y1 + x1)e  + (y2 + x2)e  + (y3 + x3)e
--R                  1             2             3
--R                  Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 26

--S 27 of 36
x * y + y * x
--R
--R
--R   (9)  0
--R                  Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 27

--S 28 of 36
dual2 a == coefficient(a,[2,3]) * i + coefficient(a,[3,1]) * j + coefficient(a,[1,2]) * k
--R
--R                                                                   Type: Void
--E 28

--S 29 of 36
dual2(x*y)
--R
--R   Compiling function dual2 with type CliffordAlgebra(3,Fraction
--R      Polynomial Integer,MATRIX) -> CliffordAlgebra(3,Fraction
--R      Polynomial Integer,MATRIX)
--R
--R   (11)  (x2 y3 - x3 y2)e  + (- x1 y3 + x3 y1)e  + (x1 y2 - x2 y1)e
--R                         1                     2                  3
--R                  Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 29
)clear all

--S 30 of 36
K := Fraction Integer
--R
--R
--R   (1)  Fraction Integer
--R                                                                 Type: Domain
--E 30

--S 31 of 36
g := matrix [ [1,0,0,0], [0,-1,0,0], [0,0,-1,0], [0,0,0,-1] ]
--R
--R
--R        +1   0    0    0 +
```

```
--R        |                |
--R        |0  - 1   0    0 |
--R   (2)  |                |
--R        |0   0   - 1   0 |
--R        |                |
--R        +0   0    0   - 1+
--R                                                     Type: Matrix Integer
--E 31

--S 32 of 36
D := CliffordAlgebra(4,K, quadraticForm g)
--R
--R
--R   (3)  CliffordAlgebra(4,Fraction Integer,MATRIX)
--R                                                            Type: Domain
--E 32

--S 33 of 36
gam := [e(i)$D for i in 1..4]
--R
--R
--R   (4)  [e ,e ,e ,e ]
--R          1  2  3  4
--R                        Type: List CliffordAlgebra(4,Fraction Integer,MATRIX)
--E 33

--S 34 of 36
m := 1; n:= 2; r := 3; s := 4;
--R
--R
--R                                                     Type: PositiveInteger
--E 34

--S 35 of 36
lhs := reduce(+, [reduce(+, [ g(l,t)*gam(l)*gam(m)*gam(n)*gam(r)*gam(s)*gam(t) for l in 1..4]) for t in
--R
--R
--R   (6)  - 4e e e e
--R            1 2 3 4
--R                             Type: CliffordAlgebra(4,Fraction Integer,MATRIX)
--E 35

--S 36 of 36
rhs := 2*(gam s * gam m*gam n*gam r + gam r*gam n*gam m*gam s)
--R
--R
--R   (7)  - 4e e e e
--R            1 2 3 4
--R                             Type: CliffordAlgebra(4,Fraction Integer,MATRIX)
--E 36
```

```
)spool
)lisp (bye)
```

— **CliffordAlgebra.help** —

```
====================================================================
CliffordAlgebra examples
====================================================================

CliffordAlgebra(n,K,Q) defines a vector space of dimension 2^n over
the field K with a given quadratic form Q.  If {e1..en} is a basis for
K^n then


{ 1,
  e(i)          1 <= i <= n,
  e(i1)*e(i2)   1 <= i1 < i2 <=n,
  ...,
  e(1)*e(2)*...*e(n) }

is a basis for the Clifford algebra. The algebra is defined by the relations

  e(i)*e(i) = Q(e(i))
  e(i)*e(j) = -e(j)*e(i),  for i ^= j

Examples of Clifford Algebras are gaussians (complex numbers),
quaternions, exterior algebras and spin algebras.

====================================================================
The Complex Numbers as a Clifford Algebra
====================================================================

This is the field over which we will work, rational functions with
integer coefficients.

  K := Fraction Polynomial Integer
   Fraction Polynomial Integer
                         Type: Domain

We use this matrix for the quadratic form.

  m := matrix [ [-1] ]
   [- 1]
                         Type: Matrix Integer

We get complex arithmetic by using this domain.
```

```
  C := CliffordAlgebra(1, K, quadraticForm m)
   CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
                         Type: Domain

Here is i, the usual square root of -1.

  i: C := e(1)
   e
    1
               Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)

Here are some examples of the arithmetic.

  x := a + b * i
   a + b e
          1
               Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)

  y := c + d * i
   c + d e
          1
               Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)

  x * y
   - b d + a c + (a d + b c)e
                             1
               Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)

====================================================================
The Quaternion Numbers as a Clifford Algebra
====================================================================

This is the field over which we will work, rational functions with
integer coefficients.

  K := Fraction Polynomial Integer
   Fraction Polynomial Integer
                      Type: Domain

We use this matrix for the quadratic form.

  m := matrix [ [-1,0],[0,-1] ]
   +- 1   0 +
   |        |
   + 0   - 1+
                      Type: Matrix Integer

The resulting domain is the quaternions.

  H  := CliffordAlgebra(2, K, quadraticForm m)
```

```
   CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
                     Type: Domain
```

We use Hamilton's notation for i, j, k.

```
  i: H  := e(1)
   e
    1
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  j: H  := e(2)
   e
    2
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  k: H  := i * j
   e e
    1 2
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  x := a + b * i + c * j + d * k
   a + b e  + c e  + d e e
          1      2      1 2
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  y := e + f * i + g * j + h * k
   e + f e  + g e  + h e e
          1      2      1 2
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  x + y
   e + a + (f + b)e  + (g + c)e  + (h + d)e e
                   1           2           1 2
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  x * y
   - d h - c g - b f + a e + (c h - d g + a f + b e)e
                                                     1
   +
     (- b h + a g + d f + c e)e  + (a h + b g - c f + d e)e e
                               2                           1 2
             Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)

  y * x
   - d h - c g - b f + a e + (- c h + d g + a f + b e)e
                                                       1
   +
     (b h + a g - d f + c e)e  + (a h - b g + c f + d e)e e
                             2                           1 2
                 Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
====================================================================
The Exterior Algebra on a Three Space
====================================================================

This is the field over which we will work, rational functions with
integer coefficients.

  K := Fraction Polynomial Integer
   Fraction Polynomial Integer
                  Type: Domain

If we chose the three by three zero quadratic form, we obtain
the exterior algebra on e(1),e(2),e(3).

  Ext := CliffordAlgebra(3, K, quadraticForm 0)
   CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
                  Type: Domain

This is a three dimensional vector algebra.  We define i, j, k as the
unit vectors.

  i: Ext := e(1)
   e
    1
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

  j: Ext := e(2)
   e
    2
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

  k: Ext := e(3)
   e
    3
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

Now it is possible to do arithmetic.

  x := x1*i + x2*j + x3*k
   x1 e  + x2 e  + x3 e
       1       2       3
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

  y := y1*i + y2*j + y3*k
   y1 e  + y2 e  + y3 e
       1       2       3
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

  x + y
```

```
   (y1 + x1)e  + (y2 + x2)e  + (y3 + x3)e
             1            2            3
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

  x * y + y * x
   0
             Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
```

On an n space, a grade p form has a dual n-p form. In particular, in three space the dual of a grade two element identifies

```
  e1*e2 -> e3, e2*e3 -> e1, e3*e1 -> e2.

  dual2 a == coefficient(a,[2,3]) * i + coefficient(a,[3,1]) * j + coefficient(a,[1,2]) * k
                      Type: Void
```

The vector cross product is then given by this.

```
  dual2(x*y)
   (x2 y3 - x3 y2)e  + (- x1 y3 + x3 y1)e  + (x1 y2 - x2 y1)e
                   1                     2                  3
           Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
```

```
====================================================================
The Dirac Spin Algebra
====================================================================
```

In this section we will work over the field of rational numbers.

```
  K := Fraction Integer
   Fraction Integer
                       Type: Domain
```

We define the quadratic form to be the Minkowski space-time metric.

```
  g := matrix [ [1,0,0,0], [0,-1,0,0], [0,0,-1,0], [0,0,0,-1] ]
   +1   0    0    0 +
   |                |
   |0  - 1   0    0 |
   |                |
   |0   0   - 1   0 |
   |                |
   +0   0    0   - 1+
                       Type: Matrix Integer
```

We obtain the Dirac spin algebra used in Relativistic Quantum Field Theory.

```
  D := CliffordAlgebra(4,K, quadraticForm g)
   CliffordAlgebra(4,Fraction Integer,MATRIX)
                       Type: Domain
```

```
The usual notation for the basis is gamma with a superscript.  For
Axiom input we will use gam(i):

  gam := [e(i)$D for i in 1..4]
   [e ,e ,e ,e ]
     1  2  3  4
                 Type: List CliffordAlgebra(4,Fraction Integer,MATRIX)

There are various contraction identities of the form

  g(l,t)*gam(l)*gam(m)*gam(n)*gam(r)*gam(s)*gam(t) =
      2*(gam(s)gam(m)gam(n)gam(r) + gam(r)*gam(n)*gam(m)*gam(s))

where a sum over l and t is implied.

Verify this identity for particular values of m,n,r,s.

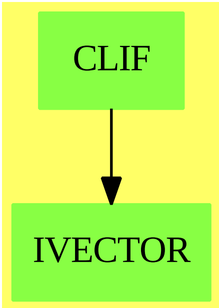
  m := 1; n:= 2; r := 3; s := 4;
                      Type: PositiveInteger

  lhs := reduce(+, [reduce(+, [ g(l,t)*gam(l)*gam(m)*gam(n)*gam(r)*gam(s)*gam(t) for l in 1..4]) for t i
   - 4e e e e
       1 2 3 4
                      Type: CliffordAlgebra(4,Fraction Integer,MATRIX)

  rhs := 2*(gam s * gam m*gam n*gam r + gam r*gam n*gam m*gam s)
   - 4e e e e
       1 2 3 4
                      Type: CliffordAlgebra(4,Fraction Integer,MATRIX)

See Also:
o )help Complex
o )help Quaternion
o )show CliffordAlgebra
o $AXIOM/doc/src/algebra/clifford.spad
```

4.5.12 CliffordAlgebra (CLIF)



See

⇒ "QuadraticForm" (QFORM) 18.1.1 on page 2114

Exports:

0	1	characteristic	coefficient	coerce
dimension	e	hash	latex	monomial
one?	recip	sample	subtractIfCan	zero?
?~=?	?*?	?**?	?^?	?+?
?-?	-?	?/?	?=?	

— domain CLIF CliffordAlgebra —

```
)abbrev domain CLIF CliffordAlgebra
++ Author: Stephen M. Watt
++ Date Created: August 1988
++ Date Last Updated: May 17, 1991
++ Basic Operations: wholeRadix, fractRadix, wholeRagits, fractRagits
++ Related Domains: QuadraticForm, Quaternion, Complex
++ Also See:
++ AMS Classifications:
++ Keywords: clifford algebra, grassman algebra, spin algebra
++ Examples:
++ References:
++
++ Description:
++ CliffordAlgebra(n, K, Q) defines a vector space of dimension \spad{2**n}
++ over K, given a quadratic form Q on \spad{K**n}.
++
++ If \spad{e[i]}, \spad{1<=i<=n} is a basis for \spad{K**n} then
++ 1, \spad{e[i]} (\spad{1<=i<=n}), \spad{e[i1]*e[i2]}
++ (\spad{1<=i1<i2<=n}),...,\spad{e[1]*e[2]*..*e[n]}
++ is a basis for the Clifford Algebra.
++
++ The algebra is defined by the relations\br
++ \tab{5}\spad{e[i]*e[j] = -e[j]*e[i]}  (\spad{i \~~= j}),\br
++ \tab{5}\spad{e[i]*e[i] = Q(e[i])}
```

```
++
++ Examples of Clifford Algebras are: gaussians, quaternions, exterior
++ algebras and spin algebras.

CliffordAlgebra(n, K, Q): T == Impl where
    n: PositiveInteger
    K: Field
    Q: QuadraticForm(n, K)

    PI ==> PositiveInteger
    NNI==> NonNegativeInteger

    T ==> Join(Ring, Algebra(K), VectorSpace(K)) with
        e: PI -> %
          ++ e(n) produces the appropriate unit element.
        monomial: (K, List PI) -> %
          ++ monomial(c,[i1,i2,...,iN]) produces the value given by
          ++ \spad{c*e(i1)*e(i2)*...*e(iN)}.
        coefficient:  (%, List PI) -> K
          ++ coefficient(x,[i1,i2,...,iN])  extracts the coefficient of
          ++ \spad{e(i1)*e(i2)*...*e(iN)} in x.
        recip: % -> Union(%, "failed")
          ++ recip(x) computes the multiplicative inverse of x or "failed"
          ++ if x is not invertible.

    Impl ==> add
        Qeelist :=  [Q unitVector(i::PositiveInteger) for i in 1..n]
        dim     :=  2**n

        Rep     := PrimitiveArray K

        New     ==> new(dim, 0$K)$Rep

        x, y, z: %
        c: K
        m: Integer

        characteristic() == characteristic()$K
        dimension()      == dim::CardinalNumber

        x = y ==
            for i in 0..dim-1 repeat
                if x.i ^= y.i then return false
            true

        x + y == (z := New; for i in 0..dim-1 repeat z.i := x.i + y.i; z)
        x - y == (z := New; for i in 0..dim-1 repeat z.i := x.i - y.i; z)
        - x   == (z := New; for i in 0..dim-1 repeat z.i := - x.i; z)
        m * x == (z := New; for i in 0..dim-1 repeat z.i := m*x.i; z)
        c * x == (z := New; for i in 0..dim-1 repeat z.i := c*x.i; z)
```

```
0            == New
1            == (z := New; z.0 := 1; z)
coerce(m): % == (z := New; z.0 := m::K; z)
coerce(c): % == (z := New; z.0 := c; z)

e b ==
    b::NNI > n => error "No such basis element"
    iz := 2**((b-1)::NNI)
    z := New; z.iz := 1; z

-- The ei*ej products could instead be precomputed in
-- a (2**n)**2 multiplication table.
addMonomProd(c1: K, b1: NNI, c2: K, b2: NNI, z: %): % ==
    c  := c1 * c2
    bz := b2
    for i in 0..n-1 | bit?(b1,i) repeat
        -- Apply rule  ei*ej = -ej*ei for i^=j
        k := 0
        for j in i+1..n-1 | bit?(b1, j) repeat k := k+1
        for j in 0..i-1   | bit?(bz, j) repeat k := k+1
        if odd? k then c := -c
        -- Apply rule  ei**2 = Q(ei)
        if bit?(bz,i) then
            c := c * Qeelist.(i+1)
            bz:= (bz - 2**i)::NNI
        else
            bz:= bz + 2**i
    z.bz := z.bz + c
    z

x * y ==
    z := New
    for ix in 0..dim-1 repeat
        if x.ix ^= 0 then for iy in 0..dim-1 repeat
            if y.iy ^= 0 then addMonomProd(x.ix,ix,y.iy,iy,z)
    z

canonMonom(c: K, lb: List PI): Record(coef: K, basel: NNI) ==
    -- 0. Check input
    for b in lb repeat b > n => error "No such basis element"

    -- 1. Apply identity ei*ej = -ej*ei, i^=j.
    -- The Rep assumes n is small so bubble sort is ok.
    -- Using bubble sort keeps the exchange info obvious.
    wasordered   := false
    exchanges := 0
    while not wasordered repeat
        wasordered := true
        for i in 1..#lb-1 repeat
```

```
            if lb.i > lb.(i+1) then
                t := lb.i; lb.i := lb.(i+1); lb.(i+1) := t
                exchanges := exchanges + 1
                wasordered := false
    if odd? exchanges then c := -c

    -- 2. Prepare the basis element
    -- Apply identity ei*ei = Q(ei).
    bz := 0
    for b in lb repeat
        bn := (b-1)::NNI
        if bit?(bz, bn) then
            c := c * Qeelist bn
            bz:= ( bz - 2**bn )::NNI
        else
            bz:= bz + 2**bn
    [c, bz::NNI]

monomial(c, lb) ==
    r := canonMonom(c, lb)
    z := New
    z r.basel := r.coef
    z
coefficient(z, lb) ==
    r := canonMonom(1, lb)
    r.coef = 0 => error "Cannot take coef of 0"
    z r.basel/r.coef

Ex ==> OutputForm

coerceMonom(c: K, b: NNI): Ex ==
    b = 0 => c::Ex
    ml := [sub("e"::Ex, i::Ex) for i in 1..n | bit?(b,i-1)]
    be := reduce("*", ml)
    c = 1 => be
    c::Ex * be

coerce(x): Ex ==
    tl := [coerceMonom(x.i,i) for i in 0..dim-1 | x.i^=0]
    null tl => "0"::Ex
    reduce("+", tl)

localPowerSets(j:NNI): List(List(PI)) ==
  l: List List PI := list []
  j = 0 => l
  Sm := localPowerSets((j-1)::NNI)
  Sn: List List PI := []
  for x in Sm repeat Sn := cons(cons(j pretend PI, x),Sn)
  append(Sn, Sm)
```

```
        powerSets(j:NNI):List List PI == map(reverse, localPowerSets j)

        Pn:List List PI := powerSets(n)

        recip(x: %): Union(%, "failed") ==
          one:% := 1
          -- tmp:c := x*yC - 1$C
          rhsEqs : List K := []
          lhsEqs: List List K := []
          lhsEqi: List K
          for pi in Pn repeat
            rhsEqs := cons(coefficient(one, pi), rhsEqs)

            lhsEqi := []
            for pj in Pn repeat
                lhsEqi := cons(coefficient(x*monomial(1,pj),pi),lhsEqi)
            lhsEqs := cons(reverse(lhsEqi),lhsEqs)
          ans := particularSolution(matrix(lhsEqs),vector(rhsEqs)_
             )$LinearSystemMatrixPackage(K, Vector K, Vector K, Matrix K)
          ans case "failed" => "failed"
          ansP := parts(ans)
          ansC:% := 0
          for pj in Pn repeat
            cj:= first ansP
            ansP := rest ansP
            ansC := ansC + cj*monomial(1,pj)
          ansC
```

———-

— CLIF.dotabb —

```
"CLIF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CLIF"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"CLIF" -> "IVECTOR"
```

———-

4.6 domain COLOR Color

— Color.input —

```
)set break resume
)sys rm -f Color.output
```

```
)spool Color.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Color
--R Color  is a domain constructor
--R Abbreviation for Color is COLOR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for COLOR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (DoubleFloat,%) -> %           ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R blue : () -> %                       coerce : % -> OutputForm
--R color : Integer -> %                 green : () -> %
--R hash : % -> SingleInteger            hue : % -> Integer
--R latex : % -> String                  red : () -> %
--R yellow : () -> %                     ?~=? : (%,%) -> Boolean
--R numberOfHues : () -> PositiveInteger
--R
--E 1

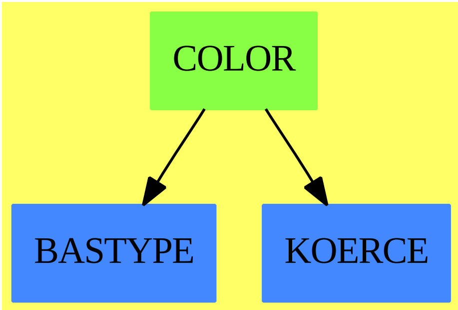
)spool
)lisp (bye)
```

— **Color.help** —

```
====================================================================
Color examples
====================================================================

See Also:
o )show Color
```

4.6.1 Color (COLOR)



See

⇒ "Palette" (PALETTE) 17.4.1 on page 1856

Exports:

blue	coerce	color	green	hash
hue	latex	numberOfHues	red	yellow
?~=?	?*?	?+?	?=?	

— **domain COLOR Color** —

```
)abbrev domain COLOR Color
++ Author: Jim Wen
++ Date Created: 10 May 1989
++ Date Last Updated: 19 Mar 1991 by Jon Steinbach
++ Basic Operations: red, yellow, green, blue, hue, numberOfHues, color, +, *, =
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Color() specifies a domain of 27 colors provided in the
++ \Language{} system (the colors mix additively).

Color(): Exports == Implementation where
  I      ==> Integer
  PI     ==> PositiveInteger
  SF     ==> DoubleFloat

  Exports ==> AbelianSemiGroup with
    "*"    : (PI, %) -> %
      ++ s * c, returns the color c, whose weighted shade has been scaled by s.
    "*"    : (SF, %) -> %
      ++ s * c, returns the color c, whose weighted shade has been scaled by s.
    "+"    : (%, %) -> %
      ++ c1 + c2 additively mixes the two colors c1 and c2.
```

```
    red    : ()      -> %
      ++ red() returns the position of the red hue from total hues.
    yellow : ()      -> %
      ++ yellow() returns the position of the yellow hue from total hues.
    green  : ()      -> %
      ++ green() returns the position of the green hue from total hues.
    blue   : ()      -> %
      ++ blue() returns the position of the blue hue from total hues.
    hue    : %       -> I
      ++ hue(c) returns the hue index of the indicated color c.
    numberOfHues : ()    -> PI
      ++ numberOfHues() returns the number of total hues, set in totalHues.
    color  : Integer -> %
      ++ color(i) returns a color of the indicated hue i.

  Implementation ==> add
    totalHues   ==> 27  --see  (header.h file) for the current number

    Rep := Record(hue:I, weight:SF)


    f:SF * c:% ==
      -- s * c returns the color c, whose weighted shade has been scaled by s
      zero? f => c
      -- 0 is the identitly function...or maybe an error is better?
      [c.hue, f * c.weight]

    x + y ==
     x.hue = y.hue => [x.hue, x.weight + y.weight]
     if y.weight > x.weight then  -- let x be color with bigger weight
       c := x
       x := y
       y := c
     diff := x.hue - y.hue
     if (xHueSmaller:= (diff < 0)) then diff := -diff
     if (moreThanHalf:=(diff > totalHues quo 2)) then diff := totalHues-diff
     offset : I := wholePart(round (diff::SF/(2::SF)**(x.weight/y.weight)) )
     if (xHueSmaller and ^moreThanHalf) or (^xHueSmaller and moreThanHalf) then
       ans := x.hue + offset
     else
       ans := x.hue - offset
     if (ans < 0) then ans := totalHues + ans
     else if (ans > totalHues) then ans := ans - totalHues
     [ans,1]

    x = y     == (x.hue = y.hue) and (x.weight = y.weight)
    red()     == [1,1]
    yellow()  == [11::I,1]
    green()   == [14::I,1]
    blue()    == [22::I,1]
```

```
    sample()  == red()
    hue c     == c.hue
    i:PositiveInteger * c:% == i::SF * c
    numberOfHues() == totalHues

    color i ==
      if (i<0) or (i>totalHues) then
       error concat("Color should be in the range 1..",totalHues::String)
      [i::I, 1]

    coerce(c:%):OutputForm ==
      hconcat ["Hue: "::OutputForm, (c.hue)::OutputForm,
               "  Weight: "::OutputForm, (c.weight)::OutputForm]
```

— COLOR.dotabb —

```
"COLOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COLOR"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"COLOR" -> "BASTYPE"
"COLOR" -> "KOERCE"
```

4.7 domain COMM Commutator

— Commutator.input —

```
)set break resume
)sys rm -f Commutator.output
)spool Commutator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Commutator
--R Commutator  is a domain constructor
--R Abbreviation for Commutator is COMM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for COMM
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean              coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R mkcomm : (%,%) -> %                 mkcomm : Integer -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

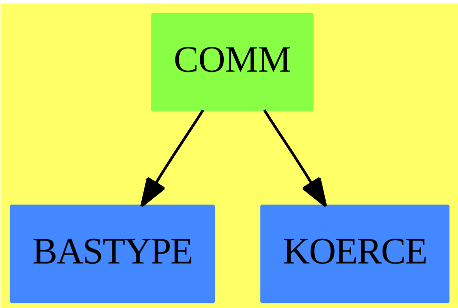
— Commutator.help —

```
====================================================================
Commutator examples
====================================================================

See Also:
o )show Commutator
```

———-

4.7.1 Commutator (COMM)



See

⇒ "OrdSetInts" (OSI) 16.20.1 on page 1825
⇒ "FreeNilpotentLie" (FNLA) 7.33.1 on page 993

Exports:
coerce hash latex mkcomm ?=? ?~=?

— domain COMM Commutator —

```
)abbrev domain COMM Commutator
++ Author : Larry Lambe
++ Date created: 30 June 1988.
++ Updated     : 10 March 1991
++ Description:
++ A type for basic commutators

Commutator: Export == Implement where
   I   ==> Integer
   OSI ==> OrdSetInts
   O   ==> OutputForm

   Export == SetCategory with
     mkcomm : I -> %
       ++ mkcomm(i) is not documented
     mkcomm : (%,%) -> %
       ++ mkcomm(i,j) is not documented

   Implement == add
     P   :=  Record(left:%,right:%)
     Rep := Union(OSI,P)
     x,y: %
     i  : I

     x = y ==
        (x case OSI) and (y case OSI) => x::OSI = y::OSI
        (x case P) and (y case P) =>
           xx:P := x::P
           yy:P := y::P
           (xx.right = yy.right) and (xx.left = yy.left)
        false

     mkcomm(i) == i::OSI
     mkcomm(x,y) == construct(x,y)$P

     coerce(x: %): O ==
        x case OSI => x::OSI::O
        xx := x::P
        bracket([xx.left::O,xx.right::O])$O
```

———-

— COMM.dotabb —

```
"COMM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COMM"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"COMM" -> "BASTYPE"
```

```
"COMM" -> "KOERCE"
```

———-

4.8 domain COMPLEX Complex

— Complex.input —

```
)set break resume
)sys rm -f Complex.output
)spool Complex.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
a := complex(4/3,5/2)
--R
--R
--R        4   5
--R   (1)  - + - %i
--R        3   2
--R                                        Type: Complex Fraction Integer
--E 1

--S 2 of 16
b := complex(4/3,-5/2)
--R
--R
--R        4   5
--R   (2)  - - - %i
--R        3   2
--R                                        Type: Complex Fraction Integer
--E 2

--S 3 of 16
a + b
--R
--R
--R        8
--R   (3)  -
--R        3
--R                                        Type: Complex Fraction Integer
--E 3

--S 4 of 16
a - b
```

```
--R
--R
--R   (4)  5%i
--R                                              Type: Complex Fraction Integer
--E 4

--S 5 of 16
a * b
--R
--R
--R        289
--R   (5)  ---
--R         36
--R                                              Type: Complex Fraction Integer
--E 5

--S 6 of 16
a / b
--R
--R
--R          161   240
--R   (6)  - --- + --- %i
--R          289   289
--R                                              Type: Complex Fraction Integer
--E 6

--S 7 of 16
% :: Fraction Complex Integer
--R
--R
--R        - 15 + 8%i
--R   (7)  ----------
--R         15 + 8%i
--R                                              Type: Fraction Complex Integer
--E 7

--S 8 of 16
3.4 + 6.7 * %i
--R
--R
--R   (8)  3.4 + 6.7 %i
--R                                                         Type: Complex Float
--E 8

--S 9 of 16
conjugate a
--R
--R
--R        4   5
--R   (9)  - - - %i
```

```
--R        3   2
--R                                               Type: Complex Fraction Integer
--E 9

--S 10 of 16
norm a
--R
--R
--R         289
--R   (10)  ---
--R          36
--R                                                       Type: Fraction Integer
--E 10

--S 11 of 16
real a
--R
--R
--R         4
--R   (11)  -
--R         3
--R                                                       Type: Fraction Integer
--E 11

--S 12 of 16
imag a
--R
--R
--R         5
--R   (12)  -
--R         2
--R                                                       Type: Fraction Integer
--E 12

--S 13 of 16
gcd(13 - 13*%i,31 + 27*%i)
--R
--R
--R   (13)  5 + %i
--R                                                        Type: Complex Integer
--E 13

--S 14 of 16
lcm(13 - 13*%i,31 + 27*%i)
--R
--R
--R   (14)  143 - 39%i
--R                                                        Type: Complex Integer
--E 14
```

```
--S 15 of 16
factor(13 - 13*%i)
--R
--R
--R   (15)  - (1 + %i)(2 + 3%i)(3 + 2%i)
--R                                                 Type: Factored Complex Integer
--E 15

--S 16 of 16
factor complex(2,0)
--R
--R
--R                        2
--R   (16)  - %i (1 + %i)
--R                                                 Type: Factored Complex Integer
--E 16
)spool
)lisp (bye)
```

— **Complex.help** —

```
====================================================================
Complex examples
====================================================================

The Complex constructor implements complex objects over a commutative
ring R.  Typically, the ring R is Integer, Fraction Integer, Float or
DoubleFloat.  R can also be a symbolic type, like Polynomial Integer.

Complex objects are created by the complex operation.

  a := complex(4/3,5/2)
   4   5
   - + - %i
   3   2
                        Type: Complex Fraction Integer

  b := complex(4/3,-5/2)
   4   5
   - - - %i
   3   2
                        Type: Complex Fraction Integer

The standard arithmetic operations are available.

  a + b
   8
```

```
 -
 3
                     Type: Complex Fraction Integer

a - b
 5%i
                     Type: Complex Fraction Integer

a * b
 289
 ---
  36
                     Type: Complex Fraction Integer
```

If R is a field, you can also divide the complex objects.

```
a / b
   161   240
 - --- + --- %i
   289   289
                     Type: Complex Fraction Integer
```

We can view the last object as a fraction of complex integers.

```
% :: Fraction Complex Integer
 - 15 + 8%i
 ----------
   15 + 8%i
                    Type: Fraction Complex Integer
```

The predefined macro %i is defined to be complex(0,1).

```
3.4 + 6.7 * %i
 3.4 + 6.7 %i
                   Type: Complex Float
```

You can also compute the conjugate and norm of a complex number.

```
conjugate a
 4   5
 - - - %i
 3   2
                   Type: Complex Fraction Integer

norm a
 289
 ---
  36
                   Type: Fraction Integer
```

```
The real and imag operations are provided to extract the real and
imaginary parts, respectively.

  real a
   4
   -
   3
                      Type: Fraction Integer

  imag a
   5
   -
   2
                      Type: Fraction Integer

The domain Complex Integer is also called the Gaussian integers.  If R
is the integers (or, more generally, a EuclideanDomain), you can compute
greatest common divisors.

  gcd(13 - 13*%i,31 + 27*%i)
   5 + %i
                      Type: Complex Integer

You can also compute least common multiples.

  lcm(13 - 13*%i,31 + 27*%i)
   143 - 39%i
                      Type: Complex Integer

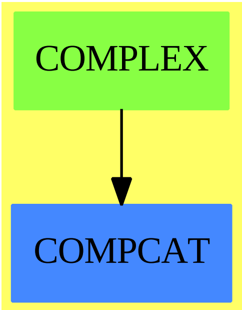
You can factor Gaussian integers.

  factor(13 - 13*%i)
   - (1 + %i)(2 + 3%i)(3 + 2%i)
                      Type: Factored Complex Integer

  factor complex(2,0)
               2
   - %i (1 + %i)
                      Type: Factored Complex Integer

See Also
o )show Complex
```

4.8.1 Complex (COMPLEX)



Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
argument	asec	asech
asin	asinh	associates?
atan	atanh	basis
characteristic	characteristicPolynomial	charthRoot
coerce	complex	conditionP
conjugate	convert	coordinates
cos	cosh	cot
coth	createPrimitiveElement	csc
csch	D	definingPolynomial
derivationCoordinates	differentiate	discreteLog
discriminant	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	factorsOfCyclicGroupSize
gcd	gcdPolynomial	generator
hash	imag	imaginary
index	init	inv
latex	lcm	lift
log	lookup	map
max	min	minimalPolynomial
multiEuclidean	nextItem	norm
nthRoot	OMwrite	one?
order	patternMatch	pi
polarCoordinates	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	rank	rational
rational?	rationalIfCan	real
recip	reduce	reducedSystem
regularRepresentation	representationType	represents
retract	retractIfCan	sample
sec	sech	sin
sinh	size	sizeLess?
solveLinearPolynomialEquation	sqrt	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
tableForDiscreteLogarithm	tan	tanh
trace	traceMatrix	unit?
unitCanonical	unitNormal	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?	?.?	?quo?
?rem?		

— domain COMPLEX Complex —

```
)abbrev domain COMPLEX Complex
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{Complex(R)} creates the domain of elements of the form
++ \spad{a + b * i} where \spad{a} and b come from the ring R,
++ and i is a new element such that \spad{i**2 = -1}.

Complex(R:CommutativeRing): ComplexCategory(R) with
     if R has OpenMath then OpenMath
   == add
       Rep := Record(real:R, imag:R)

       if R has OpenMath then
         writeOMComplex(dev: OpenMathDevice, x: %): Void ==
          OMputApp(dev)
          OMputSymbol(dev, "complex1", "complex__cartesian")
          OMwrite(dev, real x)
          OMwrite(dev, imag x)
          OMputEndApp(dev)

         OMwrite(x: %): String ==
          s: String := ""
          sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
          dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
          OMputObject(dev)
          writeOMComplex(dev, x)
          OMputEndObject(dev)
          OMclose(dev)
          s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
          s

         OMwrite(x: %, wholeObj: Boolean): String ==
          s: String := ""
          sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
          dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
          if wholeObj then
            OMputObject(dev)
          writeOMComplex(dev, x)
          if wholeObj then
```

```
          OMputEndObject(dev)
        OMclose(dev)
        s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
        s

      OMwrite(dev: OpenMathDevice, x: %): Void ==
        OMputObject(dev)
        writeOMComplex(dev, x)
        OMputEndObject(dev)

      OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
        if wholeObj then
          OMputObject(dev)
        writeOMComplex(dev, x)
        if wholeObj then
          OMputEndObject(dev)

    0                == [0, 0]
    1                == [1, 0]
    zero? x          == zero?(x.real) and zero?(x.imag)
--    one? x           == one?(x.real) and zero?(x.imag)
    one? x           == ((x.real) = 1) and zero?(x.imag)
    coerce(r:R):%    == [r, 0]
    complex(r, i)   == [r, i]
    real x           == x.real
    imag x           == x.imag
    x + y            == [x.real + y.real, x.imag + y.imag]
                          -- by re-defining this here, we save 5 fn calls
    x:% * y:% ==
      [x.real * y.real - x.imag * y.imag,
       x.imag * y.real + y.imag * x.real] -- here we save nine!


    if R has IntegralDomain then
      _exquo(x:%, y:%) == -- to correct bad defaulting problem
        zero? y.imag => x exquo y.real
        x * conjugate(y) exquo norm(y)
```

———-

— COMPLEX.dotabb —

```
"COMPLEX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COMPLEX"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"COMPLEX" -> "COMPCAT"
```

———-

4.9 domain CDFMAT ComplexDoubleFloatMatrix

— ComplexDoubleFloatMatrix.input —

```
)set break resume
)sys rm -f ComplexDoubleFloatMatrix.output
)spool ComplexDoubleFloatMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
)show ComplexDoubleFloatMatrix
--R ComplexDoubleFloatMatrix  is a domain constructor
--R Abbreviation for ComplexDoubleFloatMatrix is CDFMAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for CDFMAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (%,%) -> %
--R ?+? : (%,%) -> %                      -? : % -> %
--R ?-? : (%,%) -> %                      antisymmetric? : % -> Boolean
--R copy : % -> %                         diagonal? : % -> Boolean
--R diagonalMatrix : List % -> %          empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R horizConcat : (%,%) -> %              maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       qnew : (Integer,Integer) -> %
--R sample : () -> %                      square? : % -> Boolean
--R squareTop : % -> %                    symmetric? : % -> Boolean
--R transpose : % -> %                    vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (ComplexDoubleFloatVector,%) -> ComplexDoubleFloatVector
--R ?*? : (%,ComplexDoubleFloatVector) -> ComplexDoubleFloatVector
--R ?*? : (%,Complex DoubleFloat) -> %
--R ?*? : (Complex DoubleFloat,%) -> %
--R ?**? : (%,Integer) -> % if Complex DoubleFloat has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Complex DoubleFloat) -> % if Complex DoubleFloat has FIELD
--R ?=? : (%,%) -> Boolean if Complex DoubleFloat has SETCAT
--R any? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : ComplexDoubleFloatVector -> %
--R coerce : % -> OutputForm if Complex DoubleFloat has SETCAT
--R column : (%,Integer) -> ComplexDoubleFloatVector
--R columnSpace : % -> List ComplexDoubleFloatVector if Complex DoubleFloat has EUCDOM
--R count : (Complex DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and Complex DoubleFlo
--R count : ((Complex DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
```

```
--R determinant : % -> Complex DoubleFloat if Complex DoubleFloat has commutative *
--R diagonalMatrix : List Complex DoubleFloat -> %
--R elt : (%,List Integer,List Integer) -> %
--R elt : (%,Integer,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R elt : (%,Integer,Integer) -> Complex DoubleFloat
--R eval : (%,List Complex DoubleFloat,List Complex DoubleFloat) -> % if Complex DoubleFloat
--R eval : (%,Complex DoubleFloat,Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB
--R eval : (%,Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPLEX DO
--R eval : (%,List Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPL
--R every? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,Complex DoubleFloat) -> Union(%,"failed") if Complex DoubleFloat has INTDOM
--R fill! : (%,Complex DoubleFloat) -> %
--R hash : % -> SingleInteger if Complex DoubleFloat has SETCAT
--R inverse : % -> Union(%,"failed") if Complex DoubleFloat has FIELD
--R latex : % -> String if Complex DoubleFloat has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R listOfLists : % -> List List Complex DoubleFloat
--R map : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,%,Complex Dou
--R map : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,%) -> %
--R map : ((Complex DoubleFloat -> Complex DoubleFloat),%) -> %
--R map! : ((Complex DoubleFloat -> Complex DoubleFloat),%) -> %
--R matrix : List List Complex DoubleFloat -> %
--R member? : (Complex DoubleFloat,%) -> Boolean if $ has finiteAggregate and Complex DoubleF
--R members : % -> List Complex DoubleFloat if $ has finiteAggregate
--R minordet : % -> Complex DoubleFloat if Complex DoubleFloat has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,Complex DoubleFloat) -> %
--R nullSpace : % -> List ComplexDoubleFloatVector if Complex DoubleFloat has INTDOM
--R nullity : % -> NonNegativeInteger if Complex DoubleFloat has INTDOM
--R parts : % -> List Complex DoubleFloat
--R pfaffian : % -> Complex DoubleFloat if Complex DoubleFloat has COMRING
--R qelt : (%,Integer,Integer) -> Complex DoubleFloat
--R qsetelt! : (%,Integer,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R rank : % -> NonNegativeInteger if Complex DoubleFloat has INTDOM
--R row : (%,Integer) -> ComplexDoubleFloatVector
--R rowEchelon : % -> % if Complex DoubleFloat has EUCDOM
--R scalarMatrix : (NonNegativeInteger,Complex DoubleFloat) -> %
--R setColumn! : (%,Integer,ComplexDoubleFloatVector) -> %
--R setRow! : (%,Integer,ComplexDoubleFloatVector) -> %
--R setelt : (%,List Integer,List Integer,%) -> %
--R setelt : (%,Integer,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R setsubMatrix! : (%,Integer,Integer,%) -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R subMatrix : (%,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (%,Integer,Integer) -> %
--R swapRows! : (%,Integer,Integer) -> %
--R transpose : ComplexDoubleFloatVector -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
--R ?~=? : (%,%) -> Boolean if Complex DoubleFloat has SETCAT
--R
```

```
--E 1

--S 2 of 6
a:CDFMAT:=qnew(2,3)
--R
--R        +0.  0.  0.+
--R   (1)  |          |
--R        +0.  0.  0.+
--R                                               Type: ComplexDoubleFloatMatrix
--E 2

--S 3 of 6
qsetelt!(a,1,1,1.0+2*%i)
--R
--R   (2)  1. + 2. %i
--R                                                    Type: Complex DoubleFloat
--E 3

--S 4 of 6
a
--R
--R        +0.      0.      0.+
--R   (3)  |                  |
--R        +0.  1. + 2. %i  0.+
--R                                               Type: ComplexDoubleFloatMatrix
--E 4

--S 5 of 6
qsetelt!(a,0,0,2.0+4*%i)
--R
--R   (4)  2. + 4. %i
--R                                                    Type: Complex DoubleFloat
--E 5

--S 6 of 6
a
--R
--R        +2. + 4. %i      0.      0.+
--R   (5)  |                          |
--R        +    0.      1. + 2. %i  0.+
--R                                               Type: ComplexDoubleFloatMatrix
--E 6

)spool
)lisp (bye)
```

————

— ComplexDoubleFloatMatrix.help —

```
====================================================================
ComplexDoubleFloatMatrix examples
====================================================================
This domain creates a lisp simple array of machine doublefloats.
It provides one new function called qnew which takes an integer
that gives the array length.

NOTE: Unlike normal Axiom arrays the ComplexDoubleFloatMatrix arrays
are 0-based so the first element is 0. Axiom arrays normally
start at 1.

a:CDFMAT:=qnew(2,3)
        +0.  0.  0.+
        |          |
        +0.  0.  0.+

qsetelt!(a,1,1,1.0+2*%i)
        1. + 2. %i

a
        +0.      0.      0.+
        |                  |
        +0.  1. + 2. %i  0.+

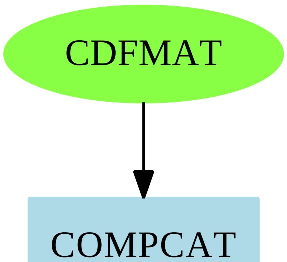
qsetelt!(a,0,0,2.0+4*%i)
        2. + 4. %i

a
        +2. + 4. %i      0.      0.+
        |                          |
        +    0.      1. + 2. %i  0.+

See Also:
o )help Float
o )help DoubleFloat
o )show ComplexDoubleFloatMatrix
```

————

4.9.1 ComplexDoubleFloatMatrix (CDFMAT)



Exports:

#?	-?	?**?	?*?
?+?	?-?	?/?	?=?
?~=?	antisymmetric?	any?	coerce
column	columnSpace	copy	count
determinant	diagonal?	diagonalMatrix	elt
empty	empty?	eq?	eval
every?	exquo	fill!	hash
horizConcat	inverse	latex	less?
listOfLists	map	map!	matrix
maxColIndex	maxRowIndex	member?	members
minColIndex	minRowIndex	minordet	more?
ncols	new	nrows	nullSpace
nullity	parts	pfaffian	qelt
qnew	qsetelt!	rank	row
rowEchelon	sample	scalarMatrix	setColumn!
setRow!	setelt	setsubMatrix!	size?
square?	squareTop	subMatrix	swapColumns!
swapRows!	symmetric?	transpose	vertConcat
zero			

— domain CDFMAT ComplexDoubleFloatMatrix —

```
)abbrev domain CDFMAT ComplexDoubleFloatMatrix
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements matrices
++ (two dimensional arrays) of complex double precision floating point
++ numbers.  Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).

ComplexDoubleFloatMatrix : MatrixCategory(Complex DoubleFloat,
                         ComplexDoubleFloatVector,
                         ComplexDoubleFloatVector) with
    qnew : (Integer, Integer) -> %
```

```
      ++ qnew(n, m) creates a new uninitialized n by m matrix.
      ++
      ++X t1:CDFMAT:=qnew(3,4)

  == add

    NNI ==> Integer
    Qelt2 ==> CDAREF2$Lisp
    Qsetelt2 ==> CDSETAREF2$Lisp
    Qnrows ==> CDANROWS$Lisp
    Qncols ==> CDANCOLS$Lisp
    Qnew ==> MAKE_-CDOUBLE_-MATRIX$Lisp

    minRowIndex x == 0
    minColIndex x == 0
    nrows x == Qnrows(x)
    ncols x == Qncols(x)
    maxRowIndex x == Qnrows(x) - 1
    maxColIndex x == Qncols(x) - 1

    qelt(m, i, j) == Qelt2(m, i, j)
    qsetelt_!(m, i, j, r) == Qsetelt2(m, i, j, r)

    empty() == Qnew(0$Integer, 0$Integer)
    qnew(rows, cols) == Qnew(rows, cols)
    new(rows, cols, a) ==
        res := Qnew(rows, cols)
        for i in 0..(rows - 1) repeat
            for j in 0..(cols - 1) repeat
                Qsetelt2(res, i, j, a)
        res
```

— **CDFMAT.dotabb** —

```
"CDFMAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CDFMAT",
          shape=ellipse]
"COMPCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"CDFMAT" -> "COMPCAT"
```

4.10 domain CDFVEC ComplexDoubleFloatVector

— ComplexDoubleFloatVector.input —

```
)set break resume
)sys rm -f ComplexDoubleFloatVector.output
)spool ComplexDoubleFloatVector.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
)show ComplexDoubleFloatVector
--R ComplexDoubleFloatVector  is a domain constructor
--R Abbreviation for ComplexDoubleFloatVector is CDFVEC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for CDFVEC
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R copy : % -> %                         delete : (%,Integer) -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R qnew : Integer -> %                   reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,Complex DoubleFloat) -> % if Complex DoubleFloat has MONOID
--R ?*? : (Complex DoubleFloat,%) -> % if Complex DoubleFloat has MONOID
--R ?*? : (Integer,%) -> % if Complex DoubleFloat has ABELGRP
--R ?+? : (%,%) -> % if Complex DoubleFloat has ABELSG
--R ?-? : (%,%) -> % if Complex DoubleFloat has ABELGRP
--R -? : % -> % if Complex DoubleFloat has ABELGRP
--R ?<? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R ?<=? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R ?=? : (%,%) -> Boolean if Complex DoubleFloat has SETCAT
--R ?>? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R ?>=? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R any? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Complex DoubleFloat has SETCAT
--R concat : (Complex DoubleFloat,%) -> %
--R concat : (%,Complex DoubleFloat) -> %
--R construct : List Complex DoubleFloat -> %
--R convert : % -> InputForm if Complex DoubleFloat has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Complex DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and Complex DoubleFlo
--R count : ((Complex DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (%,%) -> % if Complex DoubleFloat has RING
```

```
--R delete : (%,UniversalSegment Integer) -> %
--R dot : (%,%) -> Complex DoubleFloat if Complex DoubleFloat has RING
--R ?.? : (%,UniversalSegment Integer) -> %
--R ?.? : (%,Integer) -> Complex DoubleFloat
--R elt : (%,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R entries : % -> List Complex DoubleFloat
--R entry? : (Complex DoubleFloat,%) -> Boolean if $ has finiteAggregate and Complex DoubleF
--R eval : (%,List Complex DoubleFloat,List Complex DoubleFloat) -> % if Complex DoubleFloat
--R eval : (%,Complex DoubleFloat,Complex DoubleFloat) -> % if Complex DoubleFloat has EVALA
--R eval : (%,Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPLEX D
--R eval : (%,List Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMP
--R every? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Complex DoubleFloat) -> % if $ has shallowlyMutable
--R find : ((Complex DoubleFloat -> Boolean),%) -> Union(Complex DoubleFloat,"failed")
--R first : % -> Complex DoubleFloat if Integer has ORDSET
--R hash : % -> SingleInteger if Complex DoubleFloat has SETCAT
--R insert : (Complex DoubleFloat,%,Integer) -> %
--R latex : % -> String if Complex DoubleFloat has SETCAT
--R length : % -> Complex DoubleFloat if Complex DoubleFloat has RADCAT and Complex DoubleFl
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> Complex DoubleFloat if Complex DoubleFloat has RADCAT and Complex Doubl
--R map : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,%) -> %
--R map : ((Complex DoubleFloat -> Complex DoubleFloat),%) -> %
--R map! : ((Complex DoubleFloat -> Complex DoubleFloat),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if Complex DoubleFloat has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Complex DoubleFloat,%) -> Boolean if $ has finiteAggregate and Complex Double
--R members : % -> List Complex DoubleFloat if $ has finiteAggregate
--R merge : (%,%) -> % if Complex DoubleFloat has ORDSET
--R merge : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Complex DoubleFloat has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Complex DoubleFloat) -> %
--R outerProduct : (%,%) -> Matrix Complex DoubleFloat if Complex DoubleFloat has RING
--R parts : % -> List Complex DoubleFloat if $ has finiteAggregate
--R position : (Complex DoubleFloat,%,Integer) -> Integer if Complex DoubleFloat has SETCAT
--R position : (Complex DoubleFloat,%) -> Integer if Complex DoubleFloat has SETCAT
--R position : ((Complex DoubleFloat -> Boolean),%) -> Integer
--R qelt : (%,Integer) -> Complex DoubleFloat
--R qsetelt! : (%,Integer,Complex DoubleFloat) -> Complex DoubleFloat if $ has shallowlyMuta
--R reduce : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%) -> Compl
--R reduce : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,Complex D
--R reduce : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,Complex D
--R remove : ((Complex DoubleFloat -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Complex DoubleFloat,%) -> % if $ has finiteAggregate and Complex DoubleFloat h
--R removeDuplicates : % -> % if $ has finiteAggregate and Complex DoubleFloat has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Complex DoubleFloat -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Complex DoubleFloat) -> Complex DoubleFloat if $ ha
```

```
--R setelt : (%,Integer,Complex DoubleFloat) -> Complex DoubleFloat if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Complex DoubleFloat has ORDSET
--R sort : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and Complex DoubleFloat has ORDSET
--R sort! : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Complex DoubleFloat has ORDSET
--R sorted? : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R vector : List Complex DoubleFloat -> %
--R zero : NonNegativeInteger -> % if Complex DoubleFloat has ABELMON
--R ?~=? : (%,%) -> Boolean if Complex DoubleFloat has SETCAT
--R
--E 1

--S 2 of 6
t1:CDFVEC:=qnew(5)
--R
--R   (1)  [0.,0.,0.,0.,0.]
--R                                                 Type: ComplexDoubleFloatVector
--E 2

-- NOTE: CDFVEC arrays are 0-based, normal Axiom arrays are 1-based
--S 3 of 6
t1.1:=1.0+2*%i
--R
--R   (2)  1. + 2. %i
--R                                                      Type: Complex DoubleFloat
--E 3

--S 4 of 6
t1
--R
--R   (3)  [0.,1. + 2. %i,0.,0.,0.]
--R                                                 Type: ComplexDoubleFloatVector
--E 4

--S 5 of 6
t1.0:=3.0+4.0*%i
--R
--R   (4)  3. + 4. %i
--R                                                      Type: Complex DoubleFloat
--E 5

--S 6 of 6
t1
--R
--R   (5)  [3. + 4. %i,1. + 2. %i,0.,0.,0.]
--R                                                 Type: ComplexDoubleFloatVector
--E 6
```

```
)spool
)lisp (bye)
```

———-

— ComplexDoubleFloatVector.help —

```
====================================================================
ComplexDoubleFloatVector examples
====================================================================
This domain creates a lisp simple array of machine complex doublefloats.
It provides one new function called qnew which takes an integer
that gives the array length.

NOTE: Unlike normal Axiom arrays the ComplexDoubleFloatVector arrays
are 0-based so the first element is 0. Axiom arrays normally
start at 1.

t1:CDFVEC:=qnew(5)
     [0.,0.,0.,0.,0.]

t1.1:=1.0+2*%i
     1. + 2. %i

t1
     [0.,1. + 2. %i,0.,0.,0.]

t1.0:=3.0+4.0*%i
     3. + 4. %i

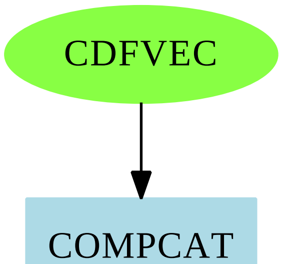
t1

     [3. + 4. %i,1. + 2. %i,0.,0.,0.]

See Also:
o )help Float
o )help DoubleFloat
o )show ComplexDoubleFloatVector
```

———-

4.10.1 ComplexDoubleFloatVector (CDFVEC)



Exports:

#?	-?	?*?	?+?
?-?	?.?	?<=?	?<?
?=?	?>=?	?>?	?~=?
any?	coerce	concat	construct
convert	copy	copyInto!	count
cross	delete	dot	elt
empty	empty?	entries	entry?
eq?	eval	every?	fill!
find	first	hash	index?
indices	insert	latex	length
less?	magnitude	map	map!
max	maxIndex	member?	members
merge	min	minIndex	more?
new	outerProduct	parts	position
qelt	qnew	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!
sample	select	setelt	size?
sort	sort!	sorted?	swap!
vector	zero		

— domain CDFVEC ComplexDoubleFloatVector —

```
)abbrev domain CDFVEC ComplexDoubleFloatVector
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements vectors
++ (one dimensional arrays) of complex double precision floating point
++ numbers.  Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).
ComplexDoubleFloatVector : VectorCategory Complex DoubleFloat with
    qnew : Integer -> %
      ++ qnew(n) creates a new uninitialized vector of length n.
      ++
      ++X t1:CDFVEC:=qnew 7
```

```
    vector: List Complex DoubleFloat -> %
      ++ vector(l) converts the list l to a vector.
      ++
      ++X t1:List(Complex(DoubleFloat)):=[1+2*%i,3+4*%i,-5-6*%i]
      ++X t2:CDFVEC:=vector(t1)
  == add

    Qelt1 ==> CDELT$Lisp
    Qsetelt1 ==> CDSETELT$Lisp

    qelt(x, i) == Qelt1(x, i)
    qsetelt_!(x, i, s) == Qsetelt1(x, i, s)
    Qsize ==> CDLEN$Lisp
    Qnew ==> MAKE_-CDOUBLE_-VECTOR$Lisp

    #x                         == Qsize x
    minIndex x                 == 0
    empty()                    == Qnew(0$Lisp)
    qnew(n)                    == Qnew(n)
    new(n, x)                  ==
        res := Qnew(n)
        fill_!(res, x)
    qelt(x, i)                 == Qelt1(x, i)
    elt(x:%, i:Integer)        == Qelt1(x, i)
    qsetelt_!(x, i, s)         == Qsetelt1(x, i, s)
    setelt(x : %, i : Integer, s : Complex DoubleFloat) ==
        Qsetelt1(x, i, s)
    fill_!(x, s)       ==
        for i in 0..((Qsize(x)) - 1) repeat Qsetelt1(x, i, s)
        x
```

— **CDFVEC.dotabb** —

```
"CDFVEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CDFVEC",
          shape=ellipse]
"COMPCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"CDFVEC" -> "COMPCAT"
```

4.11 domain CONTFRAC ContinuedFraction

— **ContinuedFraction.input** —

```
)set break resume
)sys rm -f ContinuedFraction.output
)spool ContinuedFraction.output
)set message test on
)set message auto off
)clear all
--S 1 of 22
c := continuedFraction(314159/100000)
--R
--R
--R              1 |     1  |     1 |     1  |     1 |     1 |     1 |
--R   (1)  3 + +---+ + +----+ + +---+ + +----+ + +---+ + +---+ + +---+
--R            | 7     | 15     | 1     | 25     | 1     | 7     | 4
--R                                                 Type: ContinuedFraction Integer
--E 1

--S 2 of 22
partialQuotients c
--R
--R
--R   (2)  [3,7,15,1,25,1,7,4]
--R                                                            Type: Stream Integer
--E 2

--S 3 of 22
convergents c
--R
--R
--R           22 333 355 9208 9563 76149 314159
--R   (3)  [3,--,---,---,----,----,-----,------]
--R            7 106 113 2931 3044 24239 100000
--R                                                   Type: Stream Fraction Integer
--E 3

--S 4 of 22
approximants c
--R
--R
--R                                      ------
--R           22 333 355 9208 9563 76149 314159
--R   (4)  [3,--,---,---,----,----,-----,------]
--R            7 106 113 2931 3044 24239 100000
--R                                                   Type: Stream Fraction Integer
--E 4

--S 5 of 22
pq := partialQuotients(1/c)
--R
--R
--R   (5)  [0,3,7,15,1,25,1,7,4]
```

```
--R                                                        Type: Stream Integer
--E 5

--S 6 of 22
continuedFraction(first pq,repeating [1],rest pq)
--R
--R
--R           1 |     1 |     1  |     1 |     1  |     1 |     1 |     1 |
--R   (6)  +---+ + +---+ + +----+ + +---+ + +----+ + +---+ + +---+ + +---+
--R        | 3     | 7     | 15     | 1     | 25     | 1     | 7     | 4
--R                                                 Type: ContinuedFraction Integer
--E 6

--S 7 of 22
z:=continuedFraction(3,repeating [1],repeating [3,6])
--R
--R
--R   (7)
--R           1 |     1 |     1 |     1 |     1 |     1 |     1 |     1 |     1 |
--R     3 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
--R         | 3     | 6     | 3     | 6     | 3     | 6     | 3     | 6     | 3
--R   +
--R       1 |
--R     +---+ + ...
--R     | 6
--R                                                 Type: ContinuedFraction Integer
--E 7

--S 8 of 22
dens:Stream Integer := cons(1,generate((x+->x+4),6))
--R
--R
--R   (8)  [1,6,10,14,18,22,26,30,34,38,...]
--R                                                        Type: Stream Integer
--E 8

--S 9 of 22
cf := continuedFraction(0,repeating [1],dens)
--R
--R
--R   (9)
--R       1 |     1 |     1  |     1  |     1  |     1  |     1  |     1  |
--R     +---+ + +---+ + +----+ + +----+ + +----+ + +----+ + +----+ + +----+
--R     | 1     | 6     | 10     | 14     | 18     | 22     | 26     | 30
--R   +
--R       1  |     1  |
--R     +----+ + +----+ + ...
--R     | 34     | 38
--R                                                 Type: ContinuedFraction Integer
--E 9
```

```
--S 10 of 22
ccf := convergents cf
--R
--R
--R              6 61  860 15541 342762  8927353 268163352  9126481321
--R   (10)  [0,1,-,--,----,-----,------,--------,---------,----------,...]
--R              7 71 1001 18089 398959 10391023 312129649 10622799089
--R                                             Type: Stream Fraction Integer
--E 10

--S 11 of 22
eConvergents := [2*e + 1 for e in ccf]
--R
--R
--R              19 193 2721 49171 1084483 28245729 848456353 28875761731
--R   (11)  [1,3,--,---,----,-----,-------,--------,---------,-----------,...]
--R               7  71 1001 18089  398959 10391023 312129649 10622799089
--R                                             Type: Stream Fraction Integer
--E 11

--S 12 of 22
eConvergents :: Stream Float
--R
--R
--R   (12)
--R   [1.0, 3.0, 2.7142857142 857142857, 2.7183098591 549295775,
--R    2.7182817182 817182817, 2.7182818287 356957267, 2.7182818284 585634113,
--R    2.7182818284 590458514, 2.7182818284 590452348, 2.7182818284 590452354,
--R    ...]
--R                                                        Type: Stream Float
--E 12

--S 13 of 22
exp 1.0
--R
--R
--R   (13)  2.7182818284 590452354
--R                                                               Type: Float
--E 13

--S 14 of 22
cf := continuedFraction(1,[(2*i+1)**2 for i in 0..],repeating [2])
--R
--R
--R   (14)
--R          1 |     9 |     25 |     49 |     81 |     121 |     169 |     225 |
--R     1 + +---+ + +---+ + +----+ + +----+ + +----+ + +-----+ + +-----+ + +-----+
--R         | 2     | 2     | 2      | 2      | 2      |  2      |  2      |  2
--R   +
```

```
--R       289 |     361 |
--R     +-----+ + +-----+ + ...
--R     |  2      |  2
--R                                               Type: ContinuedFraction Integer
--E 14

--S 15 of 22
ccf := convergents cf
--R
--R
--R           3 15 105 315 3465 45045 45045 765765 14549535
--R   (15)  [1,-,--,---,---,----,-----,-----,------,--------,...]
--R           2 13  76 263 2578 36979 33976 622637 11064338
--R                                                  Type: Stream Fraction Integer
--E 15

--S 16 of 22
piConvergents := [4/p for p in ccf]
--R
--R
--R           8 52 304 1052 10312 147916 135904 2490548 44257352
--R   (16)  [4,-,--,---,----,-----,------,------,-------,--------,...]
--R           3 15 105  315  3465  45045  45045  765765 14549535
--R                                                  Type: Stream Fraction Integer
--E 16

--S 17 of 22
piConvergents :: Stream Float
--R
--R
--R   (17)
--R   [4.0, 2.6666666666 666666667, 3.4666666666 666666667,
--R    2.8952380952 380952381, 3.3396825396 825396825, 2.9760461760 461760462,
--R    3.2837384837 384837385, 3.0170718170 718170718, 3.2523659347 188758953,
--R    3.0418396189 294022111, ...]
--R                                                            Type: Stream Float
--E 17

--S 18 of 22
continuedFraction((- 122 + 597*%i)/(4 - 4*%i))
--R
--R
--R                              1    |         1     |
--R   (18)  - 90 + 59%i + +---------+ + +-----------+
--R                        | 1 - 2%i     | - 1 + 2%i
--R                                       Type: ContinuedFraction Complex Integer
--E 18

--S 19 of 22
r : Fraction UnivariatePolynomial(x,Fraction Integer)
```

```
--R
--R                                                                    Type: Void
--E 19

--S 20 of 22
r := ((x - 1) * (x - 2)) / ((x-3) * (x-4))
--R
--R
--R           2
--R          x  - 3x + 2
--R   (20)  ------------
--R          2
--R         x  - 7x + 12
--R                        Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 20

--S 21 of 22
continuedFraction r
--R
--R
--R                  1    |         1     |
--R   (21)  1 + +---------+ + +-----------+
--R             | 1     9     | 16     40
--R             | - x - -     | -- x - --
--R             | 4     8     |  3      3
--R             Type: ContinuedFraction UnivariatePolynomial(x,Fraction Integer)
--E 21

--S 22 of 22
[i*i for i in convergents(z) :: Stream Float]
--R
--R
--R   (22)
--R   [9.0, 11.1111111111 11111111, 10.9944598337 9501385, 11.0002777777 77777778,
--R    10.9999860763 98799786, 11.0000006979 29731039, 10.9999999650 15834446,
--R    11.0000000017 53603304, 10.9999999999 12099531, 11.0000000000 04406066,
--R    ...]
--R                                                             Type: Stream Float
--E 22
)spool
)lisp (bye)
```

———

— ContinuedFraction.help —

```
====================================================================
ContinuedFraction examples
====================================================================
```

```
Continued fractions have been a fascinating and useful tool in
mathematics for well over three hundred years.  Axiom implements
continued fractions for fractions of any Euclidean domain.  In
practice, this usually means rational numbers.  In this section we
demonstrate some of the operations available for manipulating both
finite and infinite continued fractions.

The ContinuedFraction domain is a field and therefore you can add,
subtract, multiply and divide the fractions.

The continuedFraction operation converts its fractional argument to a
continued fraction.

  c := continuedFraction(314159/100000)
         1 |     1  |     1 |     1  |     1 |     1 |     1 |
   3 + +---+ + +----+ + +---+ + +----+ + +---+ + +---+ + +---+
       | 7     | 15     | 1     | 25     | 1     | 7     | 4
                        Type: ContinuedFraction Integer

This display is a compact form of the bulkier

        3 +                1
            -------------------------------
            7 +              1
                ---------------------------
                15 +            1
                     ----------------------
                     1 +          1
                         ------------------
                         25 +       1
                              -------------
                              1 +     1
                                  ---------
                                  7 +   1
                                      -----
                                        4

You can write any rational number in a similar form.  The fraction
will be finite and you can always take the "numerators" to be 1.
That is, any rational number can be written as a simple, finite
continued fraction of the form

        a(1) +           1
               -------------------------
               a(2) +          1
                      --------------------
                      a(3) +
                             .
                              .
```

```
                  .
                        1
                  -------------
                  a(n-1) +  1
                           ----
                           a(n)
```

The a(i) are called partial quotients and the operation partialQuotients creates a stream of them.

```
  partialQuotients c
   [3,7,15,1,25,1,7,4]
                         Type: Stream Integer
```

By considering more and more of the fraction, you get the convergents. For example, the first convergent is a(1), the second is a(1) + 1/a(2) and so on.

```
  convergents c
       22 333 355 9208 9563 76149 314159
   [3,--,---,---,----,----,-----,------]
        7 106 113 2931 3044 24239 100000
                          Type: Stream Fraction Integer
```

Since this is a finite continued fraction, the last convergent is the original rational number, in reduced form. The result of approximants is always an infinite stream, though it may just repeat the "last" value.

```
  approximants c
                                    ------
       22 333 355 9208 9563 76149 314159
   [3,--,---,---,----,----,-----,------]
        7 106 113 2931 3044 24239 100000
                          Type: Stream Fraction Integer
```

Inverting c only changes the partial quotients of its fraction by inserting a 0 at the beginning of the list.

```
  pq := partialQuotients(1/c)
   [0,3,7,15,1,25,1,7,4]
                          Type: Stream Integer
```

Do this to recover the original continued fraction from this list of partial quotients. The three-argument form of the continuedFraction operation takes an element which is the whole part of the fraction, a stream of elements which are the numerators of the fraction, and a stream of elements which are the denominators of the fraction.

```
  continuedFraction(first pq,repeating [1],rest pq)
```

```
   1 |     1 |     1  |     1 |     1  |     1 |     1 |     1 |
 +---+ + +---+ + +----+ + +---+ + +----+ + +---+ + +---+ + +---+
 | 3     | 7     | 15     | 1     | 25     | 1     | 7     | 4
                         Type: ContinuedFraction Integer
```

The streams need not be finite for continuedFraction. Can you guess which irrational number has the following continued fraction? See the end of this section for the answer.

```
z:=continuedFraction(3,repeating [1],repeating [3,6])
        1 |     1 |     1 |     1 |     1 |     1 |     1 |     1 |     1 |
  3 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
      | 3     | 6     | 3     | 6     | 3     | 6     | 3     | 6     | 3
 +
    1 |
  +---+ + ...
  | 6
                         Type: ContinuedFraction Integer
```

In 1737 Euler discovered the infinite continued fraction expansion

```
e - 1            1
----- = ---------------------
  2     1 +         1
            -----------------
            6 +       1
                -------------
                10 +    1
                     --------
                     14 + ...
```

We use this expansion to compute rational and floating point approximations of e. For this and other interesting expansions, see C. D. Olds, Continued Fractions, New Mathematical Library, (New York: Random House, 1963), pp. 134--139.}

By looking at the above expansion, we see that the whole part is 0 and the numerators are all equal to 1. This constructs the stream of denominators.

```
dens:Stream Integer := cons(1,generate((x+->x+4),6))
 [1,6,10,14,18,22,26,30,34,38,...]
                        Type: Stream Integer
```

Therefore this is the continued fraction expansion for (e - 1) / 2.

```
cf := continuedFraction(0,repeating [1],dens)
    1 |     1 |     1  |     1  |     1  |     1  |     1  |     1  |
  +---+ + +---+ + +----+ + +----+ + +----+ + +----+ + +----+ + +----+
```

```
    | 1     | 6     | 10     | 14     | 18     | 22     | 26     | 30
 +
     1  |     1  |
   +----+ + +----+ + ...
   | 34     | 38
                  Type: ContinuedFraction Integer
```

These are the rational number convergents.

```
ccf := convergents cf
      6 61  860 15541 342762  8927353 268163352  9126481321
 [0,1,-,--,----,-----,------,--------,---------,-----------,...]
      7 71 1001 18089 398959 10391023 312129649 10622799089
                  Type: Stream Fraction Integer
```

You can get rational convergents for e by multiplying by 2 and adding 1.

```
eConvergents := [2*e + 1 for e in ccf]
       19 193 2721 49171 1084483 28245729 848456353 28875761731
  [1,3,--,---,----,-----,-------,--------,---------,-----------,...]
        7  71 1001 18089  398959 10391023 312129649 10622799089
                  Type: Stream Fraction Integer
```

You can also compute the floating point approximations to these convergents.

```
eConvergents :: Stream Float
 [1.0, 3.0, 2.7142857142 857142857, 2.7183098591 549295775,
  2.7182817182 817182817, 2.7182818287 356957267, 2.7182818284 585634113,
  2.7182818284 590458514, 2.7182818284 590452348, 2.7182818284 590452354,
  ...]
                     Type: Stream Float
```

Compare this to the value of e computed by the exp operation in Float.

```
exp 1.0
 2.7182818284 590452354
                     Type: Float
```

In about 1658, Lord Brouncker established the following expansion
for 4 / pi,

```
       1 +           1
           -----------------------
           2 +         9
               -------------------
               2 +       25
                   ---------------
                   2 +      49
                       -----------
                       2 +    81
```

```
                            -------
                            2 + ...
```

Let's use this expansion to compute rational and floating point approximations for pi.

```
  cf := continuedFraction(1,[(2*i+1)**2 for i in 0..],repeating [2])
          1 |     9 |     25 |     49 |     81 |     121 |     169 |     225 |
     1 + +---+ + +---+ + +----+ + +----+ + +----+ + +-----+ + +-----+ + +-----+
         | 2     | 2     | 2      | 2      | 2      |  2      |  2      |  2
   +
       289 |     361 |
     +-----+ + +-----+ + ...
     |  2      |  2
                       Type: ContinuedFraction Integer

  ccf := convergents cf
      3 15 105 315 3465 45045 45045 765765 14549535
   [1,-,--,---,---,----,-----,-----,------,--------,...]
      2 13  76 263 2578 36979 33976 622637 11064338
                       Type: Stream Fraction Integer

  piConvergents := [4/p for p in ccf]
      8 52 304 1052 10312 147916 135904 2490548 44257352
   [4,-,--,---,----,-----,------,------,-------,--------,...]
      3 15 105  315  3465  45045  45045  765765 14549535
                       Type: Stream Fraction Integer
```

As you can see, the values are converging to pi = 3.14159265358979323846..., but not very quickly.

```
  piConvergents :: Stream Float
   [4.0, 2.6666666666 666666667, 3.4666666666 666666667,
    2.8952380952 380952381, 3.3396825396 825396825, 2.9760461760 461760462,
    3.2837384837 384837385, 3.0170718170 718170718, 3.2523659347 188758953,
    3.0418396189 294022111, ...]
                      Type: Stream Float
```

You need not restrict yourself to continued fractions of integers. Here is an expansion for a quotient of Gaussian integers.

```
  continuedFraction((- 122 + 597*%i)/(4 - 4*%i))
                      1    |         1     |
   - 90 + 59%i + +---------+ + +-----------+
                 | 1 - 2%i     | - 1 + 2%i
                      Type: ContinuedFraction Complex Integer
```

This is an expansion for a quotient of polynomials in one variable with rational number coefficients.

```
  r : Fraction UnivariatePolynomial(x,Fraction Integer)
                     Type: Void

  r := ((x - 1) * (x - 2)) / ((x-3) * (x-4))
     2
    x  - 3x + 2
   ------------
    2
   x  - 7x + 12
                     Type: Fraction UnivariatePolynomial(x,Fraction Integer)

  continuedFraction r
            1    |         1     |
   1 + +---------+ + +-----------+
       | 1     9     | 16     40
       | - x - -     | -- x - --
       | 4     8     |  3      3
            Type: ContinuedFraction UnivariatePolynomial(x,Fraction Integer)
```

To conclude this section, we give you evidence that

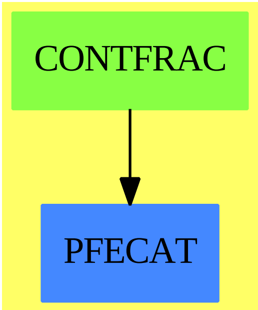
```
    z = 3 +           1
            -----------------------
            3 +          1
                -------------------
                6 +        1
                    ---------------
                    3 +      1
                        -----------
                        6 +    1
                            -------
                            3 + ...
```

is the expansion of sqrt(11).

```
  [i*i for i in convergents(z) :: Stream Float]
   [9.0, 11.1111111111 11111111, 10.9944598337 9501385, 11.0002777777 77777778,
    10.9999860763 98799786, 11.0000006979 29731039, 10.9999999650 15834446,
    11.0000000017 53603304, 10.9999999999 12099531, 11.0000000000 04406066,
    ...]
                       Type: Stream Float
```

See Also:
o)help Stream
o)show ContinuedFraction

4.11.1 ContinuedFraction (CONTFRAC)



Exports:

0	1	approximants
associates?	characteristic	coerce
complete	continuedFraction	convergents
denominators	divide	extend
euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	gcd
gcdPolynomial	hash	inv
latex	lcm	multiEuclidean
numerators	one?	partialDenominators
partialNumerators	partialQuotients	prime?
principalIdeal	recip	reducedContinuedFraction
reducedForm	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan
unit?	unitCanonical	unitNormal
wholePart	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain CONTFRAC ContinuedFraction —

```
)abbrev domain CONTFRAC ContinuedFraction
++ Author: Stephen M. Watt
++ Date Created: January 1987
++ Change History:
++   11 April   1990
++    7 October 1991 -- SMW: Treat whole part specially.  Added comments.
++ Basic Operations:
++   (Field), (Algebra),
++   approximants, complete, continuedFraction, convergents, denominators,
++   extend, numerators, partialDenominators, partialNumerators,
++   partialQuotients, reducedContinuedFraction, reducedForm, wholePart
```

```
++ Related Constructors:
++ Also See: Fraction
++ AMS Classifications: 11A55 11J70 11K50 11Y65 30B70 40A15
++ Keywords: continued fraction, convergent
++ References:
++ Description:
++ \spadtype{ContinuedFraction} implements general
++ continued fractions.  This version is not restricted to simple,
++ finite fractions and uses the \spadtype{Stream} as a
++ representation.  The arithmetic functions assume that the
++ approximants alternate below/above the convergence point.
++ This is enforced by ensuring the partial numerators and partial
++ denominators are greater than 0 in the Euclidean domain view of \spad{R}
++ (i.e. \spad{sizeLess?(0, x)}).

ContinuedFraction(R): Exports == Implementation where
  R :     EuclideanDomain
  Q   ==> Fraction R
  MT  ==> MoebiusTransform Q
  OUT ==> OutputForm

  Exports ==> Join(Algebra R,Algebra Q,Field) with
    continuedFraction:        Q -> %
      ++ continuedFraction(r) converts the fraction \spadvar{r} with
      ++ components of type \spad{R} to a continued fraction over
      ++ \spad{R}.

    continuedFraction:        (R, Stream R, Stream R) -> %
      ++ continuedFraction(b0,a,b) constructs a continued fraction in
      ++ the following way:  if \spad{a = [a1,a2,...]} and \spad{b =
      ++ [b1,b2,...]} then the result is the continued fraction
      ++ \spad{b0 + a1/(b1 + a2/(b2 + ...))}.

    reducedContinuedFraction: (R, Stream R) -> %
      ++ reducedContinuedFraction(b0,b) constructs a continued
      ++ fraction in the following way:  if \spad{b = [b1,b2,...]}
      ++ then the result is the continued fraction \spad{b0 + 1/(b1 +
      ++ 1/(b2 + ...))}.  That is, the result is the same as
      ++ \spad{continuedFraction(b0,[1,1,1,...],[b1,b2,b3,...])}.

    partialNumerators:   % -> Stream R
      ++ partialNumerators(x) extracts the numerators in \spadvar{x}.
      ++ That is, if \spad{x = continuedFraction(b0, [a1,a2,a3,...],
      ++ [b1,b2,b3,...])}, then \spad{partialNumerators(x) =
      ++ [a1,a2,a3,...]}.

    partialDenominators: % -> Stream R
      ++ partialDenominators(x) extracts the denominators in
      ++ \spadvar{x}.  That is, if \spad{x = continuedFraction(b0,
      ++ [a1,a2,a3,...], [b1,b2,b3,...])}, then
```

```
    ++ \spad{partialDenominators(x) = [b1,b2,b3,...]}.

  partialQuotients:    % -> Stream R
    ++ partialQuotients(x) extracts the partial quotients in
    ++ \spadvar{x}.  That is, if \spad{x = continuedFraction(b0,
    ++ [a1,a2,a3,...], [b1,b2,b3,...])}, then
    ++ \spad{partialQuotients(x) = [b0,b1,b2,b3,...]}.

  wholePart:           % -> R
    ++ wholePart(x) extracts the whole part of \spadvar{x}.  That
    ++ is, if \spad{x = continuedFraction(b0, [a1,a2,a3,...],
    ++ [b1,b2,b3,...])}, then \spad{wholePart(x) = b0}.

  reducedForm:         % -> %
    ++ reducedForm(x) puts the continued fraction \spadvar{x} in
    ++ reduced form, i.e.  the function returns an equivalent
    ++ continued fraction of the form
    ++ \spad{continuedFraction(b0,[1,1,1,...],[b1,b2,b3,...])}.

  approximants:        % -> Stream Q
    ++ approximants(x) returns the stream of approximants of the
    ++ continued fraction \spadvar{x}. If the continued fraction is
    ++ finite, then the stream will be infinite and periodic with
    ++ period 1.

  convergents:         % -> Stream Q
    ++ convergents(x) returns the stream of the convergents of the
    ++ continued fraction \spadvar{x}. If the continued fraction is
    ++ finite, then the stream will be finite.

  numerators:          % -> Stream R
    ++ numerators(x) returns the stream of numerators of the
    ++ approximants of the continued fraction \spadvar{x}. If the
    ++ continued fraction is finite, then the stream will be finite.

  denominators:        % -> Stream R
    ++ denominators(x) returns the stream of denominators of the
    ++ approximants of the continued fraction \spadvar{x}. If the
    ++ continued fraction is finite, then the stream will be finite.

  extend:              (%,Integer) -> %
    ++ extend(x,n) causes the first \spadvar{n} entries in the
    ++ continued fraction \spadvar{x} to be computed.  Normally
    ++ entries are only computed as needed.

  complete:            % -> %
    ++ complete(x) causes all entries in \spadvar{x} to be computed.
    ++ Normally entries are only computed as needed.  If \spadvar{x}
    ++ is an infinite continued fraction, a user-initiated interrupt is
    ++ necessary to stop the computation.
```

```
 Implementation ==> add

-- isOrdered  ==> R is Integer
   isOrdered  ==> R has OrderedRing and R has multiplicativeValuation
   canReduce? ==> isOrdered or R has additiveValuation

   Rec ==> Record(num: R, den: R)
   Str ==> Stream Rec
   Rep :=  Record(value: Record(whole: R, fract: Str), reduced?: Boolean)

   import Str

   genFromSequence:     Stream Q -> %
   genReducedForm:      (Q, Stream Q, MT)    -> Stream Rec
   genFractionA:        (Stream R,Stream R)  -> Stream Rec
   genFractionB:        (Stream R,Stream R)  -> Stream Rec
   genNumDen:           (R,R, Stream Rec)    -> Stream R

   genApproximants:     (R,R,R,R,Stream Rec) -> Stream Q
   genConvergents:      (R,R,R,R,Stream Rec) -> Stream Q
   iGenApproximants:    (R,R,R,R,Stream Rec) -> Stream Q
   iGenConvergents:     (R,R,R,R,Stream Rec) -> Stream Q

   reducedForm c ==
       c.reduced? => c
       explicitlyFinite? c.value.fract =>
                     continuedFraction last complete convergents c
       canReduce? => genFromSequence approximants c
       error "Reduced form not defined for this continued fraction."

   eucWhole(a: Q): R == numer a quo denom a

   eucWhole0(a: Q): R ==
       isOrdered =>
           n := numer a
           d := denom a
           q := n quo d
           r := n - q*d
           if r < 0 then q := q - 1
           q
       eucWhole a

   x = y ==
       x := reducedForm x
       y := reducedForm y

       x.value.whole ^= y.value.whole => false

       xl := x.value.fract; yl := y.value.fract
```

```
    while not empty? xl and not empty? yl repeat
        frst.xl.den ^= frst.yl.den => return false
        xl := rst xl; yl := rst yl
    empty? xl and empty? yl

continuedFraction q == q :: %

if isOrdered then
    continuedFraction(wh,nums,dens) == [[wh,genFractionA(nums,dens)],false]

    genFractionA(nums,dens) ==
        empty? nums or empty? dens => empty()
        n := frst nums
        d := frst dens
        n < 0 => error "Numerators must be greater than 0."
        d < 0 => error "Denominators must be greater than 0."
        concat([n,d]$Rec, delay genFractionA(rst nums,rst dens))
else
    continuedFraction(wh,nums,dens) == [[wh,genFractionB(nums,dens)],false]

    genFractionB(nums,dens) ==
        empty? nums or empty? dens => empty()
        n := frst nums
        d := frst dens
        concat([n,d]$Rec, delay genFractionB(rst nums,rst dens))

reducedContinuedFraction(wh,dens) ==
    continuedFraction(wh, repeating [1], dens)

coerce(n:Integer):% == [[n::R,empty()], true]
coerce(r:R):%       == [[r,   empty()], true]

coerce(a: Q): % ==
  wh := eucWhole0 a
  fr := a - wh::Q
  zero? fr => [[wh, empty()], true]

  l : List Rec := empty()
  n := numer fr
  d := denom fr
  while not zero? d repeat
    qr := divide(n,d)
    l  := concat([1,qr.quotient],l)
    n  := d
    d  := qr.remainder
  [[wh, construct rest reverse_! l], true]

characteristic() == characteristic()$Q
```

```
genFromSequence apps ==
    lo := first apps; apps := rst apps
    hi := first apps; apps := rst apps
    while eucWhole0 lo ^= eucWhole0 hi repeat
        lo := first apps; apps := rst apps
        hi := first apps; apps := rst apps
    wh := eucWhole0 lo
    [[wh, genReducedForm(wh::Q, apps, moebius(1,0,0,1))], canReduce?]

genReducedForm(wh0, apps, mt) ==
    lo: Q := first apps - wh0; apps := rst apps
    hi: Q := first apps - wh0; apps := rst apps
    lo = hi and zero? eval(mt, lo) => empty()
    mt  := recip mt
    wlo := eucWhole eval(mt, lo)
    whi := eucWhole eval(mt, hi)
    while wlo ^= whi repeat
        wlo := eucWhole eval(mt, first apps - wh0); apps := rst apps
        whi := eucWhole eval(mt, first apps - wh0); apps := rst apps
    concat([1,wlo], delay genReducedForm(wh0, apps, shift(mt, -wlo::Q)))

wholePart c ==
  c.value.whole
partialNumerators c ==
  map(x1+->x1.num, c.value.fract)$StreamFunctions2(Rec,R)
partialDenominators c ==
  map(x1+->x1.den, c.value.fract)$StreamFunctions2(Rec,R)
partialQuotients c ==
  concat(c.value.whole, partialDenominators c)

approximants c ==
  empty? c.value.fract => repeating [c.value.whole::Q]
  genApproximants(1,0,c.value.whole,1,c.value.fract)
convergents c ==
  empty? c.value.fract => concat(c.value.whole::Q, empty())
  genConvergents (1,0,c.value.whole,1,c.value.fract)
numerators c ==
  empty? c.value.fract => concat(c.value.whole, empty())
  genNumDen(1,c.value.whole,c.value.fract)
denominators c ==
  genNumDen(0,1,c.value.fract)

extend(x,n) == (extend(x.value.fract,n); x)
complete(x) == (complete(x.value.fract); x)

iGenApproximants(pm2,qm2,pm1,qm1,fr) == delay
  nd := frst fr
  pm := nd.num*pm2 + nd.den*pm1
  qm := nd.num*qm2 + nd.den*qm1
```

```
  genApproximants(pm1,qm1,pm,qm,rst fr)

genApproximants(pm2,qm2,pm1,qm1,fr) ==
  empty? fr => repeating [pm1/qm1]
  concat(pm1/qm1,iGenApproximants(pm2,qm2,pm1,qm1,fr))

iGenConvergents(pm2,qm2,pm1,qm1,fr) == delay
  nd := frst fr
  pm := nd.num*pm2 + nd.den*pm1
  qm := nd.num*qm2 + nd.den*qm1
  genConvergents(pm1,qm1,pm,qm,rst fr)

genConvergents(pm2,qm2,pm1,qm1,fr) ==
  empty? fr => concat(pm1/qm1, empty())
  concat(pm1/qm1,iGenConvergents(pm2,qm2,pm1,qm1,fr))

genNumDen(m2,m1,fr) ==
  empty? fr => concat(m1,empty())
  concat(m1,delay genNumDen(m1,m2*frst(fr).num + m1*frst(fr).den,rst fr))

gen  ==> genFromSequence
apx  ==> approximants

c, d: %
a: R
q: Q
n: Integer

0 == (0$R) :: %
1 == (1$R) :: %

c + d   == genFromSequence map((x,y) +-> x + y, apx c, apx d)
c - d   == genFromSequence map((x,y) +-> x - y, apx c, rest apx d)
- c     == genFromSequence map(x +-> - x, rest apx c)
c * d   == genFromSequence map((x,y) +-> x * y, apx c, apx d)
a * d   == genFromSequence map(x +-> a * x, apx d)
q * d   == genFromSequence map(x +-> q * x, apx d)
n * d   == genFromSequence map(x +-> n * x, apx d)
c / d   == genFromSequence map((x,y) +-> x / y, apx c, rest apx d)
recip c ==(c = 0 => "failed";
   genFromSequence map(x +-> 1/x, rest apx c))

showAll?: () -> Boolean
showAll?() ==
  NULL(_$streamsShowAll$Lisp)$Lisp => false
  true

zagRec(t:Rec):OUT == zag(t.num :: OUT,t.den :: OUT)

coerce(c:%): OUT ==
```

```
      wh := c.value.whole
      fr := c.value.fract
      empty? fr => wh :: OUT
      count : NonNegativeInteger := _$streamCount$Lisp
      l : List OUT := empty()
      for n in 1..count while not empty? fr repeat
        l  := concat(zagRec frst fr,l)
        fr := rst fr
      if showAll?() then
        for n in (count + 1).. while explicitEntries? fr repeat
          l  := concat(zagRec frst fr,l)
          fr := rst fr
      if not explicitlyEmpty? fr then l := concat("..." :: OUT,l)
      l := reverse_! l
      e := reduce("+",l)
      zero? wh => e
      (wh :: OUT) + e
```

— CONTFRAC.dotabb —

```
"CONTFRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CONTFRAC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"CONTFRAC" -> "PFECAT"
```

Chapter 5

Chapter D

5.1 domain DBASE Database

— Database.input —

```
)set break resume
)sys rm -f Database.output
)spool Database.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Database
--R Database S where
--R   S: OrderedSet with
--R       ?.? : (%,Symbol) -> String
--R       display : % -> Void
--R       fullDisplay : % -> Void  is a domain constructor
--R Abbreviation for Database is DBASE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DBASE
--R
--R------------------------------- Operations --------------------------------
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                coerce : List S -> %
--R coerce : % -> OutputForm              display : % -> Void
--R ?.? : (%,QueryEquation) -> %          fullDisplay : % -> Void
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ?.? : (%,Symbol) -> DataList String
--R fullDisplay : (%,PositiveInteger,PositiveInteger) -> Void
```

```
--R
--E 1

)spool
)lisp (bye)
```

———-

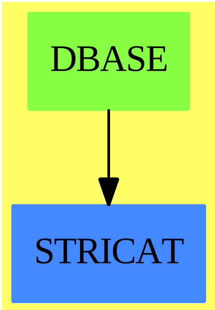
— **Database.help** —

```
====================================================================
Database examples
====================================================================

See Also:
o )show Database
```

———-

5.1.1 Database (DBASE)



See

⇒ "DataList" (DLIST) 5.2.1 on page 445
⇒ "IndexCard" (ICARD) 10.2.1 on page 1159
⇒ "QueryEquation" (QEQUAT) 18.4.1 on page 2129

Exports:

coerce	display	fullDisplay	hash	latex
?+?	?-?	?=?	?~=?	?.?

— **domain DBASE Database** —

```
)abbrev domain DBASE Database
++ Author: Mark Botch
++ Description:
```

```
++ This domain implements a simple view of a database whose fields are
++ indexed by symbols

Database(S): Exports == Implementation where
  S: OrderedSet with
    elt: (%,Symbol) -> String
      ++ elt(x,s) returns an element of x indexed by s
    display: % -> Void
      ++ display(x) displays x in some form
    fullDisplay: % -> Void
      ++ fullDisplay(x) displays x in detail
  Exports == SetCategory with
    elt: (%,QueryEquation) -> %
      ++ elt(db,q) returns all elements of \axiom{db} which satisfy \axiom{q}.
    elt: (%,Symbol) -> DataList String
      ++ elt(db,s) returns the \axiom{s} field of each element of \axiom{db}.
    _+: (%,%) -> %
      ++ db1+db2 returns the merge of databases db1 and db2
    _-: (%,%) -> %
      ++ db1-db2 returns the difference of databases db1 and db2 i.e. consisting
      ++ of elements in db1 but not in db2
    coerce: List S -> %
      ++ coerce(l) makes a database out of a list
    display: % -> Void
      ++ display(db) prints a summary line for each entry in \axiom{db}.
    fullDisplay: % -> Void
      ++ fullDisplay(db) prints full details of each entry in \axiom{db}.
    fullDisplay: (%,PositiveInteger,PositiveInteger) -> Void
      ++ fullDisplay(db,start,end ) prints full details of entries in the range
      ++ \axiom{start..end} in \axiom{db}.
  Implementation == List S add
    s: Symbol
    Rep := List S
    coerce(u: List S):% == u@%
    elt(data: %,s: Symbol) == [x.s for x in data] :: DataList(String)
    elt(data: %,eq: QueryEquation) ==
      field := variable eq
      val := value eq
      [x for x in data | stringMatches?(val,x.field)$Lisp]
    x+y==removeDuplicates_! merge(x,y)
    x-y==mergeDifference(copy(x::Rep),y::Rep)$MergeThing(S)
    coerce(data): OutputForm == (#data):: OutputForm
    display(data) ==  for x in data repeat display x
    fullDisplay(data) == for x in data repeat fullDisplay x
    fullDisplay(data,n,m) == for x in data for i in 1..m repeat
      if i >= n then fullDisplay x
```

———-

— DBASE.dotabb —

```
"DBASE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DBASE"]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"DBASE" -> "STRICAT"
```

———-

5.2 domain DLIST DataList

— DataList.input —

```
)set break resume
)sys rm -f DataList.output
)spool DataList.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DataList
--R DataList S: OrderedSet  is a domain constructor
--R Abbreviation for DataList is DLIST
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DLIST
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %              coerce : % -> List S
--R coerce : List S -> %                concat : (%,S) -> %
--R concat : List % -> %                concat : (S,%) -> %
--R concat : (%,%) -> %                 concat! : (%,S) -> %
--R concat! : (%,%) -> %                construct : List S -> %
--R copy : % -> %                       cycleEntry : % -> %
--R cycleTail : % -> %                  cyclic? : % -> Boolean
--R datalist : List S -> %              delete : (%,Integer) -> %
--R delete! : (%,Integer) -> %          distance : (%,%) -> Integer
--R ?.sort : (%,sort) -> %              ?.unique : (%,unique) -> %
--R elt : (%,Integer,S) -> S            ?.? : (%,Integer) -> S
--R ?.last : (%,last) -> S              ?.rest : (%,rest) -> %
--R ?.first : (%,first) -> S            ?.value : (%,value) -> S
--R empty : () -> %                     empty? : % -> Boolean
--R entries : % -> List S               eq? : (%,%) -> Boolean
--R explicitlyFinite? : % -> Boolean    first : % -> S
--R index? : (Integer,%) -> Boolean     indices : % -> List Integer
--R insert : (S,%,Integer) -> %         insert : (%,%,Integer) -> %
```

```
--R insert! : (S,%,Integer) -> %         insert! : (%,%,Integer) -> %
--R last : % -> S                        leaf? : % -> Boolean
--R leaves : % -> List S                 list : S -> %
--R map : (((S,S) -> S),%,%) -> %        map : ((S -> S),%) -> %
--R new : (NonNegativeInteger,S) -> %    nodes : % -> List %
--R possiblyInfinite? : % -> Boolean     qelt : (%,Integer) -> S
--R rest : % -> %                        reverse : % -> %
--R sample : () -> %                     second : % -> S
--R tail : % -> %                        third : % -> S
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R ?.count : (%,count) -> NonNegativeInteger
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
```

```
--R merge! : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R position : ((S -> Boolean),%) -> Integer
--R position : (S,%) -> Integer if S has SETCAT
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,%) -> % if S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates! : % -> % if S has SETCAT
--R rest : (%,NonNegativeInteger) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((S,S) -> Boolean),%) -> %
--R sort : % -> % if S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if S has ORDSET
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

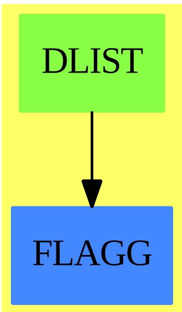
)spool
)lisp (bye)
```

— DataList.help —

```
====================================================================
DataList examples
====================================================================

See Also:
o )show DataList
```

5.2.1 DataList (DLIST)



See

⇒ "IndexCard" (ICARD) 10.2.1 on page 1159
⇒ "Database" (DBASE) 5.1.1 on page 440
⇒ "QueryEquation" (QEQUAT) 18.4.1 on page 2129

Exports:

any?	child?	children	coerce
concat	concat!	construct	convert
copy	copyInto!	count	cycleEntry
cycleLength	cycleSplit!	cycleTail	cyclic?
datalist	delete	delete!	distance
elt	empty	empty?	entries
entry?	eq?	eval	every?
explicitlyFinite?	fill!	find	first
hash	index?	indices	insert
insert!	last	latex	leaf?
leaves	less?	list	map
map!	max	maxIndex	member?
members	merge	merge!	min
minIndex	more?	new	node?
nodes	parts	position	possiblyInfinite?
qelt	qsetelt!	reduce	remove
remove!	removeDuplicates	removeDuplicates!	rest
reverse	reverse!	sample	second
select	select!	setchildren!	setelt
setfirst!	setlast!	setrest!	setvalue!
size?	sort	sort!	sorted?
split!	swap!	tail	third
value	#?	?<?	?<=?
?=?	?>?	?>=?	?.?
?~=?	?.count	?.sort	?.unique
?.last	?.rest	?.first	?.value

— domain DLIST DataList —

```
)abbrev domain DLIST DataList
++ Author: Mark Botch
++ Description:
++ This domain provides some nice functions on lists

DataList(S:OrderedSet) : Exports == Implementation where
  Exports == ListAggregate(S) with
    coerce: List S -> %
      ++ coerce(l) creates a datalist from l
    coerce: % -> List S
      ++ coerce(x) returns the list of elements in x
    datalist: List S -> %
      ++ datalist(l) creates a datalist from l
    elt: (%,"unique") -> %
      ++ \axiom{l.unique} returns \axiom{l} with duplicates removed.
      ++ Note: \axiom{l.unique = removeDuplicates(l)}.
    elt: (%,"sort") -> %
      ++ \axiom{l.sort} returns \axiom{l} with elements sorted.
```

```
      ++ Note: \axiom{l.sort = sort(l)}
    elt: (%,"count") -> NonNegativeInteger
      ++ \axiom{l."count"} returns the number of elements in \axiom{l}.
  Implementation == List(S) add
    elt(x,"unique") == removeDuplicates(x)
    elt(x,"sort") == sort(x)
    elt(x,"count") == #x
    coerce(x:List S) == x pretend %
    coerce(x:%):List S == x pretend (List S)
    coerce(x:%): OutputForm == (x :: List S) :: OutputForm
    datalist(x:List S) == x::%
```

———-

— DLIST.dotabb —

```
"DLIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DLIST"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"DLIST" -> "FLAGG"
```

———-

5.3 domain DECIMAL DecimalExpansion

— DecimalExpansion.input —

```
)set break resume
)sys rm -f DecimalExpansion.output
)spool DecimalExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
r := decimal(22/7)
--R
--R
--R              ______
--R   (1)  3.142857
--R                                                         Type: DecimalExpansion
--E 1

--S 2 of 7
r + decimal(6/7)
--R
```

```
--R
--R   (2)  4
--R                                                         Type: DecimalExpansion
--E 2

--S 3 of 7
[decimal(1/i) for i in 350..354]
--R
--R
--R   (3)
--R          ------    ------         --    --------------------------------
--R   [0.00285714, 0.002849, 0.0028409, 0.00283286118980169971671388101983,
--R         ----------------------------------------------------------
--R    0.00282485875706214689265536723163841807909604519774011299435]
--R                                                    Type: List DecimalExpansion
--E 3

--S 4 of 7
decimal(1/2049)
--R
--R
--R   (4)
--R   0.
--R     OVERBAR
--R        00048804294777940458760370912640312347486578818936066373840897999023914
--R          10444119082479258174719375305026842362127867252318204001952171791176
--R          18350414836505612493899463152757442654953635919960956564177647632991 7
--R          03269887750122010736944851146900927281600780868716447047340165934602 2
--R          4499755978526110297706198145436798438262567105905319668130795 51
--R                                                         Type: DecimalExpansion
--E 4

--S 5 of 7
p := decimal(1/4)*x**2 + decimal(2/3)*x + decimal(4/9)
--R
--R
--R             2     _      _
--R   (5)  0.25x  + 0.6x + 0.4
--R                                              Type: Polynomial DecimalExpansion
--E 5

--S 6 of 7
q := differentiate(p, x)
--R
--R
--R                    _
--R   (6)  0.5x + 0.6
--R                                              Type: Polynomial DecimalExpansion
--E 6
```

```
--S 7 of 7
g := gcd(p, q)
--R
--R
--R                -
--R   (7)  x + 1.3
--R                                           Type: Polynomial DecimalExpansion
--E 7
)spool
)lisp (bye)
```

———–

— DecimalExpansion.help —

```
====================================================================
DecimalExpansion examples
====================================================================

All rationals have repeating decimal expansions.  Operations to access
the individual digits of a decimal expansion can be obtained by
converting the value to RadixExpansion(10).

The operation decimal is used to create this expansion of type
DecimalExpansion.

  r := decimal(22/7)
        ------
      3.142857
                      Type: DecimalExpansion

Arithmetic is exact.

  r + decimal(6/7)
   4
                      Type: DecimalExpansion

The period of the expansion can be short or long ...

  [decimal(1/i) for i in 350..354]
        ------    ------         --    --------------------------------
    [0.00285714, 0.002849, 0.0028409, 0.00283286118980169971671388101983,
        ----------------------------------------------------------
      0.00282485875706214689265536723163841807909604519774011299435]
                      Type: List DecimalExpansion

or very long.

  decimal(1/2049)
```

```
    -----------------------------------------------------------------------
  0.00048804294777940458760370912640312347486578818936066373840897999023914

    ---------------------------------------------------------------------
    104441190824792581747193753050268423621278672523182040019521717911176

    ---------------------------------------------------------------------
    183504148365056124938994631527574426549536359199609565641776476329917

    ---------------------------------------------------------------------
    032698877501220107369448511469009272816007808687164470473401659346022

    ---------------------------------------------------------------
    449975597852611029770619814543679843826256710590531966813079551
                    Type: DecimalExpansion
```

These numbers are bona fide algebraic objects.

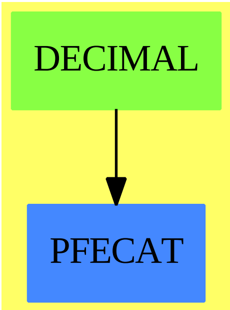
```
  p := decimal(1/4)*x**2 + decimal(2/3)*x + decimal(4/9)
        2     _      _
  0.25x  + 0.6x + 0.4
                    Type: Polynomial DecimalExpansion

  q := differentiate(p, x)
              _
  0.5x + 0.6
                    Type: Polynomial DecimalExpansion

  g := gcd(p, q)
           _
  x + 1.3
                    Type: Polynomial DecimalExpansion
```

See Also:
o)help RadixExpansion
o)help BinaryExpansion
o)help HexadecimalExpansion
o)show DecimalExpansion

5.3.1 DecimalExpansion (DECIMAL)



See

⇒ "RadixExpansion" (RADIX) 19.2.1 on page 2165
⇒ "BinaryExpansion" (BINARY) 3.7.1 on page 274
⇒ "HexadecimalExpansion" (HEXADEC) 9.3.1 on page 1108

Exports:

0	1	abs
associates?	ceiling	characteristic
charthRoot	coerce	conditionP
convert	D	decimal
denom	denominator	differentiate
divide	euclideanSize	eval
expressIdealMember	exquo	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractionPart	gcd
gcdPolynomial	hash	init
inv	latex	lcm
map	max	min
multiEuclidean	negative?	nextItem
numer	numerator	one?
patternMatch	positive?	prime?
principalIdeal	random	recip
reducedSystem	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
zero?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?.?
?quo?	?rem?	

— domain DECIMAL DecimalExpansion —

```
)abbrev domain DECIMAL DecimalExpansion
++ Author: Stephen M. Watt
++ Date Created: October, 1986
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains: RadixExpansion
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, repeating decimal
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ decimal expansions.

DecimalExpansion(): Exports == Implementation where
  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
      ++ coerce(d) converts a decimal expansion to a rational number.
    coerce: % -> RadixExpansion(10)
      ++ coerce(d) converts a decimal expansion to a radix expansion
      ++ with base 10.
    fractionPart: % -> Fraction Integer
      ++ fractionPart(d) returns the fractional part of a decimal expansion.
    decimal: Fraction Integer -> %
      ++ decimal(r) converts a rational number to a decimal expansion.

  Implementation ==> RadixExpansion(10) add
    decimal r == r :: %
    coerce(x:%): RadixExpansion(10) == x pretend RadixExpansion(10)
```

— DECIMAL.dotabb —

```
"DECIMAL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DECIMAL"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DECIMAL" -> "PFECAT"
```

5.4 Denavit-Hartenberg Matrices

5.4.1 Homogeneous Transformations

The study of robot manipulation is concerned with the relationship between objects, and between objects and manipulators. In this chapter we will develop the representation necessary to describe these relationships. Similar problems of representation have already been solved in the field of computer graphics, where the relationship between objects must also be described. Homogeneous transformations are used in this field and in computer vision [Duda] [Robserts63] [Roberts65]. These transformations were employed by Denavit to describe linkages [Denavit] and are now used to describe manipulators [Pieper] [Paul72] [Paul77b].

We will first establish notation for vectors and planes and then introduce transformations on them. These transformations consist primarily of translation and rotation. We will then show that these transformations can also be considered as coordinate frames in which to represent objects, including the manipulator. The inverse transformation will then be introduced. A later section describes the general rotation transformation representing a rotation about a vector. An algorithm is then described to find the equivalent axis and angle of rotations represented by any given transformation. A brief section on stretching and scaling transforms is included together with a section on the perspective transformation. The chapter concludes with a section on transformation equations.

5.4.2 Notation

In describing the relationship between objects we will make use of point vectors, planes, and coordinate frames. Point vectors are denoted by lower case, bold face characters. Planes are denoted by script characters, and coordinate frames by upper case, bold face characters. For example:

vectors	$\mathbf{v}$, $\mathbf{x1}$, $\mathbf{x}$
planes	$\mathcal{P}$, $\mathcal{Q}$
coordinate frames	$\mathbf{I}$, $\mathbf{A}$, $\mathbf{CONV}$

We will use point vectors, planes, and coordinate frames as variables which have associated values. For example, a point vector has as value its three Cartesian coordinate components.

If we wish to describe a point in space, which we will call p, with respect to a coordinate frame $\mathbf{E}$, we will use a vector which we will call $\mathbf{v}$. We will write this as

$$^{E}\mathbf{v}$$

The leading superscript describes the defining coordinate frame.

We might also wish to describe this same point, p, with respect to a different coordinate frame, for example $\mathbf{H}$, using a vector $\mathbf{w}$ as

$$^{H}\mathbf{w}$$

$\mathbf{v}$ and $\mathbf{w}$ are two vectors which probably have different component values and $\mathbf{v} \neq \mathbf{w}$ even though both vectors describe the same point p. The case might also exist of a vector $\mathbf{a}$ describing a point 3 inches above any frame

$$^{F^1}\mathbf{a} \qquad ^{F^2}\mathbf{a}$$

In this case the vectors are identical but describe different points. Frequently, the defining frame will be obvious from the text and the superscripts will be left off. In many cases the name of the vector will be the same as the name of the object described, for example, the tip of a pin might be described by a vector **tip** with respect to a frame **BASE** as

$$^{BASE}\mathbf{tip}$$

If it were obvious from the text that we were describing the vector with respect to **BASE** then we might simply write

$$\mathbf{tip}$$

If we also wish to describe this point with respect to another coordinate frame say, **HAND**, then we must use another vector to describe this relationship, for example

$$^{HAND}\mathbf{tv}$$

$^{HAND}\mathbf{tv}$ and **tip** both describe the same feature but have different values. In order to refer to individual components of coordinate frames, point vectors, or planes, we add subscripts to indicate the particular component. For example, the vector $^{HAND}\mathbf{tv}$ has components $^{HAND}\mathbf{tv_x}$, $^{HAND}\mathbf{tv_y}$, $^{HAND}\mathbf{tv_z}$.

5.4.3 Vectors

The homogeneous coordinate representation of objects in n-space is an $(n+1)$-space entity such that a particular perspective projection recreates the n-space. This can also be viewed as the addition of an extra coordinate to each vector, a scale factor, such that the vector has the same meaning if each component, including the scale factor, is multiplied by a constant.

A point vector

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} \tag{1.1}$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along the x, y, and z coordinate axes, respectively, is represented in homogeneous coordinates as a column matrix

$$\mathbf{v} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \\ \mathbf{w} \end{bmatrix} \tag{1.2}$$

where

$$\begin{aligned}\mathbf{a} &= \mathbf{x}/\mathbf{w}\\ \mathbf{b} &= \mathbf{y}/\mathbf{w}\\ \mathbf{c} &= \mathbf{z}/\mathbf{w}\end{aligned} \tag{1.3}$$

Thus the vector $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ can be represented as $[3, 4, 5, 1]^{\mathrm{T}}$ or as $[6, 8, 10, 2]^{\mathrm{T}}$ or again as $[-30, -40, -50, -10]^{\mathrm{T}}$, etc. The superscript T indicates the transpose of the row vector into a column vector. The vector at the origin, the null vector, is represented as $[0, 0, 0, n]^{\mathrm{T}}$ where n is any non-zero scale factor. The vector $[0, 0, 0, 0]^{\mathrm{T}}$ is undefined. Vectors of the form $[a, b, c, 0]^{\mathrm{T}}$ represent vectors at infinity and are used to represent directions; the addition of any other finite vector does not change their value in any way.

We will also make use of the vector dot and cross products. Given two vectors

$$\begin{aligned}\mathbf{a} &= a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \qquad (1.4)\\ \mathbf{b} &= b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}\end{aligned}$$

we define the vector dot product, indicated by "·" as

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z \tag{1.5}$$

The dot product of two vectors is a scalar. The cross product, indicated by an "×", is another vector perpendicular to the plane formed by the vectors of the product and is defined by

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{i} + (a_z b_x - a_x b_z)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k} \tag{1.6}$$

This definition is easily remembered as the expansion of the determinant

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \tag{1.7}$$

5.4.4 Planes

A plane is represented as a row matrix

$$\mathcal{P} = [a, b, c, d] \tag{1.8}$$

such that if a point $\mathbf{v}$ lies in a plane $\mathcal{P}$ the matrix product

$$\mathcal{P}\ \mathbf{v} = 0 \tag{1.9}$$

or in expanded form

$$xa + yb + zc + wd = 0 \tag{1.10}$$

If we define a constant

$$m = +\sqrt{a^2 + b^2 + c^2} \tag{1.11}$$

and divide Equation 1.10 by wm we obtain

$$\frac{x}{w}\frac{a}{m} + \frac{y}{w}\frac{b}{m} + \frac{z}{w}\frac{c}{m} = -\frac{d}{m} \tag{1.12}$$

The left hand side of Equation 1.12 is the vector dot product of two vectors $(x/w)\mathbf{i} + (y/w)\mathbf{j} + (z/w)\mathbf{k}$ and $(a/m)\mathbf{i} + (b/m)\mathbf{j} + (c/m)\mathbf{k}$ and represents the directed distance of the point $(x/w)\mathbf{i} + (y/w)\mathbf{j} + (z/w)\mathbf{k}$ along the vector
$(a/m)\mathbf{i} + (b/m)\mathbf{j} + (c/m)\mathbf{k}$. The vector $(a/m)\mathbf{i} + (b/m)\mathbf{j} + (c/m)\mathbf{k}$ can be interpreted as the outward pointing normal of a plane situated a distance $-d/m$ from the origin in the direction of the normal. Thus a plane $\mathcal{P}$ parallel to the x,y plane, one unit along the z axis, is represented as

$$\mathcal{P} = [0, 0, 1, -1] \tag{1.13}$$

$$\text{or as} \quad \mathcal{P} = [0, 0, 2, -2] \tag{1.14}$$

$$\text{or as} \quad \mathcal{P} = [0, 0, -100, 100] \tag{1.15}$$

A point $\mathbf{v} = [10, 20, 1, 1]$ should lie in this plane

$$[0, 0, -100, 100] \begin{bmatrix} 10 \\ 20 \\ 1 \\ 1 \end{bmatrix} = 0 \tag{1.16}$$

or

$$[0, 0, 1, -1] \begin{bmatrix} -5 \\ -10 \\ -.5 \\ -.5 \end{bmatrix} = 0 \tag{1.17}$$

The point $\mathbf{v} = [0, 0, 2, 1]$ lies above the plane

$$[0, 0, 2, -2] \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = 2 \tag{1.18}$$

and $\mathcal{P}$ **v** is indeed positive, indicating that the point is outside the plane in the direction of the outward pointing normal. A point $\mathbf{v} = [0, 0, 0, 1]$ lies below the plane

$$[0, 0, 1, -1] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = -1 \tag{1.19}$$

The plane $[0, 0, 0, 0]$ is undefined.

5.4.5 Transformations

A transformation of the space **H** is a 4x4 matrix and can represent translation, rotation, stretching, and perspective transformations. Given a point **u**, its transformation **v** is represented by the matrix product

$$\mathbf{v} = \mathbf{H}\mathbf{u} \tag{1.20}$$

The corresponding plane transformation $\mathcal{P}$ to $\mathcal{Q}$ is

$$\mathcal{Q}{=}\mathcal{P}\ \mathbf{H^{-1}} \tag{1.21}$$

as we requre that the condition

$$\mathcal{Q}\ \mathbf{v} = \mathcal{P}\ \mathbf{u} \tag{1.22}$$

is invariant under all transformations. To verify this we substitute from Equations 1.20 and 1.21 into the left hand side of 1.22 and we obtain on the right hand side $\mathbf{H^{-1}H}$ which is the identity matrix **I**

$$\mathcal{P}\ \mathbf{H^{-1}Hu} = \mathcal{P}\ \mathbf{u} \tag{1.23}$$

5.4.6 Translation Transformation

The transformation **H** corresponding to a translation by a vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is

$$\mathbf{H} = \mathbf{Trans(a, b, c)} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.24}$$

Given a vector $\mathbf{u} = [x, y, z, w]^{\mathrm{T}}$ the transformed vector **v** is given by

$$\mathbf{H} = \mathbf{Trans(a, b, c)} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \tag{1.25}$$

$$\mathbf{v} = \begin{bmatrix} x + aw \\ y + bw \\ z + cw \\ w \end{bmatrix} = \begin{bmatrix} x/w + a \\ y/w + b \\ z/w + c \\ 1 \end{bmatrix} \tag{1.26}$$

The translation may also be interpreted as the addition of the two vectors $(x/w)\mathbf{i} + (y/w)\mathbf{j} + (z/w)\mathbf{k}$ and $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

Every element of a transformation matrix may be multiplied by a non-zero constant without changing the transformation, in the same manner as points and planes. Consider the vector $2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ translated by, or added to
4**i** - 3**j** + 7**k**

$$\begin{bmatrix} 6 \\ 0 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix} \tag{1.27}$$

If we multiply the transmation matrix elements by, say, -5, and the vector elements by 2, we obtain

$$\begin{bmatrix} -60 \\ 0 \\ -90 \\ -10 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 & -20 \\ 0 & -5 & 0 & 15 \\ 0 & 0 & -5 & -35 \\ 0 & 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 4 \\ 2 \end{bmatrix} \tag{1.28}$$

which corresponds to the vector $[6, 0, 9, 1]^{\mathrm{T}}$ as before. The point $[2, 3, 2, 1]$ lies in the plane $[1, 0, 0, -2]$

$$[1, 0, 0, -2] \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix} = 0 \tag{1.29}$$

The transformed point is, as we have already found, $[6, 0, 9, 1]^{\mathrm{T}}$. We will now compute the transformed plane. The inverse of the transform is

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the transformed plane

$$[1\ 0\ 0\ -6] = [1\ 0\ 0\ -2] \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.30}$$

Once again the transformed point lies in the transformed plane

$$[1\ 0\ 0\ -6] \begin{bmatrix} 6 \\ 0 \\ 9 \\ 1 \end{bmatrix} = 0 \tag{1.31}$$

The general translation operation can be represented in Axiom as

— **translate** —

```
translate(x,y,z) ==
 matrix(_
  [[1,0,0,x],_
   [0,1,0,y],_
   [0,0,1,z],_
   [0,0,0,1]])
```

———-

5.4.7 Rotation Transformations

The transformations corresponding to rotations about the x, y, and z axes by an angle θ are

$$\mathbf{Rot}(\mathbf{x}, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\ \theta & -sin\ \theta & 0 \\ 0 & sin\ \theta & cos\ \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.32}$$

Rotations can be described in Axiom as functions that return matrices. We can define a function for each of the rotation matrices that correspond to the rotations about each axis. Note that the sine and cosine functions in Axiom expect their argument to be in radians rather than degrees. This conversion is

$$radians = \frac{degrees * \pi}{180}$$

The Axiom code for $\mathbf{Rot}(\mathbf{x}, \mathbf{degree})$ is

— **rotatex** —

```
rotatex(degree) ==
 angle := degree * pi() / 180::R
 cosAngle := cos(angle)
 sinAngle := sin(angle)
 matrix(_
  [[1,     0,            0,       0], _
   [0, cosAngle, -sinAngle, 0], _
   [0, sinAngle,  cosAngle, 0], _
   [0,     0,            0,       1]])
```

————

$$\mathbf{Rot}(\mathbf{y},\theta) = \begin{bmatrix} cos\ \theta & 0 & sin\ \theta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\ \theta & 0 & cos\ \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.33}$$

The Axiom code for $\mathbf{Rot}(\mathbf{y}, \mathbf{degree})$ is

— **rotatey** —

```
rotatey(degree) ==
 angle := degree * pi() / 180::R
 cosAngle := cos(angle)
 sinAngle := sin(angle)
 matrix(_
  [[ cosAngle, 0, sinAngle, 0], _
   [    0,        1,     0,       0], _
   [-sinAngle, 0, cosAngle, 0], _
   [    0,        0,     0,       1]])
```

————

$$\mathbf{Rot}(\mathbf{z},\theta) = \begin{bmatrix} cos\ \theta & -sin\ \theta & 0 & 0 \\ sin\ \theta & cos\ \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.34}$$

And the Axiom code for $\mathbf{Rot}(\mathbf{z}, \mathbf{degree})$ is

— **rotatez** —

```
rotatez(degree) ==
 angle := degree * pi() / 180::R
 cosAngle := cos(angle)
 sinAngle := sin(angle)
 matrix(_
```

```
[[cosAngle, -sinAngle, 0, 0], _
 [sinAngle,  cosAngle, 0, 0], _
 [   0,          0,       1, 0], _
 [   0,          0,       0, 1]])
```

Let us interpret these rotations by means of an example. Given a point $\mathbf{u} = 7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ what is the effect of rotating it 90° about the $\mathbf{z}$ axis to $\mathbf{v}$? The transform is obtained from Equation 1.34 with $sin\ \theta = 1$ and $cos\ \theta = 0$.

$$\begin{bmatrix} -3 \\ 7 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \tag{1.35}$$

Let us now rotate $\mathbf{v}$ 90° about the y axis to $\mathbf{w}$. The transform is obtained from Equation 1.33 and we have

$$\begin{bmatrix} 2 \\ 7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 7 \\ 2 \\ 1 \end{bmatrix} \tag{1.36}$$

If we combine these two rotations we have

$$\mathbf{v} = \mathbf{Rot(z, 90)u} \tag{1.37}$$

$$\text{and} \quad \mathbf{w} = \mathbf{Rot(y, 90)v} \tag{1.38}$$

Substituting for $\mathbf{v}$ from Equation 1.37 into Equation 1.38 we obtain

$$\mathbf{w} = \mathbf{Rot(y, 90)}\ \mathbf{Rot(z, 90)}\ \mathbf{u} \tag{1.39}$$

$$\mathbf{Rot(y, 90)}\ \mathbf{Rot(z, 90)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.40}$$

$$\mathbf{Rot(y, 90)}\ \mathbf{Rot(z, 90)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.41}$$

thus

$$\mathbf{w} = \begin{bmatrix} 2 \\ 7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \tag{1.42}$$

as we obtained before.

If we reverse the order of rotations and first rotate 90° about the y axis and then 90° about the z axis, we obtain a different position

$$\mathbf{Rot(z, 90)Rot(y, 90)} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.43}$$

and the point $\mathbf{u}$ transforms into $\mathbf{w}$ as

$$\begin{bmatrix} -3 \\ 2 \\ -7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \tag{1.44}$$

We should expect this, as matrix multiplication is noncommutative.

$$\mathbf{AB} \neq \mathbf{BA} \tag{1.45}$$

We will now combine the original rotation with a translation $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$. We obtain the translation from Equation 1.27 and the rotation from Equation 1.41. The matrix expression is

$$\mathbf{Trans(4, -3, 7)Rot(y, 90)Rot(z, 90)} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.46}$$

and our point $\mathbf{w} = 7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ transforms into $\mathbf{x}$ as

$$\begin{bmatrix} 6 \\ 4 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \tag{1.47}$$

5.4.8 Coordinate Frames

We can interpret the elements of the homogeneous transformation as four vectors describing a second coordinate frame. The vector $[0,0,0,1]^{\mathrm{T}}$ lies at the origin of the second coordinate frame. Its transformation corresponds to the right hand column of the transformation matrix. Consider the transform in Equation 1.47

$$\begin{bmatrix} 4 \\ -3 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{1.48}$$

The transform of the null vector is $[4,-3,7,1]^{\mathrm{T}}$, the right hand column. If we transform vectors corresponding to unit vectors along the x, y, and z axes, we obtain $[4,-2,7,1]^{\mathrm{T}}$, $[4,-3,8,1]^{\mathrm{T}}$, and $[5,-3,7,1]^{\mathrm{T}}$, respectively. Those four vectors form a coordinate frame.

The direction of these unit vectors is formed by subtracting the vector representing the origin of this coordinate frame and extending the vectors to infinity by reducing their scale factors to zero. The direction of the x, y, and z axes of this frame are $[0,1,0,0]^{\mathrm{T}}$, $[0,0,1,0]^{\mathrm{T}}$, and $[1,0,0,0]^{\mathrm{T}}$, respectively. These direction vectors correspond to the first three columns of the transformation matrix. The transformation matrix thus describes the three axis directions and the position of the origin of a coordinate frame rotated and translated away from the reference coordinate frame. When a vector is transformed, as in Equation 1.47, the original vector can be considered as a vector described in the coordinate frame. The transformed vector is the same vector described with respect to the reference coordinate frame.

5.4.9 Relative Transformations

The rotations and translations we have been describing have all been made with respect to the fixed reference coordinate frame. Thus, in the example given,

$$\mathbf{Trans(4,-3,7)Rot(y,90)Rot(z,90)} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.49}$$

the frame is first rotated around the reference z axis by $90°$, then rotated $90°$ around the reference y axis, and finally translated by $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$. We may also interpret the operation in the reverse order, from left to right, as follows: the object is first translated by $4\mathbf{i}-3\mathbf{j}+7\mathbf{k}$; it is then rotated $90°$ around the current frames axes, which in this case are the same as the reference axes; it is then rotated $90°$ about the newly rotated (current) frames axes.

In general, if we postmultiply a transform representing a frame by a second transformation describing a rotation and/or translation, we make that translation and/or rotation with respect to the frame axes described by the first transformation. If we premultiply the frame transformation by a transformation representing a translation and/or rotation, then that translation and/or rotation is made with respect to the base reference coordinate frame.

Thus, given a frame **C** and a transformation **T**, corresponding to a rotation of 90° about the z axis, and a translation of 10 units in the x direction, we obtain a new position **X** when the change is made in the base coordinates $\mathbf{X} = \mathbf{TC}$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 10 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.50}$$

and a new position **Y** when the change is made relative to the frame axes as $\mathbf{Y} = \mathbf{CT}$

$$\begin{bmatrix} 0 & -1 & 0 & 30 \\ 0 & 0 & -1 & 10 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 10 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.51}$$

5.4.10 Objects

Transformations are used to describe the position and orientation of objects. An object is described by six points with respect to a coordinate frame fixed in the object.

If we rotate the object 90° about the z axis and then 90° about the y axis, followed by a translation of four units in the x direction, we can describe the transformation as

$$\mathbf{Trans(4,0,0)Rot(y,90)Rot(z,90)} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.52}$$

The transformation matrix represents the operation of rotation and translation on a coordinate frame originally aligned with the reference coordinate frame. We may transform the six points of the object as

$$\begin{bmatrix} 4 & 4 & 6 & 6 & 4 & 4 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{1.53}$$

It can be seen that the object described bears the same fixed relationship to its coordinate frame, whose position and orientation are described by the transformation. Given an object described by a reference coordinate frame, and a transformation representing the position and orientation of the object's axes, the object can be simply reconstructed, without the necessity of transforming all the points, by noting the direction and orientation of key features with respect to the describing frame's coordinate axes. By drawing the transformed coordinate frame, the object can be related to the new axis directions.

5.4.11 Inverse Transformations

We are now in a position to develop the inverse transformation as the transform which carries the transformed coordinate frame back to the original frame. This is simply the description of the reference coordinate frame with respect to the transformed frame. Suppose the direction of the reference frame x axis is $[0, 0, 1, 0]^{\mathrm{T}}$ with respect to the transformed frame. The y and z axes are $[1, 0, 0, 0]^{\mathrm{T}}$ and $[0, 1, 0, 0]^{\mathrm{T}}$, respectively. The location of the origin is $[0, 0, -4, 1]^{\mathrm{T}}$ with respect to the transformed frame and thus the inverse transformation is

$$\mathbf{T^{-1}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.54}$$

That this is indeed the tranform inverse is easily verifyed by multiplying it by the transform $\mathbf{T}$ to obtain the identity transform

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.55}$$

In general, given a transform with elements

$$\mathbf{T} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.56}$$

then the inverse is

$$\mathbf{T^{-1}} = \begin{bmatrix} n_x & n_y & n_z & -\mathbf{p}\cdot\mathbf{n} \\ o_x & o_y & o_z & -\mathbf{p}\cdot\mathbf{o} \\ a_x & a_y & a_z & -\mathbf{p}\cdot\mathbf{a} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.57}$$

where $\mathbf{p}$, $\mathbf{n}$, $\mathbf{o}$, and $\mathbf{a}$ are the four column vectors and "$\cdot$" represents the vector dot product. This result is easily verified by postmultiplying Equation 1.56 by Equation 1.57.

5.4.12 General Rotation Transformation

We state the rotation transformations for rotations about the x, y, and z axes (Equations 1.32, 1.33 and 1.34). These transformations have a simple geometric interpretation. For example, in the case of a rotation about the z axis, the column representing the z axis will remain constant, while the column elements representing the x and y axes will vary.

We will now develop the transformation matrix representing a rotation around an arbitrary vector $\mathbf{k}$ located at the origin. In order to do this we will imagine that $\mathbf{k}$ is the z axis unit vector of a coordinate frame $\mathbf{C}$

$$\mathbf{C} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.58}$$

$$\mathbf{k} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \tag{1.59}$$

Rotating around the vector $\mathbf{k}$ is then equivalent to rotating around the z axis of the frame $\mathbf{C}$.

$$\mathbf{Rot}(\mathbf{k}, \theta) = \mathbf{Rot}(^{\mathbf{C}}\mathbf{z}, \theta) \tag{1.60}$$

If we are given a frame $\mathbf{T}$ described with respect to the reference coordinate frame, we can find a frame $\mathbf{X}$ which describes the same frame with respect to frame $\mathbf{C}$ as

$$\mathbf{T} = \mathbf{CX} \tag{1.61}$$

where $\mathbf{X}$ describes the position of $\mathbf{T}$ with respect to frame $\mathbf{C}$. Solving for $\mathbf{X}$ we obtain

$$\mathbf{X} = \mathbf{C^{-1}T} \tag{1.62}$$

Rotation $\mathbf{T}$ around $\mathbf{k}$ is equivalent to rotating $\mathbf{X}$ around the z axis of frame $\mathbf{C}$

$$\mathbf{Rot}(\mathbf{k}, \theta)\mathbf{T} = \mathbf{CRot}(\mathbf{z}, \theta)\mathbf{X} \tag{1.63}$$

$$\mathbf{Rot}(\mathbf{k}, \theta)\mathbf{T} = \mathbf{CRot}(\mathbf{z}, \theta)\mathbf{C^{-1}T}. \tag{1.64}$$

Thus

$$\mathbf{Rot}(\mathbf{k}, \theta) = \mathbf{CRot}(\mathbf{z}, \theta)\mathbf{C^{-1}} \tag{1.65}$$

However, we have only $\mathbf{k}$, the z axis of the frame $\mathbf{C}$. By expanding equation 1.65 we will discover that $\mathbf{CRot}(\mathbf{z}, \theta)\mathbf{C^{-1}}$ is a function of $\mathbf{k}$ only.

Multiplying $\mathbf{Rot}(\mathbf{z}, \theta)$ on the right by $\mathbf{C^{-1}}$ we obtain

$$\mathbf{Rot}(\mathbf{z}, \theta)\mathbf{C^{-1}} = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & n_y & n_z & 0 \\ o_x & o_x & o_z & 0 \\ a_x & a_y & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x cos\theta - o_x sin\theta & n_y cos\theta - o_y sin\theta & n_z cos\theta - o_z sin\theta & 0 \\ n_x sin\theta + o_x cos\theta & n_y sin\theta + o_y cos\theta & n_z sin\theta + o_z cos\theta & 0 \\ a_x & a_y & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.66}$$

premultiplying by

$$\mathbf{C} = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.67}$$

we obtain $\mathbf{CRot}(\mathbf{z}, \theta)\mathbf{C^{-1}}$

$$\begin{bmatrix} n_x n_x cos\theta - n_x o_x sin\theta + n_x o_x sin\theta + o_x o_x cos\theta + a_x a_x \\ n_y n_x cos\theta - n_y o_x sin\theta + n_x o_y sin\theta + o_x o_y cos\theta + a_y a_x \\ n_z n_x cos\theta - n_z o_x sin\theta + n_x o_z sin\theta + o_x o_z cos\theta + a_z a_x \\ 0 \end{bmatrix.$$

$$\begin{matrix} n_x n_y cos\theta - n_x o_y sin\theta + n_y o_x sin\theta + o_y o_x cos\theta + a_x a_y \\ n_y n_y cos\theta - n_y o_y sin\theta + n_y o_y sin\theta + o_y o_y cos\theta + a_y a_y \\ n_z n_y cos\theta - n_z o_y sin\theta + n_y o_z sin\theta + o_y o_z cos\theta + a_z a_y \\ 0 \end{matrix} \tag{1.68}$$

$$\left. \begin{matrix} n_x n_z cos\theta - n_x o_z sin\theta + n_z o_x sin\theta + o_z o_x cos\theta + a_x a_x & 0 \\ n_y n_z cos\theta - n_y o_z sin\theta + n_z o_y sin\theta + o_z o_y cos\theta + a_y a_z & 0 \\ n_z n_z cos\theta - n_z o_z sin\theta + n_z o_z sin\theta + o_z o_z cos\theta + a_z a_z & 0 \\ 0 & 1 \end{matrix} \right]$$

Simplifying, using the following relationships:
the dot product of any row or column of $\mathbf{C}$ with any other row or column is zero, as the vectors are orthogonal;
the dot product of any row or column of $\mathbf{C}$ with itself is $\mathbf{1}$ as the vectors are of unit magnitude;
the z unit vector is the vector cross product of the x and y vectors or

$$\mathbf{a} = \mathbf{n} \times \mathbf{o} \tag{1.69}$$

which has components

$$a_x = n_y o_z - n_z o_y$$

$$a_y = n_z o_x - n_x o_z$$

$$a_z = n_x o_y - n_y o_x$$

the versine, abbreviated **vers** θ, is defined as **vers** $\theta = (1 - cos\ \theta)$, $k_x = a_x$, $k_y = a_y$ and $k_z = a_z$. We obtain $\mathbf{Rot}(\mathbf{k}, \theta) =$

$$\begin{bmatrix} k_xk_xvers\theta + cos\theta & k_yk_xvers\theta - k_zsin\theta & k_zk_xvers\theta + k_ysin\theta & 0 \\ k_xk_yvers\theta + k_zsin\theta & k_yk_yvers\theta + cos\theta & k_zk_yvers\theta - k_xsin\theta & 0 \\ k_xk_zvers\theta - k_ysin\theta & k_yk_zvers\theta + k_xsin\theta & k_zk_zvers\theta + cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.70}$$

This is an important result and should be thoroughly understood before proceeding further.

From this general rotation transformation we can obtain each of the elementary rotation transforms. For example $\mathbf{Rot}(\mathbf{x}, \theta)$ is $\mathbf{Rot}(\mathbf{k}, \theta)$ where $k_x = 1$, $k_y = 0$, and $k_z = 0$. Substituting these values of $\mathbf{k}$ into Equation 1.70 we obtain

$$\mathbf{Rot}(\mathbf{x}, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.71}$$

as before.

5.4.13 Equivalent Angle and Axis of Rotation

Given any arbitrary rotational transformation, we can use Equation 1.70 to obtain an axis about which an equivalent rotation θ is made as follows. Given a rotational transformation **R**

$$\mathbf{R} = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.72}$$

we may equate **R** to **Rot(k,θ)**

$$\begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} k_xk_xvers\theta + cos\theta & k_yk_xvers\theta - k_zsin\theta & k_zk_xvers\theta + k_ysin\theta & 0 \\ k_xk_yvers\theta + k_zsin\theta & k_yk_yvers\theta + cos\theta & k_zk_yvers\theta - k_xsin\theta & 0 \\ k_xk_zvers\theta - k_ysin\theta & k_yk_zvers\theta + k_xsin\theta & k_zk_zvers\theta + cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.73}$$

Summing the diagonal terms of Equation 1.73 we obtain

$$n_x + o_y + a_z + 1 = k_x^2vers\theta + cos\theta + k_y^2vers\theta + cos\theta + k_z^2vers\theta + cos\theta + 1 \tag{1.74}$$

$$\begin{aligned} n_x + o_y + a_z &= (k_x^2 + k_y^2 + k_z^2) vers\theta + 3cos\theta \\ &= 1 + 2cos\theta \end{aligned} \tag{1.75}$$

and the cosine of the angle of rotation is

$$cos\theta = \frac{1}{2}(n_x + o_y + a_z - 1) \tag{1.76}$$

Differencing pairs of off-diagonal terms in Equation 1.73 we obtain

$$o_z - a_y = 2k_x sin\theta \tag{1.77}$$

$$a_x - n_z = 2k_y sin\theta \tag{1.78}$$

$$n_y - o_x = 2k_z sin\theta \tag{1.79}$$

Squaring and adding Equations 1.77-1.79 we obtain an expression for $sin\theta$

$$(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2 = 4sin^2\theta \tag{1.80}$$

and the sine of the angle of rotation is

$$sin\ \theta = \pm\frac{1}{2}\sqrt{(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2} \tag{1.81}$$

We may define the rotation to be positive about the vector **k** such that $0 \leq \theta \leq 180°$. In this case the + sign is appropriate in Equation 1.81 and thus the angle of rotation θ is uniquely defined as

$$tan\ \theta = \frac{\sqrt{(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2}}{(n_x + o_y + a_z - 1)} \tag{1.82}$$

The components of **k** may be obtained from Equations 1.77-1.79 as

$$k_x = \frac{o_z - a_y}{2sin\theta} \tag{1.83}$$

$$k_y = \frac{a_x - n_z}{2sin\theta} \tag{1.84}$$

$$k_z = \frac{n_y - o_x}{2sin\theta} \tag{1.85}$$

When the angle of rotation is very small, the axis of rotation is physically not well defined due to the small magnitude of both numerator and denominator in Equations 1.83-1.85. If the resulting angle is small, the vector **k** should be renormalized to ensure that $|\mathbf{k}| = 1$. When the angle of rotation approaches 180° the vector **k** is once again poorly defined by Equation 1.83-1.85 as the magnitude of the sine is again decreasing. The axis of rotation is, however, physically well defined in this case. When $\theta < 150°$, the denominator of Equations

1.83-1.85 is less than 1. As the angle increases to 180° the rapidly decreasing magnitude of both numerator and denominator leads to considerable inaccuracies in the determination of **k**. At $\theta = 180°$, Equations 1.83-1.85 are of the form 0/0, yielding no information at all about a physically well defined vector **k**. If the angle of rotation is greater than 90°, then we must follow a different approach in determining **k**. Equating the diagonal elements of Equation 1.73 we obtain

$$k_x^2 vers\theta + cos\theta = n_x \tag{1.86}$$

$$k_y^2 vers\theta + cos\theta = o_y \tag{1.87}$$

$$k_z^2 vers\theta + cos\theta = a_z \tag{1.88}$$

Substituting for $cos\theta$ and $vers\theta$ from Equation 1.76 and solving for the elements of **k** we obtain further

$$k_x = \pm\sqrt{\frac{n_x - cos\theta}{1 - cos\theta}} \tag{1.89}$$

$$k_y = \pm\sqrt{\frac{o_y - cos\theta}{1 - cos\theta}} \tag{1.90}$$

$$k_z = \pm\sqrt{\frac{a_z - cos\theta}{1 - cos\theta}} \tag{1.91}$$

The largest component of **k** defined by Equations 1.89-1.91 corresponds to the most positive component of n_x, o_y, and a_z. For this largest element, the sign of the radical can be obtained from Equations 1.77-1.79. As the sine of the angle of rotation θ must be positive, then the sign of the component of **k** defined by Equations 1.77-1.79 must be the same as the sign of the left hand side of these equations. Thus we may combine Equations 1.89-1.91 with the information contained in Equations 1.77-1.79 as follows

$$k_x = sgn(o_z - a_y)\sqrt{\frac{(n_x - cos\theta)}{1 - cos\theta}} \tag{1.92}$$

$$k_y = sgn(a_x - n_z)\sqrt{\frac{(o_y - cos\theta)}{1 - cos\theta}} \tag{1.93}$$

$$k_z = sgn(n_y - o_x)\sqrt{\frac{(a_z - cos\theta)}{1 - cos\theta}} \tag{1.94}$$

where $sgn(e) = +1$ if $e \geq 0$ and $sgn(e) = -1$ if $e \leq 0$.

Only the largest element of **k** is determined from Equations 1.92-1.94, corresponding to the most positive element of n_x, o_y, and a_z. The remaining elements are more accurately determined by the following equations formed by summing pairs of off-diagonal elements of Equation 1.73

$$n_y + o_x = 2k_xk_yvers\theta \tag{1.95}$$

$$o_z + a_y = 2k_yk_zvers\theta \tag{1.96}$$

$$n_z + a_x = 2k_zk_xvers\theta \tag{1.97}$$

If k_x is largest then

$$k_y = \frac{n_y + o_x}{2k_xvers\theta} \quad \text{from Equation 1.95} \tag{1.98}$$

$$k_z = \frac{a_x + n_z}{2k_xvers\theta} \quad \text{from Equation 1.97} \tag{1.99}$$

If k_y is largest then

$$k_x = \frac{n_y + o_x}{2k_yvers\theta} \quad \text{from Equation 1.95} \tag{1.100}$$

$$k_z = \frac{o_z + a_y}{2k_yvers\theta} \quad \text{from Equation 1.96} \tag{1.101}$$

If k_z is largest then

$$k_x = \frac{a_x + n_z}{2k_zvers\theta} \quad \text{from Equation 1.97} \tag{1.102}$$

$$k_y = \frac{o_z + a_y}{2k_zvers\theta} \quad \text{from Equation 1.96} \tag{1.103}$$

5.4.14 Example 1.1

Determine the equivalent axis and angle of rotation for the matrix given in Equations 1.41

$$\mathbf{Rot(y, 90)Rot(z, 90)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.104}$$

We first determine $\mathbf{cos}\ \theta$ from Equation 1.76

$$cos\theta = \frac{1}{2}(0 + 0 + 0 - 1) = -\frac{1}{2} \tag{1.105}$$

and $sin\ \theta$ from Equation 1.81

$$sin\theta = \frac{1}{2}\sqrt{(1-0)^2+(1-0)^2+(1-0)^2} = \frac{\sqrt{3}}{2} \tag{1.106}$$

Thus

$$\theta = tan^{-1}\left(\frac{\sqrt{3}}{2}\Bigg/\frac{-1}{2}\right) = 120^\circ \tag{1.107}$$

As $\theta > 90$, we determine the largest component of **k** corresponding to the largest element on the diagonal. As all diagonal elements are equal in this example we may pick any one. We will pick k_x given by Equation 1.92

$$k_x = +\sqrt{(0+\frac{1}{2})\Bigg/(1+\frac{1}{2})} = \frac{1}{\sqrt{3}} \tag{1.108}$$

As we have determined k_x we may now determine k_y and k_z from Equations 1.98 and 1.99, respectively

$$k_y = \frac{1+0}{\sqrt{3}} = \frac{1}{\sqrt{3}} \tag{1.109}$$

$$k_z = \frac{1+0}{\sqrt{3}} = \frac{1}{\sqrt{3}} \tag{1.110}$$

In summary, then

$$\mathbf{Rot(y,90)Rot(z,90)} = \mathbf{Rot(k,120)} \tag{1.111}$$

where

$$\mathbf{k} = \frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k} \tag{1.112}$$

Any combination of rotations is always equivalent to a single rotation about some axis **k** by an angle θ, an important result that we will make use of later.

5.4.15 Stretching and Scaling

A transform **T**

$$\mathbf{T} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.113}$$

will stretch objects uniformly along the x axis by a factor a, along the y axis by a factor b, and along the z axis by a factor c. Consider any point on an object $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$; its tranform is

$$\begin{bmatrix} ax \\ by \\ cz \\ 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{1.114}$$

indicating stretching as stated. Thus a cube could be transformed into a rectangular parallelepiped by such a transform.

The Axiom code to perform this scale change is:

— **scale** —

```
scale(scalex, scaley, scalez) ==
 matrix(_
  [[scalex, 0      ,0     , 0], _
   [0     , scaley ,0     , 0], _
   [0     , 0,      scalez, 0], _
   [0     , 0,      0     , 1]])
```

The transform **S** where

$$\mathbf{S} = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.115}$$

will scale any object by the factor s.

5.4.16 Perspective Transformations

Consider the image formed of an object by a simple lens.

The axis of the lens is along the y axis for convenience. An object point x,y,z is imaged at x',y',z' if the lens has a focal length f (f is considered positive). y' represents the image distance and varies with object distance y. If we plot points on a plane perpendicular to the y axis located at y' (the film plane in a camera), then a perspective image is formed.

We will first obtain values of x', y', and z', then introduce a perspective transformation and show that the same values are obtained.

Based on the fact that a ray passing through the center of the lens is undeviated we may write

$$\frac{z}{y} = \frac{z'}{y'} \tag{1.116}$$

$$\text{and } \frac{x}{y} = \frac{x'}{y'} \tag{1.117}$$

Based on the additional fact that a ray parallel to the lens axis passes through the focal point f, we may write

$$\frac{z}{f} = \frac{z'}{y' + f} \tag{1.118}$$

$$\text{and } \frac{x}{f} = \frac{x'}{y' + f} \tag{1.119}$$

Notice that x', y', and z' are negative and that f is positive. Eliminating y' between Equations 1.116 and 1.118 we obtain

$$\frac{z}{f} = \frac{z'}{(\frac{z'y}{z} + f)} \tag{1.120}$$

and solving for z' we obtain the result

$$z' = \frac{z}{(1 - \frac{y}{f})} \tag{1.121}$$

Working with Equations 1.117 and 1.119 we can similarly obtain

$$x' = \frac{x}{(1 - \frac{y}{f})} \tag{1.122}$$

In order to obtain the image distance y' we rewrite Equations 1.116 and 1.118 as

$$\frac{z}{z'} = \frac{y}{y'} \tag{1.123}$$

and

$$\frac{z}{z'} = \frac{f}{y' + f} \tag{1.124}$$

thus

$$\frac{y}{y'} = \frac{f}{y' + f} \tag{1.125}$$

and solving for y' we obtain the result

$$y' = \frac{y}{(1 - \frac{y}{f})} \tag{1.126}$$

The homogeneous transformation **P** which produces the same result is

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} \tag{1.127}$$

as any point $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ transforms as

$$\begin{bmatrix} x \\ y \\ z \\ 1 - \frac{y}{f} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{1.128}$$

The image point x', y',, z', obtained by dividing through by the weight factor $(1 - \frac{y}{f})$, is

$$\frac{x}{(1 - \frac{y}{f})}\mathbf{i} + \frac{y}{(1 - \frac{y}{f})}\mathbf{j} + \frac{z}{(1 - \frac{y}{f})}\mathbf{k} \tag{1.129}$$

This is the same result that we obtained above.

A transform similar to **P** but with $-\frac{1}{f}$ at the bottom of the first column produces a perspective transformation along the x axis. If the $-\frac{1}{f}$ term is in the third column then the projection is along the z axis.

5.4.17 Transform Equations

We will frequently be required to deal with transform equations in which a coordinate frame is described in two or more ways. A manipulator is positioned with respect to base coordinates by a transform **Z**. The end of the manipulator is described by a transform ${}^{Z}\mathbf{T}_6$, and the end effector is described by ${}^{T_6}\mathbf{E}$. An object is positioned with respect to base coordinates by a transform **B**, and finally the manipulator end effector is positioned with respect to the object by ${}^{B}\mathbf{G}$. We have two descriptions of the position of the end effector, one with respect to the object and one with respect to the manipulator. As both positions are the same, we may equate the two descriptions

$$\mathbf{Z}{}^{Z}\mathbf{T}_6{}^{T_6}\mathbf{E} = \mathbf{B}{}^{B}\mathbf{G} \tag{1.130}$$

If we wish to solve Equation 1.130 for the manipulator transform $\mathbf{T}_6$ we must premultiply Equation 1.130 by $\mathbf{Z}^{-1}$ and postmultiply by $\mathbf{E}^{-1}$ to obtain

$$\mathbf{T}_6 = \mathbf{Z}^{-1}\mathbf{B}\mathbf{G}\mathbf{E}^{-1} \tag{1.131}$$

As a further example, consider that the position of the object **B** is unknown, but that the manipulator is moved such that the end effector is positioned over the object correctly. We may then solve for **B** from Equation 1.130 by postmultiplying by $\mathbf{G}^{-1}$.

$$\mathbf{B} = \mathbf{Z}\mathbf{T}_6\mathbf{E}\mathbf{G}^{-1} \tag{1.133}$$

5.4.18 Summary

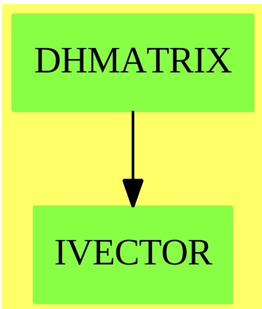
Homogeneous transformations may be readily used to describe the positions and orientations of coordinate frames in space. If a coordinate frame is embedded in an object then the position and orientation of the object are also readily described.

The description of object A in terms of object B by means of a homogeneous transformation may be inverted to obtain the description of object B in terms of object A. This is not a property of a simple vector description of the relative displacement of one object with respect to another.

Transformations may be interpreted as a product of rotation and translation transformations. If they are intrepreted from left to right, then the rotations and translations are in terms of the currently defined coordinate frame. If they are interpreted from right to left, then the rotations and translations are described with respect to the reference coordinate frame.

Homogeneous transformations describe coordinate frames in terms of rectangular components, which are the sines and cosines of angles. This description may be related to rotations in which case the description is in terms of a vector and angle of rotation.

5.4.19 DenavitHartenbergMatrix (DHMATRIX)



Exports:

antisymmetric?	any?	coerce	column	copy
count	determinant	diagonal?	diagonalMatrix	elt
empty	empty?	eq?	eval	every?
exquo	fill!	hash	horizConcat	identity
inverse	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minordet	minRowIndex	more?
ncols	new	nrows	nullSpace	nullity
parts	qelt	qsetelt!	rank	rotatex
rotatey	rotatez	row	rowEchelon	sample
scalarMatrix	scale	setColumn!	setRow!	setelt
setsubMatrix!	size?	square?	squareTop	subMatrix
swapColumns!	swapRows!	symmetric?	translate	transpose
vertConcat	zero	#?	?**?	?/?
?=?	?~=?	?*?	?+?	-?
?-?				

— domain DHMATRIX DenavitHartenbergMatrix —

```
)abbrev domain DHMATRIX DenavitHartenbergMatrix

++ Author: Timothy Daly
++ Date Created: June 26, 1991
++ Date Last Updated: 26 June 1991
++ Description:
++ 4x4 Matrices for coordinate transformations\br
++ This package contains functions to create 4x4 matrices
++ useful for rotating and transforming coordinate systems.
++ These matrices are useful for graphics and robotics.
++ (Reference: Robot Manipulators Richard Paul MIT Press 1981)
++
++ A Denavit-Hartenberg Matrix is a 4x4 Matrix of the form:\br
++ \tab{5}\spad{nx ox ax px}\br
++ \tab{5}\spad{ny oy ay py}\br
++ \tab{5}\spad{nz oz az pz}\br
++ \tab{5}\spad{0  0  0  1}\br
++ (n, o, and a are the direction cosines)

DenavitHartenbergMatrix(R): Exports == Implementation where
  R : Join(Field,  TranscendentalFunctionCategory)

-- for the implementation of dhmatrix
  minrow ==> 1
  mincolumn ==> 1
--
  nx ==> x(1,1)::R
  ny ==> x(2,1)::R
```

```
  nz ==> x(3,1)::R
  ox ==> x(1,2)::R
  oy ==> x(2,2)::R
  oz ==> x(3,2)::R
  ax ==> x(1,3)::R
  ay ==> x(2,3)::R
  az ==> x(3,3)::R
  px ==> x(1,4)::R
  py ==> x(2,4)::R
  pz ==> x(3,4)::R
  row ==> Vector(R)
  col ==> Vector(R)
  radians ==> pi()/180

  Exports ==> MatrixCategory(R,row,col) with
   "*": (%, Point R) -> Point R
     ++ t*p applies the dhmatrix t to point p
   identity: () -> %
     ++ identity() create the identity dhmatrix
   rotatex: R -> %
     ++ rotatex(r) returns a dhmatrix for rotation about axis x for r degrees
   rotatey: R -> %
     ++ rotatey(r) returns a dhmatrix for rotation about axis y for r degrees
   rotatez: R -> %
     ++ rotatez(r) returns a dhmatrix for rotation about axis z for r degrees
   scale: (R,R,R) -> %
     ++ scale(sx,sy,sz) returns a dhmatrix for scaling in the x, y and z
     ++ directions
   translate: (R,R,R) -> %
     ++ translate(x,y,z) returns a dhmatrix for translation by x, y, and z

  Implementation ==> Matrix(R) add

    identity() == matrix([[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]])

--    inverse(x) == (inverse(x pretend (Matrix R))$Matrix(R)) pretend %
--    dhinverse(x) == matrix( _
--        [[nx,ny,nz,-(px*nx+py*ny+pz*nz)],_
--         [ox,oy,oz,-(px*ox+py*oy+pz*oz)],_
--         [ax,ay,az,-(px*ax+py*ay+pz*az)],_
--         [ 0, 0, 0, 1]])

    d * p ==
       v := p pretend Vector R
       v := concat(v, 1$R)
       v := d * v
       point ([v.1, v.2, v.3]$List(R))

\getchunk{rotatex}
```

```
\getchunk{rotatey}

\getchunk{rotatez}

\getchunk{scale}

\getchunk{translate}
```

———-

— DHMATRIX.dotabb —

```
"DHMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DHMATRIX"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"DHMATRIX" -> "IVECTOR"
```

———-

5.5 domain DEQUEUE Dequeue

— Dequeue.input —

```
)set break resume
)sys rm -f Dequeue.output
)spool Dequeue.output
)set message test on
)set message auto off
)clear all

--S 1 of 63
a:Dequeue INT:= dequeue [1,2,3,4,5]
--R
--R
--R   (1)  [1,2,3,4,5]
--R                                                     Type: Dequeue Integer
--E 1

--S 2 of 63
dequeue! a
--R
--R
--R   (2)  1
--R                                                     Type: PositiveInteger
--E 2
```

```
--S 3 of 63
a
--R
--R
--R   (3)  [2,3,4,5]
--R                                                     Type: Dequeue Integer
--E 3

--S 4 of 63
extract! a
--R
--R
--R   (4)  2
--R                                                     Type: PositiveInteger
--E 4

--S 5 of 63
a
--R
--R
--R   (5)  [3,4,5]
--R                                                     Type: Dequeue Integer
--E 5

--S 6 of 63
enqueue!(9,a)
--R
--R
--R   (6)  9
--R                                                     Type: PositiveInteger
--E 6

--S 7 of 63
a
--R
--R
--R   (7)  [3,4,5,9]
--R                                                     Type: Dequeue Integer
--E 7

--S 8 of 63
insert!(8,a)
--R
--R
--R   (8)  [3,4,5,9,8]
--R                                                     Type: Dequeue Integer
--E 8

--S 9 of 63
```

```
a
--R
--R
--R   (9)  [3,4,5,9,8]
--R                                                      Type: Dequeue Integer
--E 9

--S 10 of 63
front a
--R
--R
--R   (10)  3
--R                                                      Type: PositiveInteger
--E 10

--S 11 of 63
back a
--R
--R
--R   (11)  8
--R                                                      Type: PositiveInteger
--E 11

--S 12 of 63
bottom! a
--R
--R
--R   (12)  8
--R                                                      Type: PositiveInteger
--E 12

--S 13 of 63
a
--R
--R
--R   (13)  [3,4,5,9]
--R                                                      Type: Dequeue Integer
--E 13

--S 14 of 63
depth a
--R
--R
--R   (14)  4
--R                                                      Type: PositiveInteger
--E 14

--S 15 of 63
height a
--R
```

```
--R
--R   (15)  4
--R                                                    Type: PositiveInteger
--E 15

--S 16 of 63
insertBottom!(6,a)
--R
--R
--R   (16)  6
--R                                                    Type: PositiveInteger
--E 16

--S 17 of 63
a
--R
--R
--R   (17)  [3,4,5,9,6]
--R                                                    Type: Dequeue Integer
--E 17

--S 18 of 63
extractBottom! a
--R
--R
--R   (18)  6
--R                                                    Type: PositiveInteger
--E 18

--S 19 of 63
a
--R
--R
--R   (19)  [3,4,5,9]
--R                                                    Type: Dequeue Integer
--E 19

--S 20 of 63
insertTop!(7,a)
--R
--R
--R   (20)  7
--R                                                    Type: PositiveInteger
--E 20

--S 21 of 63
a
--R
--R
--R   (21)  [7,3,4,5,9]
```

```
--R                                                   Type: Dequeue Integer
--E 21

--S 22 of 63
extractTop! a
--R
--R
--R   (22)  7
--R                                                   Type: PositiveInteger
--E 22

--S 23 of 63
a
--R
--R
--R   (23)  [3,4,5,9]
--R                                                   Type: Dequeue Integer
--E 23

--S 24 of 63
top a
--R
--R
--R   (24)  3
--R                                                   Type: PositiveInteger
--E 24

--S 25 of 63
a
--R
--R
--R   (25)  [3,4,5,9]
--R                                                   Type: Dequeue Integer
--E 25

--S 26 of 63
top! a
--R
--R
--R   (26)  3
--R                                                   Type: PositiveInteger
--E 26

--S 27 of 63
a
--R
--R
--R   (27)  [4,5,9]
--R                                                   Type: Dequeue Integer
--E 27
```

```
--S 28 of 63
reverse! a
--R
--R
--R   (28)  [9,5,4]
--R                                                    Type: Dequeue Integer
--E 28

--S 29 of 63
rotate! a
--R
--R
--R   (29)  [5,4,9]
--R                                                    Type: Dequeue Integer
--E 29

--S 30 of 63
inspect a
--R
--R
--R   (30)  5
--R                                                    Type: PositiveInteger
--E 30

--S 31 of 63
empty? a
--R
--R
--R   (31)  false
--R                                                            Type: Boolean
--E 31

--S 32 of 63
#a
--R
--R
--R   (32)  3
--R                                                    Type: PositiveInteger
--E 32

--S 33 of 63
length a
--R
--R
--R   (33)  3
--R                                                    Type: PositiveInteger
--E 33

--S 34 of 63
```

```
less?(a,9)
--R
--R
--R   (34)  true
--R                                                             Type: Boolean
--E 34

--S 35 of 63
more?(a,9)
--R
--R
--R   (35)  false
--R                                                             Type: Boolean
--E 35

--S 36 of 63
size?(a,#a)
--R
--R
--R   (36)  true
--R                                                             Type: Boolean
--E 36

--S 37 of 63
size?(a,9)
--R
--R
--R   (37)  false
--R                                                             Type: Boolean
--E 37

--S 38 of 63
parts a
--R
--R
--R   (38)  [5,4,9]
--R                                                        Type: List Integer
--E 38

--S 39 of 63
bag([1,2,3,4,5])$Dequeue(INT)
--R
--R
--R   (39)  [1,2,3,4,5]
--R                                                     Type: Dequeue Integer
--E 39

--S 40 of 63
b:=empty()$(Dequeue INT)
--R
```

```
--R
--R   (40)  []
--R                                                    Type: Dequeue Integer
--E 40

--S 41 of 63
empty? b
--R
--R
--R   (41)  true
--R                                                            Type: Boolean
--E 41

--S 42 of 63
sample()$Dequeue(INT)
--R
--R
--R   (42)  []
--R                                                    Type: Dequeue Integer
--E 42

--S 43 of 63
c:=copy a
--R
--R
--R   (43)  [5,4,9]
--R                                                    Type: Dequeue Integer
--E 43

--S 44 of 63
eq?(a,c)
--R
--R
--R   (44)  false
--R                                                            Type: Boolean
--E 44

--S 45 of 63
eq?(a,a)
--R
--R
--R   (45)  true
--R                                                            Type: Boolean
--E 45

--S 46 of 63
(a=c)@Boolean
--R
--R
--R   (46)  true
```

```
--R                                                         Type: Boolean
--E 46

--S 47 of 63
(a=a)@Boolean
--R
--R
--R   (47)  true
--R                                                         Type: Boolean
--E 47

--S 48 of 63
a~=c
--R
--R
--R   (48)  false
--R                                                         Type: Boolean
--E 48

--S 49 of 63
any?(x+->(x=4),a)
--R
--R
--R   (49)  true
--R                                                         Type: Boolean
--E 49

--S 50 of 63
any?(x+->(x=11),a)
--R
--R
--R   (50)  false
--R                                                         Type: Boolean
--E 50

--S 51 of 63
every?(x+->(x=11),a)
--R
--R
--R   (51)  false
--R                                                         Type: Boolean
--E 51

--S 52 of 63
count(4,a)
--R
--R
--R   (52)  1
--R                                                 Type: PositiveInteger
--E 52
```

```
--S 53 of 63
count(x+->(x>2),a)
--R
--R
--R   (53)  3
--R                                                 Type: PositiveInteger
--E 53

--S 54 of 63
map(x+->x+10,a)
--R
--R
--R   (54)  [15,14,19]
--R                                                 Type: Dequeue Integer
--E 54

--S 55 of 63
a
--R
--R
--R   (55)  [5,4,9]
--R                                                 Type: Dequeue Integer
--E 55

--S 56 of 63
map!(x+->x+10,a)
--R
--R
--R   (56)  [15,14,19]
--R                                                 Type: Dequeue Integer
--E 56

--S 57 of 63
a
--R
--R
--R   (57)  [15,14,19]
--R                                                 Type: Dequeue Integer
--E 57

--S 58 of 63
members a
--R
--R
--R   (58)  [15,14,19]
--R                                                    Type: List Integer
--E 58

--S 59 of 63
```

```
member?(14,a)
--R
--R
--R   (59)  true
--R                                                            Type: Boolean
--E 59

--S 60 of 63
coerce a
--R
--R
--R   (60)  [15,14,19]
--R                                                         Type: OutputForm
--E 60

--S 61 of 63
hash a
--R
--R
--I   (61)  4999531
--R                                                      Type: SingleInteger
--E 61

--S 62 of 63
latex a
--R
--R
--R   (62)  "\mbox{\bf Unimplemented}"
--R                                                             Type: String
--E 62

--S 63 of 63
)show Dequeue
--R
--R Dequeue S: SetCategory  is a domain constructor
--R Abbreviation for Dequeue is DEQUEUE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DEQUEUE
--R
--R------------------------------- Operations --------------------------------
--R back : % -> S                          bag : List S -> %
--R bottom! : % -> S                       copy : % -> %
--R depth : % -> NonNegativeInteger        dequeue : List S -> %
--R dequeue : () -> %                      dequeue! : % -> S
--R empty : () -> %                        empty? : % -> Boolean
--R enqueue! : (S,%) -> S                  eq? : (%,%) -> Boolean
--R extract! : % -> S                      extractBottom! : % -> S
--R extractTop! : % -> S                   front : % -> S
--R height : % -> NonNegativeInteger       insert! : (S,%) -> %
--R insertBottom! : (S,%) -> S             insertTop! : (S,%) -> S
```

```
--R inspect : % -> S                    length : % -> NonNegativeInteger
--R map : ((S -> S),%) -> %             pop! : % -> S
--R push! : (S,%) -> S                  reverse! : % -> %
--R rotate! : % -> %                    sample : () -> %
--R top : % -> S                        top! : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 63

)spool
)lisp (bye)
```

———-

— Dequeue.help —

```
====================================================================
Dequeue examples
====================================================================

A Dequeue is a double-ended queue so elements can be added to
either end.

Here we create an dequeue of integers from a list. Notice that the
order in the list is the order in the dequeue.

   a:Dequeue INT:= dequeue [1,2,3,4,5]
      [1,2,3,4,5]
```

```
We can remove the top of the dequeue using dequeue!:

   dequeue! a
      1

Notice that the use of dequeue! is destructive (destructive operations
in Axiom usually end with ! to indicate that the underlying data
structure is changed).

   a
      [2,3,4,5]

The extract! operation is another name for the dequeue! operation and
has the same effect. This operation treats the dequeue as a BagAggregate:

   extract! a
      2

and you can see that it also has destructively modified the dequeue:

   a
      [3,4,5]

Next we use enqueue! to add a new element to the end of the dequeue:

   enqueue!(9,a)
      9

Again, the enqueue! operation is destructive so the dequeue is changed:

   a
      [3,4,5,9]

Another name for enqueue! is insert!, which treats the dequeue as a
BagAggregate:

   insert!(8,a)
      [3,4,5,9,8]

and it modifies the dequeue:

   a
      [3,4,5,9,8]

The front operation returns the item at the front of the dequeue:

   front a
      3
```

The back operation returns the item at the back of the dequeue:

```
back a
   8
```

The bottom! operation returns the item at the back of the dequeue:

```
bottom! a
   8
```

and it modifies the dequeue:

```
a
   [3,4,5,9]
```

The depth function returns the number of elements in the dequeue:

```
depth a
   4
```

The height function returns the number of elements in the dequeue:

```
height a
   4
```

The insertBottom! function adds the element at the end:

```
insertBottom!(6,a)
   6
```

and it modifies the dequeue:

```
a
   [3,4,5,9,6]
```

The extractBottom! function removes the element at the end:

```
extractBottom! a
   6
```

and it modifies the dequeue:

```
a
   [3,4,5,9]
```

The insertTop! function adds the element at the top:

```
insertTop!(7,a)
   7
```

and it modifies the dequeue:

```
a
   [7,3,4,5,9]
```

The extractTop! function adds the element at the top:

```
extractTop! a
   7
```

and it modifies the dequeue:

```
a
   [3,4,5,9]
```

The top function returns the top element:

```
top a
   3
```

and it does not modifies the dequeue:

```
a
   [3,4,5,9]
```

The top! function returns the top element:

```
top! a
   3
```

and it modifies the dequeue:

```
a
   [4,5,9]
```

The reverse! operation destructively reverses the elements of the dequeue:

```
reverse! a
   [9,5,4]
```

The rotate! operation moves the top element to the bottom:

```
rotate! a
   [5,4,9]
```

The inspect function returns the top of the dequeue without modification, viewed as a BagAggregate:

```
inspect a
```

```
      5
```

The empty? operation returns true only if there are no element on the dequeue, otherwise it returns false:

```
   empty? a
      false
```

The # (length) operation:

```
   #a
       3
```

The length operation does the same thing:

```
   length a
       3
```

The less? predicate will compare the dequeue length to an integer:

```
   less?(a,9)
        true
```

The more? predicate will compare the dequeue length to an integer:

```
   more?(a,9)
        false
```

The size? operation will compare the dequeue length to an integer:

```
   size?(a,#a)
        true
```

and since the last computation must alwasy be true we try:

```
   size?(a,9)
        false
```

The parts function will return the dequeue as a list of its elements:

```
   parts a
        [5,4,9]
```

If we have a BagAggregate of elements we can use it to construct a dequeue:

```
   bag([1,2,3,4,5])$Dequeue(INT)
        [1,2,3,4,5]
```

The empty function will construct an empty dequeue of a given type:

```
   b:=empty()$(Deequeue INT)
        []
```

and the empty? predicate allows us to find out if a dequeue is empty:

```
   empty? b
        true
```

The sample function returns a sample, empty dequeue:

```
   sample()$Dequeue(INT)
        []
```

We can copy a dequeue and it does not share storage so subsequent
modifications of the original dequeue will not affect the copy:

```
   c:=copy a
        [5,4,9]
```

The eq? function is only true if the lists are the same reference,
so even though c is a copy of a, they are not the same:

```
   eq?(a,c)
        false
```

However, a clearly shares a reference with itself:

```
   eq?(a,a)
        true
```

But we can compare a and c for equality:

```
   (a=c)@Boolean
        true
```

and clearly a is equal to itself:

```
   (a=a)@Boolean
        true
```

and since a and c are equal, they are clearly NOT not-equal:

```
   a~=c
        false
```

We can use the any? function to see if a predicate is true for any element:

```
   any?(x+->(x=4),a)
        true
```

or false for every element:

```
any?(x+->(x=11),a)
     false
```

We can use the every? function to check every element satisfies a predicate:

```
every?(x+->(x=11),a)
     false
```

We can count the elements that are equal to an argument of this type:

```
count(4,a)
     1
```

or we can count against a boolean function:

```
count(x+->(x>2),a)
     3
```

You can also map a function over every element, returning a new dequeue:

```
map(x+->x+10,a)
     [15,14,19]
```

Notice that the orignal dequeue is unchanged:

```
a
     [5,4,9]
```

You can use map! to map a function over every element and change
the original dequeue since map! is destructive:

```
map!(x+->x+10,a)
    [15,14,19]
```

Notice that the orignal dequeue has been changed:

```
a
    [15,14,19]
```

The member function can also get the element of the dequeue as a list:

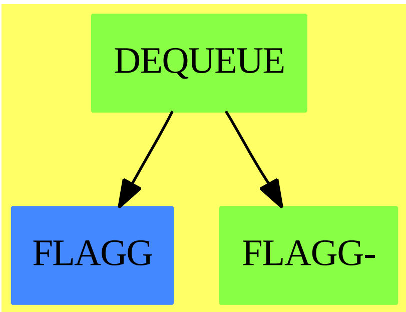
```
members a
    [15,14,19]
```

and using member? we can test if the dequeue holds a given element:

```
member?(14,a)
    true
```

```
See Also:
o )show Stack
o )show ArrayStack
o )show Queue
o )show Dequeue
o )show Heap
o )show BagAggregate
```

5.5.1 Dequeue (DEQUEUE)



See

⇒ “Stack” (STACK) 20.28.1 on page 2521
⇒ “ArrayStack” (ASTACK) 2.10.1 on page 65
⇒ “Queue” (QUEUE) 18.5.1 on page 2143
⇒ “Heap” (HEAP) 9.2.1 on page 1100

Exports:

any?	back	bag	bottom!	coerce
copy	count	depth	dequeue	dequeue!
empty	empty?	enqueue!	eq?	eval
every?	extract!	extractBottom!	extractTop!	front
height	hash	insert!	insertBottom!	insertTop!
inspect	latex	length	less?	map
map!	member?	members	more?	parts
pop!	push!	reverse!	rotate!	sample
size?	top	top!	#?	?=?
?~=?				

— domain DEQUEUE Dequeue —

```
)abbrev domain DEQUEUE Dequeue
++ Author: Michael Monagan and Stephen Watt
++ Date Created:June 86 and July 87
++ Date Last Updated:Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Linked list implementation of a Dequeue

Dequeue(S:SetCategory): DequeueAggregate S with
    dequeue: List S -> %
      ++ dequeue([x,y,...,z]) creates a dequeue with first (top or front)
      ++ element x, second element y,...,and last (bottom or back) element z.
      ++
      ++E g:Dequeue INT:= dequeue [1,2,3,4,5]

 -- Inherited Signatures repeated for examples documentation

    dequeue_! : % -> S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X dequeue! a
      ++X a
    extract_! : % -> S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X extract! a
      ++X a
    enqueue_! : (S,%) -> S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X enqueue! (9,a)
      ++X a
    insert_! : (S,%) -> %
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X insert! (8,a)
      ++X a
    inspect : % -> S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X inspect a
    front : % -> S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
```

```
    ++X front a
  back : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X back a
  rotate_! : % -> %
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X rotate! a
  length : % -> NonNegativeInteger
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X length a
  less? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X less?(a,9)
  more? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X more?(a,9)
  size? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X size?(a,5)
  bag : List S -> %
    ++
    ++X bag([1,2,3,4,5])$Dequeue(INT)
  empty? : % -> Boolean
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X empty? a
  empty : () -> %
    ++
    ++X b:=empty()$(Dequeue INT)
  sample : () -> %
    ++
    ++X sample()$Dequeue(INT)
  copy : % -> %
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X copy a
  eq? : (%,%) -> Boolean
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X b:=copy a
    ++X eq?(a,b)
  map :  ((S -> S),%) -> %
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
```

```
    ++X map(x+->x+10,a)
    ++X a
  depth : % -> NonNegativeInteger
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X depth a
  dequeue : () -> %
    ++
    ++X a:Dequeue INT:= dequeue ()
  height : % -> NonNegativeInteger
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X height a
  top : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X top a
  bottom_! : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X bottom! a
    ++X a
  extractBottom_! : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X extractBottom! a
    ++X a
  extractTop_! : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X extractTop! a
    ++X a
  insertBottom_! : (S,%) -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X insertBottom! a
    ++X a
  insertTop_! : (S,%) -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X insertTop! a
    ++X a
  pop_! : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X pop! a
    ++X a
  push_! : (S,%) -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
```

```
    ++X push! a
    ++X a
  reverse_! : % -> %
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X reverse! a
    ++X a
  top_! : % -> S
    ++
    ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
    ++X top! a
    ++X a
  if $ has shallowlyMutable then
    map! :  ((S -> S),%) -> %
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X map!(x+->x+10,a)
      ++X a
  if S has SetCategory then
    latex : % -> String
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X latex a
    hash : % -> SingleInteger
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X hash a
    coerce : % -> OutputForm
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X coerce a
    "=": (%,%) -> Boolean
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X b:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X (a=b)@Boolean
    "~=" : (%,%) -> Boolean
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X b:=copy a
      ++X (a~=b)
  if % has finiteAggregate then
    every? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X every?(x+->(x=4),a)
    any? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X any?(x+->(x=4),a)
```

```
    count :  ((S -> Boolean),%) -> NonNegativeInteger
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X count(x+->(x>2),a)
    _# : % -> NonNegativeInteger
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X #a
    parts : % -> List S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X parts a
    members : % -> List S
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X members a
  if % has finiteAggregate and S has SetCategory then
    member? : (S,%) -> Boolean
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X member?(3,a)
    count : (S,%) -> NonNegativeInteger
      ++
      ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
      ++X count(4,a)

== Queue S add
  Rep := Reference List S
  bottom! d == extractBottom! d
  dequeue d == ref copy d
  extractBottom! d ==
      if empty? d then error "empty dequeue"
      p := deref d
      n := maxIndex p
      n = 1 =>
         r := first p
         setref(d,[])
         r
      q := rest(p,(n-2)::NonNegativeInteger)
      r := first rest q
      q.rest := []
      r
  top! d == extractTop! d
  extractTop! d ==
      if empty? d then error "empty dequeue"
      e := top d
      setref(d,rest deref d)
      e
  height d == # deref d
  depth d == # deref d
```

```
    insertTop!(e,d) == (setref(d,cons(e,deref d)); e)
    lastTail==> LAST$Lisp
    insertBottom!(e,d) ==
        if empty? d then setref(d, list e)
        else lastTail.(deref d).rest := list e
        e
    top d == if empty? d then error "empty dequeue" else first deref d
    reverse! d == (setref(d,reverse deref d); d)
```

— **DEQUEUE.dotabb** —

```
"DEQUEUE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DEQUEUE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"DEQUEUE" -> "FLAGG-"
"DEQUEUE" -> "FLAGG"
```

5.6 domain DERHAM DeRhamComplex

— **DeRhamComplex.input** —

```
)set break resume
)sys rm -f DeRhamComplex.output
)spool DeRhamComplex.output
)set message test on
)set message auto off
)clear all
--S 1 of 34
coefRing := Integer
--R
--R
--R   (1)  Integer
--R                                                            Type: Domain
--E 1

--S 2 of 34
lv : List Symbol := [x,y,z]
--R
--R
--R   (2)  [x,y,z]
```

```
--R                                                        Type: List Symbol
--E 2

--S 3 of 34
der := DERHAM(coefRing,lv)
--R
--R
--R   (3)  DeRhamComplex(Integer,[x,y,z])
--R                                                             Type: Domain
--E 3

--S 4 of 34
R := Expression coefRing
--R
--R
--R   (4)  Expression Integer
--R                                                             Type: Domain
--E 4

--S 5 of 34
f : R := x**2*y*z-5*x**3*y**2*z**5
--R
--R
--R            3 2 5    2
--R   (5)  - 5x y z  + x y z
--R                                                 Type: Expression Integer
--E 5

--S 6 of 34
g : R := z**2*y*cos(z)-7*sin(x**3*y**2)*z**2
--R
--R
--R            2     3 2       2
--R   (6)  - 7z sin(x y ) + y z cos(z)
--R                                                 Type: Expression Integer
--E 6

--S 7 of 34
h : R :=x*y*z-2*x**3*y*z**2
--R
--R
--R            3   2
--R   (7)  - 2x y z  + x y z
--R                                                 Type: Expression Integer
--E 7

--S 8 of 34
dx : der := generator(1)
--R
--R
```

```
--R   (8)  dx
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 8

--S 9 of 34
dy : der := generator(2)
--R
--R
--R   (9)  dy
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 9

--S 10 of 34
dz : der := generator(3)
--R
--R
--R   (10)  dz
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 10

--S 11 of 34
[dx,dy,dz] := [generator(i)$der for i in 1..3]
--R
--R
--R   (11)  [dx,dy,dz]
--R                                    Type: List DeRhamComplex(Integer,[x,y,z])
--E 11

--S 12 of 34
alpha : der := f*dx + g*dy + h*dz
--R
--R
--R   (12)
--R            3   2                   2     3 2       2
--R     (- 2x y z  + x y z)dz + (- 7z sin(x y ) + y z cos(z))dy
--R   +
--R            3 2 5    2
--R     (- 5x y z  + x y z)dx
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 12

--S 13 of 34
beta  : der := cos(tan(x*y*z)+x*y*z)*dx + x*dy
--R
--R
--R   (13)  x dy + cos(tan(x y z) + x y z)dx
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 13

--S 14 of 34
```

```
exteriorDifferential alpha
--R
--R
--R   (14)
--R         2                  3 2                   3 2
--R     (y z sin(z) + 14z sin(x y ) - 2y z cos(z) - 2x z  + x z)dy dz
--R   +
--R         3 2 4     2   2           2
--R     (25x y z  - 6x y z  + y z - x y)dx dz
--R   +
--R           2 2 2     3 2       3   5    2
--R     (- 21x y z cos(x y ) + 10x y z  - x z)dx dy
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 14

--S 15 of 34
exteriorDifferential %
--R
--R
--R   (15)  0
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 15

--S 16 of 34
gamma := alpha * beta
--R
--R
--R   (16)
--R        4   2    2               3   2
--R     (2x y z  - x y z)dy dz + (2x y z  - x y z)cos(tan(x y z) + x y z)dx dz
--R   +
--R       2     3 2       2                                  4 2 5    3
--R   ((7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z  + x y z)dx dy
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 16

--S 17 of 34
exteriorDifferential(gamma) - (exteriorDifferential(alpha)*beta - alpha * exteriorDifferenti
--R
--R
--R   (17)  0
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 17

--S 18 of 34
a : BOP := operator('a)
--R
--R
--R   (18)  a
--R                                                          Type: BasicOperator
```

```
--E 18

--S 19 of 34
b : BOP := operator('b)
--R
--R
--R   (19)  b
--R                                                        Type: BasicOperator
--E 19

--S 20 of 34
c : BOP := operator('c)
--R
--R
--R   (20)  c
--R                                                        Type: BasicOperator
--E 20

--S 21 of 34
sigma := a(x,y,z) * dx + b(x,y,z) * dy + c(x,y,z) * dz
--R
--R
--R   (21)  c(x,y,z)dz + b(x,y,z)dy + a(x,y,z)dx
--R                                       Type: DeRhamComplex(Integer,[x,y,z])
--E 21

--S 22 of 34
theta  := a(x,y,z) * dx * dy + b(x,y,z) * dx * dz + c(x,y,z) * dy * dz
--R
--R
--R   (22)  c(x,y,z)dy dz + b(x,y,z)dx dz + a(x,y,z)dx dy
--R                                       Type: DeRhamComplex(Integer,[x,y,z])
--E 22

--S 23 of 34
totalDifferential(a(x,y,z))$der
--R
--R
--R   (23)  a  (x,y,z)dz + a  (x,y,z)dy + a  (x,y,z)dx
--R          ,3             ,2             ,1
--R                                       Type: DeRhamComplex(Integer,[x,y,z])
--E 23

--S 24 of 34
exteriorDifferential sigma
--R
--R
--R   (24)
--R     (c  (x,y,z) - b  (x,y,z))dy dz + (c  (x,y,z) - a  (x,y,z))dx dz
--R        ,2           ,3                  ,1           ,3
```

```
--R   +
--R     (b  (x,y,z) - a  (x,y,z))dx dy
--R       ,1           ,2
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 24

--S 25 of 34
exteriorDifferential theta
--R
--R
--R   (25)  (c  (x,y,z) - b  (x,y,z) + a  (x,y,z))dx dy dz
--R           ,1           ,2           ,3
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 25

--S 26 of 34
one : der := 1
--R
--R
--R   (26)  1
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 26

--S 27 of 34
g1 : der := a([x,t,y,u,v,z,e]) * one
--R
--R
--R   (27)  a(x,t,y,u,v,z,e)
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 27

--S 28 of 34
h1 : der := a([x,y,x,t,x,z,y,r,u,x]) * one
--R
--R
--R   (28)  a(x,y,x,t,x,z,y,r,u,x)
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 28

--S 29 of 34
exteriorDifferential g1
--R
--R
--R   (29)  a  (x,t,y,u,v,z,e)dz + a  (x,t,y,u,v,z,e)dy + a  (x,t,y,u,v,z,e)dx
--R          ,6                      ,3                      ,1
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 29

--S 30 of 34
exteriorDifferential h1
```

```
--R
--R
--R   (30)
--R     a  (x,y,x,t,x,z,y,r,u,x)dz
--R      ,6
--R   +
--R     (a  (x,y,x,t,x,z,y,r,u,x) + a  (x,y,x,t,x,z,y,r,u,x))dy
--R       ,7                          ,2
--R   +
--R         a   (x,y,x,t,x,z,y,r,u,x) + a  (x,y,x,t,x,z,y,r,u,x)
--R          ,10                          ,5
--R       +
--R         a  (x,y,x,t,x,z,y,r,u,x) + a  (x,y,x,t,x,z,y,r,u,x)
--R          ,3                          ,1
--R    *
--R       dx
--R                                            Type: DeRhamComplex(Integer,[x,y,z])
--E 30

--S 31 of 34
coefficient(gamma, dx*dy)
--R
--R
--R            2     3 2       2                                   4 2 5    3
--R   (31)  (7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z  + x y z
--R                                                        Type: Expression Integer
--E 31

--S 32 of 34
coefficient(gamma, one)
--R
--R
--R   (32)  0
--R                                                        Type: Expression Integer
--E 32

--S 33 of 34
coefficient(g1,one)
--R
--R
--R   (33)  a(x,t,y,u,v,z,e)
--R                                                        Type: Expression Integer
--E 33

--S 34 of 34
gamma := alpha * beta
--R
--R
--R   (34)
--R         4   2    2                3   2
```

```
--R     (2x y z  - x y z)dy dz + (2x y z  - x y z)cos(tan(x y z) + x y z)dx dz
--R   +
--R       2     3 2       2                                  4 2 5    3
--R   ((7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z  + x y z)dx dy
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 34
)spool
)lisp (bye)
```

— DeRhamComplex.help —

```
====================================================================
DeRhamComplex examples
====================================================================

The domain constructor DeRhamComplex creates the class of differential
forms of arbitrary degree over a coefficient ring.  The De Rham
complex constructor takes two arguments: a ring, coefRing, and a list
of coordinate variables.

This is the ring of coefficients.

  coefRing := Integer
   Integer
                     Type: Domain

These are the coordinate variables.

  lv : List Symbol := [x,y,z]
   [x,y,z]
                     Type: List Symbol


This is the De Rham complex of Euclidean three-space using coordinates
x, y and z.

  der := DERHAM(coefRing,lv)
   DeRhamComplex(Integer,[x,y,z])
                     Type: Domain

This complex allows us to describe differential forms having
expressions of integers as coefficients.  These coefficients can
involve any number of variables, for example, f(x,t,r,y,u,z).  As
we've chosen to work with ordinary Euclidean three-space, expressions
involving these forms are treated as functions of x, y and z with the
additional arguments t, r and u regarded as symbolic constants.
```

Here are some examples of coefficients.

```
  R := Expression coefRing
   Expression Integer
                     Type: Domain

  f : R := x**2*y*z-5*x**3*y**2*z**5
        3 2 5    2
   - 5x y z  + x y z
                     Type: Expression Integer

  g : R := z**2*y*cos(z)-7*sin(x**3*y**2)*z**2
        2     3 2       2
   - 7z sin(x y ) + y z cos(z)
                     Type: Expression Integer

  h : R :=x*y*z-2*x**3*y*z**2
        3   2
   - 2x y z  + x y z
                     Type: Expression Integer
```

We now define the multiplicative basis elements for the exterior algebra over R.

```
  dx : der := generator(1)
   dx
                     Type: DeRhamComplex(Integer,[x,y,z])

  dy : der := generator(2)
   dy
                     Type: DeRhamComplex(Integer,[x,y,z])

  dz : der := generator(3)
   dz
                     Type: DeRhamComplex(Integer,[x,y,z])
```

This is an alternative way to give the above assignments.

```
  [dx,dy,dz] := [generator(i)$der for i in 1..3]
    [dx,dy,dz]
                     Type: List DeRhamComplex(Integer,[x,y,z])
```

Now we define some one-forms.

```
  alpha : der := f*dx + g*dy + h*dz
           3   2                  2     3 2       2
     (- 2x y z  + x y z)dz + (- 7z sin(x y ) + y z cos(z))dy
   +
           3 2 5    2
     (- 5x y z  + x y z)dx
```

```
                    Type: DeRhamComplex(Integer,[x,y,z])

  beta  : der := cos(tan(x*y*z)+x*y*z)*dx + x*dy
    x dy + cos(tan(x y z) + x y z)dx
                    Type: DeRhamComplex(Integer,[x,y,z])
```

A well-known theorem states that the composition of exteriorDifferential with itself is the zero map for continuous forms. Let's verify this theorem for alpha.

```
  exteriorDifferential alpha
         2                  3 2                     3 2
     (y z sin(z) + 14z sin(x y ) - 2y z cos(z) - 2x z  + x z)dy dz
   +
         3 2 4     2   2          2
     (25x y z  - 6x y z  + y z - x y)dx dz
   +
           2 2 2     3 2       3   5    2
     (- 21x y z cos(x y ) + 10x y z  - x z)dx dy
                     Type: DeRhamComplex(Integer,[x,y,z])
```

We see a lengthy output of the last expression, but nevertheless, the composition is zero.

```
  exteriorDifferential %
    0
                     Type: DeRhamComplex(Integer,[x,y,z])
```

Now we check that exteriorDifferential is a "graded derivation" D, that is, D satisfies:

```
  D(a*b) = D(a)*b + (-1)**degree(a)*a*D(b)

  gamma := alpha * beta
        4   2    2                3   2
     (2x y z  - x y z)dy dz + (2x y z  - x y z)cos(tan(x y z) + x y z)dx dz
   +
       2     3 2       2                                   4 2 5    3
   ((7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z  + x y z)dx dy
                     Type: DeRhamComplex(Integer,[x,y,z])
```

We try this for the one-forms alpha and beta.

```
  exteriorDifferential(gamma) - (exteriorDifferential(alpha)*beta - alpha * exteriorDifferent
    0
                     Type: DeRhamComplex(Integer,[x,y,z])
```

Now we define some "basic operators"

```
  a : BOP := operator('a)
```

```
    a
                       Type: BasicOperator

  b : BOP := operator('b)
    b
                       Type: BasicOperator

  c : BOP := operator('c)
    c
                       Type: BasicOperator
```

We also define some indeterminate one- and two-forms using these operators.

```
  sigma := a(x,y,z) * dx + b(x,y,z) * dy + c(x,y,z) * dz
    c(x,y,z)dz + b(x,y,z)dy + a(x,y,z)dx
                       Type: DeRhamComplex(Integer,[x,y,z])

  theta  := a(x,y,z) * dx * dy + b(x,y,z) * dx * dz + c(x,y,z) * dy * dz
    c(x,y,z)dy dz + b(x,y,z)dx dz + a(x,y,z)dx dy
                       Type: DeRhamComplex(Integer,[x,y,z])
```

This allows us to get formal definitions for the "gradient" ...

```
  totalDifferential(a(x,y,z))$der
   (23)  a  (x,y,z)dz + a  (x,y,z)dy + a  (x,y,z)dx
          ,3             ,2             ,1
                       Type: DeRhamComplex(Integer,[x,y,z])
```

the "curl" ...

```
  exteriorDifferential sigma
     (c  (x,y,z) - b  (x,y,z))dy dz + (c  (x,y,z) - a  (x,y,z))dx dz
       ,2            ,3                 ,1            ,3
   +
     (b  (x,y,z) - a  (x,y,z))dx dy
       ,1            ,2
                        Type: DeRhamComplex(Integer,[x,y,z])
```

and the "divergence."

```
  exteriorDifferential theta
    (c  (x,y,z) - b  (x,y,z) + a  (x,y,z))dx dy dz
      ,1            ,2            ,3
                        Type: DeRhamComplex(Integer,[x,y,z])
```

Note that the De Rham complex is an algebra with unity. This element 1 is the basis for elements for zero-forms, that is, functions in our space.

```
  one : der := 1
```

```
    1
                       Type: DeRhamComplex(Integer,[x,y,z])
```

To convert a function to a function lying in the De Rham complex, multiply the function by "one."

```
  g1 : der := a([x,t,y,u,v,z,e]) * one
    a(x,t,y,u,v,z,e)
                       Type: DeRhamComplex(Integer,[x,y,z])
```

A current limitation of Axiom forces you to write functions with more than four arguments using square brackets in this way.

```
  h1 : der := a([x,y,x,t,x,z,y,r,u,x]) * one
    a(x,y,x,t,x,z,y,r,u,x)
                        Type: DeRhamComplex(Integer,[x,y,z])
```

Now note how the system keeps track of where your coordinate functions are located in expressions.

```
  exteriorDifferential g1
    a  (x,t,y,u,v,z,e)dz + a  (x,t,y,u,v,z,e)dy + a  (x,t,y,u,v,z,e)dx
     ,6                     ,3                     ,1
                        Type: DeRhamComplex(Integer,[x,y,z])

  exteriorDifferential h1
     a  (x,y,x,t,x,z,y,r,u,x)dz
      ,6
   +
     (a  (x,y,x,t,x,z,y,r,u,x) + a  (x,y,x,t,x,z,y,r,u,x))dy
       ,7                         ,2
   +
         a   (x,y,x,t,x,z,y,r,u,x) + a  (x,y,x,t,x,z,y,r,u,x)
          ,10                         ,5
       +
         a  (x,y,x,t,x,z,y,r,u,x) + a  (x,y,x,t,x,z,y,r,u,x)
          ,3                         ,1
    *
       dx
                       Type: DeRhamComplex(Integer,[x,y,z])
```

In this example of Euclidean three-space, the basis for the De Rham complex consists of the eight forms: 1, dx, dy, dz, dx*dy, dx*dz, dy*dz, and dx*dy*dz.

```
  coefficient(gamma, dx*dy)
       2     3 2       2                                   4 2 5    3
    (7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z  + x y z
                       Type: Expression Integer

  coefficient(gamma, one)
```

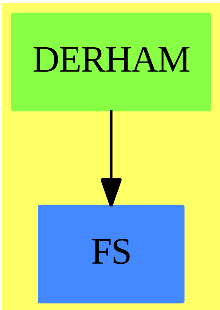
```
    0
                        Type: Expression Integer

  coefficient(g1,one)
    a(x,t,y,u,v,z,e)
                        Type: Expression Integer

See Also:
o )help Operator
o )show DeRhamComplex
```

———-

5.6.1 DeRhamComplex (DERHAM)



See

⇒ "ExtAlgBasis" (EAB) 6.8.1 on page 711
⇒ "AntiSymm" (ANTISYM) 2.8.1 on page 40

Exports:

0	1	characteristic	coefficient
coerce	degree	exteriorDifferential	generator
hash	homogeneous?	latex	leadingBasisTerm
leadingCoefficient	map	one?	recip
reductum	retract	retractable?	retractIfCan
sample	subtractIfCan	totalDifferential	zero?
?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?

— domain DERHAM DeRhamComplex —

```
)abbrev domain DERHAM DeRhamComplex
++ Author: Larry A. Lambe
++ Date    : 01/26/91.
```

```
++ Revised : 12/01/91.
++
++ based on code from '89 (AntiSymmetric)
++
++ Needs: LeftAlgebra, ExtAlgBasis, FreeMod(Ring,OrderedSet)
++
++ Description:
++ The deRham complex of Euclidean space, that is, the
++ class of differential forms of arbitary degree over a coefficient ring.
++ See Flanders, Harley, Differential Forms, With Applications to the Physical
++ Sciences, New York, Academic Press, 1963.

DeRhamComplex(CoefRing,listIndVar:List Symbol): Export == Implement where
  CoefRing :  Join(Ring, OrderedSet)
  ASY     ==> AntiSymm(R,listIndVar)
  DIFRING ==> DifferentialRing
  LALG    ==> LeftAlgebra
  FMR     ==> FreeMod(R,EAB)
  I       ==> Integer
  L       ==> List
  EAB     ==> ExtAlgBasis  -- these are exponents of basis elements in order
  NNI     ==> NonNegativeInteger
  O       ==> OutputForm
  R       ==> Expression(CoefRing)

  Export == Join(LALG(R), RetractableTo(R)) with
      leadingCoefficient : %           -> R
        ++ leadingCoefficient(df) returns the leading
        ++ coefficient of differential form df.
      leadingBasisTerm   : %           -> %
        ++ leadingBasisTerm(df) returns the leading
        ++ basis term of differential form df.
      reductum           : %           -> %
        ++ reductum(df), where df is a differential form,
        ++ returns df minus the leading
        ++ term of df if df has two or more terms, and
        ++ 0 otherwise.
      coefficient        : (%,%)     -> R
        ++ coefficient(df,u), where df is a differential form,
        ++ returns the coefficient of df containing the basis term u
        ++ if such a term exists, and 0 otherwise.
      generator          : NNI         -> %
        ++ generator(n) returns the nth basis term for a differential form.
      homogeneous?       : %           -> Boolean
        ++  homogeneous?(df) tests if all of the terms of
        ++  differential form df have the same degree.
      retractable?       : %           -> Boolean
        ++  retractable?(df) tests if differential form df is a 0-form,
        ++  i.e., if degree(df) = 0.
      degree             : %           -> I
```

```
        ++  degree(df) returns the homogeneous degree of differential form df.
      map                : (R -> R, %) -> %
        ++  map(f,df) replaces each coefficient x of differential
        ++  form df by \spad{f(x)}.
      totalDifferential    : R -> %
        ++  totalDifferential(x) returns the total differential
        ++  (gradient) form for element x.
      exteriorDifferential : % -> %
        ++  exteriorDifferential(df) returns the exterior
        ++  derivative (gradient, curl, divergence, ...) of
        ++  the differential form df.

  Implement == ASY add
      Rep := ASY

      dim := #listIndVar

      totalDifferential(f) ==
        divs:=[differentiate(f,listIndVar.i)*generator(i)$ASY for i in 1..dim]
        reduce("+",divs)

      termDiff : (R, %) -> %
      termDiff(r,e) ==
        totalDifferential(r) * e

      exteriorDifferential(x) ==
        x = 0 => 0
        termDiff(leadingCoefficient(x)$Rep,leadingBasisTerm x) + exteriorDifferential(reductum x)

      lv := [concat("d",string(liv))$String::Symbol for liv in listIndVar]

      displayList:EAB -> O
      displayList(x):O ==
        le: L I := exponents(x)$EAB
--      reduce(_*,[(lv.i)::O for i in 1..dim | le.i = 1])$L(O)
--        reduce(_*,[(lv.i)::O for i in 1..dim | one?(le.i)])$L(O)
        reduce(_*,[(lv.i)::O for i in 1..dim | ((le.i) = 1)])$L(O)

      makeTerm:(R,EAB) -> O
      makeTerm(r,x) ==
      -- we know that r ^= 0
        x = Nul(dim)$EAB  => r::O
--        one? r => displayList(x)
        (r = 1) => displayList(x)
--      r = 1 => displayList(x)
        r::O * displayList(x)

      terms : % -> List Record(k: EAB, c: R)
      terms(a) ==
        -- it is the case that there are at least two terms in a
```

```
        a pretend List Record(k: EAB, c: R)

      coerce(a):O ==
        a           = O$Rep => O$I::O
        ta := terms a
--      reductum(a) = O$Rep => makeTerm(leadingCoefficient a, a.first.k)
        null ta.rest => makeTerm(ta.first.c, ta.first.k)
        reduce(_+,[makeTerm(t.c,t.k) for t in ta])$L(O)
```

— DERHAM.dotabb —

```
"DERHAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DERHAM"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"DERHAM" -> "FS"
```

5.7 domain DSTREE DesingTree

— DesingTree.input —

```
)set break resume
)sys rm -f DesingTree.output
)spool DesingTree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DesingTree
--R DesingTree S: SetCategory  is a domain constructor
--R Abbreviation for DesingTree is DSTREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DSTREE
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %                copy : % -> %
--R cyclic? : % -> Boolean                distance : (%,%) -> Integer
--R ?.value : (%,value) -> S              empty : () -> %
--R empty? : % -> Boolean                 encode : % -> String
--R eq? : (%,%) -> Boolean                fullOut : % -> OutputForm
--R fullOutput : () -> Boolean            fullOutput : Boolean -> Boolean
```

```
--R leaf? : % -> Boolean                  leaves : % -> List S
--R map : ((S -> S),%) -> %               nodes : % -> List %
--R sample : () -> %                      tree : List S -> %
--R tree : S -> %                         tree : (S,List %) -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———

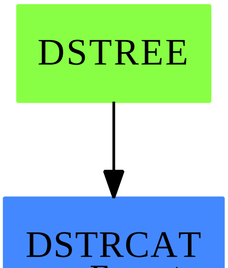
— DesingTree.help —

```
====================================================================
DesingTree examples
====================================================================

See Also:
o )show DesingTree
```

———

5.7.1 DesingTree (DSTREE)



Exports:

#?	?.value	?=?	?~=?	any?
child?	children	coerce	copy	count
cyclic?	distance	empty	empty?	encode
eq?	eval	every?	fullOut	fullOutput
hash	latex	leaf?	leaves	less?
map	map!	member?	members	more?
node?	nodes	parts	sample	setchildren!
setelt	setvalue!	size?	tree	value

— **domain DSTREE DesingTree** —

```
)abbrev domain DSTREE DesingTree
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This category is part of the PAFF package
DesingTree(S: SetCategory): T==C where

 T == DesingTreeCategory(S) with

    encode: % -> String
       ++ encode(t) returns a string indicating the "shape" of the tree

    fullOut: % -> OutputForm
      ++ fullOut(tr) yields a full output of tr (see function fullOutput).

    fullOutput: Boolean -> Boolean
      ++ fullOutput(b) sets a flag such that when true,
      ++ a coerce to OutputForm yields the full output of
      ++ tr, otherwise encode(tr) is output (see encode function).
```

```
      ++ The default is false.

    fullOutput: () -> Boolean
      ++ fullOutput returns the value of the flag set by fullOutput(b).

 C == add
    Rep ==> Record(value: S, args: List %)

    fullOut(t:%): OutputForm ==
      empty? children t => (value t) ::OutputForm
      prefix((value t)::OutputForm, [fullOut(tr) for tr in children t])

    fullOutputFlag:Boolean:=false()

    fullOutput(f)== fullOutputFlag:=f

    fullOutput == fullOutputFlag

    leaves(t)==
      empty?(chdr:=children(t)) => list(value(t))
      concat([leaves(subt) for subt in chdr])

    t1=t2 == value t1 = value t2 and children t1 = children t2

    coerce(t:%):OutputForm==
      ^fullOutput() => encode(t) :: OutputForm
      fullOut(t)

    tree(s,ls) == ([s,ls]:Rep):%

    tree(s:S) == ([s,[]]:Rep):%

    tree(ls:List(S))==
      empty?(ls) =>
        error "Cannot create a tree with an empty list"
      f:=first(ls)
      empty?(rs:=rest(ls)) =>
        tree(f)
      tree(f,[tree(rs)])

    value t == (t:Rep).value

    children t == ((t:Rep).args):List %

    setchildren_!(t,ls) == ((t:Rep).args:=ls;t pretend %)

    setvalue_!(t,s) == ((t:Rep).value:=s;s)

    encode(t)==
      empty?(chtr:=children(t)) => empty()$String
```

```
concat([concat(["U",encode(arb),"."]) for arb in chtr])
```

———-

— DSTREE.dotabb —

```
"DSTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DSTREE"];
"DSTRCAT"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=DSTRCAT"]
"DSTREE" -> "DSTRCAT"
```

———-

5.8 domain DSMP DifferentialSparseMultivariatePolynomial

— DifferentialSparseMultivariatePolynomial.input —

```
)set break resume
)sys rm -f DifferentialSparseMultivariatePolynomial.output
)spool DifferentialSparseMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialSparseMultivariatePolynomial
--R DifferentialSparseMultivariatePolynomial(R: Ring,S: OrderedSet,V: DifferentialVariableCat
--R Abbreviation for DifferentialSparseMultivariatePolynomial is DSMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DSMP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                       ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %         ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                  D : % -> % if R has DIFRING
--R D : (%,List V) -> %                    D : (%,V) -> %
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,PositiveInteger) -> %         coefficients : % -> List R
--R coerce : S -> %                        coerce : V -> %
--R coerce : R -> %                        coerce : Integer -> %
```

```
--R coerce : % -> OutputForm             degree : % -> IndexedExponents V
--R differentiate : (%,List V) -> %      differentiate : (%,V) -> %
--R eval : (%,List V,List %) -> %        eval : (%,V,%) -> %
--R eval : (%,List V,List R) -> %        eval : (%,V,R) -> %
--R eval : (%,List %,List %) -> %        eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %           eval : (%,List Equation %) -> %
--R ground : % -> R                      ground? : % -> Boolean
--R hash : % -> SingleInteger            initial : % -> %
--R isobaric? : % -> Boolean             latex : % -> String
--R leader : % -> V                      leadingCoefficient : % -> R
--R leadingMonomial : % -> %             map : ((R -> R),%) -> %
--R monomial? : % -> Boolean             monomials : % -> List %
--R one? : % -> Boolean                  order : % -> NonNegativeInteger
--R primitiveMonomials : % -> List %     recip : % -> Union(%,"failed")
--R reductum : % -> %                    retract : % -> S
--R retract : % -> V                     retract : % -> R
--R sample : () -> %                     separant : % -> %
--R variables : % -> List V              weight : % -> NonNegativeInteger
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,List V,List NonNegativeInteger) -> %
--R D : (%,V,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List V,List NonNegativeInteger) -> %
--R coefficient : (%,V,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents V) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseMultivariatePolynomial(R,S) -> %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,V) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
```

```
--R convert : % -> InputForm if R has KONVERT INFORM and V has KONVERT INFORM
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT and V has KONVERT PATTERN INT
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT and V has KONVERT PATTERN FLO
--R degree : (%,S) -> NonNegativeInteger
--R degree : (%,List V) -> List NonNegativeInteger
--R degree : (%,V) -> NonNegativeInteger
--R differentialVariables : % -> List S
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,List V,List NonNegativeInteger) -> %
--R differentiate : (%,V,NonNegativeInteger) -> %
--R discriminant : (%,V) -> % if R has COMRING
--R eval : (%,List S,List R) -> % if R has DIFRING
--R eval : (%,S,R) -> % if R has DIFRING
--R eval : (%,List S,List %) -> % if R has DIFRING
--R eval : (%,S,%) -> % if R has DIFRING
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(V,"failed")
--R makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
--R makeVariable : S -> (NonNegativeInteger -> %)
--R mapExponents : ((IndexedExponents V -> IndexedExponents V),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List V) -> List NonNegativeInteger
--R minimumDegree : (%,V) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents V
--R monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
--R monomial : (%,List V,List NonNegativeInteger) -> %
--R monomial : (%,V,NonNegativeInteger) -> %
--R monomial : (R,IndexedExponents V) -> %
--R multivariate : (SparseUnivariatePolynomial %,V) -> %
--R multivariate : (SparseUnivariatePolynomial R,V) -> %
```

```
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,S) -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has
--R pomopo! : (%,R,IndexedExponents V,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,V) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%,V) -> % if R has COMRING
--R retract : % -> SparseMultivariatePolynomial(R,S)
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseMultivariatePolynomial(R,S),"failed")
--R retractIfCan : % -> Union(S,"failed")
--R retractIfCan : % -> Union(V,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R ha
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List V) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,V) -> SparseUnivariatePolynomial %
--R weight : (%,S) -> NonNegativeInteger
--R weights : (%,S) -> List NonNegativeInteger
--R weights : % -> List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

— DifferentialSparseMultivariatePolynomial.help —

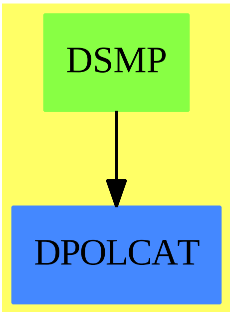
```
====================================================================
DifferentialSparseMultivariatePolynomial examples
```

```
====================================================================

See Also:
o )show DifferentialSparseMultivariatePolynomial
```

———-

5.8.1 DifferentialSparseMultivariatePolynomial (DSMP)



See

⇒ “OrderlyDifferentialVariable” (ODVAR) 16.17.1 on page 1816
⇒ “SequentialDifferentialVariable” (SDVAR) 20.7.1 on page 2348
⇒ “OrderlyDifferentialPolynomial” (ODPOL) 16.16.1 on page 1813
⇒ “SequentialDifferentialPolynomial” (SDPOL) 20.6.1 on page 2345

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentialVariables
differentiate	discriminant	eval
exquo	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	hash
initial	isExpt	isobaric?
isPlus	isTimes	latex
lcm	leader	leadingCoefficient
leadingMonomial	makeVariable	map
mapExponents	max	min
minimumDegree	monicDivide	monomial
monomials	monomial?	multivariate
numberOfMonomials	one?	order
patternMatch	pomopo!	prime?
primitiveMonomials	primitivePart	recip
reducedSystem	reductum	resultant
retract	retractIfCan	sample
separant	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
weight	weights	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

— domain DSMP DifferentialSparseMultivariatePolynomial —

```
)abbrev domain DSMP DifferentialSparseMultivariatePolynomial
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:DifferentialPolynomialCategory
++ Related Constructors:
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++           order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
```

```
++ \spadtype{DifferentialSparseMultivariatePolynomial} implements
++ an ordinary differential polynomial ring by combining a
++ domain belonging to the category \spadtype{DifferentialVariableCategory}
++ with the domain \spadtype{SparseMultivariatePolynomial}.

DifferentialSparseMultivariatePolynomial(R, S, V):
     Exports == Implementation where
  R: Ring
  S: OrderedSet
  V: DifferentialVariableCategory S
  E   ==> IndexedExponents(V)
  PC  ==> PolynomialCategory(R,IndexedExponents(V),V)
  PCL ==> PolynomialCategoryLifting
  P   ==> SparseMultivariatePolynomial(R, V)
  SUP ==> SparseUnivariatePolynomial
  SMP ==> SparseMultivariatePolynomial(R, S)

  Exports ==> Join(DifferentialPolynomialCategory(R,S,V,E),
                  RetractableTo SMP)

  Implementation ==> P add
    retractIfCan(p:$):Union(SMP, "failed") ==
      zero? order p =>
        map(x+->retract(x)@S :: SMP,y+->y::SMP, p)$PCL(
                                IndexedExponents V, V, R, $, SMP)
      "failed"

    coerce(p:SMP):$ ==
      map(x+->x::V::$, y+->y::$, p)$PCL(IndexedExponents S, S, R, SMP, $)
```

— **DSMP.dotabb** —

```
"DSMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DSMP"]
"DPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DPOLCAT"]
"DSMP" -> "DPOLCAT"
```

5.9 domain DIRPROD DirectProduct

— **DirectProduct.input** —

```
)set break resume
```

```
)sys rm -f DirectProduct.output
)spool DirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProduct
--R DirectProduct(dim: NonNegativeInteger,R: Type)  is a domain constructor
--R Abbreviation for DirectProduct is DIRPROD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DIRPROD
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has RING            1 : () -> % if R has MONOID
--R 0 : () -> % if R has CABMON           coerce : % -> Vector R
--R copy : % -> %                         directProduct : Vector R -> %
--R ?.? : (%,Integer) -> R                elt : (%,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List R                 eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R map : ((R -> R),%) -> %               qelt : (%,Integer) -> R
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (PositiveInteger,%) -> % if R has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if R has CABMON
--R ?*? : (R,%) -> % if R has RING
--R ?*? : (%,R) -> % if R has RING
--R ?*? : (%,%) -> % if R has MONOID
--R ?*? : (Integer,%) -> % if R has RING
--R ?**? : (%,PositiveInteger) -> % if R has MONOID
--R ?**? : (%,NonNegativeInteger) -> % if R has MONOID
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has RING
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has OAMONS or R has ORDRING
--R ?<=? : (%,%) -> Boolean if R has OAMONS or R has ORDRING
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has OAMONS or R has ORDRING
--R ?>=? : (%,%) -> Boolean if R has OAMONS or R has ORDRING
--R D : (%,(R -> R)) -> % if R has RING
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL and R has RING
--R D : (%,Symbol) -> % if R has PDRING SYMBOL and R has RING
--R D : (%,NonNegativeInteger) -> % if R has DIFRING and R has RING
--R D : % -> % if R has DIFRING and R has RING
--R ?^? : (%,PositiveInteger) -> % if R has MONOID
--R ?^? : (%,NonNegativeInteger) -> % if R has MONOID
```

```
--R abs : % -> % if R has ORDRING
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if R has RING
--R coerce : R -> % if R has SETCAT
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT and R has SETCAT
--R coerce : Integer -> % if R has RETRACT INT and R has SETCAT or R has RING
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(R -> R)) -> % if R has RING
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL and R
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING and R has RING
--R differentiate : % -> % if R has DIFRING and R has RING
--R dimension : () -> CardinalNumber if R has FIELD
--R dot : (%,%) -> R if R has RING
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R index : PositiveInteger -> % if R has FINITE
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if R has FINITE
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has OAMONS or R has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R min : (%,%) -> % if R has OAMONS or R has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has MONOID
--R parts : % -> List R if $ has finiteAggregate
--R positive? : % -> Boolean if R has ORDRING
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R random : () -> % if R has FINITE
--R recip : % -> Union(%,"failed") if R has MONOID
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R) if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
```

```
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT and R has RING
--R retract : % -> R if R has SETCAT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT and R has SETCAT
--R retract : % -> Integer if R has RETRACT INT and R has SETCAT
--R retractIfCan : % -> Union(R,"failed") if R has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT and R has SETCAT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT and R has SETCAT
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R sign : % -> Integer if R has ORDRING
--R size : () -> NonNegativeInteger if R has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has CABMON
--R sup : (%,%) -> % if R has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if R has RING
--R zero? : % -> Boolean if R has CABMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———

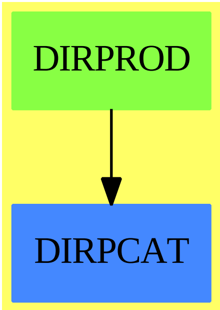
— DirectProduct.help —

```
====================================================================
DirectProduct examples
====================================================================

See Also:
o )show DirectProduct
```

———

5.9.1 DirectProduct (DIRPROD)



Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
#?	?*?	?**?	?+?	?-?
?/?	?<?	?<=?	?=?	?>?
?>=?	?^?	?~=?	-?	?.?

— domain DIRPROD DirectProduct —

```
)abbrev domain DIRPROD DirectProduct
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, IndexedVector
++ Also See: OrderedDirectProduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
++ underlying component type. This contrasts with simple vectors in that
++ the members can be viewed as having constant length. Thus many
++ categorical properties can by lifted from the underlying component type.
```

```
++ Component extraction operations are provided but no updating operations.
++ Thus new direct product elements can either be created by converting
++ vector elements using the \spadfun{directProduct} function
++ or by taking appropriate linear combinations of basis vectors provided
++ by the \spad{unitVector} operation.

DirectProduct(dim:NonNegativeInteger, R:Type):
  DirectProductCategory(dim, R) == Vector R add

      Rep := Vector R

      coerce(z:%):Vector(R)        == copy(z)$Rep pretend Vector(R)
      coerce(r:R):%                == new(dim, r)$Rep

      parts x == VEC2LIST(x)$Lisp

      directProduct z ==
        size?(z, dim) => copy(z)$Rep
        error "Not of the correct length"


      if R has SetCategory then
        same?: % -> Boolean
        same? z == every?(x +-> x = z(minIndex z), z)

        x = y == _and/[qelt(x,i)$Rep = qelt(y,i)$Rep for i in 1..dim]

        retract(z:%):R ==
          same? z => z(minIndex z)
          error "Not retractable"

        retractIfCan(z:%):Union(R, "failed") ==
          same? z => z(minIndex z)
          "failed"


      if R has AbelianSemiGroup then
        u:% + v:% == map(_+ , u, v)$Rep

      if R has AbelianMonoid then
        0 == zero(dim)$Vector(R) pretend %

      if R has Monoid then
        1 == new(dim, 1)$Vector(R) pretend %
        u:% * r:R       == map(x +-> x * r, u)
        r:R * u:%       == map(x +-> r * x, u)
        x:% * y:% == [x.i * y.i for i in 1..dim]$Vector(R) pretend %

      if R has CancellationAbelianMonoid then
        subtractIfCan(u:%, v:%):Union(%,"failed") ==
```

```
          w := new(dim,0)$Vector(R)
          for i in 1..dim repeat
            (c:=subtractIfCan(qelt(u, i)$Rep, qelt(v,i)$Rep)) case "failed" =>
                    return "failed"
            qsetelt_!(w, i, c::R)$Rep
          w pretend %

      if R has Ring then

        u:% * v:%                    == map(_* , u, v)$Rep

        recip z ==
          w := new(dim,0)$Vector(R)
          for i in minIndex w .. maxIndex w repeat
            (u := recip qelt(z, i)) case "failed" => return "failed"
            qsetelt_!(w, i, u::R)
          w pretend %

        unitVector i ==
          v:= new(dim,0)$Vector(R)
          v.i := 1
          v pretend %

      if R has OrderedSet then
        x < y ==
          for i in 1..dim repeat
             qelt(x,i) < qelt(y,i) => return true
             qelt(x,i) > qelt(y,i) => return false
          false

      if R has OrderedAbelianMonoidSup then sup(x, y) == map(sup, x, y)

--)bo $noSubsumption := false
```

— **DIRPROD.dotabb** —

```
"DIRPROD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DIRPROD"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"DIRPROD" -> "DIRPCAT"
```

5.10 domain DPMM DirectProductMatrixModule

— DirectProductMatrixModule.input —

```
)set break resume
)sys rm -f DirectProductMatrixModule.output
)spool DirectProductMatrixModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProductMatrixModule
--R DirectProductMatrixModule(n: PositiveInteger,R: Ring,M: SquareMatrixCategory(n,R,DirectProduct(n,R),
--R Abbreviation for DirectProductMatrixModule is DPMM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DPMM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (M,%) -> %                      ?*? : (PositiveInteger,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (R,%) -> %
--R ?+? : (%,%) -> %                      -? : % -> %
--R ?-? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R coerce : % -> Vector S                copy : % -> %
--R directProduct : Vector S -> %         ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List S
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R latex : % -> String                   map : ((S -> S),%) -> %
--R qelt : (%,Integer) -> S               sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,%) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has MONOID
--R ?*? : (S,%) -> % if S has RING
--R ?*? : (%,S) -> % if S has RING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING o
--R ?**? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RIN
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : % -> % if S has DIFRING and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
```

```
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R 1 : () -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has M
--R ?^? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R ?^? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : S -> % if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : % -> % if S has DIFRING and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R index : PositiveInteger -> % if S has FINITE
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has DIFRING and S has RING or S has LINEXP INT and S has RING or
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
```

```
--R random : () -> % if S has FINITE
--R recip : % -> Union(%,"failed") if S has DIFRING and S has RING or S has LINEXP INT and S has RING or
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retract : % -> S if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S,"failed") if S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R sup : (%,%) -> % if S has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R
--E 1

)spool
)lisp (bye)
```

———-

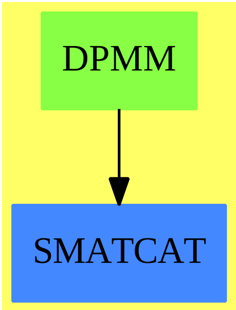
— DirectProductMatrixModule.help —

```
====================================================================
DirectProductMatrixModule examples
====================================================================

See Also:
o )show DirectProductMatrixModule
```

———-

5.10.1 DirectProductMatrixModule (DPMM)



See

⇒ "OppositeMonogenicLinearOperator" (OMLO) 16.11.1 on page 1768
⇒ "OrdinaryDifferentialRing" (ODR) 16.18.1 on page 1820
⇒ "DirectProductModule" (DPMO) 5.11.1 on page 542

Exports:

0	1	coerce	copy	directProduct
elt	empty	empty?	entries	eq?
hash	index?	indices	latex	map
qelt	sample	zero?	D	abs
any?	characteristic	coerce	count	differentiate
dimension	dot	entry?	eval	every?
fill!	first	index	less?	lookup
map!	max	maxIndex	member?	members
min	minIndex	more?	negative?	one?
parts	positive?	qsetelt!	random	recip
reducedSystem	retract	retractIfCan	setelt	sign
size	size?	subtractIfCan	sup	swap!
unitVector	#?	?*?	?**?	?/?
?<?	?<=?	?>?	?>=?	?^?
?^?	-?	?-?	?=?	?.?
?~=?				

— domain DPMM DirectProductMatrixModule —

```
)abbrev domain DPMM DirectProductMatrixModule
++ Author:  Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
```

```
++ References:
++ Description:
++ This constructor provides a direct product type with a
++ left matrix-module view.

DirectProductMatrixModule(n, R, M, S): DPcategory == DPcapsule where
    n: PositiveInteger
    R: Ring
    RowCol ==> DirectProduct(n,R)
    M: SquareMatrixCategory(n,R,RowCol,RowCol)
    S: LeftModule(R)

    DPcategory == Join(DirectProductCategory(n,S), LeftModule(R), LeftModule(M))

    DPcapsule == DirectProduct(n, S) add
        Rep := Vector(S)
        r:R * x:$ == [r*x.i for i in 1..n]
        m:M * x:$ == [ +/[m(i,j)*x.j for j in 1..n] for i in 1..n]
```

— **DPMM.dotabb** —

```
"DPMM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DPMM"]
"SMATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SMATCAT"]
"DPMM" -> "SMATCAT"
```

5.11 domain DPMO DirectProductModule

— **DirectProductModule.input** —

```
)set break resume
)sys rm -f DirectProductModule.output
)spool DirectProductModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProductModule
--R DirectProductModule(n: NonNegativeInteger,R: Ring,S: LeftModule R)  is a domain constructor
--R Abbreviation for DirectProductModule is DPMO
```

```
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DPMO
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?*? : (Integer,%) -> %
--R ?*? : (R,%) -> %                     ?+? : (%,%) -> %
--R -? : % -> %                          ?-? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               0 : () -> %
--R coerce : % -> OutputForm             coerce : % -> Vector S
--R copy : % -> %                        directProduct : Vector S -> %
--R ?.? : (%,Integer) -> S               elt : (%,Integer,S) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List S                eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger            index? : (Integer,%) -> Boolean
--R indices : % -> List Integer          latex : % -> String
--R map : ((S -> S),%) -> %              qelt : (%,Integer) -> S
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,%) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S
--R ?*? : (S,%) -> % if S has RING
--R ?*? : (%,S) -> % if S has RING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R ?**? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT an
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : % -> % if S has DIFRING and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R 1 : () -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has N
--R ?^? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R ?^? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : S -> % if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : % -> % if S has DIFRING and S has RING
```

```
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R index : PositiveInteger -> % if S has FINITE
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has MONO
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(%,"failed") if S has DIFRING and S has RING or S has LINEXP INT and S has RING or
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retract : % -> S if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S,"failed") if S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
```

```
--R sup : (%,%) -> % if S has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R
--E 1

)spool
)lisp (bye)
```

———-

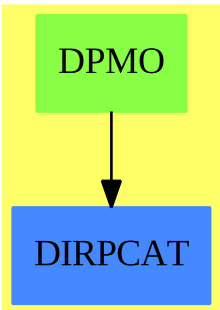
— **DirectProductModule.help** —

```
====================================================================
DirectProductModule examples
====================================================================

See Also:
o )show DirectProductModule
```

———-

5.11.1 DirectProductModule (DPMO)



See

⇒ "OppositeMonogenicLinearOperator" (OMLO) 16.11.1 on page 1768
⇒ "OrdinaryDifferentialRing" (ODR) 16.18.1 on page 1820
⇒ "DirectProductMatrixModule" (DPMM) 5.10.1 on page 538

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entries	entry?	eval	every?
eq?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
#?	?*?	?**?	?/?	?<?
?<=?	?>?	?>=?	?^?	?+?
-?	?-?	?=?	?.?	?~=?

— domain DPMO DirectProductModule —

```
)abbrev domain DPMO DirectProductModule
++ Author:  Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This constructor provides a direct product of R-modules
++ with an R-module view.

DirectProductModule(n, R, S): DPcategory == DPcapsule where
    n: NonNegativeInteger
    R: Ring
    S: LeftModule(R)

    DPcategory == Join(DirectProductCategory(n,S), LeftModule(R))
    --  with if S has Algebra(R) then Algebra(R)
    --  <above line leads to matchMmCond: unknown form of condition>

    DPcapsule == DirectProduct(n,S) add
        Rep := Vector(S)
        r:R * x:$ == [r * x.i for i in 1..n]
```

————

— **DPMO.dotabb** —

```
"DPMO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DPMO"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"DPMO" -> "DIRPCAT"
```

5.12 domain DIRRING DirichletRing

The Dirichlet Ring is the ring of arithmetical functions

$$f : \mathbb{N}_+ \to R$$

(see http://en.wikipedia.org/wiki/Arithmetic_function) together with the Dirichlet convolution (see http://en.wikipedia.org/wiki/Dirichlet_convolution) as multiplication and component-wise addition. Since we can consider the values an arithmetic functions assumes as the coefficients of a Dirichlet generating series, we call R the coefficient ring of a function.

In general we only assume that the coefficient ring R is a ring. If R happens to be commutative, then so is the Dirichlet ring, and in this case it is even an algebra.

Apart from the operations inherited from those categories, we only provide some convenient coercion functions.

— **DirichletRing.input** —

```
)set break resume
)sys rm -f DirichletRing.output
)spool DirichletRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 21
t1:DIRRING INT := (n:PI):INT +-> moebiusMu n
--R
--R
--R   (1)  [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R                                                     Type: DirichletRing Integer
--E 1

--S 2 of 21
[t1.i for i in 1..4]
--R
--R
```

```
--R   (2)  [1,- 1,- 1,0]
--R                                                          Type: List Integer
--E 2

--S 3 of 21
t2:DIRRING INT := [moebiusMu n for n in 1..]
--R
--R
--R   (3)  [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R                                                 Type: DirichletRing Integer
--E 3

--S 4 of 21
[t2.i for i in 1..4]
--R
--R
--R   (4)  [1,- 1,- 1,0]
--R                                                          Type: List Integer
--E 4

--S 5 of 21
DIRRING INT has CommutativeRing
--R
--R
--R   (5)  true
--R                                                             Type: Boolean
--E 5

--S 6 of 21
mu:DIRRING FRAC INT := (n:PI):FRAC INT +-> moebiusMu n
--R
--R
--R   (6)  [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R                                        Type: DirichletRing Fraction Integer
--E 6

--S 7 of 21
phi:DIRRING FRAC INT := (n:PI):FRAC INT +-> eulerPhi n
--R
--R
--R   (7)  [1,1,2,2,4,2,6,4,6,4,...]
--R                                        Type: DirichletRing Fraction Integer
--E 7

--S 8 of 21
t3:=[(recip mu * phi).n for n in 1..10]
--R
--R
--R   (8)  [1,2,3,4,5,6,7,8,9,10]
--R                                                 Type: List Fraction Integer
```

```
--E 8

--S 9 of 21
t4:=[(phi * recip mu).n for n in 1..10]
--R
--R
--R   (9)  [1,2,3,4,5,6,7,8,9,10]
--R                                                 Type: List Fraction Integer
--E 9

--S 10 of 21
reduce(_and,[(x = y)@Boolean for x in t3 for y in t4])
--R
--R
--R   (10)  true
--R                                                              Type: Boolean
--E 10

--S 11 of 21
DIRRING FRAC INT has Algebra FRAC INT
--R
--R
--R   (11)  true
--R                                                              Type: Boolean
--E 11

--S 12 of 21
t5:=[(1/2 * phi).n for n in 1..10]
--R
--R
--R          1 1
--R   (12)  [-,-,1,1,2,1,3,2,3,2]
--R          2 2
--R                                                 Type: List Fraction Integer
--E 12

--S 13 of 21
t6:=[eulerPhi n/2 for n in 1..10]
--R
--R
--R          1 1
--R   (13)  [-,-,1,1,2,1,3,2,3,2]
--R          2 2
--R                                                 Type: List Fraction Integer
--E 13

--S 14 of 21
reduce(_and,[(x = y)@Boolean for x in t5 for y in t6])
--R
--R
```

```
--R   (14)  true
--R                                                             Type: Boolean
--E 14

--S 15 of 21
t7:=[(recip mu).n for n in 1..10]
--R
--R
--R   (15)  [1,1,1,1,1,1,1,1,1,1]
--R                                               Type: List Fraction Integer
--E 15

--S 16 of 21
t8:=[1 for n in 1..10]
--R
--R
--R   (16)  [1,1,1,1,1,1,1,1,1,1]
--R                                                Type: List PositiveInteger
--E 16

--S 17 of 21
reduce(_and,[(x = y)@Boolean for x in t7 for y in t8])
--R
--R
--R   (17)  true
--R                                                             Type: Boolean
--E 17

--S 18 of 21
t9:=[(recip mu * phi).n for n in 1..10]
--R
--R
--R   (18)  [1,2,3,4,5,6,7,8,9,10]
--R                                               Type: List Fraction Integer
--E 18

--S 19 of 21
t10:=[n for n in 1..10]
--R
--R
--R   (19)  [1,2,3,4,5,6,7,8,9,10]
--R                                                Type: List PositiveInteger
--E 19

--S 20 of 21
reduce(_and,[(x = y)@Boolean for x in t9 for y in t10])
--R
--R
--R   (20)  true
--R                                                             Type: Boolean
```

```
--E 20

--S 21 of 21
)show DirichletRing
--R DirichletRing Coef: Ring  is a domain constructor
--R Abbreviation for DirichletRing is DIRRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DIRRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> Stream Coef
--R coerce : Stream Coef -> %             coerce : Integer -> %
--R coerce : % -> OutputForm              ?.? : (%,PositiveInteger) -> Coef
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R zeta : () -> %                        ?~=? : (%,%) -> Boolean
--R ?*? : (%,Coef) -> % if Coef has COMRING
--R ?*? : (Coef,%) -> % if Coef has COMRING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R additive? : (%,PositiveInteger) -> Boolean
--R associates? : (%,%) -> Boolean if Coef has COMRING
--R characteristic : () -> NonNegativeInteger
--R coerce : % -> % if Coef has COMRING
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> (PositiveInteger -> Coef)
--R coerce : (PositiveInteger -> Coef) -> %
--R exquo : (%,%) -> Union(%,"failed") if Coef has COMRING
--R multiplicative? : (%,PositiveInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if Coef has COMRING
--R unitCanonical : % -> % if Coef has COMRING
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has COMRING
--R
--E 21

)spool
)lisp (bye)
```

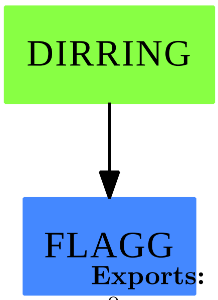
— **DirichletRing.help** —

```
====================================================================
DirichletRing examples
====================================================================

See Also:
o )show DirichletRing
```

———-

5.12.1 DirichletRing (DIRRING)



Exports:

0	1	?*?	?**?	?+?
?-?	-?	?=?	?^?	?.?
? =?	additive?	associates?	characteristic	coerce
exquo	hash	latex	multiplicative?	one?
recip	sample	subtractIfCan	unit?	unitCanonical
unitNormal	zero?	zeta		

— **domain DIRRING DirichletRing** —

```
)abbrev domain DIRRING DirichletRing
++ Author: Martin Rubey
++ Description: DirichletRing is the ring of arithmetical functions
++ with Dirichlet convolution as multiplication
DirichletRing(Coef: Ring):
            Exports == Implementation where

    PI ==> PositiveInteger
    FUN ==> PI -> Coef

    Exports ==> Join(Ring, Eltable(PI, Coef)) with

      if Coef has CommutativeRing then
          IntegralDomain
```

```
  if Coef has CommutativeRing then
      Algebra Coef

  coerce: FUN -> %
  coerce: % -> FUN
  coerce: Stream Coef -> %
  coerce: % -> Stream Coef

  zeta: constant -> %
    ++ zeta() returns the function which is constantly one

  multiplicative?: (%, PI) -> Boolean
    ++ multiplicative?(a, n) returns true if the first
    ++ n coefficients of a are multiplicative

  additive?: (%, PI) -> Boolean
    ++ additive?(a, n) returns true if the first
    ++ n coefficients of a are additive

Implementation ==> add

    Rep := Record(function: FUN)

    per(f: Rep): % == f pretend %
    rep(a: %): Rep == a pretend Rep

    elt(a: %, n: PI): Coef ==
        f: FUN := (rep a).function
        f n

    coerce(a: %): FUN == (rep a).function

    coerce(f: FUN): % == per [f]

    indices: Stream Integer
            := integers(1)$StreamTaylorSeriesOperations(Integer)

    coerce(a: %): Stream Coef ==
        f: FUN := (rep a).function
        map((n: Integer): Coef +-> f(n::PI), indices)
           $StreamFunctions2(Integer, Coef)

    coerce(f: Stream Coef): % ==
        ((n: PI): Coef +-> f.(n::Integer))::%

    coerce(f: %): OutputForm == f::Stream Coef::OutputForm

    1: % ==
        ((n: PI): Coef +-> (if one? n then 1$Coef else 0$Coef))::%
```

```
0: % ==
    ((n: PI): Coef +-> 0$Coef)::%

zeta: % ==
    ((n: PI): Coef +-> 1$Coef)::%

(f: %) + (g: %) ==
    ((n: PI): Coef +-> f(n)+g(n))::%

- (f: %) ==
    ((n: PI): Coef +-> -f(n))::%

(a: Integer) * (f: %) ==
    ((n: PI): Coef +-> a*f(n))::%

(a: Coef) * (f: %) ==
    ((n: PI): Coef +-> a*f(n))::%

import IntegerNumberTheoryFunctions

(f: %) * (g: %) ==
  conv := (n: PI): Coef +-> _
    reduce((a: Coef, b: Coef): Coef +-> a + b, _
      [f(d::PI) * g((n quo d)::PI) for d in divisors(n::Integer)], 0)
                 $ListFunctions2(Coef, Coef)
  conv::%

unit?(a: %): Boolean == not (recip(a(1$PI))$Coef case "failed")

qrecip: (%, Coef, PI) -> Coef
qrecip(f: %, f1inv: Coef, n: PI): Coef ==
  if one? n then f1inv
  else
      -f1inv * reduce(_+, [f(d::PI) * qrecip(f, f1inv, (n quo d)::PI) _
                           for d in rest divisors(n)], 0) _
                    $ListFunctions2(Coef, Coef)

recip f ==
    if (f1inv := recip(f(1$PI))$Coef) case "failed" then "failed"
    else
        mp := (n: PI): Coef +-> qrecip(f, f1inv, n)

        mp::%::Union(%, "failed")

multiplicative?(a, n) ==
    for i in 2..n repeat
        fl := factors(factor i)$Factored(Integer)
        rl := [a.(((f.factor)::PI)**((f.exponent)::PI)) for f in fl]
        if a.(i::PI) ~= reduce((r:Coef, s:Coef):Coef +-> r*s, rl)
```

```
            then
                output(i::OutputForm)$OutputPackage
                output(rl::OutputForm)$OutputPackage
                return false
        true

    additive?(a, n) ==
        for i in 2..n repeat
            fl := factors(factor i)$Factored(Integer)
            rl := [a.(((f.factor)::PI)**((f.exponent)::PI)) for f in fl]
            if a.(i::PI) ~= reduce((r:Coef, s:Coef):Coef +-> r+s, rl)
            then
                output(i::OutputForm)$OutputPackage
                output(rl::OutputForm)$OutputPackage
                return false
        true
```

———-

— **DIRRING.dotabb** —

```
"DIRRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DIRRING"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"DIRRING" -> "FLAGG"
```

———-

5.13 domain DMP DistributedMultivariatePolynomial

— **DistributedMultivariatePolynomial.input** —

```
)set break resume
)sys rm -f DistributedMultivariatePolynomial.output
)spool DistributedMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
--R
--R                                                              Type: Void
--E 1

--S 2 of 10
```

```
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
--R
--R
--R                 2       2
--R   (2)  - 4z + 4y x + 16x  + 1
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 2

--S 3 of 10
d2 := 2*z*y**2 + 4*x + 1
--R
--R
--R            2
--R   (3)  2z y  + 4x + 1
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 3

--S 4 of 10
d3 := 2*z*x**2 - 2*y**2 - x
--R
--R
--R            2     2
--R   (4)  2z x  - 2y  - x
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 4

--S 5 of 10
groebner [d1,d2,d3]
--R
--R
--R   (5)
--R        1568  6   1264  5    6   4   182  3   2047  2    103      2857
--R   [z - ---- x  - ---- x  + --- x  + --- x  - ---- x  - ---- x - -----,
--R        2745       305      305      549       610      2745     10980
--R     2    112  6    84  5   1264  4    13  3    84  2   1772       2
--R    y  + ---- x  - --- x  - ---- x  - --- x  + --- x  + ---- x + ----,
--R         2745      305       305      549      305      2745     2745
--R     7   29  6   17  4   11  3    1  2   15     1
--R    x  + -- x  - -- x  - -- x  + -- x  + -- x + -]
--R          4      16       8      32      16     4
--R       Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 5

--S 6 of 10
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
--R
--R                                                                   Type: Void
--E 6

--S 7 of 10
```

```
n1 := d1
--R
--R
--R          2       2
--R   (7)  4y x + 16x  - 4z + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 7

--S 8 of 10
n2 := d2
--R
--R
--R            2
--R   (8)  2z y  + 4x + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 8

--S 9 of 10
n3 := d3
--R
--R
--R            2     2
--R   (9)  2z x  - 2y  - x
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 9

--S 10 of 10
groebner [n1,n2,n3]
--R
--R
--R   (10)
--R     4      3   3  2   1     1   4   29  3   1  2   7        9     1
--R   [y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
--R               2      2     8        4      8      4       16     4
--R       2        1   2      2       1     2    2   1
--R    z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
--R                2                  4              2
--R     2     2     2   1     3
--R    z  - 4y  + 2x  - - z - - x]
--R                     4     2
--RType: List HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 10
)spool
)lisp (bye)
```

— DistributedMultivariatePolynomial.help —

```
====================================================================
MultivariatePolynomial
DistributedMultivariatePolynomial
HomogeneousDistributedMultivariatePolynomial
GeneralDistributedMultivariatePolynomial
====================================================================

DistributedMultivariatePolynomial which is abbreviated as DMP and
HomogeneousDistributedMultivariatePolynomial, which is abbreviated
as HDMP, are very similar to MultivariatePolynomial except that
they are represented and displayed in a non-recursive manner.

  (d1,d2,d3) : DMP([z,y,x],FRAC INT)
                    Type: Void

The constructor DMP orders its monomials lexicographically while
HDMP orders them by total order refined by reverse lexicographic
order.

  d1 := -4*z + 4*y**2*x + 16*x**2 + 1
            2       2
   - 4z + 4y x + 16x  + 1
            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  d2 := 2*z*y**2 + 4*x + 1
       2
   2z y  + 4x + 1
            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  d3 := 2*z*x**2 - 2*y**2 - x
       2     2
   2z x  - 2y  - x
            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

These constructors are mostly used in Groebner basis calculations.

  groebner [d1,d2,d3]
        1568  6   1264  5    6   4   182  3   2047  2    103      2857
   [z - ---- x  - ---- x  + --- x  + --- x  - ---- x  - ---- x - -----,
        2745       305      305      549       610      2745     10980
     2    112  6    84  5   1264  4    13  3    84  2   1772       2
    y  + ---- x  - --- x  - ---- x  - --- x  + --- x  + ---- x + ----,
         2745      305       305      549      305      2745     2745
     7    29  6   17  4   11  3    1  2   15     1
    x  + -- x  - -- x  - -- x  + -- x  + -- x + -]
          4      16       8      32      16     4
       Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  (n1,n2,n3) : HDMP([z,y,x],FRAC INT)
                    Type: Void
```

```
  n1 := d1
     2        2
   4y x + 16x  - 4z + 1
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  n2 := d2
       2
   2z y  + 4x + 1
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  n3 := d3
       2     2
   2z x  - 2y  - x
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

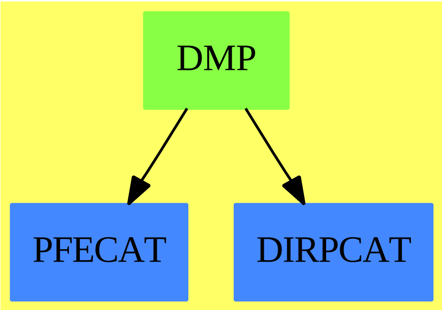
Note that we get a different Groebner basis when we use the HDMP polynomials, as expected.

```
  groebner [n1,n2,n3]
     4     3   3  2   1     1   4   29  3   1  2   7        9     1
   [y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
               2      2     8        4      8      4       16     4
       2        1   2      2       1     2    2   1
    z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
                2                  4              2
     2     2     2   1     3
    z  - 4y  + 2x  - - z - - x]
                     4     2
      Type: List HomogeneousDistributedMultivariatePolynomial([z,y,x],
                                                       Fraction Integer)
```

GeneralDistributedMultivariatePolynomial is somewhat more flexible in the sense that as well as accepting a list of variables to specify the variable ordering, it also takes a predicate on exponent vectors to specify the term ordering. With this polynomial type the user can experiment with the effect of using completely arbitrary term orderings. This flexibility is mostly important for algorithms such as Groebner basis calculations which can be very sensitive to term ordering.

See Also:
o)help Polynomial
o)help UnivariatePolynomial
o)help MultivariatePolynomial
o)help HomogeneousDistributedMultivariatePolynomial
o)help GeneralDistributedMultivariatePolynomial
o)show DistributedMultivariatePolynomial

5.13.1 DistributedMultivariatePolynomial (DMP)



See

⇒ "GeneralDistributedMultivariatePolynomial" (GDMP) 8.1.1 on page 1018
⇒ "HomogeneousDistributedMultivariatePolynomial" (HDMP) 9.10.1 on page 1145

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	convert	D
degree	differentiate	discriminant
eval	exquo	factor
factorPolynomial	factorSquareFreePolynomial	gcd
gcdPolynomial	ground	ground?
hash	isExpt	isPlus
isTimes	latex	lcm
leadingCoefficient	leadingMonomial	mainVariable
map	mapExponents	max
min	minimumDegree	monicDivide
monomial	monomial?	monomials
multivariate	numberOfMonomials	one?
patternMatch	pomopo!	prime?
primitiveMonomials	primitivePart	recip
reducedSystem	reductum	resultant
retract	retractIfCan	reorder
retract	solveLinearPolynomialEquation	sample
squareFree	squareFreePolynomial	squareFreePart
subtractIfCan	totalDegree	unit?
unitCanonical	unitNormal	univariate
variables	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?/?	?<?
?<=?	?>?	?>=?

— domain DMP DistributedMultivariatePolynomial —

```
)abbrev domain DMP DistributedMultivariatePolynomial
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd, leadingCoefficient
++ Related Constructors: GeneralDistributedMultivariatePolynomial,
++ HomogeneousDistributedMultivariatePolynomial
++ Also See: Polynomial
++ AMS Classifications:
++ Keywords: polynomial, multivariate, distributed
++ References:
++ Description:
++ This type supports distributed multivariate polynomials
++ whose variables are from a user specified list of symbols.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.
```

```
++ The term ordering is lexicographic specified by the variable
++ list parameter with the most significant variable first in the list.

DistributedMultivariatePolynomial(vl,R): public == private where
  vl : List Symbol
  R  : Ring
  E   ==> DirectProduct(#vl,NonNegativeInteger)
  OV  ==> OrderedVariableList(vl)
  public == PolynomialCategory(R,E,OV) with
      reorder: (%,List Integer) -> %
        ++ reorder(p, perm) applies the permutation perm to the variables
        ++ in a polynomial and returns the new correctly ordered polynomial

  private ==
    GeneralDistributedMultivariatePolynomial(vl,R,E)
```

— **DMP.dotabb** —

```
"DMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DMP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"DMP" -> "PFECAT"
"DMP" -> "DIRPCAT"
```

5.14 domain DIV Divisor

— **Divisor.input** —

```
)set break resume
)sys rm -f Divisor.output
)spool Divisor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Divisor
--R Divisor S: SetCategoryWithDegree  is a domain constructor
--R Abbreviation for Divisor is DIV
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DIV
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,S) -> %                ?*? : (Integer,%) -> %
--R ?*? : (%,Integer) -> %                ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (S,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coefficient : (S,%) -> Integer        coerce : S -> %
--R coerce : % -> OutputForm              collect : % -> %
--R concat : (%,%) -> %                   degree : % -> Integer
--R divOfPole : % -> %                    divOfZero : % -> %
--R effective? : % -> Boolean             hash : % -> SingleInteger
--R incr : % -> %                         latex : % -> String
--R mapGen : ((S -> S),%) -> %            nthCoef : (%,Integer) -> Integer
--R nthFactor : (%,Integer) -> S          reductum : % -> %
--R retract : % -> S                      sample : () -> %
--R size : % -> NonNegativeInteger        split : % -> List %
--R supp : % -> List S                    suppOfPole : % -> List S
--R suppOfZero : % -> List S              zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R head : % -> Record(gen: S,exp: Integer)
--R highCommonTerms : (%,%) -> % if Integer has OAMON
--R mapCoef : ((Integer -> Integer),%) -> %
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(gen: S,exp: Integer)
--R
--E 1

)spool
)lisp (bye)
```

———

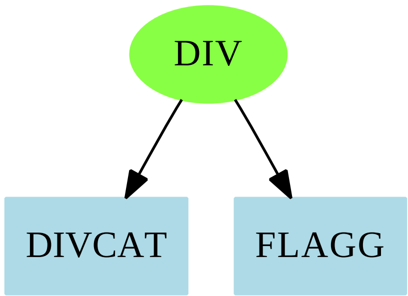
— **Divisor.help** —

```
====================================================================
Divisor examples
====================================================================

See Also:
o )show Divisor
```

———

5.14.1 Divisor (DIV)



Exports:

0	-?	?*?	?+?
?-?	?<=?	?=?	?~=?
coefficient	coerce	collect	concat
degree	divOfPole	divOfZero	effective?
hash	head	highCommonTerms	incr
latex	mapCoef	mapGen	nthCoef
nthFactor	reductum	retract	retractIfCan
sample	size	split	subtractIfCan
supp	suppOfPole	suppOfZero	terms
zero?			

— domain DIV Divisor —

```
)abbrev domain DIV Divisor
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
Divisor(S:SetCategoryWithDegree):Exports == Implementation where

  PT      ==> Record(gen:S,exp: Integer)
  INT     ==> Integer
  BOOLEAN ==> Boolean
  LIST    ==> List

  Exports == DivisorCategory(S) with

    head: % -> PT

    reductum: % -> %
```

```
Implementation == List PT add

  Rep := List PT

  incr(d)==
    [ [ pt.gen , pt.exp + 1 ] for pt in d ]

  inOut: PT -> OutputForm

  inOut(pp)==
    one?(pp.exp) => pp.gen :: OutputForm
    bl:OutputForm:= " " ::OutputForm
    (pp.exp :: OutputForm) * hconcat(bl,pp.gen :: OutputForm)

  coerce(d:%):OutputForm==
    zero?(d) => ("0"::OutputForm)
    ll:List OutputForm:=[inOut df  for df in d]
    reduce("+",ll)

  reductum(d)==
    zero?(d) => d
    dl:Rep:= d pretend Rep
    dlr := rest dl
    empty?(dlr) => 0
    dlr

  head(d)==
    zero?(d) => error "Cannot take head of zero"
    dl:Rep:= d pretend Rep
    first dl

  coerce(s:S) == [[s,1]$PT]::%

  split(a) ==
    zero?(a) => []
    [[r]::% for r in a]

  coefficient(s,a)==
    r:INT:=0
    for pt in terms(a) repeat
      if pt.gen=s then
        r:=pt.exp
    r

  terms(a)==a::Rep

  0==empty()$Rep

  supp(a)==
```

```
    aa:=terms(collect(a))
    [p.gen for p in aa | ^zero?(p.exp)]

  suppOfZero(a)==
    aa:=terms(collect(a))
    [p.gen for p in aa | (p.exp) > 0 ]

  suppOfPole(a)==
    aa:=terms(collect(a))
    [p.gen for p in aa | p.exp < 0 ]

  divOfZero(a)==
    aa:=terms(collect(a))
    [p for p in aa | (p.exp) > 0 ]::%

  divOfPole(a)==
    aa:=terms(collect(a))
    [p for p in aa | p.exp < 0 ]::%

  zero?(a)==
    ((collect(a)::Rep)=empty()$Rep)::BOOLEAN

  collect(d)==
    a:=d::Rep
    empty?(a) => 0
    t:Rep:=empty()
    ff:PT:=first(a)
    one?(#(a)) =>
      if zero?(ff.exp) then
        t::%
      else
        a::%
    inList?:Boolean:=false()
    newC:INT
    restred:=terms(collect((rest(a)::%)))
    zero?(ff.exp) =>
      restred::%
    for bb in restred repeat
      b:=bb::PT
      if b.gen=ff.gen then
        newC:=b.exp+ff.exp
        if ^zero?(newC) then
          t:=concat(t,[b.gen,newC]$PT)
        inList?:=true()
      else
        t:=concat(t,b)
    if ^inList? then
      t:=cons(ff,t)
    t::%
```

```
a:% + b:% ==
  collect(concat(a pretend Rep,b pretend Rep))

a:% - b:% ==
  a + (-1)*b

-a:% == (-1)*a

n:INT * a:% ==
  zero?(n) => 0
  t:Rep:=empty()
  for p in a pretend Rep repeat
    t:=concat(t,[ p.gen, n*p.exp]$PT)
  t::%

a:% <= b:% ==
  bma:= b - a
  effective? bma => true
  false

effective?(a)== empty?(suppOfPole(a))

degree(d:%):Integer==
  reduce("+",[(p.exp * degree(p.gen)) for p in d @ Rep],0$INT)
```

————

— DIV.dotabb —

```
"DIV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DIV",shape=ellipse]
"DIVCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVCAT"];
"FLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"DIV" -> "DIVCAT"
"DIV" -> "FLAGG"
```

————

5.15 domain DFLOAT DoubleFloat

Greg Vanuxem has added some functionality to allow the user to modify the printed format of floating point numbers. The format of the numbers follows the common lisp format specification for floats. First we include Greg's email to show the use of this feature:

```
PS: For those who use the Doublefloat domain
    there is an another (undocumented) patch that adds a
    lisp format to the DoubleFloat output routine. Copy
```

```
    int/algebra/DFLOAT.spad to your working directory,
    patch it, compile it and ")lib" it when necessary.


(1) -> )boot $useBFasDefault:=false

(SPADLET |$useBFasDefault| NIL)
Value = NIL
(1) -> a:= matrix [ [0.5978,0.2356], [0.4512,0.2355] ]

        +      0.5978              0.2356      +
   (1)  |                                      |
        +0.45119999999999999  0.23549999999999999+
                               Type: Matrix DoubleFloat
(2) -> )lib DFLOAT
   DoubleFloat is now explicitly exposed in frame initial
   DoubleFloat will be automatically loaded when needed
from /home/greg/Axiom/DFLOAT.nrlib/code
(2) -> doubleFloatFormat("~,4,,F")

   (2)  "~G"
                               Type: String
(3) -> a

        +0.5978  0.2356+
   (3)  |              |
        +0.4512  0.2355+
                               Type: Matrix DoubleFloat
```

So it is clear that he has added a new function called `doubleFloatFormat` which takes a string argument that specifies the common lisp format control string (¨~,4,,F¨). For reference we quote from the common lisp manual [**?**]. On page 582 we find:

> A format directive consists of a tilde (~), optional prefix parameters separated by commas, optional colon (:) and at-sign (@) modifiers, and a single character indicating what kind of directive this is. The alphabetic case of the directive character is ignored. The prefix parameters are generally integers, notated as optionally signed decimal numbers.
>
> X3J13 voted in June 1987 (80) to specify that if both colon and at-sign modifiers are present, they may appear in either order; thus ~:@R and ~@:R mean the same thing. However, it is traditional to put the colon first, and all examples in the book put colon before at-signs.

On page 588 we find:

> ~F

Fixed-format floating-point. The next *arg* is printed as a floating point number.

The full form is ~*w,d,k,overfowchar,padchar*F. The parameter *w* is the width of the filed to be printed; *d* is the number of digits to print after the decimal point; *k* is a scale factor that defaults to zero.

Exactly *w* characters will be output. First, leading copies of the character *padchar* (which defaults to a space) are printed, if necessary, to pad the field on the left. If the *arg* is negative, then a minus sign is printed; if the *arg* is not negative, then a plus signed is printed if and only if the @ modifier was specified. Then a sequence of digits, containing a single embedded decimal point, is printed; this represents the magnitude of the value of *arg* times 10^k, rounded to *d* fractional digits. (When rounding up and rounding down would produce printed values equidistant from the scaled value of *arg*, then the implementation is free to use either one. For example, printing the argument 6.375 using the format ~4.2F may correctly produce either 6.37 or 6.38.) Leading zeros are not permitted, except that a single zero digit is output before the decimal point if the printed value is less than 1, and this single zero digit is not output after all if $w = d + 1$.

If it is impossible to print the value in the required format in the field of width *w*, then one of two actions is taken. If the parameter *overflowchar* is specified, then *w* copies of that parameter are printed instead of the scaled value of *arg*. If the *overflowchar* parameter is omitted, then the scaled value is printed using more than *w* characters, as many more as may be needed.

If the *w* parameter is omitted, then the field is of variable width. In effect, a value is chosen for *w* in such a way that no leading pad characters need to be printed and exactly *d* characters will follow the decimal point. For example, the directive ~,2F will print exactly two digits after the decimal point and as many as necessary before the decimal point.

If the parameter *d* is omitted, then there is no constraint on the number of digits to appear after the decimal point. A value is chosen for *d* in such a way that as many digits as possible may be printed subject to the width constraint imposed by the parameter *w* and the constraint that no trailing zero digits may appear in the fraction, except that if the fraction to be printed is zero, then a single zero digit should appear after the decimal point if permitted by the width constraint.

If both *w* and *d* are omitted, then the effect is to print the value using ordinary free-format output; `prin1` uses this format for any number whose magnitude is either zero or between 10^{-3} (inclusive) and 10^7 (exclusive).

If *w* is omitted, then if the magnitude of *arg* is so large (or, if *d* is also omitted, so small) that more than 100 digits would have to be printed, then an implementation is free, at its discretion, to print the number using exponential notation instead, as if by the directive ~E (with all parameters of ~E defaulted, not taking their valued from the ~F directive).

If *arg* is a rational number, then it is coerced to be a `single-float` and then printed. (Alternatively, an implementation is permitted to process a rational number by any other method that has essentially the same behavior but avoids such hazards as loss of precision or overflow because of the coercion. However, note that if *w* and *d* are unspecified and the number has no exact decimal representation, for example 1/3, some precision cutoff must be chosen by the implementation; only a finite number of digits may be printed.)

If *arg* is a complex number or some non-numeric object, then it is printed using the format directive ˜*w*D, thereby printing it in decimal radix and a minimum field width of *w*. (If it is desired to print each of the real part and imaginary part of a complex number using a ˜F directive, then this must be done explicitly with two ˜F directives and code to extract the two parts of the complex number.)

A key difficulty is creating test suites which generate DoubleFloats. We need to be bit-exact in the results. This can be achieved using the common lisp `integer-decode-float` function. We cover that function in two different ways, with `machineFraction` which translates the results into an Axiom **Fraction(Integer)** and `integerDecode` which just returns a list of the multiple values from the `integer-decode-float` function.

— DoubleFloat.input —

```
)set break resume
)sys rm -f DoubleFloat.output
)spool DoubleFloat.output
)set message test on
)set message auto off
)clear all

--S 1 of 13
2.71828
--R
--R
--R   (1)  2.71828
--R                                                                Type: Float
--E 1

--S 2 of 13
2.71828@DoubleFloat
--R
--R
--R   (2)  2.71828
--R                                                          Type: DoubleFloat
--E 2

--S 3 of 13
2.71828 :: DoubleFloat
--R
--R
--R   (3)  2.71828
--R                                                          Type: DoubleFloat
--E 3
```

```
--S 4 of 13
eApprox : DoubleFloat := 2.71828
--R
--R
--R   (4)  2.71828
--R                                                          Type: DoubleFloat
--E 4

--S 5 of 13
avg : List DoubleFloat -> DoubleFloat
--R
--R                                                                 Type: Void
--E 5

--S 6 of 13
avg l ==
  empty? l => 0 :: DoubleFloat
  reduce(_+,l) / #l
--R
--R                                                                 Type: Void
--E 6

--S 7 of 13
avg []
--R
--R   Compiling function avg with type List DoubleFloat -> DoubleFloat
--R
--R   (7)  0.
--R                                                          Type: DoubleFloat
--E 7

--S 8 of 13
avg [3.4,9.7,-6.8]
--R
--R
--R   (8)  2.0999999999999996
--R                                                          Type: DoubleFloat
--E 8

--S 9 of 13
cos(3.1415926)$DoubleFloat
--R
--R
--R   (9)  - 0.99999999999999856
--R                                                          Type: DoubleFloat
--E 9

--S 10 of 13
cos(3.1415926 :: DoubleFloat)
```

```
--R
--R
--R   (10)  - 0.99999999999999856
--R                                                            Type: DoubleFloat
--E 10

--S 11 of 13
a:DFLOAT := -1.0/3.0
--R
--R
--R   (11)  - 0.33333333333333337
--R                                                            Type: DoubleFloat
--E 11

--S 12 of 13
integerDecode a
--R
--R
--R   (12)  [6004799503160662,- 54,- 1]
--R                                                           Type: List Integer
--E 12

--S 13 of 13
machineFraction a
--R
--R
--R           3002399751580331
--R   (13)  - ----------------
--R           9007199254740992
--R                                                       Type: Fraction Integer
--E 13

)spool
)lisp (bye)
```

— DoubleFloat.help —

```
====================================================================
DoubleFloat examples
====================================================================

Axiom provides two kinds of floating point numbers.  The domain Float
(abbreviation FLOAT) implements a model of arbitrary precision
floating point numbers.  The domain DoubleFloat (abbreviation DFLOAT)
is intended to make available hardware floating point arithmetic in
Axiom.  The actual model of floating point DoubleFloat that provides
```

```
is system-dependent.  For example, on the IBM system 370 Axiom uses
IBM double precision which has fourteen hexadecimal digits of
precision or roughly sixteen decimal digits.  Arbitrary precision
floats allow the user to specify the precision at which arithmetic
operations are computed.  Although this is an attractive facility, it
comes at a cost. Arbitrary-precision floating-point arithmetic
typically takes twenty to two hundred times more time than hardware
floating point.

The usual arithmetic and elementary functions are available for
DoubleFloat.  By default, floating point numbers that you enter into
Axiom are of type Float.

  2.71828
   2.71828
                      Type: Float

You must therefore tell Axiom that you want to use DoubleFloat values
and operations.  The following are some conservative guidelines for
getting Axiom to use DoubleFloat.

To get a value of type DoubleFloat, use a target with @, ...

  2.71828@DoubleFloat
   2.71828
                      Type: DoubleFloat

a conversion, ...

  2.71828 :: DoubleFloat
   2.71828
                      Type: DoubleFloat

or an assignment to a declared variable.  It is more efficient if you
use a target rather than an explicit or implicit conversion.

  eApprox : DoubleFloat := 2.71828
   2.71828
                      Type: DoubleFloat

You also need to declare functions that work with DoubleFloat.

  avg : List DoubleFloat -> DoubleFloat
                      Type: Void

  avg l ==
    empty? l => 0 :: DoubleFloat
    reduce(_+,l) / #l
                      Type: Void
```

```
  avg []
   0.
                      Type: DoubleFloat

  avg [3.4,9.7,-6.8]
   2.1000000000000001
                      Type: DoubleFloat
```

Use package-calling for operations from DoubleFloat unless the
arguments themselves are already of type DoubleFloat.

```
  cos(3.1415926)$DoubleFloat
   -0.99999999999999856
                      Type: DoubleFloat

  cos(3.1415926 :: DoubleFloat)
   -0.99999999999999856
                      Type: DoubleFloat
```

By far, the most common usage of DoubleFloat is for functions to be
graphed.

You can get the exact machine-specific bits of a DoubleFloat in two ways.
The first is to use the integerDecode function to break the DoubleFloat
into components.

```
  a := -1.0/3.0

    -0.33333333333333331

  integerDecode a

    [6004799503160661,- 54,- 1]
```

This is the mantissa, exact to 54 bits, the power of 2, and the sign.
Thus it is:

```
    6004799503160661 * 2^-54
```

You can get the same information as a fraction with

```
   machineFraction a

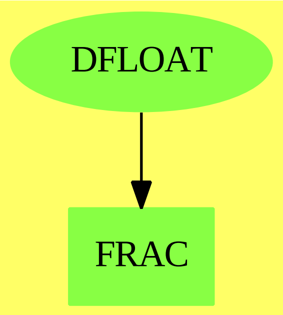
     6004799503160661
  - -----------------
    18014398509481984
```

where the denominator 18014398509481984 is 2^54

```
See Also:
o )help Float
o )show DoubleFloat
```

———-

5.15.1 DoubleFloat (DFLOAT)



Exports:

0	1	abs	acos
acosh	acot	acoth	acsc
acsch	airyAi	airyBi	asec
asech	asin	asinh	associates?
atan	atanh	base	besselI
besselJ	besselK	besselY	Beta
bits	ceiling	characteristic	coerce
convert	cos	cosh	cot
coth	csc	csch	D
decreasePrecision	differentiate	digamma	digits
divide	doubleFloatFormat	euclideanSize	exp
expressIdealMember	exp1	exponent	exquo
extendedEuclidean	factor	float	floor
fractionPart	Gamma	gcd	gcdPolynomial
hash	increasePrecision	integerDecode	inv
latex	lcm	log	log10
log2	machineFraction	mantissa	max
min	multiEuclidean	negative?	norm
nthRoot	OMwrite	one?	order
patternMatch	pi	polygamma	positive?
precision	prime?	principalIdeal	rationalApproximation
recip	retract	retractIfCan	round
sample	sec	sech	sign
sin	sinh	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	tan
tanh	truncate	unit?	unitCanonical
unitNormal	wholePart	zero?	?*?
?**?	?+?	?-?	-?
?/?	?<?	?<=?	?=?
?>?	?>=?	?^?	?quo?
?rem?	?~=?		

— domain DFLOAT DoubleFloat —

```
)abbrev domain DFLOAT DoubleFloat
++ Author: Michael Monagan
++ Date Created:
++   January 1988
++ Change History:
++ Basic Operations: exp1, hash, log2, log10, rationalApproximation, / , **
++ Related Constructors:
++ Keywords: small float
++ Description:
++ \spadtype{DoubleFloat} is intended to make accessible
++ hardware floating point arithmetic in Axiom, either native double
++ precision, or IEEE. On most machines, there will be hardware support for
```

```
++ the arithmetic operations: ++ +, *, / and possibly also the
++ sqrt operation.
++ The operations exp, log, sin, cos, atan are normally coded in
++ software based on minimax polynomial/rational approximations.
++
++ Some general comments about the accuracy of the operations:
++ the operations +, *, / and sqrt are expected to be fully accurate.
++ The operations exp, log, sin, cos and atan are not expected to be
++ fully accurate.  In particular, sin and cos
++ will lose all precision for large arguments.
++
++ The Float domain provides an alternative to the DoubleFloat domain.
++ It provides an arbitrary precision model of floating point arithmetic.
++ This means that accuracy problems like those above are eliminated
++ by increasing the working precision where necessary.  \spadtype{Float}
++ provides some special functions such as erf, the error function
++ in addition to the elementary functions.  The disadvantage of Float is that
++ it is much more expensive than small floats when the latter can be used.

DoubleFloat(): Join(FloatingPointSystem, DifferentialRing, OpenMath,
   TranscendentalFunctionCategory, SpecialFunctionCategory, _
   ConvertibleTo InputForm) with
      _/   : (%, Integer) -> %
        ++ x / i computes the division from x by an integer i.
      _*_* : (%,%) -> %
        ++ x ** y returns the yth power of x (equal to \spad{exp(y log x)}).
      exp1 : () -> %
        ++ exp1() returns the natural log base \spad{2.718281828...}.
      hash : % -> Integer
        ++ hash(x) returns the hash key for x
      log2 :  % -> %
        ++ log2(x) computes the logarithm with base 2 for x.
      log10: % -> %
        ++ log10(x) computes the logarithm with base 10 for x.
      atan : (%,%) -> %
        ++ atan(x,y) computes the arc tangent from x with phase y.
      Gamma: % -> %
        ++ Gamma(x) is the Euler Gamma function.
      Beta : (%,%) -> %
        ++ Beta(x,y) is \spad{Gamma(x) * Gamma(y)/Gamma(x+y)}.
      doubleFloatFormat : String -> String
        ++ change the output format for doublefloats using lisp format strings
      rationalApproximation: (%, NonNegativeInteger) -> Fraction Integer
        ++ rationalApproximation(f, n) computes a rational approximation
        ++ r to f with relative error \spad{< 10**(-n)}.
      rationalApproximation: (%, NonNegativeInteger, NonNegativeInteger) -> _
           Fraction Integer
        ++ rationalApproximation(f, n, b) computes a rational
        ++ approximation r to f with relative error \spad{< b**(-n)}
        ++ (that is, \spad{|(r-f)/f| < b**(-n)}).
```

```
    machineFraction : % -> Fraction Integer
      ++ machineFraction(x) returns a bit-exact fraction of the machine
      ++ floating point number using the common lisp integer-decode-float
      ++ function. See Steele, ISBN 0-13-152414-3 p354
      ++ This function can be used to print results which do not depend
      ++ on binary-to-decimal conversions
      ++
      ++X a:DFLOAT:=-1.0/3.0
      ++X machineFraction a
    integerDecode : % -> List Integer
      ++ integerDecode(x) returns the multiple values of the  common
      ++ lisp integer-decode-float function.
      ++ See Steele, ISBN 0-13-152414-3 p354. This function can be used
      ++ to ensure that the results are bit-exact and do not depend on
      ++ the binary-to-decimal conversions.
      ++
      ++X a:DFLOAT:=-1.0/3.0
      ++X integerDecode a

== add
  format: String := "~G"
  MER ==> Record(MANTISSA:Integer,EXPONENT:Integer)

  manexp: % -> MER

  doubleFloatFormat(s:String): String ==
    ss: String := format
    format := s
    ss

  OMwrite(x: %): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    OMputObject(dev)
    OMputFloat(dev, convert x)
    OMputEndObject(dev)
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

  OMwrite(x: %, wholeObj: Boolean): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    if wholeObj then
      OMputObject(dev)
    OMputFloat(dev, convert x)
    if wholeObj then
      OMputEndObject(dev)
```

```
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  OMputFloat(dev, convert x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
  OMputFloat(dev, convert x)
  if wholeObj then
    OMputEndObject(dev)

checkComplex(x:%):% == C_-TO_-R(x)$Lisp
-- In AKCL we used to have to make the arguments to ASIN ACOS ACOSH ATANH
-- complex to get the correct behaviour.
--makeComplex(x: %):% == COMPLEX(x, 0$%)$Lisp

machineFraction(df:%):Fraction(Integer) ==
  numer:Integer:=INTEGER_-DECODE_-FLOAT_-NUMERATOR(df)$Lisp
  denom:Integer:=INTEGER_-DECODE_-FLOAT_-DENOMINATOR(df)$Lisp
  sign:Integer:=INTEGER_-DECODE_-FLOAT_-SIGN(df)$Lisp
  sign*numer/denom

integerDecode(df:%):List(Integer) ==
  numer:Integer:=INTEGER_-DECODE_-FLOAT_-NUMERATOR(df)$Lisp
  exp:Integer:=INTEGER_-DECODE_-FLOAT_-EXPONENT(df)$Lisp
  sign:Integer:=INTEGER_-DECODE_-FLOAT_-SIGN(df)$Lisp
  [numer,exp,sign]

base()           == FLOAT_-RADIX(0$%)$Lisp
mantissa x       == manexp(x).MANTISSA
exponent x       == manexp(x).EXPONENT
precision()      == FLOAT_-DIGITS(0$%)$Lisp
bits()           ==
  base() = 2 => precision()
  base() = 16 => 4*precision()
  wholePart(precision()*log2(base()::%))::PositiveInteger
max()            == MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp
min()            == MOST_-NEGATIVE_-DOUBLE_-FLOAT$Lisp
order(a) == precision() + exponent a - 1
0                == FLOAT(0$Lisp,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
1                == FLOAT(1$Lisp,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
-- rational approximation to e accurate to 23 digits
exp1()  == FLOAT(534625820200,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp / _
           FLOAT(196677847971,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
pi()    == FLOAT(PI$Lisp,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
```

```
coerce(x:%):OutputForm ==
  x >= 0 => message(FORMAT(NIL$Lisp,format,x)$Lisp pretend String)
  - (message(FORMAT(NIL$Lisp,format,-x)$Lisp pretend String))
convert(x:%):InputForm == convert(x pretend DoubleFloat)$InputForm
x < y            == DFLESSTHAN(x,y)$Lisp
- x              == DFUNARYMINUS(x)$Lisp
x + y            == DFADD(x,y)$Lisp
x:% - y:%        == DFSUBTRACT(x,y)$Lisp
x:% * y:%        == DFMULTIPLY(x,y)$Lisp
i:Integer * x:%  == DFINTEGERMULTIPLY(i,x)$Lisp
max(x,y)         == DFMAX(x,y)$Lisp
min(x,y)         == DFMIN(x,y)$Lisp
x = y            == DFEQL(x,y)$Lisp
x:% / i:Integer  == DFINTEGERDIVIDE(x,i)$Lisp
sqrt x           == checkComplex DFSQRT(x)$Lisp
log10 x          == checkComplex DFLOG(x,10)$Lisp
x:% ** i:Integer == DFINTEGEREXPT(x,i)$Lisp
x:% ** y:%       == checkComplex DFEXPT(x,y)$Lisp
coerce(i:Integer):% == FLOAT(i,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
exp x            == DFEXP(x)$Lisp
log x            == checkComplex DFLOGE(x)$Lisp
log2 x           == checkComplex DFLOG(x,2)$Lisp
sin x            == DFSIN(x)$Lisp
cos x            == DFCOS(x)$Lisp
tan x            == DFTAN(x)$Lisp
cot x            == COT(x)$Lisp
sec x            == SEC(x)$Lisp
csc x            == CSC(x)$Lisp
asin x           == checkComplex DFASIN(x)$Lisp -- can be complex
acos x           == checkComplex DFACOS(x)$Lisp -- can be complex
atan x           == DFATAN(x)$Lisp
acsc x           == checkComplex ACSC(x)$Lisp
acot x           == ACOT(x)$Lisp
asec x           == checkComplex ASEC(x)$Lisp
sinh x           == SINH(x)$Lisp
cosh x           == COSH(x)$Lisp
tanh x           == TANH(x)$Lisp
csch x           == CSCH(x)$Lisp
coth x           == COTH(x)$Lisp
sech x           == SECH(x)$Lisp
asinh x          == DFASINH(x)$Lisp
acosh x          == checkComplex DFACOSH(x)$Lisp -- can be complex
atanh x          == checkComplex DFATANH(x)$Lisp -- can be complex
acsch x          == ACSCH(x)$Lisp
acoth x          == checkComplex ACOTH(x)$Lisp
asech x          == checkComplex ASECH(x)$Lisp
x:% / y:%        == DFDIVIDE(x,y)$Lisp
negative? x      == DFMINUSP(x)$Lisp
zero? x          == ZEROP(x)$Lisp
hash x           == HASHEQ(x)$Lisp
```

```
   recip(x)         == (zero? x => "failed"; 1 / x)
   differentiate x  == 0

   SFSFUN           ==> DoubleFloatSpecialFunctions()
   sfx              ==> x pretend DoubleFloat
   sfy              ==> y pretend DoubleFloat
   airyAi x         == airyAi(sfx)$SFSFUN pretend %
   airyBi x         == airyBi(sfx)$SFSFUN pretend %
   besselI(x,y)     == besselI(sfx,sfy)$SFSFUN pretend %
   besselJ(x,y)     == besselJ(sfx,sfy)$SFSFUN pretend %
   besselK(x,y)     == besselK(sfx,sfy)$SFSFUN pretend %
   besselY(x,y)     == besselY(sfx,sfy)$SFSFUN pretend %
   Beta(x,y)        == Beta(sfx,sfy)$SFSFUN pretend %
   digamma x        == digamma(sfx)$SFSFUN pretend %
   Gamma x          == Gamma(sfx)$SFSFUN pretend %
-- not implemented in SFSFUN
--   Gamma(x,y)       == Gamma(sfx,sfy)$SFSFUN pretend %
   polygamma(x,y)   ==
       if (n := retractIfCan(x:%):Union(Integer, "failed")) case Integer _
          and n >= 0
       then polygamma(n::Integer::NonNegativeInteger,sfy)$SFSFUN pretend %
       else error "polygamma: first argument should be a nonnegative integer"

   wholePart x            == TRUNCATE(x)$Lisp
   float(ma,ex,b)   == ma*(b::%)**ex
   convert(x:%):DoubleFloat == x pretend DoubleFloat
   convert(x:%):Float == convert(x pretend DoubleFloat)$Float
   rationalApproximation(x, d) == rationalApproximation(x, d, 10)

   atan(x,y) ==
      x = 0 =>
         y > 0 => pi()/2
         y < 0 => -pi()/2
         0
      -- Only count on first quadrant being on principal branch.
      theta := atan abs(y/x)
      if x < 0 then theta := pi() - theta
      if y < 0 then theta := - theta
      theta

   retract(x:%):Fraction(Integer) ==
     rationalApproximation(x, (precision() - 1)::NonNegativeInteger, base())

   retractIfCan(x:%):Union(Fraction Integer, "failed") ==
     rationalApproximation(x, (precision() - 1)::NonNegativeInteger, base())

   retract(x:%):Integer ==
     x = ((n := wholePart x)::%) => n
     error "Not an integer"
```

```
   retractIfCan(x:%):Union(Integer, "failed") ==
     x = ((n := wholePart x)::%) => n
     "failed"

   sign(x) == retract FLOAT_-SIGN(x,1)$Lisp

   abs x   == FLOAT_-SIGN(1,x)$Lisp

   manexp(x) ==
      zero? x => [0,0]
      s := sign x; x := abs x
      if x > max()$% then return [s*mantissa(max())+1,exponent max()]
      me:Record(man:%,exp:Integer) := MANEXP(x)$Lisp
      two53:= base()**precision()
      [s*wholePart(two53 * me.man ),me.exp-precision()]

-- rationalApproximation(y,d,b) ==
--    this is the quotient remainder algorithm (requires wholePart operation)
--    x := y
--    if b < 2 then error "base must be > 1"
--    tol := (b::%)**d
--    p0,p1,q0,q1 : Integer
--    p0 := 0; p1 := 1; q0 := 1; q1 := 0
--    repeat
--       a := wholePart x
--       x := fractionPart x
--       p2 := p0+a*p1
--       q2 := q0+a*q1
--       if x = 0 or tol*abs(q2*y-(p2::%)) < abs(q2*y) then
--          return (p2/q2)
--       (p0,p1) := (p1,p2)
--       (q0,q1) := (q1,q2)
--       x := 1/x

   rationalApproximation(f,d,b) ==
      -- this algorithm expresses f as n / d where d = BASE ** k
      -- then all arithmetic operations are done over the integers
      (nu, ex) := manexp f
      BASE := base()
      ex >= 0 => (nu * BASE ** (ex::NonNegativeInteger))::Fraction(Integer)
      de :Integer := BASE**((-ex)::NonNegativeInteger)
      b < 2 => error "base must be > 1"
      tol := b**d
      s := nu; t := de
      p0:Integer := 0; p1:Integer := 1; q0:Integer := 1; q1:Integer := 0
      repeat
         (q,r) := divide(s, t)
         p2 := q*p1+p0
         q2 := q*q1+q0
         r = 0 or tol*abs(nu*q2-de*p2) < de*abs(p2) => return(p2/q2)
```

```
        (p0,p1) := (p1,p2)
        (q0,q1) := (q1,q2)
        (s,t) := (t,r)

   x:% ** r:Fraction Integer ==
      zero? x =>
         zero? r => error "0**0 is undefined"
         negative? r => error "division by 0"
         0
--     zero? r or one? x => 1
      zero? r or (x = 1) => 1
--     one?  r => x
      (r = 1) => x
      n := numer r
      d := denom r
      negative? x =>
         odd? d =>
            odd? n => return -((-x)**r)
            return ((-x)**r)
         error "negative root"
      d = 2 => sqrt(x) ** n
      x ** (n::% / d::%)
```

———-

— **DFLOAT.dotabb** —

```
"DFLOAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DFLOAT",
          shape=ellipse]
"FRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRAC"]
"DFLOAT" -> "FRAC"
```

———-

5.16 domain DFMAT DoubleFloatMatrix

— **DoubleFloatMatrix.input** —

```
)set break resume
)sys rm -f DoubleFloatMatrix.output
)spool DoubleFloatMatrix.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 6
)show DoubleFloatMatrix
--R DoubleFloatMatrix  is a domain constructor
--R Abbreviation for DoubleFloatMatrix is DFMAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DFMAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %               ?*? : (%,DoubleFloat) -> %
--R ?*? : (DoubleFloat,%) -> %           ?*? : (%,%) -> %
--R ?+? : (%,%) -> %                     -? : % -> %
--R ?-? : (%,%) -> %                     antisymmetric? : % -> Boolean
--R coerce : DoubleFloatVector -> %      copy : % -> %
--R diagonal? : % -> Boolean             diagonalMatrix : List % -> %
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               fill! : (%,DoubleFloat) -> %
--R horizConcat : (%,%) -> %             maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer           minColIndex : % -> Integer
--R minRowIndex : % -> Integer           ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger      parts : % -> List DoubleFloat
--R qnew : (Integer,Integer) -> %        sample : () -> %
--R square? : % -> Boolean               squareTop : % -> %
--R symmetric? : % -> Boolean            transpose : % -> %
--R vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (DoubleFloatVector,%) -> DoubleFloatVector
--R ?*? : (%,DoubleFloatVector) -> DoubleFloatVector
--R ?**? : (%,Integer) -> % if DoubleFloat has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,DoubleFloat) -> % if DoubleFloat has FIELD
--R ?=? : (%,%) -> Boolean if DoubleFloat has SETCAT
--R any? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if DoubleFloat has SETCAT
--R column : (%,Integer) -> DoubleFloatVector
--R columnSpace : % -> List DoubleFloatVector if DoubleFloat has EUCDOM
--R count : (DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and DoubleFloat has SETCAT
--R count : ((DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> DoubleFloat if DoubleFloat has commutative *
--R diagonalMatrix : List DoubleFloat -> %
--R elt : (%,List Integer,List Integer) -> %
--R elt : (%,Integer,Integer,DoubleFloat) -> DoubleFloat
--R elt : (%,Integer,Integer) -> DoubleFloat
--R eval : (%,List DoubleFloat,List DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat h
--R eval : (%,DoubleFloat,DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,List Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCA
--R every? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,DoubleFloat) -> Union(%,"failed") if DoubleFloat has INTDOM
--R hash : % -> SingleInteger if DoubleFloat has SETCAT
```

```
--R inverse : % -> Union(%,"failed") if DoubleFloat has FIELD
--R latex : % -> String if DoubleFloat has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R listOfLists : % -> List List DoubleFloat
--R map : (((DoubleFloat,DoubleFloat) -> DoubleFloat),%,%,DoubleFloat) -> %
--R map : (((DoubleFloat,DoubleFloat) -> DoubleFloat),%,%) -> %
--R map : ((DoubleFloat -> DoubleFloat),%) -> %
--R map! : ((DoubleFloat -> DoubleFloat),%) -> %
--R matrix : List List DoubleFloat -> %
--R member? : (DoubleFloat,%) -> Boolean if $ has finiteAggregate and DoubleFloat has SETCAT
--R members : % -> List DoubleFloat if $ has finiteAggregate
--R minordet : % -> DoubleFloat if DoubleFloat has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,DoubleFloat) -> %
--R nullSpace : % -> List DoubleFloatVector if DoubleFloat has INTDOM
--R nullity : % -> NonNegativeInteger if DoubleFloat has INTDOM
--R pfaffian : % -> DoubleFloat if DoubleFloat has COMRING
--R qelt : (%,Integer,Integer) -> DoubleFloat
--R qsetelt! : (%,Integer,Integer,DoubleFloat) -> DoubleFloat
--R rank : % -> NonNegativeInteger if DoubleFloat has INTDOM
--R row : (%,Integer) -> DoubleFloatVector
--R rowEchelon : % -> % if DoubleFloat has EUCDOM
--R scalarMatrix : (NonNegativeInteger,DoubleFloat) -> %
--R setColumn! : (%,Integer,DoubleFloatVector) -> %
--R setRow! : (%,Integer,DoubleFloatVector) -> %
--R setelt : (%,List Integer,List Integer,%) -> %
--R setelt : (%,Integer,Integer,DoubleFloat) -> DoubleFloat
--R setsubMatrix! : (%,Integer,Integer,%) -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R subMatrix : (%,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (%,Integer,Integer) -> %
--R swapRows! : (%,Integer,Integer) -> %
--R transpose : DoubleFloatVector -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
--R ?~=? : (%,%) -> Boolean if DoubleFloat has SETCAT
--R
--E 1

--S 2 of 6
a:DFMAT:=qnew(2,3)
--R
--R        +0.  0.  0.+
--R   (1)  |          |
--R        +0.  0.  0.+
--R                                                        Type: DoubleFloatMatrix
--E 2

--S 3 of 6
qsetelt!(a,1,1,1.0)
--R
```

```
--R   (2)  1.
--R                                                      Type: DoubleFloat
--E 3

--S 4 of 6
a
--R
--R        +0.  0.  0.+
--R   (3)  |          |
--R        +0.  1.  0.+
--R                                                Type: DoubleFloatMatrix
--E 4

--S 5 of 6
qsetelt!(a,0,0,2.0)
--R
--R   (4)  2.
--R                                                      Type: DoubleFloat
--E 5

--S 6 of 6
a
--R
--R        +2.  0.  0.+
--R   (5)  |          |
--R        +0.  1.  0.+
--R                                                Type: DoubleFloatMatrix
--E 6

)spool
)lisp (bye)
```

———–

— DoubleFloatMatrix.help —

```
====================================================================
DoubleFloatMatrix examples
====================================================================
This domain creates a lisp simple array of machine doublefloats.
It provides one new function called qnew which takes an integer
that gives the array length.

NOTE: Unlike normal Axiom arrays the DoubleFloatMatrix arrays
are 0-based so the first element is 0. Axiom arrays normally
start at 1.

a:DFMAT:=qnew(2,3)
```

```
        +0.  0.  0.+
        |          |
        +0.  0.  0.+

qsetelt!(a,1,1,1.0)
        1.

a
        +0.  0.  0.+
        |          |
        +0.  1.  0.+

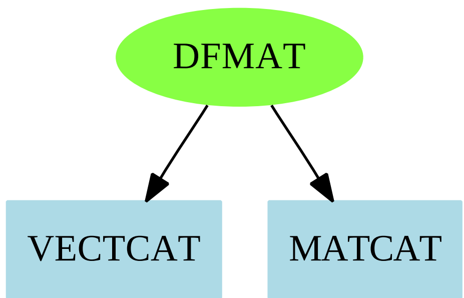
qsetelt!(a,0,0,2.0)
        2.

a
        +2.  0.  0.+
        |          |
        +0.  1.  0.+


See Also:
o )help Float
o )help DoubleFloat
o )show DoubleFloatMatrix
```

———–

5.16.1 DoubleFloatMatrix (DFMAT)



Exports:

#?	-?	?**?	?*?
?+?	?-?	?/?	?=?
?~=?	antisymmetric?	any?	coerce
coerce	column	columnSpace	copy
count	count	determinant	diagonal?
diagonalMatrix	diagonalMatrix	elt	elt
elt	empty	empty?	eq?
eval	eval	eval	eval
every?	exquo	fill!	hash
horizConcat	inverse	latex	less?
listOfLists	map	map	map
map!	matrix	maxColIndex	maxRowIndex
member?	members	minColIndex	minRowIndex
minordet	more?	ncols	new
nrows	nullSpace	nullity	parts
pfaffian	qelt	qnew	qsetelt!
rank	row	rowEchelon	sample
scalarMatrix	setColumn!	setRow!	setelt
setelt	setsubMatrix!	size?	square?
squareTop	subMatrix	swapColumns!	swapRows!
symmetric?	transpose	transpose	vertConcat
zero			

— domain DFMAT DoubleFloatMatrix —

```
)abbrev domain DFMAT DoubleFloatMatrix
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements matrices
++ (two dimensional arrays) of double precision floating point
++ numbers.  Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).
DoubleFloatMatrix : MatrixCategory(DoubleFloat,
                         DoubleFloatVector,
                         DoubleFloatVector) with
    qnew : (Integer, Integer) -> %
      ++ qnew(n, m) creates a new uninitialized n by m matrix.
      ++
      ++X t1:DFMAT:=qnew(3,4)

  == add

    Qelt2 ==> DAREF2$Lisp
    Qsetelt2 ==> DSETAREF2$Lisp
    Qnrows ==> DANROWS$Lisp
    Qncols ==> DANCOLS$Lisp
    Qnew ==> MAKE_-DOUBLE_-MATRIX$Lisp
    Qnew1 ==> MAKE_-DOUBLE_-MATRIX1$Lisp
```

```
    minRowIndex x == 0
    minColIndex x == 0
    nrows x == Qnrows(x)
    ncols x == Qncols(x)
    maxRowIndex x == Qnrows(x) - 1
    maxColIndex x == Qncols(x) - 1

    qelt(m, i, j) == Qelt2(m, i, j)
    qsetelt_!(m, i, j, r) == Qsetelt2(m, i, j, r)

    empty() == Qnew(0$Integer, 0$Integer)
    qnew(rows, cols) == Qnew(rows, cols)
    new(rows, cols, a) == Qnew1(rows, cols, a)
```

———-

— DFMAT.dotabb —

```
"DFMAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DFMAT",
          shape=ellipse]
"VECTCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"MATCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
"DFMAT" -> "VECTCAT"
"DFMAT" -> "MATCAT"
```

———-

5.17 domain DFVEC DoubleFloatVector

— DoubleFloatVector.input —

```
)set break resume
)sys rm -f DoubleFloatVector.output
)spool DoubleFloatVector.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
)show DoubleFloatVector
--R DoubleFloatVector  is a domain constructor
--R Abbreviation for DoubleFloatVector is DFVEC
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DFVEC
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                   concat : (%,%) -> %
--R concat : (DoubleFloat,%) -> %         concat : (%,DoubleFloat) -> %
--R construct : List DoubleFloat -> %     copy : % -> %
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> DoubleFloat
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List DoubleFloat       eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           qelt : (%,Integer) -> DoubleFloat
--R qnew : Integer -> %                   reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,DoubleFloat) -> % if DoubleFloat has MONOID
--R ?*? : (DoubleFloat,%) -> % if DoubleFloat has MONOID
--R ?*? : (Integer,%) -> % if DoubleFloat has ABELGRP
--R ?+? : (%,%) -> % if DoubleFloat has ABELSG
--R ?-? : (%,%) -> % if DoubleFloat has ABELGRP
--R -? : % -> % if DoubleFloat has ABELGRP
--R ?<? : (%,%) -> Boolean if DoubleFloat has ORDSET
--R ?<=? : (%,%) -> Boolean if DoubleFloat has ORDSET
--R ?=? : (%,%) -> Boolean if DoubleFloat has SETCAT
--R ?>? : (%,%) -> Boolean if DoubleFloat has ORDSET
--R ?>=? : (%,%) -> Boolean if DoubleFloat has ORDSET
--R any? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if DoubleFloat has SETCAT
--R convert : % -> InputForm if DoubleFloat has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and DoubleFloat has SETCAT
--R count : ((DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (%,%) -> % if DoubleFloat has RING
--R delete : (%,UniversalSegment Integer) -> %
--R dot : (%,%) -> DoubleFloat if DoubleFloat has RING
--R ?.? : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,DoubleFloat) -> DoubleFloat
--R entry? : (DoubleFloat,%) -> Boolean if $ has finiteAggregate and DoubleFloat has SETCAT
--R eval : (%,List DoubleFloat,List DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat h
--R eval : (%,DoubleFloat,DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,List Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCA
--R every? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,DoubleFloat) -> % if $ has shallowlyMutable
--R find : ((DoubleFloat -> Boolean),%) -> Union(DoubleFloat,"failed")
--R first : % -> DoubleFloat if Integer has ORDSET
--R hash : % -> SingleInteger if DoubleFloat has SETCAT
--R insert : (DoubleFloat,%,Integer) -> %
--R latex : % -> String if DoubleFloat has SETCAT
--R length : % -> DoubleFloat if DoubleFloat has RADCAT and DoubleFloat has RING
--R less? : (%,NonNegativeInteger) -> Boolean
```

```
--R magnitude : % -> DoubleFloat if DoubleFloat has RADCAT and DoubleFloat has RING
--R map : (((DoubleFloat,DoubleFloat) -> DoubleFloat),%,%) -> %
--R map : ((DoubleFloat -> DoubleFloat),%) -> %
--R map! : ((DoubleFloat -> DoubleFloat),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if DoubleFloat has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (DoubleFloat,%) -> Boolean if $ has finiteAggregate and DoubleFloat has SETCAT
--R members : % -> List DoubleFloat if $ has finiteAggregate
--R merge : (%,%) -> % if DoubleFloat has ORDSET
--R merge : (((DoubleFloat,DoubleFloat) -> Boolean),%,%) -> %
--R min : (%,%) -> % if DoubleFloat has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,DoubleFloat) -> %
--R outerProduct : (%,%) -> Matrix DoubleFloat if DoubleFloat has RING
--R parts : % -> List DoubleFloat if $ has finiteAggregate
--R position : (DoubleFloat,%,Integer) -> Integer if DoubleFloat has SETCAT
--R position : (DoubleFloat,%) -> Integer if DoubleFloat has SETCAT
--R position : ((DoubleFloat -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,DoubleFloat) -> DoubleFloat if $ has shallowlyMutable
--R reduce : (((DoubleFloat,DoubleFloat) -> DoubleFloat),%) -> DoubleFloat if $ has finiteAgg
--R reduce : (((DoubleFloat,DoubleFloat) -> DoubleFloat),%,DoubleFloat) -> DoubleFloat if $ h
--R reduce : (((DoubleFloat,DoubleFloat) -> DoubleFloat),%,DoubleFloat,DoubleFloat) -> Double
--R remove : ((DoubleFloat -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (DoubleFloat,%) -> % if $ has finiteAggregate and DoubleFloat has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and DoubleFloat has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((DoubleFloat -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,DoubleFloat) -> DoubleFloat if $ has shallowlyMutabl
--R setelt : (%,Integer,DoubleFloat) -> DoubleFloat if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if DoubleFloat has ORDSET
--R sort : (((DoubleFloat,DoubleFloat) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and DoubleFloat has ORDSET
--R sort! : (((DoubleFloat,DoubleFloat) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if DoubleFloat has ORDSET
--R sorted? : (((DoubleFloat,DoubleFloat) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if DoubleFloat has ABELMON
--R ?~=? : (%,%) -> Boolean if DoubleFloat has SETCAT
--R
--E 1

--S 2 of 6
t1:DFVEC:=qnew 17
--R
--R   (1)  [0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]
--R                                                         Type: DoubleFloatVector
--E 2
```

```
-- NOTE: DFVEC arrays are 0-based, normal Axiom arrays are 1-based
--S 3 of 6
t1.1:=1.0

   (3)  1.
                                                      Type: DoubleFloat
--E 3

--S 4 of 6
t1

   (4)  [0.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]
                                                Type: DoubleFloatVector
--E 4

--S 5 of 6
t1.0:=2.0

   (5)  2.
                                                      Type: DoubleFloat
--E 5

--S 6 of 6
t1

   (6)  [2.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]
                                                Type: DoubleFloatVector
--E 6

)spool
)lisp (bye)
```

— **DoubleFloatVector.help** —

```
====================================================================
DoubleFloatVector examples
====================================================================
This domain creates a lisp simple array of machine doublefloats.
It provides one new function called qnew which takes an integer
that gives the array length.

NOTE: Unlike normal Axiom arrays the DoubleFloatVector arrays
are 0-based so the first element is 0. Axiom arrays normally
start at 1.

a:DFVEC:=qnew 17
```

```
     [0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]

a.1:=1.0
     1.

a
     [0.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]

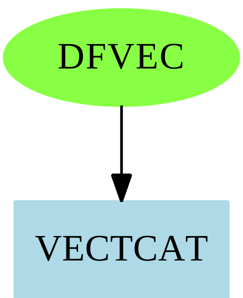
a.0:=2.0
     2.

a
     [2.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]


See Also:
o )help Float
o )help DoubleFloat
o )show DoubleFloatVector
```

———–

5.17.1 DoubleFloatVector (DFVEC)



Exports:

#?	-?	?*?	?+?	?-?
?.?	?<=?	?<?	?=?	?>=?
?>?	?~=?	any?	coerce	concat
construct	convert	copy	copyInto!	count
cross	delete	dot	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	find	first	hash
index?	indices	insert	latex	length
less?	magnitude	map	map!	max
maxIndex	member?	members	merge	min
minIndex	more?	new	outerProduct	parts
position	qelt	qnew	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	zero		

— domain DFVEC DoubleFloatVector —

```
)abbrev domain DFVEC DoubleFloatVector
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements vectors
++ (one dimensional arrays) of double precision floating point
++ numbers.  Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).
DoubleFloatVector : VectorCategory DoubleFloat with
    qnew : Integer -> %
      ++ qnew(n) creates a new uninitialized vector of length n.
      ++
      ++X t1:DFVEC:=qnew(7)
  == add

    Qelt1 ==> DELT$Lisp
    Qsetelt1 ==> DSETELT$Lisp

    qelt(x, i) == Qelt1(x, i)
    qsetelt_!(x, i, s) == Qsetelt1(x, i, s)
    Qsize ==> DLEN$Lisp
    Qnew ==> MAKE_-DOUBLE_-VECTOR$Lisp
    Qnew1 ==> MAKE_-DOUBLE_-VECTOR1$Lisp

    #x                          == Qsize x
    minIndex x                  == 0
    empty()                     == Qnew(0$Lisp)
    qnew(n)                     == Qnew(n)
    new(n, x)                   == Qnew1(n, x)
    qelt(x, i)                  == Qelt1(x, i)
    elt(x:%, i:Integer)         == Qelt1(x, i)
    qsetelt_!(x, i, s)          == Qsetelt1(x, i, s)
```

```
    setelt(x : %, i : Integer, s : DoubleFloat) == Qsetelt1(x, i, s)
    fill_!(x, s)       ==
        for i in 0..((Qsize(x)) - 1) repeat Qsetelt1(x, i, s)
        x
```

———-

— DFVEC.dotabb —

```
"DFVEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DFVEC",
          shape=ellipse]
"VECTCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"DFVEC" -> "VECTCAT"
```

———-

5.18 domain DROPT DrawOption

— DrawOption.input —

```
)set break resume
)sys rm -f DrawOption.output
)spool DrawOption.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DrawOption
--R DrawOption  is a domain constructor
--R Abbreviation for DrawOption is DROPT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DROPT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean               adaptive : Boolean -> %
--R clip : List Segment Float -> %       clip : Boolean -> %
--R coerce : % -> OutputForm             curveColor : Palette -> %
--R curveColor : Float -> %              hash : % -> SingleInteger
--R latex : % -> String                  pointColor : Palette -> %
--R pointColor : Float -> %              range : List Segment Float -> %
--R ranges : List Segment Float -> %     style : String -> %
--R title : String -> %                  toScale : Boolean -> %
--R tubePoints : PositiveInteger -> %    tubeRadius : Float -> %
```

```
--R unit : List Float -> %                var1Steps : PositiveInteger -> %
--R var2Steps : PositiveInteger -> %      ?~=? : (%,%) -> Boolean
--R colorFunction : ((DoubleFloat,DoubleFloat,DoubleFloat) -> DoubleFloat) -> %
--R colorFunction : ((DoubleFloat,DoubleFloat) -> DoubleFloat) -> %
--R colorFunction : (DoubleFloat -> DoubleFloat) -> %
--R coord : (Point DoubleFloat -> Point DoubleFloat) -> %
--R coordinates : (Point DoubleFloat -> Point DoubleFloat) -> %
--R option : (List %,Symbol) -> Union(Any,"failed")
--R option? : (List %,Symbol) -> Boolean
--R range : List Segment Fraction Integer -> %
--R space : ThreeSpace DoubleFloat -> %
--R viewpoint : Record(theta: DoubleFloat,phi: DoubleFloat,scale: DoubleFloat,scaleX: DoubleFloat,scaleY
--R
--E 1

)spool
)lisp (bye)
```

———-

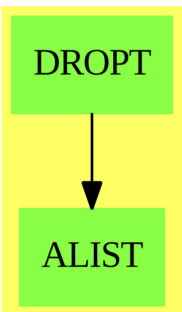
— DrawOption.help —

```
====================================================================
DrawOption examples
====================================================================

See Also:
o )show DrawOption
```

———-

5.18.1 DrawOption (DROPT)



Exports:

adaptive	clip	coerce	colorFunction	coord
coordinates	curveColor	hash	latex	option
option?	pointColor	range	ranges	space
style	title	toScale	tubePoints	tubeRadius
unit	var1Steps	var2Steps	viewpoint	?=?
?~=?				

— domain DROPT DrawOption —

```
)abbrev domain DROPT DrawOption
++ Author: Stephen Watt
++ Date Created: 1 March 1990
++ Date Last Updated: 31 Oct 1990, Jim Wen
++ Basic Operations: adaptive, clip, title, style, toScale, coordinates,
++ pointColor, curveColor, colorFunction, tubeRadius, range, ranges,
++ var1Steps, var2Steps, tubePoints, unit
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ DrawOption allows the user to specify defaults for the
++ creation and rendering of plots.

DrawOption(): Exports == Implementation where
 RANGE ==> List Segment Float
 UNIT  ==> List Float
 PAL   ==> Palette
 POINT ==> Point(DoubleFloat)
 SEG   ==> Segment Float
 SF     ==> DoubleFloat
 SPACE3 ==> ThreeSpace(DoubleFloat)
 VIEWPT ==> Record(theta:SF, phi:SF, scale:SF, scaleX:SF, scaleY:SF, scaleZ:SF, deltaX:SF, de

 Exports ==> SetCategory with
  adaptive : Boolean -> %
    ++ adaptive(b) turns adaptive 2D plotting on if b is true, or off if b is
    ++ false. This option is expressed in the form \spad{adaptive == b}.
  clip : Boolean -> %
    ++ clip(b) turns 2D clipping on if b is true, or off if b is false.
    ++ This option is expressed in the form \spad{clip == b}.
  viewpoint : VIEWPT -> %
    ++ viewpoint(vp) creates a viewpoint data structure corresponding to the
    ++ list of values. The values are interpreted as [theta, phi, scale,
    ++ scaleX, scaleY, scaleZ, deltaX, deltaY].  This option is expressed
    ++ in the form \spad{viewpoint == ls}.
  title  : String -> %
```

```
    ++ title(s) specifies a title for a plot by the indicated string s.
    ++ This option is expressed in the form \spad{title == s}.
  style  : String -> %
    ++ style(s) specifies the drawing style in which the graph will be plotted
    ++ by the indicated string s. This option is expressed in the
    ++ form \spad{style == s}.
  toScale : Boolean -> %
    ++ toScale(b) specifies whether or not a plot is to be drawn to scale;
    ++ if b is true it is drawn to scale, if b is false it is not. This option
    ++ is expressed in the form \spad{toScale == b}.
  clip : List SEG -> %
    ++ clip([l]) provides ranges for user-defined clipping as specified
    ++ in the list l. This option is expressed in the form \spad{clip == [l]}.
  coordinates : (POINT -> POINT) -> %
    ++ coordinates(p) specifies a change of coordinate systems of point p.
    ++ This option is expressed in the form \spad{coordinates == p}.
  pointColor : Float -> %
    ++ pointColor(v) specifies a color, v, for 2D graph points. This option
    ++ is expressed in the form \spad{pointColor == v}.
  pointColor : PAL -> %
    ++ pointColor(p) specifies a color index for 2D graph points from the
    ++ spadcolors palette p. This option is expressed in the
    ++ form \spad{pointColor == p}.
  curveColor : Float -> %
    ++ curveColor(v) specifies a color, v, for 2D graph curves.
    ++ This option is expressed in the form \spad{curveColor == v}.
  curveColor : PAL -> %
    ++ curveColor(p) specifies a color index for 2D graph curves from the
    ++ spadcolors palette p.
    ++ This option is expressed in the form \spad{curveColor ==p}.
  colorFunction : (SF -> SF) -> %
    ++ colorFunction(f(z)) specifies the color based upon the z-component of
    ++ three dimensional plots. This option is expressed in the
    ++ form \spad{colorFunction == f(z)}.
  colorFunction : ((SF,SF) -> SF) -> %
    ++ colorFunction(f(u,v)) specifies the color for three dimensional plots
    ++ as a function based upon the two parametric variables. This option
    ++ is expressed in the form \spad{colorFunction == f(u,v)}.
  colorFunction : ((SF,SF,SF) -> SF) -> %
    ++ colorFunction(f(x,y,z)) specifies the color for three dimensional
    ++ plots as a function of x, y, and z coordinates. This option is
    ++ expressed in the form \spad{colorFunction == f(x,y,z)}.
  tubeRadius : Float -> %
    ++ tubeRadius(r) specifies a radius, r, for a tube plot around a 3D curve;
    ++ is expressed in the form \spad{tubeRadius == 4}.
  range : List SEG -> %
    ++ range([l]) provides a user-specified range l.
    ++ This option is expressed in the form \spad{range == [l]}.
  range : List Segment Fraction Integer -> %
    ++ range([i]) provides a user-specified range i.
```

```
    ++ This option is expressed in the form \spad{range == [i]}.
  ranges : RANGE -> %
    ++ ranges(l) provides a list of user-specified ranges l.
    ++ This option is expressed in the form \spad{ranges == l}.
  space : SPACE3 -> %
    ++ space specifies the space into which we will draw.  If none is given
    ++ then a new space is created.
  var1Steps : PositiveInteger -> %
    ++ var1Steps(n) indicates the number of subdivisions, n, of the first
    ++ range variable. This option is expressed in the
    ++ form \spad{var1Steps == n}.
  var2Steps : PositiveInteger -> %
    ++ var2Steps(n) indicates the number of subdivisions, n,  of the second
    ++ range variable. This option is expressed in the
    ++ form \spad{var2Steps == n}.
  tubePoints : PositiveInteger -> %
    ++ tubePoints(n) specifies the number of points, n, defining the circle
    ++ which creates the tube around a 3D curve, the default is 6.
    ++ This option is expressed in the form \spad{tubePoints == n}.
  coord : (POINT->POINT) -> %
    ++ coord(p) specifies a change of coordinates of point p.
    ++ This option is expressed in the form \spad{coord == p}.
  unit   : UNIT -> %
    ++ unit(lf) will mark off the units according to the indicated list lf.
    ++ This option is expressed in the form \spad{unit == [f1,f2]}.
  option : (List %, Symbol) -> Union(Any, "failed")
    ++ option() is not to be used at the top level;
    ++ option determines internally which drawing options are indicated in
    ++ a draw command.
  option?: (List %, Symbol) -> Boolean
    ++ option?() is not to be used at the top level;
    ++ option? internally returns true for drawing options which are
    ++ indicated in a draw command, or false for those which are not.
 Implementation ==> add
  import AnyFunctions1(String)
  import AnyFunctions1(Segment Float)
  import AnyFunctions1(VIEWPT)
  import AnyFunctions1(List Segment Float)
  import AnyFunctions1(List Segment Fraction Integer)
  import AnyFunctions1(List Integer)
  import AnyFunctions1(PositiveInteger)
  import AnyFunctions1(Boolean)
  import AnyFunctions1(RANGE)
  import AnyFunctions1(UNIT)
  import AnyFunctions1(Float)
  import AnyFunctions1(POINT -> POINT)
  import AnyFunctions1(SF -> SF)
  import AnyFunctions1((SF,SF) -> SF)
  import AnyFunctions1((SF,SF,SF) -> SF)
  import AnyFunctions1(POINT)
```

```
import AnyFunctions1(PAL)
import AnyFunctions1(SPACE3)

Rep := Record(keyword:Symbol, value:Any)

length:List SEG -> NonNegativeInteger
-- these lists will become tuples in a later version
length tup == # tup

lengthR:List Segment Fraction Integer -> NonNegativeInteger
-- these lists will become tuples in a later version
lengthR tup == # tup

lengthI:List Integer -> NonNegativeInteger
-- these lists will become tuples in a later version
lengthI tup == # tup

viewpoint vp ==
  ["viewpoint"::Symbol, vp::Any]

title s == ["title"::Symbol, s::Any]
style s == ["style"::Symbol, s::Any]
toScale b == ["toScale"::Symbol, b::Any]
clip(b:Boolean) == ["clipBoolean"::Symbol, b::Any]
adaptive b == ["adaptive"::Symbol, b::Any]

pointColor(x:Float) == ["pointColorFloat"::Symbol, x::Any]
pointColor(c:PAL) == ["pointColorPalette"::Symbol, c::Any]
curveColor(x:Float) == ["curveColorFloat"::Symbol, x::Any]
curveColor(c:PAL) == ["curveColorPalette"::Symbol, c::Any]
colorFunction(f:SF -> SF) == ["colorFunction1"::Symbol, f::Any]
colorFunction(f:(SF,SF) -> SF) == ["colorFunction2"::Symbol, f::Any]
colorFunction(f:(SF,SF,SF) -> SF) == ["colorFunction3"::Symbol, f::Any]
clip(tup:List SEG) ==
  length tup > 3 =>
    error "clip: at most 3 segments may be specified"
  ["clipSegment"::Symbol, tup::Any]
coordinates f == ["coordinates"::Symbol, f::Any]
tubeRadius x == ["tubeRadius"::Symbol, x::Any]
range(tup:List Segment Float) ==
  ((n := length tup) > 3) =>
    error "range: at most 3 segments may be specified"
  n < 2 =>
    error "range: at least 2 segments may be specified"
  ["rangeFloat"::Symbol, tup::Any]
range(tup:List Segment Fraction Integer) ==
  ((n := lengthR tup) > 3) =>
    error "range: at most 3 segments may be specified"
  n < 2 =>
    error "range: at least 2 segments may be specified"
```

```
    ["rangeRat"::Symbol, tup::Any]

  ranges s               == ["ranges"::Symbol, s::Any]
  space s                == ["space"::Symbol, s::Any]
  var1Steps s            == ["var1Steps"::Symbol, s::Any]
  var2Steps s            == ["var2Steps"::Symbol, s::Any]
  tubePoints s           == ["tubePoints"::Symbol, s::Any]
  coord s                == ["coord"::Symbol, s::Any]
  unit s                 == ["unit"::Symbol, s::Any]
  coerce(x:%):OutputForm == x.keyword::OutputForm = x.value::OutputForm
  x:% = y:%              == x.keyword = y.keyword and x.value = y.value

  option?(l, s) ==
    for x in l repeat
      x.keyword = s => return true
    false

  option(l, s) ==
    for x in l repeat
      x.keyword = s => return(x.value)
    "failed"
```

———-

— DROPT.dotabb —

```
"DROPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DROPT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"DROPT" -> "ALIST"
```

———-

5.19 domain D01AJFA d01ajfAnnaType

— d01ajfAnnaType.input —

```
)set break resume
)sys rm -f d01ajfAnnaType.output
)spool d01ajfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show d01ajfAnnaType
--R d01ajfAnnaType  is a domain constructor
--R Abbreviation for d01ajfAnnaType is D01AJFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AJFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Doub
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1

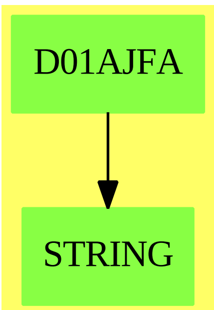
)spool
)lisp (bye)
```

— **d01ajfAnnaType.help** —

```
====================================================================
d01ajfAnnaType examples
====================================================================

See Also:
o )show d01ajfAnnaType
```

5.19.1 d01ajfAnnaType (D01AJFA)



Exports:
coerce hash latex measure numericalIntegration ?~=? ?=?

— domain D01AJFA d01ajfAnnaType —

```
)abbrev domain D01AJFA d01ajfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01ajfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01AJF, a general numerical integration routine which
++ can handle some singularities in the input function.  The function
++ \axiomFun{measure} measures the usefulness of the routine D01AJF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01ajfAnnaType(): NumericalIntegrationCategory == Result add
  EF2   ==> ExpressionFunctions2
  EDF   ==> Expression DoubleFloat
  LDF   ==> List DoubleFloat
  SDF   ==> Stream DoubleFloat
  DF    ==> DoubleFloat
  FI    ==> Fraction Integer
  EFI   ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA   ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT   ==> Integer
  BOP   ==> BasicOperator
  S     ==> Symbol
  ST    ==> String
  LST   ==> List String
  RT    ==> RoutinesTable
  Rep:=Result
  import Rep, NagIntegrationPackage, d01AgentsPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    pp:SDF := singularitiesOf(args)
    not (empty?(pp)$SDF) =>
      [0.1,"d01ajf: There is a possible problem at the following point(s): "
           commaSeparate(sdf2lst(pp)) ,ext]
    [getMeasure(R,d01ajf :: S)$RT,
       "The general routine d01ajf is our default",ext]
```

```
  numericalIntegration(args:NIA,hints:Result) ==
    ArgsFn := map(x+->convert(x)$DF,args.fn)$EF2(DF,Float)
    b:Float := getButtonValue("d01ajf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
    iw:INT := 75*fEvals
    f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
    d01ajf(getlo(args.range),gethi(args.range),args.abserr,_
           args.relerr,4*iw,iw,-1,f)
```

———-

— D01AJFA.dotabb —

```
"D01AJFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AJFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AJFA" -> "STRING"
```

———-

5.20 domain D01AKFA d01akfAnnaType

— d01akfAnnaType.input —

```
)set break resume
)sys rm -f d01akfAnnaType.output
)spool d01akfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01akfAnnaType
--R d01akfAnnaType  is a domain constructor
--R Abbreviation for d01akfAnnaType is D01AKFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AKFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Doub
```

```
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Orde
--R
--E 1

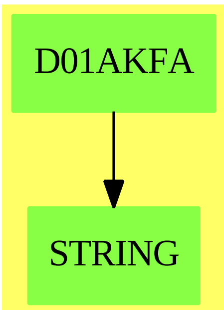
)spool
)lisp (bye)
```

— d01akfAnnaType.help —

```
====================================================================
d01akfAnnaType examples
====================================================================

See Also:
o )show d01akfAnnaType
```

5.20.1 d01akfAnnaType (D01AKFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01AKFA d01akfAnnaType —

```
)abbrev domain D01AKFA d01akfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
```

```
++ Description:
++ \axiomType{d01akfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01AKF, a numerical integration routine which is
++ is suitable for oscillating, non-singular functions.  The function
++ \axiomFun{measure} measures the usefulness of the routine D01AKF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01akfAnnaType(): NumericalIntegrationCategory == Result add
  EF2   ==> ExpressionFunctions2
  EDF   ==> Expression DoubleFloat
  LDF   ==> List DoubleFloat
  SDF   ==> Stream DoubleFloat
  DF    ==> DoubleFloat
  FI    ==> Fraction Integer
  EFI   ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA   ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT   ==> Integer
  BOP   ==> BasicOperator
  S     ==> Symbol
  ST    ==> String
  LST   ==> List String
  RT    ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    pp:SDF := singularitiesOf(args)
    not (empty?(pp)$SDF) =>
      [0.0,"d01akf: There is a possible problem at the following point(s): "
             commaSeparate(sdf2lst(pp)) ,ext]
    o:Float := functionIsOscillatory(args)
    one := 1.0
    m:Float := (getMeasure(R,d01akf@S)$RT)*(one-one/(one+sqrt(o)))**2
    m > 0.8 => [m,"d01akf: The expression shows much oscillation",ext]
    m > 0.6 => [m,"d01akf: The expression shows some oscillation",ext]
    m > 0.5 => [m,"d01akf: The expression shows little oscillation",ext]
    [m,"d01akf: The expression shows little or no oscillation",ext]

  numericalIntegration(args:NIA,hints:Result) ==
    ArgsFn := map(x+->convert(x)$DF,args.fn)$EF2(DF,Float)
    b:Float := getButtonValue("d01akf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
    iw:INT := 75*fEvals
    f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
    d01akf(getlo(args.range),gethi(args.range),args.abserr,_
```

```
args.relerr,4*iw,iw,-1,f)
```

———-

— D01AKFA.dotabb —

```
"D01AKFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AKFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AKFA" -> "STRING"
```

———-

5.21 domain D01ALFA d01alfAnnaType

— d01alfAnnaType.input —

```
)set break resume
)sys rm -f d01alfAnnaType.output
)spool d01alfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01alfAnnaType
--R d01alfAnnaType  is a domain constructor
--R Abbreviation for d01alfAnnaType is D01ALFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01ALFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCo
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCom
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
--E 1

)spool
)lisp (bye)
```

———-

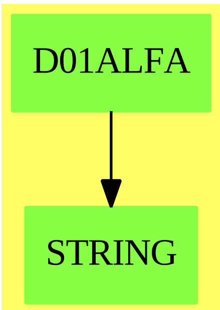
— d01alfAnnaType.help —

```
====================================================================
d01alfAnnaType examples
====================================================================

See Also:
o )show d01alfAnnaType
```

———-

5.21.1 d01alfAnnaType (D01ALFA)



Exports:
coerce hash latex measure numericalIntegration ?~=? ?=?

— domain D01ALFA d01alfAnnaType —

```
)abbrev domain D01ALFA d01alfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01alfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01ALF, a general numerical integration routine which
++ can handle a list of singularities.  The
++ function \axiomFun{measure} measures the usefulness of the routine D01ALF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.
```

```
d01alfAnnaType(): NumericalIntegrationCategory == Result add
  EF2   ==> ExpressionFunctions2
  EDF   ==> Expression DoubleFloat
  LDF   ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    streamOfZeros:SDF := singularitiesOf(args)
    listOfZeros:LST := removeDuplicates!(sdf2lst(streamOfZeros))
    numberOfZeros:INT := # listOfZeros
    (numberOfZeros > 15)@Boolean =>
      [0.0,"d01alf: The list of singularities is too long", ext]
    positive?(numberOfZeros) =>
      l:LDF := entries(complete(streamOfZeros)$SDF)$SDF
      lany:Any := coerce(l)$AnyFunctions1(LDF)
      ex:Record(key:S,entry:Any) := [d01alfextra@S,lany]
      ext := insert!(ex,ext)$Result
      st:ST := "Recommended is d01alf with the singularities "
                   commaSeparate(listOfZeros)
      m :=
--        one?(numberOfZeros) => 0.4
        (numberOfZeros = 1) => 0.4
        getMeasure(R,d01alf@S)$RT
      [m, st, ext]
    [0.0, "d01alf: A list of suitable singularities has not been found", ext]

  numericalIntegration(args:NIA,hints:Result) ==
    la:Any := coerce(search((d01alfextra@S),hints)$Result)@Any
    listOfZeros:LDF := retract(la)$AnyFunctions1(LDF)
    l:= removeDuplicates(listOfZeros)$LDF
    n:Integer := (#(l))$List(DF)
    M:Matrix DF := matrix([l])$(Matrix DF)
    b:Float := getButtonValue("d01alf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
```

```
    iw:INT := 75*fEvals
    ArgsFn := map(x+->convert(x)$DF,args.fn)$EF2(DF,Float)
    f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
    d01alf(getlo(args.range),gethi(args.range),n,M,_
           args.abserr,args.relerr,2*n*iw,n*iw,-1,f)
```

———-

— **D01ALFA.dotabb** —

```
"D01ALFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01ALFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01ALFA" -> "STRING"
```

———-

5.22 domain D01AMFA d01amfAnnaType

— **d01amfAnnaType.input** —

```
)set break resume
)sys rm -f d01amfAnnaType.output
)spool d01amfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01amfAnnaType
--R d01amfAnnaType  is a domain constructor
--R Abbreviation for d01amfAnnaType is D01AMFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AMFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Doub
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

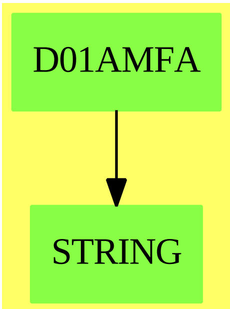
— d01amfAnnaType.help —

```
====================================================================
d01amfAnnaType examples
====================================================================

See Also:
o )show d01amfAnnaType
```

———-

5.22.1 d01amfAnnaType (D01AMFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01AMFA d01amfAnnaType —

```
)abbrev domain D01AMFA d01amfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01amfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
```

```
++ for the NAG routine D01AMF, a general numerical integration routine which
++ can handle infinite or semi-infinite range of the input function.  The
++ function \axiomFun{measure} measures the usefulness of the routine D01AMF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01amfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    Range:=rangeIsFinite(args)
    pp:SDF := singularitiesOf(args)
    not (empty?(pp)$SDF) =>
      [0.0,"d01amf: There is a possible problem at the following point(s): "
                   commaSeparate(sdf2lst(pp)), ext]
    [getMeasure(R,d01amf@S)$RT, "d01amf is a reasonable choice if the "
         "integral is infinite or semi-infinite and d01transform cannot "
           "do better than using general routines",ext]

  numericalIntegration(args:NIA,hints:Result) ==
    r:INT
    bound:DF
    ArgsFn := map(x+->convert(x)$DF,args.fn)$EF2(DF,Float)
    b:Float := getButtonValue("d01amf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
    iw:INT := 150*fEvals
    f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
    Range:=rangeIsFinite(args)
    if (Range case upperInfinite) then
      bound := getlo(args.range)
      r := 1
    else if (Range case lowerInfinite) then
```

```
      bound := gethi(args.range)
      r := -1
    else
      bound := 0$DF
      r := 2
    d01amf(bound,r,args.abserr,args.relerr,4*iw,iw,-1,f)
```

— D01AMFA.dotabb —

```
"D01AMFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AMFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AMFA" -> "STRING"
```

5.23 domain D01ANFA d01anfAnnaType

— d01anfAnnaType.input —

```
)set break resume
)sys rm -f d01anfAnnaType.output
)spool d01anfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01anfAnnaType
--R d01anfAnnaType  is a domain constructor
--R Abbreviation for d01anfAnnaType is D01ANFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01ANFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCo
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

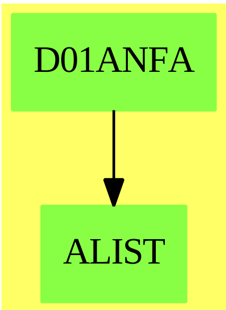
— d01anfAnnaType.help —

```
====================================================================
d01anfAnnaType examples
====================================================================

See Also:
o )show d01anfAnnaType
```

———-

5.23.1 d01anfAnnaType (D01ANFA)



Exports:
coerce hash latex measure numericalIntegration ?~=? ?=?

— domain D01ANFA d01anfAnnaType —

```
)abbrev domain D01ANFA d01anfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01anfAnnaType} is a domain of
```

```
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01ANF, a numerical integration routine which can
++ handle weight functions of the form cos(\omega x) or sin(\omega x).  The
++ function \axiomFun{measure} measures the usefulness of the routine D01ANF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01anfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, d01WeightsPackage, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    weight:Union(Record(op:BOP,w:DF),"failed") :=
      exprHasWeightCosWXorSinWX(args)
    weight case "failed" =>
      [0.0,"d01anf: A suitable weight has not been found", ext]
    weight case Record(op:BOP,w:DF) =>
      wany := coerce(weight)$AnyFunctions1(Record(op:BOP,w:DF))
      ex:Record(key:S,entry:Any) := [d01anfextra@S,wany]
      ext := insert!(ex,ext)$Result
      ws:ST := string(name(weight.op)$BOP)$S "(" df2st(weight.w)
                        string(args.var)$S ")"
      [getMeasure(R,d01anf@S)$RT,
        "d01anf: The expression has a suitable weight:- " ws, ext]

  numericalIntegration(args:NIA,hints:Result) ==
    a:INT
    r:Any := coerce(search((d01anfextra@S),hints)$Result)@Any
    rec:Record(op:BOP,w:DF) := retract(r)$AnyFunctions1(Record(op:BOP,w:DF))
    Var := args.var :: EDF
    o:BOP := rec.op
    den:EDF := o((rec.w*Var)$EDF)
    Argsfn:EDF := args.fn/den
```

```
    if (name(o) = cos@S)@Boolean then a := 1
    else a := 2
    b:Float := getButtonValue("d01anf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
    iw:INT := 75*fEvals
    ArgsFn := map(x+->convert(x)$DF,Argsfn)$EF2(DF,Float)
    f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
    d01anf(getlo(args.range),gethi(args.range),rec.w,a,_
           args.abserr,args.relerr,4*iw,iw,-1,f)
```

———-

— D01ANFA.dotabb —

```
"D01ANFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01ANFA"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"D01ANFA" -> "ALIST"
```

———-

5.24 domain D01APFA d01apfAnnaType

— d01apfAnnaType.input —

```
)set break resume
)sys rm -f d01apfAnnaType.output
)spool d01apfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01apfAnnaType
--R d01apfAnnaType  is a domain constructor
--R Abbreviation for d01apfAnnaType is D01APFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01APFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
```

```
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Orde
--R
--E 1

)spool
)lisp (bye)
```

———-

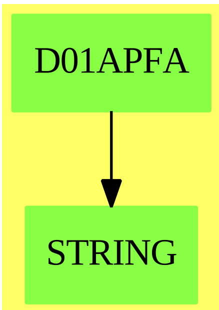
— d01apfAnnaType.help —

```
====================================================================
d01apfAnnaType examples
====================================================================

See Also:
o )show d01apfAnnaType
```

———-

5.24.1 d01apfAnnaType (D01APFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01APFA d01apfAnnaType —

```
)abbrev domain D01APFA d01apfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
```

```
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01apfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01APF, a general numerical integration routine which
++ can handle end point singularities of the algebraico-logarithmic form
++ w(x) = (x-a)^c * (b-x)^d.  The
++ function \axiomFun{measure} measures the usefulness of the routine D01APF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01apfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, NagIntegrationPackage, d01AgentsPackage, d01WeightsPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    d := (c := 0$DF)
    if ((a := exprHasAlgebraicWeight(args)) case LDF) then
      if  (a.1 > -1) then c := a.1
      if  (a.2 > -1) then d := a.2
    l:INT := exprHasLogarithmicWeights(args)
--    (zero? c) and (zero? d) and (one? l) =>
    (zero? c) and (zero? d) and (l = 1) =>
        [0.0,"d01apf: A suitable singularity has not been found", ext]
    out:LDF := [c,d,l :: DF]
    outany:Any := coerce(out)$AnyFunctions1(LDF)
    ex:Record(key:S,entry:Any) := [d01apfextra@S,outany]
    ext := insert!(ex,ext)$Result
    st:ST :=  "Recommended is d01apf with c = " df2st(c) ", d = "
                            df2st(d) " and l = " string(l)$ST
    [getMeasure(R,d01apf@S)$RT, st, ext]

  numericalIntegration(args:NIA,hints:Result) ==
```

```
    Var:EDF := coerce(args.var)$EDF
    la:Any := coerce(search((d01apfextra@S),hints)$Result)@Any
    list:LDF := retract(la)$AnyFunctions1(LDF)
    Fac1:EDF := (Var - (getlo(args.range) :: EDF))$EDF
    Fac2:EDF := ((gethi(args.range) :: EDF) - Var)$EDF
    c := first(list)$LDF
    d := second(list)$LDF
    l := (retract(third(list)$LDF)@INT)$DF
    thebiz:EDF := (Fac1**(c :: EDF))*(Fac2**(d :: EDF))
    if l > 1 then
      if l = 2 then
        thebiz := thebiz*log(Fac1)
      else if l = 3 then
        thebiz := thebiz*log(Fac2)
      else
        thebiz := thebiz*log(Fac1)*log(Fac2)
    Fn :=  (args.fn/thebiz)$EDF
    ArgsFn := map(x+->convert(x)$DF,Fn)$EF2(DF,Float)
    b:Float := getButtonValue("d01apf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
    iw:INT := 75*fEvals
    f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
    d01apf(getlo(args.range),gethi(args.range),c,d,l,_
           args.abserr,args.relerr,4*iw,iw,-1,f)
```

———-

— **D01APFA.dotabb** —

```
"D01APFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01APFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01APFA" -> "STRING"
```

———-

5.25 domain D01AQFA d01aqfAnnaType

— **d01aqfAnnaType.input** —

```
)set break resume
)sys rm -f d01aqfAnnaType.output
)spool d01aqfAnnaType.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show d01aqfAnnaType
--R d01aqfAnnaType  is a domain constructor
--R Abbreviation for d01aqfAnnaType is D01AQFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AQFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Doub
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1

)spool
)lisp (bye)
```

———-

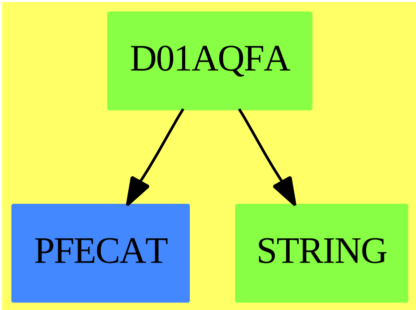
— d01aqfAnnaType.help —

```
====================================================================
d01aqfAnnaType examples
====================================================================

See Also:
o )show d01aqfAnnaType
```

———-

5.25.1 d01aqfAnnaType (D01AQFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01AQFA d01aqfAnnaType —

```
)abbrev domain D01AQFA d01aqfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01aqfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01AQF, a general numerical integration routine which
++ can solve an integral of the form
++ /home/bjd/Axiom/anna/hypertex/bitmaps/d01aqf.xbm
++ The function \axiomFun{measure} measures the usefulness of the routine
++ D01AQF for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01aqfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
```

```
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    Den := denominator(args.fn)
--    one? Den =>
    (Den = 1) =>
      [0.0,"d01aqf: A suitable weight function has not been found", ext]
    listOfZeros:LDF := problemPoints(args.fn,args.var,args.range)
    numberOfZeros := (#(listOfZeros))$LDF
    zero?(numberOfZeros) =>
      [0.0,"d01aqf: A suitable weight function has not been found", ext]
    numberOfZeros = 1 =>
      s:SDF := singularitiesOf(args)
      more?(s,1)$SDF =>
        [0.0,"d01aqf: Too many singularities have been found", ext]
      cFloat:Float := (convert(first(listOfZeros)$LDF)@Float)$DF
      cString:ST := (convert(cFloat)@ST)$Float
      lany:Any := coerce(listOfZeros)$AnyFunctions1(LDF)
      ex:Record(key:S,entry:Any) := [d01aqfextra@S,lany]
      ext := insert!(ex,ext)$Result
      [getMeasure(R,d01aqf@S)$RT, "Recommended is d01aqf with the "
        "hilbertian weight function of 1/(x-c) where c = " cString, ext]
    [0.0,"d01aqf: More than one factor has been found and so does not "
                "have a suitable weight function",ext]

  numericalIntegration(args:NIA,hints:Result) ==
    Args := copy args
    ca:Any :=  coerce(search((d01aqfextra@S),hints)$Result)@Any
    c:DF := first(retract(ca)$AnyFunctions1(LDF))$LDF
    ci:FI := df2fi(c)$ExpertSystemToolsPackage
    Var:EFI := Args.var :: EFI
    Gx:EFI := (Var-(ci::EFI))*(edf2efi(Args.fn)$ExpertSystemToolsPackage)
    ArgsFn := map(x+->convert(x)$FI,Gx)$EF2(FI,Float)
    b:Float := getButtonValue("d01aqf","functionEvaluations")$AttributeButtons
    fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
    iw:INT := 75*fEvals
    f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
    d01aqf(getlo(Args.range),gethi(Args.range),c,_
           Args.abserr,Args.relerr,4*iw,iw,-1,f)
```

———

— **D01AQFA.dotabb** —

```
"D01AQFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AQFA"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AQFA" -> "STRING"
"D01AQFA" -> "PFECAT"
```

————

5.26 domain D01ASFA d01asfAnnaType

— d01asfAnnaType.input —

```
)set break resume
)sys rm -f d01asfAnnaType.output
)spool d01asfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01asfAnnaType
--R d01asfAnnaType  is a domain constructor
--R Abbreviation for d01asfAnnaType is D01ASFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01ASFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCom
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Orde
--R
--E 1

)spool
)lisp (bye)
```

————

— d01asfAnnaType.help —

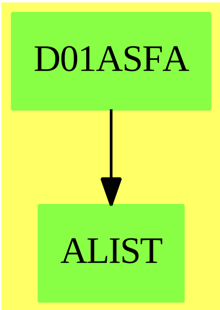
```
====================================================================
```

```
d01asfAnnaType examples
====================================================================

See Also:
o )show d01asfAnnaType
```

———-

5.26.1 d01asfAnnaType (D01ASFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01ASFA d01asfAnnaType —

```
)abbrev domain D01ASFA d01asfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01asfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01ASF, a numerical integration routine which can
++ handle weight functions of the form cos(\omega x) or sin(\omega x) on an
++ semi-infinite range.  The
++ function \axiomFun{measure} measures the usefulness of the routine D01ASF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01asfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
```

```
LDF  ==> List DoubleFloat
SDF  ==> Stream DoubleFloat
DF  ==> DoubleFloat
FI  ==> Fraction Integer
EFI  ==> Expression Fraction Integer
SOCDF  ==> Segment OrderedCompletion DoubleFloat
NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
INT  ==> Integer
BOP  ==> BasicOperator
S  ==> Symbol
ST  ==> String
LST  ==> List String
RT  ==> RoutinesTable
Rep:=Result
import Rep, d01WeightsPackage, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  Range := rangeIsFinite(args)
  not(Range case upperInfinite) =>
    [0.0,"d01asf is not a suitable routine for infinite integrals",ext]
  weight: Union(Record(op:BOP,w:DF),"failed") :=
    exprHasWeightCosWXorSinWX(args)
  weight case "failed" =>
    [0.0,"d01asf: A suitable weight has not been found", ext]
  weight case Record(op:BOP,w:DF) =>
    wany := coerce(weight)$AnyFunctions1(Record(op:BOP,w:DF))
    ex:Record(key:S,entry:Any) := [d01asfextra@S,wany]
    ext := insert!(ex,ext)$Result
    ws:ST := string(name(weight.op)$BOP)$S "(" df2st(weight.w)
                    string(args.var)$S ")"
    [getMeasure(R,d01asf@S)$RT,
      "d01asf: A suitable weight has been found:- " ws, ext]

numericalIntegration(args:NIA,hints:Result) ==
  i:INT
  r:Any := coerce(search((d01asfextra@S),hints)$Result)@Any
  rec:Record(op:BOP,w:DF) := retract(r)$AnyFunctions1(Record(op:BOP,w:DF))
  Var := args.var :: EDF
  o:BOP := rec.op
  den:EDF := o((rec.w*Var)$EDF)
  Argsfn:EDF := args.fn/den
  if (name(o) = cos@S)@Boolean then i := 1
  else i := 2
  b:Float := getButtonValue("d01asf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
  iw:INT := 75*fEvals
  ArgsFn := map(x +-> convert(x)$DF,Argsfn)$EF2(DF,Float)
  f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
```

```
    err :=
      positive?(args.abserr) => args.abserr
      args.relerr
    d01asf(getlo(args.range),rec.w,i,err,50,4*iw,2*iw,-1,f)
```

— D01ASFA.dotabb —

```
"D01ASFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01ASFA"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"D01ASFA" -> "ALIST"
```

5.27 domain D01FCFA d01fcfAnnaType

— d01fcfAnnaType.input —

```
)set break resume
)sys rm -f d01fcfAnnaType.output
)spool d01fcfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01fcfAnnaType
--R d01fcfAnnaType  is a domain constructor
--R Abbreviation for d01fcfAnnaType is D01FCFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01FCFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Doub
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

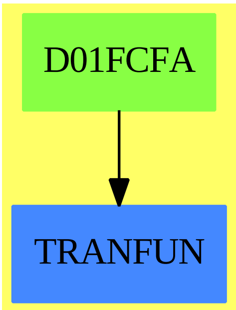
— d01fcfAnnaType.help —

```
====================================================================
d01fcfAnnaType examples
====================================================================

See Also:
o )show d01fcfAnnaType
```

———-

5.27.1 d01fcfAnnaType (D01FCFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01FCFA d01fcfAnnaType —

```
)abbrev domain D01FCFA d01fcfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01fcfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01FCF, a numerical integration routine which can
```

```
++ handle multi-dimensional quadrature over a finite region.  The
++ function \axiomFun{measure} measures the usefulness of the routine D01GBF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01fcfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:MDNIA) ==
    ext:Result := empty()$Result
    segs := args.range
    vars := variables(args.fn)$EDF
    for i in 1..# vars repeat
      nia:NIA := [vars.i,args.fn,segs.i,args.abserr,args.relerr]
      not rangeIsFinite(nia) case finite => return
        [0.0,"d01fcf is not a suitable routine for infinite integrals",ext]
    [getMeasure(R,d01fcf@S)$RT, "Recommended is d01fcf", ext]

  numericalIntegration(args:MDNIA,hints:Result) ==
    import Integer
    segs := args.range
    dim := # segs
    err := args.relerr
    low:Matrix DF := matrix([[getlo(segs.i) for i in 1..dim]])$(Matrix DF)
    high:Matrix DF := matrix([[gethi(segs.i) for i in 1..dim]])$(Matrix DF)
    b:Float := getButtonValue("d01fcf","functionEvaluations")$AttributeButtons
    a:Float:= exp(1.1513*(1.0/(2.0*(1.0-b))))
    alpha:INT := 2**dim+2*dim**2+2*dim+1
    d:Float := max(1.e-3,nthRoot(convert(err)@Float,4))$Float
    minpts:INT := (fEvals := wholePart(a))*wholePart(alpha::Float/d)
    maxpts:INT := 5*minpts
    lenwrk:INT := (dim+2)*(1+(33*fEvals))
    ArgsFn := map(x+->convert(x)$DF,args.fn)$EF2(DF,Float)
```

```
    f : Union(fn:FileName,fp:Asp4(FUNCTN)) := [retract(ArgsFn)$Asp4(FUNCTN)]
    out:Result := d01fcf(dim,low,high,maxpts,err,lenwrk,minpts,-1,f)
    changeName(finval@Symbol,result@Symbol,out)
```

———-

— D01FCFA.dotabb —

```
"D01FCFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01FCFA"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"D01FCFA" -> "TRANFUN"
```

———-

5.28 domain D01GBFA d01gbfAnnaType

— d01gbfAnnaType.input —

```
)set break resume
)sys rm -f d01gbfAnnaType.output
)spool d01gbfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01gbfAnnaType
--R d01gbfAnnaType  is a domain constructor
--R Abbreviation for d01gbfAnnaType is D01GBFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01GBFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCo
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Orde
--R
--E 1

)spool
)lisp (bye)
```

———-

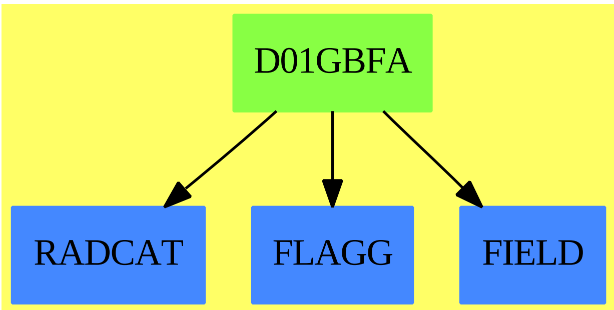
— d01gbfAnnaType.help —

```
====================================================================
d01gbfAnnaType examples
====================================================================

See Also:
o )show d01gbfAnnaType
```

———-

5.28.1 d01gbfAnnaType (D01GBFA)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01GBFA d01gbfAnnaType —

```
)abbrev domain D01GBFA d01gbfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01gbfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01GBF, a numerical integration routine which can
++ handle multi-dimensional quadrature over a finite region.  The
++ function \axiomFun{measure} measures the usefulness of the routine D01GBF
++ for the given problem.  The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.
```

```
d01gbfAnnaType(): NumericalIntegrationCategory == Result add
  EF2  ==> ExpressionFunctions2
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
  DF  ==> DoubleFloat
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA  ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT  ==> Integer
  BOP  ==> BasicOperator
  S  ==> Symbol
  ST  ==> String
  LST  ==> List String
  RT  ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

  measure(R:RT,args:MDNIA) ==
    ext:Result := empty()$Result
    (rel := args.relerr) < 0.01 :: DF =>
      [0.1, "d01gbf: The relative error requirement is too small",ext]
    segs := args.range
    vars := variables(args.fn)$EDF
    for i in 1..# vars repeat
      nia:NIA := [vars.i,args.fn,segs.i,args.abserr,rel]
      not rangeIsFinite(nia) case finite => return
        [0.0,"d01gbf is not a suitable routine for infinite integrals",ext]
    [getMeasure(R,d01gbf@S)$RT, "Recommended is d01gbf", ext]

  numericalIntegration(args:MDNIA,hints:Result) ==
    import Integer
    segs := args.range
    dim:INT := # segs
    low:Matrix DF := matrix([[getlo(segs.i) for i in 1..dim]])$(Matrix DF)
    high:Matrix DF := matrix([[gethi(segs.i) for i in 1..dim]])$(Matrix DF)
    b:Float := getButtonValue("d01gbf","functionEvaluations")$AttributeButtons
    a:Float:= exp(1.1513*(1.0/(2.0*(1.0-b))))
    maxcls:INT := 1500*(dim+1)*(fEvals:INT := wholePart(a))
    mincls:INT := 300*fEvals
    c:Float := nthRoot((maxcls::Float)/4.0,dim)$Float
    lenwrk:INT := 3*dim*(d:INT := wholePart(c))+10*dim
    wrkstr:Matrix DF := matrix([[0$DF for i in 1..lenwrk]])$(Matrix DF)
    ArgsFn := map(x+->convert(x)$DF,args.fn)$EF2(DF,Float)
    f : Union(fn:FileName,fp:Asp4(FUNCTN)) := [retract(ArgsFn)$Asp4(FUNCTN)]
    out:Result := _
       d01gbf(dim,low,high,maxcls,args.relerr,lenwrk,mincls,wrkstr,-1,f)
```

```
changeName(finest@Symbol,result@Symbol,out)
```

———-

— D01GBFA.dotabb —

```
"D01GBFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01GBFA"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"D01GBFA" -> "FIELD"
"D01GBFA" -> "RADCAT"
"D01GBFA" -> "FLAGG"
```

———-

5.29 domain D01TRNS d01TransformFunctionType

— d01TransformFunctionType.input —

```
)set break resume
)sys rm -f d01TransformFunctionType.output
)spool d01TransformFunctionType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01TransformFunctionType
--R d01TransformFunctionType  is a domain constructor
--R Abbreviation for d01TransformFunctionType is D01TRNS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01TRNS
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComplet
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Doub
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

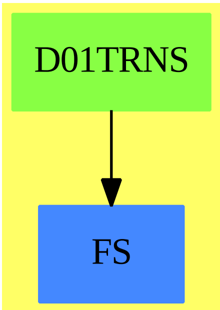
— d01TransformFunctionType.help —

```
====================================================================
d01TransformFunctionType examples
====================================================================

See Also:
o )show d01TransformFunctionType
```

———-

5.29.1 d01TransformFunctionType (D01TRNS)



Exports:
coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01TRNS d01TransformFunctionType —

```
)abbrev domain D01TRNS d01TransformFunctionType
++ Author: Brian Dupee
++ Date Created: April 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ Since an infinite integral cannot be evaluated numerically
++ it is necessary to transform the integral onto finite ranges.
```

```
++ \axiomType{d01TransformFunctionType} uses the mapping \spad{x -> 1/x}
++ and contains the functions \axiomFun{measure} and
++ \axiomFun{numericalIntegration}.

EDF  ==> Expression DoubleFloat
EEDF  ==> Equation Expression DoubleFloat
FI  ==> Fraction Integer
EFI  ==> Expression Fraction Integer
EEFI  ==> Equation Expression Fraction Integer
EF2  ==> ExpressionFunctions2
DF  ==> DoubleFloat
F  ==> Float
SOCDF  ==> Segment OrderedCompletion DoubleFloat
OCDF  ==> OrderedCompletion DoubleFloat
NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
INT     ==> Integer
PI  ==> PositiveInteger
HINT  ==> Record(str:String,fn:EDF,range:SOCDF,ext:Result)
S  ==> Symbol
ST  ==> String
LST  ==> List String
Measure  ==> Record(measure:F,explanations:ST,extra:Result)
MS  ==> Record(measure:F,name:ST,explanations:LST,extra:Result)

d01TransformFunctionType():NumericalIntegrationCategory == Result add
  Rep:=Result
  import d01AgentsPackage,Rep

  rec2any(re:Record(str:ST,fn:EDF,range:SOCDF)):Any ==
    coerce(re)$AnyFunctions1(Record(str:ST,fn:EDF,range:SOCDF))

  changeName(ans:Result,name:ST):Result ==
    sy:S := coerce(name "Answer")$S
    anyAns:Any := coerce(ans)$AnyFunctions1(Result)
    construct([[sy,anyAns]])$Result

  getIntegral(args:NIA,hint:HINT) : Result ==
   Args := copy args
   Args.fn := hint.fn
   Args.range := hint.range
   integrate(Args::NumericalIntegrationProblem)$AnnaNumericalIntegrationPackage

  transformFunction(args:NIA) : NIA ==
    Args := copy args
    Var := Args.var :: EFI                 -- coerce Symbol to EFI
    NewVar:EFI := inv(Var)$EFI             -- invert it
    VarEqn:EEFI:=equation(Var,NewVar)$EEFI -- turn it into an equation
    Afn:EFI := edf2efi(Args.fn)$ExpertSystemToolsPackage
    Afn := subst(Afn,VarEqn)$EFI           -- substitute into function
    Var2:EFI := Var**2
```

```
  Afn:= simplify(Afn/Var2)$TranscendentalManipulations(FI,EFI)
  Args.fn:= map(x+->convert(x)$FI,Afn)$EF2(FI,DF)
  Args

doit(seg:SOCDF,args:NIA):MS ==
  Args := copy args
  Args.range := seg
  measure(Args::NumericalIntegrationProblem)$AnnaNumericalIntegrationPackage

transform(c:Boolean,args:NIA):Measure ==
  if c then
    l := coerce(recip(lo(args.range)))@OCDF
    Seg:SOCDF := segment(0$OCDF,l)
  else
    h := coerce(recip(hi(args.range)))@OCDF
    Seg:SOCDF := segment(h,0$OCDF)
  Args := transformFunction(args)
  m:MS := doit(Seg,Args)
  out1:ST :=
     "The recommendation is to transform the function and use " m.name
  out2:List(HINT) := [[m.name,Args.fn,Seg,m.extra]]
  out2Any:Any := coerce(out2)$AnyFunctions1(List(HINT))
  ex:Record(key:S,entry:Any) := [d01transformextra@S,out2Any]
  extr:Result := construct([ex])$Result
  [m.measure,out1,extr]

split(c:PI,args:NIA):Measure ==
  Args := copy args
  Args.relerr := Args.relerr/2
  Args.abserr := Args.abserr/2
  if (c = 1)@Boolean then
    seg1:SOCDF := segment(-1$OCDF,1$OCDF)
  else if (c = 2)@Boolean then
    seg1 := segment(lo(Args.range),1$OCDF)
  else
    seg1 := segment(-1$OCDF,hi(Args.range))
  m1:MS := doit(seg1,Args)
  Args := transformFunction Args
  if (c = 2)@Boolean then
    seg2:SOCDF := segment(0$OCDF,1$OCDF)
  else if (c = 3)@Boolean then
    seg2 := segment(-1$OCDF,0$OCDF)
  else seg2 := seg1
  m2:MS := doit(seg2,Args)
  m1m:F := m1.measure
  m2m:F := m2.measure
  m:F := m1m*m2m/((m1m*m2m)+(1.0-m1m)*(1.0-m2m))
  out1:ST := "The recommendation is to transform the function and use "
                           m1.name " and " m2.name
  out2:List(HINT) :=
```

```
          [[m1.name,args.fn,seg1,m1.extra],[m2.name,Args.fn,seg2,m2.extra]]
  out2Any:Any := coerce(out2)$AnyFunctions1(List(HINT))
  ex:Record(key:S,entry:Any) := [d01transformextra@S,out2Any]
  extr:Result := construct([ex])$Result
  [m,out1,extr]

measure(R:RoutinesTable,args:NIA) ==
  Range:=rangeIsFinite(args)
  Range case bothInfinite => split(1,args)
  Range case upperInfinite =>
    positive?(lo(args.range))$OCDF =>
      transform(true,args)
    split(2,args)
  Range case lowerInfinite =>
    negative?(hi(args.range))$OCDF =>
      transform(false,args)
    split(3,args)

numericalIntegration(args:NIA,hints:Result) ==
  mainResult:DF := mainAbserr:DF := 0$DF
  ans:Result := empty()$Result
  hla:Any := coerce(search((d01transformextra@S),hints)$Result)@Any
  hintList := retract(hla)$AnyFunctions1(List(HINT))
  methodName:ST := empty()$ST
  repeat
    if (empty?(hintList)$(List(HINT)))
      then leave
    item := first(hintList)$List(HINT)
    a:Result := getIntegral(args,item)
    anyRes := coerce(search((result@S),a)$Result)@Any
    midResult := retract(anyRes)$AnyFunctions1(DF)
    anyErr := coerce(search((abserr pretend S),a)$Result)@Any
    midAbserr := retract(anyErr)$AnyFunctions1(DF)
    mainResult := mainResult+midResult
    mainAbserr := mainAbserr+midAbserr
    if (methodName = item.str)@Boolean then
      methodName := concat([item.str,"1"])$ST
    else
      methodName := item.str
    ans := concat(ans,changeName(a,methodName))$ExpertSystemToolsPackage
    hintList := rest(hintList)$(List(HINT))
  anyResult := coerce(mainResult)$AnyFunctions1(DF)
  anyAbserr := coerce(mainAbserr)$AnyFunctions1(DF)
  recResult:Record(key:S,entry:Any):=[result@S,anyResult]
  recAbserr:Record(key:S,entry:Any):=[abserr pretend S,anyAbserr]
  insert!(recAbserr,insert!(recResult,ans))$Result
```

———-

— D01TRNS.dotabb —

```
"D01TRNS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01TRNS"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"D01TRNS" -> "FS"
```

5.30 domain D02BBFA d02bbfAnnaType

— d02bbfAnnaType.input —

```
)set break resume
)sys rm -f d02bbfAnnaType.output
)spool d02bbfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02bbfAnnaType
--R d02bbfAnnaType  is a domain constructor
--R Abbreviation for d02bbfAnnaType is D02BBFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02BBFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expressic
--R
--E 1

)spool
)lisp (bye)
```

— d02bbfAnnaType.help —

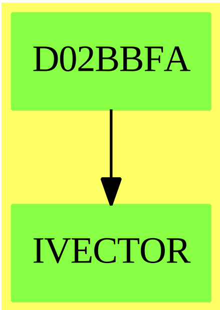
```
====================================================================
d02bbfAnnaType examples
```

```
====================================================================

See Also:
o )show d02bbfAnnaType
```

———-

5.30.1 d02bbfAnnaType (D02BBFA)



Exports:
coerce hash latex measure ODESolve ?=? ?~=?

— **domain D02BBFA d02bbfAnnaType** —

```
)abbrev domain D02BBFA d02bbfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02bbfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02BBF, a ODE routine which uses an
++ Runge-Kutta method to solve a system of differential
++ equations.  The function \axiomFun{measure} measures the
++ usefulness of the routine D02BBF for the given problem.  The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02bbfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
  -- Runge Kutta

  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
```

```
MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
SOCDF ==> Segment OrderedCompletion DoubleFloat
VEDF ==> Vector Expression DoubleFloat
VEF ==> Vector Expression Float
EF ==> Expression Float
VDF ==> Vector DoubleFloat
VMF ==> Vector MachineFloat
MF ==> MachineFloat
ODEA ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
                    g:EDF,abserr:DF,relerr:DF)
RSS ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT ==> Integer
EF2 ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage
import AttributeButtons

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02bbf","accuracy")$AttributeButtons
  accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.999 => 0$F
  0.8*exp(-((10*accuracyIntensityValue)**3)$F/266)$F

stiffnessCF(stiffnessIntensityValue:F):F ==
  b := getButtonValue("d02bbf","stiffness")$AttributeButtons
  0.5*exp(-(2*combineFeatureCompatibility(b,stiffnessIntensityValue))**2)$F

stabilityCF(stabilityIntensityValue:F):F ==
  b := getButtonValue("d02bbf","stability")$AttributeButtons
  0.5 * cos(combineFeatureCompatibility(b,stabilityIntensityValue))$F

expenseOfEvaluationCF(ode:ODEA):F ==
  b := getButtonValue("d02bbf","expense")$AttributeButtons
  expenseOfEvaluationIntensityValue :=
    combineFeatureCompatibility(b,expenseOfEvaluationIF(ode))
  0.35+0.2*exp(-(2.0*expenseOfEvaluationIntensityValue)**3)$F

measure(R:RoutinesTable,args:ODEA) ==
  m := getMeasure(R,d02bbf :: Symbol)$RoutinesTable
  ssf := stiffnessAndStabilityOfODEIF args
  m := combineFeatureCompatibility(m,[accuracyCF(args),
          stiffnessCF(ssf.stiffnessFactor),
            expenseOfEvaluationCF(args),
              stabilityCF(ssf.stabilityFactor)])
  [m,"Runge-Kutta Merson method"]
```

```
  ODESolve(ode:ODEA) ==
    i:LDF := ode.intvals
    M := inc(# i)$INT
    irelab := 0$INT
    if positive?(a := ode.abserr) then
      inc(irelab)$INT
    if positive?(r := ode.relerr) then
      inc(irelab)$INT
    if positive?(a+r) then
      tol:DF := a + r
    else
      tol := float(1,-4,10)$DF
    asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
      [retract(vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
    asp8:Union(fn:FileName,fp:Asp8(OUTPUT)) :=
      [coerce(ldf2vmf(i)$ExpertSystemToolsPackage)$Asp8(OUTPUT)]
    d02bbf(ode.xend,M,# ode.fn,irelab,ode.xinit,matrix([ode.yinit])$MDF,
             tol,-1,asp7,asp8)
```

— **D02BBFA.dotabb** —

```
"D02BBFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02BBFA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"D02BBFA" -> "IVECTOR"
```

5.31 domain D02BHFA d02bhfAnnaType

— **d02bhfAnnaType.input** —

```
)set break resume
)sys rm -f d02bhfAnnaType.output
)spool d02bhfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02bhfAnnaType
--R d02bhfAnnaType  is a domain constructor
--R Abbreviation for d02bhfAnnaType is D02BHFA
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02BHFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expressi
--R
--E 1

)spool
)lisp (bye)
```

———-

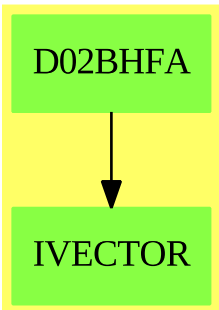
— **d02bhfAnnaType.help** —

```
====================================================================
d02bhfAnnaType examples
====================================================================

See Also:
o )show d02bhfAnnaType
```

———-

5.31.1 d02bhfAnnaType (D02BHFA)



Exports:
coerce hash latex measure ODESolve ?=? ?~=?

— **domain D02BHFA d02bhfAnnaType** —

```
)abbrev domain D02BHFA d02bhfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02bhfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02BHF, a ODE routine which uses an
++ Runge-Kutta method to solve a system of differential
++ equations.  The function \axiomFun{measure} measures the
++ usefulness of the routine D02BHF for the given problem.  The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02bhfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
  -- Runge Kutta
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  MDF  ==> Matrix DoubleFloat
  DF  ==> DoubleFloat
  F  ==> Float
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  VEDF  ==> Vector Expression DoubleFloat
  VEF  ==> Vector Expression Float
  EF  ==> Expression Float
  VDF  ==> Vector DoubleFloat
  VMF  ==> Vector MachineFloat
  MF  ==> MachineFloat
  ODEA  ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
                    g:EDF,abserr:DF,relerr:DF)
  RSS  ==> Record(stiffnessFactor:F,stabilityFactor:F)
  INT  ==> Integer
  EF2  ==> ExpressionFunctions2

  import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage
  import AttributeButtons

  accuracyCF(ode:ODEA):F ==
    b := getButtonValue("d02bhf","accuracy")$AttributeButtons
    accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
    accuracyIntensityValue > 0.999 => 0$F
    0.8*exp(-((10*accuracyIntensityValue)**3)$F/266)$F

  stiffnessCF(stiffnessIntensityValue:F):F ==
    b := getButtonValue("d02bhf","stiffness")$AttributeButtons
    0.5*exp(-(2*combineFeatureCompatibility(b,stiffnessIntensityValue))**2)$F
```

```
  stabilityCF(stabilityIntensityValue:F):F ==
    b := getButtonValue("d02bhf","stability")$AttributeButtons
    0.5 * cos(combineFeatureCompatibility(b,stabilityIntensityValue))$F

  expenseOfEvaluationCF(ode:ODEA):F ==
    b := getButtonValue("d02bhf","expense")$AttributeButtons
    expenseOfEvaluationIntensityValue :=
      combineFeatureCompatibility(b,expenseOfEvaluationIF(ode))
    0.35+0.2*exp(-(2.0*expenseOfEvaluationIntensityValue)**3)$F

  measure(R:RoutinesTable,args:ODEA) ==
    m := getMeasure(R,d02bhf :: Symbol)$RoutinesTable
    ssf := stiffnessAndStabilityOfODEIF args
    m := combineFeatureCompatibility(m,[accuracyCF(args),
            stiffnessCF(ssf.stiffnessFactor),
              expenseOfEvaluationCF(args),
                stabilityCF(ssf.stabilityFactor)])
    [m,"Runge-Kutta Merson method"]

  ODESolve(ode:ODEA) ==
    irelab := 0$INT
    if positive?(a := ode.abserr) then
      inc(irelab)$INT
    if positive?(r := ode.relerr) then
      inc(irelab)$INT
    if positive?(a+r) then
      tol := max(a,r)$DF
    else
      tol:DF := float(1,-4,10)$DF
    asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
      [retract(e:VEF := vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
    asp9:Union(fn:FileName,fp:Asp9(G)) :=
      [retract(edf2ef(ode.g)$ExpertSystemToolsPackage)$Asp9(G)]
    d02bhf(ode.xend,# e,irelab,0$DF,ode.xinit,matrix([ode.yinit])$MDF,
             tol,-1,asp9,asp7)
```

———-

— **D02BHFA.dotabb** —

```
"D02BHFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02BHFA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"D02BHFA" -> "IVECTOR"
```

———-

5.32 domain D02CJFA d02cjfAnnaType

— d02cjfAnnaType.input —

```
)set break resume
)sys rm -f d02cjfAnnaType.output
)spool d02cjfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02cjfAnnaType
--R d02cjfAnnaType  is a domain constructor
--R Abbreviation for d02cjfAnnaType is D02CJFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02CJFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat,yinit: List
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFlo
--R
--E 1

)spool
)lisp (bye)
```

———

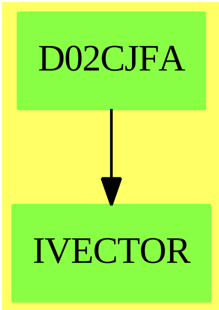
— d02cjfAnnaType.help —

```
====================================================================
d02cjfAnnaType examples
====================================================================

See Also:
o )show d02cjfAnnaType
```

———

5.32.1 d02cjfAnnaType (D02CJFA)



Exports:
coerce hash latex measure ODESolve ?=? ?~=?

— domain D02CJFA d02cjfAnnaType —

```
)abbrev domain D02CJFA d02cjfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02cjfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02CJF, a ODE routine which uses an
++ Adams-Moulton-Bashworth method to solve a system of differential
++ equations.  The function \axiomFun{measure} measures the
++ usefulness of the routine D02CJF for the given problem.  The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02cjfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
  -- Adams
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  MDF  ==> Matrix DoubleFloat
  DF  ==> DoubleFloat
  F  ==> Float
  FI  ==> Fraction Integer
  EFI  ==> Expression Fraction Integer
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  VEDF  ==> Vector Expression DoubleFloat
  VEF  ==> Vector Expression Float
  EF  ==> Expression Float
  VDF  ==> Vector DoubleFloat
  VMF  ==> Vector MachineFloat
```

```
MF  ==> MachineFloat
ODEA  ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
                    g:EDF,abserr:DF,relerr:DF)
RSS  ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT  ==> Integer
EF2  ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02cjf","accuracy")$AttributeButtons
  accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.9999 => 0$F
  0.6*(cos(accuracyIntensityValue*(pi()$F)/2)$F)**0.755

stiffnessCF(ode:ODEA):F ==
  b := getButtonValue("d02cjf","stiffness")$AttributeButtons
  ssf := stiffnessAndStabilityOfODEIF ode
  stiffnessIntensityValue :=
    combineFeatureCompatibility(b,ssf.stiffnessFactor)
  0.5*exp(-(1.1*stiffnessIntensityValue)**3)$F

measure(R:RoutinesTable,args:ODEA) ==
  m := getMeasure(R,d02cjf :: Symbol)$RoutinesTable
  m := combineFeatureCompatibility(m,[accuracyCF(args), stiffnessCF(args)])
  [m,"Adams method"]

ODESolve(ode:ODEA) ==
  i:LDF := ode.intvals
  if empty?(i) then
    i := [ode.xend]
  M := inc(# i)$INT
  if positive?((a := ode.abserr)*(r := ode.relerr))$DF then
    ire:String := "D"
  else
    if positive?(a) then
      ire:String := "A"
    else
      ire:String := "R"
  tol := max(a,r)$DF
  asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
    [retract(e:VEF := vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
  asp8:Union(fn:FileName,fp:Asp8(OUTPUT)) :=
    [coerce(ldf2vmf(i)$ExpertSystemToolsPackage)$Asp8(OUTPUT)]
  asp9:Union(fn:FileName,fp:Asp9(G)) :=
    [retract(edf2ef(ode.g)$ExpertSystemToolsPackage)$Asp9(G)]
  d02cjf(ode.xend,M,# e,tol,ire,ode.xinit,matrix([ode.yinit])$MDF,
          -1,asp9,asp7,asp8)
```

— D02CJFA.dotabb —

```
"D02CJFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02CJFA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"D02CJFA" -> "IVECTOR"
```

5.33 domain D02EJFA d02ejfAnnaType

— d02ejfAnnaType.input —

```
)set break resume
)sys rm -f d02ejfAnnaType.output
)spool d02ejfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02ejfAnnaType
--R d02ejfAnnaType  is a domain constructor
--R Abbreviation for d02ejfAnnaType is D02EJFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02EJFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expressio
--R
--E 1

)spool
)lisp (bye)
```

— d02ejfAnnaType.help —

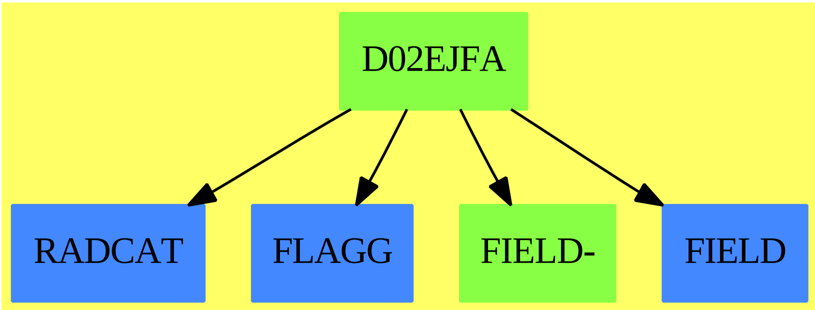
```
====================================================================
```

```
d02ejfAnnaType examples
====================================================================

See Also:
o )show d02ejfAnnaType
```

———-

5.33.1 d02ejfAnnaType (D02EJFA)



Exports:
coerce hash latex measure ODESolve ?=? ?~=?

— **domain D02EJFA d02ejfAnnaType** —

```
)abbrev domain D02EJFA d02ejfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02ejfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02EJF, a ODE routine which uses a backward
++ differentiation formulae method to handle a stiff system
++ of differential equations.  The function \axiomFun{measure} measures
++ the usefulness of the routine D02EJF for the given problem.  The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02ejfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
  -- BDF "Stiff"
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
```

```
MDF  ==> Matrix DoubleFloat
DF  ==> DoubleFloat
F  ==> Float
FI  ==> Fraction Integer
EFI  ==> Expression Fraction Integer
SOCDF  ==> Segment OrderedCompletion DoubleFloat
VEDF  ==> Vector Expression DoubleFloat
VEF  ==> Vector Expression Float
EF  ==> Expression Float
VDF  ==> Vector DoubleFloat
VMF  ==> Vector MachineFloat
MF  ==> MachineFloat
ODEA  ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
                    g:EDF,abserr:DF,relerr:DF)
RSS  ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT  ==> Integer
EF2  ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02ejf","accuracy")$AttributeButtons
  accuracyIntensityValue :=  combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.999 => 0$F
  0.5*exp(-((10*accuracyIntensityValue)**3)$F/250)$F

intermediateResultsCF(ode:ODEA):F ==
  intermediateResultsIntensityValue := intermediateResultsIF(ode)
  i := 0.5 * exp(-(intermediateResultsIntensityValue/1.649)**3)$F
  a := accuracyCF(ode)
  i+(0.5-i)*(0.5-a)

stabilityCF(ode:ODEA):F ==
  b := getButtonValue("d02ejf","stability")$AttributeButtons
  ssf := stiffnessAndStabilityOfODEIF ode
  stabilityIntensityValue :=
    combineFeatureCompatibility(b,ssf.stabilityFactor)
  0.68 - 0.5 * exp(-(stabilityIntensityValue)**3)$F

expenseOfEvaluationCF(ode:ODEA):F ==
  b := getButtonValue("d02ejf","expense")$AttributeButtons
  expenseOfEvaluationIntensityValue :=
    combineFeatureCompatibility(b,expenseOfEvaluationIF(ode))
  0.5 * exp(-(1.7*expenseOfEvaluationIntensityValue)**3)$F

systemSizeCF(args:ODEA):F ==
  (1$F - systemSizeIF(args))/2.0

measure(R:RoutinesTable,args:ODEA) ==
  arg := copy args
```

```
    m := getMeasure(R,d02ejf :: Symbol)$RoutinesTable
    m := combineFeatureCompatibility(m,[intermediateResultsCF(arg),
           accuracyCF(arg),
             systemSizeCF(arg),
               expenseOfEvaluationCF(arg),
                 stabilityCF(arg)])
    [m,"BDF method for Stiff Systems"]

  ODESolve(ode:ODEA) ==
    i:LDF := ode.intvals
    m := inc(# i)$INT
    if positive?((a := ode.abserr)*(r := ode.relerr))$DF then
      ire:String := "D"
    else
      if positive?(a) then
        ire:String := "A"
      else
        ire:String := "R"
    if positive?(a+r)$DF then
      tol := max(a,r)$DF
    else
      tol := float(1,-4,10)$DF
    asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
      [retract(e:VEF := vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
    asp31:Union(fn:FileName,fp:Asp31(PEDERV)) :=
      [retract(e)$Asp31(PEDERV)]
    asp8:Union(fn:FileName,fp:Asp8(OUTPUT)) :=
      [coerce(ldf2vmf(i)$ExpertSystemToolsPackage)$Asp8(OUTPUT)]
    asp9:Union(fn:FileName,fp:Asp9(G)) :=
      [retract(edf2ef(ode.g)$ExpertSystemToolsPackage)$Asp9(G)]
    n:INT := # ode.yinit
    iw:INT := (12+n)*n+50
    ans := d02ejf(ode.xend,m,n,ire,iw,ode.xinit,matrix([ode.yinit])$MDF,
             tol,-1,asp9,asp7,asp31,asp8)
```

— D02EJFA.dotabb —

```
"D02EJFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02EJFA"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FIELD-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FIELD"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"D02EJFA" -> "FLAGG"
"D02EJFA" -> "FIELD-"
"D02EJFA" -> "FIELD"
"D02EJFA" -> "RADCAT"
```

5.34 domain D03EEFA d03eefAnnaType

— d03eefAnnaType.input —

```
)set break resume
)sys rm -f d03eefAnnaType.output
)spool d03eefAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d03eefAnnaType
--R d03eefAnnaType  is a domain constructor
--R Abbreviation for d03eefAnnaType is D03EEFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D03EEFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R PDESolve : Record(pde: List Expression DoubleFloat,constraints: List Record(start: Double
--R measure : (RoutinesTable,Record(pde: List Expression DoubleFloat,constraints: List Record
--R
--E 1

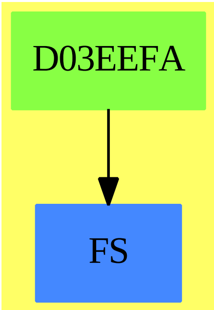
)spool
)lisp (bye)
```

— d03eefAnnaType.help —

```
====================================================================
d03eefAnnaType examples
====================================================================

See Also:
o )show d03eefAnnaType
```

5.34.1 d03eefAnnaType (D03EEFA)



Exports:
coerce hash latex measure PDESolve ?=? ?~=?

— **domain D03EEFA d03eefAnnaType** —

```
)abbrev domain D03EEFA d03eefAnnaType
++ Author: Brian Dupee
++ Date Created: June 1996
++ Date Last Updated: June 1996
++ Basic Operations:
++ Description:
++ \axiomType{d03eefAnnaType} is a domain of
++ \axiomType{PartialDifferentialEquationsSolverCategory}
++ for the NAG routines D03EEF/D03EDF.

d03eefAnnaType():PartialDifferentialEquationsSolverCategory == Result add
  -- 2D Elliptic PDE
  LEDF  ==> List Expression DoubleFloat
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  MDF  ==> Matrix DoubleFloat
  DF  ==> DoubleFloat
  F  ==> Float
  FI  ==> Fraction Integer
  VEF  ==> Vector Expression Float
  EF  ==> Expression Float
  MEF  ==> Matrix Expression Float
  NNI  ==> NonNegativeInteger
  INT  ==> Integer
  PDEC  ==> Record(start:DF, finish:DF, grid:NNI, boundaryType:INT,
                    dStart:MDF, dFinish:MDF)
  PDEB  ==> Record(pde:LEDF, constraints:List PDEC,
                    f:List LEDF, st:String, tol:DF)

  import d03AgentsPackage, NagPartialDifferentialEquationsPackage
```

```
import ExpertSystemToolsPackage

measure(R:RoutinesTable,args:PDEB) ==
  (# (args.constraints) > 2)@Boolean =>
    [0$F,"d03eef/d03edf is unsuitable for PDEs of order more than 2"]
  elliptic?(args) =>
    m := getMeasure(R,d03eef :: Symbol)$RoutinesTable
    [m,"d03eef/d03edf is suitable"]
  [0$F,"d03eef/d03edf is unsuitable for hyperbolic or parabolic PDEs"]

PDESolve(args:PDEB) ==
  xcon := first(args.constraints)
  ycon := second(args.constraints)
  nx := xcon.grid
  ny := ycon.grid
  p := args.pde
  x1 := xcon.start
  x2 := xcon.finish
  y1 := ycon.start
  y2 := ycon.finish
  lda := ((4*(nx+1)*(ny+1)+2) quo 3)$INT
  scheme:String :=
   central?((x2-x1)/2,(y2-y1)/2,args.pde) => "C"
   "U"
  asp73:Union(fn:FileName,fp:Asp73(PDEF)) :=
   [retract(vector([edf2ef u for u in p])$VEF)$Asp73(PDEF)]
  asp74:Union(fn:FileName,fp:Asp74(BNDY)) :=
   [retract(matrix([[edf2ef v for v in w] for w in args.f])$MEF)$Asp74(BNDY)]
  fde := d03eef(x1,x2,y1,y2,nx,ny,lda,scheme,-1,asp73,asp74)
  ub := new(1,nx*ny,0$DF)$MDF
  A := search(a::Symbol,fde)$Result
  A case "failed" => empty()$Result
  AA := A::Any
  fdea := retract(AA)$AnyFunctions1(MDF)
  r := search(rhs::Symbol,fde)$Result
  r case "failed" => empty()$Result
  rh := r::Any
  fderhs := retract(rh)$AnyFunctions1(MDF)
  d03edf(nx,ny,lda,15,args.tol,0,fdea,fderhs,ub,-1)
```

———-

— D03EEFA.dotabb —

```
"D03EEFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D03EEFA"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"D03EEFA" -> "FS"
```

———-

5.35 domain D03FAFA d03fafAnnaType

— d03fafAnnaType.input —

```
)set break resume
)sys rm -f d03fafAnnaType.output
)spool d03fafAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d03fafAnnaType
--R d03fafAnnaType  is a domain constructor
--R Abbreviation for d03fafAnnaType is D03FAFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D03FAFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R PDESolve : Record(pde: List Expression DoubleFloat,constraints: List Record(start: DoubleFloat,finis
--R measure : (RoutinesTable,Record(pde: List Expression DoubleFloat,constraints: List Record(start: Dou
--R
--E 1

)spool
)lisp (bye)
```

———-

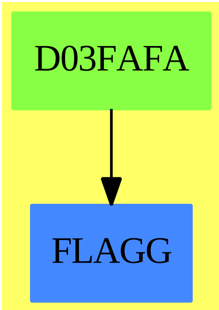
— d03fafAnnaType.help —

```
====================================================================
d03fafAnnaType examples
====================================================================

See Also:
o )show d03fafAnnaType
```

———-

5.35.1 d03fafAnnaType (D03FAFA)



Exports:
coerce hash latex measure PDESolve ?=? ?~=?

— domain D03FAFA d03fafAnnaType —

```
)abbrev domain D03FAFA d03fafAnnaType
++ Author: Brian Dupee
++ Date Created: July 1996
++ Date Last Updated: July 1996
++ Basic Operations:
++ Description:
++ \axiomType{d03fafAnnaType} is a domain of
++ \axiomType{PartialDifferentialEquationsSolverCategory}
++ for the NAG routine D03FAF.

d03fafAnnaType():PartialDifferentialEquationsSolverCategory == Result add
  -- 3D Helmholtz PDE
  LEDF  ==> List Expression DoubleFloat
  EDF  ==> Expression DoubleFloat
  LDF  ==> List DoubleFloat
  MDF  ==> Matrix DoubleFloat
  DF  ==> DoubleFloat
  F  ==> Float
  FI  ==> Fraction Integer
  VEF  ==> Vector Expression Float
  EF  ==> Expression Float
  MEF  ==> Matrix Expression Float
  NNI  ==> NonNegativeInteger
  INT  ==> Integer
  PDEC  ==> Record(start:DF, finish:DF, grid:NNI, boundaryType:INT,
                    dStart:MDF, dFinish:MDF)
  PDEB  ==> Record(pde:LEDF, constraints:List PDEC,
                    f:List LEDF, st:String, tol:DF)

  import d03AgentsPackage, NagPartialDifferentialEquationsPackage
```

```
  import ExpertSystemToolsPackage

  measure(R:RoutinesTable,args:PDEB) ==
    (# (args.constraints) < 3)@Boolean =>
      [0$F,"d03faf is unsuitable for PDEs of order other than 3"]
    [0$F,"d03faf isn't finished"]
```

———-

— **D03FAFAs.dotabb** —

```
"D03FAFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D03FAFA"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"D03FAFA" -> "FLAGG"
```

———-

Chapter 6

Chapter E

6.1 domain EQ Equation

— **Equation.input** —

```
)set break resume
)sys rm -f Equation.output
)spool Equation.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
eq1 := 3*x + 4*y = 5
--R
--R
--R   (1)  4y + 3x= 5
--R                                              Type: Equation Polynomial Integer
--E 1

--S 2 of 12
eq2 := 2*x + 2*y = 3
--R
--R
--R   (2)  2y + 2x= 3
--R                                              Type: Equation Polynomial Integer
--E 2

--S 3 of 12
lhs eq1
--R
--R
--R   (3)  4y + 3x
```

```
--R                                                  Type: Polynomial Integer
--E 3

--S 4 of 12
rhs eq1
--R
--R
--R   (4)  5
--R                                                  Type: Polynomial Integer
--E 4

--S 5 of 12
eq1 + eq2
--R
--R
--R   (5)  6y + 5x= 8
--R                                         Type: Equation Polynomial Integer
--E 5

--S 6 of 12
eq1 * eq2
--R
--R
--R          2              2
--R   (6)  8y  + 14x y + 6x = 15
--R                                         Type: Equation Polynomial Integer
--E 6

--S 7 of 12
2*eq2 - eq1
--R
--R
--R   (7)  x= 1
--R                                         Type: Equation Polynomial Integer
--E 7

--S 8 of 12
eq1**2
--R
--R
--R           2              2
--R   (8)  16y  + 24x y + 9x = 25
--R                                         Type: Equation Polynomial Integer
--E 8

--S 9 of 12
if x+1 = y then "equal" else "unequal"
--R
--R
--R   (9)  "unequal"
```

```
--R                                                              Type: String
--E 9

--S 10 of 12
eqpol := x+1 = y
--R
--R
--R   (10)  x + 1= y
--R                                            Type: Equation Polynomial Integer
--E 10

--S 11 of 12
if eqpol then "equal" else "unequal"
--R
--R
--R   (11)  "unequal"
--R                                                              Type: String
--E 11

--S 12 of 12
eqpol::Boolean
--R
--R
--R   (12)  false
--R                                                             Type: Boolean
--E 12
)spool
)lisp (bye)
```

— **Equation.help** —

```
====================================================================
Equation examples
====================================================================

The Equation domain provides equations as mathematical objects.  These
are used, for example, as the input to various solve operations.

Equations are created using the equals symbol, =.

  eq1 := 3*x + 4*y = 5
   4y + 3x= 5
                         Type: Equation Polynomial Integer

  eq2 := 2*x + 2*y = 3
   2y + 2x= 3
                         Type: Equation Polynomial Integer
```

The left- and right-hand sides of an equation are accessible using the operations lhs and rhs.

```
  lhs eq1
   4y + 3x
                         Type: Polynomial Integer

  rhs eq1
   5
                         Type: Polynomial Integer
```

Arithmetic operations are supported and operate on both sides of the equation.

```
  eq1 + eq2
   6y + 5x= 8
                         Type: Equation Polynomial Integer

  eq1 * eq2
     2            2
   8y  + 14x y + 6x = 15
                         Type: Equation Polynomial Integer

  2*eq2 - eq1
   x= 1
                         Type: Equation Polynomial Integer
```

Equations may be created for any type so the arithmetic operations will be defined only when they make sense. For example, exponentiation is not defined for equations involving non-square matrices.

```
  eq1**2
      2             2
   16y  + 24x y + 9x = 25
                          Type: Equation Polynomial Integer
```

Note that an equals symbol is also used to test for equality of values in certain contexts. For example, x+1 and y are unequal as polynomials.

```
  if x+1 = y then "equal" else "unequal"
   "unequal"
                           Type: String

  eqpol := x+1 = y
   x + 1= y
                           Type: Equation Polynomial Integer
```

If an equation is used where a Boolean value is required, then it is evaluated using the equality test from the operand type.

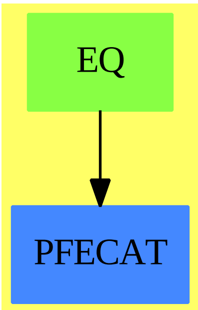
```
  if eqpol then "equal" else "unequal"
   "unequal"
                              Type: String

If one wants a Boolean value rather than an equation, all one has to
do is ask!

  eqpol::Boolean
   false
                              Type: Boolean

See Also:
o )show Equation
```

6.1.1 Equation (EQ)



Exports:

0	1	characteristic	coerce	commutator
conjugate	D	differentiate	dimension	equation
eval	factorAndSplit	hash	inv	latex
leftOne	leftZero	lhs	map	one?
recip	rhs	rightOne	rightZero	sample
subst	subtractIfCan	swap	zero?	?~=?
-?	?=?	?*?	?**?	?+?
?-?	?/?	?=?	?^?	

— **domain EQ Equation** —

```
)abbrev domain EQ Equation
--FOR THE BENEFIT  OF LIBAXO GENERATION
```

```
++ Author: Stephen M. Watt, enhancements by Johannes Grabmeier
++ Date Created: April 1985
++ Date Last Updated: June 3, 1991; September 2, 1992
++ Basic Operations: =
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ Equations as mathematical objects.  All properties of the basis domain,
++ e.g. being an abelian group are carried over the equation domain, by
++ performing the structural operations on the left and on the
++ right hand side.
--   The interpreter translates "=" to "equation".  Otherwise, it will
--   find a modemap for "=" in the domain of the arguments.

Equation(S: Type): public == private where
  Ex ==> OutputForm
  public ==> Type with
    "=": (S, S) -> $
        ++ a=b creates an equation.
    equation: (S, S) -> $
        ++ equation(a,b) creates an equation.
    swap: $ -> $
        ++ swap(eq) interchanges left and right hand side of equation eq.
    lhs: $ -> S
        ++ lhs(eqn) returns the left hand side of equation eqn.
    rhs: $ -> S
        ++ rhs(eqn) returns the right hand side of equation eqn.
    map: (S -> S, $) -> $
        ++ map(f,eqn) constructs a new equation by applying f to both
        ++ sides of eqn.
    if S has InnerEvalable(Symbol,S) then
             InnerEvalable(Symbol,S)
    if S has SetCategory then
        SetCategory
        CoercibleTo Boolean
        if S has Evalable(S) then
           eval: ($, $) -> $
               ++ eval(eqn, x=f) replaces x by f in equation eqn.
           eval: ($, List $) -> $
               ++ eval(eqn, [x1=v1, ... xn=vn]) replaces xi by vi in equation eqn.
    if S has AbelianSemiGroup then
        AbelianSemiGroup
        "+": (S, $) -> $
            ++ x+eqn produces a new equation by adding x to both sides of
            ++ equation eqn.
        "+": ($, S) -> $
```

```
        ++ eqn+x produces a new equation by adding x to  both sides of
        ++ equation eqn.
if S has AbelianGroup then
    AbelianGroup
    leftZero : $ -> $
      ++ leftZero(eq) subtracts the left hand side.
    rightZero : $ -> $
      ++ rightZero(eq) subtracts the right hand side.
    "-": (S, $) -> $
        ++ x-eqn produces a new equation by subtracting both sides of
        ++ equation eqn from x.
    "-": ($, S) -> $
        ++ eqn-x produces a new equation by subtracting x from  both sides of
        ++ equation eqn.
if S has SemiGroup then
    SemiGroup
    "*": (S, $) -> $
        ++ x*eqn produces a new equation by multiplying both sides of
        ++ equation eqn by x.
    "*": ($, S) -> $
        ++ eqn*x produces a new equation by multiplying both sides of
        ++ equation eqn by x.
if S has Monoid then
    Monoid
    leftOne : $ -> Union($,"failed")
      ++ leftOne(eq) divides by the left hand side, if possible.
    rightOne : $ -> Union($,"failed")
      ++ rightOne(eq) divides by the right hand side, if possible.
if S has Group then
    Group
    leftOne : $ -> Union($,"failed")
      ++ leftOne(eq) divides by the left hand side.
    rightOne : $ -> Union($,"failed")
      ++ rightOne(eq) divides by the right hand side.
if S has Ring then
  Ring
  BiModule(S,S)
if S has CommutativeRing then
  Module(S)
  --Algebra(S)
if S has IntegralDomain then
  factorAndSplit : $ -> List $
    ++ factorAndSplit(eq) make the right hand side 0 and
    ++ factors the new left hand side. Each factor is equated
    ++ to 0 and put into the resulting list without repetitions.
if S has PartialDifferentialRing(Symbol) then
  PartialDifferentialRing(Symbol)
if S has Field then
  VectorSpace(S)
  "/": ($, $) -> $
```

```
        ++ e1/e2 produces a new equation by dividing the left and right
        ++ hand sides of equations e1 and e2.
    inv: $ -> $
        ++ inv(x) returns the multiplicative inverse of x.
  if S has ExpressionSpace then
      subst: ($, $) -> $
           ++ subst(eq1,eq2) substitutes eq2 into both sides of eq1
           ++ the lhs of eq2 should be a kernel

private ==> add
  Rep := Record(lhs: S, rhs: S)
  eq1,eq2: $
  s : S
  if S has IntegralDomain then
      factorAndSplit eq ==
        (S has factor : S -> Factored S) =>
          eq0 := rightZero eq
          [equation(rcf.factor,0) for rcf in factors factor lhs eq0]
        [eq]
  l:S = r:S      == [l, r]
  equation(l, r) == [l, r]    -- hack!  See comment above.
  lhs eqn        == eqn.lhs
  rhs eqn        == eqn.rhs
  swap eqn     == [rhs eqn, lhs eqn]
  map(fn, eqn)   == equation(fn(eqn.lhs), fn(eqn.rhs))

  if S has InnerEvalable(Symbol,S) then
      s:Symbol
      ls:List Symbol
      x:S
      lx:List S
      eval(eqn,s,x) == eval(eqn.lhs,s,x) = eval(eqn.rhs,s,x)
      eval(eqn,ls,lx) == eval(eqn.lhs,ls,lx) = eval(eqn.rhs,ls,lx)
  if S has Evalable(S) then
      eval(eqn1:$, eqn2:$):$ ==
         eval(eqn1.lhs, eqn2 pretend Equation S) =
             eval(eqn1.rhs, eqn2 pretend Equation S)
      eval(eqn1:$, leqn2:List $):$ ==
         eval(eqn1.lhs, leqn2 pretend List Equation S) =
             eval(eqn1.rhs, leqn2 pretend List Equation S)
  if S has SetCategory then
      eq1 = eq2 == (eq1.lhs = eq2.lhs)@Boolean and
                   (eq1.rhs = eq2.rhs)@Boolean
      coerce(eqn:$):Ex == eqn.lhs::Ex = eqn.rhs::Ex
      coerce(eqn:$):Boolean == eqn.lhs = eqn.rhs
  if S has AbelianSemiGroup then
      eq1 + eq2 == eq1.lhs + eq2.lhs = eq1.rhs + eq2.rhs
      s + eq2 == [s,s] + eq2
      eq1 + s == eq1 + [s,s]
  if S has AbelianGroup then
```

```
      - eq == (- lhs eq) = (-rhs eq)
      s - eq2 == [s,s] - eq2
      eq1 - s == eq1 - [s,s]
      leftZero eq == 0 = rhs eq - lhs eq
      rightZero eq == lhs eq - rhs eq = 0
      0 == equation(0$S,0$S)
      eq1 - eq2 == eq1.lhs - eq2.lhs = eq1.rhs - eq2.rhs
  if S has SemiGroup then
      eq1:$ * eq2:$ == eq1.lhs * eq2.lhs = eq1.rhs * eq2.rhs
      l:S   * eqn:$ == l       * eqn.lhs = l       * eqn.rhs
      l:S * eqn:$  ==  l * eqn.lhs    =    l * eqn.rhs
      eqn:$ * l:S  ==  eqn.lhs * l    =    eqn.rhs * l
      -- We have to be a bit careful here: raising to a +ve integer is OK
      -- (since it's the equivalent of repeated multiplication)
      -- but other powers may cause contradictions
      -- Watch what else you add here! JHD 2/Aug 1990
  if S has Monoid then
      1 == equation(1$S,1$S)
      recip eq ==
        (lh := recip lhs eq) case "failed" => "failed"
        (rh := recip rhs eq) case "failed" => "failed"
        [lh :: S, rh :: S]
      leftOne eq ==
        (re := recip lhs eq) case "failed" => "failed"
        1 = rhs eq * re
      rightOne eq ==
        (re := recip rhs eq) case "failed" => "failed"
        lhs eq * re = 1
  if S has Group then
      inv eq == [inv lhs eq, inv rhs eq]
      leftOne eq == 1 = rhs eq * inv rhs eq
      rightOne eq == lhs eq * inv rhs eq = 1
  if S has Ring then
      characteristic() == characteristic()$S
      i:Integer * eq:$ == (i::S) * eq
  if S has IntegralDomain then
      factorAndSplit eq ==
        (S has factor : S -> Factored S) =>
          eq0 := rightZero eq
          [equation(rcf.factor,0) for rcf in factors factor lhs eq0]
        (S has Polynomial Integer) =>
          eq0 := rightZero eq
          MF ==> MultivariateFactorize(Symbol, IndexedExponents Symbol, _
             Integer, Polynomial Integer)
          p : Polynomial Integer := (lhs eq0) pretend Polynomial Integer
          [equation((rcf.factor) pretend S,0) for rcf in factors factor(p)$MF]
        [eq]
  if S has PartialDifferentialRing(Symbol) then
      differentiate(eq:$, sym:Symbol):$ ==
         [differentiate(lhs eq, sym), differentiate(rhs eq, sym)]
```

```
    if S has Field then
        dimension() == 2 :: CardinalNumber
        eq1:$ / eq2:$ == eq1.lhs / eq2.lhs = eq1.rhs / eq2.rhs
        inv eq == [inv lhs eq, inv rhs eq]
    if S has ExpressionSpace then
        subst(eq1,eq2) ==
            eq3 := eq2 pretend Equation S
            [subst(lhs eq1,eq3),subst(rhs eq1,eq3)]
```

— EQ.dotabb —

```
"EQ" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EQ"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"EQ" -> "PFECAT"
```

6.2 domain EQTBL EqTable

— EqTable.input —

```
)set break resume
)sys rm -f EqTable.output
)spool EqTable.output
)set message test on
)set message auto off
)clear all
--S 1 of 6
e: EqTable(List Integer, Integer) := table()
--R
--R
--R   (1)  table()
--R                                          Type: EqTable(List Integer,Integer)
--E 1

--S 2 of 6
l1 := [1,2,3]
--R
--R
--R   (2)  [1,2,3]
--R                                                   Type: List PositiveInteger
--E 2
```

```
--S 3 of 6
l2 := [1,2,3]
--R
--R
--R   (3)  [1,2,3]
--R                                                   Type: List PositiveInteger
--E 3

--S 4 of 6
e.l1 := 111
--R
--R
--R   (4)  111
--R                                                        Type: PositiveInteger
--E 4

--S 5 of 6
e.l2 := 222
--R
--R
--R   (5)  222
--R                                                        Type: PositiveInteger
--E 5

--S 6 of 6
e.l1
--R
--R
--R   (6)  111
--R                                                        Type: PositiveInteger
--E 6
)spool
)lisp (bye)
```

— EqTable.help —

```
====================================================================
EqTable examples
====================================================================

The EqTable domain provides tables where the keys are compared using
eq?.  Keys are considered equal only if they are the same instance of
a structure.  This is useful if the keys are themselves updatable
structures.  Otherwise, all operations are the same as for type Table.

The operation table is here used to create a table where the keys are
```

```
lists of integers.

  e: EqTable(List Integer, Integer) := table()
   table()
                    Type: EqTable(List Integer,Integer)

These two lists are equal according to =, but not according to eq?.

  l1 := [1,2,3]
   [1,2,3]
                    Type: List PositiveInteger

  l2 := [1,2,3]
   [1,2,3]
                    Type: List PositiveInteger
Because the two lists are not eq?, separate values can be stored under
each.

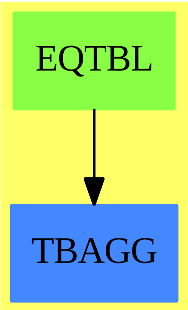
  e.l1 := 111
   111
                    Type: PositiveInteger

  e.l2 := 222
   222
                    Type: PositiveInteger

  e.l1
   111
                    Type: PositiveInteger

See Also:
o )help Table
o )show EqTable
```

6.2.1 EqTable (EQTBL)



See

⇒ "HashTable" (HASHTBL) 9.1.1 on page 1085
⇒ "InnerTable" (INTABL) 10.27.1 on page 1299
⇒ "Table" (TABLE) 21.1.1 on page 2621
⇒ "StringTable" (STRTBL) 20.32.1 on page 2569
⇒ "GeneralSparseTable" (GSTBL) 8.5.1 on page 1044
⇒ "SparseTable" (STBL) 20.16.1 on page 2409

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	table	#?	?=?	?~=?
?.?				

— domain EQTBL EqTable —

```
)abbrev domain EQTBL EqTable
++ Author: Stephen M. Watt
++ Date Created:
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: HashTable, Table, StringTable
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
```

```
++ Description:
++ This domain provides tables where the keys are compared using
++ \spadfun{eq?}.  Thus keys are considered equal only if they
++ are the same instance of a structure.

EqTable(Key: SetCategory, Entry: SetCategory) ==
      HashTable(Key, Entry, "EQ")
```

———-

— EQTBL.dotabb —

```
"EQTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EQTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"EQTBL" -> "TBAGG"
```

———-

6.3 domain EMR EuclideanModularRing

— EuclideanModularRing.input —

```
)set break resume
)sys rm -f EuclideanModularRing.output
)spool EuclideanModularRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show EuclideanModularRing
--R EuclideanModularRing(S: CommutativeRing,R: UnivariatePolynomialCategory S,Mod: AbelianMon
--R Abbreviation for EuclideanModularRing is EMR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EMR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> R                       coerce : % -> %
```

```
--R coerce : Integer -> %                coerce : % -> OutputForm
--R ?.? : (%,R) -> R                     gcd : List % -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R modulus : % -> Mod                   one? : % -> Boolean
--R ?quo? : (%,%) -> %                   recip : % -> Union(%,"failed")
--R reduce : (R,Mod) -> %                ?rem? : (%,%) -> %
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R exQuo : (%,%) -> Union(%,"failed")
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———

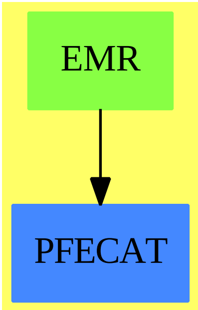
— **EuclideanModularRing.help** —

```
====================================================================
EuclideanModularRing examples
====================================================================

See Also:
o )show EuclideanModularRing
```

———

6.3.1 EuclideanModularRing (EMR)



See

⇒ "ModularRing" (MODRING) 14.10.1 on page 1604
⇒ "ModularField" (MODFIELD) 14.9.1 on page 1602

Exports:

0	1	associates?	characteristic	coerce
divide	euclideanSize	exQuo	expressIdealMember	exquo
extendedEuclidean	gcd	gcdPolynomial	hash	inv
latex	lcm	modulus	multiEuclidean	one?
principalIdeal	recip	reduce	sample	sizeLess?
subtractIfCan	unit?	unitCanonical	unitNormal	zero?
?~=?	?*?	?**?	?^?	?+?
?-?	-?	?=?	?.?	?quo?
?rem?				

— domain EMR EuclideanModularRing —

```
)abbrev domain EMR EuclideanModularRing
++ Author: Mark Botch
++ Description:
++ These domains are used for the factorization and gcds
++ of univariate polynomials over the integers in order to work modulo
++ different  primes.
++ See \spadtype{ModularRing}, \spadtype{ModularField}

EuclideanModularRing(S,R,Mod,reduction:(R,Mod) -> R,
                     merge:(Mod,Mod) -> Union(Mod,"failed"),
                      exactQuo : (R,R,Mod) -> Union(R,"failed")) : C == T
 where
  S    :  CommutativeRing
  R    :  UnivariatePolynomialCategory S
  Mod  :  AbelianMonoid

  C == EuclideanDomain with
   modulus :   %     -> Mod
```

```
    ++ modulus(x) is not documented
  coerce  :   %     -> R
    ++ coerce(x) is not documented
  reduce  : (R,Mod) -> %
    ++ reduce(r,m) is not documented
  exQuo   :  (%,%)  -> Union(%,"failed")
    ++ exQuo(x,y) is not documented
  recip   :    %    -> Union(%,"failed")
    ++ recip(x) is not documented
  inv     :    %    -> %
    ++ inv(x) is not documented
  elt     : (%, R)  -> R
    ++ elt(x,r) or x.r is not documented

 T == ModularRing(R,Mod,reduction,merge,exactQuo) add

   --representation
     Rep:= Record(val:R,modulo:Mod)
   --declarations
     x,y,z: %

     divide(x,y) ==
       t:=merge(x.modulo,y.modulo)
       t case "failed" => error "incompatible moduli"
       xm:=t::Mod
       yv:=y.val
       invlcy:R
--       if one? leadingCoefficient yv then invlcy:=1
       if (leadingCoefficient yv = 1) then invlcy:=1
       else
         invlcy:=(inv reduce((leadingCoefficient yv)::R,xm)).val
         yv:=reduction(invlcy*yv,xm)
       r:=monicDivide(x.val,yv)
       [reduce(invlcy*r.quotient,xm),reduce(r.remainder,xm)]

     if R has fmecg:(R,NonNegativeInteger,S,R)->R
        then x rem y  ==
          t:=merge(x.modulo,y.modulo)
          t case "failed" => error "incompatible moduli"
          xm:=t::Mod
          yv:=y.val
          invlcy:R
--          if not one? leadingCoefficient yv then
          if not (leadingCoefficient yv = 1) then
            invlcy:=(inv reduce((leadingCoefficient yv)::R,xm)).val
            yv:=reduction(invlcy*yv,xm)
          dy:=degree yv
          xv:=x.val
          while (d:=degree xv - dy)>=0 repeat
                xv:=reduction(fmecg(xv,d::NonNegativeInteger,
```

```
                                    leadingCoefficient xv,yv),xm)
                  xv = 0 => return [xv,xm]$Rep
            [xv,xm]$Rep
          else x rem y  ==
            t:=merge(x.modulo,y.modulo)
            t case "failed" => error "incompatible moduli"
            xm:=t::Mod
            yv:=y.val
            invlcy:R
--            if not one? leadingCoefficient yv then
            if not (leadingCoefficient yv = 1) then
              invlcy:=(inv reduce((leadingCoefficient yv)::R,xm)).val
              yv:=reduction(invlcy*yv,xm)
            r:=monicDivide(x.val,yv)
            reduce(r.remainder,xm)

      euclideanSize x == degree x.val

      unitCanonical x ==
        zero? x => x
        degree(x.val) = 0 => 1
--        one? leadingCoefficient(x.val) => x
        (leadingCoefficient(x.val) = 1) => x
        invlcx:%:=inv reduce((leadingCoefficient(x.val))::R,x.modulo)
        invlcx * x

      unitNormal x ==
--        zero?(x) or one?(leadingCoefficient(x.val)) => [1, x, 1]
        zero?(x) or ((leadingCoefficient(x.val)) = 1) => [1, x, 1]
        lcx := reduce((leadingCoefficient(x.val))::R,x.modulo)
        invlcx:=inv lcx
        degree(x.val) = 0 => [lcx, 1, invlcx]
        [lcx, invlcx * x, invlcx]

      elt(x : %,s : R) : R == reduction(elt(x.val,s),x.modulo)
```

——————

— EMR.dotabb —

```
"EMR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EMR"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"EMR" -> "PFECAT"
```

——————

6.4 domain EXIT Exit

— Exit.input —

```
)set break resume
)sys rm -f Exit.output
)spool Exit.output
)set message test on
)set message auto off
)clear all
--S 1
n := 0
--R
--R
--R   (1)  0
--R                                                     Type: NonNegativeInteger
--E 1

--S 2
gasp(): Exit ==
  free n
  n := n + 1
  error "Oh no!"
--R
--R   Function declaration gasp : () -> Exit has been added to workspace.
--R                                                                    Type: Void
--E 2

--S 3
half(k) ==
  if odd? k then gasp()
  else k quo 2
--R
--R                                                                    Type: Void
--E 3

--S 4
half 4
--R
--R   Compiling function gasp with type () -> Exit
--R   Compiling function half with type PositiveInteger -> Integer
--R
--R   (4)  2
--R                                                        Type: PositiveInteger
--E 4

--S 5
half 3
```

```
--R
--R
--RDaly Bug
--R   Error signalled from user code in function gasp:
--R      Oh no!
--E 5

--S 6
n
--R
--R
--R   (5)  1
--R                                                        Type: NonNegativeInteger
--E 6
)spool
)lisp (bye)
```

———-

— Exit.help —

```
====================================================================
Exit examples
====================================================================

A function that does not return directly to its caller has Exit as its
return type.  The operation error is an example of one which does not
return to its caller.  Instead, it causes a return to top-level.

  n := 0

The function gasp is given return type Exit since it is guaranteed
never to return a value to its caller.

  gasp(): Exit ==
    free n
    n := n + 1
    error "Oh no!"

The return type of half is determined by resolving the types of the
two branches of the if.

  half(k) ==
    if odd? k then gasp()
    else k quo 2

Because gasp has the return type Exit, the type of if in half is
resolved to be Integer.
```

```
  half 4

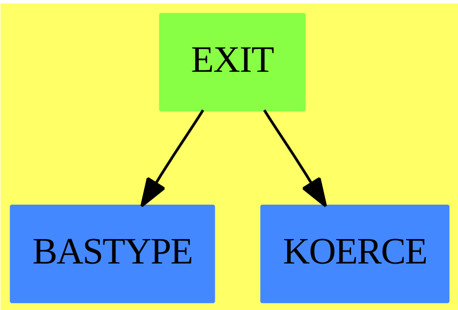
  half 3

  n

See Also:
o )show Exit
```

———-

6.4.1 Exit (EXIT)



Exports:
coerce hash latex ?=? ?~=?

— domain EXIT Exit —

```
)abbrev domain EXIT Exit
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains: ErrorFunctions, ResolveLatticeCompletion, Void
++ Also See:
++ AMS Classifications:
++ Keywords: exit, throw, error, non-local return
++ Examples:
++ References:
++ Description:
++ A function which does not return directly to its caller should
++ have Exit as its return type.
++
```

```
++ Note that It is convenient to have a formal \spad{coerce} into each type
++ from type Exit. This allows, for example, errors to be raised in
++ one half of a type-balanced \spad{if}.

Exit: SetCategory == add
        coerce(n:%) == error "Cannot use an Exit value."
        n1 = n2     == error "Cannot use an Exit value."
```

————

— EXIT.dotabb —

```
"EXIT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXIT"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"EXIT" -> "BASTYPE"
"EXIT" -> "KOERCE"
```

————

6.5 domain EXPEXPAN ExponentialExpansion

— ExponentialExpansion.input —

```
)set break resume
)sys rm -f ExponentialExpansion.output
)spool ExponentialExpansion.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExponentialExpansion
--R ExponentialExpansion(R: Join(OrderedSet,RetractableTo Integer,LinearlyExplicitRingOver In
--R Abbreviation for ExponentialExpansion is EXPEXPAN
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EXPEXPAN
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
```

```
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              denominator : % -> %
--R factor : % -> Factored %              gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R numerator : % -> %                    one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> %
--R ?*? : (UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen),%) -> %
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen),UnivariatePuiseuxSeriesWithEx
--R ?<? : (%,%) -> Boolean if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ORDSET
--R ?<=? : (%,%) -> Boolean if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ORDSE
--R ?>? : (%,%) -> Boolean if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ORDSET
--R ?>=? : (%,%) -> Boolean if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ORDSE
--R D : (%,(UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> UnivariatePuiseuxSeriesWi
--R D : (%,(UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> UnivariatePuiseuxSeriesWi
--R D : (%,List Symbol,List NonNegativeInteger) -> % if UnivariatePuiseuxSeriesWithExponentialSingularit
--R D : (%,Symbol,NonNegativeInteger) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var
--R D : (%,List Symbol) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has PDRI
--R D : (%,Symbol) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has PDRING SY
--R D : (%,NonNegativeInteger) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) h
--R D : % -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has DIFRING
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has OINTDOM
--R ceiling : % -> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) if UnivariatePuiseuxS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and UnivariatePuiseuxSeriesWithExponentialSingul
--R coerce : UnivariatePuiseuxSeries(FE,var,cen) -> %
--R coerce : Symbol -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has RETRACT
--R coerce : UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and UnivariatePuiseuxSeriesWithExp
--R convert : % -> DoubleFloat if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has RE
--R convert : % -> Float if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has REAL
--R convert : % -> InputForm if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has KONV
--R convert : % -> Pattern Float if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has
--R convert : % -> Pattern Integer if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) ha
--R denom : % -> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : (%,(UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> UnivariatePui
```

```
--R differentiate : (%,(UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> Un
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if UnivariatePuiseuxSeriesWi
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if UnivariatePuiseuxSeriesWithExponent
--R differentiate : (%,List Symbol) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity
--R differentiate : (%,Symbol) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE
--R differentiate : (%,NonNegativeInteger) -> % if UnivariatePuiseuxSeriesWithExponentialSing
--R differentiate : % -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if Univari
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if
--R eval : (%,List Symbol,List UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen
--R eval : (%,List Equation UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen))
--R eval : (%,Equation UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> %
--R eval : (%,UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen),UnivariatePuise
--R eval : (%,List UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen),List Univa
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R floor : % -> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) if Univariat
--R fractionPart : % -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R init : () -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has STE
--R limitPlus : % -> Union(OrderedCompletion FE,"failed")
--R map : ((UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> UnivariatePuis
--R max : (%,%) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has O
--R min : (%,%) -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has O
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,ce
--R nextItem : % -> Union(%,"failed") if UnivariatePuiseuxSeriesWithExponentialSingularity(R
--R numer : % -> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R positive? : % -> Boolean if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,ce
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has I
--R reducedSystem : Matrix % -> Matrix UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix UnivariatePuiseuxSeriesWithExpo
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if
--R reducedSystem : Matrix % -> Matrix Integer if UnivariatePuiseuxSeriesWithExponentialSingu
--R retract : % -> UnivariatePuiseuxSeries(FE,var,cen)
--R retract : % -> Integer if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retract : % -> Fraction Integer if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE
--R retract : % -> Symbol if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retract : % -> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retractIfCan : % -> Union(UnivariatePuiseuxSeries(FE,var,cen),"failed")
--R retractIfCan : % -> Union(Integer,"failed") if UnivariatePuiseuxSeriesWithExponentialSing
```

```
--R retractIfCan : % -> Union(Fraction Integer,"failed") if UnivariatePuiseuxSeriesWithExponentialSingul
--R retractIfCan : % -> Union(Symbol,"failed") if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE
--R retractIfCan : % -> Union(UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen),"failed")
--R sign : % -> Integer if UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Univ
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R wholePart : % -> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen) if UnivariatePuiseu
--R
--E 1

)spool
)lisp (bye)
```

———-

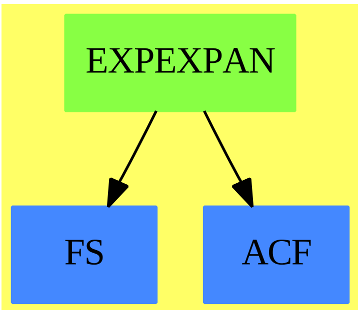
— ExponentialExpansion.help —

```
====================================================================
ExponentialExpansion examples
====================================================================

See Also:
o )show ExponentialExpansion
```

———-

6.5.1 ExponentialExpansion (EXPEXPAN)



See

⇒ "ExponentialOfUnivariatePuiseuxSeries" (EXPUPXS) 6.7.1 on page 707
⇒ "UnivariatePuiseuxSeriesWithExponentialSingularity" (UPXSSING) 22.7.1 on page 2809

Exports:

0	1	associates?
abs	ceiling	characteristic
charthRoot	coerce	conditionP
convert	D	denom
denominator	differentiate	divide
euclideanSize	expressIdealMember	eval
exquo	extendedEuclidean	factor
factorSquareFreePolynomial	factorPolynomial	floor
fractionPart	gcd	gcdPolynomial
hash	init	inv
latex	lcm	limitPlus
map	max	min
multiEuclidean	negative?	nextItem
numer	numerator	one?
patternMatch	positive?	prime?
principalIdeal	random	recip
reducedSystem	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
zero?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?.?
?quo?	?rem?	

— domain EXPEXPAN ExponentialExpansion —

```
)abbrev domain EXPEXPAN ExponentialExpansion
++ Author: Clifton J. Williamson
++ Date Created: 13 August 1992
++ Date Last Updated: 27 August 1992
++ Basic Operations:
++ Related Domains: UnivariatePuiseuxSeries(FE,var,cen),
++                  ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: limit, functional expression, power series
++ Examples:
++ References:
++ Description:
++ UnivariatePuiseuxSeriesWithExponentialSingularity is a domain used to
++ represent essential singularities of functions.  Objects in this domain
++ are quotients of sums, where each term in the sum is a univariate Puiseux
++ series times the exponential of a univariate Puiseux series.
```

```
ExponentialExpansion(R,FE,var,cen): Exports == Implementation where
  R   : Join(OrderedSet,RetractableTo Integer,_
             LinearlyExplicitRingOver Integer,GcdDomain)
  FE  : Join(AlgebraicallyClosedField,TranscendentalFunctionCategory,_
             FunctionSpace R)
  var : Symbol
  cen : FE
  RN       ==> Fraction Integer
  UPXS     ==> UnivariatePuiseuxSeries(FE,var,cen)
  EXPUPXS  ==> ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)
  UPXSSING ==> UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen)
  OFE      ==> OrderedCompletion FE
  Result   ==> Union(OFE,"failed")
  PxRec    ==> Record(k: Fraction Integer,c:FE)
  Term     ==> Record(%coef:UPXS,%expon:EXPUPXS,%expTerms:List PxRec)
  TypedTerm ==> Record(%term:Term,%type:String)
  SIGNEF   ==> ElementaryFunctionSign(R,FE)

  Exports ==> Join(QuotientFieldCategory UPXSSING,RetractableTo UPXS) with
    limitPlus : % -> Union(OFE,"failed")
      ++ limitPlus(f(var)) returns \spad{limit(var -> a+,f(var))}.
    coerce: UPXS -> %
      ++ coerce(f) converts a \spadtype{UnivariatePuiseuxSeries} to
      ++ an \spadtype{ExponentialExpansion}.

  Implementation ==> Fraction(UPXSSING) add
    coeff : Term -> UPXS
    exponent : Term -> EXPUPXS
    upxssingIfCan : % -> Union(UPXSSING,"failed")
    seriesQuotientLimit: (UPXS,UPXS) -> Union(OFE,"failed")
    seriesQuotientInfinity: (UPXS,UPXS) -> Union(OFE,"failed")

    Rep := Fraction UPXSSING

    ZEROCOUNT : RN := 1000/1

    coeff term == term.%coef
    exponent term == term.%expon

    --!! why is this necessary?
    --!! code can run forever in retractIfCan if original assignment
    --!! for 'ff' is used
    upxssingIfCan f ==
--      one? denom f => numer f
      (denom f = 1) => numer f
      "failed"

    retractIfCan(f:%):Union(UPXS,"failed") ==
      --ff := (retractIfCan$Rep)(f)@Union(UPXSSING,"failed")
```

```
    --ff case "failed" => "failed"
    (ff := upxssingIfCan f) case "failed" => "failed"
    (fff := retractIfCan(ff::UPXSSING)@Union(UPXS,"failed")) case "failed" =>
      "failed"
    fff :: UPXS

  f:UPXSSING / g:UPXSSING ==
    (rec := recip g) case "failed" => f /$Rep g
    f * (rec :: UPXSSING) :: %

  f:% / g:% ==
    (rec := recip numer g) case "failed" => f /$Rep g
    (rec :: UPXSSING) * (denom g) * f

  coerce(f:UPXS) == f :: UPXSSING :: %

  seriesQuotientLimit(num,den) ==
    -- limit of the quotient of two series
    series := num / den
    (ord := order(series,1)) > 0 => 0
    coef := coefficient(series,ord)
    member?(var,variables coef) => "failed"
    ord = 0 => coef :: OFE
    (sig := sign(coef)$SIGNEF) case "failed" => return "failed"
    (sig :: Integer) = 1 => plusInfinity()
    minusInfinity()

  seriesQuotientInfinity(num,den) ==
    -- infinite limit: plus or minus?
    -- look at leading coefficients of series to tell
    (numOrd := order(num,ZEROCOUNT)) = ZEROCOUNT => "failed"
    (denOrd := order(den,ZEROCOUNT)) = ZEROCOUNT => "failed"
    cc := coefficient(num,numOrd)/coefficient(den,denOrd)
    member?(var,variables cc) => "failed"
    (sig := sign(cc)$SIGNEF) case "failed" => return "failed"
    (sig :: Integer) = 1 => plusInfinity()
    minusInfinity()

  limitPlus f ==
    zero? f => 0
    (den := denom f) = 1 => limitPlus numer f
    (numerTerm := dominantTerm(num := numer f)) case "failed" => "failed"
    numType := (numTerm := numerTerm :: TypedTerm).%type
    (denomTerm := dominantTerm den) case "failed" => "failed"
    denType := (denTerm := denomTerm :: TypedTerm).%type
    numExpon := exponent numTerm.%term; denExpon := exponent denTerm.%term
    numCoef := coeff numTerm.%term; denCoef := coeff denTerm.%term
    -- numerator tends to zero exponentially
    (numType = "zero") =>
      -- denominator tends to zero exponentially
```

```
        (denType = "zero") =>
          (exponDiff := numExpon - denExpon) = 0 =>
            seriesQuotientLimit(numCoef,denCoef)
          expCoef := coefficient(exponDiff,order exponDiff)
          (sig := sign(expCoef)$SIGNEF) case "failed" => return "failed"
          (sig :: Integer) = -1 => 0
          seriesQuotientInfinity(numCoef,denCoef)
        0 -- otherwise limit is zero
      -- numerator is a Puiseux series
      (numType = "series") =>
        -- denominator tends to zero exponentially
        (denType = "zero") =>
          seriesQuotientInfinity(numCoef,denCoef)
        -- denominator is a series
        (denType = "series") => seriesQuotientLimit(numCoef,denCoef)
        0
      -- remaining case: numerator tends to infinity exponentially
      -- denominator tends to infinity exponentially
      (denType = "infinity") =>
        (exponDiff := numExpon - denExpon) = 0 =>
          seriesQuotientLimit(numCoef,denCoef)
        expCoef := coefficient(exponDiff,order exponDiff)
        (sig := sign(expCoef)$SIGNEF) case "failed" => return "failed"
        (sig :: Integer) = -1 => 0
        seriesQuotientInfinity(numCoef,denCoef)
      -- denominator tends to zero exponentially or is a series
      seriesQuotientInfinity(numCoef,denCoef)
```

— **EXPEXPAN.dotabb** —

```
"EXPEXPAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXPEXPAN"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"EXPEXPAN" -> "ACF"
"EXPEXPAN" -> "FS"
```

6.6 domain EXPR Expression

— **Expression.input** —

```
)set break resume
```

```
)sys rm -f Expression.output
)spool Expression.output
)set message test on
)set message auto off
)clear all
--S 1 of 23
sin(x) + 3*cos(x)**2
--R
--R
--R                        2
--R   (1)  sin(x) + 3cos(x)
--R                                                    Type: Expression Integer
--E 1

--S 2 of 23
tan(x) - 3.45*x
--R
--R
--R   (2)  tan(x) - 3.45 x
--R                                                      Type: Expression Float
--E 2

--S 3 of 23
(tan sqrt 7 - sin sqrt 11)**2 / (4 - cos(x - y))
--R
--R
--R                 +-+ 2         +--+      +-+         +--+ 2
--R          - tan(\|7 )  + 2sin(\|11 )tan(\|7 ) - sin(\|11 )
--R   (3)  -------------------------------------------------
--R                           cos(y - x) - 4
--R                                                    Type: Expression Integer
--E 3

--S 4 of 23
log(exp  x)@Expression(Integer)
--R
--R
--R   (4)  x
--R                                                    Type: Expression Integer
--E 4

--S 5 of 23
log(exp  x)@Expression(Complex Integer)
--R
--R
--R              x
--R   (5)  log(%e )
--R                                            Type: Expression Complex Integer
--E 5
```

```
--S 6 of 23
sqrt 3 + sqrt(2 + sqrt(-5))
--R
--R
--R         +----------+
--R         | +---+          +-+
--R   (6)  \|\|- 5  + 2  + \|3
--R                                                      Type: AlgebraicNumber
--E 6

--S 7 of 23
% :: Expression Integer
--R
--R
--R         +----------+
--R         | +---+          +-+
--R   (7)  \|\|- 5  + 2  + \|3
--R                                                    Type: Expression Integer
--E 7

--S 8 of 23
height mainKernel sin(x + 4)
--R
--R
--R   (8)  2
--R                                                      Type: PositiveInteger
--E 8

--S 9 of 23
e := (sin(x) - 4)**2 / ( 1 - 2*y*sqrt(- y) )
--R
--R
--R                2
--R        - sin(x)  + 8sin(x) - 16
--R   (9)  ------------------------
--R                 +---+
--R              2y\|- y  - 1
--R                                                    Type: Expression Integer
--E 9

--S 10 of 23
numer e
--R
--R
--R                 2
--R   (10)  - sin(x)  + 8sin(x) - 16
--R        Type: SparseMultivariatePolynomial(Integer,Kernel Expression Integer)
--E 10

--S 11 of 23
```

```
denom e
--R
--R
--R            +---+
--R   (11)  2y\|- y  - 1
--R        Type: SparseMultivariatePolynomial(Integer,Kernel Expression Integer)
--E 11

--S 12 of 23
D(e, x)
--R
--R
--R                                     +---+
--R         (4y cos(x)sin(x) - 16y cos(x))\|- y  - 2cos(x)sin(x) + 8cos(x)
--R   (12)  --------------------------------------------------------------
--R                                +---+     3
--R                             4y\|- y  + 4y  - 1
--R                                                     Type: Expression Integer
--E 12

--S 13 of 23
D(e, [x, y], [1, 2])
--R
--R
--R   (13)
--R               7       4                          7        4          +---+
--R       ((- 2304y  + 960y )cos(x)sin(x) + (9216y  - 3840y )cos(x))\|- y
--R     +
--R             9        6       3
--R       (- 960y  + 2160y  - 180y  - 3)cos(x)sin(x)
--R     +
--R            9        6       3
--R       (3840y  - 8640y  + 720y  + 12)cos(x)
--R  /
--R            12        9        6       3      +---+        11        8       5
--R       (256y   - 1792y  + 1120y  - 112y  + 1)\|- y  - 1024y   + 1792y  - 448y
--R     +
--R          2
--R       16y
--R                                                     Type: Expression Integer
--E 13

--S 14 of 23
complexNumeric(cos(2 - 3*%i))
--R
--R
--R   (14)  - 4.1896256909 688072301 + 9.1092278937 55336598 %i
--R                                                        Type: Complex Float
--E 14
```

```
--S 15 of 23
numeric(tan 3.8)
--R
--R
--R   (15)  0.7735560905 0312607286
--R                                                            Type: Float
--E 15

--S 16 of 23
e2 := cos(x**2 - y + 3)
--R
--R
--R                  2
--R   (16)  cos(y - x  - 3)
--R                                                  Type: Expression Integer
--E 16

--S 17 of 23
e3 := asin(e2) - %pi/2
--R
--R
--R                2
--R   (17)  - y + x  + 3
--R                                                  Type: Expression Integer
--E 17

--S 18 of 23
e3 :: Polynomial Integer
--R
--R
--R                2
--R   (18)  - y + x  + 3
--R                                                  Type: Polynomial Integer
--E 18

--S 19 of 23
e3 :: DMP([x, y], Integer)
--R
--R
--R          2
--R   (19)  x  - y + 3
--R                          Type: DistributedMultivariatePolynomial([x,y],Integer)
--E 19

--S 20 of 23
sin %pi
--R
--R
--R   (20)  0
--R                                                  Type: Expression Integer
```

```
--E 20

--S 21 of 23
cos(%pi / 4)
--R
--R
--R           +-+
--R          \|2
--R   (21)  ----
--R           2
--R                                            Type: Expression Integer
--E 21

--S 22 of 23
tan(x)**6 + 3*tan(x)**4 + 3*tan(x)**2 + 1
--R
--R
--R                 6          4          2
--R   (22)  tan(x)  + 3tan(x)  + 3tan(x)  + 1
--R                                            Type: Expression Integer
--E 22

--S 23 of 23
simplify %
--R
--R
--R              1
--R   (23)  -------
--R               6
--R         cos(x)
--R                                            Type: Expression Integer
--E 23
)spool
)lisp (bye)
```

— **Expression.help** —

```
====================================================================
Expression examples
====================================================================

Expression is a constructor that creates domains whose objects
can have very general symbolic forms.  Here are some examples:

This is an object of type Expression Integer.

  sin(x) + 3*cos(x)**2
```

```
This is an object of type Expression Float.

  tan(x) - 3.45*x

This object contains symbolic function applications, sums,
products, square roots, and a quotient.

  (tan sqrt 7 - sin sqrt 11)**2 / (4 - cos(x - y))

As you can see, Expression actually takes an argument domain.  The
coefficients of the terms within the expression belong to the argument
domain.  Integer and Float, along with Complex Integer and Complex
Float are the most common coefficient domains.

The choice of whether to use a Complex coefficient domain or not is
important since Axiom can perform some simplifications on real-valued
objects

  log(exp  x)@Expression(Integer)

... which are not valid on complex ones.

  log(exp  x)@Expression(Complex Integer)

Many potential coefficient domains, such as AlgebraicNumber, are not
usually used because Expression can subsume them.

  sqrt 3 + sqrt(2 + sqrt(-5))

  % :: Expression Integer

Note that we sometimes talk about "an object of type Expression."
This is not really correct because we should say, for example, "an
object of type Expression Integer" or "an object of type Expression
Float."  By a similar abuse of language, when we refer to an
"expression" in this section we will mean an object of type Expression
R for some domain R.

The Axiom documentation contains many examples of the use of
Expression.  For the rest of this section, we'll give you some
pointers to those examples plus give you some idea of how to
manipulate expressions.

It is important for you to know that Expression creates domains that
have category Field.  Thus you can invert any non-zero expression and
you shouldn't expect an operation like factor to give you much
information.  You can imagine expressions as being represented as
quotients of "multivariate" polynomials where the "variables" are
kernels. A kernel can either be a symbol such as x or a symbolic
```

function application like sin(x + 4). The second example is actually a nested kernel since the argument to sin contains the kernel x.

```
height mainKernel sin(x + 4)
```

Actually, the argument to sin is an expression, and so the structure of Expression is recursive. See Kernel which demonstrates how to extract the kernels in an expression.

Use the HyperDoc Browse facility to see what operations are applicable to expression. At the time of this writing, there were 262 operations with 147 distinct name in Expression Integer. For example, numer and denom extract the numerator and denominator of an expression.

```
e := (sin(x) - 4)**2 / ( 1 - 2*y*sqrt(- y) )
```

```
numer e
```

```
denom e
```

Use D to compute partial derivatives.

```
D(e, x)
```

```
D(e, [x, y], [1, 2])
```

When an expression involves no ‘‘symbol kernels’’ (for example, x), it may be possible to numerically evaluate the expression.

If you suspect the evaluation will create a complex number, use complexNumeric.

```
complexNumeric(cos(2 - 3*%i))
```

If you know it will be real, use numeric.

```
numeric(tan 3.8)
```

The numeric operation will display an error message if the evaluation yields a calue with an non-zero imaginary part. Both of these operations have an optional second argument n which specifies that the accuracy of the approximation be up to n decimal places.

When an expression involves no "symbolic application" kernels, it may be possible to convert it a polynomial or rational function in the variables that are present.

```
e2 := cos(x**2 - y + 3)
```

```
e3 := asin(e2) - %pi/2
```

```
  e3 :: Polynomial Integer

This also works for the polynomial types where specific variables
and their ordering are given.

  e3 :: DMP([x, y], Integer)

Finally, a certain amount of simplication takes place as expressions
are constructed.

  sin %pi

  cos(%pi / 4)

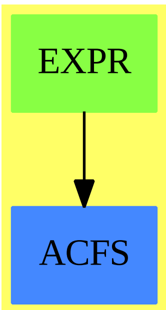
For simplications that involve multiple terms of the expression, use
simplify.

  tan(x)**6 + 3*tan(x)**4 + 3*tan(x)**2 + 1

  simplify %

See Also:
o )show Kernel
o )show Expression
```

6.6.1 Expression (EXPR)



See

⇒ "Pi" (HACKPI) 17.16.1 on page 1937

Exports:

0	1	abs	acos
acosh	acot	acoth	acsc
acsch	airyAi	airyBi	applyQuote
asec	asech	asin	asinh
associates?	atan	atanh	belong?
besselI	besselJ	besselK	besselY
Beta	binomial	box	characteristic
charthRoot	Ci	coerce	commutator
conjugate	convert	cos	cosh
cot	coth	csc	csch
D	definingPolynomial	denom	denominator
differentiate	digamma	dilog	distribute
divide	Ei	elt	erf
euclideanSize	eval	even?	exp
expressIdealMember	exquo	extendedEuclidean	factor
factorPolynomial	factorial	factorials	freeOf?
Gamma	gcd	gcdPolynomial	ground
ground?	hash	height	integral
inv	is?	isExpt	isMult
isPlus	isPower	isTimes	kernel
kernels	latex	lcm	li
log	mainKernel	map	max
min	minPoly	multiEuclidean	nthRoot
number?	numer	numerator	odd?
one?	operator	operators	paren
patternMatch	permutation	pi	polygamma
prime?	principalIdeal	product	recip
reduce	reducedSystem	retract	retractIfCan
rootOf	rootsOf	sample	sec
sech	Si	simplifyPower	sin
sinh	sizeLess?	sqrt	squareFree
squareFreePart	squareFreePolynomial	subst	subtractIfCan
summation	tan	tanh	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	zeroOf	zerosOf
?<?	?<=?	?=?	?>?
?>=?	?~=?	?*?	?**?
?+?	-?	?-?	?/?
?^?	?quo?	?rem?	

— domain EXPR Expression —

```
)abbrev domain EXPR Expression
++ Author: Manuel Bronstein
++ Date Created: 19 July 1988
```

```
++ Date Last Updated: October 1993 (P.Gianni), February 1995 (MB)
++ Keywords: operator, kernel, function.
++ Description:
++ Top-level mathematical expressions involving symbolic functions.

Expression(R:OrderedSet): Exports == Implementation where
  Q   ==> Fraction Integer
  K   ==> Kernel %
  MP  ==> SparseMultivariatePolynomial(R, K)
  AF  ==> AlgebraicFunction(R, %)
  EF  ==> ElementaryFunction(R, %)
  CF  ==> CombinatorialFunction(R, %)
  LF  ==> LiouvillianFunction(R, %)
  AN  ==> AlgebraicNumber
  KAN ==> Kernel AN
  FSF ==> FunctionalSpecialFunction(R, %)
  ESD ==> ExpressionSpace_&(%)
  FSD ==> FunctionSpace_&(%, R)
  SYMBOL ==> "%symbol"
  ALGOP  ==> "%alg"
  POWER  ==> "%power"::Symbol
  SUP    ==> SparseUnivariatePolynomial

  Exports ==> FunctionSpace R with
    if R has IntegralDomain then
      AlgebraicallyClosedFunctionSpace R
      TranscendentalFunctionCategory
      CombinatorialOpsCategory
      LiouvillianFunctionCategory
      SpecialFunctionCategory
      reduce: % -> %
        ++ reduce(f) simplifies all the unreduced algebraic quantities
        ++ present in f by applying their defining relations.
      number?: % -> Boolean
        ++ number?(f) tests if f is rational
      simplifyPower: (%,Integer) -> %
        ++ simplifyPower?(f,n) is not documented
      if R has GcdDomain then
        factorPolynomial : SUP  % -> Factored SUP %
          ++ factorPolynomial(p) is not documented
        squareFreePolynomial : SUP % -> Factored SUP %
          ++ squareFreePolynomial(p) is not documented
      if R has RetractableTo Integer then RetractableTo AN

  Implementation ==> add
    import KernelFunctions2(R, %)

    retNotUnit     : % -> R
    retNotUnitIfCan: % -> Union(R, "failed")
```

```
    belong? op == true

    retNotUnit x ==
      (u := constantIfCan(k := retract(x)@K)) case R => u::R
      error "Not retractable"

    retNotUnitIfCan x ==
      (r := retractIfCan(x)@Union(K,"failed")) case "failed" => "failed"
      constantIfCan(r::K)

    if R has IntegralDomain then
      reduc  : (%, List Kernel %) -> %
      commonk   : (%, %) -> List K
      commonk0  : (List K, List K) -> List K
      toprat    : % -> %
      algkernels: List K -> List K
      evl       : (MP, K, SparseUnivariatePolynomial %) -> Fraction MP
      evl0      : (MP, K) -> SparseUnivariatePolynomial Fraction MP

      Rep := Fraction MP
      0                == 0$Rep
      1                == 1$Rep
--      one? x           == one?(x)$Rep
      one? x           == (x = 1)$Rep
      zero? x          == zero?(x)$Rep
      - x:%            == -$Rep x
      n:Integer * x:%  == n *$Rep x
      coerce(n:Integer) ==  coerce(n)$Rep@Rep::%
      x:% * y:%        == reduc(x *$Rep y, commonk(x, y))
      x:% + y:%        == reduc(x +$Rep y, commonk(x, y))
      (x:% - y:%):%    == reduc(x -$Rep y, commonk(x, y))
      x:% / y:%        == reduc(x /$Rep y, commonk(x, y))

      number?(x:%):Boolean ==
        if R has RetractableTo(Integer) then
          ground?(x) or ((retractIfCan(x)@Union(Q,"failed")) case Q)
        else
          ground?(x)

      simplifyPower(x:%,n:Integer):% ==
        k : List K := kernels x
        is?(x,POWER) =>
          -- Look for a power of a number in case we can do a simplification
          args : List % := argument first k
          not(#args = 2) => error "Too many arguments to **"
          number?(args.1) =>
             reduc((args.1) **$Rep n, algkernels kernels (args.1))**(args.2)
          (first args)**(n*second(args))
        reduc(x **$Rep n, algkernels k)
```

```
      x:% ** n:NonNegativeInteger ==
        n = 0 => 1$%
        n = 1 => x
        simplifyPower(numerator x,n pretend Integer) /
           simplifyPower(denominator x,n pretend Integer)

      x:% ** n:Integer ==
        n = 0 => 1$%
        n = 1 => x
        n = -1 => 1/x
        simplifyPower(numerator x,n) /
           simplifyPower(denominator x,n)

      x:% ** n:PositiveInteger ==
        n = 1 => x
        simplifyPower(numerator x,n pretend Integer) /
           simplifyPower(denominator x,n pretend Integer)

      x:% < y:%        == x <$Rep y
      x:% = y:%        == x =$Rep y
      numer x          == numer(x)$Rep
      denom x          == denom(x)$Rep
      coerce(p:MP):%   == coerce(p)$Rep
      reduce x         == reduc(x, algkernels kernels x)
      commonk(x, y)    == commonk0(algkernels kernels x, algkernels kernels y)
      algkernels l     == select_!(x +-> has?(operator x, ALGOP), l)
      toprat f == ratDenom(f,algkernels kernels f)$AlgebraicManipulations(R, %)

      x:MP / y:MP ==
       reduc(x /$Rep y,commonk0(algkernels variables x,algkernels variables y))

-- since we use the reduction from FRAC SMP which asssumes that the
-- variables are independent, we must remove algebraic from the denominators
      reducedSystem(m:Matrix %):Matrix(R) ==
        mm:Matrix(MP) := reducedSystem(map(toprat, m))$Rep
        reducedSystem(mm)$MP

-- since we use the reduction from FRAC SMP which asssumes that the
-- variables are independent, we must remove algebraic from the denominators
      reducedSystem(m:Matrix %, v:Vector %):
       Record(mat:Matrix R, vec:Vector R) ==
        r:Record(mat:Matrix MP, vec:Vector MP) :=
          reducedSystem(map(toprat, m), map(toprat, v))$Rep
        reducedSystem(r.mat, r.vec)$MP

-- The result MUST be left sorted deepest first   MB 3/90
      commonk0(x, y) ==
        ans := empty()$List(K)
        for k in reverse_! x repeat if member?(k, y) then ans := concat(k, ans)
        ans
```

```
rootOf(x:SparseUnivariatePolynomial %, v:Symbol) == rootOf(x,v)$AF
pi()                      == pi()$EF
exp x                     == exp(x)$EF
log x                     == log(x)$EF
sin x                     == sin(x)$EF
cos x                     == cos(x)$EF
tan x                     == tan(x)$EF
cot x                     == cot(x)$EF
sec x                     == sec(x)$EF
csc x                     == csc(x)$EF
asin x                    == asin(x)$EF
acos x                    == acos(x)$EF
atan x                    == atan(x)$EF
acot x                    == acot(x)$EF
asec x                    == asec(x)$EF
acsc x                    == acsc(x)$EF
sinh x                    == sinh(x)$EF
cosh x                    == cosh(x)$EF
tanh x                    == tanh(x)$EF
coth x                    == coth(x)$EF
sech x                    == sech(x)$EF
csch x                    == csch(x)$EF
asinh x                   == asinh(x)$EF
acosh x                   == acosh(x)$EF
atanh x                   == atanh(x)$EF
acoth x                   == acoth(x)$EF
asech x                   == asech(x)$EF
acsch x                   == acsch(x)$EF

abs x                     == abs(x)$FSF
Gamma x                   == Gamma(x)$FSF
Gamma(a, x)               == Gamma(a, x)$FSF
Beta(x,y)                 == Beta(x,y)$FSF
digamma x                 == digamma(x)$FSF
polygamma(k,x)            == polygamma(k,x)$FSF
besselJ(v,x)              == besselJ(v,x)$FSF
besselY(v,x)              == besselY(v,x)$FSF
besselI(v,x)              == besselI(v,x)$FSF
besselK(v,x)              == besselK(v,x)$FSF
airyAi x                  == airyAi(x)$FSF
airyBi x                  == airyBi(x)$FSF

x:% ** y:%                == x **$CF y
factorial x               == factorial(x)$CF
binomial(n, m)            == binomial(n, m)$CF
permutation(n, m)         == permutation(n, m)$CF
factorials x              == factorials(x)$CF
factorials(x, n)          == factorials(x, n)$CF
summation(x:%, n:Symbol)           == summation(x, n)$CF
```

```
summation(x:%, s:SegmentBinding %) == summation(x, s)$CF
product(x:%, n:Symbol)             == product(x, n)$CF
product(x:%, s:SegmentBinding %)   == product(x, s)$CF

erf x                              == erf(x)$LF
Ei x                               == Ei(x)$LF
Si x                               == Si(x)$LF
Ci x                               == Ci(x)$LF
li x                               == li(x)$LF
dilog x                            == dilog(x)$LF
fresnelS x                         == fresnelS(x)$LF
fresnelC x                         == fresnelC(x)$LF
integral(x:%, n:Symbol)            == integral(x, n)$LF
integral(x:%, s:SegmentBinding %)  == integral(x, s)$LF

operator op ==
  belong?(op)$AF  => operator(op)$AF
  belong?(op)$EF  => operator(op)$EF
  belong?(op)$CF  => operator(op)$CF
  belong?(op)$LF  => operator(op)$LF
  belong?(op)$FSF => operator(op)$FSF
  belong?(op)$FSD => operator(op)$FSD
  belong?(op)$ESD => operator(op)$ESD
  nullary? op and has?(op, SYMBOL) => operator(kernel(name op)$K)
  (n := arity op) case "failed" => operator name op
  operator(name op, n::NonNegativeInteger)

reduc(x, l) ==
  for k in l repeat
    p := minPoly k
    x := evl(numer x, k, p) /$Rep evl(denom x, k, p)
  x

evl0(p, k) ==
  numer univariate(p::Fraction(MP),
             k)$PolynomialCategoryQuotientFunctions(IndexedExponents K,
                                                      K,R,MP,Fraction MP)

-- uses some operations from Rep instead of % in order not to
-- reduce recursively during those operations.
evl(p, k, m) ==
  degree(p, k) < degree m => p::Fraction(MP)
  (((evl0(p, k) pretend SparseUnivariatePolynomial($)) rem m)
     pretend SparseUnivariatePolynomial Fraction MP) (k::MP::Fraction(MP))

if R has GcdDomain then
  noalg?: SUP % -> Boolean

  noalg? p ==
    while p ^= 0 repeat
```

```
        not empty? algkernels kernels leadingCoefficient p => return false
        p := reductum p
      true

    gcdPolynomial(p:SUP %, q:SUP %) ==
      noalg? p and noalg? q => gcdPolynomial(p, q)$Rep
      gcdPolynomial(p, q)$GcdDomain_&(%)

    factorPolynomial(x:SUP %) : Factored SUP % ==
      uf:= factor(x pretend SUP(Rep))$SupFractionFactorizer(
                                IndexedExponents K,K,R,MP)
      uf pretend Factored SUP %

    squareFreePolynomial(x:SUP %) : Factored SUP % ==
      uf:= squareFree(x pretend SUP(Rep))$SupFractionFactorizer(
                                IndexedExponents K,K,R,MP)
      uf pretend Factored SUP %

  if R is AN then
    -- this is to force the coercion R -> EXPR R to be used
    -- instead of the coercioon AN -> EXPR R which loops.
    -- simpler looking code will fail! MB 10/91
    coerce(x:AN):% == (monomial(x, 0$IndexedExponents(K))$MP)::%

  if (R has RetractableTo Integer) then
    x:% ** r:Q                          == x **$AF r
    minPoly k                           == minPoly(k)$AF
    definingPolynomial x                == definingPolynomial(x)$AF
    retract(x:%):Q                      == retract(x)$Rep
    retractIfCan(x:%):Union(Q, "failed") == retractIfCan(x)$Rep

    if not(R is AN) then
      k2expr  : KAN -> %
      smp2expr: SparseMultivariatePolynomial(Integer, KAN) -> %
      R2AN    : R  -> Union(AN, "failed")
      k2an    : K  -> Union(AN, "failed")
      smp2an  : MP -> Union(AN, "failed")


      coerce(x:AN):% == smp2expr(numer x) / smp2expr(denom x)
      k2expr k       == map(x+->x::%, k)$ExpressionSpaceFunctions2(AN, %)

      smp2expr p ==
        map(k2expr,x+->x::%,p)_
          $PolynomialCategoryLifting(IndexedExponents KAN,
               KAN, Integer, SparseMultivariatePolynomial(Integer, KAN), %)

      retractIfCan(x:%):Union(AN, "failed") ==
        ((n:= smp2an numer x) case AN) and ((d:= smp2an denom x) case AN)
             => (n::AN) / (d::AN)
```

```
        "failed"

      R2AN r ==
        (u := retractIfCan(r::%)@Union(Q, "failed")) case Q => u::Q::AN
        "failed"

      k2an k ==
        not(belong?(op := operator k)$AN) => "failed"
        arg:List(AN) := empty()
        for x in argument k repeat
          if (a := retractIfCan(x)@Union(AN, "failed")) case "failed" then
            return "failed"
          else arg := concat(a::AN, arg)
        (operator(op)$AN) reverse_!(arg)

      smp2an p ==
        (x1 := mainVariable p) case "failed" => R2AN leadingCoefficient p
        up := univariate(p, k := x1::K)
        (t  := k2an k) case "failed" => "failed"
        ans:AN := 0
        while not ground? up repeat
          (c:=smp2an leadingCoefficient up) case "failed" => return "failed"
          ans := ans + (c::AN) * (t::AN) ** (degree up)
          up  := reductum up
        (c := smp2an leadingCoefficient up) case "failed" => "failed"
        ans + c::AN

  if R has ConvertibleTo InputForm then
    convert(x:%):InputForm == convert(x)$Rep
    import MakeUnaryCompiledFunction(%, %, %)
    eval(f:%, op: BasicOperator, g:%, x:Symbol):% ==
      eval(f,[op],[g],x)
    eval(f:%, ls:List BasicOperator, lg:List %, x:Symbol) ==
      -- handle subsrcipted symbols by renaming -> eval -> renaming back
      llsym:List List Symbol:=[variables g for g in lg]
      lsym:List Symbol:= removeDuplicates concat llsym
      lsd:List Symbol:=select (scripted?,lsym)
      empty? lsd=> eval(f,ls,[compiledFunction(g, x) for g in lg])
      ns:List Symbol:=[new()$Symbol for i in lsd]
      lforwardSubs:List Equation % := [(i::%)= (j::%) for i in lsd for j in ns]
      lbackwardSubs:List Equation % := [(j::%)= (i::%) for i in lsd for j in ns]
      nlg:List % :=[subst(g,lforwardSubs) for g in lg]
      res:% :=eval(f, ls, [compiledFunction(g, x) for g in nlg])
      subst(res,lbackwardSubs)
  if R has PatternMatchable Integer then
    patternMatch(x:%, p:Pattern Integer,
     l:PatternMatchResult(Integer, %)) ==
      patternMatch(x, p, l)$PatternMatchFunctionSpace(Integer, R, %)

  if R has PatternMatchable Float then
```

```
    patternMatch(x:%, p:Pattern Float,
     l:PatternMatchResult(Float, %)) ==
      patternMatch(x, p, l)$PatternMatchFunctionSpace(Float, R, %)

else  -- R is not an integral domain
  operator op ==
    belong?(op)$FSD => operator(op)$FSD
    belong?(op)$ESD => operator(op)$ESD
    nullary? op and has?(op, SYMBOL) => operator(kernel(name op)$K)
    (n := arity op) case "failed" => operator name op
    operator(name op, n::NonNegativeInteger)

  if R has Ring then
    Rep := MP
    0              == 0$Rep
    1              == 1$Rep
    - x:%          == -$Rep x
    n:Integer *x:% == n *$Rep x
    x:% * y:%      == x *$Rep y
    x:% + y:%      == x +$Rep y
    x:% = y:%      == x =$Rep y
    x:% < y:%      == x <$Rep y
    numer x        == x@Rep
    coerce(p:MP):% == p

    reducedSystem(m:Matrix %):Matrix(R) ==
      reducedSystem(m)$Rep

    reducedSystem(m:Matrix %, v:Vector %):
     Record(mat:Matrix R, vec:Vector R) ==
      reducedSystem(m, v)$Rep

    if R has ConvertibleTo InputForm then
      convert(x:%):InputForm == convert(x)$Rep

    if R has PatternMatchable Integer then
      kintmatch: (K,Pattern Integer,PatternMatchResult(Integer,Rep))
                                  -> PatternMatchResult(Integer, Rep)

      kintmatch(k, p, l) ==
        patternMatch(k, p, l pretend PatternMatchResult(Integer, %)
          )$PatternMatchKernel(Integer, %)
            pretend PatternMatchResult(Integer, Rep)

      patternMatch(x:%, p:Pattern Integer,
       l:PatternMatchResult(Integer, %)) ==
        patternMatch(x@Rep, p,
                     l pretend PatternMatchResult(Integer, Rep),
                      kintmatch
                       )$PatternMatchPolynomialCategory(Integer,
```

```
                      IndexedExponents K, K, R, Rep)
                        pretend PatternMatchResult(Integer, %)

  if R has PatternMatchable Float then
    kfltmatch: (K, Pattern Float, PatternMatchResult(Float, Rep))
                               -> PatternMatchResult(Float, Rep)

    kfltmatch(k, p, l) ==
      patternMatch(k, p, l pretend PatternMatchResult(Float, %)
        )$PatternMatchKernel(Float, %)
          pretend PatternMatchResult(Float, Rep)

    patternMatch(x:%, p:Pattern Float,
     l:PatternMatchResult(Float, %)) ==
      patternMatch(x@Rep, p,
                   l pretend PatternMatchResult(Float, Rep),
                    kfltmatch
                     )$PatternMatchPolynomialCategory(Float,
                      IndexedExponents K, K, R, Rep)
                        pretend PatternMatchResult(Float, %)

else   -- R is not even a ring
  if R has AbelianMonoid then
    import ListToMap(K, %)

    kereval        : (K, List K, List %) -> %
    subeval        : (K, List K, List %) -> %

    Rep := FreeAbelianGroup K

    0              == 0$Rep
    x:% + y:%      == x +$Rep y
    x:% = y:%      == x =$Rep y
    x:% < y:%      == x <$Rep y
    coerce(k:K):%  == coerce(k)$Rep
    kernels x      == [f.gen for f in terms x]
    coerce(x:R):%  == (zero? x => 0; constantKernel(x)::%)
    retract(x:%):R == (zero? x => 0; retNotUnit x)
    coerce(x:%):OutputForm == coerce(x)$Rep
    kereval(k, lk, lv) ==
     match(lk, lv, k, (x2:K):% +-> map(x1 +-> eval(x1, lk, lv), x2))

    subeval(k, lk, lv) ==
      match(lk, lv, k,
       (x:K):% +->
         kernel(operator x, [subst(a, lk, lv) for a in argument x]))

    isPlus x ==
      empty?(l := terms x) or empty? rest l => "failed"
      [t.exp *$Rep t.gen for t in l]$List(%)
```

```
          isMult x ==
            empty?(l := terms x) or not empty? rest l => "failed"
            t := first l
            [t.exp, t.gen]

          eval(x:%, lk:List K, lv:List %) ==
            _+/[t.exp * kereval(t.gen, lk, lv) for t in terms x]

          subst(x:%, lk:List K, lv:List %) ==
            _+/[t.exp * subeval(t.gen, lk, lv) for t in terms x]

          retractIfCan(x:%):Union(R, "failed") ==
            zero? x => 0
            retNotUnitIfCan x

          if R has AbelianGroup then -(x:%) == -$Rep x

--      else      -- R is not an AbelianMonoid
--        if R has SemiGroup then
--    Rep := FreeGroup K
--    1              == 1$Rep
--    x:% * y:%      == x *$Rep y
--    x:% = y:%      == x =$Rep y
--    coerce(k:K):%  == k::Rep
--    kernels x      == [f.gen for f in factors x]
--    coerce(x:R):%  == (one? x => 1; constantKernel x)
--    retract(x:%):R == (one? x => 1; retNotUnit x)
--    coerce(x:%):OutputForm == coerce(x)$Rep

--    retractIfCan(x:%):Union(R, "failed") ==
--      one? x => 1
--      retNotUnitIfCan x

--    if R has Group then inv(x:%):% == inv(x)$Rep

        else   -- R is nothing
            import ListToMap(K, %)

            Rep := K

            x:% < y:%      == x <$Rep y
            x:% = y:%      == x =$Rep y
            coerce(k:K):%  == k
            kernels x      == [x pretend K]
            coerce(x:R):%  == constantKernel x
            retract(x:%):R == retNotUnit x
            retractIfCan(x:%):Union(R, "failed") == retNotUnitIfCan x
            coerce(x:%):OutputForm               == coerce(x)$Rep
```

```
            eval(x:%, lk:List K, lv:List %) ==
              match(lk, lv, x pretend K,
                (x1:K):% +-> map(x2 +-> eval(x2, lk, lv), x1))

            subst(x, lk, lv) ==
              match(lk, lv, x pretend K,
                (x1:K):% +->
                  kernel(operator x1, [subst(a, lk, lv) for a in argument x1]))

            if R has ConvertibleTo InputForm then
              convert(x:%):InputForm == convert(x)$Rep

--          if R has PatternMatchable Integer then
--            convert(x:%):Pattern(Integer) == convert(x)$Rep
--
--            patternMatch(x:%, p:Pattern Integer,
--             l:PatternMatchResult(Integer, %)) ==
--              patternMatch(x pretend K,p,l)$PatternMatchKernel(Integer, %)
--
--          if R has PatternMatchable Float then
--            convert(x:%):Pattern(Float) == convert(x)$Rep
--
--            patternMatch(x:%, p:Pattern Float,
--             l:PatternMatchResult(Float, %)) ==
--              patternMatch(x pretend K, p, l)$PatternMatchKernel(Float, %)
```

———

— EXPR.dotabb —

```
"EXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXPR"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"EXPR" -> "ACFS"
```

———

6.7 domain EXPUPXS ExponentialOfUnivariatePuiseuxSeries

— ExponentialOfUnivariatePuiseuxSeries.input —

```
)set break resume
)sys rm -f ExponentialOfUnivariatePuiseuxSeries.output
)spool ExponentialOfUnivariatePuiseuxSeries.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show ExponentialOfUnivariatePuiseuxSeries
--R ExponentialOfUnivariatePuiseuxSeries(FE: Join(Field,OrderedSet),var: Symbol,cen: FE)  is
--R Abbreviation for ExponentialOfUnivariatePuiseuxSeries is EXPUPXS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EXPUPXS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (FE,%) -> %                     ?*? : (%,FE) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> FE
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Fraction Integer
--R ?.? : (%,Fraction Integer) -> FE      hash : % -> SingleInteger
--R inv : % -> % if FE has FIELD          latex : % -> String
--R leadingCoefficient : % -> FE          leadingMonomial : % -> %
--R map : ((FE -> FE),%) -> %             max : (%,%) -> %
--R min : (%,%) -> %                      monomial? : % -> Boolean
--R one? : % -> Boolean                   order : % -> Fraction Integer
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if FE has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if FE has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if FE has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if FE has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if FE has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if FE has FIELD
--R ?/? : (%,FE) -> % if FE has FIELD
--R D : % -> % if FE has *: (Fraction Integer,FE) -> FE
--R D : (%,NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE
--R D : (%,Symbol) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBOL
--R D : (%,List Symbol) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBO
--R D : (%,Symbol,NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE and FE ha
--R D : (%,List Symbol,List NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE
--R ?^? : (%,Integer) -> % if FE has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if FE has ALGEBRA FRAC INT
```

```
--R acosh : % -> % if FE has ALGEBRA FRAC INT
--R acot : % -> % if FE has ALGEBRA FRAC INT
--R acoth : % -> % if FE has ALGEBRA FRAC INT
--R acsc : % -> % if FE has ALGEBRA FRAC INT
--R acsch : % -> % if FE has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> FE if FE has **: (FE,Fraction Integer) -> FE and FE has coerce
--R asec : % -> % if FE has ALGEBRA FRAC INT
--R asech : % -> % if FE has ALGEBRA FRAC INT
--R asin : % -> % if FE has ALGEBRA FRAC INT
--R asinh : % -> % if FE has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if FE has INTDOM
--R atan : % -> % if FE has ALGEBRA FRAC INT
--R atanh : % -> % if FE has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if FE has CHARNZ
--R coefficient : (%,Fraction Integer) -> FE
--R coerce : % -> % if FE has INTDOM
--R coerce : Fraction Integer -> % if FE has ALGEBRA FRAC INT
--R coerce : FE -> % if FE has COMRING
--R cos : % -> % if FE has ALGEBRA FRAC INT
--R cosh : % -> % if FE has ALGEBRA FRAC INT
--R cot : % -> % if FE has ALGEBRA FRAC INT
--R coth : % -> % if FE has ALGEBRA FRAC INT
--R csc : % -> % if FE has ALGEBRA FRAC INT
--R csch : % -> % if FE has ALGEBRA FRAC INT
--R differentiate : % -> % if FE has *: (Fraction Integer,FE) -> FE
--R differentiate : (%,NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE
--R differentiate : (%,Symbol) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMB
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE and FE h
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> F
--R divide : (%,%) -> Record(quotient: %,remainder: %) if FE has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if FE has FIELD
--R eval : (%,FE) -> Stream FE if FE has **: (FE,Fraction Integer) -> FE
--R exp : % -> % if FE has ALGEBRA FRAC INT
--R exponent : % -> UnivariatePuiseuxSeries(FE,var,cen)
--R exponential : UnivariatePuiseuxSeries(FE,var,cen) -> %
--R exponentialOrder : % -> Fraction Integer
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if FE has FIELD
--R exquo : (%,%) -> Union(%,"failed") if FE has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if FE has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if FE has FIELD
--R factor : % -> Factored % if FE has FIELD
--R gcd : (%,%) -> % if FE has FIELD
--R gcd : List % -> % if FE has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R integrate : (%,Symbol) -> % if FE has integrate: (FE,Symbol) -> FE and FE has variables: FE -> List
--R integrate : % -> % if FE has ALGEBRA FRAC INT
```

```
--R lcm : (%,%) -> % if FE has FIELD
--R lcm : List % -> % if FE has FIELD
--R log : % -> % if FE has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (FE,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if FE has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if FE has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if FE has ALGEBRA FRAC INT
--R prime? : % -> Boolean if FE has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if FE has FIELD
--R ?quo? : (%,%) -> % if FE has FIELD
--R ?rem? : (%,%) -> % if FE has FIELD
--R sec : % -> % if FE has ALGEBRA FRAC INT
--R sech : % -> % if FE has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: FE)) -> %
--R sin : % -> % if FE has ALGEBRA FRAC INT
--R sinh : % -> % if FE has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if FE has FIELD
--R sqrt : % -> % if FE has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if FE has FIELD
--R squareFreePart : % -> % if FE has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if FE has ALGEBRA FRAC INT
--R tanh : % -> % if FE has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: FE)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if FE has INTDOM
--R unitCanonical : % -> % if FE has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if FE has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

————

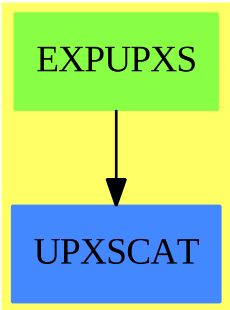
— ExponentialOfUnivariatePuiseuxSeries.help —

```
====================================================================
ExponentialOfUnivariatePuiseuxSeries examples
====================================================================
```

```
See Also:
o )show ExponentialOfUnivariatePuiseuxSeries
```

———

6.7.1 ExponentialOfUnivariatePuiseuxSeries (EXPUPXS)



See

⇒ "UnivariatePuiseuxSeriesWithExponentialSingularity" (UPXSSING) 22.7.1 on page 2809
⇒ "ExponentialExpansion" (EXPEXPAN) 6.5.1 on page 679

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	exponent
exponential	exponentialOrder	expressIdealMember	exquo
extend	extendedEuclidean	factor	gcd
gcdPolynomial	hash	integrate	inv
latex	lcm	leadingCoefficient	leadingMonomial
log	map	max	min
monomial	monomial?	multiEuclidean	multiplyExponents
multiplyExponents	nthRoot	one?	order
pi	pole?	prime?	principalIdeal
recip	reductum	sample	sec
sech	series	sin	sinh
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	tan	tanh	terms
truncate	unit?	unitCanonical	unitNormal
variable	variables	zero?	?*?
?**?	?+?	?-?	-?
?<?	?<=?	?=?	?>?
?>=?	?^?	?.?	?~=?
?/?	?quo?	?rem?	

— domain EXPUPXS ExponentialOfUnivariatePuiseuxSeries —

```
)abbrev domain EXPUPXS ExponentialOfUnivariatePuiseuxSeries
++ Author: Clifton J. Williamson
++ Date Created: 4 August 1992
++ Date Last Updated: 27 August 1992
++ Basic Operations:
++ Related Domains: UnivariatePuiseuxSeries(FE,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: limit, functional expression, power series, essential singularity
++ Examples:
++ References:
++ Description:
++ ExponentialOfUnivariatePuiseuxSeries is a domain used to represent
++ essential singularities of functions.  An object in this domain is a
++ function of the form \spad{exp(f(x))}, where \spad{f(x)} is a Puiseux
++ series with no terms of non-negative degree.  Objects are ordered
++ according to order of singularity, with functions which tend more
```

```
++ rapidly to zero or infinity considered to be larger.  Thus, if
++ \spad{order(f(x)) < order(g(x))}, i.e. the first non-zero term of
++ \spad{f(x)} has lower degree than the first non-zero term of \spad{g(x)},
++ then \spad{exp(f(x)) > exp(g(x))}.  If \spad{order(f(x)) = order(g(x))},
++ then the ordering is essentially random.  This domain is used
++ in computing limits involving functions with essential singularities.

ExponentialOfUnivariatePuiseuxSeries(FE,var,cen):_
      Exports == Implementation where
  FE  : Join(Field,OrderedSet)
  var : Symbol
  cen : FE
  UPXS ==> UnivariatePuiseuxSeries(FE,var,cen)

  Exports ==> Join(UnivariatePuiseuxSeriesCategory(FE),OrderedAbelianMonoid) _
        with
    exponential : UPXS -> %
      ++ exponential(f(x)) returns \spad{exp(f(x))}.
      ++ Note: the function does NOT check that \spad{f(x)} has no
      ++ non-negative terms.
    exponent : % -> UPXS
      ++ exponent(exp(f(x))) returns \spad{f(x)}
    exponentialOrder: % -> Fraction Integer
      ++ exponentialOrder(exp(c * x **(-n) + ...)) returns \spad{-n}.
      ++ exponentialOrder(0) returns \spad{0}.

  Implementation ==> UPXS add

    Rep := UPXS

    exponential f == complete f
    exponent f == f pretend UPXS
    exponentialOrder f == order(exponent f,0)

    zero? f == empty? entries complete terms f

    f = g ==
    -- we redefine equality because we know that we are dealing with
    -- a FINITE series, so there is no danger in computing all terms
      (entries complete terms f) = (entries complete terms g)

    f < g ==
      zero? f => not zero? g
      zero? g => false
      (ordf := exponentialOrder f) > (ordg := exponentialOrder g) => true
      ordf < ordg => false
      (fCoef := coefficient(f,ordf)) = (gCoef := coefficient(g,ordg)) =>
        reductum(f) < reductum(g)
      fCoef < gCoef  -- this is "random" if FE is EXPR INT
```

```
coerce(f:%):OutputForm ==
  ("%e" :: OutputForm) ** ((coerce$Rep)(complete f)@OutputForm)
```

———-

— EXPUPXS.dotabb —

```
"EXPUPXS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXPUPXS"]
"UPXSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=UPXSCAT"]
"EXPUPXS" -> "UPXSCAT"
```

———-

6.8 domain EAB ExtAlgBasis

— ExtAlgBasis.input —

```
)set break resume
)sys rm -f ExtAlgBasis.output
)spool ExtAlgBasis.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExtAlgBasis
--R ExtAlgBasis  is a domain constructor
--R Abbreviation for ExtAlgBasis is EAB
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EAB
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                 ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                 ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean                Nul : NonNegativeInteger -> %
--R coerce : List Integer -> %             coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger       exponents : % -> List Integer
--R hash : % -> SingleInteger              latex : % -> String
--R max : (%,%) -> %                       min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

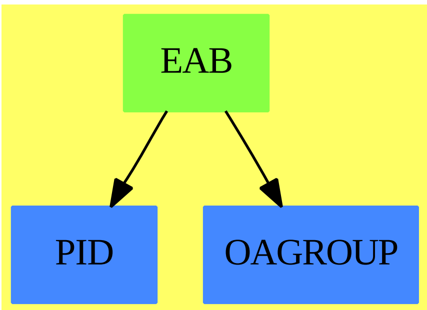
— **ExtAlgBasis.help** —

```
====================================================================
ExtAlgBasis examples
====================================================================

See Also:
o )show ExtAlgBasis
```

———-

6.8.1 ExtAlgBasis (EAB)



See

⇒ "AntiSymm" (ANTISYM) 2.8.1 on page 40
⇒ "DeRhamComplex" (DERHAM) 5.6.1 on page 515

Exports:

coerce	degree	exponents	hash	latex
max	min	Nul	?~=?	?<?
?<=?	?=?	?>?	?>=?	

— **domain EAB ExtAlgBasis** —

```
)abbrev domain EAB ExtAlgBasis
++ Author: Larry Lambe
++ Date created: 03/14/89
++ Description:
++ A domain used in the construction of the exterior algebra on a set
++ X over a ring R.  This domain represents the set of all ordered
++ subsets of the set X, assumed to be in correspondance with
++ {1,2,3, ...}.  The ordered subsets are themselves ordered
++ lexicographically and are in bijective correspondance with an ordered
```

```
++ basis of the exterior algebra.  In this domain we are dealing strictly
++ with the exponents of basis elements which can only be 0 or 1.
-- Thus we really have L({0,1}).
++
++ The multiplicative identity element of the exterior algebra corresponds
++ to the empty subset of X.  A coerce from List Integer to an
++ ordered basis element is provided to allow the convenient input of
++ expressions. Another exported function forgets the ordered structure
++ and simply returns the list corresponding to an ordered subset.

ExtAlgBasis(): Export == Implement where
   I   ==> Integer
   L   ==> List
   NNI ==> NonNegativeInteger

   Export == OrderedSet with
     coerce     : L I -> %
        ++ coerce(l) converts a list of 0's and 1's into a basis
        ++ element, where 1 (respectively 0) designates that the
        ++ variable of the corresponding index of l is (respectively, is not)
        ++ present.
        ++ Error: if an element of l is not 0 or 1.
     degree     : %   -> NNI
        ++ degree(x) gives the numbers of 1's in x, i.e., the number
        ++ of non-zero exponents in the basis element that x represents.
     exponents  : %   -> L I
        ++ exponents(x) converts a domain element into a list of zeros
        ++ and ones corresponding to the exponents in the basis element
        ++ that x represents.
--   subscripts : %   -> L I
        -- subscripts(x) looks at the exponents in x and converts
        -- them to the proper subscripts
     Nul        : NNI -> %
        ++ Nul() gives the basis element 1 for the algebra generated
        ++ by n generators.

   Implement == add
     Rep := L I
     x,y :  %

     x = y == x =$Rep y

     x < y ==
       null x            => not null y
       null y            => false
       first x = first y => rest x < rest y
       first x > first y

     coerce(li:(L I)) ==
       for x in li repeat
```

```
        if x ^= 1 and x ^= 0 then error "coerce: values can only be 0 and 1"
      li

     degree x         == (_+/x)::NNI

     exponents x      == copy(x @ Rep)

--   subscripts x     ==
--      cntr:I := 1
--      result: L I := []
--      for j in x repeat
--        if j = 1 then result := cons(cntr,result)
--        cntr:=cntr+1
--      reverse_! result

     Nul n            == [0 for i in 1..n]

     coerce x         == coerce(x @ Rep)$(L I)
```

— EAB.dotabb —

```
"EAB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EAB"]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"EAB" -> "PID"
"EAB" -> "OAGROUP"
```

6.9 domain E04DGFA e04dgfAnnaType

— e04dgfAnnaType.input —

```
)set break resume
)sys rm -f e04dgfAnnaType.output
)spool e04dgfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04dgfAnnaType
```

```
--R e04dgfAnnaType  is a domain constructor
--R Abbreviation for e04dgfAnnaType is E04DGFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04DGFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat))
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: Lis
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -
--R
--E 1

)spool
)lisp (bye)
```

———-

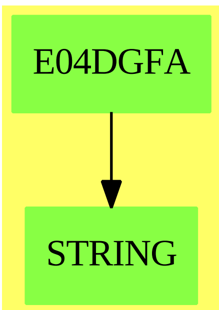
— **e04dgfAnnaType.help** —

```
====================================================================
e04dgfAnnaType examples
====================================================================

See Also:
o )show e04dgfAnnaType
```

———-

6.9.1 e04dgfAnnaType (E04DGFA)



See

⇒ "E04FDFA" (e04fdfAnnaType) 6.10.1 on page 718
⇒ "E04GCFA" (e04gcfAnnaType) 6.11.1 on page 721
⇒ "E04JAFA" (e04jafAnnaType) 6.12.1 on page 726
⇒ "E04MBFA" (e04mbfAnnaType) 6.13.1 on page 729
⇒ "E04NAFA" (e04nafAnnaType) 6.14.1 on page 733
⇒ "E04UCFA" (e04ucfAnnaType) 6.15.1 on page 736

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04DGFA e04dgfAnnaType —

```
)abbrev domain E04DGFA e04dgfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04dgfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04DGF, a general optimization routine which
++ can handle some singularities in the input function.  The function
++ \axiomFun{measure} measures the usefulness of the routine E04DGF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04dgfAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
  MDF  ==> Matrix DoubleFloat
  MPDF  ==> Matrix Polynomial DoubleFloat
  MF  ==> Matrix Float
  MEF  ==> Matrix Expression Float
  LEDF  ==> List Expression DoubleFloat
  VEF  ==> Vector Expression Float
  NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA  ==> Record(lfn:LEDF, init:LDF)
  EF2  ==> ExpressionFunctions2
  MI  ==> Matrix Integer
  INT  ==> Integer
  F  ==> Float
  NNI  ==> NonNegativeInteger
  S  ==> Symbol
  LS  ==> List Symbol
```

```
MVCF  ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF  ==> Stream DoubleFloat
LSDF  ==> List Stream DoubleFloat
SOCDF  ==> Segment OrderedCompletion DoubleFloat
OCDF  ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage, ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  string:String := "e04dgf is "
  positive?(#(args.cf) + #(args.lb) + #(args.ub)) =>
    string := concat(string,"unsuitable for constrained problems. ")
    [0.0,string]
  string := concat(string,"recommended")
  [getMeasure(R,e04dgf@Symbol)$RoutinesTable, string]

numericalOptimization(args:NOA) ==
  argsFn:EDF := args.fn
  n:NNI := #(variables(argsFn)$EDF)
  fu:DF := float(4373903597,-24,10)$DF
  it:INT := max(50,5*n)
  lin:DF := float(9,-1,10)$DF
  ma:DF := float(1,20,10)$DF
  op:DF := float(326,-14,10)$DF
  x:MDF := mat(args.init,n)
  ArgsFn:Expression Float := edf2ef(argsFn)
  f:Union(fn:FileName,fp:Asp49(OBJFUN)) := [retract(ArgsFn)$Asp49(OBJFUN)]
  e04dgf(n,1$DF,fu,it,lin,true,ma,op,1,1,n,0,x,-1,f)
```

— **E04DGFA.dotabb** —

```
"E04DGFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04DGFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04DGFA" -> "STRING"
```

6.10 domain E04FDFA e04fdfAnnaType

— **e04fdfAnnaType.input** —

```
)set break resume
)sys rm -f e04fdfAnnaType.output
)spool e04fdfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04fdfAnnaType
--R e04fdfAnnaType  is a domain constructor
--R Abbreviation for e04fdfAnnaType is E04FDFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04FDFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)
```

———-

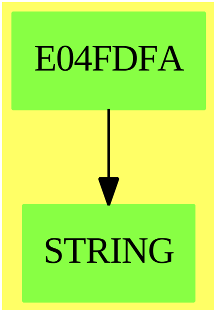
— e04fdfAnnaType.help —

```
====================================================================
e04fdfAnnaType examples
====================================================================

See Also:
o )show e04fdfAnnaType
```

———-

6.10.1 e04fdfAnnaType (E04FDFA)



See

⇒ "E04DGFA" (e04dgfAnnaType) 6.9.1 on page 714
⇒ "E04GCFA" (e04gcfAnnaType) 6.11.1 on page 721
⇒ "E04JAFA" (e04jafAnnaType) 6.12.1 on page 726
⇒ "E04MBFA" (e04mbfAnnaType) 6.13.1 on page 729
⇒ "E04NAFA" (e04nafAnnaType) 6.14.1 on page 733
⇒ "E04UCFA" (e04ucfAnnaType) 6.15.1 on page 736

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04FDFA e04fdfAnnaType —

```
)abbrev domain E04FDFA e04fdfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04fdfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04FDF, a general optimization routine which
++ can handle some singularities in the input function.  The function
++ \axiomFun{measure} measures the usefulness of the routine E04FDF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04fdfAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
```

```
MDF  ==> Matrix DoubleFloat
MPDF  ==> Matrix Polynomial DoubleFloat
MF  ==> Matrix Float
MEF  ==> Matrix Expression Float
LEDF  ==> List Expression DoubleFloat
VEF  ==> Vector Expression Float
NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
LSA  ==> Record(lfn:LEDF, init:LDF)
EF2  ==> ExpressionFunctions2
MI  ==> Matrix Integer
INT  ==> Integer
F  ==> Float
NNI  ==> NonNegativeInteger
S  ==> Symbol
LS  ==> List Symbol
MVCF  ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF  ==> Stream DoubleFloat
LSDF  ==> List Stream DoubleFloat
SOCDF  ==> Segment OrderedCompletion DoubleFloat
OCDF  ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  argsFn := args.fn
  string:String := "e04fdf is "
  positive?(#(args.cf) + #(args.lb) + #(args.ub)) =>
    string := concat(string,"unsuitable for constrained problems. ")
    [0.0,string]
  n:NNI := #(variables(argsFn)$EDF)
  (n>1)@Boolean =>
    string := concat(string,"unsuitable for single instances of multivariate problems. ")
    [0.0,string]
  sumOfSquares(argsFn) case "failed" =>
    string := concat(string,"unsuitable.")
    [0.0,string]
  string := concat(string,"recommended since the function is a sum of squares.")
  [getMeasure(R,e04fdf@Symbol)$RoutinesTable, string]

measure(R:RoutinesTable,args:LSA) ==
  string:String := "e04fdf is recommended"
  [getMeasure(R,e04fdf@Symbol)$RoutinesTable, string]

numericalOptimization(args:NOA) ==
  argsFn := args.fn
  lw:INT := 14
  x := mat(args.init,1)
```

```
    (a := sumOfSquares(argsFn)) case EDF =>
      ArgsFn := vector([edf2ef(a)])$VEF
      f : Union(fn:FileName,fp:Asp50(LSFUN1)) := [retract(ArgsFn)$Asp50(LSFUN1)]
      out:Result := e04fdf(1,1,1,lw,x,-1,f)
      changeNameToObjf(fsumsq@Symbol,out)
    empty()$Result

  numericalOptimization(args:LSA) ==
    argsFn := copy args.lfn
    m:INT := #(argsFn)
    n:NNI := #(variables(args))
    nn:INT := n
    lw:INT :=
--      one?(nn) => 9+5*m
      (nn = 1) => 9+5*m
      nn*(7+n+2*m+((nn-1) quo 2)$INT)+3*m
    x := mat(args.init,n)
    ArgsFn := vector([edf2ef(i)$ExpertSystemToolsPackage for i in argsFn])$VEF
    f : Union(fn:FileName,fp:Asp50(LSFUN1)) := [retract(ArgsFn)$Asp50(LSFUN1)]
    out:Result := e04fdf(m,n,1,lw,x,-1,f)
    changeNameToObjf(fsumsq@Symbol,out)
```

— E04FDFA.dotabb —

```
"E04FDFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04FDFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04FDFA" -> "STRING"
```

6.11 domain E04GCFA e04gcfAnnaType

— e04gcfAnnaType.input —

```
)set break resume
)sys rm -f e04gcfAnnaType.output
)spool e04gcfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show e04gcfAnnaType
--R e04gcfAnnaType  is a domain constructor
--R Abbreviation for e04gcfAnnaType is E04GCFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04GCFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)
```

———-

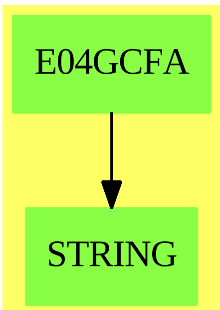
— e04gcfAnnaType.help —

```
====================================================================
e04gcfAnnaType examples
====================================================================

See Also:
o )show e04gcfAnnaType
```

———-

6.11.1 e04gcfAnnaType (E04GCFA)



See

⇒ "E04DGFA" (e04dgfAnnaType) 6.9.1 on page 714
⇒ "E04FDFA" (e04fdfAnnaType) 6.10.1 on page 718
⇒ "E04JAFA" (e04jafAnnaType) 6.12.1 on page 726
⇒ "E04MBFA" (e04mbfAnnaType) 6.13.1 on page 729
⇒ "E04NAFA" (e04nafAnnaType) 6.14.1 on page 733
⇒ "E04UCFA" (e04ucfAnnaType) 6.15.1 on page 736

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04GCFA e04gcfAnnaType —

```
)abbrev domain E04GCFA e04gcfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04gcfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04GCF, a general optimization routine which
++ can handle some singularities in the input function.  The function
++ \axiomFun{measure} measures the usefulness of the routine E04GCF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04gcfAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
  MDF  ==> Matrix DoubleFloat
  MPDF  ==> Matrix Polynomial DoubleFloat
  MF  ==> Matrix Float
  MEF  ==> Matrix Expression Float
  LEDF  ==> List Expression DoubleFloat
  VEF  ==> Vector Expression Float
  NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA  ==> Record(lfn:LEDF, init:LDF)
  EF2  ==> ExpressionFunctions2
  MI  ==> Matrix Integer
  INT  ==> Integer
  F  ==> Float
  NNI  ==> NonNegativeInteger
  S  ==> Symbol
```

```
LS  ==> List Symbol
MVCF  ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF  ==> Stream DoubleFloat
LSDF  ==> List Stream DoubleFloat
SOCDF  ==> Segment OrderedCompletion DoubleFloat
OCDF  ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage,ExpertSystemContinuityPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  argsFn:EDF := args.fn
  string:String := "e04gcf is "
  positive?(#(args.cf) + #(args.lb) + #(args.ub)) =>
    string := concat(string,"unsuitable for constrained problems. ")
    [0.0,string]
  n:NNI := #(variables(argsFn)$EDF)
  (n>1)@Boolean =>
    string := concat(string,"unsuitable for single instances of multivariate problems. ")
    [0.0,string]
  a := coerce(float(10,0,10))$OCDF
  seg:SOCDF := -a..a
  sings := singularitiesOf(argsFn,variables(argsFn)$EDF,seg)
  s := #(sdf2lst(sings))
  positive? s =>
    string := concat(string,"not recommended for discontinuous functions.")
    [0.0,string]
  sumOfSquares(args.fn) case "failed" =>
    string := concat(string,"unsuitable.")
    [0.0,string]
  string := concat(string,"recommended since the function is a sum of squares.")
  [getMeasure(R,e04gcf@Symbol)$RoutinesTable, string]

measure(R:RoutinesTable,args:LSA) ==
  string:String := "e04gcf is "
  a := coerce(float(10,0,10))$OCDF
  seg:SOCDF := -a..a
  sings := concat([singularitiesOf(i,variables(args),seg) for i in args.lfn])$SDF
  s := #(sdf2lst(sings))
  positive? s =>
    string := concat(string,"not recommended for discontinuous functions.")
    [0.0,string]
  string := concat(string,"recommended.")
  m := getMeasure(R,e04gcf@Symbol)$RoutinesTable
  m := m-(1-exp(-(expenseOfEvaluation(args))**3))
  [m, string]

numericalOptimization(args:NOA) ==
```

```
    argsFn:EDF := args.fn
    lw:INT := 16
    x := mat(args.init,1)
    (a := sumOfSquares(argsFn)) case EDF =>
      ArgsFn := vector([edf2ef(a)$ExpertSystemToolsPackage])$VEF
      f : Union(fn:FileName,fp:Asp19(LSFUN2)) := [retract(ArgsFn)$Asp19(LSFUN2)]
      out:Result := e04gcf(1,1,1,lw,x,-1,f)
      changeNameToObjf(fsumsq@Symbol,out)
    empty()$Result

  numericalOptimization(args:LSA) ==
    argsFn := copy args.lfn
    m:NNI := #(argsFn)
    n:NNI := #(variables(args))
    lw:INT :=
--      one?(n) => 11+5*m
      (n = 1) => 11+5*m
      2*n*(4+n+m)+3*m
    x := mat(args.init,n)
    ArgsFn := vector([edf2ef(i)$ExpertSystemToolsPackage for i in argsFn])$VEF
    f : Union(fn:FileName,fp:Asp19(LSFUN2)) := [retract(ArgsFn)$Asp19(LSFUN2)]
    out:Result := e04gcf(m,n,1,lw,x,-1,f)
    changeNameToObjf(fsumsq@Symbol,out)
```

— E04GCFA.dotabb —

```
"E04GCFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04GCFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04GCFA" -> "STRING"
```

6.12 domain E04JAFA e04jafAnnaType

— e04jafAnnaType.input —

```
)set break resume
)sys rm -f e04jafAnnaType.output
)spool e04jafAnnaType.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show e04jafAnnaType
--R e04jafAnnaType  is a domain constructor
--R Abbreviation for e04jafAnnaType is E04JAFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04JAFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)
```

———-

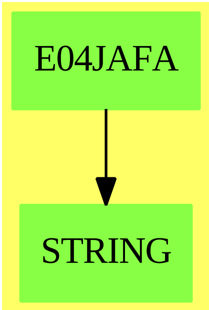
— **e04jafAnnaType.help** —

```
====================================================================
e04jafAnnaType examples
====================================================================

See Also:
o )show e04jafAnnaType
```

———-

6.12.1 e04jafAnnaType (E04JAFA)



See

⇒ "E04DGFA" (e04dgfAnnaType) 6.9.1 on page 714
⇒ "E04FDFA" (e04fdfAnnaType) 6.10.1 on page 718
⇒ "E04GCFA" (e04gcfAnnaType) 6.11.1 on page 721
⇒ "E04MBFA" (e04mbfAnnaType) 6.13.1 on page 729
⇒ "E04NAFA" (e04nafAnnaType) 6.14.1 on page 733
⇒ "E04UCFA" (e04ucfAnnaType) 6.15.1 on page 736

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04JAFA e04jafAnnaType —

```
)abbrev domain E04JAFA e04jafAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04jafAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04JAF, a general optimization routine which
++ can handle some singularities in the input function.  The function
++ \axiomFun{measure} measures the usefulness of the routine E04JAF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04jafAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
```

```
  MDF  ==> Matrix DoubleFloat
  MPDF  ==> Matrix Polynomial DoubleFloat
  MF  ==> Matrix Float
  MEF  ==> Matrix Expression Float
  LEDF  ==> List Expression DoubleFloat
  VEF  ==> Vector Expression Float
  NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA  ==> Record(lfn:LEDF, init:LDF)
  EF2  ==> ExpressionFunctions2
  MI  ==> Matrix Integer
  INT  ==> Integer
  F  ==> Float
  NNI  ==> NonNegativeInteger
  S  ==> Symbol
  LS  ==> List Symbol
  MVCF  ==> MultiVariableCalculusFunctions
  ESTOOLS2 ==> ExpertSystemToolsPackage2
  SDF  ==> Stream DoubleFloat
  LSDF  ==> List Stream DoubleFloat
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  OCDF  ==> OrderedCompletion DoubleFloat

  Rep:=Result
  import Rep, NagOptimisationPackage
  import e04AgentsPackage,ExpertSystemToolsPackage

  bound(a:LOCDF,b:LOCDF):Integer ==
    empty?(concat(a,b)) => 1
--    one?(#(removeDuplicates(a))) and  zero?(first(a)) => 2
    (#(removeDuplicates(a)) = 1) and  zero?(first(a)) => 2
--    one?(#(removeDuplicates(a))) and one?(#(removeDuplicates(b))) => 3
    (#(removeDuplicates(a)) = 1) and (#(removeDuplicates(b)) = 1) => 3
    0

  measure(R:RoutinesTable,args:NOA) ==
    string:String := "e04jaf is "
    if positive?(#(args.cf)) then
      if not simpleBounds?(args.cf) then
        string :=
          concat(string,"suitable for simple bounds only, not constraint functions.")
    (# string) < 20 =>
      if zero?(#(args.lb) + #(args.ub)) then
        string := concat(string, "usable if there are no constraints")
        [getMeasure(R,e04jaf@Symbol)$RoutinesTable*0.5,string]
      else
        string := concat(string,"recommended")
        [getMeasure(R,e04jaf@Symbol)$RoutinesTable, string]
    [0.0,string]

  numericalOptimization(args:NOA) ==
```

```
    argsFn:EDF := args.fn
    n:NNI := #(variables(argsFn)$EDF)
    ibound:INT := bound(args.lb,args.ub)
    m:INT := n
    lw:INT := max(13,12 * m + ((m * (m - 1)) quo 2)$INT)$INT
    bl := mat(finiteBound(args.lb,float(1,6,10)$DF),n)
    bu := mat(finiteBound(args.ub,float(1,6,10)$DF),n)
    x := mat(args.init,n)
    ArgsFn:EF := edf2ef(argsFn)
    fr:Union(fn:FileName,fp:Asp24(FUNCT1)) := [retract(ArgsFn)$Asp24(FUNCT1)]
    out:Result := e04jaf(n,ibound,n+2,lw,bl,bu,x,-1,fr)
    changeNameToObjf(f@Symbol,out)
```

—— E04JAFA.dotabb ——

```
"E04JAFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04JAFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04JAFA" -> "STRING"
```

6.13 domain E04MBFA e04mbfAnnaType

—— e04mbfAnnaType.input ——

```
)set break resume
)sys rm -f e04mbfAnnaType.output
)spool e04mbfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04mbfAnnaType
--R e04mbfAnnaType  is a domain constructor
--R Abbreviation for e04mbfAnnaType is E04MBFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04MBFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
```

```
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)
```

———-

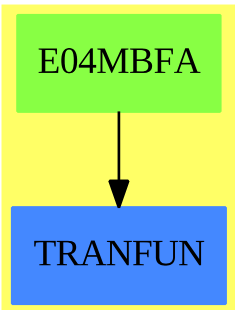
— **e04mbfAnnaType.help** —

```
====================================================================
e04mbfAnnaType examples
====================================================================

See Also:
o )show e04mbfAnnaType
```

———-

6.13.1 e04mbfAnnaType (E04MBFA)



See

⇒ “E04DGFA” (e04dgfAnnaType) 6.9.1 on page 714
⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718
⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721
⇒ “E04JAFA” (e04jafAnnaType) 6.12.1 on page 726
⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733
⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04MBFA e04mbfAnnaType —

```
)abbrev domain E04MBFA e04mbfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04mbfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04MBF, an optimization routine for Linear functions.
++ The function
++ \axiomFun{measure} measures the usefulness of the routine E04MBF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04mbfAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
  MDF  ==> Matrix DoubleFloat
  MPDF  ==> Matrix Polynomial DoubleFloat
  MF  ==> Matrix Float
  MEF  ==> Matrix Expression Float
  LEDF  ==> List Expression DoubleFloat
  VEF  ==> Vector Expression Float
  NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA  ==> Record(lfn:LEDF, init:LDF)
  EF2  ==> ExpressionFunctions2
  MI  ==> Matrix Integer
  INT  ==> Integer
  F  ==> Float
  NNI  ==> NonNegativeInteger
  S  ==> Symbol
  LS  ==> List Symbol
  MVCF  ==> MultiVariableCalculusFunctions
  ESTOOLS2 ==> ExpertSystemToolsPackage2
  SDF  ==> Stream DoubleFloat
  LSDF  ==> List Stream DoubleFloat
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  OCDF  ==> OrderedCompletion DoubleFloat
```

```
  Rep:=Result
  import Rep, NagOptimisationPackage
  import e04AgentsPackage,ExpertSystemToolsPackage

  measure(R:RoutinesTable,args:NOA) ==
    (not linear?([args.fn])) or (not linear?(args.cf)) =>
      [0.0,"e04mbf is for a linear objective function and constraints only."]
    [getMeasure(R,e04mbf@Symbol)$RoutinesTable,"e04mbf is recommended" ]

  numericalOptimization(args:NOA) ==
    argsFn:EDF := args.fn
    c := args.cf
    listVars:List LS := concat(variables(argsFn)$EDF,[variables(z)$EDF for z in c])
    n:NNI := #(v := removeDuplicates(concat(listVars)$LS)$LS)
    A:MDF := linearMatrix(args.cf,n)
    nclin:NNI := # linearPart(c)
    nrowa:NNI := max(1,nclin)
    bl:MDF := mat(finiteBound(args.lb,float(1,21,10)$DF),n)
    bu:MDF := mat(finiteBound(args.ub,float(1,21,10)$DF),n)
    cvec:MDF := mat(coefficients(retract(argsFn)@PDF)$PDF,n)
    x := mat(args.init,n)
    lwork:INT :=
      nclin < n => 2*nclin*(nclin+4)+2+6*n+nrowa
      2*(n+3)*n+4*nclin+nrowa
    out:Result := e04mbf(20,1,n,nclin,n+nclin,nrowa,A,bl,bu,cvec,true,2*n,lwork,x,-1)
    changeNameToObjf(objlp@Symbol,out)
```

— E04MBFA.dotabb —

```
"E04MBFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04MBFA"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"E04MBFA" -> "TRANFUN"
```

6.14 domain E04NAFA e04nafAnnaType

— e04nafAnnaType.input —

```
)set break resume
)sys rm -f e04nafAnnaType.output
)spool e04nafAnnaType.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04nafAnnaType
--R e04nafAnnaType  is a domain constructor
--R Abbreviation for e04nafAnnaType is E04NAFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04NAFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat))
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: Lis
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -
--R
--E 1

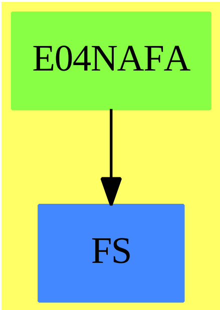
)spool
)lisp (bye)
```

— e04nafAnnaType.help —

```
====================================================================
e04nafAnnaType examples
====================================================================

See Also:
o )show e04nafAnnaType
```

6.14.1 e04nafAnnaType (E04NAFA)



See

⇒ "E04DGFA" (e04dgfAnnaType) 6.9.1 on page 714
⇒ "E04FDFA" (e04fdfAnnaType) 6.10.1 on page 718
⇒ "E04GCFA" (e04gcfAnnaType) 6.11.1 on page 721
⇒ "E04JAFA" (e04jafAnnaType) 6.12.1 on page 726
⇒ "E04MBFA" (e04mbfAnnaType) 6.13.1 on page 729
⇒ "E04UCFA" (e04ucfAnnaType) 6.15.1 on page 736

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04NAFA e04nafAnnaType —

```
)abbrev domain E04NAFA e04nafAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04nafAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04NAF, an optimization routine for Quadratic functions.
++ The function
++ \axiomFun{measure} measures the usefulness of the routine E04NAF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04nafAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
```

```
MDF  ==> Matrix DoubleFloat
MPDF  ==> Matrix Polynomial DoubleFloat
MF  ==> Matrix Float
MEF  ==> Matrix Expression Float
LEDF  ==> List Expression DoubleFloat
VEF  ==> Vector Expression Float
NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
LSA  ==> Record(lfn:LEDF, init:LDF)
EF2  ==> ExpressionFunctions2
MI  ==> Matrix Integer
INT  ==> Integer
F  ==> Float
NNI  ==> NonNegativeInteger
S  ==> Symbol
LS  ==> List Symbol
MVCF  ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF  ==> Stream DoubleFloat
LSDF  ==> List Stream DoubleFloat
SOCDF  ==> Segment OrderedCompletion DoubleFloat
OCDF  ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  string:String := "e04naf is "
  argsFn:EDF := args.fn
  if not (quadratic?(argsFn) and linear?(args.cf)) then
    string :=
      concat(string,"for a quadratic function with linear constraints only.")
  (# string) < 20 =>
    string := concat(string,"recommended")
    [getMeasure(R,e04naf@Symbol)$RoutinesTable, string]
  [0.0,string]

numericalOptimization(args:NOA) ==
  argsFn:EDF := args.fn
  c := args.cf
  listVars:List LS := concat(variables(argsFn)$EDF,[variables(z)$EDF for z in c])
  n:NNI := #(v := sort(removeDuplicates(concat(listVars)$LS)$LS)$LS)
  A:MDF := linearMatrix(c,n)
  nclin:NNI := # linearPart(c)
  nrowa:NNI := max(1,nclin)
  big:DF := float(1,10,10)$DF
  fea:MDF := new(1,n+nclin,float(1053,-11,10)$DF)$MDF
  bl:MDF := mat(finiteBound(args.lb,float(1,21,10)$DF),n)
  bu:MDF := mat(finiteBound(args.ub,float(1,21,10)$DF),n)
  alin:EDF := splitLinear(argsFn)
```

```
    p:PDF := retract(alin)@PDF
    pl:List PDF := [coefficient(p,i,1)$PDF for i in v]
    cvec:MDF := mat([pdf2df j for j in pl],n)
    h1:MPDF := hessian(p,v)$MVCF(S,PDF,VPDF,LS)
    hess:MDF := map(pdf2df,h1)$ESTOOLS2(PDF,DF)
    h2:MEF := map(df2ef,hess)$ESTOOLS2(DF,EF)
    x := mat(args.init,n)
    istate:MI := zero(1,n+nclin)$MI
    lwork:INT := 2*n*(n+2*nclin)+nrowa
    qphess:Union(fn:FileName,fp:Asp20(QPHESS)) := [retract(h2)$Asp20(QPHESS)]
    out:Result := e04naf(20,1,n,nclin,n+nclin,nrowa,n,n,big,A,bl,bu,cvec,fea,
                         hess,true,false,true,2*n,lwork,x,istate,-1,qphess)
    changeNameToObjf(obj@Symbol,out)
```

———

— **E04NAFA.dotabb** —

```
"E04NAFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04NAFA"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"E04NAFA" -> "FS"
```

———

6.15 domain E04UCFA e04ucfAnnaType

— **e04ucfAnnaType.input** —

```
)set break resume
)sys rm -f e04ucfAnnaType.output
)spool e04ucfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04ucfAnnaType
--R e04ucfAnnaType  is a domain constructor
--R Abbreviation for e04ucfAnnaType is E04UCFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04UCFA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: Lis
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -
--R
--E 1

)spool
)lisp (bye)
```

———-

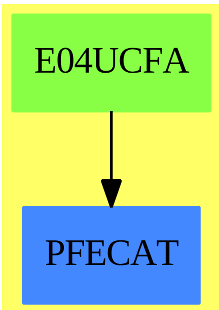
— e04ucfAnnaType.help —

```
====================================================================
e04ucfAnnaType examples
====================================================================

See Also:
o )show e04ucfAnnaType
```

———-

6.15.1 e04ucfAnnaType (E04UCFA)



See

⇒ "E04DGFA" (e04dgfAnnaType) 6.9.1 on page 714
⇒ "E04FDFA" (e04fdfAnnaType) 6.10.1 on page 718
⇒ "E04GCFA" (e04gcfAnnaType) 6.11.1 on page 721
⇒ "E04JAFA" (e04jafAnnaType) 6.12.1 on page 726
⇒ "E04MBFA" (e04mbfAnnaType) 6.13.1 on page 729
⇒ "E04NAFA" (e04nafAnnaType) 6.14.1 on page 733

Exports:
coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04UCFA e04ucfAnnaType —

```
)abbrev domain E04UCFA e04ucfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: November 1997
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04ucfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04UCF, a general optimization routine which
++ can handle some singularities in the input function.  The function
++ \axiomFun{measure} measures the usefulness of the routine E04UCF
++ for the given problem.  The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04ucfAnnaType(): NumericalOptimizationCategory == Result add
  DF  ==> DoubleFloat
  EF  ==> Expression Float
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  VPDF  ==> Vector Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  LOCDF  ==> List OrderedCompletion DoubleFloat
  MDF  ==> Matrix DoubleFloat
  MPDF  ==> Matrix Polynomial DoubleFloat
  MF  ==> Matrix Float
  MEF  ==> Matrix Expression Float
  LEDF  ==> List Expression DoubleFloat
  VEF  ==> Vector Expression Float
  NOA  ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA  ==> Record(lfn:LEDF, init:LDF)
  EF2  ==> ExpressionFunctions2
  MI  ==> Matrix Integer
  INT  ==> Integer
  F  ==> Float
  NNI  ==> NonNegativeInteger
  S  ==> Symbol
  LS  ==> List Symbol
  MVCF  ==> MultiVariableCalculusFunctions
  ESTOOLS2 ==> ExpertSystemToolsPackage2
  SDF  ==> Stream DoubleFloat
  LSDF  ==> List Stream DoubleFloat
  SOCDF  ==> Segment OrderedCompletion DoubleFloat
  OCDF  ==> OrderedCompletion DoubleFloat
```

```
Rep:=Result
import Rep,NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  zero?(#(args.lb) + #(args.ub)) =>
    [0.0,"e04ucf is not recommended if there are no bounds specified"]
  zero?(#(args.cf)) =>
    string:String := "e04ucf is usable but not always recommended if there are no constrai
    [getMeasure(R,e04ucf@Symbol)$RoutinesTable*0.5,string]
  [getMeasure(R,e04ucf@Symbol)$RoutinesTable,"e04ucf is recommended"]

numericalOptimization(args:NOA) ==
  Args := sortConstraints(args)
  argsFn := Args.fn
  c := Args.cf
  listVars:List LS := concat(variables(argsFn)$EDF,[variables(z)$EDF for z in c])
  n:NNI := #(v := sort(removeDuplicates(concat(listVars)$LS)$LS)$LS)
  lin:NNI := #(linearPart(c))
  nlcf := nonLinearPart(c)
  nonlin:NNI := #(nlcf)
  if empty?(nlcf) then
    nlcf := new(n,coerce(first(v)$LS)$EDF)$LEDF
  nrowa:NNI := max(1,lin)
  nrowj:NNI := max(1,nonlin)
  A:MDF := linearMatrix(c,n)
  bl:MDF := mat(finiteBound(Args.lb,float(1,25,10)$DF),n)
  bu:MDF := mat(finiteBound(Args.ub,float(1,25,10)$DF),n)
  liwork:INT := 3*n+lin+2*nonlin
  lwork:INT :=
    zero?(lin+nonlin) => 20*n
    zero?(nonlin) => 2*n*(n+10)+11*lin
    2*n*(n+nonlin+10)+(11+n)*lin + 21*nonlin
  cra:DF := float(1,-2,10)$DF
  fea:DF := float(1053671201,-17,10)$DF
  fun:DF := float(4373903597,-24,10)$DF
  infb:DF := float(1,15,10)$DF
  lint:DF := float(9,-1,10)$DF
  maji:INT := max(50,3*(n+lin)+10*nonlin)
  mini:INT := max(50,3*(n+lin+nonlin))
  nonf:DF := float(105,-10,10)$DF
  opt:DF := float(326,-10,10)$DF
  ste:DF := float(2,0,10)$DF
  istate:MI := zero(1,n+lin+nonlin)$MI
  cjac:MDF :=
    positive?(nonlin) => zero(nrowj,n)$MDF
    zero(nrowj,1)$MDF
  clambda:MDF := zero(1,n+lin+nonlin)$MDF
  r:MDF := zero(n,n)$MDF
  x:MDF := mat(Args.init,n)
```

```
    VectCF:VEF := vector([edf2ef e for e in nlcf])$VEF
    ArgsFn:EF := edf2ef(argsFn)
    fasp : Union(fn:FileName,fp:Asp49(OBJFUN)) := [retract(ArgsFn)$Asp49(OBJFUN)]
    casp : Union(fn:FileName,fp:Asp55(CONFUN)) := [retract(VectCF)$Asp55(CONFUN)]
    e04ucf(n,lin,nonlin,nrowa,nrowj,n,A,bl,bu,liwork,lwork,false,cra,3,fea,
            fun,true,infb,infb,fea,lint,true,maji,1,mini,0,-1,nonf,opt,ste,1,
             1,n,n,3,istate,cjac,clambda,r,x,-1,casp,fasp)
```

— **E04UCFA.dotabb** —

```
"E04UCFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04UCFA"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"E04UCFA" -> "PFECAT"
```

Chapter 7

Chapter F

7.1 domain FR Factored

— **Factored.input** —

```
)set break resume
)sys rm -f Factored.output
)spool Factored.output
)set message test on
)set message auto off
)clear all
--S 1 of 38
g := factor(4312)
--R
--R
--R         3 2
--R   (1)  2 7 11
--R                                                    Type: Factored Integer
--E 1

--S 2 of 38
unit(g)
--R
--R
--R   (2)  1
--R                                                     Type: PositiveInteger
--E 2

--S 3 of 38
numberOfFactors(g)
--R
--R
```

```
--R   (3)  3
--R                                                      Type: PositiveInteger
--E 3

--S 4 of 38
[nthFactor(g,i) for i in 1..numberOfFactors(g)]
--R
--R
--R   (4)  [2,7,11]
--R                                                        Type: List Integer
--E 4

--S 5 of 38
[nthExponent(g,i) for i in 1..numberOfFactors(g)]
--R
--R
--R   (5)  [3,2,1]
--R                                                        Type: List Integer
--E 5

--S 6 of 38
[nthFlag(g,i) for i in 1..numberOfFactors(g)]
--R
--R
--R   (6)  ["prime","prime","prime"]
--R                                Type: List Union("nil","sqfr","irred","prime")
--E 6

--S 7 of 38
factorList(g)
--R
--R
--R   (7)
--R   [[flg= "prime",fctr= 2,xpnt= 3], [flg= "prime",fctr= 7,xpnt= 2],
--R    [flg= "prime",fctr= 11,xpnt= 1]]
--RType: List Record(flg: Union("nil","sqfr","irred","prime"),fctr: Integer,xpnt: Integer)
--E 7

--S 8 of 38
factors(g)
--R
--R
--R   (8)
--R   [[factor= 2,exponent= 3],[factor= 7,exponent= 2],[factor= 11,exponent= 1]]
--R                          Type: List Record(factor: Integer,exponent: Integer)
--E 8

--S 9 of 38
first(%).factor
--R
```

```
--R
--R   (9)  2
--R                                                        Type: PositiveInteger
--E 9

--S 10 of 38
g := factor(4312)
--R
--R
--R          3 2
--R   (10)  2 7 11
--R                                                       Type: Factored Integer
--E 10

--S 11 of 38
expand(g)
--R
--R
--R   (11)  4312
--R                                                        Type: PositiveInteger
--E 11

--S 12 of 38
reduce(*,[t.factor for t in factors(g)])
--R
--R
--R   (12)  154
--R                                                        Type: PositiveInteger
--E 12

--S 13 of 38
g := factor(4312)
--R
--R
--R          3 2
--R   (13)  2 7 11
--R                                                       Type: Factored Integer
--E 13

--S 14 of 38
f := factor(246960)
--R
--R
--R          4 2   3
--R   (14)  2 3 5 7
--R                                                       Type: Factored Integer
--E 14

--S 15 of 38
f * g
```

```
--R
--R
--R          7 2   5
--R   (15)  2 3 5 7 11
--R                                                     Type: Factored Integer
--E 15

--S 16 of 38
f**500
--R
--R
--R          2000 1000 500 1500
--R   (16)  2    3    5   7
--R                                                     Type: Factored Integer
--E 16

--S 17 of 38
gcd(f,g)
--R
--R
--R          3 2
--R   (17)  2 7
--R                                                     Type: Factored Integer
--E 17

--S 18 of 38
lcm(f,g)
--R
--R
--R          4 2   3
--R   (18)  2 3 5 7 11
--R                                                     Type: Factored Integer
--E 18

--S 19 of 38
f + g
--R
--R
--R          3 2
--R   (19)  2 7 641
--R                                                     Type: Factored Integer
--E 19

--S 20 of 38
f - g
--R
--R
--R          3 2
--R   (20)  2 7 619
--R                                                     Type: Factored Integer
```

```
--E 20

--S 21 of 38
zero?(factor(0))
--R
--R
--R   (21)  true
--R                                                  Type: Boolean
--E 21

--S 22 of 38
zero?(g)
--R
--R
--R   (22)  false
--R                                                  Type: Boolean
--E 22

--S 23 of 38
one?(factor(1))
--R
--R
--R   (23)  true
--R                                                  Type: Boolean
--E 23

--S 24 of 38
one?(f)
--R
--R
--R   (24)  false
--R                                                  Type: Boolean
--E 24

--S 25 of 38
0$Factored(Integer)
--R
--R
--R   (25)  0
--R                                         Type: Factored Integer
--E 25

--S 26 of 38
1$Factored(Integer)
--R
--R
--R   (26)  1
--R                                         Type: Factored Integer
--E 26
```

```
--S 27 of 38
nilFactor(24,2)
--R
--R
--R          2
--R   (27)  24
--R                                                    Type: Factored Integer
--E 27

--S 28 of 38
nthFlag(%,1)
--R
--R
--R   (28)  "nil"
--R                                                    Type: Union("nil",...)
--E 28

--S 29 of 38
sqfrFactor(30,2)
--R
--R
--R          2
--R   (29)  30
--R                                                    Type: Factored Integer
--E 29

--S 30 of 38
irreducibleFactor(13,10)
--R
--R
--R          10
--R   (30)  13
--R                                                    Type: Factored Integer
--E 30

--S 31 of 38
primeFactor(11,5)
--R
--R
--R          5
--R   (31)  11
--R                                                    Type: Factored Integer
--E 31

--S 32 of 38
h := factor(-720)
--R
--R
--R           4 2
--R   (32)  - 2 3 5
```

```
--R                                                   Type: Factored Integer
--E 32

--S 33 of 38
h - makeFR(unit(h),factorList(h))
--R
--R
--R   (33)  0
--R                                                   Type: Factored Integer
--E 33

--S 34 of 38
p := (4*x*x-12*x+9)*y*y + (4*x*x-12*x+9)*y + 28*x*x - 84*x + 63
--R
--R
--R            2            2      2                  2
--R   (34)  (4x  - 12x + 9)y  + (4x  - 12x + 9)y + 28x  - 84x + 63
--R                                                 Type: Polynomial Integer
--E 34

--S 35 of 38
fp := factor(p)
--R
--R
--R                 2  2
--R   (35)  (2x - 3) (y  + y + 7)
--R                                        Type: Factored Polynomial Integer
--E 35

--S 36 of 38
D(p,x)
--R
--R
--R                   2
--R   (36)  (8x - 12)y  + (8x - 12)y + 56x - 84
--R                                                 Type: Polynomial Integer
--E 36

--S 37 of 38
D(fp,x)
--R
--R
--R                    2
--R   (37)  4(2x - 3)(y  + y + 7)
--R                                        Type: Factored Polynomial Integer
--E 37

--S 38 of 38
numberOfFactors(%)
--R
```

```
--R
--R   (38)  3
--R                                                   Type: PositiveInteger
--E 38
)spool
)lisp (bye)
```

— Factored.help —

```
====================================================================
Factored examples
====================================================================

Factored creates a domain whose objects are kept in factored form as
long as possible.  Thus certain operations like * (multiplication) and
gcd are relatively easy to do.  Others, such as addition, require
somewhat more work, and the result may not be completely factored
unless the argument domain R provides a factor operation.  Each object
consists of a unit and a list of factors, where each factor consists
of a member of R (the base), an exponent, and a flag indicating what
is known about the base.  A flag may be one of "nil", "sqfr", "irred"
or "prime", which mean that nothing is known about the base, it is
square-free, it is irreducible, or it is prime, respectively.  The
current restriction to factored objects of integral domains allows
simplification to be performed without worrying about multiplication
order.

====================================================================
Decomposing Factored Objects
====================================================================

In this section we will work with a factored integer.

  g := factor(4312)
     3 2
    2 7 11
                         Type: Factored Integer

Let's begin by decomposing g into pieces.  The only possible
units for integers are 1 and -1.

  unit(g)
    1
                         Type: PositiveInteger

There are three factors.
```

```
  numberOfFactors(g)
    3
                         Type: PositiveInteger
```

We can make a list of the bases, ...

```
  [nthFactor(g,i) for i in 1..numberOfFactors(g)]
    [2,7,11]
                         Type: List Integer
```

and the exponents, ...

```
  [nthExponent(g,i) for i in 1..numberOfFactors(g)]
    [3,2,1]
                         Type: List Integer
```

and the flags. You can see that all the bases (factors) are prime.

```
  [nthFlag(g,i) for i in 1..numberOfFactors(g)]
    ["prime","prime","prime"]
                         Type: List Union("nil","sqfr","irred","prime")
```

A useful operation for pulling apart a factored object into a list
of records of the components is factorList.

```
  factorList(g)
    [[flg= "prime",fctr= 2,xpnt= 3], [flg= "prime",fctr= 7,xpnt= 2],
     [flg= "prime",fctr= 11,xpnt= 1]]
           Type: List Record(flg: Union("nil","sqfr","irred","prime"),
                             fctr: Integer,xpnt: Integer)
```

If you don't care about the flags, use factors.

```
  factors(g)
    [[factor= 2,exponent= 3],[factor= 7,exponent= 2],[factor= 11,exponent= 1]]
                         Type: List Record(factor: Integer,exponent: Integer)
```

Neither of these operations returns the unit.

```
  first(%).factor
    2
                         Type: PositiveInteger
```

==
Expanding Factored Objects
==

Recall that we are working with this factored integer.

```
  g := factor(4312)
```

```
   3 2
  2 7 11
                         Type: Factored Integer
```

To multiply out the factors with their multiplicities, use expand.

```
expand(g)
  4312
                         Type: PositiveInteger
```

If you would like, say, the distinct factors multiplied together but with multiplicity one, you could do it this way.

```
reduce(*,[t.factor for t in factors(g)])
  154
                         Type: PositiveInteger
```

```
====================================================================
Arithmetic with Factored Objects
====================================================================
```

We're still working with this factored integer.

```
g := factor(4312)
   3 2
  2 7 11
                         Type: Factored Integer
```

We'll also define this factored integer.

```
f := factor(246960)
   4 2   3
  2 3 5 7
                         Type: Factored Integer
```

Operations involving multiplication and division are particularly easy with factored objects.

```
f * g
   7 2   5
  2 3 5 7 11
                         Type: Factored Integer

f**500
   2000 1000 500 1500
  2    3    5   7
                         Type: Factored Integer

gcd(f,g)
   3 2
```

```
  2 7
                     Type: Factored Integer
```

```
lcm(f,g)
   4 2   3
  2 3 5 7 11
                     Type: Factored Integer
```

If we use addition and subtraction things can slow down because we may need to compute greatest common divisors.

```
f + g
   3 2
  2 7 641
                     Type: Factored Integer
```

```
f - g
   3 2
  2 7 619
                     Type: Factored Integer
```

Test for equality with 0 and 1 by using zero? and one?, respectively.

```
zero?(factor(0))
  true
                     Type: Boolean
```

```
zero?(g)
  false
                     Type: Boolean
```

```
one?(factor(1))
  true
                     Type: Boolean
```

```
one?(f)
  false
                     Type: Boolean
```

Another way to get the zero and one factored objects is to use package calling.

```
0$Factored(Integer)
  0
                     Type: Factored Integer
```

```
1$Factored(Integer)
  1
                     Type: Factored Integer
```

```
====================================================================
Creating New Factored Objects
====================================================================

The map operation is used to iterate across the unit and bases of a
factored object.

The following four operations take a base and an exponent and create a
factored object.  They differ in handling the flag component.

  nilFactor(24,2)
      2
    24
                         Type: Factored Integer

This factor has no associated information.

  nthFlag(%,1)
    "nil"
                         Type: Union("nil",...)

This factor is asserted to be square-free.

  sqfrFactor(30,2)
      2
    30
                         Type: Factored Integer

This factor is asserted to be irreducible.

  irreducibleFactor(13,10)
      10
    13
                         Type: Factored Integer

This factor is asserted to be prime.

  primeFactor(11,5)
      5
    11
                         Type: Factored Integer

A partial inverse to factorList is makeFR.

  h := factor(-720)
       4 2
    - 2 3 5
                         Type: Factored Integer

The first argument is the unit and the second is a list of records as
```

```
returned by factorList.

  h - makeFR(unit(h),factorList(h))
    0
                         Type: Factored Integer

====================================================================
Factored Objects with Variables
====================================================================

Some of the operations available for polynomials are also available
for factored polynomials.

  p := (4*x*x-12*x+9)*y*y + (4*x*x-12*x+9)*y + 28*x*x - 84*x + 63
       2           2      2                  2
    (4x  - 12x + 9)y  + (4x  - 12x + 9)y + 28x  - 84x + 63
                         Type: Polynomial Integer

  fp := factor(p)
            2  2
    (2x - 3) (y  + y + 7)
                         Type: Factored Polynomial Integer

You can differentiate with respect to a variable.

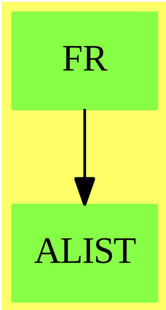
  D(p,x)
              2
    (8x - 12)y  + (8x - 12)y + 56x - 84
                         Type: Polynomial Integer

  D(fp,x)
               2
    4(2x - 3)(y  + y + 7)
                         Type: Factored Polynomial Integer

  numberOfFactors(%)
    3
                         Type: PositiveInteger

See Also:
o )help FactoredFunctions2
o )show Factored
```

7.1.1 Factored (FR)



Exports:

0	1	associates?	characteristic	coerce
convert	D	differentiate	eval	expand
exponent	exquo	factor	factorList	factors
flagFactor	gcd	gcdPolynomial	hash	irreducibleFactor
latex	lcm	makeFR	map	nilFactor
nthExponent	nthFactor	nthFlag	numberOfFactors	one?
prime?	primeFactor	rational	rational?	rationalIfCan
recip	retract	retractIfCan	sample	sqfrFactor
squareFree	squareFreePart	subtractIfCan	unit	unit?
unitCanonical	unitNormal	unitNormalize	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?.?		

— **domain FR Factored** —

```
)abbrev domain FR Factored
++ Author: Robert S. Sutor
++ Date Created: 1985
++ Change History:
++   21 Jan 1991   J Grabmeier     Corrected a bug in exquo.
++   16 Aug 1994   R S Sutor       Improved convert to InputForm
++ Basic Operations:
++   expand, exponent, factorList, factors, flagFactor, irreducibleFactor,
++   makeFR, map, nilFactor, nthFactor, nthFlag, numberOfFactors,
++   primeFactor, sqfrFactor, unit, unitNormalize,
++ Related Constructors: FactoredFunctionUtilities, FactoredFunctions2
++ Also See:
++ AMS Classifications: 11A51, 11Y05
++ Keywords: factorization, prime, square-free, irreducible, factor
++ References:
++ Description:
++ \spadtype{Factored} creates a domain whose objects are kept in
++ factored form as long as possible.  Thus certain operations like
```

```
++ multiplication and gcd are relatively easy to do.  Others, like
++ addition require somewhat more work, and unless the argument
++ domain provides a factor function, the result may not be
++ completely factored.  Each object consists of a unit and a list of
++ factors, where a factor has a member of R (the "base"), and
++ exponent and a flag indicating what is known about the base.  A
++ flag may be one of "nil", "sqfr", "irred" or "prime", which respectively mean
++ that nothing is known about the base, it is square-free, it is
++ irreducible, or it is prime.  The current
++ restriction to integral domains allows simplification to be
++ performed without worrying about multiplication order.

Factored(R: IntegralDomain): Exports == Implementation where
  fUnion ==> Union("nil", "sqfr", "irred", "prime")
  FF     ==> Record(flg: fUnion, fctr: R, xpnt: Integer)
  SRFE   ==> Set(Record(factor:R, exponent:Integer))

  Exports ==> Join(IntegralDomain, DifferentialExtension R, Algebra R,
                 FullyEvalableOver R, FullyRetractableTo R) with
   expand: % -> R
    ++ expand(f) multiplies the unit and factors together, yielding an
    ++ "unfactored" object. Note: this is purposely not called
    ++ \spadfun{coerce} which would cause the interpreter to do this
    ++ automatically.
    ++
    ++X f:=nilFactor(y-x,3)
    ++X expand(f)

   exponent:  % -> Integer
    ++ exponent(u) returns the exponent of the first factor of
    ++ \spadvar{u}, or 0 if the factored form consists solely of a unit.
    ++
    ++X f:=nilFactor(y-x,3)
    ++X exponent(f)

   makeFR  : (R, List FF) -> %
    ++ makeFR(unit,listOfFactors) creates a factored object (for
    ++ use by factoring code).
    ++
    ++X f:=nilFactor(x-y,3)
    ++X g:=factorList f
    ++X makeFR(z,g)

   factorList : % -> List FF
    ++ factorList(u) returns the list of factors with flags (for
    ++ use by factoring code).
    ++
    ++X f:=nilFactor(x-y,3)
    ++X factorList f
```

```
nilFactor: (R, Integer) -> %
 ++ nilFactor(base,exponent) creates a factored object with
 ++ a single factor with no information about the kind of
 ++ base (flag = "nil").
 ++
 ++X nilFactor(24,2)
 ++X nilFactor(x-y,3)

factors: % -> List Record(factor:R, exponent:Integer)
 ++ factors(u) returns a list of the factors in a form suitable
 ++ for iteration. That is, it returns a list where each element
 ++ is a record containing a base and exponent.  The original
 ++ object is the product of all the factors and the unit (which
 ++ can be extracted by \axiom{unit(u)}).
 ++
 ++X f:=x*y^3-3*x^2*y^2+3*x^3*y-x^4
 ++X factors f
 ++X g:=makeFR(z,factorList f)
 ++X factors g

irreducibleFactor: (R, Integer) -> %
 ++ irreducibleFactor(base,exponent) creates a factored object with
 ++ a single factor whose base is asserted to be irreducible
 ++ (flag = "irred").
 ++
 ++X a:=irreducibleFactor(3,1)
 ++X nthFlag(a,1)

nthExponent: (%, Integer) -> Integer
 ++ nthExponent(u,n) returns the exponent of the nth factor of
 ++ \spadvar{u}.  If \spadvar{n} is not a valid index for a factor
 ++ (for example, less than 1 or too big), 0 is returned.
 ++
 ++X a:=factor 9720000
 ++X nthExponent(a,2)

nthFactor:  (%,Integer) -> R
 ++ nthFactor(u,n) returns the base of the nth factor of
 ++ \spadvar{u}.  If \spadvar{n} is not a valid index for a factor
 ++ (for example, less than 1 or too big), 1 is returned.  If
 ++ \spadvar{u} consists only of a unit, the unit is returned.
 ++
 ++X a:=factor 9720000
 ++X nthFactor(a,2)

nthFlag:    (%,Integer) -> fUnion
 ++ nthFlag(u,n) returns the information flag of the nth factor of
 ++ \spadvar{u}.  If \spadvar{n} is not a valid index for a factor
 ++ (for example, less than 1 or too big), "nil" is returned.
 ++
```

```
 ++X a:=factor 9720000
 ++X nthFlag(a,2)

numberOfFactors : %  -> NonNegativeInteger
 ++ numberOfFactors(u) returns the number of factors in \spadvar{u}.
 ++
 ++X a:=factor 9720000
 ++X numberOfFactors a

primeFactor:   (R,Integer) -> %
 ++ primeFactor(base,exponent) creates a factored object with
 ++ a single factor whose base is asserted to be prime
 ++ (flag = "prime").
 ++
 ++X a:=primeFactor(3,4)
 ++X nthFlag(a,1)

sqfrFactor:   (R,Integer) -> %
 ++ sqfrFactor(base,exponent) creates a factored object with
 ++ a single factor whose base is asserted to be square-free
 ++ (flag = "sqfr").
 ++
 ++X a:=sqfrFactor(3,5)
 ++X nthFlag(a,1)

flagFactor: (R,Integer, fUnion) -> %
 ++ flagFactor(base,exponent,flag) creates a factored object with
 ++ a single factor whose base is asserted to be properly
 ++ described by the information flag.

unit:    % -> R
 ++ unit(u) extracts the unit part of the factorization.
 ++
 ++X f:=x*y^3-3*x^2*y^2+3*x^3*y-x^4
 ++X unit f
 ++X g:=makeFR(z,factorList f)
 ++X unit g

unitNormalize: % -> %
 ++ unitNormalize(u) normalizes the unit part of the factorization.
 ++ For example, when working with factored integers, this operation will
 ++ ensure that the bases are all positive integers.

map:     (R -> R, %) -> %
 ++ map(fn,u) maps the function \userfun{fn} across the factors of
 ++ \spadvar{u} and creates a new factored object. Note: this clears
 ++ the information flags (sets them to "nil") because the effect of
 ++ \userfun{fn} is clearly not known in general.
 ++
 ++X m(a:Factored Polynomial Integer):Factored Polynomial Integer == a^2
```

```
    ++X f:=x*y^3-3*x^2*y^2+3*x^3*y-x^4
    ++X map(m,f)
    ++X g:=makeFR(z,factorList f)
    ++X map(m,g)

    -- the following operations are conditional on R

   if R has GcdDomain then GcdDomain
   if R has RealConstant then RealConstant
   if R has UniqueFactorizationDomain then UniqueFactorizationDomain

   if R has ConvertibleTo InputForm then ConvertibleTo InputForm

   if R has IntegerNumberSystem then
      rational?    : % -> Boolean
        ++ rational?(u) tests if \spadvar{u} is actually a
        ++ rational number (see \spadtype{Fraction Integer}).
      rational     : % -> Fraction Integer
        ++ rational(u) assumes spadvar{u} is actually a rational number
        ++ and does the conversion to rational number
        ++ (see \spadtype{Fraction Integer}).
      rationalIfCan: % -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(u) returns a rational number if u
        ++ really is one, and "failed" otherwise.

   if R has Eltable(%, %) then Eltable(%, %)
   if R has Evalable(%) then Evalable(%)
   if R has InnerEvalable(Symbol, %) then InnerEvalable(Symbol, %)

  Implementation ==> add

  -- Representation:
    -- Note: exponents are allowed to be integers so that some special cases
    -- may be used in simplications
    Rep := Record(unt:R, fct:List FF)

    if R has ConvertibleTo InputForm then
      convert(x:%):InputForm ==
        empty?(lf := reverse factorList x) => convert(unit x)@InputForm
        l := empty()$List(InputForm)
        for rec in lf repeat
--          one?(rec.fctr) => l
          ((rec.fctr) = 1) => l
          iFactor : InputForm := binary( convert("::" :: Symbol)@InputForm, [convert(rec.fct
          iExpon  : InputForm := convert(rec.xpnt)@InputForm
          iFun    : List InputForm :=
            rec.flg case "nil" =>
               [convert("nilFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
            rec.flg case "sqfr" =>
               [convert("sqfrFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
```

```
            rec.flg case "prime" =>
               [convert("primeFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
            rec.flg case "irred" =>
               [convert("irreducibleFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
            nil$List(InputForm)
          l := concat( iFun pretend InputForm, l )
--        one?(rec.xpnt) =>
--          l := concat(convert(rec.fctr)@InputForm, l)
--        l := concat(convert(rec.fctr)@InputForm ** rec.xpnt, l)
        empty? l => convert(unit x)@InputForm
        if unit x ^= 1 then l := concat(convert(unit x)@InputForm,l)
        empty? rest l => first l
        binary(convert(_*::Symbol)@InputForm, l)@InputForm

    orderedR? := R has OrderedSet

  -- Private function signatures:
    reciprocal             : % -> %
    qexpand                : % -> R
    negexp?                : % -> Boolean
    SimplifyFactorization  : List FF -> List FF
    LispLessP              : (FF, FF) -> Boolean
    mkFF                   : (R, List FF) -> %
    SimplifyFactorization1 : (FF, List FF) -> List FF
    stricterFlag           : (fUnion, fUnion) -> fUnion

    nilFactor(r, i)      == flagFactor(r, i, "nil")
    sqfrFactor(r, i)     == flagFactor(r, i, "sqfr")
    irreducibleFactor(r, i)      == flagFactor(r, i, "irred")
    primeFactor(r, i)    == flagFactor(r, i, "prime")
    unit? u              == (empty? u.fct) and (not zero? u.unt)
    factorList u         == u.fct
    unit u               == u.unt
    numberOfFactors u    == # u.fct
    0                    == [1, [["nil", 0, 1]$FF]]
    zero? u              == # u.fct = 1 and
                             (first u.fct).flg case "nil" and
                              zero? (first u.fct).fctr and
--                               one? u.unt
                               (u.unt = 1)
    1                    == [1, empty()]
    one? u               == empty? u.fct and u.unt = 1
    mkFF(r, x)           == [r, x]
    coerce(j:Integer):%  == (j::R)::%
    characteristic()     == characteristic()$R
    i:Integer * u:%      == (i :: %) * u
    r:R * u:%            == (r :: %) * u
    factors u            == [[fe.fctr, fe.xpnt] for fe in factorList u]
    expand u             == retract u
    negexp? x           == "or"/[negative?(y.xpnt) for y in factorList x]
```

```
    makeFR(u, l) ==
-- normalizing code to be installed when contents are handled better
-- current squareFree returns the content as a unit part.
--        if (not unit?(u)) then
--            l := cons(["nil", u, 1]$FF,l)
--            u := 1
        unitNormalize mkFF(u, SimplifyFactorization l)

    if R has IntegerNumberSystem then
      rational? x     == true
      rationalIfCan x == rational x

      rational x ==
        convert(unit x)@Integer *
           _*/[(convert(f.fctr)@Integer)::Fraction(Integer)
                                 ** f.xpnt for f in factorList x]

    if R has Eltable(R, R) then
      elt(x:%, v:%) == x(expand v)

    if R has Evalable(R) then
      eval(x:%, l:List Equation %) ==
        eval(x,[expand lhs e = expand rhs e for e in l]$List(Equation R))

    if R has InnerEvalable(Symbol, R) then
      eval(x:%, ls:List Symbol, lv:List %) ==
        eval(x, ls, [expand v for v in lv]$List(R))

    if R has RealConstant then
  --! negcount and rest commented out since RealConstant doesn't support
  --! positive? or negative?
  --  negcount: % -> Integer
  --  positive?(x:%):Boolean == not(zero? x) and even?(negcount x)
  --  negative?(x:%):Boolean == not(zero? x) and odd?(negcount x)
  --  negcount x ==
  --    n := count(negative?(#1.fctr), factorList x)$List(FF)
  --    negative? unit x => n + 1
  --    n

      convert(x:%):Float ==
        convert(unit x)@Float *
                _*/[convert(f.fctr)@Float ** f.xpnt for f in factorList x]

      convert(x:%):DoubleFloat ==
        convert(unit x)@DoubleFloat *
          _*/[convert(f.fctr)@DoubleFloat ** f.xpnt for f in factorList x]

    u:% * v:% ==
      zero? u or zero? v => 0
```

```
--      one? u => v
      (u = 1) => v
--      one? v => u
      (v = 1) => u
      mkFF(unit u * unit v,
          SimplifyFactorization concat(factorList u, copy factorList v))

    u:% ** n:NonNegativeInteger ==
      mkFF(unit(u)**n, [[x.flg, x.fctr, n * x.xpnt] for x in factorList u])

    SimplifyFactorization x ==
      empty? x => empty()
      x := sort_!(LispLessP, x)
      x := SimplifyFactorization1(first x, rest x)
      if orderedR? then x := sort_!(LispLessP, x)
      x

    SimplifyFactorization1(f, x) ==
      empty? x =>
        zero?(f.xpnt) => empty()
        list f
      f1 := first x
      f.fctr = f1.fctr =>
        SimplifyFactorization1([stricterFlag(f.flg, f1.flg),
                                 f.fctr, f.xpnt + f1.xpnt], rest x)
      l := SimplifyFactorization1(first x, rest x)
      zero?(f.xpnt) => l
      concat(f, l)


    coerce(x:%):OutputForm ==
      empty?(lf := reverse factorList x) => (unit x)::OutputForm
      l := empty()$List(OutputForm)
      for rec in lf repeat
--        one?(rec.fctr) => l
        ((rec.fctr) = 1) => l
--        one?(rec.xpnt) =>
        ((rec.xpnt) = 1) =>
          l := concat(rec.fctr :: OutputForm, l)
        l := concat(rec.fctr::OutputForm ** rec.xpnt::OutputForm, l)
      empty? l => (unit x) :: OutputForm
      e :=
        empty? rest l => first l
        reduce(_*, l)
      1 = unit x => e
      (unit x)::OutputForm * e

    retract(u:%):R ==
      negexp? u =>  error "Negative exponent in factored object"
      qexpand u
```

```
    qexpand u ==
      unit u *
         _*/[y.fctr ** (y.xpnt::NonNegativeInteger) for y in factorList u]

    retractIfCan(u:%):Union(R, "failed") ==
      negexp? u => "failed"
      qexpand u

    LispLessP(y, y1) ==
      orderedR? => y.fctr < y1.fctr
      GGREATERP(y.fctr, y1.fctr)$Lisp => false
      true

    stricterFlag(fl1, fl2) ==
      fl1 case "prime"   => fl1
      fl1 case "irred"   =>
        fl2 case "prime" => fl2
        fl1
      fl1 case "sqfr"    =>
        fl2 case "nil"   => fl1
        fl2
      fl2

    if R has IntegerNumberSystem
      then
        coerce(r:R):% ==
          factor(r)$IntegerFactorizationPackage(R) pretend %
      else
        if R has UniqueFactorizationDomain
          then
            coerce(r:R):% ==
              zero? r => 0
              unit? r => mkFF(r, empty())
              unitNormalize(squareFree(r) pretend %)
          else
            coerce(r:R):% ==
--              one? r => 1
              (r = 1) => 1
              unitNormalize mkFF(1, [["nil", r, 1]$FF])

    u = v ==
      (unit u = unit v) and # u.fct = # v.fct and
        set(factors u)$SRFE =$SRFE set(factors v)$SRFE

    - u ==
      zero? u => u
      mkFF(- unit u, factorList u)

    recip u  ==
```

```
      not empty? factorList u => "failed"
      (r := recip unit u) case "failed" => "failed"
      mkFF(r::R, empty())

    reciprocal u ==
      mkFF((recip unit u)::R,
                    [[y.flg, y.fctr, - y.xpnt]$FF for y in factorList u])

    exponent u ==  -- exponent of first factor
      empty?(fl := factorList u) or zero? u => 0
      first(fl).xpnt

    nthExponent(u, i) ==
      l := factorList u
      zero? u or i < 1 or i > #l => 0
      (l.(minIndex(l) + i - 1)).xpnt

    nthFactor(u, i) ==
      zero? u => 0
      zero? i => unit u
      l := factorList u
      negative? i or i > #l => 1
      (l.(minIndex(l) + i - 1)).fctr

    nthFlag(u, i) ==
      l := factorList u
      zero? u or i < 1 or i > #l => "nil"
      (l.(minIndex(l) + i - 1)).flg

    flagFactor(r, i, fl) ==
      zero? i => 1
      zero? r => 0
      unitNormalize mkFF(1, [[fl, r, i]$FF])

    differentiate(u:%, deriv: R -> R) ==
      ans := deriv(unit u) * ((u exquo unit(u)::%)::%)
      ans + (_+/[fact.xpnt * deriv(fact.fctr) *
       ((u exquo nilFactor(fact.fctr, 1))::%) for fact in factorList u])

    map(fn, u) ==
     fn(unit u) * _*/[irreducibleFactor(fn(f.fctr),f.xpnt) for f in factorList u]

    u exquo v ==
      empty?(x1 := factorList v) =>  unitNormal(retract v).associate *  u
      empty? factorList u => "failed"
      v1 := u * reciprocal v
      goodQuotient:Boolean := true
      while (goodQuotient and (not empty? x1)) repeat
        if x1.first.xpnt < 0
          then goodQuotient := false
```

```
          else x1 := rest x1
        goodQuotient => v1
        "failed"

      unitNormal u == -- does a bunch of work, but more canonical
        (ur := recip(un := unit u)) case "failed" => [1, u, 1]
        as := ur::R
        vl := empty()$List(FF)
        for x in factorList u repeat
          ucar := unitNormal(x.fctr)
          e := abs(x.xpnt)::NonNegativeInteger
          if x.xpnt < 0
            then  --  associate is recip of unit
              un := un * (ucar.associate ** e)
              as := as * (ucar.unit ** e)
            else
              un := un * (ucar.unit ** e)
              as := as * (ucar.associate ** e)
--          if not one?(ucar.canonical) then
          if not ((ucar.canonical) = 1) then
            vl := concat([x.flg, ucar.canonical, x.xpnt], vl)
        [mkFF(un, empty()), mkFF(1, reverse_! vl), mkFF(as, empty())]

      unitNormalize u ==
        uca := unitNormal u
        mkFF(unit(uca.unit)*unit(uca.canonical),factorList(uca.canonical))

      if R has GcdDomain then
        u + v ==
          zero? u => v
          zero? v => u
          v1 := reciprocal(u1 := gcd(u, v))
          (expand(u * v1) + expand(v * v1)) * u1

        gcd(u, v) ==
--          one? u or one? v => 1
          (u = 1) or (v = 1) => 1
          zero? u => v
          zero? v => u
          f1 := empty()$List(Integer)  -- list of used factor indices in x
          f2 := f1      -- list of indices corresponding to a given factor
          f3 := empty()$List(List Integer)    -- list of f2-like lists
          x := concat(factorList u, factorList v)
          for i in minIndex x .. maxIndex x repeat
            if not member?(i, f1) then
              f1 := concat(i, f1)
              f2 := [i]
              for j in i+1..maxIndex x repeat
                if x.i.fctr = x.j.fctr then
                    f1 := concat(j, f1)
```

```
                    f2 := concat(j, f2)
              f3 := concat(f2, f3)
          x1 := empty()$List(FF)
          while not empty? f3 repeat
            f1 := first f3
            if #f1 > 1 then
              i  := first f1
              y  := copy x.i
              f1 := rest f1
              while not empty? f1 repeat
                i := first f1
                if x.i.xpnt < y.xpnt then y.xpnt := x.i.xpnt
                f1 := rest f1
              x1 := concat(y, x1)
            f3 := rest f3
          if orderedR? then x1 := sort_!(LispLessP, x1)
          mkFF(1, x1)

      else   -- R not a GCD domain
        u + v ==
          zero? u => v
          zero? v => u
          irreducibleFactor(expand u + expand v, 1)

      if R has UniqueFactorizationDomain then
        prime? u ==
          not(empty?(l := factorList u)) and (empty? rest l) and
--                         one?(l.first.xpnt) and (l.first.flg case "prime")
                         ((l.first.xpnt) = 1) and (l.first.flg case "prime")
```

———–

— FR.dotabb —

```
"FR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FR" -> "ALIST"
```

———–

7.2 domain FILE File

— File.input —

```
)set break resume
```

```
)sys rm -f File.output
)spool File.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
ifile:File List Integer:=open("jazz1","output")
--R
--R
--R   (1)  "jazz1"
--R                                                  Type: File List Integer
--E 1

--S 2 of 12
write!(ifile, [-1,2,3])
--R
--R
--R   (2)  [- 1,2,3]
--R                                                       Type: List Integer
--E 2

--S 3 of 12
write!(ifile, [10,-10,0,111])
--R
--R
--R   (3)  [10,- 10,0,111]
--R                                                       Type: List Integer
--E 3

--S 4 of 12
write!(ifile, [7])
--R
--R
--R   (4)  [7]
--R                                                       Type: List Integer
--E 4

--S 5 of 12
reopen!(ifile, "input")
--R
--R
--R   (5)  "jazz1"
--R                                                  Type: File List Integer
--E 5

--S 6 of 12
read! ifile
--R
--R
--R   (6)  [- 1,2,3]
```

```
--R                                                    Type: List Integer
--E 6

--S 7 of 12
read! ifile
--R
--R
--R   (7)  [10,- 10,0,111]
--R                                                    Type: List Integer
--E 7

--S 8 of 12
readIfCan! ifile
--R
--R
--R   (8)  [7]
--R                                         Type: Union(List Integer,...)
--E 8

--S 9 of 12
readIfCan! ifile
--R
--R
--R   (9)  "failed"
--R                                            Type: Union("failed",...)
--E 9

--S 10 of 12
iomode ifile
--R
--R
--R   (10)  "input"
--R                                                           Type: String
--E 10

--S 11 of 12
name ifile
--R
--R
--R   (11)  "jazz1"
--R                                                         Type: FileName
--E 11

--S 12 of 12
close! ifile
--R
--R
--R   (12)  "jazz1"
--R                                               Type: File List Integer
--E 12
```

```
)system rm jazz1
)spool
)lisp (bye)
```

———-

— **File.help** —

```
====================================================================
File examples
====================================================================

The File(S) domain provides a basic interface to read and write values
of type S in files.

Before working with a file, it must be made accessible to Axiom with
the open operation.

  ifile:File List Integer:=open("/tmp/jazz1","output")
    "jazz1"
                         Type: File List Integer

The open function arguments are a FileNam} and a String specifying the
mode.  If a full pathname is not specified, the current default
directory is assumed.  The mode must be one of "input" or "output".
If it is not specified, "input" is assumed.  Once the file has been
opened, you can read or write data.

The operations read and write are provided.

  write!(ifile, [-1,2,3])
    [- 1,2,3]
                         Type: List Integer

  write!(ifile, [10,-10,0,111])
    [10,- 10,0,111]
                         Type: List Integer

  write!(ifile, [7])
    [7]
                         Type: List Integer

You can change from writing to reading (or vice versa) by reopening a file.

  reopen!(ifile, "input")
    "jazz1"
                         Type: File List Integer

  read! ifile
```

```
  [- 1,2,3]
                     Type: List Integer

read! ifile
  [10,- 10,0,111]
                     Type: List Integer
```

The read operation can cause an error if one tries to read more data than is in the file. To guard against this possibility the readIfCan operation should be used.

```
readIfCan! ifile
  [7]
                     Type: Union(List Integer,...)

readIfCan! ifile
  "failed"
                     Type: Union("failed",...)
```

You can find the current mode of the file, and the file's name.

```
iomode ifile
  "input"
                     Type: String

name ifile
  "jazz1"
                     Type: FileName
```

When you are finished with a file, you should close it.

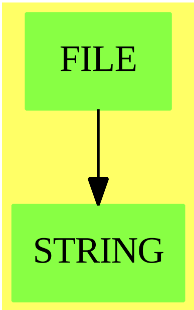
```
close! ifile
  "jazz1"
                     Type: File List Integer

)system rm /tmp/jazz1
```

A limitation of the underlying LISP system is that not all values can be represented in a file. In particular, delayed values containing compiled functions cannot be saved.

See Also:
o)help TextFile
o)help KeyedAccessFile
o)help Library
o)help Filename
o)show File

7.2.1 File (FILE)



See

⇒ "TextFile" (TEXTFILE) 21.5.1 on page 2651
⇒ "BinaryFile" (BINFILE) 3.8.1 on page 277
⇒ "KeyedAccessFile" (KAFILE) 12.2.1 on page 1377
⇒ "Library" (LIB) 13.2.1 on page 1392

Exports:

close!	coerce	flush	hash	iomode
latex	name	open	readIfCan!	read!
reopen!	write!	?=?	?~=?	

— **domain FILE File** —

```
)abbrev domain FILE File
++ Author: Stephen M. Watt, Victor Miller
++ Date Created: 1984
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides a basic model of files to save arbitrary values.
++ The operations provide sequential access to the contents.

File(S:SetCategory): FileCategory(FileName, S) with
        readIfCan_!: % -> Union(S, "failed")
            ++ readIfCan!(f) returns a value from the file f, if possible.
            ++ If f is not open for reading, or if f is at the end of file
            ++ then \spad{"failed"} is the result.
```

```
== add
    FileState ==> SExpression
    IOMode    ==> String

    Rep:=Record(fileName:    FileName,   _
                fileState:   FileState,  _
                fileIOmode:  IOMode)

    defstream(fn: FileName, mode: IOMode): FileState ==
        mode = "input"  =>
          not readable? fn => error ["File is not readable", fn]
          MAKE_-INSTREAM(fn::String)$Lisp
        mode = "output" =>
          not writable? fn => error ["File is not writable", fn]
          MAKE_-OUTSTREAM(fn::String)$Lisp
        error ["IO mode must be input or output", mode]

    f1 = f2 ==
        f1.fileName = f2.fileName
    coerce(f: %): OutputForm ==
        f.fileName::OutputForm

    open fname ==
        open(fname, "input")
    open(fname, mode) ==
        fstream := defstream(fname, mode)
        [fname, fstream, mode]
    reopen_!(f, mode) ==
        fname := f.fileName
        f.fileState := defstream(fname, mode)
        f.fileIOmode:= mode
        f
    close_! f ==
        SHUT(f.fileState)$Lisp
        f.fileIOmode := "closed"
        f
    name f ==
        f.fileName
    iomode f ==
        f.fileIOmode
    read_! f ==
        f.fileIOmode ^= "input" =>
            error "File not in read state"
        x := VMREAD(f.fileState)$Lisp
        PLACEP(x)$Lisp =>
            error "End of file"
        x
    readIfCan_! f ==
        f.fileIOmode ^= "input" =>
            error "File not in read state"
```

```
            x: S := VMREAD(f.fileState)$Lisp
            PLACEP(x)$Lisp => "failed"
            x
        write_!(f, x) ==
            f.fileIOmode ^= "output" =>
                error "File not in write state"
            z := PRINT_-FULL(x, f.fileState)$Lisp
            TERPRI(f.fileState)$Lisp
            x

        flush f ==
            f.fileIOmode ^= "output" => error "File not in write state"
            FORCE_-OUTPUT(f.fileState)$Lisp
```

— FILE.dotabb —

```
"FILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FILE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FILE" -> "STRING"
```

7.3 domain FNAME FileName

— FileName.input —

```
)set break resume
)sys rm -f FileName.output
)spool FileName.output
)set message test on
)set message auto off
)clear all
--S 1 of 18
fn: FileName
--R
--R                                                                      Type: Void
--E 1

--S 2 of 18
fn := "fname.input"
--R
--R
```

```
--R   (2)  "fname.input"
--R                                                    Type: FileName
--E 2

--S 3 of 18
directory fn
--R
--R
--R   (3)  ""
--R                                                      Type: String
--E 3

--S 4 of 18
name fn
--R
--R
--R   (4)  "fname"
--R                                                      Type: String
--E 4

--S 5 of 18
extension fn
--R
--R
--R   (5)  "input"
--R                                                      Type: String
--E 5

--S 6 of 18
fn := filename("/tmp", "fname", "input")
--R
--R
--R   (6)  "/tmp/fname.input"
--R                                                    Type: FileName
--E 6

--S 7 of 18
objdir := "/tmp"
--R
--R
--R   (7)  "/tmp"
--R                                                      Type: String
--E 7

--S 8 of 18
fn := filename(objdir, "table", "spad")
--R
--R
--R   (8)  "/tmp/table.spad"
--R                                                    Type: FileName
```

```
--E 8

--S 9 of 18
fn := filename("", "letter", "")
--R
--R
--R   (9)  "letter"
--R                                                              Type: FileName
--E 9

--S 10 of 18
exists? "/etc/passwd"
--R
--R
--R   (10)  true
--R                                                               Type: Boolean
--E 10

--S 11 of 18
readable? "/etc/passwd"
--R
--R
--R   (11)  true
--R                                                               Type: Boolean
--E 11

--S 12 of 18
readable? "/etc/security/passwd"
--R
--R
--R   (12)  false
--R                                                               Type: Boolean
--E 12

--S 13 of 18
readable? "/ect/passwd"
--R
--R
--R   (13)  false
--R                                                               Type: Boolean
--E 13

--S 14 of 18
writable? "/etc/passwd"
--R
--R
--R   (14)  true
--R                                                               Type: Boolean
--E 14
```

```
--S 15 of 18
writable? "/dev/null"
--R
--R
--R   (15)  true
--R                                                                Type: Boolean
--E 15

--S 16 of 18
writable? "/etc/DoesNotExist"
--R
--R
--R   (16)  true
--R                                                                Type: Boolean
--E 16

--S 17 of 18
writable? "/tmp/DoesNotExist"
--R
--R
--R   (17)  true
--R                                                                Type: Boolean
--E 17

--S 18 of 18
fn := new(objdir, "xxx", "yy")
--R
--R
--I   (18)  "/tmp/xxx1419.yy"
--R                                                               Type: FileName
--E 18
)spool
)lisp (bye)
```

— FileName.help —

```
====================================================================
FileName examples
====================================================================

The FileName domain provides an interface to the computer's file
system.  Functions are provided to manipulate file names and to test
properties of files.

The simplest way to use file names in the Axiom interpreter is to rely
on conversion to and from strings.  The syntax of these strings
depends on the operating system.
```

```
  fn: FileName
                      Type: Void
```

On Linux, this is a proper file syntax:

```
  fn := "fname.input"
    "fname.input"
                      Type: FileName
```

Although it is very convenient to be able to use string notation for file names in the interpreter, it is desirable to have a portable way of creating and manipulating file names from within programs.

A measure of portability is obtained by considering a file name to consist of three parts: the directory, the name, and the extension.

```
  directory fn
    ""
                     Type: String

  name fn
    "fname"
                     Type: String

  extension fn
    "input"
                     Type: String
```

The meaning of these three parts depends on the operating system. For example, on CMS the file "SPADPROF INPUT M" would have directory "M", name "SPADPROF" and extension "INPUT".

It is possible to create a filename from its parts.

```
  fn := filename("/tmp", "fname", "input")
    "/tmp/fname.input"
                     Type: FileName
```

When writing programs, it is helpful to refer to directories via variables.

```
  objdir := "/tmp"
    "/tmp"
                     Type: String

  fn := filename(objdir, "table", "spad")
    "/tmp/table.spad"
                     Type: FileName
```

If the directory or the extension is given as an empty string, then a default is used. On AIX, the defaults are the current directory and no extension.

```
  fn := filename("", "letter", "")
    "letter"
                        Type: FileName
```

Three tests provide information about names in the file system.

The exists? operation tests whether the named file exists.

```
  exists? "/etc/passwd"
    true
                        Type: Boolean
```

The operation readable? tells whether the named file can be read. If the file does not exist, then it cannot be read.

```
  readable? "/etc/passwd"
    true
                        Type: Boolean

  readable? "/etc/security/passwd"
    false
                        Type: Boolean

  readable? "/ect/passwd"
    false
                        Type: Boolean
```

Likewise, the operation writable? tells whether the named file can be written. If the file does not exist, the test is determined by the properties of the directory.

```
  writable? "/etc/passwd"
    true
                        Type: Boolean

  writable? "/dev/null"
    true
                        Type: Boolean

  writable? "/etc/DoesNotExist"
    true
                        Type: Boolean

  writable? "/tmp/DoesNotExist"
    true
                        Type: Boolean
```

```
The new operation constructs the name of a new writable file.  The
argument sequence is the same as for filename, except that the name
part is actually a prefix for a constructed unique name.

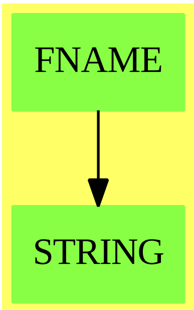
The resulting file is in the specified directory with the given
extension, and the same defaults are used.

  fn := new(objdir, "xxx", "yy")
    "/tmp/xxx1419.yy"
                      Type: FileName

See Also:
o )show FileName
```

———-

7.3.1 FileName (FNAME)



Exports:

coerce	directory	exists?	extension	filename
hash	latex	name	new	readable?
writable?	?=?	?~=?		

— domain FNAME FileName —

```
)abbrev domain FNAME FileName
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 20, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides an interface to names in the file system.

FileName(): FileNameCategory == add

        f1 = f2                  == EQUAL(f1, f2)$Lisp
        coerce(f: %): OutputForm == f::String::OutputForm

        coerce(f: %): String     == NAMESTRING(f)$Lisp
        coerce(s: String): %     == PARSE_-NAMESTRING(s)$Lisp

        filename(d,n,e)          == fnameMake(d,n,e)$Lisp

        directory(f:%): String   == fnameDirectory(f)$Lisp
        name(f:%): String        == fnameName(f)$Lisp
        extension(f:%): String   == fnameType(f)$Lisp

        exists? f                == fnameExists?(f)$Lisp
        readable? f              == fnameReadable?(f)$Lisp
        writable? f              == fnameWritable?(f)$Lisp

        new(d,pref,e)            == fnameNew(d,pref,e)$Lisp
```

— **FNAME.dotabb** —

```
"FNAME" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FNAME"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FNAME" -> "STRING"
```

7.4 domain FDIV FiniteDivisor

— **FiniteDivisor.input** —

```
)set break resume
)sys rm -f FiniteDivisor.output
)spool FiniteDivisor.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show FiniteDivisor
--R FiniteDivisor(F: Field,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePolynomialCateg
--R Abbreviation for FiniteDivisor is FDIV
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FDIV
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R divisor : (R,UP,UP,UP,F) -> %         divisor : (F,F,Integer) -> %
--R divisor : (F,F) -> %                  divisor : R -> %
--R finiteBasis : % -> Vector R           hash : % -> SingleInteger
--R lSpaceBasis : % -> Vector R           latex : % -> String
--R principal? : % -> Boolean             reduce : % -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R decompose : % -> Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
--R divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
--R generator : % -> Union(R,"failed")
--R ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

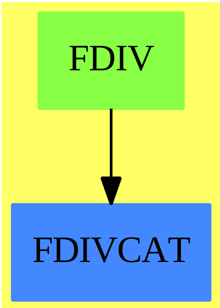
— **FiniteDivisor.help** —

```
====================================================================
FiniteDivisor examples
====================================================================

See Also:
o )show FiniteDivisor
```

———

7.4.1 FiniteDivisor (FDIV)



See

⇒ "FractionalIdeal" (FRIDEAL) 7.25.1 on page 961
⇒ "FramedModule" (FRMOD) 7.26.1 on page 967
⇒ "HyperellipticFiniteDivisor" (HELLFDIV) 9.11.1 on page 1149

Exports:

0	coerce	decompose	divisor	finiteBasis
generator	hash	ideal	lSpaceBasis	latex
principal?	reduce	sample	subtractIfCan	zero?
?~=?	?*?	?+?	?-?	-?
?=?				

— domain FDIV FiniteDivisor —

```
)abbrev domain FDIV FiniteDivisor
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 29 July 1993
++ Description:
++ This domains implements finite rational divisors on a curve, that
++ is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.
++ Keywords: divisor, algebraic, curve.
++ Examples: )r FDIV INPUT

FiniteDivisor(F, UP, UPUP, R): Exports == Implementation where
  F    : Field
  UP   : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP
  R    : FunctionFieldCategory(F, UP, UPUP)

  N    ==> NonNegativeInteger
  RF   ==> Fraction UP
  ID   ==> FractionalIdeal(UP, RF, UPUP, R)

  Exports ==> FiniteDivisorCategory(F, UP, UPUP, R) with
```

```
    finiteBasis: % -> Vector R
      ++ finiteBasis(d) returns a basis for d as a module over K[x].
    lSpaceBasis: % -> Vector R
      ++ lSpaceBasis(d) returns a basis for \spad{L(d) = {f | (f) >= -d}}
      ++ as a module over \spad{K[x]}.

  Implementation ==> add
    if hyperelliptic()$R case UP then
      Rep := HyperellipticFiniteDivisor(F, UP, UPUP, R)

      0                        == 0$Rep
      coerce(d:$):OutputForm  == coerce(d)$Rep
      d1 = d2                  == d1 =$Rep d2
      n * d                    == n *$Rep d
      d1 + d2                  == d1 +$Rep d2
      - d                      == -$Rep d
      ideal d                  == ideal(d)$Rep
      reduce d                 == reduce(d)$Rep
      generator d              == generator(d)$Rep
      decompose d              == decompose(d)$Rep
      divisor(i:ID)            == divisor(i)$Rep
      divisor(f:R)             == divisor(f)$Rep
      divisor(a, b)            == divisor(a, b)$Rep
      divisor(a, b, n)         == divisor(a, b, n)$Rep
      divisor(h, d, dp, g, r) == divisor(h, d, dp, g, r)$Rep

    else
      Rep := Record(id:ID, fbasis:Vector(R))

      import CommonDenominator(UP, RF, Vector RF)
      import UnivariatePolynomialCommonDenominator(UP, RF, UPUP)

      makeDivisor : (UP, UPUP, UP) -> %
      intReduce   : (R, UP) -> R

      ww := integralBasis()$R

      0                        == [1, empty()]
      divisor(i:ID)            == [i, empty()]
      divisor(f:R)             == divisor ideal [f]
      coerce(d:%):OutputForm  == ideal(d)::OutputForm
      ideal d                  == d.id
      decompose d              == [ideal d, 1]
      d1 = d2                  == basis(ideal d1) = basis(ideal d2)
      n * d                    == divisor(ideal(d) ** n)
      d1 + d2                  == divisor(ideal d1 * ideal d2)
      - d                      == divisor inv ideal d
      divisor(h, d, dp, g, r) == makeDivisor(d, lift h - (r * dp)::RF::UPUP, g)

      intReduce(h, b) ==
```

```
        v := integralCoordinates(h).num
        integralRepresents(
                      [qelt(v, i) rem b for i in minIndex v .. maxIndex v], 1)

      divisor(a, b) ==
        x := monomial(1, 1)$UP
        not ground? gcd(d := x - a::UP, retract(discriminant())@UP) =>
                                          error "divisor: point is singular"
        makeDivisor(d, monomial(1, 1)$UPUP - b::UP::RF::UPUP, 1)

      divisor(a, b, n) ==
        not(ground? gcd(d := monomial(1, 1)$UP - a::UP,
            retract(discriminant())@UP)) and
                ((n exquo rank()) case "failed") =>
                                  error "divisor: point is singular"
        m:N :=
          n < 0 => (-n)::N
          n::N
        g := makeDivisor(d**m,(monomial(1,1)$UPUP - b::UP::RF::UPUP)**m,1)
        n < 0 => -g
        g

      reduce d ==
        (i := minimize(j := ideal d)) = j => d
        #(n := numer i) ^= 2 => divisor i
        cd := splitDenominator lift n(1 + minIndex n)
        b  := gcd(cd.den * retract(retract(n minIndex n)@RF)@UP,
                  retract(norm reduce(cd.num))@UP)
        e  := cd.den * denom i
        divisor ideal([(b / e)::R,
          reduce map((s:RF):RF+->(retract(s)@UP rem b)/e, cd.num)]$Vector(R))

      finiteBasis d ==
        if empty?(d.fbasis) then
          d.fbasis := normalizeAtInfinity
                        basis module(ideal d)$FramedModule(UP, RF, UPUP, R, ww)
        d.fbasis

      generator d ==
        bsis := finiteBasis d
        for i in minIndex bsis .. maxIndex bsis repeat
          integralAtInfinity? qelt(bsis, i) =>
            return primitivePart qelt(bsis,i)
        "failed"

      lSpaceBasis d ==
        map_!(primitivePart, reduceBasisAtInfinity finiteBasis(-d))

-- b = center, hh = integral function, g = gcd(b, discriminant)
      makeDivisor(b, hh, g) ==
```

```
      b := gcd(b, retract(norm(h := reduce hh))@UP)
      h := intReduce(h, b)
      if not ground? gcd(g, b) then h := intReduce(h ** rank(), b)
      divisor ideal [b::RF::R, h]$Vector(R)
```

———-

— FDIV.dotabb —

```
"FDIV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FDIV"]
"FDIVCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FDIVCAT"]
"FDIV" -> "FDIVCAT"
```

———-

7.5 domain FF FiniteField

— FiniteField.input —

```
)set break resume
)sys rm -f FiniteField.output
)spool FiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteField
--R FiniteField(p: PositiveInteger,n: PositiveInteger)  is a domain constructor
--R Abbreviation for FiniteField is FF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PrimeField p,%) -> %          ?*? : (%,PrimeField p) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,PrimeField p) -> %          ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %       algebraic? : % -> Boolean
```

```
--R associates? : (%,%) -> Boolean        basis : () -> Vector %
--R coerce : PrimeField p -> %            coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R norm : % -> PrimeField p              one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> PrimeField p           sample : () -> %
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               trace : % -> PrimeField p
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R D : % -> % if PrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R Frobenius : % -> % if PrimeField p has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R charthRoot : % -> % if PrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if PrimeField p has FINITE
--R coordinates : Vector % -> Matrix PrimeField p
--R coordinates : % -> Vector PrimeField p
--R createNormalElement : () -> % if PrimeField p has FINITE
--R createPrimitiveElement : () -> % if PrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial PrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R differentiate : % -> % if PrimeField p has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PrimeField p has CHARNZ or PrimeField p
--R discreteLog : % -> NonNegativeInteger if PrimeField p has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if PrimeField p has
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
```

```
--R generator : () -> % if PrimeField p has FINITE
--R index : PositiveInteger -> % if PrimeField p has FINITE
--R init : () -> % if PrimeField p has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial PrimeField p) -> % if PrimeField p ha
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial PrimeField p,"failed") i
--R linearAssociatedLog : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has F
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has
--R lookup : % -> PositiveInteger if PrimeField p has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if PrimeField p h
--R minimalPolynomial : % -> SparseUnivariatePolynomial PrimeField p
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if PrimeField p has FINITE
--R norm : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R normal? : % -> Boolean if PrimeField p has FINITE
--R normalElement : () -> % if PrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if PrimeField p has CHARNZ or PrimeField
--R order : % -> PositiveInteger if PrimeField p has FINITE
--R primeFrobenius : % -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if PrimeField p has CHARNZ or PrimeField p h
--R primitive? : % -> Boolean if PrimeField p has FINITE
--R primitiveElement : () -> % if PrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if PrimeField p has F
--R represents : Vector PrimeField p -> %
--R retractIfCan : % -> Union(PrimeField p,"failed")
--R size : () -> NonNegativeInteger if PrimeField p has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Prime
--R trace : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———

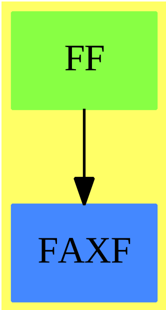
— **FiniteField.help** —

```
====================================================================
FiniteField examples
====================================================================

See Also:
o )show FiniteField
```

——–

7.5.1 FiniteField (FF)



See

⇒ "FiniteFieldExtensionByPolynomial" (FFP) 7.10.1 on page 818
⇒ "FiniteFieldExtension" (FFX) 7.9.1 on page 813
⇒ "InnerFiniteField" (IFF) 10.21.1 on page 1247

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	index	inGroundField?
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain FF FiniteField —

```
)abbrev domain FF FiniteField
++ Author: Mark Botch
++ Date Created: ???
++ Date Last Updated: 29 May 1990
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++  finite extension, finite field, Galois field
++ Reference:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
```

```
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteField(p,n) implements finite fields with p**n elements.
++ This packages checks that p is prime.
++ For a non-checking version, see \spadtype{InnerFiniteField}.

FiniteField(p:PositiveInteger, n:PositiveInteger): _
   FiniteAlgebraicExtensionField(PrimeField p) ==_
   FiniteFieldExtensionByPolynomial(PrimeField p,_
       createIrreduciblePoly(n)$FiniteFieldPolynomialPackage(PrimeField p))
  -- old code for generating irreducible polynomials:
  -- now "better" order (sparse polys first)
  -- generateIrredPoly(n)$IrredPolyOverFiniteField(GF))
```

— **FF.dotabb** —

```
"FF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FF"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FF" -> "FAXF"
```

7.6 domain FFCG FiniteFieldCyclicGroup

— **FiniteFieldCyclicGroup.input** —

```
)set break resume
)sys rm -f FiniteFieldCyclicGroup.output
)spool FiniteFieldCyclicGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldCyclicGroup
--R FiniteFieldCyclicGroup(p: PositiveInteger,extdeg: PositiveInteger)  is a domain constructor
--R Abbreviation for FiniteFieldCyclicGroup is FFCG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFCG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PrimeField p,%) -> %           ?*? : (%,PrimeField p) -> %
```

```
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %         ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                       -? : % -> %
--R ?/? : (%,PrimeField p) -> %            ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                 1 : () -> %
--R 0 : () -> %                            ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %         algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean         basis : () -> Vector %
--R coerce : PrimeField p -> %             coerce : Fraction Integer -> %
--R coerce : % -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm               degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber       factor : % -> Factored %
--R gcd : List % -> %                      gcd : (%,%) -> %
--R hash : % -> SingleInteger              inGroundField? : % -> Boolean
--R inv : % -> %                           latex : % -> String
--R lcm : List % -> %                      lcm : (%,%) -> %
--R norm : % -> PrimeField p               one? : % -> Boolean
--R prime? : % -> Boolean                  ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")         ?rem? : (%,%) -> %
--R retract : % -> PrimeField p            sample : () -> %
--R sizeLess? : (%,%) -> Boolean           squareFree : % -> Factored %
--R squareFreePart : % -> %                trace : % -> PrimeField p
--R transcendent? : % -> Boolean           unit? : % -> Boolean
--R unitCanonical : % -> %                 zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R D : % -> % if PrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R Frobenius : % -> % if PrimeField p has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R charthRoot : % -> % if PrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if PrimeField p has FINITE
--R coordinates : Vector % -> Matrix PrimeField p
--R coordinates : % -> Vector PrimeField p
--R createNormalElement : () -> % if PrimeField p has FINITE
--R createPrimitiveElement : () -> % if PrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial PrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R differentiate : % -> % if PrimeField p has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PrimeField p has CHARNZ or P
--R discreteLog : % -> NonNegativeInteger if PrimeField p has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
```

```
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if PrimeField p has
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if PrimeField p has FINITE
--R getZechTable : () -> PrimitiveArray SingleInteger
--R index : PositiveInteger -> % if PrimeField p has FINITE
--R init : () -> % if PrimeField p has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial PrimeField p) -> % if PrimeField p has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial PrimeField p,"failed") if PrimeField
--R linearAssociatedLog : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R lookup : % -> PositiveInteger if PrimeField p has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if PrimeField p has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial PrimeField p
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if PrimeField p has FINITE
--R norm : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R normal? : % -> Boolean if PrimeField p has FINITE
--R normalElement : () -> % if PrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if PrimeField p has CHARNZ or PrimeField p has FINIT
--R order : % -> PositiveInteger if PrimeField p has FINITE
--R primeFrobenius : % -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primitive? : % -> Boolean if PrimeField p has FINITE
--R primitiveElement : () -> % if PrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if PrimeField p has FINITE
--R represents : Vector PrimeField p -> %
--R retractIfCan : % -> Union(PrimeField p,"failed")
--R size : () -> NonNegativeInteger if PrimeField p has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if PrimeField p has
--R trace : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———

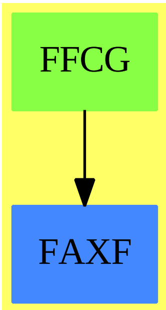
— **FiniteFieldCyclicGroup.help** —

```
====================================================================
FiniteFieldCyclicGroup examples
====================================================================

See Also:
o )show FiniteFieldCyclicGroup
```

———-

7.6.1 FiniteFieldCyclicGroup (FFCG)



See

⇒ "FiniteFieldCyclicGroupExtensionByPolynomial" (FFCGP) 7.8.1 on page 802
⇒ "FiniteFieldCyclicGroupExtension" (FFCGX) 7.7.1 on page 797

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
getZechTable	hash	index
inGroundField?	init	inv
latex	lcm	linearAssociatedExp
linearAssociatedLog	linearAssociatedOrder	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	normal?	normalElement
one?	order	prime?
primeFrobenius	primitive?	primitiveElement
principalIdeal	random	recip
representationType	represents	retract
retractIfCan	sample	size
sizeLess?	squareFree	squareFreePart
subtractIfCan	tableForDiscreteLogarithm	trace
transcendenceDegree	transcendent?	unit?
unitCanonical	unitNormal	zero?
?*?	?**?	?+?
?-?	-?	?/?
?=?	?^?	?~=?
?quo?	?rem?	

— domain FFCG FiniteFieldCyclicGroup —

```
)abbrev domain FFCG FiniteFieldCyclicGroup
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 04.04.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors:  FiniteFieldCyclicGroupExtensionByPolynomial,
++  FiniteFieldPolynomialPackage
++ Also See: FiniteField, FiniteFieldNormalBasis
++ AMS Classifications:
++ Keywords: finite field, primitive elements, cyclic group
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ Description:
```

```
++ FiniteFieldCyclicGroup(p,n) implements a finite field extension of degee n
++ over the prime field with p elements. Its elements are represented by
++ powers of a primitive element, i.e. a generator of the multiplicative
++ (cyclic) group. As primitive element we choose the root of the extension
++ polynomial, which is created by createPrimitivePoly from
++ \spadtype{FiniteFieldPolynomialPackage}. The Zech logarithms are stored
++ in a table of size half of the field size, and use \spadtype{SingleInteger}
++ for representing field elements, hence, there are restrictions
++ on the size of the field.

FiniteFieldCyclicGroup(p,extdeg):_
  Exports == Implementation where
  p : PositiveInteger
  extdeg   : PositiveInteger
  PI       ==> PositiveInteger
  FFPOLY         ==> FiniteFieldPolynomialPackage(PrimeField(p))
  SI       ==> SingleInteger
  Exports  ==> FiniteAlgebraicExtensionField(PrimeField(p)) with
    getZechTable:() -> PrimitiveArray(SingleInteger)
      ++ getZechTable() returns the zech logarithm table of the field.
      ++ This table is used to perform additions in the field quickly.
  Implementation ==> FiniteFieldCyclicGroupExtensionByPolynomial(PrimeField(p),_
                          createPrimitivePoly(extdeg)$FFPOLY)
```

— FFCG.dotabb —

```
"FFCG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFCG"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFCG" -> "FAXF"
```

7.7 domain FFCGX FiniteFieldCyclicGroupExtension

— FiniteFieldCyclicGroupExtension.input —

```
)set break resume
)sys rm -f FiniteFieldCyclicGroupExtension.output
)spool FiniteFieldCyclicGroupExtension.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show FiniteFieldCyclicGroupExtension
--R FiniteFieldCyclicGroupExtension(GF: FiniteFieldCategory,extdeg: PositiveInteger)  is a domain constr
--R Abbreviation for FiniteFieldCyclicGroupExtension is FFCGX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFCGX
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (GF,%) -> %                    ?*? : (%,GF) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,GF) -> %                    ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               D : % -> % if GF has FINITE
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean            associates? : (%,%) -> Boolean
--R basis : () -> Vector %               coerce : GF -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R coordinates : % -> Vector GF         degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R norm : % -> GF                       one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R represents : Vector GF -> %          retract : % -> GF
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R trace : % -> GF                      transcendent? : % -> Boolean
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
```

```
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINI
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R generator : () -> % if GF has FINITE
--R getZechTable : () -> PrimitiveArray SingleInteger
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FI
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if GF has FINITE
--R norm : (%,PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF,"failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF ha
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

```
--R
--E 1

)spool
)lisp (bye)
```

———-

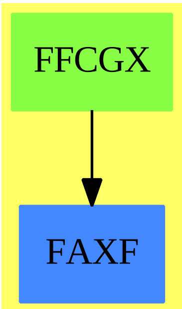
— **FiniteFieldCyclicGroupExtension.help** —

```
====================================================================
FiniteFieldCyclicGroupExtension examples
====================================================================

See Also:
o )show FiniteFieldCyclicGroupExtension
```

———-

7.7.1 FiniteFieldCyclicGroupExtension (FFCGX)



See

⇒ "FiniteFieldCyclicGroupExtensionByPolynomial" (FFCGP) 7.8.1 on page 802
⇒ "FiniteFieldCyclicGroup" (FFCG) 7.6.1 on page 792

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
getZechTable	hash	index
inGroundField?	init	inv
latex	lcm	linearAssociatedExp
linearAssociatedLog	linearAssociatedOrder	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	normal?	normalElement
one?	order	prime?
primeFrobenius	primitive?	primitiveElement
principalIdeal	random	recip
representationType	represents	retract
retractIfCan	sample	size
sizeLess?	squareFree	squareFreePart
subtractIfCan	tableForDiscreteLogarithm	trace
transcendenceDegree	transcendent?	unit?
unitCanonical	unitNormal	zero?
?*?	?**?	?+?
?-?	-?	?/?
?=?	?^?	?~=?
?quo?	?rem?	

— domain FFCGX FiniteFieldCyclicGroupExtension —

```
)abbrev domain FFCGX FiniteFieldCyclicGroupExtension
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 04.04.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors:  FiniteFieldCyclicGroupExtensionByPolynomial,
++  FiniteFieldPolynomialPackage
++ Also See: FiniteFieldExtension, FiniteFieldNormalBasisExtension
++ AMS Classifications:
++ Keywords: finite field, primitive elements, cyclic group
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
```

```
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldCyclicGroupExtension(GF,n)  implements a extension of degree n
++ over the ground field GF. Its elements are represented by powers of
++ a primitive element, i.e. a generator of the multiplicative (cyclic) group.
++ As primitive element we choose the root of the extension polynomial, which
++ is created by createPrimitivePoly from
++ \spadtype{FiniteFieldPolynomialPackage}. Zech logarithms are stored
++ in a table of size half of the field size, and use \spadtype{SingleInteger}
++ for representing field elements, hence, there are restrictions
++ on the size of the field.

FiniteFieldCyclicGroupExtension(GF,extdeg):_
  Exports == Implementation where
  GF       : FiniteFieldCategory
  extdeg   : PositiveInteger
  PI       ==> PositiveInteger
  FFPOLY         ==> FiniteFieldPolynomialPackage(GF)
  SI       ==> SingleInteger
  Exports  ==> FiniteAlgebraicExtensionField(GF) with
    getZechTable:() -> PrimitiveArray(SingleInteger)
      ++ getZechTable() returns the zech logarithm table of the field.
      ++ This table is used to perform additions in the field quickly.
  Implementation ==> FiniteFieldCyclicGroupExtensionByPolynomial(GF,_
                          createPrimitivePoly(extdeg)$FFPOLY)
```

———-

— **FFCGX.dotabb** —

```
"FFCGX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFCGX"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFCGX" -> "FAXF"
```

———-

7.8 domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial

— **FiniteFieldCyclicGroupExtensionByPolynomial.input** —

```
)set break resume
)sys rm -f FiniteFieldCyclicGroupExtensionByPolynomial.output
```

```
)spool FiniteFieldCyclicGroupExtensionByPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldCyclicGroupExtensionByPolynomial
--R FiniteFieldCyclicGroupExtensionByPolynomial(GF: FiniteFieldCategory,defpol: SparseUnivari
--R Abbreviation for FiniteFieldCyclicGroupExtensionByPolynomial is FFCGP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFCGP
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (GF,%) -> %                    ?*? : (%,GF) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,GF) -> %                    ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               D : % -> % if GF has FINITE
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean            associates? : (%,%) -> Boolean
--R basis : () -> Vector %               coerce : GF -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R coordinates : % -> Vector GF         degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R norm : % -> GF                       one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R represents : Vector GF -> %          retract : % -> GF
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R trace : % -> GF                      transcendent? : % -> Boolean
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
```

```
--R charthRoot : % -> Union(%,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if GF has FINITE
--R getZechTable : () -> PrimitiveArray SingleInteger
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if GF has FINITE
--R norm : (%,PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF,"failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
```

```
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF ha
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

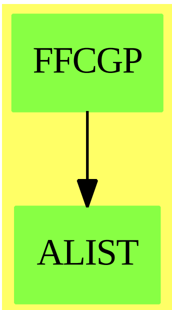
— FiniteFieldCyclicGroupExtensionByPolynomial.help —

```
====================================================================
FiniteFieldCyclicGroupExtensionByPolynomial examples
====================================================================

See Also:
o )show FiniteFieldCyclicGroupExtensionByPolynomial
```

———-

7.8.1 FiniteFieldCyclicGroupExtensionByPolynomial (FFCGP)



See

⇒ "FiniteFieldCyclicGroupExtension" (FFCGX) 7.7.1 on page 797
⇒ "FiniteFieldCyclicGroup" (FFCG) 7.6.1 on page 792

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
getZechTable	hash	index
inGroundField?	init	inv
latex	lcm	linearAssociatedExp
linearAssociatedLog	linearAssociatedOrder	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	normal?	normalElement
one?	order	prime?
primeFrobenius	primitive?	primitiveElement
principalIdeal	random	recip
representationType	represents	retract
retractIfCan	sample	size
sizeLess?	squareFree	squareFreePart
subtractIfCan	tableForDiscreteLogarithm	trace
transcendenceDegree	transcendent?	unit?
unitCanonical	unitNormal	zero?
?*?	?**?	?+?
?-?	-?	?/?
?=?	?^?	?~=?
?quo?	?rem?	

— domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial —

```
)abbrev domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors: FiniteFieldFunctions
++ Also See: FiniteFieldExtensionByPolynomial,
++  FiniteFieldNormalBasisExtensionByPolynomial
++ AMS Classifications:
++ Keywords: finite field, primitive elements, cyclic group
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
```

```
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldCyclicGroupExtensionByPolynomial(GF,defpol)  implements a
++ finite extension field of the ground field GF. Its elements are
++ represented by powers of a primitive element, i.e. a generator of the
++ multiplicative (cyclic) group. As primitive
++ element we choose the root of the extension polynomial defpol,
++ which MUST be primitive (user responsibility). Zech logarithms are stored
++ in a table of size half of the field size, and use \spadtype{SingleInteger}
++ for representing field elements, hence, there are restrictions
++ on the size of the field.

FiniteFieldCyclicGroupExtensionByPolynomial(GF,defpol):_
  Exports == Implementation where
  GF    : FiniteFieldCategory                -- the ground field
  defpol: SparseUnivariatePolynomial GF      -- the extension polynomial
  -- the root of defpol is used as the primitive element

  PI    ==> PositiveInteger
  NNI   ==> NonNegativeInteger
  I     ==> Integer
  SI    ==> SingleInteger
  SUP   ==> SparseUnivariatePolynomial
  SAE   ==> SimpleAlgebraicExtension(GF,SUP GF,defpol)
  V     ==> Vector GF
  FFP   ==> FiniteFieldExtensionByPolynomial(GF,defpol)
  FFF   ==> FiniteFieldFunctions(GF)
  OUT   ==> OutputForm
  ARR   ==> PrimitiveArray(SI)
  TBL   ==> Table(PI,NNI)


  Exports ==> FiniteAlgebraicExtensionField(GF)  with

    getZechTable:() -> ARR
      ++ getZechTable() returns the zech logarithm table of the field
      ++ it is used to perform additions in the field quickly.
  Implementation ==>  add

-- global variables ===================================================

    Rep:= SI
    -- elements are represented by small integers in the range
    -- (-1)..(size()-2). The (-1) representing the field element zero,
    -- the other small integers representing the corresponding power
    -- of the primitive element, the root of the defining polynomial

    -- it would be very nice if we could use the representation
    -- Rep:= Union("zero", IntegerMod(size()$GF ** degree(defpol) -1)),
```

```
    -- why doesn't the compiler like this ?

    extdeg:NNI  :=degree(defpol)$(SUP GF)::NNI
    -- the extension degree

    sizeFF:NNI:=(size()$GF ** extdeg) pretend NNI
    -- the size of the field

    if sizeFF > 2**20 then
      error "field too large for this representation"

    sizeCG:SI:=(sizeFF - 1) pretend SI
    -- the order of the cyclic group

    sizeFG:SI:=(sizeCG quo (size()$GF-1)) pretend SI
    -- the order of the factor group


    zechlog:ARR:=new(((sizeFF+1) quo 2)::NNI,-1::SI)$ARR
    -- the table for the zech logarithm

    alpha :=new()$Symbol :: OutputForm
    -- get a new symbol for the output representation of
    -- the elements

    primEltGF:GF:=
      odd?(extdeg)$I => -$GF coefficient(defpol,0)$(SUP GF)
      coefficient(defpol,0)$(SUP GF)
    -- the corresponding primitive element of the groundfield
    -- equals the trace of the primitive element w.r.t. the groundfield

    facOfGroupSize := nil()$(List Record(factor:Integer,exponent:Integer))
    -- the factorization of sizeCG

    initzech?:Boolean:=true
    -- gets false after initialization of the zech logarithm array

    initelt?:Boolean:=true
    -- gets false after initialization of the normal element

    normalElt:SI:=0
    -- the global variable containing a normal element

-- functions ===========================================================

    -- for completeness we have to give a dummy implementation for
    -- 'tableForDiscreteLogarithm', although this function is not
    -- necessary in the cyclic group representation case

    tableForDiscreteLogarithm(fac) == table()$TBL
```

```
    getZechTable() == zechlog
    initializeZech:() -> Void
    initializeElt: () -> Void

    order(x:$):PI ==
      zero?(x) =>
        error"order: order of zero undefined"
      (sizeCG quo gcd(sizeCG,x pretend NNI))::PI

    primitive?(x:$) ==
--      zero?(x) or one?(x) => false
      zero?(x) or (x = 1) => false
      gcd(x::Rep,sizeCG)$Rep = 1$Rep => true
      false

    coordinates(x:$) ==
      x=0 => new(extdeg,0)$(Vector GF)
      primElement:SAE:=convert(monomial(1,1)$(SUP GF))$SAE
-- the primitive element in the corresponding algebraic extension
      coordinates(primElement **$SAE (x pretend SI))$SAE

    x:$ + y:$ ==
      if initzech? then initializeZech()
      zero? x => y
      zero? y => x
      d:Rep:=positiveRemainder(y -$Rep x,sizeCG)$Rep
      (d pretend SI) <= shift(sizeCG,-$SI (1$SI)) =>
        zechlog.(d pretend SI) =$SI -1::SI => 0
        addmod(x,zechlog.(d pretend SI) pretend Rep,sizeCG)$Rep
      --d:Rep:=positiveRemainder(x -$Rep y,sizeCG)$Rep
      d:Rep:=(sizeCG -$SI d)::Rep
      addmod(y,zechlog.(d pretend SI) pretend Rep,sizeCG)$Rep
      --positiveRemainder(x +$Rep zechlog.(d pretend SI) -$Rep d,sizeCG)$Rep


    initializeZech() ==
      zechlog:=createZechTable(defpol)$FFF
      -- set initialization flag
      initzech? := false
      void()$Void

    basis(n:PI) ==
      extensionDegree() rem n ^= 0 =>
        error("argument must divide extension degree")
      m:=sizeCG quo (size()$GF**n-1)
      [index((1+i*m) ::PI) for i in 0..(n-1)]::Vector $

    n:I * x:$ == ((n::GF)::$) * x
```

```
minimalPolynomial(a) ==
  f:SUP $:=monomial(1,1)$(SUP $) - monomial(a,0)$(SUP $)
  u:$:=Frobenius(a)
  while not(u = a) repeat
    f:=f * (monomial(1,1)$(SUP $) - monomial(u,0)$(SUP $))
    u:=Frobenius(u)
  p:SUP GF:=0$(SUP GF)
  while not zero?(f)$(SUP $) repeat
    g:GF:=retract(leadingCoefficient(f)$(SUP $))
    p:=p+monomial(g,_
                 degree(f)$(SUP $))$(SUP GF)
    f:=reductum(f)$(SUP $)
  p

factorsOfCyclicGroupSize() ==
  if empty? facOfGroupSize then initializeElt()
  facOfGroupSize

representationType() == "cyclic"

definingPolynomial() == defpol

random() ==
  positiveRemainder(random()$Rep,sizeFF pretend Rep)$Rep -$Rep 1$Rep

represents(v) ==
  u:FFP:=represents(v)$FFP
  u =$FFP 0$FFP => 0
  discreteLog(u)$FFP pretend Rep



coerce(e:GF):$ ==
  zero?(e)$GF => 0
  log:I:=discreteLog(primEltGF,e)$GF::NNI *$I sizeFG
  -- version before 10.20.92: log pretend Rep
  -- 1$GF is coerced to sizeCG pretend Rep by old version
  -- now 1$GF is coerced to 0$Rep which is correct.
  positiveRemainder(log,sizeCG) pretend Rep


retractIfCan(x:$) ==
  zero? x => 0$GF
  u:= (x::Rep) exquo$Rep (sizeFG pretend Rep)
  u = "failed" => "failed"
  primEltGF **$GF ((u::$) pretend SI)

retract(x:$) ==
  a:=retractIfCan(x)
```

```
    a="failed" => error "element not in groundfield"
    a :: GF

  basis() == [index(i :: PI) for i in 1..extdeg]::Vector $


  inGroundField?(x) ==
    zero? x=> true
    positiveRemainder(x::Rep,sizeFG pretend Rep)$Rep =$Rep 0$Rep => true
    false

  discreteLog(b:$,x:$) ==
    zero? x => "failed"
    e:= extendedEuclidean(b,sizeCG,x)$Rep
    e = "failed" => "failed"
    e1:Record(coef1:$,coef2:$) := e :: Record(coef1:$,coef2:$)
    positiveRemainder(e1.coef1,sizeCG)$Rep pretend NNI

  - x:$ ==
      zero? x => 0
      characteristic() =$I 2 => x
      addmod(x,shift(sizeCG,-1)$SI pretend Rep,sizeCG)

  generator() == 1$SI
  createPrimitiveElement() == 1$SI
  primitiveElement() == 1$SI

  discreteLog(x:$) ==
    zero? x => error "discrete logarithm error"
    x pretend NNI

  normalElement() ==
    if initelt? then initializeElt()
    normalElt::$

  initializeElt() ==
    facOfGroupSize := factors(factor(sizeCG)$Integer)
    normalElt:=createNormalElement() pretend SI
    initelt?:=false
    void()$Void

  extensionDegree() == extdeg pretend PI

  characteristic() == characteristic()$GF

  lookup(x:$) ==
    x =$Rep (-$Rep 1$Rep) => sizeFF pretend PI
    (x +$Rep 1$Rep) pretend PI

  index(a:PI) ==
```

```
      positiveRemainder(a,sizeFF)$I pretend Rep -$Rep 1$Rep

    0 == (-$Rep 1$Rep)

    1 == 0$Rep

-- to get a "exponent like" output form
    coerce(x:$):OUT ==
      x =$Rep (-$Rep 1$Rep) => "0"::OUT
      x =$Rep 0$Rep => "1"::OUT
      y:I:=lookup(x)-1
      alpha **$OUT (y::OUT)

    x:$ = y:$ ==  x =$Rep y

    x:$ * y:$ ==
      x = 0 => 0
      y = 0 => 0
      addmod(x,y,sizeCG)$Rep

    a:GF * x:$ == coerce(a)@$ * x
    x:$/a:GF == x/coerce(a)@$

--    x:$ / a:GF ==
--      a = 0$GF => error "division by zero"
--      x * inv(coerce(a))

    inv(x:$)  ==
      zero?(x) => error "inv: not invertible"
--      one?(x) => 1
      (x = 1) => 1
      sizeCG -$Rep x

    x:$ ** n:PI == x ** n::I

    x:$ ** n:NNI == x ** n::I

    x:$ ** n:I ==
      m:Rep:=positiveRemainder(n,sizeCG)$I pretend Rep
      m =$Rep 0$Rep => 1
      x = 0 => 0
      mulmod(m,x,sizeCG::Rep)$Rep
```

— **FFCGP.dotabb** —

```
"FFCGP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFCGP"]
```

```
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FFCGP" -> "ALIST"
```

———-

7.9 domain FFX FiniteFieldExtension

— FiniteFieldExtension.input —

```
)set break resume
)sys rm -f FiniteFieldExtension.output
)spool FiniteFieldExtension.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldExtension
--R FiniteFieldExtension(GF: FiniteFieldCategory,n: PositiveInteger)  is a domain constructor
--R Abbreviation for FiniteFieldExtension is FFX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFX
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (GF,%) -> %                    ?*? : (%,GF) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,GF) -> %                    ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               D : % -> % if GF has FINITE
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean            associates? : (%,%) -> Boolean
--R basis : () -> Vector %               coerce : GF -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R coordinates : % -> Vector GF         degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R norm : % -> GF                       one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
```

```
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R represents : Vector GF -> %           retract : % -> GF
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R trace : % -> GF                       transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if GF has FINITE
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if GF has FINITE
--R norm : (%,PositiveInteger) -> % if GF has FINITE
```

```
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF,"failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF ha
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

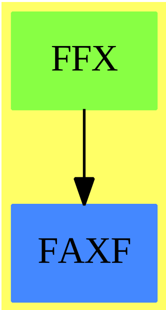
— FiniteFieldExtension.help —

```
====================================================================
FiniteFieldExtension examples
====================================================================

See Also:
o )show FiniteFieldExtension
```

———-

7.9.1 FiniteFieldExtension (FFX)



See

⇒ "FiniteFieldExtensionByPolynomial" (FFP) 7.10.1 on page 818
⇒ "InnerFiniteField" (IFF) 10.21.1 on page 1247
⇒ "FiniteField" (FF) 7.5.1 on page 787

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	index	inGroundField?
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain FFX FiniteFieldExtension —

```
)abbrev domain FFX FiniteFieldExtension
++ Authors: R.Sutor, J. Grabmeier, A. Scheerhorn
++ Date Created:
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors: FiniteFieldExtensionByPolynomial,
++  FiniteFieldPolynomialPackage
++ Also See: FiniteFieldCyclicGroupExtension,
++  FiniteFieldNormalBasisExtension
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++   finite extension, finite field, Galois field
++ Reference:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
```

```
++   Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++   AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldExtensionByPolynomial(GF, n) implements an extension
++ of the finite field GF of degree n generated by the extension
++ polynomial constructed by createIrreduciblePoly from
++ \spadtype{FiniteFieldPolynomialPackage}.

FiniteFieldExtension(GF, n): Exports == Implementation where
  GF: FiniteFieldCategory
  n : PositiveInteger
  Exports ==> FiniteAlgebraicExtensionField(GF)
  -- MonogenicAlgebra(GF, SUP) with  -- have to check this
  Implementation ==> FiniteFieldExtensionByPolynomial(GF,
       createIrreduciblePoly(n)$FiniteFieldPolynomialPackage(GF))
  -- old code for generating irreducible polynomials:
  -- now "better" order (sparse polys first)
  -- generateIrredPoly(n)$IrredPolyOverFiniteField(GF))
```

— **FFX.dotabb** —

```
"FFX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFX"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFX" -> "FAXF"
```

7.10 domain FFP FiniteFieldExtensionByPolynomial

— **FiniteFieldExtensionByPolynomial.input** —

```
)set break resume
)sys rm -f FiniteFieldExtensionByPolynomial.output
)spool FiniteFieldExtensionByPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldExtensionByPolynomial
--R FiniteFieldExtensionByPolynomial(GF: FiniteFieldCategory,defpol: SparseUnivariatePolynomial GF)  is
```

```
--R Abbreviation for FiniteFieldExtensionByPolynomial is FFP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (GF,%) -> %                     ?*? : (%,GF) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,GF) -> %                     ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : % -> % if GF has FINITE
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean             associates? : (%,%) -> Boolean
--R basis : () -> Vector %                coerce : GF -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R coordinates : % -> Vector GF          degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R norm : % -> GF                        one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R represents : Vector GF -> %           retract : % -> GF
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R trace : % -> GF                       transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
```

```
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if GF has FINITE
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if GF has FINITE
--R norm : (%,PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF,"failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF has FINITE
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

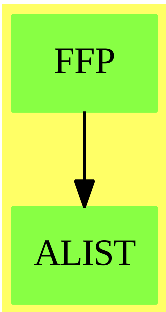
)spool
)lisp (bye)
```

— **FiniteFieldExtensionByPolynomial.help** —

```
====================================================================
FiniteFieldExtensionByPolynomial examples
====================================================================

See Also:
o )show FiniteFieldExtensionByPolynomial
```

7.10.1 FiniteFieldExtensionByPolynomial (FFP)



See

⇒ “FiniteFieldExtension” (FFX) 7.9.1 on page 813
⇒ “InnerFiniteField” (IFF) 10.21.1 on page 1247
⇒ “FiniteField” (FF) 7.5.1 on page 787

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	index	inGroundField?
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?/?
?=?	?^?	?~=?
?quo?	?rem?	

— domain FFP FiniteFieldExtensionByPolynomial —

```
)abbrev domain FFP FiniteFieldExtensionByPolynomial
++ Authors: R.Sutor, J. Grabmeier, O. Gschnitzer, A. Scheerhorn
++ Date Created:
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors:
++ Also See: FiniteFieldCyclicGroupExtensionByPolynomial,
++  FiniteFieldNormalBasisExtensionByPolynomial
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++   finite extension, finite field, Galois field
++ Reference:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
++   Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
```

```
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++   AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldExtensionByPolynomial(GF, defpol) implements the extension
++ of the finite field GF generated by the extension polynomial
++ defpol which MUST be irreducible.
++ Note: the user has the responsibility to ensure that
++ defpol is irreducible.

FiniteFieldExtensionByPolynomial(GF:FiniteFieldCategory,_
  defpol:SparseUnivariatePolynomial GF): Exports == Implementation where
--  GF     : FiniteFieldCategory
--  defpol : SparseUnivariatePolynomial GF

  PI   ==> PositiveInteger
  NNI  ==> NonNegativeInteger
  SUP  ==> SparseUnivariatePolynomial
  I    ==> Integer
  R    ==> Record(key:PI,entry:NNI)
  TBL  ==> Table(PI,NNI)
  SAE  ==> SimpleAlgebraicExtension(GF,SUP GF,defpol)
  OUT  ==> OutputForm

  Exports ==> FiniteAlgebraicExtensionField(GF)

  Implementation ==>  add

-- global variables ====================================================

    Rep:=SAE

    extdeg:PI        := degree(defpol)$(SUP GF) pretend PI
    -- the extension degree

    alpha            := new()$Symbol :: OutputForm
    -- a new symbol for the output form of field elements

    sizeCG:Integer := size()$GF**extdeg - 1
    -- the order of the multiplicative group

    facOfGroupSize := nil()$(List Record(factor:Integer,exponent:Integer))
    -- the factorization of sizeCG

    normalElt:PI:=1
    -- for the lookup of the normal Element computed by
    -- createNormalElement

    primitiveElt:PI:=1
    -- for the lookup of the primitive Element computed by
    -- createPrimitiveElement()
```

```
    initlog?:Boolean:=true
    -- gets false after initialization of the discrete logarithm table

    initelt?:Boolean:=true
    -- gets false after initialization of the primitive and the
    -- normal element


    discLogTable:Table(PI,TBL):=table()$Table(PI,TBL)
    -- tables indexed by the factors of sizeCG,
    -- discLogTable(factor) is a table  with keys
    -- primitiveElement() ** (i * (sizeCG quo factor)) and entries i for
    -- i in 0..n-1, n computed in initialize() in order to use
    -- the minimal size limit 'limit' optimal.

-- functions ===========================================================

--    createNormalElement() ==
--      a:=primitiveElement()
--      nElt:=generator()
--      for i in 1.. repeat
--        normal? nElt => return nElt
--        nElt:=nElt*a
--      nElt

    generator() == reduce(monomial(1,1)$SUP(GF))$Rep
    norm x   == resultant(defpol, lift x)

    initializeElt: () -> Void
    initializeLog: () -> Void
    basis(n:PI) ==
      (extdeg rem n) ^= 0 => error "argument must divide extension degree"
      a:$:=norm(primitiveElement(),n)
      vector [a**i for i in 0..n-1]

    degree(x) ==
      y:$:=1
      m:=zero(extdeg,extdeg+1)$(Matrix GF)
      for i in 1..extdeg+1 repeat
        setColumn_!(m,i,coordinates(y))$(Matrix GF)
        y:=y*x
      rank(m)::PI

    minimalPolynomial(x:$) ==
      y:$:=1
      m:=zero(extdeg,extdeg+1)$(Matrix GF)
      for i in 1..extdeg+1 repeat
        setColumn_!(m,i,coordinates(y))$(Matrix GF)
        y:=y*x
```

```
      v:=first nullSpace(m)$(Matrix GF)
      +/[monomial(v.(i+1),i)$(SUP GF) for i in 0..extdeg]


    normal?(x) ==
      l:List List GF:=[entries coordinates x]
      a:=x
      for i in 2..extdeg repeat
        a:=Frobenius(a)
        l:=concat(l,entries coordinates a)$(List List GF)
      ((rank matrix(l)$(Matrix GF)) = extdeg::NNI) => true
      false


    a:GF * x:$ == a *$Rep x
    n:I * x:$ == n *$Rep x
    -x == -$Rep x
    random() == random()$Rep
    coordinates(x:$) == coordinates(x)$Rep
    represents(v) == represents(v)$Rep
    coerce(x:GF):$ == coerce(x)$Rep
    definingPolynomial() == defpol
    retract(x) == retract(x)$Rep
    retractIfCan(x) == retractIfCan(x)$Rep
    index(x) == index(x)$Rep
    lookup(x) == lookup(x)$Rep
    x:$/y:$ == x /$Rep y
    x:$/a:GF == x/coerce(a)
--    x:$ / a:GF ==
--      a = 0$GF => error "division by zero"
--      x * inv(coerce(a))
    x:$ * y:$ == x *$Rep y
    x:$ + y:$ == x +$Rep y
    x:$ - y:$ == x -$Rep y
    x:$ = y:$ == x =$Rep y
    basis() == basis()$Rep
    0 == 0$Rep
    1 == 1$Rep

    factorsOfCyclicGroupSize() ==
      if empty? facOfGroupSize then initializeElt()
      facOfGroupSize

    representationType() == "polynomial"

    tableForDiscreteLogarithm(fac) ==
      if initlog? then initializeLog()
      tbl:=search(fac::PI,discLogTable)$Table(PI,TBL)
      tbl case "failed" =>
        error "tableForDiscreteLogarithm: argument must be prime divisor_
```

```
 of the order of the multiplicative group"
      tbl pretend TBL

    primitiveElement() ==
      if initelt? then initializeElt()
      index(primitiveElt)

    normalElement() ==
      if initelt? then initializeElt()
      index(normalElt)

    initializeElt() ==
      facOfGroupSize:=factors(factor(sizeCG)$Integer)
      -- get a primitive element
      pE:=createPrimitiveElement()
      primitiveElt:=lookup(pE)
      -- create a normal element
      nElt:=generator()
      while not normal? nElt repeat
        nElt:=nElt*pE
      normalElt:=lookup(nElt)
      -- set elements initialization flag
      initelt? := false
      void()$Void

    initializeLog() ==
      if initelt? then initializeElt()
-- set up tables for discrete logarithm
      limit:Integer:=30
    -- the minimum size for the discrete logarithm table
      for f in facOfGroupSize repeat
        fac:=f.factor
        base:$:=primitiveElement() ** (sizeCG quo fac)
        l:Integer:=length(fac)$Integer
        n:Integer:=0
        if odd?(l)$Integer then n:=shift(fac,-(l quo 2))
                           else n:=shift(1,(l quo 2))
        if n < limit then
          d:=(fac-1) quo limit + 1
          n:=(fac-1) quo d + 1
        tbl:TBL:=table()$TBL
        a:$:=1
        for i in (0::NNI)..(n-1)::NNI repeat
          insert_!([lookup(a),i::NNI]$R,tbl)$TBL
          a:=a*base
        insert_!([fac::PI,copy(tbl)$TBL]_
               $Record(key:PI,entry:TBL),discLogTable)$Table(PI,TBL)
      -- set logarithm initialization flag
      initlog? := false
      -- tell user about initialization
```

```
      --print("discrete logarithm tables initialized"::OUT)
      void()$Void

    coerce(e:$):OutputForm == outputForm(lift(e),alpha)

    extensionDegree() == extdeg

    size() == (sizeCG + 1) pretend NNI

--  sizeOfGroundField() == size()$GF

    inGroundField?(x) ==
      retractIfCan(x) = "failed" => false
      true

    characteristic() == characteristic()$GF
```

— FFP.dotabb —

```
"FFP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FFP" -> "ALIST"
```

7.11 domain FFNB FiniteFieldNormalBasis

— FiniteFieldNormalBasis.input —

```
)set break resume
)sys rm -f FiniteFieldNormalBasis.output
)spool FiniteFieldNormalBasis.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldNormalBasis
--R FiniteFieldNormalBasis(p: PositiveInteger,extdeg: PositiveInteger)  is a domain construct
--R Abbreviation for FiniteFieldNormalBasis is FFNB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFNB
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PrimeField p,%) -> %          ?*? : (%,PrimeField p) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,PrimeField p) -> %          ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %       algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean       basis : () -> Vector %
--R coerce : PrimeField p -> %           coerce : Fraction Integer -> %
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R norm : % -> PrimeField p             one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R retract : % -> PrimeField p          sample : () -> %
--R sizeLess? : (%,%) -> Boolean         squareFree : % -> Factored %
--R squareFreePart : % -> %              trace : % -> PrimeField p
--R transcendent? : % -> Boolean         unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R D : % -> % if PrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R Frobenius : % -> % if PrimeField p has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R charthRoot : % -> % if PrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if PrimeField p has FINITE
--R coordinates : Vector % -> Matrix PrimeField p
--R coordinates : % -> Vector PrimeField p
--R createNormalElement : () -> % if PrimeField p has FINITE
--R createPrimitiveElement : () -> % if PrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial PrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R differentiate : % -> % if PrimeField p has FINITE
```

```
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PrimeField p has CHARNZ or P
--R discreteLog : % -> NonNegativeInteger if PrimeField p has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if PrimeF
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R generator : () -> % if PrimeField p has FINITE
--R getMultiplicationMatrix : () -> Matrix PrimeField p
--R getMultiplicationTable : () -> Vector List Record(value: PrimeField p,index: SingleIntege
--R index : PositiveInteger -> % if PrimeField p has FINITE
--R init : () -> % if PrimeField p has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial PrimeField p) -> % if PrimeField p ha
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial PrimeField p,"failed") if
--R linearAssociatedLog : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FI
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has
--R lookup : % -> PositiveInteger if PrimeField p has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if PrimeField p h
--R minimalPolynomial : % -> SparseUnivariatePolynomial PrimeField p
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if PrimeField p has FINITE
--R norm : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R normal? : % -> Boolean if PrimeField p has FINITE
--R normalElement : () -> % if PrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if PrimeField p has CHARNZ or PrimeField
--R order : % -> PositiveInteger if PrimeField p has FINITE
--R primeFrobenius : % -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if PrimeField p has CHARNZ or PrimeField p h
--R primitive? : % -> Boolean if PrimeField p has FINITE
--R primitiveElement : () -> % if PrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if PrimeField p has FI
--R represents : Vector PrimeField p -> %
--R retractIfCan : % -> Union(PrimeField p,"failed")
--R size : () -> NonNegativeInteger if PrimeField p has FINITE
--R sizeMultiplication : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Prime
--R trace : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

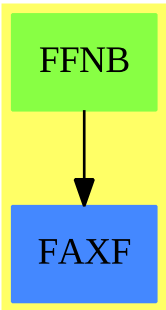
— FiniteFieldNormalBasis.help —

```
====================================================================
FiniteFieldNormalBasis examples
====================================================================

See Also:
o )show FiniteFieldNormalBasis
```

———-

7.11.1 FiniteFieldNormalBasis (FFNB)



See

⇒ "FiniteFieldNormalBasisExtensionByPolynomial" (FFNBP) 7.13.1 on page 838
⇒ "FiniteFieldNormalBasisExtension" (FFNBX) 7.12.1 on page 832

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
getMultiplicationMatrix	getMultiplicationTable	hash
index	inGroundField?	init
inv	latex	lcm
linearAssociatedExp	linearAssociatedLog	linearAssociatedOrder
lookup	minimalPolynomial	multiEuclidean
nextItem	norm	normal?
normalElement	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
recip	representationType	represents
retract	retractIfCan	sample
size	sizeLess?	sizeMultiplication
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain FFNB FiniteFieldNormalBasis —

```
)abbrev domain FFNB FiniteFieldNormalBasis
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FiniteFieldNormalBasisExtensionByPolynomial,
++  FiniteFieldPolynomialPackage
++ Also See: FiniteField, FiniteFieldCyclicGroup
++ AMS Classifications:
++ Keywords: finite field, normal basis
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
```

```
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++   AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldNormalBasis(p,n) implements a
++ finite extension field of degree n over the prime field with p elements.
++ The elements are represented by coordinate vectors with respect to
++ a normal basis,
++ i.e. a basis consisting of the conjugates (q-powers) of an element, in
++ this case called normal element.
++ This is chosen as a root of the extension polynomial
++ created by createNormalPoly

FiniteFieldNormalBasis(p,extdeg):_
  Exports == Implementation where
  p : PositiveInteger
  extdeg: PositiveInteger                    -- the extension degree
  NNI     ==> NonNegativeInteger
  FFF  ==> FiniteFieldFunctions(PrimeField(p))
  TERM    ==> Record(value:PrimeField(p),index:SingleInteger)
  Exports ==> FiniteAlgebraicExtensionField(PrimeField(p))  with
    getMultiplicationTable: ()   -> Vector List TERM
      ++ getMultiplicationTable() returns the multiplication
      ++ table for the normal basis of the field.
      ++ This table is used to perform multiplications between field elements.
    getMultiplicationMatrix: ()  -> Matrix PrimeField(p)
      ++ getMultiplicationMatrix() returns the multiplication table in
      ++ form of a matrix.
    sizeMultiplication:()   -> NNI
      ++ sizeMultiplication() returns the number of entries in the
      ++ multiplication table of the field. Note: The time of multiplication
      ++ of field elements depends on this size.

  Implementation ==> FiniteFieldNormalBasisExtensionByPolynomial(PrimeField(p),_
                    createLowComplexityNormalBasis(extdeg)$FFF)
```

———

— FFNB.dotabb —

```
"FFNB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFNB"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFNB" -> "FAXF"
```

———

7.12 domain FFNBX FiniteFieldNormalBasisExtension

— FiniteFieldNormalBasisExtension.input —

```
)set break resume
)sys rm -f FiniteFieldNormalBasisExtension.output
)spool FiniteFieldNormalBasisExtension.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldNormalBasisExtension
--R FiniteFieldNormalBasisExtension(GF: FiniteFieldCategory,extdeg: PositiveInteger)  is a do
--R Abbreviation for FiniteFieldNormalBasisExtension is FFNBX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFNBX
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (GF,%) -> %                     ?*? : (%,GF) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,GF) -> %                     ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : % -> % if GF has FINITE
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean             associates? : (%,%) -> Boolean
--R basis : () -> Vector %                coerce : GF -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R coordinates : % -> Vector GF          degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R norm : % -> GF                        one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R represents : Vector GF -> %           retract : % -> GF
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R trace : % -> GF                       transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
```

```
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if GF has FINITE
--R getMultiplicationMatrix : () -> Matrix GF
--R getMultiplicationTable : () -> Vector List Record(value: GF,index: SingleInteger)
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if GF has FINITE
--R norm : (%,PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
```

```
--R primeFrobenius : (%,NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF,"failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R sizeMultiplication : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF ha
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

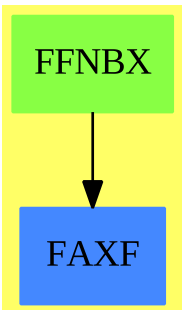
)spool
)lisp (bye)
```

— **FiniteFieldNormalBasisExtension.help** —

```
====================================================================
FiniteFieldNormalBasisExtension examples
====================================================================

See Also:
o )show FiniteFieldNormalBasisExtension
```

7.12.1 FiniteFieldNormalBasisExtension (FFNBX)



See

⇒ "FiniteFieldNormalBasisExtensionByPolynomial" (FFNBP) 7.13.1 on page 838
⇒ "FiniteFieldNormalBasis" (FFNB) 7.11.1 on page 827

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
getMultiplicationMatrix	getMultiplicationTable	hash
index	inGroundField?	init
inv	latex	lcm
linearAssociatedExp	linearAssociatedLog	linearAssociatedOrder
lookup	minimalPolynomial	multiEuclidean
nextItem	norm	normal?
normalElement	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
recip	representationType	represents
retract	retractIfCan	sample
size	sizeLess?	sizeMultiplication
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain FFNBX FiniteFieldNormalBasisExtension —

```
)abbrev domain FFNBX FiniteFieldNormalBasisExtension
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FiniteFieldNormalBasisExtensionByPolynomial,
++  FiniteFieldPolynomialPackage
++ Also See: FiniteFieldExtension, FiniteFieldCyclicGroupExtension
++ AMS Classifications:
++ Keywords: finite field, normal basis
```

```
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++   AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldNormalBasisExtensionByPolynomial(GF,n)  implements a
++ finite extension field of degree n over the ground field GF.
++ The elements are represented by coordinate vectors with respect
++ to a normal basis,
++ i.e. a basis consisting of the conjugates (q-powers) of an element, in
++ this case called normal element. This is chosen as a root of the extension
++ polynomial, created by createNormalPoly from
++ \spadtype{FiniteFieldPolynomialPackage}

FiniteFieldNormalBasisExtension(GF,extdeg):_
  Exports == Implementation where
  GF    : FiniteFieldCategory                -- the ground field
  extdeg: PositiveInteger                    -- the extension degree
  NNI     ==> NonNegativeInteger
  FFF         ==> FiniteFieldFunctions(GF)
  TERM    ==> Record(value:GF,index:SingleInteger)
  Exports ==> FiniteAlgebraicExtensionField(GF)  with
    getMultiplicationTable: ()   -> Vector List TERM
      ++ getMultiplicationTable() returns the multiplication
      ++ table for the normal basis of the field.
      ++ This table is used to perform multiplications between field elements.
    getMultiplicationMatrix: ()  -> Matrix GF
      ++ getMultiplicationMatrix() returns the multiplication table in
      ++ form of a matrix.
    sizeMultiplication:()   -> NNI
      ++ sizeMultiplication() returns the number of entries in the
      ++ multiplication table of the field. Note: the time of multiplication
      ++ of field elements depends on this size.

  Implementation ==> FiniteFieldNormalBasisExtensionByPolynomial(GF,_
                  createLowComplexityNormalBasis(extdeg)$FFF)
```

———-

— **FFNBX.dotabb** —

```
"FFNBX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFNBX"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFNBX" -> "FAXF"
```

———-

7.13 domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial

— **FiniteFieldNormalBasisExtensionByPolynomial.input** —

```
)set break resume
)sys rm -f FiniteFieldNormalBasisExtensionByPolynomial.output
)spool FiniteFieldNormalBasisExtensionByPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldNormalBasisExtensionByPolynomial
--R FiniteFieldNormalBasisExtensionByPolynomial(GF: FiniteFieldCategory,uni: Union(SparseUnivariatePolyn
--R Abbreviation for FiniteFieldNormalBasisExtensionByPolynomial is FFNBP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFNBP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (GF,%) -> %                    ?*? : (%,GF) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,GF) -> %                    ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               D : % -> % if GF has FINITE
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean            associates? : (%,%) -> Boolean
--R basis : () -> Vector %               coerce : GF -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R coordinates : % -> Vector GF         degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R norm : % -> GF                       one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R represents : Vector GF -> %          retract : % -> GF
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R trace : % -> GF                      transcendent? : % -> Boolean
--R unit? : % -> Boolean                 unitCanonical : % -> %
```

```
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINI
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R generator : () -> % if GF has FINITE
--R getMultiplicationMatrix : () -> Matrix GF
--R getMultiplicationTable : () -> Vector List Record(value: GF,index: SingleInteger)
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FI
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if GF has FINITE
--R norm : (%,PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
```

```
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF,"failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R sizeMultiplication : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF has FINITE
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———

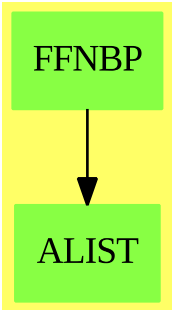
— FiniteFieldNormalBasisExtensionByPolynomial.help —

```
====================================================================
FiniteFieldNormalBasisExtensionByPolynomial examples
====================================================================

See Also:
o )show FiniteFieldNormalBasisExtensionByPolynomial
```

———

7.13.1 FiniteFieldNormalBasisExtensionByPolynomial (FFNBP)



See

⇒ "FiniteFieldNormalBasisExtension" (FFNBX) 7.12.1 on page 832
⇒ "FiniteFieldNormalBasis" (FFNB) 7.11.1 on page 827

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
getMultiplicationMatrix	getMultiplicationTable	hash
index	inGroundField?	init
inv	latex	lcm
linearAssociatedExp	linearAssociatedLog	linearAssociatedOrder
lookup	minimalPolynomial	multiEuclidean
nextItem	norm	normal?
normalElement	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	recip
random	representationType	represents
retract	retractIfCan	sample
size	sizeLess?	sizeMultiplication
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial —

```
)abbrev domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated: 08 May 1991
++ Basic Operations:
++ Related Constructors: InnerNormalBasisFieldFunctions, FiniteFieldFunctions,
++ Also See: FiniteFieldExtensionByPolynomial,
++  FiniteFieldCyclicGroupExtensionByPolynomial
++ AMS Classifications:
++ Keywords: finite field, normal basis
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics and
```

```
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM .
++   AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldNormalBasisExtensionByPolynomial(GF,uni) implements a
++ finite extension of the ground field GF. The elements are
++ represented by coordinate vectors with respect to. a normal basis,
++ i.e. a basis
++ consisting of the conjugates (q-powers) of an element, in this case
++ called normal element, where q is the size of GF.
++ The normal element is chosen as a root of the extension
++ polynomial, which MUST be normal over GF  (user responsibility)

FiniteFieldNormalBasisExtensionByPolynomial(GF,uni): Exports == _
    Implementation where
  GF    : FiniteFieldCategory            -- the ground field
  uni   : Union(SparseUnivariatePolynomial GF,_
            Vector List Record(value:GF,index:SingleInteger))

  PI   ==> PositiveInteger
  NNI  ==> NonNegativeInteger
  I    ==> Integer
  SI   ==> SingleInteger
  SUP  ==> SparseUnivariatePolynomial
  V    ==> Vector GF
  M    ==> Matrix GF
  OUT  ==> OutputForm
  TERM ==> Record(value:GF,index:SI)
  R    ==> Record(key:PI,entry:NNI)
  TBL  ==> Table(PI,NNI)
  FFF    ==> FiniteFieldFunctions(GF)
  INBFF  ==> InnerNormalBasisFieldFunctions(GF)

  Exports ==> FiniteAlgebraicExtensionField(GF)  with

      getMultiplicationTable: ()   -> Vector List TERM
        ++ getMultiplicationTable() returns the multiplication
        ++ table for the normal basis of the field.
        ++ This table is used to perform multiplications between field elements.
      getMultiplicationMatrix:()     -> M
        ++ getMultiplicationMatrix() returns the multiplication table in
        ++ form of a matrix.
      sizeMultiplication:()   -> NNI
        ++ sizeMultiplication() returns the number of entries in the
        ++ multiplication table of the field.
        ++ Note: the time of multiplication
        ++ of field elements depends on this size.
  Implementation ==> add

-- global variables ===================================================
```

```
Rep:= V     -- elements are represented by vectors over GF

alpha       :=new()$Symbol :: OutputForm
-- get a new Symbol for the output representation of the elements

initlog?:Boolean:=true
-- gets false after initialization of the logarithm table

initelt?:Boolean:=true
-- gets false after initialization of the primitive element

initmult?:Boolean:=true
-- gets false after initialization of the multiplication
-- table or the primitive element

extdeg:PI   :=1

defpol:SUP(GF):=0$SUP(GF)
-- the defining polynomial

multTable:Vector List TERM:=new(1,nil()$(List TERM))
-- global variable containing the multiplication table

if uni case (Vector List TERM) then
  multTable:=uni :: (Vector List TERM)
  extdeg:= (#multTable) pretend PI
  vv:V:=new(extdeg,0)$V
  vv.1:=1$GF
  setFieldInfo(multTable,1$GF)$INBFF
  defpol:=minimalPolynomial(vv)$INBFF
  initmult?:=false
 else
  defpol:=uni :: SUP(GF)
  extdeg:=degree(defpol)$(SUP GF) pretend PI
  multTable:Vector List TERM:=new(extdeg,nil()$(List TERM))

basisOutput : List OUT :=
  qs:OUT:=(q::Symbol)::OUT
  append([alpha, alpha **$OUT qs],_
    [alpha **$OUT (qs **$OUT i::OUT) for i in 2..extdeg-1] )


facOfGroupSize :=nil()$(List Record(factor:Integer,exponent:Integer))
-- the factorization of the cyclic group size


traceAlpha:GF:=-$GF coefficient(defpol,(degree(defpol)-1)::NNI)
-- the inverse of the trace of the normalElt
-- is computed here. It defines the imbedding of
```

```
    -- GF in the extension field

    primitiveElt:PI:=1
    -- for the lookup the primitive Element computed by createPrimitiveElement()

    discLogTable:Table(PI,TBL):=table()$Table(PI,TBL)
    -- tables indexed by the factors of sizeCG,
    -- discLogTable(factor) is a table with keys
    -- primitiveElement() ** (i * (sizeCG quo factor)) and entries i for
    -- i in 0..n-1, n computed in initialize() in order to use
    -- the minimal size limit 'limit' optimal.

-- functions ===========================================================

    initializeLog: ()     -> Void
    initializeElt: ()     -> Void
    initializeMult: ()     -> Void


    coerce(v:GF):$  == new(extdeg,v /$GF traceAlpha)$Rep
    represents(v)   ==  v::$

    degree(a) ==
      d:PI:=1
      b:= qPot(a::Rep,1)$INBFF
      while (b^=a) repeat
        b:= qPot(b::Rep,1)$INBFF
        d:=d+1
      d

    linearAssociatedExp(x,f) ==
      xm:SUP(GF):=monomial(1$GF,extdeg)$(SUP GF) - 1$(SUP GF)
      r:= (f * pol(x::Rep)$INBFF) rem xm
      vectorise(r,extdeg)$(SUP GF)
    linearAssociatedLog(x) ==  pol(x::Rep)$INBFF
    linearAssociatedOrder(x) ==
      xm:SUP(GF):=monomial(1$GF,extdeg)$(SUP GF) - 1$(SUP GF)
      xm quo gcd(xm,pol(x::Rep)$INBFF)
    linearAssociatedLog(b,x) ==
      zero? x => 0
      xm:SUP(GF):=monomial(1$GF,extdeg)$(SUP GF) - 1$(SUP GF)
      e:= extendedEuclidean(pol(b::Rep)$INBFF,xm,pol(x::Rep)$INBFF)$(SUP GF)
      e = "failed" => "failed"
      e1:= e :: Record(coef1:(SUP GF),coef2:(SUP GF))
      e1.coef1

    getMultiplicationTable() ==
      if initmult? then initializeMult()
      multTable
    getMultiplicationMatrix() ==
```

```
      if initmult? then initializeMult()
      createMultiplicationMatrix(multTable)$FFF
    sizeMultiplication() ==
      if initmult? then initializeMult()
      sizeMultiplication(multTable)$FFF

    trace(a:$) == retract trace(a,1)
    norm(a:$) == retract norm(a,1)
    generator() == normalElement(extdeg)$INBFF
    basis(n:PI) ==
      (extdeg rem n) ^= 0 => error "argument must divide extension degree"
      [Frobenius(trace(normalElement,n),i) for i in 0..(n-1)]::(Vector $)

    a:GF * x:$ == a *$Rep x

    x:$/a:GF == x/coerce(a)
--    x:$ / a:GF ==
--      a = 0$GF => error "division by zero"
--      x * inv(coerce(a))


    coordinates(x:$)  == x::Rep

    Frobenius(e) == qPot(e::Rep,1)$INBFF
    Frobenius(e,n) == qPot(e::Rep,n)$INBFF

    retractIfCan(x) ==
      inGroundField?(x) =>
        x.1 *$GF traceAlpha
      "failed"

    retract(x) ==
      inGroundField?(x) =>
        x.1 *$GF traceAlpha
      error("element not in ground field")

-- to get a "normal basis like" output form
    coerce(x:$):OUT ==
      l:List OUT:=nil()$(List OUT)
      n : PI := extdeg
--      one? n => (x.1) :: OUT
      (n = 1) => (x.1) :: OUT
      for i in 1..n for b in basisOutput repeat
        if not zero? x.i then
          mon : OUT :=
--            one? x.i => b
            (x.i = 1) => b
            ((x.i)::OUT) *$OUT b
          l:=cons(mon,l)$(List OUT)
      null(l)$(List OUT) => (0::OUT)
```

```
      r:=reduce("+",l)$(List OUT)
      r

    initializeElt() ==
      facOfGroupSize := factors factor(size()$GF**extdeg-1)$I
      -- get a primitive element
      primitiveElt:=lookup(createPrimitiveElement())
      initelt?:=false
      void()$Void

    initializeMult() ==
      multTable:=createMultiplicationTable(defpol)$FFF
      setFieldInfo(multTable,traceAlpha)$INBFF
      -- reset initialize flag
      initmult?:=false
      void()$Void

    initializeLog() ==
      if initelt? then initializeElt()
      -- set up tables for discrete logarithm
      limit:Integer:=30
      -- the minimum size for the discrete logarithm table
      for f in facOfGroupSize repeat
        fac:=f.factor
        base:$:=index(primitiveElt)**((size()$GF**extdeg -$I 1$I) quo$I fac)
        l:Integer:=length(fac)$Integer
        n:Integer:=0
        if odd?(l)$I then n:=shift(fac,-$I (l quo$I 2))$I
                     else n:=shift(1,l quo$I 2)$I
        if n <$I limit then
          d:=(fac -$I 1$I) quo$I limit +$I 1$I
          n:=(fac -$I 1$I) quo$I d +$I 1$I
        tbl:TBL:=table()$TBL
        a:$:=1
        for i in (0::NNI)..(n-1)::NNI repeat
          insert_!([lookup(a),i::NNI]$R,tbl)$TBL
          a:=a*base
        insert_!([fac::PI,copy(tbl)$TBL]_
               $Record(key:PI,entry:TBL),discLogTable)$Table(PI,TBL)
      initlog?:=false
      -- tell user about initialization
      --print("discrete logarithm table initialized"::OUT)
      void()$Void

    tableForDiscreteLogarithm(fac) ==
      if initlog? then initializeLog()
      tbl:=search(fac::PI,discLogTable)$Table(PI,TBL)
      tbl case "failed" =>
        error "tableForDiscreteLogarithm: argument must be prime _
divisor of the order of the multiplicative group"
```

```
    tbl :: TBL

  primitiveElement() ==
    if initelt? then initializeElt()
    index(primitiveElt)

  factorsOfCyclicGroupSize() ==
    if empty? facOfGroupSize then initializeElt()
    facOfGroupSize

  extensionDegree() == extdeg

  sizeOfGroundField() == size()$GF pretend NNI

  definingPolynomial() == defpol

  trace(a,d) ==
    v:=trace(a::Rep,d)$INBFF
    erg:=v
    for i in 2..(extdeg quo d) repeat
      erg:=concat(erg,v)$Rep
    erg

  characteristic() == characteristic()$GF

  random() == random(extdeg)$INBFF

  x:$ * y:$ ==
    if initmult? then initializeMult()
    setFieldInfo(multTable,traceAlpha)$INBFF
    x::Rep *$INBFF y::Rep


  1 == new(extdeg,inv(traceAlpha)$GF)$Rep

  0 == zero(extdeg)$Rep

  size() == size()$GF ** extdeg

  index(n:PI) == index(extdeg,n)$INBFF

  lookup(x:$) == lookup(x::Rep)$INBFF


  basis() ==
    a:=basis(extdeg)$INBFF
    vector([e::$ for e in entries a])


  x:$ ** e:I ==
```

```
      if initmult? then initializeMult()
      setFieldInfo(multTable,traceAlpha)$INBFF
      (x::Rep) **$INBFF e

    normal?(x) == normal?(x::Rep)$INBFF

    -(x:$) == -$Rep x
    x:$ + y:$ == x +$Rep y
    x:$ - y:$ == x -$Rep y
    x:$ = y:$ == x =$Rep y
    n:I * x:$ == x *$Rep (n::GF)




    representationType() == "normal"

    minimalPolynomial(a) ==
      if initmult? then initializeMult()
      setFieldInfo(multTable,traceAlpha)$INBFF
      minimalPolynomial(a::Rep)$INBFF

-- is x an element of the ground field GF ?
    inGroundField?(x) ==
      erg:=true
      for i in 2..extdeg repeat
        not(x.i =$GF x.1) => erg:=false
      erg

    x:$ / y:$ ==
      if initmult? then initializeMult()
      setFieldInfo(multTable,traceAlpha)$INBFF
      x::Rep /$INBFF y::Rep

    inv(a) ==
      if initmult? then initializeMult()
      setFieldInfo(multTable,traceAlpha)$INBFF
      inv(a::Rep)$INBFF

    norm(a,d) ==
      if initmult? then initializeMult()
      setFieldInfo(multTable,traceAlpha)$INBFF
      norm(a::Rep,d)$INBFF

    normalElement() == normalElement(extdeg)$INBFF
```

———-

— **FFNBP.dotabb** —

```
"FFNBP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFNBP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FFNBP" -> "ALIST"
```

———-

7.14 domain FARRAY FlexibleArray

— **FlexibleArray.input** —

```
)set break resume
)sys rm -f FlexibleArray.output
)spool FlexibleArray.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
flexibleArray [i for i in 1..6]
--R
--R
--R   (1)  [1,2,3,4,5,6]
--R                                          Type: FlexibleArray PositiveInteger
--E 1

--S 2 of 16
f : FARRAY INT := new(6,0)
--R
--R
--R   (2)  [0,0,0,0,0,0]
--R                                                  Type: FlexibleArray Integer
--E 2

--S 3 of 16
for i in 1..6 repeat f.i := i; f
--R
--R
--R   (3)  [1,2,3,4,5,6]
--R                                                  Type: FlexibleArray Integer
--E 3

--S 4 of 16
physicalLength f
--R
```

```
--R
--R   (4)  6
--R                                                      Type: PositiveInteger
--E 4

--S 5 of 16
concat!(f,11)
--R
--R
--R   (5)  [1,2,3,4,5,6,11]
--R                                                Type: FlexibleArray Integer
--E 5

--S 6 of 16
physicalLength f
--R
--R
--R   (6)  10
--R                                                      Type: PositiveInteger
--E 6

--S 7 of 16
physicalLength!(f,15)
--R
--R
--R   (7)  [1,2,3,4,5,6,11]
--R                                                Type: FlexibleArray Integer
--E 7

--S 8 of 16
concat!(f,f)
--R
--R
--R   (8)  [1,2,3,4,5,6,11,1,2,3,4,5,6,11]
--R                                                Type: FlexibleArray Integer
--E 8

--S 9 of 16
insert!(22,f,1)
--R
--R
--R   (9)  [22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
--R                                                Type: FlexibleArray Integer
--E 9

--S 10 of 16
g := f(10..)
--R
--R
--R   (10)  [2,3,4,5,6,11]
```

```
--R                                                 Type: FlexibleArray Integer
--E 10

--S 11 of 16
insert!(g,f,1)
--R
--R
--R   (11)  [2,3,4,5,6,11,22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
--R                                                 Type: FlexibleArray Integer
--E 11

--S 12 of 16
merge!(sort! f, sort! g)
--R
--R
--R   (12)  [1,1,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,11,11,11,11,22]
--R                                                 Type: FlexibleArray Integer
--E 12

--S 13 of 16
removeDuplicates! f
--R
--R
--R   (13)  [1,2,3,4,5,6,11,22]
--R                                                 Type: FlexibleArray Integer
--E 13

--S 14 of 16
select!(i +-> even? i,f)
--R
--R
--R   (14)  [2,4,6,22]
--R                                                 Type: FlexibleArray Integer
--E 14

--S 15 of 16
physicalLength f
--R
--R
--R   (15)  8
--R                                                       Type: PositiveInteger
--E 15

--S 16 of 16
shrinkable(false)$FlexibleArray(Integer)
--R
--R
--R   (16)  true
--R                                                               Type: Boolean
--E 16
```

```
)spool
)lisp (bye)
```

———-

— **FlexibleArray.help** —

```
====================================================================
FlexibleArray
====================================================================

The FlexibleArray domain constructor creates one-dimensional
arrays of elements of the same type.  Flexible arrays are an attempt
to provide a data type that has the best features of both
one-dimensional arrays (fast, random access to elements) and lists
(flexibility).  They are implemented by a fixed block of storage.
When necessary for expansion, a new, larger block of storage is
allocated and the elements from the old storage area are copied into
the new block.

Flexible arrays have available most of the operations provided by
OneDimensionalArray Vector.  Since flexible arrays are also of
category ExtensibleLinearAggregate they have operations concat!,
delete!, insert!, merge!, remove!, removeDuplicates!, and select!.  In
addition, the operations physicalLength and physicalLength! provide
user-control over expansion and contraction.

A convenient way to create a flexible array is to apply the operation
flexibleArray to a list of values.

  flexibleArray [i for i in 1..6]
    [1,2,3,4,5,6]
                     Type: FlexibleArray PositiveInteger

Create a flexible array of six zeroes.

  f : FARRAY INT := new(6,0)
    [0,0,0,0,0,0]
                     Type: FlexibleArray Integer

For i=1..6 set the i-th element to i.  Display f.

  for i in 1..6 repeat f.i := i; f
    [1,2,3,4,5,6]
                     Type: FlexibleArray Integer

Initially, the physical length is the same as the number of elements.

  physicalLength f
```

```
    6
                    Type: PositiveInteger
```

Add an element to the end of f.

```
  concat!(f,11)
    [1,2,3,4,5,6,11]
                    Type: FlexibleArray Integer
```

See that its physical length has grown.

```
  physicalLength f
    10
                    Type: PositiveInteger
```

Make f grow to have room for 15 elements.

```
  physicalLength!(f,15)
    [1,2,3,4,5,6,11]
                    Type: FlexibleArray Integer
```

Concatenate the elements of f to itself. The physical length allows room for three more values at the end.

```
  concat!(f,f)
    [1,2,3,4,5,6,11,1,2,3,4,5,6,11]
                    Type: FlexibleArray Integer
```

Use insert! to add an element to the front of a flexible array.

```
  insert!(22,f,1)
    [22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
                    Type: FlexibleArray Integer
```

Create a second flexible array from f consisting of the elements from index 10 forward.

```
  g := f(10..)
    [2,3,4,5,6,11]
                    Type: FlexibleArray Integer
```

Insert this array at the front of f.

```
  insert!(g,f,1)
    [2,3,4,5,6,11,22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
                    Type: FlexibleArray Integer
```

Merge the flexible array f into g after sorting each in place.

```
  merge!(sort! f, sort! g)
```

```
    [1,1,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,11,11,11,11,22]
                     Type: FlexibleArray Integer
```

Remove duplicates in place.

```
  removeDuplicates! f
    [1,2,3,4,5,6,11,22]
                     Type: FlexibleArray Integer
```

Remove all odd integers.

```
  select!(i +-> even? i,f)
    [2,4,6,22]
                     Type: FlexibleArray Integer
```

All these operations have shrunk the physical length of f.

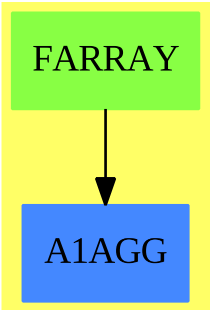
```
  physicalLength f
    8
                     Type: PositiveInteger
```

To force Axiom not to shrink flexible arrays call the shrinkable operation with the argument false. You must package call this operation. The previous value is returned.

```
  shrinkable(false)$FlexibleArray(Integer)
    true
                     Type: Boolean
```

See Also:
o)help OneDimensionalArray
o)help Vector
o)help ExtensibleLinearAggregate
o)show FlexibleArray

7.14.1 FlexibleArray (FARRAY)



See

⇒ "PrimitiveArray" (PRIMARR) 17.30.1 on page 2069
⇒ "Tuple" (TUPLE) 21.12.1 on page 2711
⇒ "IndexedFlexibleArray" (IFARRAY) 10.10.1 on page 1187
⇒ "IndexedOneDimensionalArray" (IARRAY1) 10.13.1 on page 1208
⇒ "OneDimensionalArray" (ARRAY1) 16.3.1 on page 1736

Exports:

any?	coerce	concat	concat!	construct
convert	copy	copyInto!	count	delete
delete!	elt	empty	empty?	entries
entry?	eq?	eval	every?	fill!
find	first	flexibleArray	hash	index?
indices	insert	insert!	latex	less?
map	map!	max	maxIndex	member?
members	merge	merge!	min	minIndex
more?	new	parts	physicalLength	physicalLength!
position	qelt	qsetelt!	reduce	remove
remove!	removeDuplicates	removeDuplicates!	reverse	reverse!
sample	select	select!	setelt	shrinkable
size?	sort	sort!	sorted?	swap!
#?	?<?	?<=?	?=?	?>?
?>=?	?~=?	?.?		

— domain FARRAY FlexibleArray —

```
)abbrev domain FARRAY FlexibleArray
++ Author: Mark Botch
++ Description:
++ A FlexibleArray is the notion of an array intended to allow for growth
++ at the end only.  Hence the following efficient operations
++ \spad{append(x,a)} meaning append item x at the end of the array \spad{a}
++ \spad{delete(a,n)} meaning delete the last item from the array \spad{a}
++ Flexible arrays support the other operations inherited from
++ \spadtype{ExtensibleLinearAggregate}. However, these are not efficient.
```

```
++ Flexible arrays combine the \spad{O(1)} access time property of arrays
++ with growing and shrinking at the end in \spad{O(1)} (average) time.
++ This is done by using an ordinary array which may have zero or more
++ empty slots at the end.  When the array becomes full it is copied
++ into a new larger (50% larger) array.  Conversely, when the array
++ becomes less than 1/2 full, it is copied into a smaller array.
++ Flexible arrays provide for an efficient implementation of many
++ data structures in particular heaps, stacks and sets.

FlexibleArray(S: Type) == Implementation where
  ARRAYMININDEX ==> 1       -- if you want to change this, be my guest
  Implementation ==> IndexedFlexibleArray(S, ARRAYMININDEX)
-- Join(OneDimensionalArrayAggregate S, ExtensibleLinearAggregate S)
```

— **FARRAY.dotabb** —

```
"FARRAY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FARRAY"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"FARRAY" -> "A1AGG"
```

7.15 domain FLOAT Float

As reported in bug number 4733 (rounding of negative numbers) errors were observed in operations such as

```
-> round(-3.9)
-> truncate(-3.9)
```

The problem is the unexpected behaviour of the shift with negative integer arguments.

```
-> shift(-7,-1)
```

returns -4 while the code here in float expects the value to be -3. shift uses the lisp function ASH 'arithmetic shift left' but the spad code expects an unsigned 'logical' shift. See

```
http://www.lispworks.com/reference/HyperSpec/Body/f_ash.htm#ash
```

A new internal function shift2 is defined in terms of shift to compensate for the use of ASH and provide the required function.

It is currently unknown whether the unexpected behaviour of shift for negative arguments will cause bugs in other parts of Axiom.

— **Float.input** —

```
)set break resume
)sys rm -f Float.output
)spool Float.output
)set message test on
)set message auto off
)clear all
--S 1 of 64
1.234
--R
--R
--R   (1)  1.234
--R                                                              Type: Float
--E 1

--S 2 of 64
1.234E2
--R
--R
--R   (2)  123.4
--R                                                              Type: Float
--E 2

--S 3 of 64
sqrt(1.2 + 2.3 / 3.4 ** 4.5)
--R
--R
--R   (3)  1.0996972790 671286226
--R                                                              Type: Float
--E 3

--S 4 of 64
i := 3 :: Float
--R
--R
--R   (4)  3.0
--R                                                              Type: Float
--E 4

--S 5 of 64
i :: Integer
--R
--R
--R   (5)  3
--R                                                            Type: Integer
--E 5

--S 6 of 64
i :: Fraction Integer
--R
--R
```

```
--R   (6)  3
--R                                                    Type: Fraction Integer
--E 6

--S 7 of 64
r := 3/7 :: Float
--R
--R
--R   (7)  0.4285714285 7142857143
--R                                                               Type: Float
--E 7

--S 8 of 64
r :: Fraction Integer
--R
--R
--R        3
--R   (8)  -
--R        7
--R                                                    Type: Fraction Integer
--E 8

--S 9 of 64
r :: Integer
--R
--R
--RDaly Bug
--R   Cannot convert from type Float to Integer for value
--R   0.4285714285 7142857143
--R
--E 9

--S 10 of 64
truncate 3.6
--R
--R
--R   (9)  3.0
--R                                                               Type: Float
--E 10

--S 11 of 64
round 3.6
--R
--R
--R   (10)  4.0
--R                                                               Type: Float
--E 11

--S 12 of 64
truncate(-3.6)
```

```
--R
--R
--R   (11)  - 3.0
--R                                                                  Type: Float
--E 12

--S 13 of 64
round(-3.6)
--R
--R
--R   (12)  - 4.0
--R                                                                  Type: Float
--E 13

--S 14 of 64
fractionPart 3.6
--R
--R
--R   (13)  0.6
--R                                                                  Type: Float
--E 14

--S 15 of 64
digits 40
--R
--R
--R   (14)  20
--R                                                        Type: PositiveInteger
--E 15

--S 16 of 64
sqrt 0.2
--R
--R
--R   (15)  0.4472135954 9995793928 1834733746 2552470881
--R                                                                  Type: Float
--E 16

--S 17 of 64
pi()$Float
--R
--R
--R   (16)  3.1415926535 8979323846 2643383279 502884197
--R                                                                  Type: Float
--E 17

--S 18 of 64
digits 500
--R
--R
```

```
--R   (17)  40
--R                                                        Type: PositiveInteger
--E 18

--S 19 of 64
pi()$Float
--R
--R
--R   (18)
--R  3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 592307816
--R  4 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 505822317
--R  2 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 442881097
--R  5 6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 454326648
--R  2 1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 917153643
--R  6 7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 575959195
--R  3 0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 891227938
--R  1 830119491
--R                                                                  Type: Float
--E 19

--S 20 of 64
digits 20
--R
--R
--R   (19)  500
--R                                                        Type: PositiveInteger
--E 20

--S 21 of 64
outputSpacing 0; x := sqrt 0.2
--R
--R
--R   (20)  0.44721359549995793928
--R                                                                  Type: Float
--E 21

--S 22 of 64
outputSpacing 5; x
--R
--R
--R   (21)  0.44721 35954 99957 93928
--R                                                                  Type: Float
--E 22

--S 23 of 64
y := x/10**10
--R
--R
--R   (22)  0.44721 35954 99957 93928 E -10
--R                                                                  Type: Float
```

```
--E 23

--S 24 of 64
outputFloating(); x
--R
--R
--R   (23)  0.44721 35954 99957 93928 E 0
--R                                                                  Type: Float
--E 24

--S 25 of 64
outputFixed(); y
--R
--R
--R   (24)  0.00000 00000 44721 35954 99957 93928
--R                                                                  Type: Float
--E 25

--S 26 of 64
outputFloating 2; y
--R
--R
--R   (25)  0.45 E -10
--R                                                                  Type: Float
--E 26

--S 27 of 64
outputFixed 2; x
--R
--R
--R   (26)  0.45
--R                                                                  Type: Float
--E 27

--S 28 of 64
outputGeneral()
--R
--R                                                                   Type: Void
--E 28

--S 29 of 64
a: Matrix Fraction Integer := matrix [ [1/(i+j+1) for j in 0..9] for i in 0..9]
--R
--R
--R         +    1   1   1   1   1   1   1   1    1+
--R         |1   -   -   -   -   -   -   -   -   --|
--R         |    2   3   4   5   6   7   8   9   10|
--R         |                                      |
--R         |1   1   1   1   1   1   1   1    1   1|
--R         |-   -   -   -   -   -   -   -   --  --|
```

```
--R         |2   3   4   5   6   7   8   9   10  11|
--R         |                                      |
--R         |1   1   1   1   1   1   1    1   1   1|
--R         |-   -   -   -   -   -   -   --  --  --|
--R         |3   4   5   6   7   8   9   10  11  12|
--R         |                                      |
--R         |1   1   1   1   1   1    1   1   1   1|
--R         |-   -   -   -   -   -   --  --  --  --|
--R         |4   5   6   7   8   9   10  11  12  13|
--R         |                                      |
--R         |1   1   1   1   1    1   1   1   1   1|
--R         |-   -   -   -   -   --  --  --  --  --|
--R         |5   6   7   8   9   10  11  12  13  14|
--R   (28)  |                                      |
--R         |1   1   1   1    1   1   1   1   1   1|
--R         |-   -   -   -   --  --  --  --  --  --|
--R         |6   7   8   9   10  11  12  13  14  15|
--R         |                                      |
--R         |1   1   1    1   1   1   1   1   1   1|
--R         |-   -   -   --  --  --  --  --  --  --|
--R         |7   8   9   10  11  12  13  14  15  16|
--R         |                                      |
--R         |1   1    1   1   1   1   1   1   1   1|
--R         |-   -   --  --  --  --  --  --  --  --|
--R         |8   9   10  11  12  13  14  15  16  17|
--R         |                                      |
--R         |1    1   1   1   1   1   1   1   1   1|
--R         |-   --  --  --  --  --  --  --  --  --|
--R         |9   10  11  12  13  14  15  16  17  18|
--R         |                                      |
--R         | 1   1   1   1   1   1   1   1   1   1|
--R         |--  --  --  --  --  --  --  --  --  --|
--R         +10  11  12  13  14  15  16  17  18  19+
--R                                           Type: Matrix Fraction Integer
--E 29

--S 30 of 64
d:= determinant a
--R
--R
--R                                  1
--R   (29)  -----------------------------------------------------
--R         46206893947914691316295628839036278726983680000000000
--R                                                  Type: Fraction Integer
--E 30

--S 31 of 64
d :: Float
--R
--R
```

```
--R   (30)  0.21641 79226 43149 18691 E -52
--R                                                              Type: Float
--E 31

--S 32 of 64
b: Matrix DoubleFloat := matrix [ [1/(i+j+1$DoubleFloat) for j in 0..9] for i in 0..9]
--R
--R
--R   (31)
--R   [
--R     [1., 0.5, 0.33333333333333331, 0.25, 0.20000000000000001,
--R      0.16666666666666666, 0.14285714285714285, 0.125, 0.1111111111111111,
--R      0.10000000000000001]
--R     ,
--R
--R     [0.5, 0.33333333333333331, 0.25, 0.20000000000000001, 0.16666666666666666,
--R      0.14285714285714285, 0.125, 0.1111111111111111, 0.10000000000000001,
--R      9.0909090909090912E-2]
--R     ,
--R
--R     [0.33333333333333331, 0.25, 0.20000000000000001, 0.16666666666666666,
--R      0.14285714285714285, 0.125, 0.1111111111111111, 0.10000000000000001,
--R      9.0909090909090912E-2, 8.3333333333333329E-2]
--R     ,
--R
--R     [0.25, 0.20000000000000001, 0.16666666666666666, 0.14285714285714285,
--R      0.125, 0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R      8.3333333333333329E-2, 7.6923076923076927E-2]
--R     ,
--R
--R     [0.20000000000000001, 0.16666666666666666, 0.14285714285714285, 0.125,
--R      0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R      8.3333333333333329E-2, 7.6923076923076927E-2, 7.1428571428571425E-2]
--R     ,
--R
--R     [0.16666666666666666, 0.14285714285714285, 0.125, 0.1111111111111111,
--R      0.10000000000000001, 9.0909090909090912E-2, 8.3333333333333329E-2,
--R      7.6923076923076927E-2, 7.1428571428571425E-2, 6.6666666666666666E-2]
--R     ,
--R
--R     [0.14285714285714285, 0.125, 0.1111111111111111, 0.10000000000000001,
--R      9.0909090909090912E-2, 8.3333333333333329E-2, 7.6923076923076927E-2,
--R      7.1428571428571425E-2, 6.6666666666666666E-2, 6.25E-2]
--R     ,
--R
--R     [0.125, 0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R      8.3333333333333329E-2, 7.6923076923076927E-2, 7.1428571428571425E-2,
--R      6.6666666666666666E-2, 6.25E-2, 5.8823529411764705E-2]
--R     ,
--R
```

```
--R     [0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R      8.3333333333333329E-2, 7.6923076923076927E-2, 7.1428571428571425E-2,
--R      6.6666666666666666E-2, 6.25E-2, 5.8823529411764705E-2,
--R      5.5555555555555552E-2]
--R     ,
--R
--R     [0.10000000000000001, 9.0909090909090912E-2, 8.3333333333333329E-2,
--R      7.6923076923076927E-2, 7.1428571428571425E-2, 6.6666666666666666E-2,
--R      6.25E-2, 5.8823529411764705E-2, 5.5555555555555552E-2,
--R      5.2631578947368418E-2]
--R     ]
--R                                                       Type: Matrix DoubleFloat
--E 32

--S 33 of 64
determinant b
--R
--R
--R   (32)  2.1643677945721411E-53
--R                                                              Type: DoubleFloat
--E 33

--S 34 of 64
digits 40
--R
--R
--R   (33)  20
--R                                                          Type: PositiveInteger
--E 34

--S 35 of 64
c: Matrix Float := matrix [ [1/(i+j+1$Float) for j in 0..9] for i in 0..9]
--R
--R
--R   (34)
--R   [
--R     [1.0, 0.5, 0.33333 33333 33333 33333 33333 33333 33333 33333, 0.25, 0.2,
--R      0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1]
--R     ,
--R
--R     [0.5, 0.33333 33333 33333 33333 33333 33333 33333 33333, 0.25, 0.2,
--R      0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1]
--R     ,
--R
--R     [0.33333 33333 33333 33333 33333 33333 33333 33333, 0.25, 0.2,
```

```
--R      0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4]
--R     ,
--R
--R     [0.25, 0.2, 0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2]
--R     ,
--R
--R     [0.2, 0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3]
--R     ,
--R
--R     [0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7]
--R     ,
--R
--R     [0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625]
--R     ,
--R
--R     [0.125, 0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625,
--R      0.05882 35294 11764 70588 23529 41176 47058 82352 9]
```

```
--R     ,
--R
--R     [0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625,
--R      0.05882 35294 11764 70588 23529 41176 47058 82352 9,
--R      0.05555 55555 55555 55555 55555 55555 55555 55555 6]
--R     ,
--R
--R     [0.1, 0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625,
--R      0.05882 35294 11764 70588 23529 41176 47058 82352 9,
--R      0.05555 55555 55555 55555 55555 55555 55555 55555 6,
--R      0.05263 15789 47368 42105 26315 78947 36842 10526 3]
--R    ]
--R                                                      Type: Matrix Float
--E 35

--S 36 of 64
determinant c
--R
--R
--R   (35)  0.21641 79226 43149 18690 60594 98362 26174 36159 E -52
--R                                                             Type: Float
--E 36

--S 37 of 64
digits 20
--R
--R
--R   (36)  40
--R                                                   Type: PositiveInteger
--E 37

)clear all

--S 38 of 64
outputFixed()
--R
--R                                                              Type: Void
--E 38

--S 39 of 64
a:=3.0
```

```
--R
--R
--R   (2)  3.0
--R                                                          Type: Float
--E 39

--S 40 of 64
b:=3.1
--R
--R
--R   (3)  3.1
--R                                                          Type: Float
--E 40

--S 41 of 64
c:=numeric pi()
--R
--R
--R   (4)  3.14159 26535 89793 2385
--R                                                          Type: Float
--E 41

--S 42 of 64
d:=0.0
--R
--R
--R   (5)  0.0
--R                                                          Type: Float
--E 42

--S 43 of 64
outputFixed 2
--R
--R                                                           Type: Void
--E 43

--S 44 of 64
a
--R
--R
--R   (7)  3.00
--R                                                          Type: Float
--E 44

--S 45 of 64
b
--R
--R
--R   (8)  3.10
--R                                                          Type: Float
```

```
--E 45

--S 46 of 64
c
--R
--R
--R   (9)  3.14
--R                                                            Type: Float
--E 46

--S 47 of 64
d
--R
--R
--R   (10)  0.00
--R                                                            Type: Float
--E 47

--S 48 of 64
outputFixed 0
--R
--R                                                             Type: Void
--E 48

--S 49 of 64
a
--R
--R
--R   (12)  3.0
--R                                                            Type: Float
--E 49

--S 50 of 64
b
--R
--R
--R   (13)  3.
--R                                                            Type: Float
--E 50

--S 51 of 64
c
--R
--R
--R   (14)  3.
--R                                                            Type: Float
--E 51

--S 52 of 64
31.1
```

```
--R
--R
--R   (15)  31.
--R                                                                  Type: Float
--E 52

--S 53 of 64
310.1
--R
--R
--R   (16)  310.
--R                                                                  Type: Float
--E 53

--S 54 of 64
d
--R
--R
--R   (17)  0.0
--R                                                                  Type: Float
--E 54

--S 55 of 64
outputFixed(0)
--R                                                                   Type: Void
--E 55

--S 56 of 64
1.1
--R
--R   (19)  1.
--R                                                                  Type: Float
--E 56

--S 57 of 64
3111.1
--R
--R   (20)  3111.
--R                                                                  Type: Float
--E 57

--S 58 of 64
1234567890.1
--R
--R   (21)  12345 67890.
--R                                                                  Type: Float
--E 58

--S 59 of 64
outputFixed(12)
```

```
--R                                                                   Type: Void
--E 59

--S 60 of 64
1234567890.1
--R
--R   (23)  12345 67890.09999 99999 99
--R                                                                  Type: Float
--E 60

--S 61 of 64
outputFixed(15)
--R                                                                   Type: Void
--E 61

--S 62 of 64
1234567890.1
--R
--R   (25)  12345 67890.09999 99999 98545
--R                                                                  Type: Float
--E 62

--S 63 of 64
outputFixed(2)
--R                                                                   Type: Void
--E 63

--S 64 of 64
1234567890.1
--R
--R   (27)  12345 67890.10
--R                                                                  Type: Float
--E 64

)spool
)lisp (bye)
```

———-

— Float.help —

```
====================================================================
Float
====================================================================

Axiom provides two kinds of floating point numbers.  The domain Float
implements a model of arbitrary precision floating point numbers.  The
domain DoubleFloat is intended to make available hardware floating
point arithmetic in Axiom.  The actual model of floating point that
```

```
DoubleFloat provides is system-dependent.  For example, on the IBM
system 370 Axiom uses IBM double precision which has fourteen
hexadecimal digits of precision or roughly sixteen decimal digits.
Arbitrary precision floats allow the user to specify the precision at
which arithmetic operations are computed.  Although this is an
attractive facility, it comes at a cost.  Arbitrary-precision
floating-point arithmetic typically takes twenty to two hundred times
more time than hardware floating point.

====================================================================
Introduction to Float
====================================================================

Scientific notation is supported for input and output of floating
point numbers.  A floating point number is written as a string of
digits containing a decimal point optionally followed by the letter
"E", and then the exponent.

We begin by doing some calculations using arbitrary precision floats.
The default precision is twenty decimal digits.

  1.234
   1.234
                      Type: Float

A decimal base for the exponent is assumed, so the number
1.234E2 denotes 1.234x10^2.

  1.234E2
    123.4
                      Type: Float
The normal arithmetic operations are available for floating point numbers.

  sqrt(1.2 + 2.3 / 3.4 ** 4.5)
    1.0996972790 671286226
                      Type: Float

====================================================================
Conversion Functions
====================================================================

You can use conversion to go back and forth between Integer, Fraction
Integer and Float, as appropriate.

  i := 3 :: Float
    3.0
                      Type: Float

  i :: Integer
    3
```

```
                    Type: Integer

  i :: Fraction Integer
    3
                    Type: Fraction Integer
```

Since you are explicitly asking for a conversion, you must take responsibility for any loss of exactness.

```
  r := 3/7 :: Float
    0.4285714285 7142857143
                    Type: Float

  r :: Fraction Integer
    3
    -
    7
                    Type: Fraction Integer
```

This conversion cannot be performed: use truncate or round if that is what you intend.

```
  r :: Integer
   Cannot convert from type Float to Integer for value
   0.4285714285 7142857143
```

The operations truncate and round truncate ...

```
   truncate 3.6
    3.0
                    Type: Float
```

and round to the nearest integral Float respectively.

```
  round 3.6
    4.0
                    Type: Float

  truncate(-3.6)
    - 3.0
                    Type: Float

  round(-3.6)
    - 4.0
                    Type: Float
```

The operation fractionPart computes the fractional part of x, that is, x - truncate x.

```
  fractionPart 3.6
```

```
    0.6
                     Type: Float
```

The operation digits allows the user to set the precision. It returns the previous value it was using.

```
  digits 40
    20
                     Type: PositiveInteger

  sqrt 0.2
    0.4472135954 9995793928 1834733746 2552470881
                     Type: Float

  pi()$Float
    3.1415926535 8979323846 2643383279 502884197
                     Type: Float
```

The precision is only limited by the computer memory available. Calculations at 500 or more digits of precision are not difficult.

```
  digits 500
    40
                     Type: PositiveInteger

  pi()$Float
  3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 592307816
  4 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 505822317
  2 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 442881097
  5 6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 454326648
  2 1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 917153643
  6 7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 575959195
  3 0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 891227938
  1 830119491
                     Type: Float
```

Reset digits to its default value.

```
  digits 20
    500
                     Type: PositiveInteger
```

Numbers of type Float are represented as a record of two integers, namely, the mantissa and the exponent where the base of the exponent is binary. That is, the floating point number (m,e) represents the number m x 2^e. A consequence of using a binary base is that decimal numbers can not, in general, be represented exactly.

==

```
Output Functions
====================================================================

A number of operations exist for specifying how numbers of type Float
are to be displayed.  By default, spaces are inserted every ten digits
in the output for readability. Note that you cannot include spaces in
the input form of a floating point number, though you can use
underscores.

Output spacing can be modified with the outputSpacing operation.  This
inserts no spaces and then displays the value of x.

  outputSpacing 0; x := sqrt 0.2
    0.44721359549995793928
                      Type: Float

Issue this to have the spaces inserted every 5 digits.

  outputSpacing 5; x
    0.44721 35954 99957 93928
                      Type: Float

By default, the system displays floats in either fixed format
or scientific format, depending on the magnitude of the number.

  y := x/10**10
    0.44721 35954 99957 93928 E -10
                      Type: Float

A particular format may be requested with the operations
outputFloating and outputFixed.

  outputFloating(); x
    0.44721 35954 99957 93928 E 0
                      Type: Float

  outputFixed(); y
    0.00000 00000 44721 35954 99957 93928
                      Type: Float

Additionally, you can ask for n digits to be displayed after the
decimal point.

  outputFloating 2; y
    0.45 E -10
                      Type: Float

  outputFixed 2; x
    0.45
                      Type: Float
```

```
This resets the output printing to the default behavior.

  outputGeneral()
                    Type: Void

====================================================================
An Example: Determinant of a Hilbert Matrix
====================================================================

Consider the problem of computing the determinant of a 10 by 10
Hilbert matrix.  The (i,j)-th entry of a Hilbert matrix is given
by 1/(i+j+1).

First do the computation using rational numbers to obtain the
exact result.

  a: Matrix Fraction Integer:=matrix[ [1/(i+j+1) for j in 0..9] for i in 0..9]
       +    1   1   1   1   1   1   1   1    1+
       |1   -   -   -   -   -   -   -   -   --|
       |    2   3   4   5   6   7   8   9   10|
       |                                      |
       |1   1   1   1   1   1   1   1    1   1|
       |-   -   -   -   -   -   -   -   --  --|
       |2   3   4   5   6   7   8   9   10  11|
       |                                      |
       |1   1   1   1   1   1   1    1   1   1|
       |-   -   -   -   -   -   -   --  --  --|
       |3   4   5   6   7   8   9   10  11  12|
       |                                      |
       |1   1   1   1   1   1    1   1   1   1|
       |-   -   -   -   -   -   --  --  --  --|
       |4   5   6   7   8   9   10  11  12  13|
       |                                      |
       |1   1   1   1   1    1   1   1   1   1|
       |-   -   -   -   -   --  --  --  --  --|
       |5   6   7   8   9   10  11  12  13  14|
       |                                      |
       |1   1   1   1    1   1   1   1   1   1|
       |-   -   -   -   --  --  --  --  --  --|
       |6   7   8   9   10  11  12  13  14  15|
       |                                      |
       |1   1   1    1   1   1   1   1   1   1|
       |-   -   -   --  --  --  --  --  --  --|
       |7   8   9   10  11  12  13  14  15  16|
       |                                      |
       |1   1    1   1   1   1   1   1   1   1|
       |-   -   --  --  --  --  --  --  --  --|
       |8   9   10  11  12  13  14  15  16  17|
       |                                      |
```

```
|1    1   1   1   1   1   1   1   1   1|
|-   --  --  --  --  --  --  --  --  --|
|9   10  11  12  13  14  15  16  17  18|
|                                      |
| 1   1   1   1   1   1   1   1   1   1|
|--  --  --  --  --  --  --  --  --  --|
+10  11  12  13  14  15  16  17  18  19+
                  Type: Matrix Fraction Integer
```

This version of determinant uses Gaussian elimination.

```
  d:= determinant a
                              1
    -----------------------------------------------------
    46206893947914691316295628839036278726983680000000000
                    Type: Fraction Integer

  d :: Float
    0.21641 79226 43149 18691 E -52
                    Type: Float
```

Now use hardware floats. Note that a semicolon (;) is used to prevent the display of the matrix.

```
  b: Matrix DoubleFloat:=matrix[ [1/(i+j+1\$DoubleFloat) for j in 0..9] for i in 0..9];


                    Type: Matrix DoubleFloat
```

The result given by hardware floats is correct only to four significant digits of precision. In the jargon of numerical analysis, the Hilbert matrix is said to be "ill-conditioned."

```
  determinant b
    2.1643677945721411E-53
                    Type: DoubleFloat
```

Now repeat the computation at a higher precision using Float.

```
  digits 40
    20
                    Type: PositiveInteger

  c: Matrix Float := matrix [ [1/(i+j+1\$Float) for j in 0..9] for i in 0..9];
                    Type: Matrix Float

  determinant c
    0.21641 79226 43149 18690 60594 98362 26174 36159 E -52
                    Type: Float
```

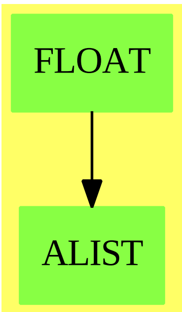
```
Reset digits to its default value.

  digits 20
    40
                     Type: PositiveInteger

See Also:
o )help DoubleFloat
o )show Float
```

———-

7.15.1 Float (FLOAT)



Exports:

0	1	abs	acos
acosh	acot	acoth	acsc
acsch	asec	asech	asin
asinh	associates?	atan	atanh
base	bits	ceiling	characteristic
coerce	convert	cos	cosh
cot	coth	csc	csch
D	decreasePrecision	differentiate	digits
divide	euclideanSize	exp	expressIdealMember
exp1	exponent	exquo	extendedEuclidean
factor	float	floor	fractionPart
gcd	gcdPolynomial	hash	increasePrecision
inv	latex	lcm	log
log10	log2	mantissa	max
min	multiEuclidean	negative?	norm
normalize	nthRoot	OMwrite	one?
order	outputFixed	outputFloating	outputGeneral
outputSpacing	patternMatch	pi	positive?
precision	prime?	principalIdeal	rationalApproximation
recip	relerror	retract	retractIfCan
round	sample	sec	sech
shift	sign	sin	sinh
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	tan	tanh	truncate
unit?	unitCanonical	unitNormal	wholePart
zero?	?*?	?**?	?+?
?-?	-?	?/?	?<?
?<=?	?=?	?>?	?>=?
?^?	?~=?	?quo?	?rem?

— domain FLOAT Float —

```
)abbrev domain FLOAT Float

B ==> Boolean
I ==> Integer
S ==> String
PI ==> PositiveInteger
RN ==> Fraction Integer
SF ==> DoubleFloat
N ==> NonNegativeInteger

++ Author: Michael Monagan
++ Date Created:
++   December 1987
++ Change History:
```

```
++   19 Jun 1990
++ Basic Operations: outputFloating, outputFixed, outputGeneral, outputSpacing,
++   atan, convert, exp1, log2, log10, normalize, rationalApproximation,
++   relerror, shift, / , **
++ Keywords: float, floating point, number
++ Description:
++ \spadtype{Float} implements arbitrary precision floating point arithmetic.
++ The number of significant digits of each operation can be set
++ to an arbitrary value (the default is 20 decimal digits).
++ The operation \spad{float(mantissa,exponent,base)} for integer
++ \spad{mantissa}, \spad{exponent} specifies the number
++ \spad{mantissa * base ** exponent}
++ The underlying representation for floats is binary
++ not decimal. The implications of this are described below.
++
++ The model adopted is that arithmetic operations are rounded to
++ to nearest unit in the last place, that is, accurate to within
++ \spad{2**(-bits)}. Also, the elementary functions and constants are
++ accurate to one unit in the last place.
++ A float is represented as a record of two integers, the mantissa
++ and the exponent.  The base of the representation is binary, hence
++ a \spad{Record(m:mantissa,e:exponent)} represents the number
++ \spad{m * 2 ** e}.
++ Though it is not assumed that the underlying integers are represented
++ with a binary base, the code will be most efficient when this is the
++ the case (this is true in most implementations of Lisp).
++ The decision to choose the base to be binary has some unfortunate
++ consequences.  First, decimal numbers like 0.3 cannot be represented
++ exactly.  Second, there is a further loss of accuracy during
++ conversion to decimal for output.  To compensate for this, if d
++ digits of precision are specified, \spad{1 + ceiling(log2 d)} bits are used.
++ Two numbers that are displayed identically may therefore be
++ not equal.  On the other hand, a significant efficiency loss would
++ be incurred if we chose to use a decimal base when the underlying
++ integer base is binary.
++
++ Algorithms used:
++ For the elementary functions, the general approach is to apply
++ identities so that the taylor series can be used, and, so
++ that it will converge within \spad{O( sqrt n )} steps.  For example,
++ using the identity \spad{exp(x) = exp(x/2)**2}, we can compute
++ \spad{exp(1/3)} to n digits of precision as follows.  We have
++ \spad{exp(1/3) = exp(2 ** (-sqrt s) / 3) ** (2 ** sqrt s)}.
++ The taylor series will converge in less than sqrt n steps and the
++ exponentiation requires sqrt n multiplications for a total of
++ \spad{2 sqrt n} multiplications.  Assuming integer multiplication costs
++ \spad{O( n**2 )} the overall running time is \spad{O( sqrt(n) n**2 )}.
++ This approach is the best known approach for precisions up to
++ about 10,000 digits at which point the methods of Brent
++ which are \spad{O( log(n) n**2 )} become competitive.  Note also that
```

```
++ summing the terms of the taylor series for the elementary
++ functions is done using integer operations.  This avoids the
++ overhead of floating point operations and results in efficient
++ code at low precisions.  This implementation makes no attempt
++ to reuse storage, relying on the underlying system to do
++ \spadgloss{garbage collection}.  I estimate that the efficiency of this
++ package at low precisions could be improved by a factor of 2
++ if in-place operations were available.
++
++ Running times: in the following, n is the number of bits of precision\br
++ \spad{*}, \spad{/}, \spad{sqrt}, \spad{pi}, \spad{exp1}, \spad{log2},
++ \spad{log10}: \spad{ O( n**2 )} \br
++ \spad{exp}, \spad{log}, \spad{sin}, \spad{atan}: \spad{O(sqrt(n) n**2)}\br
++ The other elementary functions are coded in terms of the ones above.

Float():
 Join(FloatingPointSystem, DifferentialRing, ConvertibleTo String, OpenMath,_
  CoercibleTo DoubleFloat, TranscendentalFunctionCategory, ConvertibleTo InputForm) with
   _/  : (%, I) -> %
      ++ x / i computes the division from x by an integer i.
   _*_*: (%, %) -> %
      ++ x ** y computes \spad{exp(y log x)} where \spad{x >= 0}.
   normalize: % -> %
      ++ normalize(x) normalizes x at current precision.
   relerror : (%, %) -> I
      ++ relerror(x,y) computes the absolute value of \spad{x - y} divided by
      ++ y, when \spad{y \^= 0}.
   shift: (%, I) -> %
      ++ shift(x,n) adds n to the exponent of float x.
   rationalApproximation: (%, N) -> RN
     ++ rationalApproximation(f, n) computes a rational approximation
     ++ r to f with relative error \spad{< 10**(-n)}.
   rationalApproximation: (%, N, N) -> RN
     ++ rationalApproximation(f, n, b) computes a rational
     ++ approximation r to f with relative error \spad{< b**(-n)}, that is
     ++ \spad{|(r-f)/f| < b**(-n)}.
   log2 : () -> %
      ++ log2() returns \spad{ln 2}, i.e. \spad{0.6931471805...}.
   log10: () -> %
      ++ log10() returns \spad{ln 10}: \spad{2.3025809299...}.
   exp1 : () -> %
      ++ exp1() returns  exp 1: \spad{2.7182818284...}.
   atan : (%,%) -> %
      ++ atan(x,y) computes the arc tangent from x with phase y.
   log2 : % -> %
      ++ log2(x) computes the logarithm for x to base 2.
   log10: % -> %
      ++ log10(x) computes the logarithm for x to base 10.
   convert: SF -> %
      ++ convert(x) converts a \spadtype{DoubleFloat} x to a \spadtype{Float}.
```

```
   outputFloating: () -> Void
      ++ outputFloating() sets the output mode to floating (scientific) notation, i.e.
      ++ \spad{mantissa * 10 exponent} is displayed as  \spad{0.mantissa E exponent}.
   outputFloating: N -> Void
      ++ outputFloating(n) sets the output mode to floating (scientific) notation
      ++ with n significant digits displayed after the decimal point.
   outputFixed: () -> Void
      ++ outputFixed() sets the output mode to fixed point notation;
      ++ the output will contain a decimal point.
   outputFixed: N -> Void
      ++ outputFixed(n) sets the output mode to fixed point notation,
      ++ with n digits displayed after the decimal point.
   outputGeneral: () -> Void
      ++ outputGeneral() sets the output mode (default mode) to general
      ++ notation; numbers will be displayed in either fixed or floating
      ++ (scientific) notation depending on the magnitude.
   outputGeneral: N -> Void
      ++ outputGeneral(n) sets the output mode to general notation
      ++ with n significant digits displayed.
   outputSpacing: N -> Void
      ++ outputSpacing(n) inserts a space after n (default 10) digits on output;
      ++ outputSpacing(0) means no spaces are inserted.
   arbitraryPrecision
   arbitraryExponent
  == add
   BASE ==> 2
   BITS:Reference(PI) := ref 68 -- 20 digits
   LENGTH ==> INTEGER_-LENGTH$Lisp
   ISQRT ==> approxSqrt$IntegerRoots(I)
   Rep := Record( mantissa:I, exponent:I )
   StoredConstant ==> Record( precision:PI, value:% )
   UCA ==> Record( unit:%, coef:%, associate:% )
   inc ==> increasePrecision
   dec ==> decreasePrecision

   -- local utility operations
   shift2 : (I,I) -> I          -- WSP: fix bug in shift
   times : (%,%) -> %           -- multiply x and y with no rounding
   itimes: (I,%) -> %           -- multiply by a small integer
   chop: (%,PI) -> %            -- chop x at p bits of precision
   dvide: (%,%) -> %            -- divide x by y with no rounding
   square: (%,I) -> %           -- repeated squaring with chopping
   power: (%,I) -> %            -- x ** n with chopping
   plus: (%,%) -> %             -- addition with no rounding
   sub: (%,%) -> %              -- subtraction with no rounding
   negate: % -> %               -- negation with no rounding
   ceillog10base2: PI -> PI     -- rational approximation
   floorln2: PI -> PI           -- rational approximation

   atanSeries: % -> %           -- atan(x) by taylor series |x| < 1/2
```

```
atanInverse: I -> %           -- atan(1/n) for n an integer > 1
expInverse: I -> %            -- exp(1/n) for n an integer
expSeries: % -> %             -- exp(x) by taylor series  |x| < 1/2
logSeries: % -> %             -- log(x) by taylor series 1/2 < x < 2
sinSeries: % -> %             -- sin(x) by taylor series |x| < 1/2
cosSeries: % -> %             -- cos(x) by taylor series |x| < 1/2
piRamanujan: () -> %          -- pi using Ramanujans series

writeOMFloat(dev: OpenMathDevice, x: %): Void ==
   OMputApp(dev)
   OMputSymbol(dev, "bigfloat1", "bigfloat")
   OMputInteger(dev, mantissa x)
   OMputInteger(dev, 2)
   OMputInteger(dev, exponent x)
   OMputEndApp(dev)

OMwrite(x: %): String ==
   s: String := ""
   sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
   dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
   OMputObject(dev)
   writeOMFloat(dev, x)
   OMputEndObject(dev)
   OMclose(dev)
   s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
   s

OMwrite(x: %, wholeObj: Boolean): String ==
   s: String := ""
   sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
   dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
   if wholeObj then
      OMputObject(dev)
   writeOMFloat(dev, x)
   if wholeObj then
      OMputEndObject(dev)
   OMclose(dev)
   s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
   s

OMwrite(dev: OpenMathDevice, x: %): Void ==
   OMputObject(dev)
   writeOMFloat(dev, x)
   OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
   if wholeObj then
      OMputObject(dev)
   writeOMFloat(dev, x)
   if wholeObj then
```

```
        OMputEndObject(dev)

   shift2(x,y) == sign(x)*shift(sign(x)*x,y)

   asin x ==
      zero? x => 0
      negative? x => -asin(-x)
--      one? x => pi()/2
      (x = 1) => pi()/2
      x > 1 => error "asin: argument > 1 in magnitude"
      inc 5; r := atan(x/sqrt(sub(1,times(x,x)))); dec 5
      normalize r

   acos x ==
      zero? x => pi()/2
      negative? x => (inc 3; r := pi()-acos(-x); dec 3; normalize r)
--      one? x => 0
      (x = 1) => 0
      x > 1 => error "acos: argument > 1 in magnitude"
      inc 5; r := atan(sqrt(sub(1,times(x,x)))/x); dec 5
      normalize r

   atan(x,y) ==
      x = 0 =>
         y > 0 => pi()/2
         y < 0 => -pi()/2
         0
      -- Only count on first quadrant being on principal branch.
      theta := atan abs(y/x)
      if x < 0 then theta := pi() - theta
      if y < 0 then theta := - theta
      theta

   atan x ==
      zero? x => 0
      negative? x => -atan(-x)
      if x > 1 then
         inc 4
         r := if zero? fractionPart x and x < [bits(),0] then atanInverse wholePart x
                 else atan(1/x)
         r := pi/2 - r
         dec 4
         return normalize r
      -- make |x| < O( 2**(-sqrt p) ) < 1/2 to speed series convergence
      -- by using the formula  atan(x) = 2*atan(x/(1+sqrt(1+x**2)))
      k := ISQRT (bits()-100)::I quo 5
      k := max(0,2 + k + order x)
      inc(2*k)
      for i in 1..k repeat x := x/(1+sqrt(1+x*x))
      t := atanSeries x
```

```
   dec(2*k)
   t := shift(t,k)
   normalize t

atanSeries x ==
   -- atan(x) = x (1 - x**2/3 + x**4/5 - x**6/7 + ...)  |x| < 1
   p := bits() + LENGTH bits() + 2
   s:I := d:I := shift(1,p)
   y := times(x,x)
   t := m := - shift2(y.mantissa,y.exponent+p)
   for i in 3.. by 2 while t ^= 0 repeat
      s := s + t quo i
      t := (m * t) quo d
   x * [s,-p]

atanInverse n ==
   -- compute atan(1/n) for an integer n > 1
   -- atan n = 1/n - 1/n**3/3 + 1/n**5/4 - ...
   --   pi = 16 atan(1/5) - 4 atan(1/239)
   n2 := -n*n
   e:I := bits() + LENGTH bits() + LENGTH n + 1
   s:I := shift(1,e) quo n
   t:I := s quo n2
   for k in 3.. by 2 while t ^= 0 repeat
      s := s + t quo k
      t := t quo n2
   normalize [s,-e]

sin x ==
   s := sign x; x := abs x; p := bits(); inc 4
   if x > [6,0] then (inc p; x := 2*pi*fractionPart(x/pi/2); bits p)
   if x > [3,0] then (inc p; s := -s; x := x - pi; bits p)
   if x > [3,-1] then (inc p; x := pi - x; dec p)
   -- make |x| < O( 2**(-sqrt p) ) < 1/2 to speed series convergence
   -- by using the formula  sin(3*x/3) = 3 sin(x/3) - 4 sin(x/3)**3
   -- the running time is O( sqrt p M(p) ) assuming |x| < 1
   k := ISQRT (bits()-100)::I quo 4
   k := max(0,2 + k + order x)
   if k > 0 then (inc k; x := x / 3**k::N)
   r := sinSeries x
   for i in 1..k repeat r := itimes(3,r)-shift(r**3,2)
   bits p
   s * r

sinSeries x ==
   -- sin(x) = x (1 - x**2/3! + x**4/5! - x**6/7! + ... |x| < 1/2
   p := bits() + LENGTH bits() + 2
   y := times(x,x)
   s:I := d:I := shift(1,p)
   m:I := - shift2(y.mantissa,y.exponent+p)
```

```
    t:I := m quo 6
    for i in 4.. by 2 while t ^= 0 repeat
       s := s + t
       t := (m * t) quo (i*(i+1))
       t := t quo d
    x * [s,-p]

 cos x ==
   s:I := 1; x := abs x; p := bits(); inc 4
   if x > [6,0] then (inc p; x := 2*pi*fractionPart(x/pi/2); dec p)
   if x > [3,0] then (inc p; s := -s; x := x-pi; dec p)
   if x > [1,0] then
       -- take care of the accuracy problem near pi/2
       inc p; x := pi/2-x; bits p; x := normalize x
       return (s * sin x)
    -- make |x| < O( 2**(-sqrt p) ) < 1/2 to speed series convergence
    -- by using the formula  cos(2*x/2) = 2 cos(x/2)**2 - 1
    -- the running time is O( sqrt p M(p) ) assuming |x| < 1
   k := ISQRT (bits()-100)::I quo 3
   k := max(0,2 + k + order x)
    -- need to increase precision by more than k, otherwise recursion
    -- causes loss of accuracy.
    -- Michael Monagan suggests adding a factor of log(k)
   if k > 0 then (inc(k+length(k)**2); x := shift(x,-k))
   r := cosSeries x
   for i in 1..k repeat r := shift(r*r,1)-1
   bits p
   s * r



 cosSeries x ==
    -- cos(x) = 1 - x**2/2! + x**4/4! - x**6/6! + ... |x| < 1/2
    p := bits() + LENGTH bits() + 1
    y := times(x,x)
    s:I := d:I := shift(1,p)
    m:I := - shift2(y.mantissa,y.exponent+p)
    t:I := m quo 2
    for i in 3.. by 2 while t ^= 0 repeat
       s := s + t
       t := (m * t) quo (i*(i+1))
       t := t quo d
    normalize [s,-p]

 tan x ==
    s := sign x; x := abs x; p := bits(); inc 6
    if x > [3,0] then (inc p; x := pi()*fractionPart(x/pi()); dec p)
    if x > [3,-1] then (inc p; x := pi()-x; s := -s; dec p)
    if x > 1 then (c := cos x; t := sqrt(1-c*c)/c)
    else (c := sin x; t := c/sqrt(1-c*c))
```

```
   bits p
   s * t

P:StoredConstant := [1,[1,2]]
pi() ==
   -- We use Ramanujan's identity to compute pi.
   -- The running time is quadratic in the precision.
   -- This is about twice as fast as Machin's identity on Lisp/VM
   --   pi = 16 atan(1/5) - 4 atan(1/239)
   bits() <= P.precision => normalize P.value
   (P := [bits(), piRamanujan()]) value

piRamanujan() ==
   -- Ramanujans identity for 1/pi
   -- Reference: Shanks and Wrench, Math Comp, 1962
   -- "Calculation of pi to 100,000 Decimals".
   n := bits() + LENGTH bits() + 11
   t:I := shift(1,n) quo 882
   d:I := 4*882**2
   s:I := 0
   for i in 2.. by 2 for j in 1123.. by 21460 while t ^= 0 repeat
      s := s + j*t
      m := -(i-1)*(2*i-1)*(2*i-3)
      t := (m*t) quo (d*i**3)
   1 / [s,-n-2]

sinh x ==
   zero? x => 0
   lost:I := max(- order x,0)
   2*lost > bits() => x
   inc(5+lost); e := exp x; s := (e-1/e)/2; dec(5+lost)
   normalize s

cosh x ==
   (inc 5; e := exp x; c := (e+1/e)/2; dec 5; normalize c)

tanh x ==
   zero? x => 0
   lost:I := max(- order x,0)
   2*lost > bits() => x
   inc(6+lost); e := exp x; e := e*e; t := (e-1)/(e+1); dec(6+lost)
   normalize t

asinh x ==
   p := min(0,order x)
   if zero? x or 2*p < -bits() then return x
   inc(5-p); r := log(x+sqrt(1+x*x)); dec(5-p)
   normalize r

acosh x ==
```

```
    if x < 1 then error "invalid argument to acosh"
    inc 5; r := log(x+sqrt(sub(times(x,x),1))); dec 5
    normalize r

 atanh x ==
    if x > 1 or x < -1 then error "invalid argument to atanh"
    p := min(0,order x)
    if zero? x or 2*p < -bits() then return x
    inc(5-p); r := log((x+1)/(1-x))/2; dec(5-p)
    normalize r

 log x ==
    negative? x => error "negative log"
    zero? x => error "log 0 generated"
    p := bits(); inc 5
    -- apply  log(x) = n log 2 + log(x/2**n)  so that  1/2 < x < 2
    if (n := order x) < 0 then n := n+1
    l := if n = 0 then 0 else (x := shift(x,-n); n * log2)
    -- speed the series convergence by finding m and k such that
    -- | exp(m/2**k) x - 1 |  <  1 / 2 ** O(sqrt p)
    -- write  log(exp(m/2**k) x) as m/2**k + log x
    k := ISQRT (p-100)::I quo 3
    if k > 1 then
       k := max(1,k+order(x-1))
       inc k
       ek := expInverse (2**k::N)
       dec(p quo 2); m := order square(x,k); inc(p quo 2)
       m := (6847196937 * m) quo 9878417065   -- m := m log 2
       x := x * ek ** (-m)
       l := l + [m,-k]
    l := l + logSeries x
    bits p
    normalize l

 logSeries x ==
    -- log(x) = 2 y (1 + y**2/3 + y**4/5 ...)  for  y = (x-1) / (x+1)
    -- given 1/2 < x < 2 on input we have -1/3 < y < 1/3
    p := bits() + (g := LENGTH bits() + 3)
    inc g; y := (x-1)/(x+1); dec g
    s:I := d:I := shift(1,p)
    z := times(y,y)
    t := m := shift2(z.mantissa,z.exponent+p)
    for i in 3.. by 2 while t ^= 0 repeat
       s := s + t quo i
       t := m * t quo d
    y * [s,1-p]

 L2:StoredConstant := [1,1]
 log2() ==
    --  log x  =  2 * sum( ((x-1)/(x+1))**(2*k+1)/(2*k+1), k=1.. )
```

```
   --  log 2  =  2 * sum( 1/9**k / (2*k+1), k=0..n ) / 3
   n := bits() :: N
   n <= L2.precision => normalize L2.value
   n := n + LENGTH n + 3  -- guard bits
   s:I := shift(1,n+1) quo 3
   t:I := s quo 9
   for k in 3.. by 2 while t ^= 0 repeat
      s := s + t quo k
      t := t quo 9
   L2 := [bits(),[s,-n]]
   normalize L2.value

L10:StoredConstant := [1,[1,1]]
log10() ==
   --  log x  =  2 * sum( ((x-1)/(x+1))**(2*k+1)/(2*k+1), k=0.. )
   --  log 5/4  =  2 * sum( 1/81**k / (2*k+1), k=0.. ) / 9
   n := bits() :: N
   n <= L10.precision => normalize L10.value
   n := n + LENGTH n + 5  -- guard bits
   s:I := shift(1,n+1) quo 9
   t:I := s quo 81
   for k in 3.. by 2 while t ^= 0 repeat
      s := s + t quo k
      t := t quo 81
   -- We have log 10 = log 5 + log 2 and log 5/4 = log 5 - 2 log 2
   inc 2; L10 := [bits(),[s,-n] + 3*log2]; dec 2
   normalize L10.value

log2(x) == (inc 2; r := log(x)/log2; dec 2; normalize r)
log10(x) == (inc 2; r := log(x)/log10; dec 2; normalize r)

exp(x) ==
   -- exp(n+x) = exp(1)**n exp(x) for n such that |x| < 1
   p := bits(); inc 5; e1:% := 1
   if (n := wholePart x) ^= 0 then
      inc LENGTH n; e1 := exp1 ** n; dec LENGTH n
      x := fractionPart x
   if zero? x then (bits p; return normalize e1)
   -- make |x| < O( 2**(-sqrt p) ) < 1/2 to speed series convergence
   -- by repeated use of the formula exp(2*x/2) = exp(x/2)**2
   -- results in an overall running time of O( sqrt p M(p) )
   k := ISQRT (p-100)::I quo 3
   k := max(0,2 + k + order x)
   if k > 0 then (inc k; x := shift(x,-k))
   e := expSeries x
   if k > 0 then e := square(e,k)
   bits p
   e * e1

expSeries x ==
```

```
   -- exp(x) = 1 + x + x**2/2 + ... + x**i/i!  valid for all x
   p := bits() + LENGTH bits() + 1
   s:I := d:I := shift(1,p)
   t:I := n:I := shift2(x.mantissa,x.exponent+p)
   for i in 2.. while t ^= 0 repeat
      s := s + t
      t := (n * t) quo i
      t := t quo d
   normalize [s,-p]

expInverse k ==
   -- computes exp(1/k) via continued fraction
   p0:I := 2*k+1; p1:I := 6*k*p0+1
   q0:I := 2*k-1; q1:I := 6*k*q0+1
   for i in 10*k.. by 4*k while 2 * LENGTH p0 < bits() repeat
      (p0,p1) := (p1,i*p1+p0)
      (q0,q1) := (q1,i*q1+q0)
   dvide([p1,0],[q1,0])

E:StoredConstant := [1,[1,1]]
exp1() ==
   if bits() > E.precision then E := [bits(),expInverse 1]
   normalize E.value

sqrt x ==
   negative? x => error "negative sqrt"
   m := x.mantissa; e := x.exponent
   l := LENGTH m
   p := 2 * bits() - l + 2
   if odd?(e-l) then p := p - 1
   i := shift2(x.mantissa,p)
   -- ISQRT uses a variable precision newton iteration
   i := ISQRT i
   normalize [i,(e-p) quo 2]

bits() == BITS()
bits(n) == (t := bits(); BITS() := n; t)
precision() == bits()
precision(n) == bits(n)
increasePrecision n == (b := bits(); bits((b + n)::PI); b)
decreasePrecision n == (b := bits(); bits((b - n)::PI); b)
ceillog10base2 n == ((13301 * n + 4003) quo 4004) :: PI
digits() == max(1,4004 * (bits()-1) quo 13301)::PI
digits(n) == (t := digits(); bits (1 + ceillog10base2 n); t)

order(a) == LENGTH a.mantissa + a.exponent - 1
relerror(a,b) == order((a-b)/b)
0 == [0,0]
1 == [1,0]
base() == BASE
```

```
mantissa x == x.mantissa
exponent x == x.exponent
one? a == a = 1
zero? a == zero?(a.mantissa)
negative? a == negative?(a.mantissa)
positive? a == positive?(a.mantissa)

chop(x,p) ==
   e : I := LENGTH x.mantissa - p
   if e > 0 then x := [shift2(x.mantissa,-e),x.exponent+e]
   x
float(m,e) == normalize [m,e]
float(m,e,b) ==
   m = 0 => 0
   inc 4; r := m * [b,0] ** e; dec 4
   normalize r
normalize x ==
   m := x.mantissa
   m = 0 => 0
   e : I := LENGTH m - bits()
   if e > 0 then
      y := shift2(m,1-e)
      if odd? y then
         y := (if y>0 then y+1 else y-1) quo 2
         if LENGTH y > bits() then
            y := y quo 2
            e := e+1
      else y := y quo 2
      x := [y,x.exponent+e]
   x
shift(x:%,n:I) == [x.mantissa,x.exponent+n]

x = y ==
   order x = order y and sign x = sign y and zero? (x - y)
x < y ==
   y.mantissa = 0 => x.mantissa < 0
   x.mantissa = 0 => y.mantissa > 0
   negative? x and positive? y => true
   negative? y and positive? x => false
   order x < order y => positive? x
   order x > order y => negative? x
   negative? (x-y)

abs x == if negative? x then -x else normalize x
ceiling x ==
   if negative? x then return (-floor(-x))
   if zero? fractionPart x then x else truncate x + 1
wholePart x == shift2(x.mantissa,x.exponent)
floor x == if negative? x then -ceiling(-x) else truncate x
round x == (half := [sign x,-1]; truncate(x + half))
```

```
sign x == if x.mantissa < 0 then -1 else 1
truncate x ==
   if x.exponent >= 0 then return x
   normalize [shift2(x.mantissa,x.exponent),0]
recip(x) == if x=0 then "failed" else 1/x
differentiate x == 0

- x == normalize negate x
negate x == [-x.mantissa,x.exponent]
x + y == normalize plus(x,y)
x - y == normalize plus(x,negate y)
sub(x,y) == plus(x,negate y)
plus(x,y) ==
   mx := x.mantissa; my := y.mantissa
   mx = 0 => y
   my = 0 => x
   ex := x.exponent; ey := y.exponent
   ex = ey => [mx+my,ex]
   de := ex + LENGTH mx - ey - LENGTH my
   de > bits()+1 => x
   de < -(bits()+1) => y
   if ex < ey then (mx,my,ex,ey) := (my,mx,ey,ex)
   mw := my + shift2(mx,ex-ey)
   [mw,ey]

x:% * y:% == normalize times (x,y)
x:I * y:% ==
   if LENGTH x > bits() then normalize [x,0] * y
   else normalize [x * y.mantissa,y.exponent]
x:% / y:% == normalize dvide(x,y)
x:% / y:I ==
   if LENGTH y > bits() then x / normalize [y,0] else x / [y,0]
inv x == 1 / x

times(x:%,y:%) == [x.mantissa * y.mantissa, x.exponent + y.exponent]
itimes(n:I,y:%) == [n * y.mantissa,y.exponent]

dvide(x,y) ==
   ew := LENGTH y.mantissa - LENGTH x.mantissa + bits() + 1
   mw := shift2(x.mantissa,ew) quo y.mantissa
   ew := x.exponent - y.exponent - ew
   [mw,ew]

square(x,n) ==
   ma := x.mantissa; ex := x.exponent
   for k in 1..n repeat
      ma := ma * ma; ex := ex + ex
      l:I := bits()::I - LENGTH ma
      ma := shift2(ma,l); ex := ex - l
   [ma,ex]
```

```
power(x,n) ==
   y:% := 1; z:% := x
   repeat
      if odd? n then y := chop( times(y,z), bits() )
      if (n := n quo 2) = 0 then return y
      z := chop( times(z,z), bits() )

x:% ** y:% ==
   x = 0 =>
      y = 0 => error "0**0 is undefined"
      y < 0 => error "division by 0"
      y > 0 => 0
   y = 0 => 1
   y = 1 => x
   x = 1 => 1
   p := abs order y + 5
   inc p; r := exp(y*log(x)); dec p
   normalize r

x:% ** r:RN ==
   x = 0 =>
      r = 0 => error "0**0 is undefined"
      r < 0 => error "division by 0"
      r > 0 => 0
   r = 0 => 1
   r = 1 => x
   x = 1 => 1
   n := numer r
   d := denom r
   negative? x =>
      odd? d =>
         odd? n => return -((-x)**r)
         return ((-x)**r)
      error "negative root"
   if d = 2 then
      inc LENGTH n; y := sqrt(x); y := y**n; dec LENGTH n
      return normalize y
   y := [n,0]/[d,0]
   x ** y

x:% ** n:I ==
   x = 0 =>
      n = 0 => error "0**0 is undefined"
      n < 0 => error "division by 0"
      n > 0 => 0
   n = 0 => 1
   n = 1 => x
   x = 1 => 1
   p := bits()
```

```
      bits(p + LENGTH n + 2)
      y := power(x,abs n)
      if n < 0 then y := dvide(1,y)
      bits p
      normalize y

   -- Utility routines for conversion to decimal
   ceilLength10: I -> I
   chop10: (%,I) -> %
   convert10:(%,I) -> %
   floorLength10: I -> I
   length10: I -> I
   normalize10: (%,I) -> %
   quotient10: (%,%,I) -> %
   power10: (%,I,I) -> %
   times10: (%,%,I) -> %

   convert10(x,d) ==
      m := x.mantissa; e := x.exponent
      --!! deal with bits here
      b := bits(); (q,r) := divide(abs e, b)
      b := 2**b::N; r := 2**r::N
      -- compute 2**e = b**q * r
      h := power10([b,0],q,d+5)
      h := chop10([r*h.mantissa,h.exponent],d+5)
      if e < 0 then h := quotient10([m,0],h,d)
      else times10([m,0],h,d)

   ceilLength10 n == 146 * LENGTH n quo 485 + 1
   floorLength10 n == 643 *  LENGTH n quo 2136
--   length10 n == DECIMAL_-LENGTH(n)$Lisp
   length10 n ==
      ln := LENGTH(n:=abs n)
      upper := 76573 * ln quo 254370
      lower := 21306 * (ln-1) quo 70777
      upper = lower => upper + 1
      n := n quo (10**lower::N)
      while n >= 10 repeat
         n:= n quo 10
         lower := lower + 1
      lower + 1

   chop10(x,p) ==
      e : I := floorLength10 x.mantissa - p
      if e > 0 then x := [x.mantissa quo 10**e::N,x.exponent+e]
      x
   normalize10(x,p) ==
      ma := x.mantissa
      ex := x.exponent
      e : I := length10 ma - p
```

```
   if e > 0 then
      ma := ma quo 10**(e-1)::N
      ex := ex + e
      (ma,r) := divide(ma, 10)
      if r > 4 then
         ma := ma + 1
         if ma = 10**p::N then (ma := 1; ex := ex + p)
   [ma,ex]
times10(x,y,p) == normalize10(times(x,y),p)
quotient10(x,y,p) ==
   ew := floorLength10 y.mantissa - ceilLength10 x.mantissa + p + 2
   if ew < 0 then ew := 0
   mw := (x.mantissa * 10**ew::N) quo y.mantissa
   ew := x.exponent - y.exponent - ew
   normalize10([mw,ew],p)
power10(x,n,d) ==
   x = 0 => 0
   n = 0 => 1
   n = 1 => x
   x = 1 => 1
   p:I := d + LENGTH n + 1
   e:I := n
   y:% := 1
   z:% := x
   repeat
      if odd? e then y := chop10(times(y,z),p)
      if (e := e quo 2) = 0 then return y
      z := chop10(times(z,z),p)

--------------------------------
-- Output routines for Floats --
--------------------------------
zero ==> char("0")
separator ==> space()$Character

SPACING : Reference(N) := ref 10
OUTMODE : Reference(S) := ref "general"
OUTPREC : Reference(I) := ref(-1)

fixed : % -> S
floating : % -> S
general : % -> S

padFromLeft(s:S):S ==
   zero? SPACING() => s
   n:I := #s - 1
   t := new( (n + 1 + n quo SPACING()) :: N , separator )
   for i in 0..n for j in minIndex t .. repeat
      t.j := s.(i + minIndex s)
      if (i+1) rem SPACING() = 0 then j := j+1
```

```
      t
   padFromRight(s:S):S ==
      SPACING() = 0 => s
      n:I := #s - 1
      t := new( (n + 1 + n quo SPACING()) :: N , separator )
      for i in n..0 by -1 for j in maxIndex t .. by -1 repeat
         t.j := s.(i + minIndex s)
         if (n-i+1) rem SPACING() = 0 then j := j-1
      t

   fixed f ==
      d := if OUTPREC() = -1 then digits::I else OUTPREC()
      dpos:N:= if (d > 0) then d::N else 1::N
      zero? f =>
        OUTPREC() = -1 => "0.0"
        concat("0",concat(".",padFromLeft new(dpos,zero)))
      zero? exponent f =>
        concat(padFromRight convert(mantissa f)@S,
               concat(".",padFromLeft new(dpos,zero)))
      negative? f => concat("-", fixed abs f)
      bl := LENGTH(f.mantissa) + f.exponent
      dd :=
        OUTPREC() = -1 => d
        bl > 0 => (146*bl) quo 485 + 1 + d
        d
      g := convert10(abs f,dd)
      m := g.mantissa
      e := g.exponent
      if OUTPREC() ^= -1 then
         -- round g to OUTPREC digits after the decimal point
         l := length10 m
         if -e > OUTPREC() and -e < 2*digits::I then
            g := normalize10(g,l+e+OUTPREC())
            m := g.mantissa; e := g.exponent
      s := convert(m)@S; n := #s; o := e+n
      p := if OUTPREC() = -1 then n::I else OUTPREC()
      t:S
      if e >= 0 then
         s := concat(s, new(e::N, zero))
         t := ""
      else if o <= 0 then
         t := concat(new((-o)::N,zero), s)
         s := "0"
      else
         t := s(o + minIndex s .. n + minIndex s - 1)
         s := s(minIndex s .. o + minIndex s - 1)
      n := #t
      if OUTPREC() = -1 then
         t := rightTrim(t,zero)
         if t = "" then t := "0"
```

```
   else if n > p then t := t(minIndex t .. p + minIndex t- 1)
                 else t := concat(t, new((p-n)::N,zero))
   concat(padFromRight s, concat(".", padFromLeft t))

floating f ==
   zero? f => "0.0"
   negative? f => concat("-", floating abs f)
   t:S := if zero? SPACING() then "E" else " E "
   zero? exponent f =>
     s := convert(mantissa f)@S
     concat ["0.", padFromLeft s, t, convert(#s)@S]
   -- base conversion to decimal rounded to the requested precision
   d := if OUTPREC() = -1 then digits::I else OUTPREC()
   g := convert10(f,d); m := g.mantissa; e := g.exponent
   -- I'm assuming that length10 m = # s given n > 0
   s := convert(m)@S; n := #s; o := e+n
   s := padFromLeft s
   concat ["0.", s, t, convert(o)@S]

general(f) ==
   zero? f => "0.0"
   negative? f => concat("-", general abs f)
   d := if OUTPREC() = -1 then digits::I else OUTPREC()
   zero? exponent f =>
     d := d + 1
     s := convert(mantissa f)@S
     OUTPREC() ^= -1 and (e := #s) > d =>
       t:S := if zero? SPACING() then "E" else " E "
       concat ["0.", padFromLeft s, t, convert(e)@S]
     padFromRight concat(s, ".0")
   -- base conversion to decimal rounded to the requested precision
   g := convert10(f,d); m := g.mantissa; e := g.exponent
   -- I'm assuming that length10 m = # s given n > 0
   s := convert(m)@S; n := #s; o := n + e
   -- Note: at least one digit is displayed after the decimal point
   -- and trailing zeroes after the decimal point are dropped
   if o > 0 and o <= max(n,d) then
      -- fixed format: add trailing zeroes before the decimal point
      if o > n then s := concat(s, new((o-n)::N,zero))
      t := rightTrim(s(o + minIndex s .. n + minIndex s - 1), zero)
      if t = "" then t := "0" else t := padFromLeft t
      s := padFromRight s(minIndex s .. o + minIndex s - 1)
      concat(s, concat(".", t))
   else if o <= 0 and o >= -5 then
      -- fixed format: up to 5 leading zeroes after the decimal point
      concat("0.",padFromLeft concat(new((-o)::N,zero),rightTrim(s,zero)))
   else
      -- print using E format written  0.mantissa E exponent
      t := padFromLeft rightTrim(s,zero)
      s := if zero? SPACING() then "E" else " E "
```

```
      concat ["0.", t, s, convert(e+n)@S]

   outputSpacing n == SPACING() := n
   outputFixed() == (OUTMODE() := "fixed"; OUTPREC() := -1)
   outputFixed n == (OUTMODE() := "fixed"; OUTPREC() := n::I)
   outputGeneral() == (OUTMODE() := "general"; OUTPREC() := -1)
   outputGeneral n == (OUTMODE() := "general"; OUTPREC() := n::I)
   outputFloating() == (OUTMODE() := "floating"; OUTPREC() := -1)
   outputFloating n == (OUTMODE() := "floating"; OUTPREC() := n::I)

   convert(f):S ==
      b:Integer :=
        OUTPREC() = -1 and not zero? f =>
          bits(length(abs mantissa f)::PositiveInteger)
        0
      s :=
        OUTMODE() = "fixed" => fixed f
        OUTMODE() = "floating" => floating f
        OUTMODE() = "general" => general f
        empty()$String
      if b > 0 then bits(b::PositiveInteger)
      s = empty()$String => error "bad output mode"
      s

   coerce(f):OutputForm ==
     f >= 0 => message(convert(f)@S)
     - (coerce(-f)@OutputForm)

   convert(f):InputForm ==
     convert [convert("float"::Symbol), convert mantissa f,
              convert exponent f, convert base()]$List(InputForm)

   -- Conversion routines
   convert(x:%):Float == x pretend Float
   convert(x:%):SF == makeSF(x.mantissa,x.exponent)$Lisp
   coerce(x:%):SF == convert(x)@SF
   convert(sf:SF):% == float(mantissa sf,exponent sf,base()$SF)

   retract(f:%):RN == rationalApproximation(f,(bits()-1)::N,BASE)

   retractIfCan(f:%):Union(RN, "failed") ==
     rationalApproximation(f,(bits()-1)::N,BASE)

   retract(f:%):I ==
     (f = (n := wholePart f)::%) => n
     error "Not an integer"

   retractIfCan(f:%):Union(I, "failed") ==
     (f = (n := wholePart f)::%) => n
     "failed"
```

```
rationalApproximation(f,d) == rationalApproximation(f,d,10)

rationalApproximation(f,d,b) ==
   t: Integer
   nu := f.mantissa; ex := f.exponent
   if ex >= 0 then return ((nu*BASE**(ex::N))/1)
   de := BASE**((-ex)::N)
   if b < 2 then error "base must be > 1"
   tol := b**d
   s := nu; t := de
   p0,p1,q0,q1 : Integer
   p0 := 0; p1 := 1; q0 := 1; q1 := 0
   repeat
      (q,r) := divide(s, t)
      p2 := q*p1+p0
      q2 := q*q1+q0
      if r = 0 or tol*abs(nu*q2-de*p2) < de*abs(p2) then return (p2/q2)
      (p0,p1) := (p1,p2)
      (q0,q1) := (q1,q2)
      (s,t) := (t,r)
```

———-

— FLOAT.dotabb —

```
"FLOAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLOAT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FLOAT" -> "ALIST"
```

———-

7.16 domain FC FortranCode

— FortranCode.input —

```
)set break resume
)sys rm -f FortranCode.output
)spool FortranCode.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show FortranCode
--R FortranCode  is a domain constructor
--R Abbreviation for FortranCode is FC
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for FC
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                assign : (Symbol,String) -> %
--R block : List % -> %                   call : String -> %
--R coerce : % -> OutputForm              comment : List String -> %
--R comment : String -> %                 cond : (Switch,%,%) -> %
--R cond : (Switch,%) -> %                continue : SingleInteger -> %
--R getCode : % -> SExpression            goto : SingleInteger -> %
--R hash : % -> SingleInteger             latex : % -> String
--R printCode : % -> Void                 returns : Expression Integer -> %
--R returns : Expression Float -> %       returns : () -> %
--R save : () -> %                        stop : () -> %
--R whileLoop : (Switch,%) -> %           ?~=? : (%,%) -> Boolean
--R assign : (Symbol,List Polynomial Integer,Expression Complex Float) -> %
--R assign : (Symbol,List Polynomial Integer,Expression Float) -> %
--R assign : (Symbol,List Polynomial Integer,Expression Integer) -> %
--R assign : (Symbol,Vector Expression Complex Float) -> %
--R assign : (Symbol,Vector Expression Float) -> %
--R assign : (Symbol,Vector Expression Integer) -> %
--R assign : (Symbol,Matrix Expression Complex Float) -> %
--R assign : (Symbol,Matrix Expression Float) -> %
--R assign : (Symbol,Matrix Expression Integer) -> %
--R assign : (Symbol,Expression Complex Float) -> %
--R assign : (Symbol,Expression Float) -> %
--R assign : (Symbol,Expression Integer) -> %
--R assign : (Symbol,List Polynomial Integer,Expression MachineComplex) -> %
--R assign : (Symbol,List Polynomial Integer,Expression MachineFloat) -> %
--R assign : (Symbol,List Polynomial Integer,Expression MachineInteger) -> %
--R assign : (Symbol,Vector Expression MachineComplex) -> %
--R assign : (Symbol,Vector Expression MachineFloat) -> %
--R assign : (Symbol,Vector Expression MachineInteger) -> %
--R assign : (Symbol,Matrix Expression MachineComplex) -> %
--R assign : (Symbol,Matrix Expression MachineFloat) -> %
--R assign : (Symbol,Matrix Expression MachineInteger) -> %
--R assign : (Symbol,Vector MachineComplex) -> %
--R assign : (Symbol,Vector MachineFloat) -> %
--R assign : (Symbol,Vector MachineInteger) -> %
--R assign : (Symbol,Matrix MachineComplex) -> %
--R assign : (Symbol,Matrix MachineFloat) -> %
--R assign : (Symbol,Matrix MachineInteger) -> %
--R assign : (Symbol,Expression MachineComplex) -> %
--R assign : (Symbol,Expression MachineFloat) -> %
--R assign : (Symbol,Expression MachineInteger) -> %
--R code : % -> Union(nullBranch: null,assignmentBranch: Record(var: Symbol,arrayIndex: List Polynomial
--R common : (Symbol,List Symbol) -> %
```

```
--R forLoop : (SegmentBinding Polynomial Integer,Polynomial Integer,%) -> %
--R forLoop : (SegmentBinding Polynomial Integer,%) -> %
--R operation : % -> Union(Null: null,Assignment: assignment,Conditional: conditional,Return
--R printStatement : List OutputForm -> %
--R repeatUntilLoop : (Switch,%) -> %
--R returns : Expression Complex Float -> %
--R returns : Expression MachineComplex -> %
--R returns : Expression MachineInteger -> %
--R returns : Expression MachineFloat -> %
--R setLabelValue : SingleInteger -> SingleInteger
--R
--E 1

)spool
)lisp (bye)
```

———-

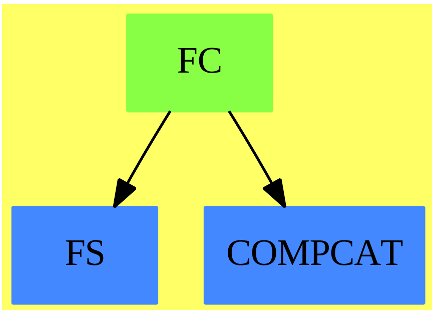
— **FortranCode.help** —

```
====================================================================
FortranCode examples
====================================================================

See Also:
o )show FortranCode
```

———-

7.16.1 FortranCode (FC)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranProgram" (FORTRAN) 7.18.1 on page 923

⇒ "ThreeDimensionalMatrix" (M3D) 21.7.1 on page 2661
⇒ "SimpleFortranProgram" (SFORT) 20.11.1 on page 2364
⇒ "Switch" (SWITCH) 20.36.1 on page 2588
⇒ "FortranTemplate" (FTEM) 7.20.1 on page 934
⇒ "FortranExpression" (FEXPR) 7.17.1 on page 914

Exports:

assign	block	call	code	coerce
comment	common	cond	continue	forLoop
getCode	goto	hash	latex	operation
printCode	printStatement	repeatUntilLoop	returns	save
setLabelValue	stop	whileLoop	?=?	? =?

— domain FC FortranCode —

```
)abbrev domain FC FortranCode
++ Author: Mike Dewar
++ Date Created: April 1991
++ Date Last Updated: 22 March 1994
++                   26 May 1994 Added common, MCD
++                   21 June 1994 Changed print to printStatement, MCD
++                   30 June 1994 Added stop, MCD
++                   12 July 1994 Added assign for String, MCD
++                    9 January 1995 Added fortran2Lines to getCall, MCD
++ Basic Operations:
++ Related Constructors: FortranProgram, Switch, FortranType
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain builds representations of program code segments for use with
++ the FortranProgram domain.

FortranCode(): public == private where
  L ==> List
  PI ==> PositiveInteger
  PIN ==> Polynomial Integer
  SEX ==> SExpression
  O ==> OutputForm
  OP ==> Union(Null:"null",
               Assignment:"assignment",
               Conditional:"conditional",
               Return:"return",
               Block:"block",
               Comment:"comment",
               Call:"call",
               For:"for",
               While:"while",
               Repeat:"repeat",
```

```
              Goto:"goto",
              Continue:"continue",
              ArrayAssignment:"arrayAssignment",
              Save:"save",
              Stop:"stop",
              Common:"common",
              Print:"print")
ARRAYASS ==> Record(var:Symbol, rand:O, ints2Floats?:Boolean)
EXPRESSION ==> Record(ints2Floats?:Boolean,expr:O)
ASS ==> Record(var:Symbol,
               arrayIndex:L PIN,
               rand:EXPRESSION
              )
COND ==> Record(switch: Switch(),
                thenClause: $,
                elseClause: $
               )
RETURN ==> Record(empty?:Boolean,value:EXPRESSION)
BLOCK ==> List $
COMMENT ==> List String
COMMON ==> Record(name:Symbol,contents:List Symbol)
CALL ==> String
FOR ==> Record(range:SegmentBinding PIN, span:PIN,  body:$)
LABEL ==> SingleInteger
LOOP ==> Record(switch:Switch(),body:$)
PRINTLIST ==> List O
OPREC ==> Union(nullBranch:"null", assignmentBranch:ASS,
                arrayAssignmentBranch:ARRAYASS,
                conditionalBranch:COND, returnBranch:RETURN,
                blockBranch:BLOCK, commentBranch:COMMENT, callBranch:CALL,
                forBranch:FOR, labelBranch:LABEL, loopBranch:LOOP,
                commonBranch:COMMON, printBranch:PRINTLIST)

public == SetCategory with
  coerce: $ -> O
    ++ coerce(f) returns an object of type OutputForm.
  forLoop: (SegmentBinding PIN,$) -> $
   ++ forLoop(i=1..10,c) creates a representation of a FORTRAN DO loop with
   ++ \spad{i} ranging over the values 1 to 10.
  forLoop: (SegmentBinding PIN,PIN,$) -> $
   ++ forLoop(i=1..10,n,c) creates a representation of a FORTRAN DO loop with
   ++ \spad{i} ranging over the values 1 to 10 by n.
  whileLoop: (Switch,$) -> $
   ++ whileLoop(s,c) creates a while loop in FORTRAN.
  repeatUntilLoop: (Switch,$) -> $
   ++ repeatUntilLoop(s,c) creates a repeat ... until loop in FORTRAN.
  goto: SingleInteger -> $
    ++ goto(l) creates a representation of a FORTRAN GOTO statement
  continue: SingleInteger -> $
    ++ continue(l) creates a representation of a FORTRAN CONTINUE labelled
```

```
    ++ with l
  comment: String -> $
    ++ comment(s) creates a representation of the String s as a single FORTRAN
    ++ comment.
  comment: List String -> $
    ++ comment(s) creates a representation of the Strings s as a multi-line
    ++ FORTRAN comment.
  call: String -> $
    ++ call(s) creates a representation of a FORTRAN CALL statement
  returns: () -> $
    ++ returns() creates a representation of a FORTRAN RETURN statement.
  returns: Expression MachineFloat -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
  returns: Expression MachineInteger -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
  returns: Expression MachineComplex -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
  returns: Expression Float -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
  returns: Expression Integer -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
  returns: Expression Complex Float -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
  cond: (Switch,$) -> $
    ++ cond(s,e) creates a representation of the FORTRAN expression
    ++ IF (s) THEN e.
  cond: (Switch,$,$) -> $
    ++ cond(s,e,f) creates a representation of the FORTRAN expression
    ++ IF (s) THEN e ELSE f.
  assign: (Symbol,String) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Expression MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Expression MachineFloat) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Expression MachineComplex) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Matrix MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
```

```
assign: (Symbol,Matrix MachineFloat) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Matrix MachineComplex) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Vector MachineInteger) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Vector MachineFloat) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Vector MachineComplex) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Matrix Expression MachineInteger) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Matrix Expression MachineFloat) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Matrix Expression MachineComplex) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Vector Expression MachineInteger) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Vector Expression MachineFloat) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Vector Expression MachineComplex) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,L PIN,Expression MachineInteger) -> $
  ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
  ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
  ++ indices).
assign: (Symbol,L PIN,Expression MachineFloat) -> $
  ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
  ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
  ++ indices).
assign: (Symbol,L PIN,Expression MachineComplex) -> $
  ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
  ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
  ++ indices).
assign: (Symbol,Expression Integer) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
  ++ x=y.
assign: (Symbol,Expression Float) -> $
  ++ assign(x,y) creates a representation of the FORTRAN expression
```

```
    ++ x=y.
  assign: (Symbol,Expression Complex Float) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Matrix Expression Integer) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Matrix Expression Float) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Matrix Expression Complex Float) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Vector Expression Integer) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Vector Expression Float) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,Vector Expression Complex Float) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
  assign: (Symbol,L PIN,Expression Integer) -> $
    ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
    ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
    ++ indices).
  assign: (Symbol,L PIN,Expression Float) -> $
    ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
    ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
    ++ indices).
  assign: (Symbol,L PIN,Expression Complex Float) -> $
    ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
    ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
    ++ indices).
  block: List($) -> $
    ++ block(l) creates a representation of the statements in l as a block.
  stop: () -> $
    ++ stop() creates a representation of a STOP statement.
  save: () -> $
    ++ save() creates a representation of a SAVE statement.
  printStatement: List O -> $
    ++ printStatement(l) creates a representation of a PRINT statement.
  common: (Symbol,List Symbol) -> $
    ++ common(name,contents) creates a representation a named common block.
  operation: $ -> OP
    ++ operation(f) returns the name of the operation represented by \spad{f}.
  code: $ -> OPREC
    ++ code(f) returns the internal representation of the object represented
    ++ by \spad{f}.
  printCode: $ -> Void
```

```
      ++ printCode(f) prints out \spad{f} in FORTRAN notation.
    getCode: $ -> SEX
      ++ getCode(f) returns a Lisp list of strings representing \spad{f}
      ++ in Fortran notation.  This is used by the FortranProgram domain.
    setLabelValue:SingleInteger -> SingleInteger
      ++ setLabelValue(i) resets the counter which produces labels to i

  private == add
    import Void
    import ASS
    import COND
    import RETURN
    import L PIN
    import O
    import SEX
    import FortranType
    import TheSymbolTable

    Rep := Record(op: OP, data: OPREC)

    -- We need to be able to generate unique labels
    labelValue:SingleInteger := 25000::SingleInteger
    setLabelValue(u:SingleInteger):SingleInteger == labelValue := u
    newLabel():SingleInteger ==
      labelValue := labelValue + 1$SingleInteger
      labelValue

    commaSep(l:List String):List(String) ==
      [(l.1),:[:[",",u] for u in rest(l)]]

    getReturn(rec:RETURN):SEX ==
      returnToken : SEX := convert("RETURN"::Symbol::O)$SEX
      elt(rec,empty?)$RETURN =>
        getStatement(returnToken,NIL$Lisp)$Lisp
      rt : EXPRESSION := elt(rec,value)$RETURN
      rv : O := elt(rt,expr)$EXPRESSION
      getStatement([returnToken,convert(rv)$SEX]$Lisp,
                   elt(rt,ints2Floats?)$EXPRESSION )$Lisp

    getStop():SEX ==
      fortran2Lines(LIST("STOP")$Lisp)$Lisp

    getSave():SEX ==
      fortran2Lines(LIST("SAVE")$Lisp)$Lisp

    getCommon(u:COMMON):SEX ==
      fortran2Lines(APPEND(LIST("COMMON"," /",string (u.name),"/ ")$Lisp,_
                    addCommas(u.contents)$Lisp)$Lisp)$Lisp

    getPrint(l:PRINTLIST):SEX ==
```

```
    ll : SEX := LIST("PRINT*")$Lisp
    for i in l repeat
      ll := APPEND(ll,CONS(",",expression2Fortran(i)$Lisp)$Lisp)$Lisp
    fortran2Lines(ll)$Lisp

  getBlock(rec:BLOCK):SEX ==
    indentFortLevel(convert(1@Integer)$SEX)$Lisp
    expr : SEX := LIST()$Lisp
    for u in rec repeat
      expr := APPEND(expr,getCode(u))$Lisp
    indentFortLevel(convert(-1@Integer)$SEX)$Lisp
    expr

  getBody(f:$):SEX ==
    operation(f) case Block => getCode f
    indentFortLevel(convert(1@Integer)$SEX)$Lisp
    expr := getCode f
    indentFortLevel(convert(-1@Integer)$SEX)$Lisp
    expr

  getElseIf(f:$):SEX ==
    rec := code f
    expr :=
     fortFormatElseIf(elt(rec.conditionalBranch,switch)$COND::O)$Lisp
    expr :=
     APPEND(expr,getBody elt(rec.conditionalBranch,thenClause)$COND)$Lisp
    elseBranch := elt(rec.conditionalBranch,elseClause)$COND
    not(operation(elseBranch) case Null) =>
      operation(elseBranch) case Conditional =>
        APPEND(expr,getElseIf elseBranch)$Lisp
      expr := APPEND(expr, getStatement(ELSE::O,NIL$Lisp)$Lisp)$Lisp
      expr := APPEND(expr, getBody elseBranch)$Lisp
    expr

  getContinue(label:SingleInteger):SEX ==
    lab : O := label::O
    if (width(lab) > 6) then error "Label too big"
    cnt : O := "CONTINUE"::O
    --sp  : O := hspace(6-width lab)
    sp  : O := hspace(_$fortIndent$Lisp -width lab)
    LIST(STRCONC(STRINGIMAGE(lab)$Lisp,sp,cnt)$Lisp)$Lisp

  getGoto(label:SingleInteger):SEX ==
   fortran2Lines(
    LIST(STRCONC("GOTO ",STRINGIMAGE(label::O)$Lisp)$Lisp)$Lisp)$Lisp

  getRepeat(repRec:LOOP):SEX ==
    sw : Switch := NOT elt(repRec,switch)$LOOP
    lab := newLabel()
    bod := elt(repRec,body)$LOOP
```

```
  APPEND(getContinue lab,getBody bod,
       fortFormatIfGoto(sw::O,lab)$Lisp)$Lisp

getWhile(whileRec:LOOP):SEX ==
  sw := NOT elt(whileRec,switch)$LOOP
  lab1 := newLabel()
  lab2 := newLabel()
  bod := elt(whileRec,body)$LOOP
  APPEND(fortFormatLabelledIfGoto(sw::O,lab1,lab2)$Lisp,
       getBody bod, getBody goto(lab1), getContinue lab2)$Lisp

getArrayAssign(rec:ARRAYASS):SEX ==
  getfortarrayexp((rec.var)::O,rec.rand,rec.ints2Floats?)$Lisp

getAssign(rec:ASS):SEX ==
  indices : L PIN := elt(rec,arrayIndex)$ASS
  if indices = []::(L PIN) then
    lhs := elt(rec,var)$ASS::O
  else
    lhs := cons(elt(rec,var)$ASS::PIN,indices)::O
    -- Must get the index brackets correct:
    lhs := (cdr car cdr convert(lhs)$SEX::SEX)::O -- Yuck!
  elt(elt(rec,rand)$ASS,ints2Floats?)$EXPRESSION =>
    assignment2Fortran1(lhs,elt(elt(rec,rand)$ASS,expr)$EXPRESSION)$Lisp
  integerAssignment2Fortran1(lhs,elt(elt(rec,rand)$ASS,expr)$EXPRESSION)$Lisp

getCond(rec:COND):SEX ==
  expr := APPEND(fortFormatIf(elt(rec,switch)$COND::O)$Lisp,
               getBody elt(rec,thenClause)$COND)$Lisp
  elseBranch := elt(rec,elseClause)$COND
  if not(operation(elseBranch) case Null) then
    operation(elseBranch) case Conditional =>
      expr := APPEND(expr,getElseIf elseBranch)$Lisp
    expr := APPEND(expr,getStatement(ELSE::O,NIL$Lisp)$Lisp,
                 getBody elseBranch)$Lisp
  APPEND(expr,getStatement(ENDIF::O,NIL$Lisp)$Lisp)$Lisp

getComment(rec:COMMENT):SEX ==
  convert([convert(concat("C     ",c)$String)@SEX for c in rec])@SEX

getCall(rec:CALL):SEX ==
  expr := concat("CALL ",rec)$String
  #expr > 1320 => error "Fortran CALL too large"
  fortran2Lines(convert([convert(expr)@SEX ])@SEX)$Lisp

getFor(rec:FOR):SEX ==
  rnge : SegmentBinding PIN := elt(rec,range)$FOR
  increment : PIN := elt(rec,span)$FOR
  lab : SingleInteger := newLabel()
  declare!(variable rnge,fortranInteger())
```

```
    expr := fortFormatDo(variable rnge, (lo segment rnge)::O,_
      (hi segment rnge)::O,increment::O,lab)$Lisp
    APPEND(expr, getBody elt(rec,body)$FOR, getContinue(lab))$Lisp

  getCode(f:$):SEX ==
    opp:OP := operation f
    rec:OPREC:= code f
    opp case Assignment => getAssign(rec.assignmentBranch)
    opp case ArrayAssignment => getArrayAssign(rec.arrayAssignmentBranch)
    opp case Conditional => getCond(rec.conditionalBranch)
    opp case Return => getReturn(rec.returnBranch)
    opp case Block => getBlock(rec.blockBranch)
    opp case Comment => getComment(rec.commentBranch)
    opp case Call => getCall(rec.callBranch)
    opp case For => getFor(rec.forBranch)
    opp case Continue => getContinue(rec.labelBranch)
    opp case Goto => getGoto(rec.labelBranch)
    opp case Repeat => getRepeat(rec.loopBranch)
    opp case While => getWhile(rec.loopBranch)
    opp case Save => getSave()
    opp case Stop => getStop()
    opp case Print => getPrint(rec.printBranch)
    opp case Common => getCommon(rec.commonBranch)
    error "Unsupported program construct."
    convert(0)@SEX

  printCode(f:$):Void ==
    displayLines1$Lisp getCode f
    void()$Void

  code (f:$):OPREC ==
    elt(f,data)$Rep

  operation (f:$):OP ==
    elt(f,op)$Rep

  common(name:Symbol,contents:List Symbol):$ ==
    [["common"]$OP,[[name,contents]$COMMON]$OPREC]$Rep

  stop():$ ==
    [["stop"]$OP,["null"]$OPREC]$Rep

  save():$ ==
    [["save"]$OP,["null"]$OPREC]$Rep

  printStatement(l:List O):$ ==
    [["print"]$OP,[l]$OPREC]$Rep

  comment(s:List String):$ ==
    [["comment"]$OP,[s]$OPREC]$Rep
```

```
comment(s:String):$ ==
  [["comment"]$OP,[list s]$OPREC]$Rep

forLoop(r:SegmentBinding PIN,body:$):$ ==
  [["for"]$OP,[[r,(incr segment r)::PIN,body]$FOR]$OPREC]$Rep

forLoop(r:SegmentBinding PIN,increment:PIN,body:$):$ ==
  [["for"]$OP,[[r,increment,body]$FOR]$OPREC]$Rep

goto(l:SingleInteger):$ ==
  [["goto"]$OP,[l]$OPREC]$Rep

continue(l:SingleInteger):$ ==
  [["continue"]$OP,[l]$OPREC]$Rep

whileLoop(sw:Switch,b:$):$ ==
  [["while"]$OP,[[sw,b]$LOOP]$OPREC]$Rep

repeatUntilLoop(sw:Switch,b:$):$ ==
  [["repeat"]$OP,[[sw,b]$LOOP]$OPREC]$Rep

returns():$ ==
  v := [false,0::O]$EXPRESSION
  [["return"]$OP,[[true,v]$RETURN]$OPREC]$Rep

returns(v:Expression MachineInteger):$ ==
  [["return"]$OP,[[false,[false,v::O]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression MachineFloat):$ ==
  [["return"]$OP,[[false,[false,v::O]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression MachineComplex):$ ==
  [["return"]$OP,[[false,[false,v::O]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression Integer):$ ==
  [["return"]$OP,[[false,[false,v::O]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression Float):$ ==
  [["return"]$OP,[[false,[false,v::O]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression Complex Float):$ ==
  [["return"]$OP,[[false,[false,v::O]$EXPRESSION]$RETURN]$OPREC]$Rep

block(l:List $):$ ==
  [["block"]$OP,[l]$OPREC]$Rep

cond(sw:Switch,thenC:$):$ ==
  [["conditional"]$OP,
   [[sw,thenC,[["null"]$OP,["null"]$OPREC]$Rep]$COND]$OPREC]$Rep
```

```
cond(sw:Switch,thenC:$,elseC:$):$ ==
  [["conditional"]$OP,[[sw,thenC,elseC]$COND]$OPREC]$Rep

coerce(f : $):O ==
  (f.op)::O

assign(v:Symbol,rhs:String):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[false,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression MachineInteger):$ ==
  [["assignment"]$OP,[[v,index,[false,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression MachineFloat):$ ==
```

```
  [["assignment"]$OP,[[v,index,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression MachineComplex):$ ==
  [["assignment"]$OP,[[v,index,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Expression MachineInteger):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[false,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Expression MachineFloat):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Expression MachineComplex):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression Integer):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression Float):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression Complex Float):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression Integer):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression Float):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression Complex Float):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::O,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression Integer):$ ==
  [["assignment"]$OP,[[v,index,[false,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression Float):$ ==
  [["assignment"]$OP,[[v,index,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression Complex Float):$ ==
  [["assignment"]$OP,[[v,index,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Expression Integer):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[false,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Expression Float):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Expression Complex Float):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[true,rhs::O]$EXPRESSION]$ASS]$OPREC]$Rep
```

```
    call(s:String):$ ==
      [["call"]$OP,[s]$OPREC]$Rep
```

———-

— FC.dotabb —

```
"FC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FC"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"FC" -> "COMPCAT"
"FC" -> "FS"
```

———-

7.17 domain FEXPR FortranExpression

— FortranExpression.input —

```
)set break resume
)sys rm -f FortranExpression.output
)spool FortranExpression.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranExpression
--R FortranExpression(basicSymbols: List Symbol,subscriptedSymbols: List Symbol,R: FortranMachineTypeCat
--R Abbreviation for FortranExpression is FEXPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FEXPR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?*? : (Integer,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     -? : % -> %
--R ?-? : (%,%) -> %                     ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean              ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean               ?>=? : (%,%) -> Boolean
--R D : (%,Symbol) -> %                  D : (%,List Symbol) -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       abs : % -> %
```

```
--R acos : % -> %                          asin : % -> %
--R atan : % -> %                          belong? : BasicOperator -> Boolean
--R box : List % -> %                      box : % -> %
--R coerce : % -> Expression R             coerce : Integer -> %
--R coerce : R -> %                        coerce : Kernel % -> %
--R coerce : % -> OutputForm               cos : % -> %
--R cosh : % -> %                          differentiate : (%,Symbol) -> %
--R distribute : (%,%) -> %                distribute : % -> %
--R elt : (BasicOperator,%,%) -> %         elt : (BasicOperator,%) -> %
--R eval : (%,List %,List %) -> %          eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %             eval : (%,List Equation %) -> %
--R eval : (%,Kernel %,%) -> %             exp : % -> %
--R freeOf? : (%,Symbol) -> Boolean        freeOf? : (%,%) -> Boolean
--R hash : % -> SingleInteger              height : % -> NonNegativeInteger
--R is? : (%,Symbol) -> Boolean            kernel : (BasicOperator,%) -> %
--R kernels : % -> List Kernel %           latex : % -> String
--R log : % -> %                           log10 : % -> %
--R map : ((% -> %),Kernel %) -> %         max : (%,%) -> %
--R min : (%,%) -> %                       one? : % -> Boolean
--R paren : List % -> %                    paren : % -> %
--R pi : () -> %                           recip : % -> Union(%,"failed")
--R retract : Symbol -> %                  retract : Expression R -> %
--R retract : % -> R                       retract : % -> Kernel %
--R sample : () -> %                       sin : % -> %
--R sinh : % -> %                          sqrt : % -> %
--R subst : (%,Equation %) -> %            tan : % -> %
--R tanh : % -> %                          tower : % -> List Kernel %
--R useNagFunctions : () -> Boolean        variables : % -> List Symbol
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,Symbol,NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R definingPolynomial : % -> % if $ has RING
--R differentiate : (%,List Symbol) -> %
--R differentiate : (%,Symbol,NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> %
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
```

```
--R eval : (%,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R is? : (%,BasicOperator) -> Boolean
--R kernel : (BasicOperator,List %) -> %
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R retract : Polynomial Float -> % if R has RETRACT FLOAT
--R retract : Fraction Polynomial Float -> % if R has RETRACT FLOAT
--R retract : Expression Float -> % if R has RETRACT FLOAT
--R retract : Polynomial Integer -> % if R has RETRACT INT
--R retract : Fraction Polynomial Integer -> % if R has RETRACT INT
--R retract : Expression Integer -> % if R has RETRACT INT
--R retractIfCan : Polynomial Float -> Union(%,"failed") if R has RETRACT FLOAT
--R retractIfCan : Fraction Polynomial Float -> Union(%,"failed") if R has RETRACT FLOAT
--R retractIfCan : Expression Float -> Union(%,"failed") if R has RETRACT FLOAT
--R retractIfCan : Polynomial Integer -> Union(%,"failed") if R has RETRACT INT
--R retractIfCan : Fraction Polynomial Integer -> Union(%,"failed") if R has RETRACT INT
--R retractIfCan : Expression Integer -> Union(%,"failed") if R has RETRACT INT
--R retractIfCan : Symbol -> Union(%,"failed")
--R retractIfCan : Expression R -> Union(%,"failed")
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R useNagFunctions : Boolean -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

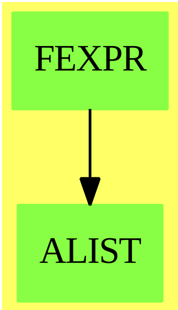
— **FortranExpression.help** —

```
====================================================================
FortranExpression examples
====================================================================

See Also:
o )show FortranExpression
```

———-

7.17.1 FortranExpression (FEXPR)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranCode" (FC) 7.16.1 on page 898
⇒ "FortranProgram" (FORTRAN) 7.18.1 on page 923
⇒ "ThreeDimensionalMatrix" (M3D) 21.7.1 on page 2661
⇒ "SimpleFortranProgram" (SFORT) 20.11.1 on page 2364
⇒ "Switch" (SWITCH) 20.36.1 on page 2588
⇒ "FortranTemplate" (FTEM) 7.20.1 on page 934

Exports:

0	1	abs	acos	asin
atan	belong?	box	characteristic	coerce
cos	cosh	D	definingPolynomial	differentiate
distribute	elt	eval	even?	exp
freeOf?	hash	height	is?	kernel
kernels	latex	log	log10	mainKernel
map	max	min	minPoly	odd?
one?	operator	operators	paren	pi
recip	retract	retractIfCan	sample	sin
sinh	sqrt	subst	subtractIfCan	tan
tanh	tower	useNagFunctions	variables	zero?
?*?	?**?	?+?	-?	?-?
?<?	?<=?	?=?	?>?	?>=?
?^?	?~=?			

— domain FEXPR FortranExpression —

```
)abbrev domain FEXPR FortranExpression
++ Author: Mike Dewar
++ Date Created:  December 1993
++ Date Last Updated: 19 May 1994
++                     7 July 1994 added %power to f77Functions
++                    12 July 1994 added RetractableTo(R)
++ Basic Operations:
```

```
++ Related Domains:
++ Also See: FortranMachineTypeCategory, MachineInteger, MachineFloat,
++  MachineComplex
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain of expressions involving functions which can be
++ translated into standard Fortran-77, with some extra extensions from
++ the NAG Fortran Library.

FortranExpression(basicSymbols,subscriptedSymbols,R):
                                Exports==Implementation where
  basicSymbols : List Symbol
  subscriptedSymbols : List Symbol
  R : FortranMachineTypeCategory

  EXPR ==> Expression
  EXF2 ==> ExpressionFunctions2
  S    ==> Symbol
  L    ==> List
  BO   ==> BasicOperator
  FRAC ==> Fraction
  POLY ==> Polynomial

  Exports ==> Join(ExpressionSpace,Algebra(R),RetractableTo(R),
                 PartialDifferentialRing(Symbol)) with
    retract : EXPR R -> $
      ++ retract(e) takes e and transforms it into a
      ++ FortranExpression checking that it contains no non-Fortran
      ++ functions, and that it only contains the given basic symbols
      ++ and subscripted symbols which correspond to scalar and array
      ++ parameters respectively.
    retractIfCan : EXPR R -> Union($,"failed")
      ++ retractIfCan(e) takes e and tries to transform it into a
      ++ FortranExpression checking that it contains no non-Fortran
      ++ functions, and that it only contains the given basic symbols
      ++ and subscripted symbols which correspond to scalar and array
      ++ parameters respectively.
    retract : S -> $
      ++ retract(e) takes e and transforms it into a FortranExpression
      ++ checking that it is one of the given basic symbols
      ++ or subscripted symbols which correspond to scalar and array
      ++ parameters respectively.
    retractIfCan : S -> Union($,"failed")
      ++ retractIfCan(e) takes e and tries to transform it into a
      ++ FortranExpression checking that it is one of the given basic symbols
      ++ or subscripted symbols which correspond to scalar and array
      ++ parameters respectively.
```

```
coerce : $ -> EXPR R
  ++ coerce(x) is not documented
if (R has RetractableTo(Integer)) then
  retract : EXPR Integer -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : EXPR Integer -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retract : FRAC POLY  Integer -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : FRAC POLY  Integer -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retract : POLY  Integer -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : POLY  Integer -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
if (R has RetractableTo(Float)) then
  retract : EXPR Float -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : EXPR Float -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
```

```
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retract : FRAC POLY  Float -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : FRAC POLY  Float -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retract : POLY  Float -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : POLY  Float -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
abs    : $ -> $
  ++ abs(x) represents the Fortran intrinsic function ABS
sqrt   : $ -> $
  ++ sqrt(x) represents the Fortran intrinsic function SQRT
exp    : $ -> $
  ++ exp(x) represents the Fortran intrinsic function EXP
log    : $ -> $
  ++ log(x) represents the Fortran intrinsic function LOG
log10  : $ -> $
  ++ log10(x) represents the Fortran intrinsic function LOG10
sin    : $ -> $
  ++ sin(x) represents the Fortran intrinsic function SIN
cos    : $ -> $
  ++ cos(x) represents the Fortran intrinsic function COS
tan    : $ -> $
  ++ tan(x) represents the Fortran intrinsic function TAN
asin   : $ -> $
  ++ asin(x) represents the Fortran intrinsic function ASIN
acos   : $ -> $
  ++ acos(x) represents the Fortran intrinsic function ACOS
atan   : $ -> $
  ++ atan(x) represents the Fortran intrinsic function ATAN
sinh   : $ -> $
  ++ sinh(x) represents the Fortran intrinsic function SINH
```

```
    cosh   : $ -> $
      ++ cosh(x) represents the Fortran intrinsic function COSH
    tanh   : $ -> $
      ++ tanh(x) represents the Fortran intrinsic function TANH
    pi     : () -> $
      ++ pi(x) represents the NAG Library function X01AAF which returns
      ++  an approximation to the value of pi
    variables : $ -> L S
      ++ variables(e) return a list of all the variables in \spad{e}.
    useNagFunctions : () -> Boolean
      ++ useNagFunctions() indicates whether NAG functions are being used
      ++  for mathematical and machine constants.
    useNagFunctions : Boolean -> Boolean
      ++ useNagFunctions(v) sets the flag which controls whether NAG functions
      ++  are being used for mathematical and machine constants.  The previous
      ++  value is returned.

  Implementation ==> EXPR R add

    -- The standard FORTRAN-77 intrinsic functions, plus nthRoot which
    -- can be translated into an arithmetic expression:
    f77Functions : L S := [abs,sqrt,exp,log,log10,sin,cos,tan,asin,acos,
                           atan,sinh,cosh,tanh,nthRoot,%power]
    nagFunctions : L S := [pi, X01AAF]
    useNagFunctionsFlag : Boolean := true

    -- Local functions to check for "unassigned" symbols etc.

    mkEqn(s1:Symbol,s2:Symbol):Equation EXPR(R) ==
      equation(s2::EXPR(R),script(s1,scripts(s2))::EXPR(R))

    fixUpSymbols(u:EXPR R):Union(EXPR R,"failed") ==
      -- If its a univariate expression then just fix it up:
      syms   : L S := variables(u)
--      one?(#basicSymbols) and zero?(#subscriptedSymbols) =>
      (#basicSymbols = 1) and zero?(#subscriptedSymbols) =>
--        not one?(#syms) => "failed"
        not (#syms = 1) => "failed"
        subst(u,equation(first(syms)::EXPR(R),first(basicSymbols)::EXPR(R)))
      -- We have one variable but it is subscripted:
--      zero?(#basicSymbols) and one?(#subscriptedSymbols) =>
      zero?(#basicSymbols) and (#subscriptedSymbols = 1) =>
        -- Make sure we don't have both X and X_i
        for s in syms repeat
          not scripted?(s) => return "failed"
--        not one?(#(syms:=removeDuplicates! [name(s) for s in syms]))=> "failed"
        not ((#(syms:=removeDuplicates! [name(s) for s in syms])) = 1)=> "failed"
        sym : Symbol := first subscriptedSymbols
        subst(u,[mkEqn(sym,i) for i in variables(u)])
      "failed"
```

```
extraSymbols?(u:EXPR R):Boolean ==
  syms   : L S := [name(v) for v in variables(u)]
  extras : L S := setDifference(syms,
                              setUnion(basicSymbols,subscriptedSymbols))
  not empty? extras

checkSymbols(u:EXPR R):EXPR(R) ==
  syms   : L S := [name(v) for v in variables(u)]
  extras : L S := setDifference(syms,
                              setUnion(basicSymbols,subscriptedSymbols))
  not empty? extras =>
    m := fixUpSymbols(u)
    m case EXPR(R) => m::EXPR(R)
    error("Extra symbols detected:",[string(v) for v in extras]$L(String))
  u

notSymbol?(v:BO):Boolean ==
  s : S := name v
  member?(s,basicSymbols) or
    scripted?(s) and member?(name s,subscriptedSymbols) => false
  true

extraOperators?(u:EXPR R):Boolean ==
  ops    : L S := [name v for v in operators(u) | notSymbol?(v)]
  if useNagFunctionsFlag then
    fortranFunctions : L S := append(f77Functions,nagFunctions)
  else
    fortranFunctions : L S := f77Functions
  extras : L S := setDifference(ops,fortranFunctions)
  not empty? extras

checkOperators(u:EXPR R):Void ==
  ops    : L S := [name v for v in operators(u) | notSymbol?(v)]
  if useNagFunctionsFlag then
    fortranFunctions : L S := append(f77Functions,nagFunctions)
  else
    fortranFunctions : L S := f77Functions
  extras : L S := setDifference(ops,fortranFunctions)
  not empty? extras =>
    error("Non FORTRAN-77 functions detected:",[string(v) for v in extras])
  void()

checkForNagOperators(u:EXPR R):$ ==
  useNagFunctionsFlag =>
    import Pi
    import PiCoercions(R)
    piOp : BasicOperator := operator X01AAF
    piSub : Equation EXPR R :=
      equation(pi()$Pi::EXPR(R),kernel(piOp,0::EXPR(R))$EXPR(R))
```

```
    subst(u,piSub) pretend $
  u pretend $

-- Conditional retractions:

if R has RetractableTo(Integer) then

  retractIfCan(u:POLY Integer):Union($,"failed") ==
    retractIfCan((u::EXPR Integer)$EXPR(Integer))@Union($,"failed")

  retract(u:POLY Integer):$ ==
    retract((u::EXPR Integer)$EXPR(Integer))@$

  retractIfCan(u:FRAC POLY Integer):Union($,"failed") ==
    retractIfCan((u::EXPR Integer)$EXPR(Integer))@Union($,"failed")

  retract(u:FRAC POLY  Integer):$ ==
    retract((u::EXPR Integer)$EXPR(Integer))@$

  int2R(u:Integer):R == u::R

  retractIfCan(u:EXPR Integer):Union($,"failed") ==
    retractIfCan(map(int2R,u)$EXF2(Integer,R))@Union($,"failed")

  retract(u:EXPR Integer):$ ==
    retract(map(int2R,u)$EXF2(Integer,R))@$

if R has RetractableTo(Float) then

  retractIfCan(u:POLY Float):Union($,"failed") ==
    retractIfCan((u::EXPR Float)$EXPR(Float))@Union($,"failed")

  retract(u:POLY Float):$ ==
    retract((u::EXPR Float)$EXPR(Float))@$

  retractIfCan(u:FRAC POLY Float):Union($,"failed") ==
    retractIfCan((u::EXPR Float)$EXPR(Float))@Union($,"failed")

  retract(u:FRAC POLY  Float):$ ==
    retract((u::EXPR Float)$EXPR(Float))@$

  float2R(u:Float):R == (u::R)

  retractIfCan(u:EXPR Float):Union($,"failed") ==
    retractIfCan(map(float2R,u)$EXF2(Float,R))@Union($,"failed")

  retract(u:EXPR Float):$ ==
    retract(map(float2R,u)$EXF2(Float,R))@$

-- Exported Functions
```

```
    useNagFunctions():Boolean == useNagFunctionsFlag
    useNagFunctions(v:Boolean):Boolean ==
      old := useNagFunctionsFlag
      useNagFunctionsFlag := v
      old

    log10(x:$):$ ==
      kernel(operator log10,x)

    pi():$ == kernel(operator X01AAF,0)

    coerce(u:$):EXPR R == u pretend EXPR(R)

    retractIfCan(u:EXPR R):Union($,"failed") ==
      if (extraSymbols? u) then
        m := fixUpSymbols(u)
        m case "failed" => return "failed"
        u := m::EXPR(R)
      extraOperators? u => "failed"
      checkForNagOperators(u)

    retract(u:EXPR R):$ ==
      u:=checkSymbols(u)
      checkOperators(u)
      checkForNagOperators(u)

    retractIfCan(u:Symbol):Union($,"failed") ==
      not (member?(u,basicSymbols) or
           scripted?(u) and member?(name u,subscriptedSymbols)) => "failed"
      (((u::EXPR(R))$(EXPR R))pretend Rep)::$

    retract(u:Symbol):$ ==
      res : Union($,"failed") := retractIfCan(u)
      res case "failed" => error("Illegal Symbol Detected:",u::String)
      res::$
```

———-

— **FEXPR.dotabb** —

```
"FEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FEXPR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FEXPR" -> "ALIST"
```

———-

7.18 domain FORTRAN FortranProgram

— **FortranProgram.input** —

```
)set break resume
)sys rm -f FortranProgram.output
)spool FortranProgram.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranProgram
--R FortranProgram(name: Symbol,returnType: Union(fst: FortranScalarType,void: void),argument
--R Abbreviation for FortranProgram is FORTRAN
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for FORTRAN
--R
--R------------------------------- Operations --------------------------------
--R coerce : Expression Float -> %        coerce : Expression Integer -> %
--R coerce : List FortranCode -> %        coerce : FortranCode -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Equation Expression Complex Float -> %
--R coerce : Equation Expression Float -> %
--R coerce : Equation Expression Integer -> %
--R coerce : Expression Complex Float -> %
--R coerce : Equation Expression MachineComplex -> %
--R coerce : Equation Expression MachineFloat -> %
--R coerce : Equation Expression MachineInteger -> %
--R coerce : Expression MachineComplex -> %
--R coerce : Expression MachineFloat -> %
--R coerce : Expression MachineInteger -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

———

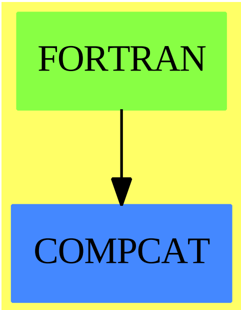
— **FortranProgram.help** —

```
====================================================================
FortranProgram examples
====================================================================

See Also:
```

```
o )show FortranProgram
```

7.18.1 FortranProgram (FORTRAN)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranCode" (FC) 7.16.1 on page 898
⇒ "ThreeDimensionalMatrix" (M3D) 21.7.1 on page 2661
⇒ "SimpleFortranProgram" (SFORT) 20.11.1 on page 2364
⇒ "Switch" (SWITCH) 20.36.1 on page 2588
⇒ "FortranTemplate" (FTEM) 7.20.1 on page 934
⇒ "FortranExpression" (FEXPR) 7.17.1 on page 914

Exports:
coerce outputAsFortran

— **domain FORTRAN FortranProgram** —

```
)abbrev domain FORTRAN FortranProgra\\
++ Author: Mike Dewar
++ Date Created: October 1992
++ Date Last Updated: 13 January 1994
++                    23 January 1995 Added support for intrinsic functions
++ Basic Operations:
++ Related Constructors: FortranType, FortranCode, Switch
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \axiomType{FortranProgram} allows the user to build and manipulate simple
++ models of FORTRAN subprograms.  These can then be transformed into
++ actual FORTRAN notation.
```

```
FortranProgram(name,returnType,arguments,symbols): Exports == Implement where
  name       : Symbol
  returnType : Union(fst:FortranScalarType,void:"void")
  arguments  : List Symbol
  symbols    : SymbolTable

  FC     ==> FortranCode
  EXPR   ==> Expression
  INT    ==> Integer
  CMPX   ==> Complex
  MINT   ==> MachineInteger
  MFLOAT ==> MachineFloat
  MCMPLX ==> MachineComplex
  REP    ==> Record(localSymbols : SymbolTable, code : List FortranCode)

  Exports ==> FortranProgramCategory with
    coerce : FortranCode -> $
        ++ coerce(fc) is not documented
    coerce : List FortranCode -> $
        ++ coerce(lfc) is not documented
    coerce : REP -> $
        ++ coerce(r) is not documented
    coerce : EXPR MINT -> $
        ++ coerce(e) is not documented
    coerce : EXPR MFLOAT -> $
        ++ coerce(e) is not documented
    coerce : EXPR MCMPLX -> $
        ++ coerce(e) is not documented
    coerce : Equation EXPR MINT -> $
        ++ coerce(eq) is not documented
    coerce : Equation EXPR MFLOAT -> $
        ++ coerce(eq) is not documented
    coerce : Equation EXPR MCMPLX -> $
        ++ coerce(eq) is not documented
    coerce : EXPR INT -> $
        ++ coerce(e) is not documented
    coerce : EXPR Float -> $
        ++ coerce(e) is not documented
    coerce : EXPR CMPX Float -> $
        ++ coerce(e) is not documented
    coerce : Equation EXPR INT -> $
        ++ coerce(eq) is not documented
    coerce : Equation EXPR Float -> $
        ++ coerce(eq) is not documented
    coerce : Equation EXPR CMPX Float -> $
        ++ coerce(eq) is not documented

  Implement ==> add
```

```
Rep := REP

import SExpression
import TheSymbolTable
import FortranCode

makeRep(b:List FortranCode):$ ==
  construct(empty()$SymbolTable,b)$REP

codeFrom(u:$):List FortranCode ==
  elt(u::Rep,code)$REP

outputAsFortran(p:$):Void ==
  setLabelValue(25000::SingleInteger)$FC
  -- Do this first to catch any extra type declarations:
  tempName := "FPTEMP"::Symbol
  newSubProgram(tempName)
  initialiseIntrinsicList()$Lisp
  body : List SExpression := [getCode(l)$FortranCode for l in codeFrom(p)]
  intrinsics : SExpression := getIntrinsicList()$Lisp
  endSubProgram()
  fortFormatHead(returnType::OutputForm, name::OutputForm, _
                 arguments::OutputForm)$Lisp
  printTypes(symbols)$SymbolTable
  printTypes((p::Rep).localSymbols)$SymbolTable
  printTypes(tempName)$TheSymbolTable
  fortFormatIntrinsics(intrinsics)$Lisp
  clearTheSymbolTable(tempName)
  for expr in body repeat displayLines1(expr)$Lisp
  dispStatement(END::OutputForm)$Lisp
  void()$Void

mkString(l:List Symbol):String ==
  unparse(convert(l::OutputForm)@InputForm)$InputForm

checkVariables(user:List Symbol,target:List Symbol):Void ==
  -- We don't worry about whether the user has subscripted the
  -- variables or not.
  setDifference(map(name$Symbol,user),target) ^= empty()$List(Symbol) =>
    s1 : String := mkString(user)
    s2 : String := mkString(target)
    error ["Incompatible variable lists:", s1, s2]
  void()$Void

coerce(u:EXPR MINT) : $ ==
  checkVariables(variables(u)$EXPR(MINT),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR MINT) : $ ==
```

```
    retractIfCan(lhs u)@Union(Kernel(EXPR MINT),"failed") case "failed" =>
      error "left hand side is not a kernel"
    vList : List Symbol := variables lhs u
    #vList ^= #arguments =>
      error "Incorrect number of arguments"
    veList : List EXPR MINT := [w::EXPR(MINT) for w in vList]
    aeList : List EXPR MINT := [w::EXPR(MINT) for w in arguments]
    eList : List Equation EXPR MINT :=
      [equation(w,v) for w in veList for v in aeList]
    (subst(rhs u,eList))::$

  coerce(u:EXPR MFLOAT) : $ ==
    checkVariables(variables(u)$EXPR(MFLOAT),arguments)
    l : List(FC) := [assign(name,u)$FC,returns()$FC]
    makeRep l

  coerce(u:Equation EXPR MFLOAT) : $ ==
    retractIfCan(lhs u)@Union(Kernel(EXPR MFLOAT),"failed") case "failed" =>
      error "left hand side is not a kernel"
    vList : List Symbol := variables lhs u
    #vList ^= #arguments =>
      error "Incorrect number of arguments"
    veList : List EXPR MFLOAT := [w::EXPR(MFLOAT) for w in vList]
    aeList : List EXPR MFLOAT := [w::EXPR(MFLOAT) for w in arguments]
    eList : List Equation EXPR MFLOAT :=
      [equation(w,v) for w in veList for v in aeList]
    (subst(rhs u,eList))::$

  coerce(u:EXPR MCMPLX) : $ ==
    checkVariables(variables(u)$EXPR(MCMPLX),arguments)
    l : List(FC) := [assign(name,u)$FC,returns()$FC]
    makeRep l

  coerce(u:Equation EXPR MCMPLX) : $ ==
    retractIfCan(lhs u)@Union(Kernel EXPR MCMPLX,"failed") case "failed"=>
      error "left hand side is not a kernel"
    vList : List Symbol := variables lhs u
    #vList ^= #arguments =>
      error "Incorrect number of arguments"
    veList : List EXPR MCMPLX := [w::EXPR(MCMPLX) for w in vList]
    aeList : List EXPR MCMPLX := [w::EXPR(MCMPLX) for w in arguments]
    eList : List Equation EXPR MCMPLX :=
      [equation(w,v) for w in veList for v in aeList]
    (subst(rhs u,eList))::$


  coerce(u:REP):$ ==
    u@Rep

  coerce(u:$):OutputForm ==
```

```
      coerce(name)$Symbol

    coerce(c:List FortranCode):$ ==
      makeRep c

    coerce(c:FortranCode):$ ==
      makeRep [c]

    coerce(u:EXPR INT) : $ ==
      checkVariables(variables(u)$EXPR(INT),arguments)
      l : List(FC) := [assign(name,u)$FC,returns()$FC]
      makeRep l

    coerce(u:Equation EXPR INT) : $ ==
      retractIfCan(lhs u)@Union(Kernel(EXPR INT),"failed") case "failed" =>
        error "left hand side is not a kernel"
      vList : List Symbol := variables lhs u
      #vList ^= #arguments =>
        error "Incorrect number of arguments"
      veList : List EXPR INT := [w::EXPR(INT) for w in vList]
      aeList : List EXPR INT := [w::EXPR(INT) for w in arguments]
      eList : List Equation EXPR INT :=
        [equation(w,v) for w in veList for v in aeList]
      (subst(rhs u,eList))::$

    coerce(u:EXPR Float) : $ ==
      checkVariables(variables(u)$EXPR(Float),arguments)
      l : List(FC) := [assign(name,u)$FC,returns()$FC]
      makeRep l

    coerce(u:Equation EXPR Float) : $ ==
      retractIfCan(lhs u)@Union(Kernel(EXPR Float),"failed") case "failed" =>
        error "left hand side is not a kernel"
      vList : List Symbol := variables lhs u
      #vList ^= #arguments =>
        error "Incorrect number of arguments"
      veList : List EXPR Float := [w::EXPR(Float) for w in vList]
      aeList : List EXPR Float := [w::EXPR(Float) for w in arguments]
      eList : List Equation EXPR Float :=
        [equation(w,v) for w in veList for v in aeList]
      (subst(rhs u,eList))::$

    coerce(u:EXPR Complex Float) : $ ==
      checkVariables(variables(u)$EXPR(Complex Float),arguments)
      l : List(FC) := [assign(name,u)$FC,returns()$FC]
      makeRep l

    coerce(u:Equation EXPR CMPX Float) : $ ==
      retractIfCan(lhs u)@Union(Kernel EXPR CMPX Float,"failed") case "failed"=>
        error "left hand side is not a kernel"
```

```
    vList : List Symbol := variables lhs u
    #vList ^= #arguments =>
      error "Incorrect number of arguments"
    veList : List EXPR CMPX Float := [w::EXPR(CMPX Float) for w in vList]
    aeList : List EXPR CMPX Float := [w::EXPR(CMPX Float) for w in arguments]
    eList : List Equation EXPR CMPX Float :=
      [equation(w,v) for w in veList for v in aeList]
    (subst(rhs u,eList))::$
```

— FORTRAN.dotabb —

```
"FORTRAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FORTRAN"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"FORTRAN" -> "COMPCAT"
```

7.19 domain FST FortranScalarType

— FortranScalarType.input —

```
)set break resume
)sys rm -f FortranScalarType.output
)spool FortranScalarType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranScalarType
--R FortranScalarType  is a domain constructor
--R Abbreviation for FortranScalarType is FST
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FST
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                character? : % -> Boolean
--R coerce : % -> SExpression             coerce : % -> Symbol
--R coerce : Symbol -> %                  coerce : String -> %
--R coerce : % -> OutputForm              complex? : % -> Boolean
--R double? : % -> Boolean                doubleComplex? : % -> Boolean
--R integer? : % -> Boolean               logical? : % -> Boolean
```

```
--R real? : % -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

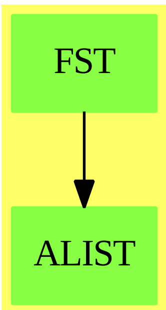
— FortranScalarType.help —

```
====================================================================
FortranScalarType examples
====================================================================

See Also:
o )show FortranScalarType
```

———-

7.19.1 FortranScalarType (FST)



See

⇒ "FortranType" (FT) 7.21.1 on page 938
⇒ "SymbolTable" (SYMTAB) 20.38.1 on page 2606
⇒ "TheSymbolTable" (SYMS) 21.6.1 on page 2655

Exports:
character? coerce complex? double? doubleComplex? integer? logical? real? ?=?

— domain FST FortranScalarType —

```
)abbrev domain FST FortranScalarType
++ Author: Mike Dewar
```

```
++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Creates and manipulates objects which correspond to the
++ basic FORTRAN data types: REAL, INTEGER, COMPLEX, LOGICAL and CHARACTER

FortranScalarType() : exports == implementation where

  exports == CoercibleTo OutputForm with
    coerce : String -> $
      ++ coerce(s) transforms the string s into an element of
      ++ FortranScalarType provided s is one of "real", "double precision",
      ++ "complex", "logical", "integer", "character", "REAL",
      ++ "COMPLEX", "LOGICAL", "INTEGER", "CHARACTER",
      ++ "DOUBLE PRECISION"
    coerce : Symbol -> $
      ++ coerce(s) transforms the symbol s into an element of
      ++ FortranScalarType provided s is one of real, complex,double precision,
      ++ logical, integer, character, REAL, COMPLEX, LOGICAL,
      ++ INTEGER, CHARACTER, DOUBLE PRECISION
    coerce : $ -> Symbol
      ++ coerce(x) returns the symbol associated with x
    coerce : $ -> SExpression
      ++ coerce(x) returns the s-expression associated with x
    real?  : $ -> Boolean
      ++ real?(t) tests whether t is equivalent to the FORTRAN type REAL.
    double? : $ -> Boolean
      ++ double?(t) tests whether t is equivalent to the FORTRAN type
      ++ DOUBLE PRECISION
    integer?  : $ -> Boolean
      ++ integer?(t) tests whether t is equivalent to the FORTRAN type INTEGER.
    complex?  : $ -> Boolean
      ++ complex?(t) tests whether t is equivalent to the FORTRAN type COMPLEX.
    doubleComplex?  : $ -> Boolean
      ++ doubleComplex?(t) tests whether t is equivalent to the (non-standard)
      ++ FORTRAN type DOUBLE COMPLEX.
    character?  : $ -> Boolean
      ++ character?(t) tests whether t is equivalent to the FORTRAN type
      ++ CHARACTER.
    logical?  : $ -> Boolean
      ++ logical?(t) tests whether t is equivalent to the FORTRAN type LOGICAL.
    "=" : ($,$) -> Boolean
      ++ x=y tests for equality
```

```
implementation == add

  U == Union(RealThing:"real",
             IntegerThing:"integer",
             ComplexThing:"complex",
             CharacterThing:"character",
             LogicalThing:"logical",
             DoublePrecisionThing:"double precision",
             DoubleComplexThing:"double complex")
  Rep := U

  doubleSymbol : Symbol := "double precision"::Symbol
  upperDoubleSymbol : Symbol := "DOUBLE PRECISION"::Symbol
  doubleComplexSymbol : Symbol := "double complex"::Symbol
  upperDoubleComplexSymbol : Symbol := "DOUBLE COMPLEX"::Symbol

  u = v ==
    u case RealThing and v case RealThing => true
    u case IntegerThing and v case IntegerThing => true
    u case ComplexThing and v case ComplexThing => true
    u case LogicalThing and v case LogicalThing => true
    u case CharacterThing and v case CharacterThing => true
    u case DoublePrecisionThing and v case DoublePrecisionThing => true
    u case DoubleComplexThing and v case DoubleComplexThing => true
    false

  coerce(t:$):OutputForm ==
    t case RealThing => coerce(REAL)$Symbol
    t case IntegerThing => coerce(INTEGER)$Symbol
    t case ComplexThing => coerce(COMPLEX)$Symbol
    t case CharacterThing => coerce(CHARACTER)$Symbol
    t case DoublePrecisionThing => coerce(upperDoubleSymbol)$Symbol
    t case DoubleComplexThing => coerce(upperDoubleComplexSymbol)$Symbol
    coerce(LOGICAL)$Symbol

  coerce(t:$):SExpression ==
    t case RealThing => convert(real::Symbol)@SExpression
    t case IntegerThing => convert(integer::Symbol)@SExpression
    t case ComplexThing => convert(complex::Symbol)@SExpression
    t case CharacterThing => convert(character::Symbol)@SExpression
    t case DoublePrecisionThing => convert(doubleSymbol)@SExpression
    t case DoubleComplexThing => convert(doubleComplexSymbol)@SExpression
    convert(logical::Symbol)@SExpression

  coerce(t:$):Symbol ==
    t case RealThing => real::Symbol
    t case IntegerThing => integer::Symbol
    t case ComplexThing => complex::Symbol
    t case CharacterThing => character::Symbol
```

```
    t case DoublePrecisionThing => doubleSymbol
    t case DoublePrecisionThing => doubleComplexSymbol
    logical::Symbol

  coerce(s:Symbol):$ ==
    s = real => ["real"]$Rep
    s = REAL => ["real"]$Rep
    s = integer => ["integer"]$Rep
    s = INTEGER => ["integer"]$Rep
    s = complex => ["complex"]$Rep
    s = COMPLEX => ["complex"]$Rep
    s = character => ["character"]$Rep
    s = CHARACTER => ["character"]$Rep
    s = logical => ["logical"]$Rep
    s = LOGICAL => ["logical"]$Rep
    s = doubleSymbol => ["double precision"]$Rep
    s = upperDoubleSymbol => ["double precision"]$Rep
    s = doubleComplexSymbol => ["double complex"]$Rep
    s = upperDoubleCOmplexSymbol => ["double complex"]$Rep

  coerce(s:String):$ ==
    s = "real" => ["real"]$Rep
    s = "integer" => ["integer"]$Rep
    s = "complex" => ["complex"]$Rep
    s = "character" => ["character"]$Rep
    s = "logical" => ["logical"]$Rep
    s = "double precision" => ["double precision"]$Rep
    s = "double complex" => ["double complex"]$Rep
    s = "REAL" => ["real"]$Rep
    s = "INTEGER" => ["integer"]$Rep
    s = "COMPLEX" => ["complex"]$Rep
    s = "CHARACTER" => ["character"]$Rep
    s = "LOGICAL" => ["logical"]$Rep
    s = "DOUBLE PRECISION" => ["double precision"]$Rep
    s = "DOUBLE COMPLEX" => ["double complex"]$Rep
    error concat([s," is invalid as a Fortran Type"])$String

  real?(t:$):Boolean == t case RealThing

  double?(t:$):Boolean == t case DoublePrecisionThing

  logical?(t:$):Boolean == t case LogicalThing

  integer?(t:$):Boolean == t case IntegerThing

  character?(t:$):Boolean == t case CharacterThing

  complex?(t:$):Boolean == t case ComplexThing

  doubleComplex?(t:$):Boolean == t case DoubleComplexThing
```

———-

— FST.dotabb —

```
"FST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FST"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FST" -> "ALIST"
```

———-

7.20 domain FTEM FortranTemplate

— FortranTemplate.input —

```
)set break resume
)sys rm -f FortranTemplate.output
)spool FortranTemplate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranTemplate
--R FortranTemplate  is a domain constructor
--R Abbreviation for FortranTemplate is FTEM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FTEM
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean               close! : % -> %
--R coerce : % -> OutputForm             flush : % -> Void
--R fortranCarriageReturn : () -> Void   fortranLiteral : String -> Void
--R hash : % -> SingleInteger            iomode : % -> String
--R latex : % -> String                  name : % -> FileName
--R open : (FileName,String) -> %        open : FileName -> %
--R read! : % -> String                  reopen! : (%,String) -> %
--R write! : (%,String) -> String        ?~=? : (%,%) -> Boolean
--R fortranLiteralLine : String -> Void
--R processTemplate : FileName -> FileName
--R processTemplate : (FileName,FileName) -> FileName
--R
--E 1
```

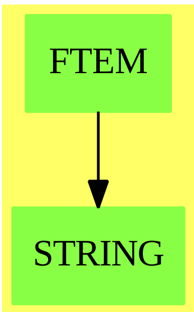
```
)spool
)lisp (bye)
```

— FortranTemplate.help —

```
====================================================================
FortranTemplate examples
====================================================================

See Also:
o )show FortranTemplate
```

7.20.1 FortranTemplate (FTEM)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranCode" (FC) 7.16.1 on page 898
⇒ "FortranProgram" (FORTRAN) 7.18.1 on page 923
⇒ "ThreeDimensionalMatrix" (M3D) 21.7.1 on page 2661
⇒ "SimpleFortranProgram" (SFORT) 20.11.1 on page 2364
⇒ "Switch" (SWITCH) 20.36.1 on page 2588
⇒ "FortranExpression" (FEXPR) 7.17.1 on page 914

Exports:

close!	coerce	fortranCarriageReturn	fortranLiteral	fortranLiteralLine
hash	iomode	latex	name	open
processTemplate	read!	reopen!	write!	?=?
?~=?				

— domain FTEM FortranTemplate —

```
)abbrev domain FTEM FortranTemplate
++ Author: Mike Dewar
++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Code to manipulate Fortran templates

FortranTemplate() : specification == implementation where

  specification == FileCategory(FileName, String) with

    processTemplate : (FileName, FileName) -> FileName
      ++ processTemplate(tp,fn) processes the template tp, writing the
      ++ result out to fn.
    processTemplate : (FileName) -> FileName
      ++ processTemplate(tp) processes the template tp, writing the
      ++ result to the current FORTRAN output stream.
    fortranLiteralLine : String -> Void
      ++ fortranLiteralLine(s) writes s to the current Fortran output stream,
      ++ followed by a carriage return
    fortranLiteral : String -> Void
      ++ fortranLiteral(s) writes s to the current Fortran output stream
    fortranCarriageReturn : () -> Void
      ++ fortranCarriageReturn() produces a carriage return on the current
      ++ Fortran output stream

  implementation == TextFile add

    import TemplateUtilities
    import FortranOutputStackPackage

    Rep := TextFile

    fortranLiteralLine(s:String):Void ==
      PRINTEXP(s,_$fortranOutputStream$Lisp)$Lisp
      TERPRI(_$fortranOutputStream$Lisp)$Lisp

    fortranLiteral(s:String):Void ==
      PRINTEXP(s,_$fortranOutputStream$Lisp)$Lisp

    fortranCarriageReturn():Void ==
      TERPRI(_$fortranOutputStream$Lisp)$Lisp
```

```
writePassiveLine!(line:String):Void ==
-- We might want to be a bit clever here and look for new SubPrograms etc.
  fortranLiteralLine line

processTemplate(tp:FileName, fn:FileName):FileName ==
  pushFortranOutputStack(fn)
  processTemplate(tp)
  popFortranOutputStack()
  fn

getLine(fp:TextFile):String ==
  line : String := stripCommentsAndBlanks readLine!(fp)
  while not empty?(line) and elt(line,maxIndex line) = char "__" repeat
    setelt(line,maxIndex line,char " ")
    line := concat(line, stripCommentsAndBlanks readLine!(fp))$String
  line

processTemplate(tp:FileName):FileName ==
  fp : TextFile := open(tp,"input")
  active : Boolean := true
  line : String
  endInput : Boolean := false
  while not (endInput or endOfFile? fp) repeat
    if active then
      line := getLine fp
      line = "endInput" => endInput := true
      if line = "beginVerbatim" then
        active := false
      else
        not empty? line => interpretString line
    else
      line := readLine!(fp)
      if line = "endVerbatim" then
        active := true
      else
        writePassiveLine! line
  close!(fp)
  if not active then
    error concat(["Missing `endVerbatim' line in ",tp::String])$String
  string(_$fortranOutputFile$Lisp)::FileName
```

———-

— FTEM.dotabb —

```
"FTEM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FTEM"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FTEM" -> "STRING"
```

———-

7.21 domain FT FortranType

— FortranType.input —

```
)set break resume
)sys rm -f FortranType.output
)spool FortranType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranType
--R FortranType  is a domain constructor
--R Abbreviation for FortranType is FT
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for FT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : FortranScalarType -> %
--R coerce : % -> OutputForm              external? : % -> Boolean
--R fortranCharacter : () -> %            fortranComplex : () -> %
--R fortranDouble : () -> %               fortranDoubleComplex : () -> %
--R fortranInteger : () -> %              fortranLogical : () -> %
--R fortranReal : () -> %                 hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R construct : (Union(fst: FortranScalarType,void: void),List Polynomial Integer,Boolean) -> %
--R construct : (Union(fst: FortranScalarType,void: void),List Symbol,Boolean) -> %
--R dimensionsOf : % -> List Polynomial Integer
--R scalarTypeOf : % -> Union(fst: FortranScalarType,void: void)
--R
--E 1

)spool
)lisp (bye)
```

———-

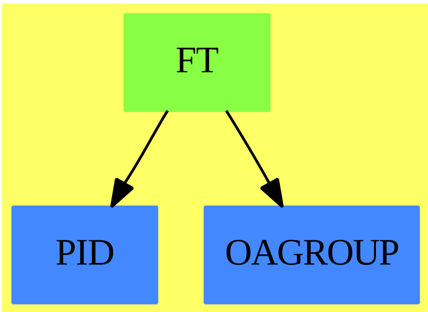
— FortranType.help —

```
====================================================================
FortranType examples
```

```
====================================================================
See Also:
o )show FortranType
```

———-

7.21.1 FortranType (FT)



See

⇒ "FortranScalarType" (FST) 7.19.1 on page 929
⇒ "SymbolTable" (SYMTAB) 20.38.1 on page 2606
⇒ "TheSymbolTable" (SYMS) 21.6.1 on page 2655

Exports:

coerce	construct	dimensionsOf	external?
fortranCharacter	fortranComplex	fortranDouble	fortranDoubleComplex
fortranInteger	fortranLogical	fortranReal	hash
latex	scalarTypeOf	?=?	?~=?

— **domain FT FortranType** —

```
)abbrev domain FT FortranType
++ Author: Mike Dewar
++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Creates and manipulates objects which correspond to FORTRAN
```

```
++ data types, including array dimensions.

FortranType() : exports == implementation where

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")

  exports == SetCategory with
    coerce : $ -> OutputForm
      ++ coerce(x) provides a printable form for x
    coerce : FST -> $
      ++ coerce(t) creates an element from a scalar type
    scalarTypeOf : $ -> FSTU
      ++ scalarTypeOf(t) returns the FORTRAN data type of t
    dimensionsOf : $ -> List Polynomial Integer
      ++ dimensionsOf(t) returns the dimensions of t
    external? : $ -> Boolean
      ++ external?(u) returns true if u is declared to be EXTERNAL
    construct : (FSTU,List Symbol,Boolean) -> $
      ++ construct(type,dims) creates an element of FortranType
    construct : (FSTU,List Polynomial Integer,Boolean) -> $
      ++ construct(type,dims) creates an element of FortranType
    fortranReal : () -> $
      ++ fortranReal() returns REAL, an element of FortranType
    fortranDouble : () -> $
      ++ fortranDouble() returns DOUBLE PRECISION, an element of FortranType
    fortranInteger : () -> $
      ++ fortranInteger() returns INTEGER, an element of FortranType
    fortranLogical : () -> $
      ++ fortranLogical() returns LOGICAL, an element of FortranType
    fortranComplex : () -> $
      ++ fortranComplex() returns COMPLEX, an element of FortranType
    fortranDoubleComplex: () -> $
      ++ fortranDoubleComplex() returns DOUBLE COMPLEX, an element of
      ++ FortranType
    fortranCharacter : () -> $
      ++ fortranCharacter() returns CHARACTER, an element of FortranType

  implementation == add

    Dims == List Polynomial Integer
    Rep := Record(type : FSTU, dimensions : Dims, external : Boolean)

    coerce(a:$):OutputForm ==
     t : OutputForm
     if external?(a) then
      if scalarTypeOf(a) case void then
        t := "EXTERNAL"::OutputForm
      else
        t := blankSeparate(["EXTERNAL"::OutputForm,
```

```
                      coerce(scalarTypeOf a)$FSTU])$OutputForm
     else
      t := coerce(scalarTypeOf a)$FSTU
     empty? dimensionsOf(a) => t
     sub(t,
         paren([u::OutputForm for u in dimensionsOf(a)])$OutputForm)$OutputForm

    scalarTypeOf(u:$):FSTU ==
      u.type

    dimensionsOf(u:$):Dims ==
      u.dimensions

    external?(u:$):Boolean ==
      u.external

    construct(t:FSTU, d:List Symbol, e:Boolean):$ ==
      e and not empty? d => error "EXTERNAL objects cannot have dimensions"
      not(e) and t case void => error "VOID objects must be EXTERNAL"
      construct(t,[l::Polynomial(Integer) for l in d],e)$Rep

    construct(t:FSTU, d:List Polynomial Integer, e:Boolean):$ ==
      e and not empty? d => error "EXTERNAL objects cannot have dimensions"
      not(e) and t case void => error "VOID objects must be EXTERNAL"
      construct(t,d,e)$Rep

    coerce(u:FST):$ ==
      construct([u]$FSTU,[]@List Polynomial Integer,false)

    fortranReal():$ == ("real"::FST)::$

    fortranDouble():$ == ("double precision"::FST)::$

    fortranInteger():$ == ("integer"::FST)::$

    fortranComplex():$ == ("complex"::FST)::$

    fortranDoubleComplex():$ == ("double complex"::FST)::$

    fortranCharacter():$ == ("character"::FST)::$

    fortranLogical():$ == ("logical"::FST)::$
```

— FT.dotabb —

```
"FT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FT"]
```

```
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"FT" -> "PID"
"FT" -> "OAGROUP"
```

———-

7.22 domain FCOMP FourierComponent

— FourierComponent.input —

```
)set break resume
)sys rm -f FourierComponent.output
)spool FourierComponent.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FourierComponent
--R FourierComponent E: OrderedSet  is a domain constructor
--R Abbreviation for FourierComponent is FCOMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FCOMP
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              argument : % -> E
--R coerce : % -> OutputForm             cos : E -> %
--R hash : % -> SingleInteger            latex : % -> String
--R max : (%,%) -> %                     min : (%,%) -> %
--R sin : E -> %                         sin? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

— FourierComponent.help —

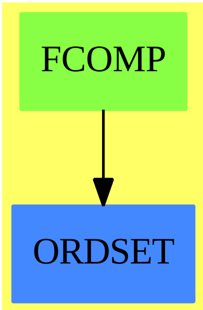
```
====================================================================
```

```
FourierComponent examples
====================================================================

See Also:
o )show FourierComponent
```

———-

7.22.1 FourierComponent (FCOMP)



See

⇒ "FourierSeries" (FSERIES) 7.23.1 on page 945

Exports:

argument	coerce	cos	hash	latex
max	min	sin	sin?	?~=?
?<?	?<=?	?=?	?>?	?>=?

— domain FCOMP FourierComponent —

```
)abbrev domain FCOMP FourierComponent
++ Author: James Davenport
++ Date Created: 17 April 1992
++ Date Last Updated: 12 June 1992
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain creates kernels for use in Fourier series

FourierComponent(E:OrderedSet):
       OrderedSet with
         sin: E -> $
```

```
        ++ sin(x) makes a sin kernel for use in Fourier series
        cos: E -> $
        ++ cos(x) makes a cos kernel for use in Fourier series
        sin?: $ -> Boolean
        ++ sin?(x) returns true if term is a sin, otherwise false
        argument: $ -> E
        ++ argument(x) returns the argument of a given sin/cos expressions
   ==
  add
   --representations
   Rep:=Record(SinIfTrue:Boolean, arg:E)
   e:E
   x,y:$
   sin e == [true,e]
   cos e == [false,e]
   sin? x == x.SinIfTrue
   argument x == x.arg
   coerce(x):OutputForm ==
     hconcat((if x.SinIfTrue then "sin" else "cos")::OutputForm,
              bracket((x.arg)::OutputForm))
   x<y ==
     x.arg < y.arg => true
     y.arg < x.arg => false
     x.SinIfTrue => false
     y.SinIfTrue
```

———-

— **FCOMP.dotabb** —

```
"FCOMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FCOMP"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"FCOMP" -> "ORDSET"
```

———-

7.23 domain FSERIES FourierSeries

— **FourierSeries.input** —

```
)set break resume
)sys rm -f FourierSeries.output
)spool FourierSeries.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show FourierSeries
--R FourierSeries(R: Join(CommutativeRing,Algebra Fraction Integer),E: Join(OrderedSet,Abeli
--R Abbreviation for FourierSeries is FSERIES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FSERIES
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                       ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %         ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,PositiveInteger) -> %         coerce : FourierComponent E -> %
--R coerce : R -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm               hash : % -> SingleInteger
--R latex : % -> String                    makeCos : (E,R) -> %
--R makeSin : (E,R) -> %                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")         sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

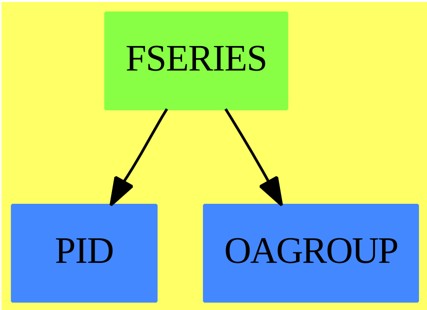
— FourierSeries.help —

```
====================================================================
FourierSeries examples
====================================================================

See Also:
o )show FourierSeries
```

———-

7.23.1 FourierSeries (FSERIES)



See

⇒ "FourierComponent" (FCOMP) 7.22.1 on page 942

Exports:

0	1	characteristic	coerce	hash
latex	makeCos	makeSin	one?	recip
sample	subtractIfCan	zero?	?~=?	?*?
?**?	?^?	?+?	?-?	-?
?=?				

— domain FSERIES FourierSeries —

```
)abbrev domain FSERIES FourierSeries
++ Author: James Davenport
++ Date Created: 17 April 1992
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain converts terms into Fourier series

FourierSeries(R:Join(CommutativeRing,Algebra(Fraction Integer)),
              E:Join(OrderedSet,AbelianGroup)):
       Algebra(R) with
         if E has canonical and R has canonical then canonical
         coerce: R -> $
           ++ coerce(r) converts coefficients into Fourier Series
         coerce: FourierComponent(E) -> $
           ++ coerce(c) converts sin/cos terms into Fourier Series
         makeSin: (E,R) -> $
           ++ makeSin(e,r) makes a sin expression with given
           ++ argument and coefficient
```

```
       makeCos: (E,R) -> $
         ++ makeCos(e,r) makes a sin expression with given
         ++argument and coefficient
  == FreeModule(R,FourierComponent(E))
add
 --representations
 Term := Record(k:FourierComponent(E),c:R)
 Rep  := List Term
 multiply : (Term,Term) -> $
 w,x1,x2:$
 t1,t2:Term
 n:NonNegativeInteger
 z:Integer
 e:FourierComponent(E)
 a:E
 r:R
 1 == [[cos 0,1]]
 coerce e ==
    sin? e and zero? argument e => 0
    if argument e < 0  then
         not sin? e => e:=cos(- argument e)
         return [[sin(- argument e),-1]]
    [[e,1]]
 multiply(t1,t2) ==
   r:=(t1.c*t2.c)*(1/2)
   s1:=argument t1.k
   s2:=argument t2.k
   sum:=s1+s2
   diff:=s1-s2
   sin? t1.k =>
     sin? t2.k =>
       makeCos(diff,r) + makeCos(sum,-r)
     makeSin(sum,r) + makeSin(diff,r)
   sin? t2.k =>
     makeSin(sum,r) + makeSin(diff,r)
   makeCos(diff,r) + makeCos(sum,r)
 x1*x2 ==
   null x1 => 0
   null x2 => 0
   +/[+/[multiply(t1,t2) for t2 in x2] for t1 in x1]
 makeCos(a,r) ==
    a<0 => [[cos(-a),r]]
    [[cos a,r]]
 makeSin(a,r) ==
    zero? a => []
    a<0 => [[sin(-a),-r]]
    [[sin a,r]]
```

———

— **FSERIES.dotabb** —

```
"FSERIES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FSERIES"]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"FSERIES" -> "PID"
"FSERIES" -> "OAGROUP"
```

——

7.24 domain FRAC Fraction

— **Fraction.input** —

```
)set break resume
)sys rm -f Fraction.output
)spool Fraction.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
a := 11/12
--R
--R
--R        11
--R   (1)  --
--R        12
--R                                                      Type: Fraction Integer
--E 1

--S 2 of 12
b := 23/24
--R
--R
--R        23
--R   (2)  --
--R        24
--R                                                      Type: Fraction Integer
--E 2

--S 3 of 12
3 - a*b**2 + a + b/a
--R
--R
--R        313271
```

```
--R   (3)  ------
--R         76032
--R                                                      Type: Fraction Integer
--E 3

--S 4 of 12
numer(a)
--R
--R
--R   (4)  11
--R                                                       Type: PositiveInteger
--E 4

--S 5 of 12
denom(b)
--R
--R
--R   (5)  24
--R                                                       Type: PositiveInteger
--E 5

--S 6 of 12
r := (x**2 + 2*x + 1)/(x**2 - 2*x + 1)
--R
--R
--R         2
--R        x  + 2x + 1
--R   (6)  -----------
--R         2
--R        x  - 2x + 1
--R                                           Type: Fraction Polynomial Integer
--E 6

--S 7 of 12
factor(r)
--R
--R
--R         2
--R        x  + 2x + 1
--R   (7)  -----------
--R         2
--R        x  - 2x + 1
--R                                  Type: Factored Fraction Polynomial Integer
--E 7

--S 8 of 12
map(factor,r)
--R
--R
--R               2
```

```
--R        (x + 1)
--R   (8)  --------
--R               2
--R        (x - 1)
--R                                  Type: Fraction Factored Polynomial Integer
--E 8

--S 9 of 12
continuedFraction(7/12)
--R
--R
--R          1 |     1 |     1 |     1 |
--R   (9)  +---+ + +---+ + +---+ + +---+
--R        | 1     | 1     | 2     | 2
--R                                             Type: ContinuedFraction Integer
--E 9

--S 10 of 12
partialFraction(7,12)
--R
--R
--R             3   1
--R   (10)  1 - -- + -
--R              2   3
--R             2
--R                                               Type: PartialFraction Integer
--E 10

--S 11 of 12
g := 2/3 + 4/5*%i
--R
--R
--R         2   4
--R   (11)  - + - %i
--R         3   5
--R                                              Type: Complex Fraction Integer
--E 11

--S 12 of 12
g :: FRAC COMPLEX INT
--R
--R
--R         10 + 12%i
--R   (12)  ---------
--R             15
--R                                              Type: Fraction Complex Integer
--E 12

)spool
)lisp (bye)
```

— Fraction.help —

```
====================================================================
Fraction examples
====================================================================

The Fraction domain implements quotients.  The elements must
belong to a domain of category IntegralDomain: multiplication
must be commutative and the product of two non-zero elements must not
be zero.  This allows you to make fractions of most things you would
think of, but don't expect to create a fraction of two matrices!  The
abbreviation for Fraction is FRAC.

Use / to create a fraction.

  a := 11/12
    11
    --
    12
                   Type: Fraction Integer

  b := 23/24
    23
    --
    24
                   Type: Fraction Integer

The standard arithmetic operations are available.

  3 - a*b**2 + a + b/a
    313271
    ------
     76032
                   Type: Fraction Integer

Extract the numerator and denominator by using numer and denom,
respectively.

  numer(a)
    11
                   Type: PositiveInteger

  denom(b)
    24
                   Type: PositiveInteger

Operations like max, min, negative?, positive? and zero?
```

are all available if they are provided for the numerators and denominators.

Don't expect a useful answer from factor, gcd or lcm if you apply them to fractions.

```
  r := (x**2 + 2*x + 1)/(x**2 - 2*x + 1)
     2
    x  + 2x + 1
    -----------
     2
    x  - 2x + 1
                  Type: Fraction Polynomial Integer
```

Since all non-zero fractions are invertible, these operations have trivial definitions.

```
  factor(r)
     2
    x  + 2x + 1
    -----------
     2
    x  - 2x + 1
                  Type: Factored Fraction Polynomial Integer
```

Use map to apply factor to the numerator and denominator, which is probably what you mean.

```
  map(factor,r)
           2
    (x + 1)
    --------
           2
    (x - 1)
                  Type: Fraction Factored Polynomial Integer
```

Other forms of fractions are available. Use continuedFraction to create a continued fraction.

```
  continuedFraction(7/12)
      1 |     1 |     1 |     1 |
    +---+ + +---+ + +---+ + +---+
    | 1     | 1     | 2     | 2
                  Type: ContinuedFraction Integer
```

Use partialFraction to create a partial fraction.

```
  partialFraction(7,12)
          3   1
     1 - -- + -
```

```
          2   3
         2
                  Type: PartialFraction Integer

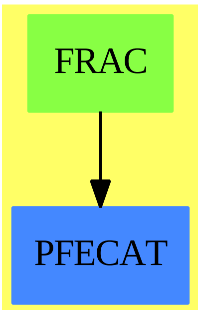
Use conversion to create alternative views of fractions with objects
moved in and out of the numerator and denominator.

  g := 2/3 + 4/5*%i
     2   4
     - + - %i
     3   5
                  Type: Complex Fraction Integer

  g :: FRAC COMPLEX INT
    10 + 12%i
    ---------
        15
                  Type: Fraction Complex Integer

See Also:
o )help ContinuedFraction
o )help PartialFraction
o )help Integer
o )show Fraction
```

7.24.1 Fraction (FRAC)



See

⇒ "Localize" (LO) 13.13.1 on page 1486
⇒ "LocalAlgebra" (LA) 13.12.1 on page 1484

Exports:

0	1	abs
associates?	characteristic	charthRoot
ceiling	coerce	conditionP
convert	D	denom
denominator	differentiate	divide
euclideanSize	eval	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	floor
fractionPart	gcd	gcdPolynomial
hash	init	inv
latex	lcm	map
max	min	multiEuclidean
negative?	nextItem	numer
numerator	OMwrite	one?
patternMatch	positive?	prime?
principalIdeal	random	recip
reducedSystem	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
zero?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?.?
?quo?	?rem?	

— domain FRAC Fraction —

```
)abbrev domain FRAC Fraction
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated: 12 February 1992
++ Basic Functions: Field, numer, denom
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: fraction, localization
++ References:
++ Description:
++ Fraction takes an IntegralDomain S and produces
++ the domain of Fractions with numerators and denominators from S.
++ If S is also a GcdDomain, then gcd's between numerator and
++ denominator will be cancelled during all operations.

Fraction(S: IntegralDomain): QuotientFieldCategory S with
```

```
       if S has IntegerNumberSystem and S has OpenMath then OpenMath
       if S has canonical and S has GcdDomain and S has canonicalUnitNormal
          then canonical
           ++ \spad{canonical} means that equal elements are in fact identical.
  == LocalAlgebra(S, S, S) add
    Rep:= Record(num:S, den:S)
    coerce(d:S):% == [d,1]
    zero?(x:%) == zero? x.num


    if S has GcdDomain and S has canonicalUnitNormal then
      retract(x:%):S ==
--        one?(x.den) => x.num
        ((x.den) = 1) => x.num
        error "Denominator not equal to 1"

      retractIfCan(x:%):Union(S, "failed") ==
--        one?(x.den) => x.num
        ((x.den) = 1) => x.num
        "failed"
    else
      retract(x:%):S ==
        (a:= x.num exquo x.den) case "failed" =>
           error "Denominator not equal to 1"
        a
      retractIfCan(x:%):Union(S,"failed") == x.num exquo x.den

    if S has EuclideanDomain then
      wholePart x ==
--        one?(x.den) => x.num
        ((x.den) = 1) => x.num
        x.num quo x.den

    if S has IntegerNumberSystem then

      floor x ==
--        one?(x.den) => x.num
        ((x.den) = 1) => x.num
        x < 0 => -ceiling(-x)
        wholePart x

      ceiling x ==
--        one?(x.den) => x.num
        ((x.den) = 1) => x.num
        x < 0 => -floor(-x)
        1 + wholePart x

      if S has OpenMath then
        -- TODO: somwhere this file does something which redefines the division
        -- operator. Doh!
```

```
      writeOMFrac(dev: OpenMathDevice, x: %): Void ==
        OMputApp(dev)
        OMputSymbol(dev, "nums1", "rational")
        OMwrite(dev, x.num, false)
        OMwrite(dev, x.den, false)
        OMputEndApp(dev)

      OMwrite(x: %): String ==
        s: String := ""
        sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
        dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
        OMputObject(dev)
        writeOMFrac(dev, x)
        OMputEndObject(dev)
        OMclose(dev)
        s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
        s

      OMwrite(x: %, wholeObj: Boolean): String ==
        s: String := ""
        sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
        dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
        if wholeObj then
          OMputObject(dev)
        writeOMFrac(dev, x)
        if wholeObj then
          OMputEndObject(dev)
        OMclose(dev)
        s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
        s

      OMwrite(dev: OpenMathDevice, x: %): Void ==
        OMputObject(dev)
        writeOMFrac(dev, x)
        OMputEndObject(dev)

      OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
        if wholeObj then
          OMputObject(dev)
        writeOMFrac(dev, x)
        if wholeObj then
          OMputEndObject(dev)

  if S has GcdDomain then
    cancelGcd: % -> S
    normalize: % -> %

    normalize x ==
      zero?(x.num) => 0
```

```
--        one?(x.den) => x
        ((x.den) = 1) => x
        uca := unitNormal(x.den)
        zero?(x.den := uca.canonical) => error "division by zero"
        x.num := x.num * uca.associate
        x

      recip x ==
        zero?(x.num) => "failed"
        normalize [x.den, x.num]

      cancelGcd x ==
--        one?(x.den) => x.den
        ((x.den) = 1) => x.den
        d := gcd(x.num, x.den)
        xn := x.num exquo d
        xn case "failed" =>
          error "gcd not gcd in QF cancelGcd (numerator)"
        xd := x.den exquo d
        xd case "failed" =>
          error "gcd not gcd in QF cancelGcd (denominator)"
        x.num := xn :: S
        x.den := xd :: S
        d

      nn:S / dd:S ==
        zero? dd => error "division by zero"
        cancelGcd(z := [nn, dd])
        normalize z

      x + y  ==
        zero? y => x
        zero? x => y
        z := [x.den,y.den]
        d := cancelGcd z
        g := [z.den * x.num + z.num * y.num, d]
        cancelGcd g
        g.den := g.den * z.num * z.den
        normalize g

      -- We can not rely on the defaulting mechanism
      -- to supply a definition for -, even though this
      -- definition would do, for thefollowing reasons:
      --  1) The user could have defined a subtraction
      --     in Localize, which would not work for
      --     QuotientField;
      --  2) even if he doesn't, the system currently
      --     places a default definition in Localize,
      --     which uses Localize's +, which does not
      --     cancel gcds
```

```
      x - y  ==
        zero? y => x
        z := [x.den, y.den]
        d := cancelGcd z
        g := [z.den * x.num - z.num * y.num, d]
        cancelGcd g
        g.den := g.den * z.num * z.den
        normalize g

      x:% * y:%  ==
        zero? x or zero? y => 0
--        one? x => y
        (x = 1) => y
--        one? y => x
        (y = 1) => x
        (x, y) := ([x.num, y.den], [y.num, x.den])
        cancelGcd x; cancelGcd y;
        normalize [x.num * y.num, x.den * y.den]

      n:Integer * x:% ==
        y := [n::S, x.den]
        cancelGcd y
        normalize [x.num * y.num, y.den]

      nn:S * x:% ==
        y := [nn, x.den]
        cancelGcd y
        normalize [x.num * y.num, y.den]

      differentiate(x:%, deriv:S -> S) ==
        y := [deriv(x.den), x.den]
        d := cancelGcd(y)
        y.num := deriv(x.num) * y.den - x.num * y.num
        (d, y.den) := (y.den, d)
        cancelGcd y
        y.den := y.den * d * d
        normalize y

      if S has canonicalUnitNormal then
        x = y == (x.num = y.num) and (x.den = y.den)
    --x / dd == (cancelGcd (z:=[x.num,dd*x.den]); normalize z)

--        one? x == one? (x.num) and one? (x.den)
        one? x == ((x.num) = 1) and ((x.den) = 1)
                  -- again assuming canonical nature of representation

    else
      nn:S/dd:S == if zero? dd then error "division by zero" else [nn,dd]

      recip x ==
```

```
        zero?(x.num) => "failed"
        [x.den, x.num]

    if (S has RetractableTo Fraction Integer) then
      retract(x:%):Fraction(Integer) == retract(retract(x)@S)

      retractIfCan(x:%):Union(Fraction Integer, "failed") ==
        (u := retractIfCan(x)@Union(S, "failed")) case "failed" => "failed"
        retractIfCan(u::S)

    else if (S has RetractableTo Integer) then
      retract(x:%):Fraction(Integer) ==
        retract(numer x) / retract(denom x)

      retractIfCan(x:%):Union(Fraction Integer, "failed") ==
        (n := retractIfCan numer x) case "failed" => "failed"
        (d := retractIfCan denom x) case "failed" => "failed"
        (n::Integer) / (d::Integer)

    QFP ==> SparseUnivariatePolynomial %
    DP ==> SparseUnivariatePolynomial S
    import UnivariatePolynomialCategoryFunctions2(%,QFP,S,DP)
    import UnivariatePolynomialCategoryFunctions2(S,DP,%,QFP)

    if S has GcdDomain then
       gcdPolynomial(pp,qq) ==
          zero? pp => qq
          zero? qq => pp
          zero? degree pp or zero? degree qq => 1
          denpp:="lcm"/[denom u for u in coefficients pp]
          ppD:DP:=map(x+->retract(x*denpp),pp)
          denqq:="lcm"/[denom u for u in coefficients qq]
          qqD:DP:=map(x+->retract(x*denqq),qq)
          g:=gcdPolynomial(ppD,qqD)
          zero? degree g => 1
--          one? (lc:=leadingCoefficient g) => map(#1::%,g)
          ((lc:=leadingCoefficient g) = 1) => map(x+->x::%,g)
          map(x+->x/lc,g)

    if (S has PolynomialFactorizationExplicit) then
       -- we'll let the solveLinearPolynomialEquations operator
       -- default from Field
       pp,qq: QFP
       lpp: List QFP
       import Factored SparseUnivariatePolynomial %
       if S has CharacteristicNonZero then
          if S has canonicalUnitNormal and S has GcdDomain then
             charthRoot x ==
               n:= charthRoot x.num
               n case "failed" => "failed"
```

```
        d:=charthRoot x.den
        d case "failed" => "failed"
        n/d
   else
      charthRoot x ==
        -- to find x = p-th root of n/d
        -- observe that xd is p-th root of n*d**(p-1)
        ans:=charthRoot(x.num *
               (x.den)**(characteristic()$%-1)::NonNegativeInteger)
        ans case "failed" => "failed"
        ans / x.den
   clear: List % -> List S
   clear l ==
      d:="lcm"/[x.den for x in l]
      [ x.num * (d exquo x.den)::S for x in l]
   mat: Matrix %
   conditionP mat ==
     matD: Matrix S
     matD:= matrix [ clear l for l in listOfLists mat ]
     ansD := conditionP matD
     ansD case "failed" => "failed"
     ansDD:=ansD :: Vector(S)
     [ ansDD(i)::% for i in 1..#ansDD]$Vector(%)

factorPolynomial(pp) ==
   zero? pp => 0
   denpp:="lcm"/[denom u for u in coefficients pp]
   ppD:DP:=map(x+->retract(x*denpp),pp)
   ff:=factorPolynomial ppD
   den1:%:=denpp::%
   lfact:List Record(flg:Union("nil", "sqfr", "irred", "prime"),
                     fctr:QFP, xpnt:Integer)
   lfact:= [[w.flg,
             if leadingCoefficient w.fctr =1 then
                    map(x+->x::%,w.fctr)
             else (lc:=(leadingCoefficient w.fctr)::%;
                    den1:=den1/lc**w.xpnt;
                     map(x+->x::%/lc,w.fctr)),
            w.xpnt] for w in factorList ff]
   makeFR(map(x+->x::%/den1,unit(ff)),lfact)
factorSquareFreePolynomial(pp) ==
   zero? pp => 0
   degree pp = 0 => makeFR(pp,empty())
   lcpp:=leadingCoefficient pp
   pp:=pp/lcpp
   denpp:="lcm"/[denom u for u in coefficients pp]
   ppD:DP:=map(x+->retract(x*denpp),pp)
   ff:=factorSquareFreePolynomial ppD
   den1:%:=denpp::%/lcpp
   lfact:List Record(flg:Union("nil", "sqfr", "irred", "prime"),
```

```
                        fctr:QFP, xpnt:Integer)
      lfact:= [[w.flg,
                if leadingCoefficient w.fctr =1 then
                       map(x+->x::%,w.fctr)
                else (lc:=(leadingCoefficient w.fctr)::%;
                       den1:=den1/lc**w.xpnt;
                        map(x+->x::%/lc,w.fctr)),
               w.xpnt] for w in factorList ff]
      makeFR(map(x+->x::%/den1,unit(ff)),lfact)
```

— FRAC.dotabb —

```
"FRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRAC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"FRAC" -> "PFECAT"
```

7.25 domain FRIDEAL FractionalIdeal

— FractionalIdeal.input —

```
)set break resume
)sys rm -f FractionalIdeal.output
)spool FractionalIdeal.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FractionalIdeal
--R FractionalIdeal(R: EuclideanDomain,F: QuotientFieldCategory R,UP: UnivariatePolynomialCa
--R Abbreviation for FractionalIdeal is FRIDEAL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FRIDEAL
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R basis : % -> Vector A                 coerce : % -> OutputForm
```

```
--R commutator : (%,%) -> %               conjugate : (%,%) -> %
--R denom : % -> R                        hash : % -> SingleInteger
--R ideal : Vector A -> %                 inv : % -> %
--R latex : % -> String                   minimize : % -> %
--R norm : % -> F                         numer : % -> Vector A
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R randomLC : (NonNegativeInteger,Vector A) -> A
--R
--E 1

)spool
)lisp (bye)
```

———-

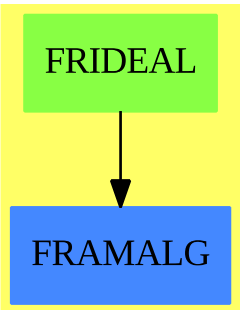
— **FractionalIdeal.help** —

```
====================================================================
FractionalIdeal examples
====================================================================

See Also:
o )show FractionalIdeal
```

———-

7.25.1 FractionalIdeal (FRIDEAL)



See

⇒ "FramedModule" (FRMOD) 7.26.1 on page 967
⇒ "HyperellipticFiniteDivisor" (HELLFDIV) 9.11.1 on page 1149
⇒ "FiniteDivisor" (FDIV) 7.4.1 on page 781

Exports:

1	basis	coerce	commutator	conjugate
denom	hash	ideal	inv	latex
minimize	norm	numer	one?	randomLC
recip	sample	?~=?	?**?	?^?
?*?	?**?	?/?	?=?	?^?

— domain FRIDEAL FractionalIdeal —

```
)abbrev domain FRIDEAL FractionalIdeal
++ Author: Manuel Bronstein
++ Date Created: 27 Jan 1989
++ Date Last Updated: 30 July 1993
++ Keywords: ideal, algebra, module.
++ Examples: )r FRIDEAL INPUT
++ Description:
++ Fractional ideals in a framed algebra.

FractionalIdeal(R, F, UP, A): Exports == Implementation where
  R : EuclideanDomain
  F : QuotientFieldCategory R
  UP: UnivariatePolynomialCategory F
  A : Join(FramedAlgebra(F, UP), RetractableTo F)

  VF  ==> Vector F
  VA  ==> Vector A
  UPA ==> SparseUnivariatePolynomial A
  QF  ==> Fraction UP

  Exports ==> Group with
    ideal   : VA -> %
      ++ ideal([f1,...,fn]) returns the ideal \spad{(f1,...,fn)}.
    basis   : %  -> VA
      ++ basis((f1,...,fn)) returns the vector \spad{[f1,...,fn]}.
    norm    : %  -> F
      ++ norm(I) returns the norm of the ideal I.
    numer   : %  -> VA
      ++ numer(1/d * (f1,...,fn)) = the vector \spad{[f1,...,fn]}.
    denom   : %  -> R
      ++ denom(1/d * (f1,...,fn)) returns d.
    minimize: %  -> %
      ++ minimize(I) returns a reduced set of generators for \spad{I}.
    randomLC: (NonNegativeInteger, VA) -> A
      ++ randomLC(n,x) should be local but conditional.

  Implementation ==> add
    import CommonDenominator(R, F, VF)
    import MatrixCommonDenominator(UP, QF)
    import InnerCommonDenominator(R, F, List R, List F)
    import MatrixCategoryFunctions2(F, Vector F, Vector F, Matrix F,
```

```
                    UP, Vector UP, Vector UP, Matrix UP)
import MatrixCategoryFunctions2(UP, Vector UP, Vector UP,
                    Matrix UP, F, Vector F, Vector F, Matrix F)
import MatrixCategoryFunctions2(UP, Vector UP, Vector UP,
                    Matrix UP, QF, Vector QF, Vector QF, Matrix QF)

Rep := Record(num:VA, den:R)

poly    : % -> UPA
invrep  : Matrix F -> A
upmat   : (A, NonNegativeInteger) -> Matrix UP
summat  : % -> Matrix UP
num2O   : VA -> OutputForm
agcd    : List A -> R
vgcd    : VF -> R
mkIdeal : (VA, R) -> %
intIdeal: (List A, R) -> %
ret?    : VA -> Boolean
tryRange: (NonNegativeInteger, VA, R, %) -> Union(%, "failed")

1               == [[1]$VA, 1]
numer i         == i.num
denom i         == i.den
mkIdeal(v, d)   == [v, d]
invrep m        == represents(transpose(m) * coordinates(1$A))
upmat(x, i)     == map(s +-> monomial(s, i)$UP, regularRepresentation x)
ret? v          == any?(s+->retractIfCan(s)@Union(F,"failed") case F, v)
x = y           == denom(x) = denom(y) and numer(x) = numer(y)
agcd l  == reduce("gcd", [vgcd coordinates a for a in l]$List(R), 0)

norm i ==
  ("gcd"/[retract(u)@R for u in coefficients determinant summat i])
          / denom(i) ** rank()$A

tryRange(range, nm, nrm, i) ==
  for j in 0..10 repeat
    a := randomLC(10 * range, nm)
    unit? gcd((retract(norm a)@R exquo nrm)::R, nrm) =>
                            return intIdeal([nrm::F::A, a], denom i)
  "failed"

summat i ==
  m := minIndex(v := numer i)
  reduce("+",
        [upmat(qelt(v, j + m), j) for j in 0..#v-1]$List(Matrix UP))

inv i ==
  m  := inverse(map(s+->s::QF, summat i))::Matrix(QF)
  cd  := splitDenominator(denom(i)::F::UP::QF * m)
  cd2 := splitDenominator coefficients(cd.den)
```

```
    invd:= cd2.den / reduce("gcd", cd2.num)
    d   := reduce("max", [degree p for p in parts(cd.num)])
    ideal
      [invd * invrep map(s+->coefficient(s, j), cd.num) for j in 0..d]$VA

  ideal v ==
    d := reduce("lcm", [commonDenominator coordinates qelt(v, i)
                        for i in minIndex v .. maxIndex v]$List(R))
    intIdeal([d::F * qelt(v, i) for i in minIndex v .. maxIndex v], d)

  intIdeal(l, d) ==
    lr := empty()$List(R)
    nr := empty()$List(A)
    for x in removeDuplicates l repeat
      if (u := retractIfCan(x)@Union(F, "failed")) case F
        then lr := concat(retract(u::F)@R, lr)
        else nr := concat(x, nr)
    r    := reduce("gcd", lr, 0)
    g    := agcd nr
    a    := (r quo (b := gcd(gcd(d, r), g)))::F::A
    d    := d quo b
    r ^= 0 and ((g exquo r) case R) => mkIdeal([a], d)
    invb := inv(b::F)
    va:VA := [invb * m for m in nr]
    zero? a => mkIdeal(va, d)
    mkIdeal(concat(a, va), d)

  vgcd v ==
    reduce("gcd",
           [retract(v.i)@R for i in minIndex v .. maxIndex v]$List(R))

  poly i ==
    m := minIndex(v := numer i)
    +/[monomial(qelt(v, i + m), i) for i in 0..#v-1]

  i1 * i2 ==
    intIdeal(coefficients(poly i1 * poly i2), denom i1 * denom i2)

  i:$ ** m:Integer ==
    m < 0 => inv(i) ** (-m)
    n := m::NonNegativeInteger
    v := numer i
    intIdeal([qelt(v, j) ** n for j in minIndex v .. maxIndex v],
             denom(i) ** n)

  num2O v ==
    paren [qelt(v, i)::OutputForm
           for i in minIndex v .. maxIndex v]$List(OutputForm)

  basis i ==
```

```
      v := numer i
      d := inv(denom(i)::F)
      [d * qelt(v, j) for j in minIndex v .. maxIndex v]

    coerce(i:$):OutputForm ==
      nm := num2O numer i
--      one? denom i => nm
      (denom i = 1) => nm
      (1::Integer::OutputForm) / (denom(i)::OutputForm) * nm

    if F has Finite then
      randomLC(m, v) ==
        +/[random()$F * qelt(v, j) for j in minIndex v .. maxIndex v]
    else
      randomLC(m, v) ==
        +/[(random()$Integer rem m::Integer) * qelt(v, j)
            for j in minIndex v .. maxIndex v]

    minimize i ==
      n := (#(nm := numer i))
--      one?(n) or (n < 3 and ret? nm) => i
      (n = 1) or (n < 3 and ret? nm) => i
      nrm    := retract(norm mkIdeal(nm, 1))@R
      for range in 1..5 repeat
        (u := tryRange(range, nm, nrm, i)) case $ => return(u::$)
      i
```

— **FRIDEAL.dotabb** —

```
"FRIDEAL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRIDEAL"]
"FRAMALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRAMALG"]
"FRIDEAL" -> "FRAMALG"
```

7.26 domain FRMOD FramedModule

— **FramedModule.input** —

```
)set break resume
)sys rm -f FramedModule.output
)spool FramedModule.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FramedModule
--R FramedModule(R: EuclideanDomain,F: QuotientFieldCategory R,UP: UnivariatePolynomialCatego
--R Abbreviation for FramedModule is FRMOD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FRMOD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,PositiveInteger) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,PositiveInteger) -> %        basis : % -> Vector A
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   module : Vector A -> %
--R norm : % -> F                         one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R module : FractionalIdeal(R,F,UP,A) -> % if A has RETRACT F
--R
--E 1

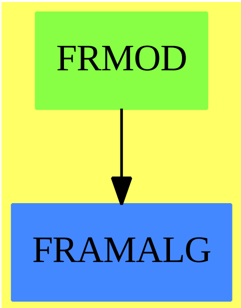
)spool
)lisp (bye)
```

— **FramedModule.help** —

```
====================================================================
FramedModule examples
====================================================================

See Also:
o )show FramedModule
```

7.26.1 FramedModule (FRMOD)



See

⇒ "FractionalIdeal" (FRIDEAL) 7.25.1 on page 961
⇒ "HyperellipticFiniteDivisor" (HELLFDIV) 9.11.1 on page 1149
⇒ "FiniteDivisor" (FDIV) 7.4.1 on page 781

Exports:

1	basis	coerce	hash	latex
module	norm	one?	recip	sample
?~=?	?**?	?^?	?*?	?**?
?=?				

— domain FRMOD FramedModule —

```
)abbrev domain FRMOD FramedModule
++ Author: Manuel Bronstein
++ Date Created: 27 Jan 1989
++ Date Last Updated: 24 Jul 1990
++ Keywords: ideal, algebra, module.
++ Description:
++ Module representation of fractional ideals.

FramedModule(R, F, UP, A, ibasis): Exports == Implementation where
  R     : EuclideanDomain
  F     : QuotientFieldCategory R
  UP    : UnivariatePolynomialCategory F
  A     : FramedAlgebra(F, UP)
  ibasis: Vector A

  VR  ==> Vector R
  VF  ==> Vector F
  VA  ==> Vector A
  M   ==> Matrix F

  Exports ==> Monoid with
    basis : %  -> VA
      ++ basis((f1,...,fn)) = the vector \spad{[f1,...,fn]}.
```

```
    norm  : %  -> F
      ++ norm(f) returns the norm of the module f.
    module: VA -> %
      ++ module([f1,...,fn]) = the module generated by \spad{(f1,...,fn)}
      ++ over R.
    if A has RetractableTo F then
      module: FractionalIdeal(R, F, UP, A) -> %
        ++ module(I) returns I viewed has a module over R.

  Implementation ==> add
    import MatrixCommonDenominator(R, F)
    import ModularHermitianRowReduction(R)

    Rep  := VA

    iflag?:Reference(Boolean) := ref true
    wflag?:Reference(Boolean) := ref true
    imat := new(#ibasis, #ibasis, 0)$M
    wmat := new(#ibasis, #ibasis, 0)$M

    rowdiv      : (VR, R)  -> VF
    vectProd    : (VA, VA) -> VA
    wmatrix     : VA -> M
    W2A         : VF -> A
    intmat      : () -> M
    invintmat   : () -> M
    getintmat   : () -> Boolean
    getinvintmat: () -> Boolean

    1                       == ibasis
    module(v:VA)            == v
    basis m                 == m pretend VA
    rowdiv(r, f)            == [r.i / f for i in minIndex r..maxIndex r]
    coerce(m:%):OutputForm == coerce(basis m)$VA
    W2A v                   == represents(v * intmat())
    wmatrix v               == coordinates(v) * invintmat()

    getinvintmat() ==
      m := inverse(intmat())::M
      for i in minRowIndex m .. maxRowIndex m repeat
        for j in minColIndex m .. maxColIndex m repeat
          imat(i, j) := qelt(m, i, j)
      false

    getintmat() ==
      m := coordinates ibasis
      for i in minRowIndex m .. maxRowIndex m repeat
        for j in minColIndex m .. maxColIndex m repeat
          wmat(i, j) := qelt(m, i, j)
      false
```

```
    invintmat() ==
      if iflag?() then iflag?() := getinvintmat()
      imat

    intmat() ==
      if wflag?() then wflag?() := getintmat()
      wmat

    vectProd(v1, v2) ==
      k := minIndex(v := new(#v1 * #v2, 0)$VA)
      for i in minIndex v1 .. maxIndex v1 repeat
        for j in minIndex v2 .. maxIndex v2 repeat
          qsetelt_!(v, k, qelt(v1, i) * qelt(v2, j))
          k := k + 1
      v pretend VA

    norm m ==
      #(basis m) ^= #ibasis => error "Module not of rank n"
      determinant(coordinates(basis m) * invintmat())

    m1 * m2 ==
      m := rowEch((cd := splitDenominator wmatrix(
                               vectProd(basis m1, basis m2))).num)
      module [u for i in minRowIndex m .. maxRowIndex m |
                        (u := W2A rowdiv(row(m, i), cd.den)) ^= 0]$VA

    if A has RetractableTo F then
      module(i:FractionalIdeal(R, F, UP, A)) ==
        module(basis i) * module(ibasis)
```

— **FRMOD.dotabb** —

```
"FRMOD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRMOD"]
"FRAMALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRAMALG"]
"FRMOD" -> "FRAMALG"
```

7.27 domain FAGROUP FreeAbelianGroup

— **FreeAbelianGroup.input** —

```
)set break resume
)sys rm -f FreeAbelianGroup.output
)spool FreeAbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeAbelianGroup
--R FreeAbelianGroup S: SetCategory  is a domain constructor
--R Abbreviation for FreeAbelianGroup is FAGROUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FAGROUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,S) -> %                ?*? : (%,Integer) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (S,%) -> %                      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coefficient : (S,%) -> Integer        coerce : S -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   mapGen : ((S -> S),%) -> %
--R nthCoef : (%,Integer) -> Integer      nthFactor : (%,Integer) -> S
--R retract : % -> S                      sample : () -> %
--R size : % -> NonNegativeInteger        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R highCommonTerms : (%,%) -> % if Integer has OAMON
--R mapCoef : ((Integer -> Integer),%) -> %
--R max : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(gen: S,exp: Integer)
--R
--E 1

)spool
)lisp (bye)
```

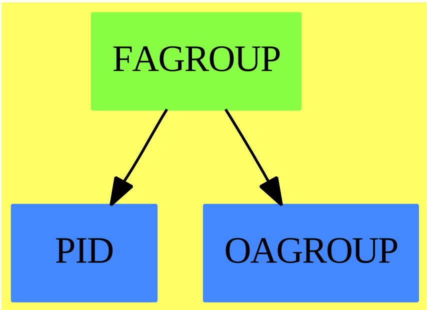
— **FreeAbelianGroup.help** —

```
====================================================================
FreeAbelianGroup examples
====================================================================

See Also:
o )show FreeAbelianGroup
```

———–

7.27.1 FreeAbelianGroup (FAGROUP)



See

⇒ "ListMonoidOps" (LMOPS) 13.10.1 on page 1473
⇒ "FreeMonoid" (FMONOID) 7.32.1 on page 987
⇒ "FreeGroup" (FGROUP) 7.29.1 on page 976
⇒ "InnerFreeAbelianMonoid" (IFAMON) 10.22.1 on page 1250
⇒ "FreeAbelianMonoid" (FAMONOID) 7.28.1 on page 974

Exports:

0	coefficient	coerce	hash	highCommonTerms
latex	mapCoef	mapGen	max	min
nthCoef	nthFactor	retract	retractIfCan	sample
size	subtractIfCan	terms	zero?	?~=?
?*?	?<?	?<=?	?>?	?>=?
?+?	?-?	-?	?=?	

— **domain FAGROUP FreeAbelianGroup** —

```
)abbrev domain FAGROUP FreeAbelianGroup
++ Free abelian group on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ The free abelian group on a set S is the monoid of finite sums of
```

```
++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are integers. The operation is commutative.

FreeAbelianGroup(S:SetCategory): Exports == Implementation where
  Exports ==> Join(AbelianGroup, Module Integer,
                   FreeAbelianMonoidCategory(S, Integer)) with
    if S has OrderedSet then OrderedSet

  Implementation ==> InnerFreeAbelianMonoid(S, Integer, 1) add
    - f == mapCoef("-", f)

    if S has OrderedSet then
      inmax: List Record(gen: S, exp: Integer) -> Record(gen: S, exp:Integer)

      inmax l ==
        mx := first l
        for t in rest l repeat
          if mx.gen < t.gen then mx := t
        mx

      -- lexicographic order
      a < b ==
        zero? a  =>
          zero? b => false
          0 < (inmax terms b).exp
        ta := inmax terms a
        zero? b => ta.exp < 0
        tb := inmax terms b
        ta.gen < tb.gen => 0 < tb.exp
        tb.gen < ta.gen => ta.exp < 0
        ta.exp < tb.exp => true
        tb.exp < ta.exp => false
        lc := ta.exp * ta.gen
        (a - lc) < (b - lc)
```

— FAGROUP.dotabb —

```
"FAGROUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FAGROUP"]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"FAGROUP" -> "PID"
"FAGROUP" -> "OAGROUP"
```

7.28 domain FAMONOID FreeAbelianMonoid

— FreeAbelianMonoid.input —

```
)set break resume
)sys rm -f FreeAbelianMonoid.output
)spool FreeAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeAbelianMonoid
--R FreeAbelianMonoid S: SetCategory  is a domain constructor
--R Abbreviation for FreeAbelianMonoid is FAMONOID
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FAMONOID
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (NonNegativeInteger,S) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (S,%) -> %                      ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : S -> %                       coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R mapGen : ((S -> S),%) -> %            nthFactor : (%,Integer) -> S
--R retract : % -> S                      sample : () -> %
--R size : % -> NonNegativeInteger        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R coefficient : (S,%) -> NonNegativeInteger
--R highCommonTerms : (%,%) -> % if NonNegativeInteger has OAMON
--R mapCoef : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R nthCoef : (%,Integer) -> NonNegativeInteger
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(gen: S,exp: NonNegativeInteger)
--R
--E 1

)spool
)lisp (bye)
```

———

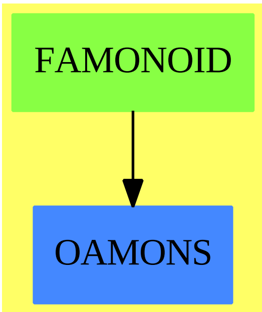
— FreeAbelianMonoid.help —

```
====================================================================
FreeAbelianMonoid examples
```

```
====================================================================

See Also:
o )show FreeAbelianMonoid
```

7.28.1 FreeAbelianMonoid (FAMONOID)



See

⇒ "ListMonoidOps" (LMOPS) 13.10.1 on page 1473
⇒ "FreeMonoid" (FMONOID) 7.32.1 on page 987
⇒ "FreeGroup" (FGROUP) 7.29.1 on page 976
⇒ "InnerFreeAbelianMonoid" (IFAMON) 10.22.1 on page 1250
⇒ "FreeAbelianGroup" (FAGROUP) 7.27.1 on page 971

Exports:

0	coefficient	coerce	hash	highCommonTerms
latex	mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size	subtractIfCan
terms	zero?	?~=?	?*?	?+?
?=?				

— **domain FAMONOID FreeAbelianMonoid** —

```
)abbrev domain FAMONOID FreeAbelianMonoid
++ Free abelian monoid on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ The free abelian monoid on a set S is the monoid of finite sums of
++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are non-negative integers. The operation is commutative.
```

```
FreeAbelianMonoid(S: SetCategory):
  FreeAbelianMonoidCategory(S, NonNegativeInteger)
    == InnerFreeAbelianMonoid(S, NonNegativeInteger, 1)
```

———-

— FAMONOID.dotabb —

```
"FAMONOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FAMONOID"]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
"FAMONOID" -> "OAMONS"
```

———-

7.29 domain FGROUP FreeGroup

— FreeGroup.input —

```
)set break resume
)sys rm -f FreeGroup.output
)spool FreeGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeGroup
--R FreeGroup S: SetCategory  is a domain constructor
--R Abbreviation for FreeGroup is FGROUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FGROUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (S,%) -> %
--R ?*? : (%,%) -> %                      ?**? : (S,Integer) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,PositiveInteger) -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : S -> %
--R coerce : % -> OutputForm              commutator : (%,%) -> %
--R conjugate : (%,%) -> %                hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R mapGen : ((S -> S),%) -> %            nthExpon : (%,Integer) -> Integer
--R nthFactor : (%,Integer) -> S          one? : % -> Boolean
```

```
--R recip : % -> Union(%,"failed")        retract : % -> S
--R sample : () -> %                      size : % -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R factors : % -> List Record(gen: S,exp: Integer)
--R mapExpon : ((Integer -> Integer),%) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

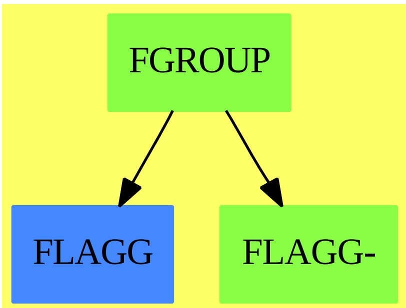
— **FreeGroup.help** —

```
====================================================================
FreeGroup examples
====================================================================

See Also:
o )show FreeGroup
```

———-

7.29.1 FreeGroup (FGROUP)



See

⇒ "ListMonoidOps" (LMOPS) 13.10.1 on page 1473
⇒ "FreeMonoid" (FMONOID) 7.32.1 on page 987
⇒ "InnerFreeAbelianMonoid" (IFAMON) 10.22.1 on page 1250
⇒ "FreeAbelianMonoid" (FAMONOID) 7.28.1 on page 974
⇒ "FreeAbelianGroup" (FAGROUP) 7.27.1 on page 971

Exports:

1	coerce	commutator	conjugate	factors
hash	inv	latex	mapExpon	mapGen
nthExpon	nthFactor	one?	recip	retract
retractIfCan	sample	size	?~=?	?**?
?^?	?*?	?/?	?=?	

— domain FGROUP FreeGroup —

```
)abbrev domain FGROUP FreeGroup
++ Free group on any set of generators
++ Author: Stephen M. Watt
++ Date Created: ???
++ Date Last Updated: 6 June 1991
++ Description:
++ The free group on a set S is the group of finite products of
++ the form \spad{reduce(*,[si ** ni])} where the si's are in S, and the ni's
++ are integers. The multiplication is not commutative.

FreeGroup(S: SetCategory): Join(Group, RetractableTo S) with
        "*":    (S, $) -> $
          ++ s * x returns the product of x by s on the left.
        "*":    ($, S) -> $
          ++ x * s returns the product of x by s on the right.
        "**"         : (S, Integer) -> $
          ++ s ** n returns the product of s by itself n times.
        size         : $ -> NonNegativeInteger
          ++ size(x) returns the number of monomials in x.
        nthExpon     : ($, Integer) -> Integer
          ++ nthExpon(x, n) returns the exponent of the n^th monomial of x.
        nthFactor    : ($, Integer) -> S
          ++ nthFactor(x, n) returns the factor of the n^th monomial of x.
        mapExpon     : (Integer -> Integer, $) -> $
          ++ mapExpon(f, a1\^e1 ... an\^en) returns
          ++ \spad{a1\^f(e1) ... an\^f(en)}.
        mapGen       : (S -> S, $) -> $
          ++ mapGen(f, a1\^e1 ... an\^en) returns
          ++ \spad{f(a1)\^e1 ... f(an)\^en}.
        factors      : $ -> List Record(gen: S, exp: Integer)
          ++ factors(a1\^e1,...,an\^en) returns \spad{[[a1, e1],...,[an, en]]}.
    == ListMonoidOps(S, Integer, 1) add
        Rep := ListMonoidOps(S, Integer, 1)

        1                       == makeUnit()
        one? f                  == empty? listOfMonoms f
        s:S ** n:Integer        == makeTerm(s, n)
        f:$ * s:S               == rightMult(f, s)
        s:S * f:$               == leftMult(s, f)
        inv f                   == reverse_! mapExpon("-", f)
        factors f               == copy listOfMonoms f
```

```
mapExpon(f, x)          == mapExpon(f, x)$Rep
mapGen(f, x)            == mapGen(f, x)$Rep
coerce(f:$):OutputForm  == outputForm(f, "*", "**", 1)

f:$ * g:$ ==
    one? f => g
    one? g => f
    r := reverse listOfMonoms f
    q := copy listOfMonoms g
    while not empty? r and not empty? q and r.first.gen = q.first.gen
        and r.first.exp = -q.first.exp repeat
             r := rest r
             q := rest q
    empty? r => makeMulti q
    empty? q => makeMulti reverse_! r
    r.first.gen = q.first.gen =>
      setlast_!(h := reverse_! r,
                        [q.first.gen, q.first.exp + r.first.exp])
      makeMulti concat_!(h, rest q)
    makeMulti concat_!(reverse_! r, q)
```

———-

— FGROUP.dotabb —

```
"FGROUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FGROUP"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FGROUP" -> "FLAGG"
"FGROUP" -> "FLAGG-"
```

———-

7.30 domain FM FreeModule

— FreeModule.input —

```
)set break resume
)sys rm -f FreeModule.output
)spool FreeModule.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show FreeModule
--R FreeModule(R: Ring,S: OrderedSet)  is a domain constructor
--R Abbreviation for FreeModule is FM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> R           leadingSupport : % -> S
--R map : ((R -> R),%) -> %               monomial : (R,S) -> %
--R reductum : % -> %                     sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

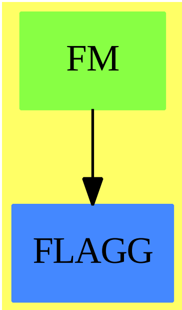
— FreeModule.help —

```
====================================================================
FreeModule examples
====================================================================

See Also:
o )show FreeModule
```

———-

7.30.1 FreeModule (FM)



See

⇒ “PolynomialRing” (PR) 17.27.1 on page 2052
⇒ “SparseUnivariatePolynomial” (SUP) 20.18.1 on page 2425
⇒ “UnivariatePolynomial” (UP) 22.4.1 on page 2784

Exports:

0	coerce	hash	latex	leadingCoefficient
leadingSupport	map	monomial	reductum	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

— domain FM FreeModule —

```
)abbrev domain FM FreeModule
++ Author: Dave Barton, James Davenport, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: BiModule(R,R)
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A bi-module is a free module
++ over a ring with generators indexed by an ordered set.
++ Each element can be expressed as a finite linear combination of
++ generators. Only non-zero terms are stored.

FreeModule(R:Ring,S:OrderedSet):
        Join(BiModule(R,R),IndexedDirectProductCategory(R,S)) with
    if R has CommutativeRing then Module(R)
 == IndexedDirectProductAbelianGroup(R,S) add
    --representations
       Term:=  Record(k:S,c:R)
       Rep:=  List Term
```

```
    --declarations
       x,y: %
       r: R
       n: Integer
       f: R -> R
       s: S
    --define
       if R has EntireRing then
         r * x  ==
             zero? r => 0
--             one? r => x
             (r = 1) => x
           --map(r*#1,x)
             [[u.k,r*u.c] for u in x ]
       else
         r * x  ==
             zero? r => 0
--             one? r => x
             (r = 1) => x
           --map(r*#1,x)
             [[u.k,a] for u in x | (a:=r*u.c) ^= 0$R]
       if R has EntireRing then
         x * r  ==
             zero? r => 0
--             one? r => x
             (r = 1) => x
           --map(r*#1,x)
             [[u.k,u.c*r] for u in x ]
       else
         x * r  ==
             zero? r => 0
--             one? r => x
             (r = 1) => x
           --map(r*#1,x)
             [[u.k,a] for u in x | (a:=u.c*r) ^= 0$R]

       coerce(x) : OutputForm ==
         null x => (0$R) :: OutputForm
         le : List OutputForm := nil
         for rec in reverse x repeat
           rec.c = 1 => le := cons(rec.k :: OutputForm, le)
           le := cons(rec.c :: OutputForm *  rec.k :: OutputForm, le)
         reduce("+",le)
```

— **FM.dotabb** —

```
"FM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FM"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FM" -> "FLAGG"
```

———-

7.31 domain FM1 FreeModule1

— FreeModule1.input —

```
)set break resume
)sys rm -f FreeModule1.output
)spool FreeModule1.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeModule1
--R FreeModule1(R: Ring,S: OrderedSet)  is a domain constructor
--R Abbreviation for FreeModule1 is FM1
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FM1
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (S,R) -> %                      ?*? : (R,S) -> %
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coefficient : (%,S) -> R
--R coefficients : % -> List R            coerce : S -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> S              map : ((R -> R),%) -> %
--R monom : (S,R) -> %                    monomial? : % -> Boolean
--R monomials : % -> List %               reductum : % -> %
--R retract : % -> S                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R leadingTerm : % -> Record(k: S,c: R)
--R listOfTerms : % -> List Record(k: S,c: R)
--R numberOfMonomials : % -> NonNegativeInteger
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

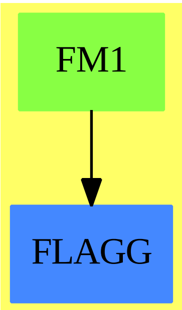
— FreeModule1.help —

```
====================================================================
FreeModule1 examples
====================================================================

See Also:
o )show FreeModule1
```

———-

7.31.1 FreeModule1 (FM1)



Exports:

0	coefficient	coefficients	coerce	hash
latex	leadingCoefficient	leadingMonomial	leadingTerm	listOfTerms
map	monom	monomial?	monomials	numberOfMonomials
reductum	retract	retractIfCan	sample	subtractIfCan
zero?	?~=?	?*?	?+?	?-?
-?	?=?			

— domain FM1 FreeModule1 —

```
)abbrev domain FM1 FreeModule1
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
```

```
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain implements linear combinations
++ of elements from the domain \spad{S} with coefficients
++ in the domain \spad{R} where \spad{S} is an ordered set
++ and \spad{R} is a ring (which may be non-commutative).
++ This domain is used by domains of non-commutative algebra such as:
++ XDistributedPolynomial, XRecursivePolynomial.

FreeModule1(R:Ring,S:OrderedSet): FMcat == FMdef where
  EX ==> OutputForm
  TERM ==> Record(k:S,c:R)

  FMcat == FreeModuleCat(R,S) with
    "*":(S,R) -> %
      ++ \spad{s*r} returns the product \spad{r*s}
      ++ used by \spadtype{XRecursivePolynomial}
  FMdef == FreeModule(R,S) add
    -- representation
      Rep := List TERM

    -- declarations
      lt: List TERM
      x : %
      r : R
      s : S

    -- define
      numberOfMonomials p ==
         # (p::Rep)

      listOfTerms(x) == x:List TERM

      leadingTerm x == x.first
      leadingMonomial x == x.first.k
      coefficients x == [t.c for t in x]
      monomials x == [ monom (t.k, t.c) for t in x]

      retractIfCan x ==
         numberOfMonomials(x) ^= 1 => "failed"
         x.first.c = 1 => x.first.k
         "failed"

      coerce(s:S):% == [[s,1$R]]
```

```
retract x ==
   (rr := retractIfCan x) case "failed" => error "FM1.retract impossible"
   rr :: S

if R has noZeroDivisors then
   r * x  ==
       r = 0 => 0
       [[u.k,r * u.c]$TERM for u in x]
   x * r  ==
       r = 0 => 0
       [[u.k,u.c * r]$TERM for u in x]
 else
   r * x  ==
       r = 0 => 0
       [[u.k,a] for u in x | not (a:=r*u.c)= 0$R]
   x * r  ==
       r = 0 => 0
       [[u.k,a] for u in x | not (a:=u.c*r)= 0$R]

r * s ==
  r = 0 => 0
  [[s,r]$TERM]

s * r ==
  r = 0 => 0
  [[s,r]$TERM]

monom(b,r):% == [[b,r]$TERM]

outTerm(r:R, s:S):EX ==
      r=1  => s::EX
      r::EX * s::EX

coerce(a:%):EX ==
      empty? a => (0$R)::EX
      reduce(_+, reverse_! [outTerm(t.c, t.k) for t in a])$List(EX)

coefficient(x,s) ==
   null x => 0$R
   x.first.k > s => coefficient(rest x,s)
   x.first.k = s => x.first.c
   0$R
```

— FM1.dotabb —

```
"FM1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FM1"]
```

```
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FM1" -> "FLAGG"
```

——–

7.32 domain FMONOID FreeMonoid

— FreeMonoid.input —

```
)set break resume
)sys rm -f FreeMonoid.output
)spool FreeMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeMonoid
--R FreeMonoid S: SetCategory  is a domain constructor
--R Abbreviation for FreeMonoid is FMONOID
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FMONOID
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (S,%) -> %
--R ?*? : (%,%) -> %                      ?**? : (%,PositiveInteger) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : S -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R hclf : (%,%) -> %                     hcrf : (%,%) -> %
--R latex : % -> String                   mapGen : ((S -> S),%) -> %
--R nthFactor : (%,Integer) -> S          one? : % -> Boolean
--R recip : % -> Union(%,"failed")        retract : % -> S
--R sample : () -> %                      size : % -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R ?**? : (S,NonNegativeInteger) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R ?^? : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Union(Record(lm: %,rm: %),"failed")
--R factors : % -> List Record(gen: S,exp: NonNegativeInteger)
--R lquo : (%,%) -> Union(%,"failed")
--R mapExpon : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
```

```
--R max : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R nthExpon : (%,Integer) -> NonNegativeInteger
--R overlap : (%,%) -> Record(lm: %,mm: %,rm: %)
--R retractIfCan : % -> Union(S,"failed")
--R rquo : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

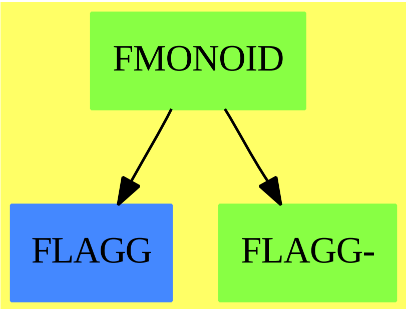
— FreeMonoid.help —

```
====================================================================
FreeMonoid examples
====================================================================

See Also:
o )show FreeMonoid
```

———-

7.32.1 FreeMonoid (FMONOID)



See

⇒ "ListMonoidOps" (LMOPS) 13.10.1 on page 1473
⇒ "FreeGroup" (FGROUP) 7.29.1 on page 976
⇒ "InnerFreeAbelianMonoid" (IFAMON) 10.22.1 on page 1250
⇒ "FreeAbelianMonoid" (FAMONOID) 7.28.1 on page 974
⇒ "FreeAbelianGroup" (FAGROUP) 7.27.1 on page 971

Exports:

1	coerce	divide	factors	hash
hclf	hcrf	latex	lquo	mapExpon
mapGen	max	min	nthExpon	nthFactor
one?	overlap	recip	rquo	retract
retractIfCan	sample	size	?~=?	?**?
?<?	?<=?	?>?	?>=?	?^?
?*?	?=?			

— domain FMONOID FreeMonoid —

```
)abbrev domain FMONOID FreeMonoid
++ Free monoid on any set of generators
++ Author: Stephen M. Watt
++ Date Created: ???
++ Date Last Updated: 6 June 1991
++ Description:
++ The free monoid on a set S is the monoid of finite products of
++ the form \spad{reduce(*,[si ** ni])} where the si's are in S, and the ni's
++ are nonnegative integers. The multiplication is not commutative.

FreeMonoid(S: SetCategory): FMcategory == FMdefinition where
    NNI ==> NonNegativeInteger
    REC ==> Record(gen: S, exp: NonNegativeInteger)
    Ex  ==> OutputForm

    FMcategory ==> Join(Monoid, RetractableTo S) with
        "*":    (S, $) -> $
          ++ s * x returns the product of x by s on the left.
        "*":    ($, S) -> $
          ++ x * s returns the product of x by s on the right.
        "**":   (S, NonNegativeInteger) -> $
          ++ s ** n returns the product of s by itself n times.
        hclf:   ($, $) -> $
          ++ hclf(x, y) returns the highest common left factor of x and y,
          ++ i.e. the largest d such that \spad{x = d a} and \spad{y = d b}.
        hcrf:   ($, $) -> $
          ++ hcrf(x, y) returns the highest common right factor of x and y,
          ++ i.e. the largest d such that \spad{x = a d} and \spad{y = b d}.
        lquo:   ($, $) -> Union($, "failed")
          ++ lquo(x, y) returns the exact left quotient of x by y i.e.
          ++ q such that \spad{x = y * q},
          ++ "failed" if x is not of the form \spad{y * q}.
        rquo:   ($, $) -> Union($, "failed")
          ++ rquo(x, y) returns the exact right quotient of x by y i.e.
          ++ q such that \spad{x = q * y},
          ++ "failed" if x is not of the form \spad{q * y}.
        divide:   ($, $) -> Union(Record(lm: $, rm: $), "failed")
          ++ divide(x, y) returns the left and right exact quotients of
```

```
          ++ x by y, i.e. \spad{[l, r]} such that \spad{x = l * y * r},
          ++ "failed" if x is not of the form \spad{l * y * r}.
        overlap: ($, $) -> Record(lm: $, mm: $, rm: $)
          ++ overlap(x, y) returns \spad{[l, m, r]} such that
          ++ \spad{x = l * m}, \spad{y = m * r} and l and r have no overlap,
          ++ i.e. \spad{overlap(l, r) = [l, 1, r]}.
        size        :   $ -> NNI
          ++ size(x) returns the number of monomials in x.
        factors      : $ -> List Record(gen: S, exp: NonNegativeInteger)
          ++ factors(a1\^e1,...,an\^en) returns \spad{[[a1, e1],...,[an, en]]}.
        nthExpon     : ($, Integer) -> NonNegativeInteger
          ++ nthExpon(x, n) returns the exponent of the n^th monomial of x.
        nthFactor    : ($, Integer) -> S
          ++ nthFactor(x, n) returns the factor of the n^th monomial of x.
        mapExpon     : (NNI -> NNI, $) -> $
          ++ mapExpon(f, a1\^e1 ... an\^en) returns \spad{a1\^f(e1) ... an\^f(en)}.
        mapGen       : (S -> S, $) -> $
          ++ mapGen(f, a1\^e1 ... an\^en) returns \spad{f(a1)\^e1 ... f(an)\^en}.
        if S has OrderedSet then OrderedSet

    FMdefinition ==> ListMonoidOps(S, NonNegativeInteger, 1) add
        Rep := ListMonoidOps(S, NonNegativeInteger, 1)

        1               == makeUnit()
        one? f          == empty? listOfMonoms f
        coerce(f:$): Ex == outputForm(f, "*", "**", 1)
        hcrf(f, g)      == reverse_! hclf(reverse f, reverse g)
        f:$ * s:S       == rightMult(f, s)
        s:S * f:$       == leftMult(s, f)
        factors f       == copy listOfMonoms f
        mapExpon(f, x)  == mapExpon(f, x)$Rep
        mapGen(f, x)    == mapGen(f, x)$Rep
        s:S ** n:NonNegativeInteger == makeTerm(s, n)

        f:$ * g:$ ==
--            one? f => g
            (f = 1) => g
--            one? g => f
            (g = 1) => f
            lg := listOfMonoms g
            ls := last(lf := listOfMonoms f)
            ls.gen = lg.first.gen =>
                setlast_!(h := copy lf,[lg.first.gen,lg.first.exp+ls.exp])
                makeMulti concat(h, rest lg)
            makeMulti concat(lf, lg)

        overlap(la, ar) ==
--            one? la or one? ar => [la, 1, ar]
            (la = 1) or (ar = 1) => [la, 1, ar]
            lla := la0 := listOfMonoms la
```

```
          lar := listOfMonoms ar
          l:List(REC) := empty()
          while not empty? lla repeat
            if lla.first.gen = lar.first.gen then
              if lla.first.exp < lar.first.exp and empty? rest lla then
                    return [makeMulti l,
                             makeTerm(lla.first.gen, lla.first.exp),
                               makeMulti concat([lar.first.gen,
                                (lar.first.exp - lla.first.exp)::NNI],
                                                            rest lar)]
              if lla.first.exp >= lar.first.exp then
                if (ru:= lquo(makeMulti rest lar,
                  makeMulti rest lla)) case $ then
                    if lla.first.exp > lar.first.exp then
                      l := concat_!(l, [lla.first.gen,
                                (lla.first.exp - lar.first.exp)::NNI])
                      m := concat([lla.first.gen, lar.first.exp],
                                                             rest lla)
                    else m := lla
                    return [makeMulti l, makeMulti m, ru::$]
            l  := concat_!(l, lla.first)
            lla := rest lla
          [makeMulti la0, 1, makeMulti lar]

      divide(lar, a) ==
--          one? a => [lar, 1]
          (a = 1) => [lar, 1]
          Na   : Integer := #(la := listOfMonoms a)
          Nlar : Integer := #(llar := listOfMonoms lar)
          l:List(REC) := empty()
          while Na <= Nlar repeat
            if llar.first.gen = la.first.gen and
               llar.first.exp >= la.first.exp then
              -- Can match a portion of this lar factor.
              -- Now match tail.
              (q:=lquo(makeMulti rest llar,makeMulti rest la))case $ =>
                 if llar.first.exp > la.first.exp then
                     l := concat_!(l, [la.first.gen,
                                (llar.first.exp - la.first.exp)::NNI])
                 return [makeMulti l, q::$]
            l    := concat_!(l, first llar)
            llar  := rest llar
            Nlar := Nlar - 1
          "failed"

      hclf(f, g) ==
          h:List(REC) := empty()
          for f0 in listOfMonoms f for g0 in listOfMonoms g repeat
              f0.gen ^= g0.gen => return makeMulti h
              h := concat_!(h, [f0.gen, min(f0.exp, g0.exp)])
```

```
            f0.exp ^= g0.exp => return makeMulti h
        makeMulti h

    lquo(aq, a) ==
        size a > #(laq := copy listOfMonoms aq) => "failed"
        for a0 in listOfMonoms a repeat
            a0.gen ^= laq.first.gen or a0.exp > laq.first.exp =>
                                                    return "failed"
            if a0.exp = laq.first.exp then laq := rest laq
            else setfirst_!(laq, [laq.first.gen,
                                 (laq.first.exp - a0.exp)::NNI])
        makeMulti laq

    rquo(qa, a) ==
        (u := lquo(reverse qa, reverse a)) case "failed" => "failed"
        reverse_!(u::$)

    if S has OrderedSet then
      a < b ==
        la := listOfMonoms a
        lb := listOfMonoms b
        na: Integer := #la
        nb: Integer := #lb
        while na > 0 and nb > 0 repeat
            la.first.gen > lb.first.gen => return false
            la.first.gen < lb.first.gen => return true
            if la.first.exp = lb.first.exp then
                la:=rest la
                lb:=rest lb
                na:=na - 1
                nb:=nb - 1
            else if la.first.exp > lb.first.exp then
                la:=concat([la.first.gen,
                       (la.first.exp - lb.first.exp)::NNI], rest lb)
                lb:=rest lb
                nb:=nb - 1
            else
                lb:=concat([lb.first.gen,
                         (lb.first.exp-la.first.exp)::NNI], rest la)
                la:=rest la
                na:=na-1
        empty? la and not empty? lb
```

— FMONOID.dotabb —

```
"FMONOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FMONOID"]
```

```
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FMONOID" -> "FLAGG-"
"FMONOID" -> "FLAGG"
```

———-

7.33 domain FNLA FreeNilpotentLie

— **FreeNilpotentLie.input** —

```
)set break resume
)sys rm -f FreeNilpotentLie.output
)spool FreeNilpotentLie.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeNilpotentLie
--R FreeNilpotentLie(n: NonNegativeInteger,class: NonNegativeInteger,R: CommutativeRing)  is
--R Abbreviation for FreeNilpotentLie is FNLA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FNLA
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antiCommutator : (%,%) -> %
--R associator : (%,%,%) -> %             coerce : % -> OutputForm
--R commutator : (%,%) -> %               deepExpand : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      shallowExpand : % -> OutputForm
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R dimension : () -> NonNegativeInteger
--R generator : NonNegativeInteger -> %
--R leftPower : (%,PositiveInteger) -> %
--R plenaryPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

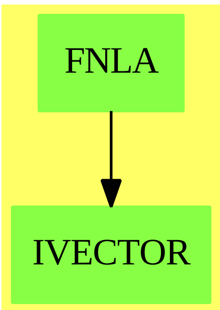
— FreeNilpotentLie.help —

```
====================================================================
FreeNilpotentLie examples
====================================================================

See Also:
o )show FreeNilpotentLie
```

———-

7.33.1 FreeNilpotentLie (FNLA)



See

⇒ "OrdSetInts" (OSI) 16.20.1 on page 1825
⇒ "Commutator" (COMM) 4.7.1 on page 395

Exports:

0	antiCommutator	associator	coerce	commutator
deepExpand	dimension	generator	hash	latex
leftPower	plenaryPower	rightPower	sample	shallowExpand
subtractIfCan	zero?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

— domain FNLA FreeNilpotentLie —

```
)abbrev domain FNLA FreeNilpotentLie
++ Author: Larry Lambe
++ Date Created: July 1988
```

```
++ Date Last Updated: March 13 1991
++ Related Constructors: OrderedSetInts, Commutator
++ AMS Classification: Primary 17B05, 17B30; Secondary 17A50
++ Keywords: free Lie algebra, Hall basis, basic commutators
++ Related Constructors:  HallBasis, FreeMod, Commutator, OrdSetInts
++ Description:
++ Generate the Free Lie Algebra over a ring R with identity;
++ A P. Hall basis is generated by a package call to HallBasis.

FreeNilpotentLie(n:NNI,class:NNI,R: CommutativeRing): Export == Implement where
   B   ==> Boolean
   Com ==> Commutator
   HB  ==> HallBasis
   I   ==> Integer
   NNI ==> NonNegativeInteger
   O   ==> OutputForm
   OSI ==> OrdSetInts
   FM  ==> FreeModule(R,OSI)
   VI  ==> Vector Integer
   VLI ==> Vector List Integer
   lC  ==> leadingCoefficient
   lS  ==> leadingSupport

   Export ==> NonAssociativeAlgebra(R) with
     dimension : () -> NNI
       ++ dimension() is the rank of this Lie algebra
     deepExpand    : %   -> O
       ++ deepExpand(x) is not documented
     shallowExpand    : %   -> O
       ++ shallowExpand(x) is not documented
     generator : NNI -> %
       ++ generator(i) is the ith Hall Basis element

   Implement ==> FM add
     Rep := FM
     f,g : %

     coms:VLI
     coms := generate(n,class)$HB

     dimension == #coms

     have : (I,I) -> %
       -- have(left,right) is a lookup function for basic commutators
       -- already generated; if the nth basic commutator is
       -- [left,wt,right], then have(left,right) = n
     have(i,j) ==
        wt:I := coms(i).2 + coms(j).2
        wt > class => 0
        lo:I := 1
```

```
     hi:I := dimension
     while hi-lo > 1 repeat
       mid:I := (hi+lo) quo 2
       if coms(mid).2 < wt then lo := mid else hi := mid
     while coms(hi).1 < i repeat hi := hi + 1
     while coms(hi).3 < j repeat hi := hi + 1
     monomial(1,hi::OSI)$FM

  generator(i) ==
    i > dimension => 0$Rep
    monomial(1,i::OSI)$FM

  putIn : I -> %
  putIn(i) ==
    monomial(1$R,i::OSI)$FM

  brkt : (I,%) -> %
  brkt(k,f) ==
    f = 0 => 0
    dg:I := value lS f
    reductum(f) = 0 =>
      k = dg  => 0
      k > dg  => -lC(f)*brkt(dg, putIn(k))
      inHallBasis?(n,k,dg,coms(dg).1) => lC(f)*have(k, dg)
      lC(f)*( brkt(coms(dg).1, _
       brkt(k,putIn coms(dg).3)) - brkt(coms(dg).3, _
        brkt(k,putIn coms(dg).1) ))
    brkt(k,monomial(lC f,lS f)$FM)+brkt(k,reductum f)

  f*g ==
    reductum(f) = 0 =>
      lC(f)*brkt(value(lS f),g)
    monomial(lC f,lS f)$FM*g + reductum(f)*g

  Fac : I -> Com
    -- an auxilliary function used for output of Free Lie algebra
    -- elements (see expand)
  Fac(m) ==
    coms(m).1 = 0 => mkcomm(m)$Com
    mkcomm(Fac coms(m).1, Fac coms(m).3)

  shallowE : (R,OSI) -> O
  shallowE(r,s) ==
    k := value s
    r = 1 =>
      k <= n => s::O
      mkcomm(mkcomm(coms(k).1)$Com,mkcomm(coms(k).3)$Com)$Com::O
    k <= n => r::O * s::O
    r::O * mkcomm(mkcomm(coms(k).1)$Com,mkcomm(coms(k).3)$Com)$Com::O
```

```
    shallowExpand(f) ==
      f = 0           => 0::O
      reductum(f) = 0 => shallowE(lC f,lS f)
      shallowE(lC f,lS f) + shallowExpand(reductum f)

    deepExpand(f) ==
      f = 0          => 0::O
      reductum(f) = 0 =>
        lC(f)=1 => Fac(value(lS f))::O
        lC(f)::O * Fac(value(lS f))::O
      lC(f)=1 => Fac(value(lS f))::O + deepExpand(reductum f)
      lC(f)::O * Fac(value(lS f))::O + deepExpand(reductum f)
```

— FNLA.dotabb —

```
"FNLA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FNLA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"FNLA" -> "IVECTOR"
```

7.34 domain FPARFRAC FullPartialFractionExpansion

— FullPartialFractionExpansion.input —

```
)set break resume
)sys rm -f FullPartialFractionExpansion.output
)spool FullPartialFractionExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
Fx := FRAC UP(x, FRAC INT)
--R
--R
--R   (1)  Fraction UnivariatePolynomial(x,Fraction Integer)
--R                                                                     Type: Domain
--E 1

--S 2 of 16
f : Fx := 36 / (x**5-2*x**4-2*x**3+4*x**2+x-2)
--R
```

```
--R
--R                     36
--R   (2)  ----------------------------
--R         5     4     3     2
--R        x  - 2x  - 2x  + 4x  + x - 2
--R                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 2

--S 3 of 16
g := fullPartialFraction f
--R
--R
--R          4       4        --+      - 3%A - 6
--R   (3)  ----- - ----- +    >        ---------
--R        x - 2   x + 1      --+              2
--R                          2         (x - %A)
--R                       %A  - 1= 0
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 3

--S 4 of 16
g :: Fx
--R
--R
--R                     36
--R   (4)  ----------------------------
--R         5     4     3     2
--R        x  - 2x  - 2x  + 4x  + x - 2
--R                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 4

--S 5 of 16
g5 := D(g, 5)
--R
--R
--R             480        480        --+      2160%A + 4320
--R   (5)  - -------- + -------- +    >        -------------
--R                 6          6      --+                  7
--R          (x - 2)    (x + 1)      2           (x - %A)
--R                               %A  - 1= 0
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 5

--S 6 of 16
f5 := D(f, 5)
--R
--R
--R   (6)
--R                 10           9            8            7            6
--R       - 544320x   + 4354560x  - 14696640x  + 28615680x  - 40085280x
```

```
--R     +
--R                5            4             3            2
--R       46656000x  - 39411360x  + 18247680x  - 5870880x  + 3317760x + 246240
--R  /
--R        20      19      18      17       16       15       14        13
--R       x   - 12x   + 53x   - 76x   - 159x   + 676x   - 391x   - 1596x
--R     +
--R            12        11         10        9        8        7        6        5
--R       2527x   + 1148x   - 4977x   + 1372x  + 4907x  - 3444x  - 2381x  + 2924x
--R     +
--R           4        3       2
--R       276x  - 1184x  + 208x  + 192x - 64
--R                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 6

--S 7 of 16
g5::Fx - f5
--R
--R
--R   (7)  0
--R                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 7

--S 8 of 16
f : Fx := (x**5 * (x-1)) / ((x**2 + x + 1)**2 * (x-2)**3)
--R
--R
--R                       6    5
--R                      x  - x
--R   (8)  -----------------------------------
--R         7     6     5     3     2
--R        x  - 4x  + 3x  + 9x  - 6x  - 4x - 8
--R                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 8

--S 9 of 16
g := fullPartialFraction f
--R
--R
--R   (9)
--R      1952       464        32                           179       135
--R      ----       ---        --                         - ---- %A + ----
--R      2401       343        49            --+           2401      2401
--R     ------ + -------- + -------- +       >           ----------------
--R      x - 2          2          3         --+              x - %A
--R              (x - 2)    (x - 2)       2
--R                                    %A  + %A + 1= 0
--R   +
--R                       37         20
--R                      ---- %A + ----
```

```
--R           --+        1029      1029
--R           >         --------------
--R           --+                    2
--R       2                   (x - %A)
--R     %A  + %A + 1= 0
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 9

--S 10 of 16
g :: Fx - f
--R
--R
--R   (10)  0
--R                         Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 10

--S 11 of 16
f : Fx := (2*x**7-7*x**5+26*x**3+8*x) / (x**8-5*x**6+6*x**4+4*x**2-8)
--R
--R
--R             7     5      3
--R           2x  - 7x  + 26x  + 8x
--R   (11)  ------------------------
--R          8     6     4     2
--R         x  - 5x  + 6x  + 4x  - 8
--R                         Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 11

--S 12 of 16
g := fullPartialFraction f
--R
--R
--R                          1                                           1
--R                          -                                           -
--R            --+         2        --+          1          --+         2
--R   (12)     >        ------ +    >        --------- +    >        ------
--R            --+      x - %A      --+              3      --+      x - %A
--R           2                   2          (x - %A)      2
--R         %A  - 2= 0          %A  - 2= 0              %A  + 1= 0
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 12

--S 13 of 16
g :: Fx - f
--R
--R
--R   (13)  0
--R                         Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 13
```

```
--S 14 of 16
f:Fx := x**3 / (x**21 + 2*x**20 + 4*x**19 + 7*x**18 + 10*x**17 + 17*x**16 + 22*x**15 + 30*x**
--R
--R
--R   (14)
--R      3
--R     x
--R  /
--R        21     20     19     18      17      16      15      14      13      12
--R       x   + 2x   + 4x   + 7x   + 10x   + 17x   + 22x   + 30x   + 36x   + 40x
--R     +
--R          11      10      9      8      7      6      5      4      3     2
--R       47x   + 46x   + 49x  + 43x  + 38x  + 32x  + 23x  + 19x  + 10x  + 7x  + 2x
--R     +
--R       1
--R                        Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 14

--S 15 of 16
g := fullPartialFraction f
--R
--R
--R   (15)
--R                  1                       1      19
--R                  - %A                    - %A - --
--R        --+       2             --+       9      27
--R         >       ------ +        >        ---------
--R        --+      x - %A         --+         x - %A
--R       2                   2
--R     %A  + 1= 0          %A  + %A + 1= 0
--R   +
--R                       1       1
--R                       -- %A - --
--R           --+         27      27
--R            >          ----------
--R           --+                 2
--R       2                (x - %A)
--R     %A  + %A + 1= 0
--R   +
--R     SIGMA
--R          5     2
--R        %A  + %A  + 1= 0
--R    ,
--R                96556567040   4   420961732891   3    59101056149   2
--R             - ------------ %A  + ------------ %A  - ------------ %A
--R               912390759099       912390759099       912390759099
--R          +
--R               373545875923      529673492498
--R             - ------------ %A + ------------
--R               912390759099      912390759099
```

```
--R       /
--R          x - %A
--R   +
--R     SIGMA
--R          5     2
--R        %A  + %A  + 1= 0
--R    ,
--R           5580868   4    2024443   3    4321919   2    84614        5070620
--R        - -------- %A  - -------- %A  + -------- %A  - ------- %A - --------
--R          94070601       94070601       94070601       1542141      94070601
--R        --------------------------------------------------------------------
--R                                                2
--R                                        (x - %A)
--R   +
--R     SIGMA
--R          5     2
--R        %A  + %A  + 1= 0
--R    ,
--R          1610957   4    2763014   3    2016775   2    266953        4529359
--R        -------- %A  + -------- %A  - -------- %A  + -------- %A + --------
--R        94070601       94070601       94070601       94070601      94070601
--R        -------------------------------------------------------------------
--R                                               3
--R                                       (x - %A)
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 15

--S 16 of 16
g :: Fx - f
--R
--R
--R   (16)  0
--R                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 16
)spool
)lisp (bye)
```

— **FullPartialFractionExpansion.help** —

```
====================================================================
FullPartialFractionExpansion expansion
====================================================================

The domain FullPartialFractionExpansion implements factor-free
conversion of quotients to full partial fractions.

Our examples will all involve quotients of univariate polynomials
```

with rational number coefficients.

```
  Fx := FRAC UP(x, FRAC INT)
    Fraction UnivariatePolynomial(x,Fraction Integer)
                    Type: Domain
```

Here is a simple-looking rational function.

```
  f : Fx := 36 / (x**5-2*x**4-2*x**3+4*x**2+x-2)
                 36
    ----------------------------
     5     4     3     2
    x  - 2x  - 2x  + 4x  + x - 2
                    Type: Fraction UnivariatePolynomial(x,Fraction Integer)
```

We use fullPartialFraction to convert it to an object of type FullPartialFractionExpansion.

```
  g := fullPartialFraction f
      4       4        --+      - 3%A - 6
    ----- - ----- +    >        ---------
    x - 2   x + 1      --+              2
                      2         (x - %A)
                    %A  - 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
                                   UnivariatePolynomial(x,Fraction Integer))
```

Use a coercion to change it back into a quotient.

```
  g :: Fx
                 36
    ----------------------------
     5     4     3     2
    x  - 2x  - 2x  + 4x  + x - 2
                  Type: Fraction UnivariatePolynomial(x,Fraction Integer)
```

Full partial fractions differentiate faster than rational functions.

```
  g5 := D(g, 5)
           480        480        --+      2160%A + 4320
    - -------- + -------- +    >        -------------
             6          6      --+                  7
      (x - 2)    (x + 1)      2           (x - %A)
                            %A  - 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
                                   UnivariatePolynomial(x,Fraction Integer))
```

```
  f5 := D(f, 5)
                10            9             8             7             6
       - 544320x   + 4354560x  - 14696640x  + 28615680x  - 40085280x
```

```
     +
               5            4            3           2
       46656000x  - 39411360x  + 18247680x  - 5870880x  + 3317760x + 246240
  /
         20      19      18      17       16       15       14        13
       x   - 12x   + 53x   - 76x   - 159x   + 676x   - 391x   - 1596x
     +
             12        11        10         9        8        7        6        5
       2527x   + 1148x   - 4977x   + 1372x  + 4907x  - 3444x  - 2381x  + 2924x
     +
           4        3       2
       276x  - 1184x  + 208x  + 192x - 64
                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
```

We can check that the two forms represent the same function.

```
  g5::Fx - f5
    0
                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
```

Here are some examples that are more complicated.

```
  f : Fx := (x**5 * (x-1)) / ((x**2 + x + 1)**2 * (x-2)**3)
                  6    5
                 x  - x
    -----------------------------------
     7     6     5     3     2
    x  - 4x  + 3x  + 9x  - 6x  - 4x - 8
                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)

  g := fullPartialFraction f
     1952       464        32                          179       135
     ----       ---        --                        - ---- %A + ----
     2401       343        49          --+             2401      2401
    ------ + -------- + -------- +      >          ----------------
     x - 2          2          3       --+              x - %A
             (x - 2)    (x - 2)       2
                                   %A  + %A + 1= 0
   +
                      37         20
                     ---- %A + ----
          --+        1029      1029
           >         --------------
          --+                    2
      2                 (x - %A)
    %A  + %A + 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
                                   UnivariatePolynomial(x,Fraction Integer))

  g :: Fx - f
```

```
   0
                   Type: Fraction UnivariatePolynomial(x,Fraction Integer)

 f : Fx := (2*x**7-7*x**5+26*x**3+8*x) / (x**8-5*x**6+6*x**4+4*x**2-8)
      7     5      3
    2x  - 7x  + 26x  + 8x
   ------------------------
    8     6     4     2
   x  - 5x  + 6x  + 4x  - 8
                   Type: Fraction UnivariatePolynomial(x,Fraction Integer)

 g := fullPartialFraction f
                 1                                            1
                 -                                            -
     --+         2        --+          1          --+         2
     >        ------ +    >        --------- +    >        ------
     --+      x - %A      --+              3      --+      x - %A
    2                    2         (x - %A)      2
   %A  - 2= 0           %A  - 2= 0              %A  + 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
                                   UnivariatePolynomial(x,Fraction Integer))

 g :: Fx - f
   0
                  Type: Fraction UnivariatePolynomial(x,Fraction Integer)

 f:Fx := x**3 / (x**21 + 2*x**20 + 4*x**19 + 7*x**18 + 10*x**17 + 17*x**16 + 22*x**15 + 30*x
    3
   x
 /
      21     20      19      18       17       16       15       14       13       12
     x   + 2x   + 4x   + 7x   + 10x   + 17x   + 22x   + 30x   + 36x   + 40x
   +
        11       10       9       8       7       6       5       4       3      2
    47x   + 46x   + 49x  + 43x  + 38x  + 32x  + 23x  + 19x  + 10x  + 7x  + 2x
   +
     1
                   Type: Fraction UnivariatePolynomial(x,Fraction Integer)

 g := fullPartialFraction f
                1                        1      19
                - %A                     - %A - --
      --+       2            --+         9      27
      >       ------ +       >          ---------
      --+     x - %A         --+          x - %A
     2                      2
    %A  + 1= 0            %A  + %A + 1= 0
  +
                       1       1
                      -- %A - --
```

```
          --+        27      27
          >          ----------
          --+                2
      2               (x - %A)
    %A  + %A + 1= 0
  +
    SIGMA
         5     2
      %A  + %A  + 1= 0
   ,
             96556567040   4   420961732891   3    59101056149   2
           - ------------ %A  + ------------ %A  - ------------ %A
             912390759099       912390759099       912390759099
         +
             373545875923      529673492498
           - ------------ %A + ------------
             912390759099      912390759099
       /
         x - %A
  +
    SIGMA
         5     2
      %A  + %A  + 1= 0
   ,
         5580868   4    2024443   3    4321919   2    84614        5070620
       - -------- %A  - -------- %A  + -------- %A  - ------- %A - --------
         94070601       94070601       94070601       1542141      94070601
       -----------------------------------------------------------------------
                                                2
                                         (x - %A)
  +
    SIGMA
         5     2
      %A  + %A  + 1= 0
   ,
       1610957   4    2763014   3    2016775   2    266953        4529359
       -------- %A  + -------- %A  - -------- %A  + -------- %A + --------
       94070601       94070601       94070601       94070601      94070601
       ----------------------------------------------------------------------
                                               3
                                        (x - %A)
Type: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
```

This verification takes much longer than the conversion to partial fractions.

```
  g :: Fx - f
    0
                      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
```

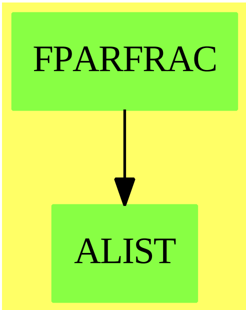
Use PartialFraction for standard partial fraction decompositions.

```
For more information, see the paper: Bronstein, M and Salvy, B.
"Full Partial Fraction Decomposition of Rational Functions,"
Proceedings of ISSAC'93, Kiev, ACM Press.

See Also:
o )help PartialFraction
o )show FullPartialFractionExpansion
```

———-

7.34.1 FullPartialFractionExpansion (FPARFRAC)



Exports:

coerce	construct	convert	D	differentiate
hash	latex	polyPart	fracPart	fullPartialFraction
?~=?	?+?	?=?		

— **domain FPARFRAC FullPartialFractionExpansion** —

```
)abbrev domain FPARFRAC FullPartialFractionExpansion
++ Author: Manuel Bronstein
++ Date Created: 9 December 1992
++ Date Last Updated: 6 October 1993
++ References: M.Bronstein & B.Salvy,
++             Full Partial Fraction Decomposition of Rational Functions,
++             in Proceedings of ISSAC'93, Kiev, ACM Press.
++ Description:
++ Full partial fraction expansion of rational functions

FullPartialFractionExpansion(F, UP): Exports == Implementation where
  F  : Join(Field, CharacteristicZero)
  UP : UnivariatePolynomialCategory F
```

```
  N   ==> NonNegativeInteger
  Q   ==> Fraction Integer
  O   ==> OutputForm
  RF  ==> Fraction UP
  SUP ==> SparseUnivariatePolynomial RF
  REC ==> Record(exponent: N, center: UP, num: UP)
  ODV ==> OrderlyDifferentialVariable Symbol
  ODP ==> OrderlyDifferentialPolynomial UP
  ODF ==> Fraction ODP
  FPF ==> Record(polyPart: UP, fracPart: List REC)

  Exports ==> Join(SetCategory, ConvertibleTo RF)  with
    "+":                 (UP, $) -> $
      ++ p + x returns the sum of p and x
    fullPartialFraction: RF -> $
      ++ fullPartialFraction(f) returns \spad{[p, [[j, Dj, Hj]...]]} such that
      ++ \spad{f = p(x) + sum_{[j,Dj,Hj] in l} sum_{Dj(a)=0} Hj(a)/(x - a)\^j}.
    polyPart:            $ -> UP
      ++ polyPart(f) returns the polynomial part of f.
    fracPart:            $  -> List REC
      ++ fracPart(f) returns the list of summands of the fractional part of f.
    construct:           List REC -> $
      ++ construct(l) is the inverse of fracPart.
    differentiate:       $ -> $
      ++ differentiate(f) returns the derivative of f.
    D:                    $ -> $
      ++ D(f) returns the derivative of f.
    differentiate:       ($, N) -> $
      ++ differentiate(f, n) returns the n-th derivative of f.
    D: ($, NonNegativeInteger) -> $
      ++ D(f, n) returns the n-th derivative of f.

  Implementation ==> add
    Rep := FPF

    fullParFrac: (UP, UP, UP, N) -> List REC
    outputexp  : (O, N) -> O
    output     : (N, UP, UP) -> O
    REC2RF     : (UP, UP, N) -> RF
    UP2SUP     : UP -> SUP
    diffrec    : REC -> REC
    FP2O       : List REC -> O

-- create a differential variable
    u  := new()$Symbol
    u0 := makeVariable(u, 0)$ODV
    alpha := u::O
    x  := monomial(1, 1)$UP
    xx := x::O
    zr := (0$N)::O
```

```
    construct l     == [0, l]
    D r             == differentiate r
    D(r, n)         == differentiate(r,n)
    polyPart f      == f.polyPart
    fracPart f      == f.fracPart
    p:UP + f:$      == [p + polyPart f, fracPart f]

    differentiate f ==
      differentiate(polyPart f) + construct [diffrec rec for rec in fracPart f]

    differentiate(r, n) ==
      for i in 1..n repeat r := differentiate r
      r

-- diffrec(sum_{rec.center(a) = 0} rec.num(a) / (x - a)^e) =
--         sum_{rec.center(a) = 0} -e rec.num(a) / (x - a)^{e+1}
--                where e = rec.exponent
    diffrec rec ==
      e := rec.exponent
      [e + 1, rec.center, - e * rec.num]

    convert(f:$):RF ==
      ans := polyPart(f)::RF
      for rec in fracPart f repeat
        ans := ans + REC2RF(rec.center, rec.num, rec.exponent)
      ans

    UP2SUP p == map((z1:F):RF +-> z1::UP::RF, p)_
        $UnivariatePolynomialCategoryFunctions2(F, UP, RF, SUP)

    -- returns Trace_k^k(a) (h(a) / (x - a)^n)  where d(a) = 0
    REC2RF(d, h, n) ==
--      one?(m := degree d) =>
      ((m := degree d) = 1) =>
        a   := - (leadingCoefficient reductum d) / (leadingCoefficient d)
        h(a)::UP / (x - a::UP)**n
      dd  := UP2SUP d
      hh  := UP2SUP h
      aa  := monomial(1, 1)$SUP
      p   := (x::RF::SUP - aa)**n rem dd
      rec := extendedEuclidean(p, dd, hh)::Record(coef1:SUP, coef2:SUP)
      t   := rec.coef1     -- we want Trace_k^k(a)(t) now
      ans := coefficient(t, 0)
      for i in 1..degree(d)-1 repeat
        t   := (t * aa) rem dd
        ans := ans + coefficient(t, i)
      ans

    fullPartialFraction f ==
```

```
  qr := divide(numer f, d := denom f)
  qr.quotient + construct concat
               [fullParFrac(qr.remainder, d, rec.factor, rec.exponent::N)
                                   for rec in factors squareFree denom f]

fullParFrac(a, d, q, n) ==
  ans:List REC := empty()
  em := e := d quo (q ** n)
  rec := extendedEuclidean(e, q, 1)::Record(coef1:UP,coef2:UP)
  bm := b := rec.coef1                  -- b = inverse of e modulo q
  lvar:List(ODV) := [u0]
  um := 1::ODP
  un := (u1 := u0::ODP)**n
  lval:List(UP)  := [q1 := q := differentiate(q0 := q)]
  h:ODF := a::ODP / (e * un)
  rec := extendedEuclidean(q1, q0, 1)::Record(coef1:UP,coef2:UP)
  c := rec.coef1                        -- c = inverse of q' modulo q
  cm := 1::UP
  cn  := (c ** n) rem q0
  for m in 1..n repeat
    p    := retract(em * un * um * h)@ODP
    pp   := retract(eval(p, lvar, lval))@UP
    h    := inv(m::Q) * differentiate h
    q    := differentiate q
    lvar := concat(makeVariable(u, m), lvar)
    lval := concat(inv((m+1)::F) * q, lval)
    qq   := q0 quo gcd(pp, q0)                    -- new center
    if (degree(qq) > 0) then
      ans  := concat([(n + 1 - m)::N, qq, (pp * bm * cn * cm) rem qq], ans)
    cm   := (c * cm) rem q0     -- cm = c**m modulo q now
    um   := u1 * um             -- um = u**m now
    em   := e * em              -- em = e**{m+1} now
    bm   := (b * bm) rem q0     -- bm = b**{m+1} modulo q now
  ans

coerce(f:$):O ==
  ans := FP2O(l := fracPart f)
  zero?(p := polyPart f) =>
    empty? l => (0$N)::O
    ans
  p::O + ans

FP2O l ==
  empty? l => empty()
  rec := first l
  ans := output(rec.exponent, rec.center, rec.num)
  for rec in rest l repeat
    ans := ans + output(rec.exponent, rec.center, rec.num)
  ans
```

```
    output(n, d, h) ==
--      one? degree d =>
      (degree d) = 1 =>
        a := - leadingCoefficient(reductum d) / leadingCoefficient(d)
        h(a)::O / outputexp((x - a::UP)::O, n)
      sum(outputForm(makeSUP h, alpha) / outputexp(xx - alpha, n),
          outputForm(makeSUP d, alpha) = zr)

    outputexp(f, n) ==
--      one? n => f
      (n = 1) => f
      f ** (n::O)
```

———–

— FPARFRAC.dotabb —

```
"FPARFRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FPARFRAC"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FPARFRAC" -> "ALIST"
```

———–

7.35 domain FUNCTION FunctionCalled

— FunctionCalled.input —

```
)set break resume
)sys rm -f FunctionCalled.output
)spool FunctionCalled.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FunctionCalled
--R FunctionCalled f: Symbol  is a domain constructor
--R Abbreviation for FunctionCalled is FUNCTION
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FUNCTION
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean              coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
```

```
--R name : % -> Symbol                    ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

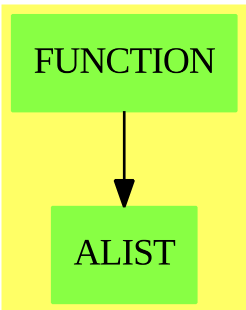
— FunctionCalled.help —

```
====================================================================
FunctionCalled examples
====================================================================

See Also:
o )show FunctionCalled
```

———-

7.35.1 FunctionCalled (FUNCTION)



Exports:
coerce hash latex name ?=? ?~=?

— domain FUNCTION FunctionCalled —

```
)abbrev domain FUNCTION FunctionCalled
++ Author: Mark Botch
++ Description:
++ This domain implements named functions

FunctionCalled(f:Symbol): SetCategory with
```

```
    name: % -> Symbol
      ++ name(x) returns the symbol
  == add
   name r                  == f
   coerce(r:%):OutputForm == f::OutputForm
   x = y                   == true
   latex(x:%):String       == latex f
```

———-

— **FUNCTION.dotabb** —

```
"FUNCTION" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FUNCTION"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FUNCTION" -> "ALIST"
```

———-

Chapter 8

Chapter G

8.1 domain GDMP GeneralDistributedMultivariatePolynomial

— GeneralDistributedMultivariatePolynomial.input —

```
)set break resume
)sys rm -f GeneralDistributedMultivariatePolynomial.output
)spool GeneralDistributedMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
--R
--R                                                                   Type: Void
--E 1

--S 2 of 10
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
--R
--R
--R                 2       2
--R   (2)  - 4z + 4y x + 16x  + 1
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 2

--S 3 of 10
d2 := 2*z*y**2 + 4*x + 1
--R
--R
```

```
--R            2
--R   (3)  2z y  + 4x + 1
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 3

--S 4 of 10
d3 := 2*z*x**2 - 2*y**2 - x
--R
--R
--R            2     2
--R   (4)  2z x  - 2y  - x
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 4

--S 5 of 10
groebner [d1,d2,d3]
--R
--R
--R   (5)
--R        1568  6   1264  5    6   4   182  3   2047  2    103      2857
--R   [z - ---- x  - ---- x  + --- x  + --- x  - ---- x  - ---- x - -----,
--R        2745       305      305      549       610      2745     10980
--R     2    112  6    84  5   1264  4    13  3    84  2   1772       2
--R    y  + ---- x  - --- x  - ---- x  - --- x  + --- x  + ---- x + ----,
--R         2745      305       305      549      305      2745     2745
--R     7   29  6   17  4   11  3    1  2   15     1
--R    x  + -- x  - -- x  - -- x  + -- x  + -- x + -]
--R          4      16       8      32      16     4
--R       Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 5

--S 6 of 10
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
--R
--R                                                                Type: Void
--E 6

--S 7 of 10
n1 := d1
--R
--R
--R          2       2
--R   (7)  4y x + 16x  - 4z + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 7

--S 8 of 10
n2 := d2
--R
--R
```

```
--R            2
--R   (8)  2z y  + 4x + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 8

--S 9 of 10
n3 := d3
--R
--R
--R            2     2
--R   (9)  2z x  - 2y  - x
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 9

--S 10 of 10
groebner [n1,n2,n3]
--R
--R
--R   (10)
--R     4     3   3  2   1     1   4   29  3   1  2   7        9     1
--R   [y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
--R               2      2     8        4      8      4       16     4
--R       2        1   2      2       1     2    2   1
--R    z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
--R                2                  4              2
--R     2     2     2   1     3
--R    z  - 4y  + 2x  - - z - - x]
--R                     4     2
--RType: List HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 10
)spool
)lisp (bye)
```

— GeneralDistributedMultivariatePolynomial.help —

```
====================================================================
MultivariatePolynomial
DistributedMultivariatePolynomial
HomogeneousDistributedMultivariatePolynomial
GeneralDistributedMultivariatePolynomial
====================================================================

DistributedMultivariatePolynomial which is abbreviated as DMP and
HomogeneousDistributedMultivariatePolynomial, which is abbreviated
as HDMP, are very similar to MultivariatePolynomial except that
they are represented and displayed in a non-recursive manner.
```

```
  (d1,d2,d3) : DMP([z,y,x],FRAC INT)
                      Type: Void
```

The constructor DMP orders its monomials lexicographically while HDMP orders them by total order refined by reverse lexicographic order.

```
  d1 := -4*z + 4*y**2*x + 16*x**2 + 1
             2       2
   - 4z + 4y x + 16x  + 1
            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  d2 := 2*z*y**2 + 4*x + 1
        2
   2z y  + 4x + 1
            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  d3 := 2*z*x**2 - 2*y**2 - x
        2     2
   2z x  - 2y  - x
            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

These constructors are mostly used in Groebner basis calculations.

```
  groebner [d1,d2,d3]
        1568  6   1264  5    6   4   182  3   2047  2    103      2857
   [z - ---- x  - ---- x  + --- x  + --- x  - ---- x  - ---- x - -----,
        2745       305      305      549       610      2745     10980
     2    112  6    84  5   1264  4    13  3    84  2   1772       2
    y  + ---- x  - --- x  - ---- x  - --- x  + --- x  + ---- x + ----,
         2745      305       305      549      305      2745     2745
     7   29  6   17  4   11  3    1  2   15     1
    x  + -- x  - -- x  - -- x  + -- x  + -- x + -]
          4      16       8      32      16     4
       Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  (n1,n2,n3) : HDMP([z,y,x],FRAC INT)
                      Type: Void

  n1 := d1
     2       2
   4y x + 16x  - 4z + 1
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  n2 := d2
        2
   2z y  + 4x + 1
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  n3 := d3
```

```
      2     2
   2z x  - 2y  - x
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

Note that we get a different Groebner basis when we use the HDMP
polynomials, as expected.

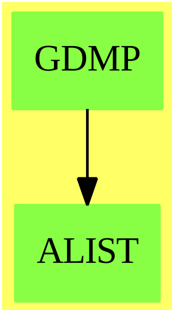
```
  groebner [n1,n2,n3]
    4     3   3  2   1     1   4   29  3   1  2   7        9     1
   [y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
             2      2     8        4      8      4       16     4
      2        1   2      2       1     2    2   1
    z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
             2                   4             2
     2     2     2   1     3
    z  - 4y  + 2x  - - z - - x]
                     4     2
      Type: List HomogeneousDistributedMultivariatePolynomial([z,y,x],
                                                     Fraction Integer)
```

GeneralDistributedMultivariatePolynomial is somewhat more flexible in
the sense that as well as accepting a list of variables to specify the
variable ordering, it also takes a predicate on exponent vectors to
specify the term ordering. With this polynomial type the user can
experiment with the effect of using completely arbitrary term orderings.
This flexibility is mostly important for algorithms such as Groebner
basis calculations which can be very sensitive to term ordering.

See Also:
o)help Polynomial
o)help UnivariatePolynomial
o)help MultivariatePolynomial
o)help HomogeneousDistributedMultivariatePolynomial
o)help DistributedMultivariatePolynomial
o)show GeneralDistributedMultivariatePolynomial

———

8.1.1 GeneralDistributedMultivariatePolynomial (GDMP)



See

⇒ "DistributedMultivariatePolynomial" (DMP) 5.13.1 on page 557
⇒ "HomogeneousDistributedMultivariatePolynomial" (HDMP) 9.10.1 on page 1145

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	D
degree	differentiate	discriminant
eval	exquo	factor
factorPolynomial	factorSquareFreePolynomial	gcd
gcdPolynomial	ground	ground?
hash	isExpt	isPlus
isTimes	latex	lcm
leadingCoefficient	leadingMonomial	mainVariable
map	mapExponents	max
min	minimumDegree	monicDivide
monomial	monomial?	monomials
multivariate	numberOfMonomials	one?
patternMatch	pomopo!	prime?
primitiveMonomials	primitivePart	recip
reducedSystem	reductum	reorder
resultant	retract	retractIfCan
sample	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
zero?	?*?	?**?
?+?	?-?	-?
?=?	?~=?	?<?
?<=?	?>?	?>=?
?^?		

— domain GDMP GeneralDistributedMultivariatePolynomial —

```
)abbrev domain GDMP GeneralDistributedMultivariatePolynomial
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd, leadingCoefficient
++ Related Constructors: DistributedMultivariatePolynomial,
++ HomogeneousDistributedMultivariatePolynomial
++ Also See: Polynomial
++ AMS Classifications:
++ Keywords: polynomial, multivariate, distributed
++ References:
++ Description:
++ This type supports distributed multivariate polynomials
++ whose variables are from a user specified list of symbols.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.
++ The term ordering is specified by its third parameter.
++ Suggested types which define term orderings include:
++ \spadtype{DirectProduct}, \spadtype{HomogeneousDirectProduct},
++ \spadtype{SplitHomogeneousDirectProduct} and finally
++ \spadtype{OrderedDirectProduct} which accepts an arbitrary user
++ function to define a term ordering.

GeneralDistributedMultivariatePolynomial(vl,R,E): public == private where
  vl: List Symbol
  R: Ring
  E: DirectProductCategory(#vl,NonNegativeInteger)
  OV  ==> OrderedVariableList(vl)
  SUP ==> SparseUnivariatePolynomial
  NNI ==> NonNegativeInteger

  public == PolynomialCategory(R,E,OV) with
      reorder: (%,List Integer) -> %
        ++ reorder(p, perm) applies the permutation perm to the variables
        ++ in a polynomial and returns the new correctly ordered polynomial

  private == PolynomialRing(R,E) add
    --representations
      Term := Record(k:E,c:R)
      Rep := List Term
      n := #vl
      Vec ==> Vector(NonNegativeInteger)
      zero?(p : %): Boolean == null(p : Rep)

      totalDegree p ==
         zero? p => 0
```

```
   "max"/[reduce("+",(t.k)::(Vector NNI), 0) for t in p]

monomial(p:%, v: OV,e: NonNegativeInteger):% ==
   locv := lookup v
   p*monomial(1,
      directProduct [if z=locv then e else 0 for z in 1..n]$Vec)

coerce(v: OV):% == monomial(1,v,1)

listCoef(p : %): List R ==
  rec : Term
  [rec.c for rec in (p:Rep)]

mainVariable(p: %) ==
   zero?(p) => "failed"
   for v in vl repeat
     vv := variable(v)::OV
     if degree(p,vv)>0 then return vv
   "failed"

ground?(p) == mainVariable(p) case "failed"

retract(p : %): R ==
    not ground? p => error "not a constant"
    leadingCoefficient p

retractIfCan(p : %): Union(R,"failed") ==
  ground?(p) => leadingCoefficient p
  "failed"

degree(p: %,v: OV) == degree(univariate(p,v))
minimumDegree(p: %,v: OV) == minimumDegree(univariate(p,v))
differentiate(p: %,v: OV) ==
      multivariate(differentiate(univariate(p,v)),v)

degree(p: %,lv: List OV) == [degree(p,v) for v in lv]
minimumDegree(p: %,lv: List OV) == [minimumDegree(p,v) for v in lv]

numberOfMonomials(p:%) ==
  l : Rep := p : Rep
  null(l) => 1
  #l

monomial?(p : %): Boolean ==
  l : Rep := p : Rep
  null(l) or null rest(l)

if R has OrderedRing then
  maxNorm(p : %): R ==
    l : List R := nil
```

```
        r,m : R
        m := 0
        for r in listCoef(p) repeat
          if r > m then m := r
          else if (-r) > m then m := -r
        m

    --trailingCoef(p : %) ==
    --  l : Rep := p : Rep
    --  null l => 0
    --  r : Term := last l
    --  r.c

    --leadingPrimitiveMonomial(p : %) ==
    --  ground?(p) => 1$%
    --  r : Term := first(p:Rep)
    --  r := [r.k,1$R]$Term     -- new cell
    -- list(r)$Rep :: %

  -- The following 2 defs are inherited from PolynomialRing

    --leadingMonomial(p : %) ==
    --  ground?(p) => p
    --  r : Term := first(p:Rep)
    --  r := [r.k,r.c]$Term     -- new cell
    --  list(r)$Rep :: %

    --reductum(p : %): % ==
    --  ground? p => 0$%
    --  (rest(p:Rep)):%

    if R has Field then
      (p : %) / (r : R) == inv(r) * p

    variables(p: %) ==
       maxdeg:Vector(NonNegativeInteger) := new(n,0)
       while not zero?(p) repeat
          tdeg := degree p
          p := reductum p
          for i in 1..n repeat
            maxdeg.i := max(maxdeg.i, tdeg.i)
       [index(i:PositiveInteger) for i in 1..n | maxdeg.i^=0]

    reorder(p: %,perm: List Integer):% ==
       #perm ^= n => error "must be a complete permutation of all vars"
       q := [[directProduct [term.k.j for j in perm]$Vec,term.c]$Term
                       for term in p]
       sort((z1,z2) +-> z1.k > z2.k,q)

    --coerce(dp:DistributedMultivariatePolynomial(vl,R)):% ==
```

```
--   q:=dp:List(Term)
--   sort(#1.k > #2.k,q):%

univariate(p: %,v: OV):SUP(%) ==
   zero?(p) => 0
   exp := degree p
   locv := lookup v
   deg:NonNegativeInteger := 0
   nexp := directProduct [if i=locv then (deg :=exp.i;0) else exp.i
                               for i in 1..n]$Vec
   monomial(monomial(leadingCoefficient p,nexp),deg)+
                univariate(reductum p,v)

eval(p: %,v: OV,val:%):% == univariate(p,v)(val)

eval(p: %,v: OV,val:R):% == eval(p,v,val::%)$%

eval(p: %,lv: List OV,lval: List R):% ==
   lv = [] => p
   eval(eval(p,first lv,(first lval)::%)$%, rest lv, rest lval)$%

-- assume Lvar are sorted correctly
evalSortedVarlist(p: %,Lvar: List OV,Lpval: List %):% ==
  v := mainVariable p
  v case "failed" => p
  pv := v:: OV
  Lvar=[] or Lpval=[] => p
  mvar := Lvar.first
  mvar > pv => evalSortedVarlist(p,Lvar.rest,Lpval.rest)
  pval := Lpval.first
  pts:SUP(%):= map(x+->evalSortedVarlist(x,Lvar,Lpval),univariate(p,pv))
  mvar=pv => pts(pval)
  multivariate(pts,pv)

eval(p:%,Lvar:List OV,Lpval:List %) ==
  nlvar:List OV := sort((x,y) +-> x > y,Lvar)
  nlpval :=
     Lvar = nlvar => Lpval
     nlpval := [Lpval.position(mvar,Lvar) for mvar in nlvar]
  evalSortedVarlist(p,nlvar,nlpval)

multivariate(p1:SUP(%),v: OV):% ==
  0=p1 => 0
  degree p1 = 0 => leadingCoefficient p1
  leadingCoefficient(p1)*(v::%)**degree(p1) +
            multivariate(reductum p1,v)

univariate(p: %):SUP(R) ==
  (v := mainVariable p) case "failed" =>
                monomial(leadingCoefficient p,0)
```

```
  q := univariate(p,v:: OV)
  ans:SUP(R) := 0
  while q ^= 0 repeat
    ans := ans + monomial(ground leadingCoefficient q,degree q)
    q := reductum q
  ans

multivariate(p:SUP(R),v: OV):% ==
  0=p => 0
  (leadingCoefficient p)*monomial(1,v,degree p) +
                 multivariate(reductum p,v)

if R has GcdDomain then
  content(p: %):R ==
    zero?(p) => 0
    "gcd"/[t.c for t in p]



  if R has EuclideanDomain and not(R has FloatingPointSystem)  then
    gcd(p: %,q:%):% ==
      gcd(p,q)$PolynomialGcdPackage(E,OV,R,%)

  else gcd(p: %,q:%):% ==
      r : R
      (pv := mainVariable(p)) case "failed" =>
        (r := leadingCoefficient p) = 0$R => q
        gcd(r,content q)::%
      (qv := mainVariable(q)) case "failed" =>
        (r := leadingCoefficient q) = 0$R => p
        gcd(r,content p)::%
      pv<qv => gcd(p,content univariate(q,qv))
      qv<pv => gcd(q,content univariate(p,pv))
      multivariate(gcd(univariate(p,pv),univariate(q,qv)),pv)

coerce(p: %) : OutputForm ==
  zero?(p) => (0$R) :: OutputForm
  l,lt : List OutputForm
  lt := nil
  vl1 := [v::OutputForm for v in vl]
  for t in reverse p repeat
    l := nil
    for i in 1..#vl1 repeat
      t.k.i = 0 => l
      t.k.i = 1 => l := cons(vl1.i,l)
      l := cons(vl1.i ** t.k.i ::OutputForm,l)
    l := reverse l
    if (t.c ^= 1) or (null l) then l := cons(t.c :: OutputForm,l)
    1 = #l => lt := cons(first l,lt)
    lt := cons(reduce("*",l),lt)
```

```
        1 = #lt => first lt
        reduce("+",lt)
```

————

— GDMP.dotabb —

```
"GDMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GDMP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"GDMP" -> "ALIST"
```

————

8.2 domain GMODPOL GeneralModulePolynomial

— GeneralModulePolynomial.input —

```
)set break resume
)sys rm -f GeneralModulePolynomial.output
)spool GeneralModulePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralModulePolynomial
--R GeneralModulePolynomial(vl: List Symbol,R: CommutativeRing,IS: OrderedSet,E: DirectProduc
--R Abbreviation for GeneralModulePolynomial is GMODPOL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GMODPOL
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                        ?*? : (%,R) -> %
--R ?*? : (%,P) -> %                        ?*? : (P,%) -> %
--R ?*? : (Integer,%) -> %                  ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                        ?-? : (%,%) -> %
--R -? : % -> %                             ?=? : (%,%) -> Boolean
--R 0 : () -> %                             build : (R,IS,E) -> %
--R coerce : % -> OutputForm                hash : % -> SingleInteger
--R latex : % -> String                     leadingCoefficient : % -> R
--R leadingExponent : % -> E                leadingIndex : % -> IS
--R multMonom : (R,E,%) -> %                reductum : % -> %
--R sample : () -> %                        unitVector : IS -> %
--R zero? : % -> Boolean                    ?~=? : (%,%) -> Boolean
```

```
--R ?*? : (NonNegativeInteger,%) -> %
--R leadingMonomial : % -> ModuleMonomial(IS,E,ff)
--R monomial : (R,ModuleMonomial(IS,E,ff)) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———–

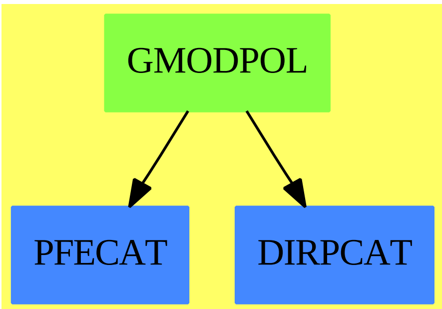
— **GeneralModulePolynomial.help** —

```
====================================================================
GeneralModulePolynomial examples
====================================================================

See Also:
o )show GeneralModulePolynomial
```

———–

8.2.1 GeneralModulePolynomial (GMODPOL)



See

⇒ "ModuleMonomial" (MODMONOM) 14.11.1 on page 1608

Exports:

0	build	coerce	hash	latex
leadingCoefficient	leadingExponent	leadingIndex	leadingMonomial	monomial
multMonom	reductum	sample	subtractIfCan	unitVector
zero?	?~=?	?*?	?+?	?-?
-?	?=?			

— domain GMODPOL GeneralModulePolynomial —

```
)abbrev domain GMODPOL GeneralModulePolynomial
++ Author: Mark Botch
++ Description:
++ This package is undocumented

GeneralModulePolynomial(vl, R, IS, E, ff, P): public  ==  private where
  vl: List(Symbol)
  R: CommutativeRing
  IS: OrderedSet
  NNI ==> NonNegativeInteger
  E: DirectProductCategory(#vl, NNI)
  MM ==> Record(index:IS, exponent:E)
  ff: (MM, MM) -> Boolean
  OV  ==> OrderedVariableList(vl)
  P: PolynomialCategory(R, E, OV)
  ModMonom ==> ModuleMonomial(IS, E, ff)


  public  ==  Join(Module(P), Module(R))  with
    leadingCoefficient: $ -> R
      ++ leadingCoefficient(x) is not documented
    leadingMonomial: $ -> ModMonom
      ++ leadingMonomial(x) is not documented
    leadingExponent: $ -> E
      ++ leadingExponent(x) is not documented
    leadingIndex: $ -> IS
      ++ leadingIndex(x) is not documented
    reductum: $ -> $
      ++ reductum(x) is not documented
    monomial: (R, ModMonom) -> $
      ++ monomial(r,x) is not documented
    unitVector: IS -> $
      ++ unitVector(x) is not documented
    build: (R, IS, E) -> $
      ++ build(r,i,e) is not documented
    multMonom: (R, E, $) -> $
      ++ multMonom(r,e,x) is not documented
    "*": (P,$) -> $
      ++ p*x is not documented


  private  ==  FreeModule(R, ModMonom)  add
        Rep:= FreeModule(R, ModMonom)
        leadingMonomial(p:$):ModMonom == leadingSupport(p)$Rep
        leadingExponent(p:$):E == exponent(leadingMonomial p)
        leadingIndex(p:$):IS == index(leadingMonomial p)
        unitVector(i:IS):$ == monomial(1,[i, 0$E]$ModMonom)
```

```
-------------------------------------------------------------------------------

       build(c:R, i:IS, e:E):$  ==  monomial(c, construct(i, e))

-------------------------------------------------------------------------------

    ----   WARNING: assumes c ^= 0

       multMonom(c:R, e:E, mp:$):$  ==
           zero? mp => mp
           monomial(c * leadingCoefficient mp, [leadingIndex mp,
                    e + leadingExponent mp]) + multMonom(c, e, reductum mp)

-------------------------------------------------------------------------------


       ((p:P) * (mp:$)):$  ==
           zero? p => 0
           multMonom(leadingCoefficient p, degree p, mp) +
              reductum(p) * mp
```

— **GMODPOL.dotabb** —

```
"GMODPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GMODPOL"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"GMODPOL" -> "PFECAT"
"GMODPOL" -> "DIRPCAT"
```

8.3 domain GCNAALG GenericNonAssociativeAlgebra

— **GenericNonAssociativeAlgebra.input** —

```
)set break resume
)sys rm -f GenericNonAssociativeAlgebra.output
)spool GenericNonAssociativeAlgebra.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show GenericNonAssociativeAlgebra
--R GenericNonAssociativeAlgebra(R: CommutativeRing,n: PositiveInteger,ls: List Symbol,gamma
--R Abbreviation for GenericNonAssociativeAlgebra is GCNAALG
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GCNAALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           alternative? : () -> Boolean
--R antiAssociative? : () -> Boolean      antiCommutative? : () -> Boolean
--R antiCommutator : (%,%) -> %           associative? : () -> Boolean
--R associator : (%,%,%) -> %             basis : () -> Vector %
--R coerce : % -> OutputForm              commutative? : () -> Boolean
--R commutator : (%,%) -> %               flexible? : () -> Boolean
--R generic : (Symbol,Vector %) -> %      generic : Vector % -> %
--R generic : Vector Symbol -> %          generic : Symbol -> %
--R generic : () -> %                     hash : % -> SingleInteger
--R jacobiIdentity? : () -> Boolean       jordanAdmissible? : () -> Boolean
--R jordanAlgebra? : () -> Boolean        latex : % -> String
--R leftAlternative? : () -> Boolean      lieAdmissible? : () -> Boolean
--R lieAlgebra? : () -> Boolean           powerAssociative? : () -> Boolean
--R rank : () -> PositiveInteger          rightAlternative? : () -> Boolean
--R sample : () -> %                      someBasis : () -> Vector %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (SquareMatrix(n,Fraction Polynomial R),%) -> %
--R ?*? : (Fraction Polynomial R,%) -> %
--R ?*? : (%,Fraction Polynomial R) -> %
--R ?*? : (NonNegativeInteger,%) -> %
--R apply : (Matrix Fraction Polynomial R,%) -> %
--R associatorDependence : () -> List Vector Fraction Polynomial R if Fraction Polynomial R I
--R coerce : Vector Fraction Polynomial R -> %
--R conditionsForIdempotents : () -> List Polynomial R if R has INTDOM
--R conditionsForIdempotents : Vector % -> List Polynomial R if R has INTDOM
--R conditionsForIdempotents : () -> List Polynomial Fraction Polynomial R
--R conditionsForIdempotents : Vector % -> List Polynomial Fraction Polynomial R
--R convert : Vector Fraction Polynomial R -> %
--R convert : % -> Vector Fraction Polynomial R
--R coordinates : Vector % -> Matrix Fraction Polynomial R
--R coordinates : % -> Vector Fraction Polynomial R
--R coordinates : (Vector %,Vector %) -> Matrix Fraction Polynomial R
--R coordinates : (%,Vector %) -> Vector Fraction Polynomial R
--R ?.? : (%,Integer) -> Fraction Polynomial R
--R generic : (Vector Symbol,Vector %) -> %
--R genericLeftDiscriminant : () -> Fraction Polynomial R if R has INTDOM
```

```
--R genericLeftMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R genericLeftNorm : % -> Fraction Polynomial R if R has INTDOM
--R genericLeftTrace : % -> Fraction Polynomial R if R has INTDOM
--R genericLeftTraceForm : (%,%) -> Fraction Polynomial R if R has INTDOM
--R genericRightDiscriminant : () -> Fraction Polynomial R if R has INTDOM
--R genericRightMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDO
--R genericRightNorm : % -> Fraction Polynomial R if R has INTDOM
--R genericRightTrace : % -> Fraction Polynomial R if R has INTDOM
--R genericRightTraceForm : (%,%) -> Fraction Polynomial R if R has INTDOM
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R
--R leftDiscriminant : () -> Fraction Polynomial R
--R leftDiscriminant : Vector % -> Fraction Polynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if Fraction Polynomial
--R leftNorm : % -> Fraction Polynomial R
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial Fraction Polynomial R if Fraction P
--R leftRecip : % -> Union(%,"failed") if Fraction Polynomial R has INTDOM
--R leftRegularRepresentation : % -> Matrix Fraction Polynomial R
--R leftRegularRepresentation : (%,Vector %) -> Matrix Fraction Polynomial R
--R leftTrace : % -> Fraction Polynomial R
--R leftTraceMatrix : () -> Matrix Fraction Polynomial R
--R leftTraceMatrix : Vector % -> Matrix Fraction Polynomial R
--R leftUnit : () -> Union(%,"failed") if Fraction Polynomial R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed")
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if Fraction Polynomial R has INTDOM
--R represents : Vector Fraction Polynomial R -> %
--R represents : (Vector Fraction Polynomial R,Vector %) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R
--R rightDiscriminant : () -> Fraction Polynomial R
--R rightDiscriminant : Vector % -> Fraction Polynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if Fraction Polynomia
--R rightNorm : % -> Fraction Polynomial R
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial Fraction Polynomial R if Fraction
--R rightRecip : % -> Union(%,"failed") if Fraction Polynomial R has INTDOM
--R rightRegularRepresentation : % -> Matrix Fraction Polynomial R
--R rightRegularRepresentation : (%,Vector %) -> Matrix Fraction Polynomial R
--R rightTrace : % -> Fraction Polynomial R
--R rightTraceMatrix : () -> Matrix Fraction Polynomial R
--R rightTraceMatrix : Vector % -> Matrix Fraction Polynomial R
--R rightUnit : () -> Union(%,"failed") if Fraction Polynomial R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed")
--R structuralConstants : () -> Vector Matrix Fraction Polynomial R
--R structuralConstants : Vector % -> Vector Matrix Fraction Polynomial R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if Fraction Polynomial R has INTDOM
```

```
--R
--E 1

)spool
)lisp (bye)
```

———-

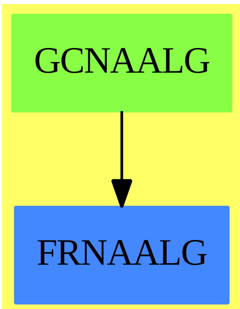
— GenericNonAssociativeAlgebra.help —

```
====================================================================
GenericNonAssociativeAlgebra examples
====================================================================

See Also:
o )show GenericNonAssociativeAlgebra
```

———-

8.3.1 GenericNonAssociativeAlgebra (GCNAALG)



Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	apply
associative?	associator
associatorDependence	basis
coerce	commutative?
commutator	conditionsForIdempotents
convert	convert
coordinates	coordinates
coordinates	coordinates
flexible?	generic
genericLeftDiscriminant	genericLeftMinimalPolynomial
genericLeftNorm	genericLeftTrace
genericLeftTraceForm	genericRightDiscriminant
genericRightMinimalPolynomial	genericRightNorm
genericRightTrace	genericRightTraceForm
hash	jacobiIdentity?
jordanAdmissible?	jordanAlgebra?
latex	leftAlternative?
leftCharacteristicPolynomial	leftDiscriminant
leftDiscriminant	leftMinimalPolynomial
leftNorm	leftPower
leftRankPolynomial	leftRankPolynomial
leftRecip	leftRegularRepresentation
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftTraceMatrix
leftUnit	leftUnits
lieAdmissible?	lieAlgebra?
noncommutativeJordanAlgebra?	plenaryPower
powerAssociative?	rank
recip	represents
rightAlternative?	rightCharacteristicPolynomial
rightDiscriminant	rightDiscriminant
rightMinimalPolynomial	rightNorm
rightPower	rightRankPolynomial
rightRankPolynomial	rightRecip
rightRegularRepresentation	rightRegularRepresentation
rightTrace	rightTraceMatrix
rightTraceMatrix	rightUnit
rightUnits	sample
someBasis	structuralConstants
structuralConstants	subtractIfCan
unit	zero?
?*?	?**?
?+?	?-?
-?	?=?
?.?	?~=?

— domain GCNAALG GenericNonAssociativeAlgebra —

```
)abbrev domain GCNAALG GenericNonAssociativeAlgebra
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 26 June 1991
++ Date Last Updated: 26 June 1991
++ Basic Operations: generic
++ Related Constructors: AlgebraPackage
++ Also See:
++ AMS Classifications:
++ Keywords: generic element. rank polynomial
++ Reference:
++  A. Woerz-Busekros: Algebra in Genetics
++  Lectures Notes in Biomathematics 36,
++  Springer-Verlag,  Heidelberg, 1980
++ Description:
++ AlgebraGenericElementPackage allows you to create generic elements
++ of an algebra, i.e. the scalars are extended to include symbolic
++ coefficients

GenericNonAssociativeAlgebra(R : CommutativeRing, n : PositiveInteger,_
  ls : List Symbol, gamma: Vector Matrix R ): public == private where

  NNI ==> NonNegativeInteger
  V   ==> Vector
  PR  ==> Polynomial R
  FPR ==> Fraction Polynomial R
  SUP ==> SparseUnivariatePolynomial
  S   ==> Symbol

  public ==> Join(FramedNonAssociativeAlgebra(FPR), _
      LeftModule(SquareMatrix(n,FPR)) ) with

    coerce : Vector FPR -> %
      ++ coerce(v) assumes that it is called with a vector
      ++ of length equal to the dimension of the algebra, then
      ++ a linear combination with the basis element is formed
    leftUnits:() -> Union(Record(particular: %, basis: List %), "failed")
      ++ leftUnits() returns the affine space of all left units of the
      ++ algebra, or \spad{"failed"} if there is none
    rightUnits:() -> Union(Record(particular: %, basis: List %), "failed")
      ++ rightUnits() returns the affine space of all right units of the
      ++ algebra, or \spad{"failed"} if there is none
    generic : () -> %
      ++ generic() returns a generic element, i.e. the linear combination
      ++ of the fixed basis with the symbolic coefficients
      ++ \spad{%x1,%x2,..}
    generic : Symbol -> %
      ++ generic(s) returns a generic element, i.e. the linear combination
```

```
    ++ of the fixed basis with the symbolic coefficients
    ++ \spad{s1,s2,..}
  generic : Vector Symbol -> %
    ++ generic(vs) returns a generic element, i.e. the linear combination
    ++ of the fixed basis with the symbolic coefficients
    ++ \spad{vs};
    ++ error, if the vector of symbols is too short
  generic : Vector % -> %
    ++ generic(ve) returns a generic element, i.e. the linear combination
    ++ of \spad{ve} basis with the symbolic coefficients
    ++ \spad{%x1,%x2,..}
  generic : (Symbol, Vector %) -> %
    ++ generic(s,v) returns a generic element, i.e. the linear combination
    ++ of v with the symbolic coefficients
    ++ \spad{s1,s2,..}
  generic : (Vector Symbol, Vector %) -> %
    ++ generic(vs,ve) returns a generic element, i.e. the linear combination
    ++ of \spad{ve} with the symbolic coefficients \spad{vs}
    ++ error, if the vector of symbols is shorter than the vector of
    ++ elements
  if R has IntegralDomain then
    leftRankPolynomial : () -> SparseUnivariatePolynomial FPR
      ++ leftRankPolynomial() returns the left minimimal polynomial
      ++ of the generic element
    genericLeftMinimalPolynomial : % -> SparseUnivariatePolynomial FPR
      ++ genericLeftMinimalPolynomial(a) substitutes the coefficients
      ++ of {em a} for the generic coefficients in
      ++ \spad{leftRankPolynomial()}
    genericLeftTrace : % -> FPR
      ++ genericLeftTrace(a) substitutes the coefficients
      ++ of \spad{a} for the generic coefficients into the
      ++ coefficient of the second highest term in
      ++ \spadfun{leftRankPolynomial} and changes the sign.
      ++  This is a linear form
    genericLeftNorm : % -> FPR
      ++ genericLeftNorm(a) substitutes the coefficients
      ++ of \spad{a} for the generic coefficients into the
      ++ coefficient of the constant term in \spadfun{leftRankPolynomial}
      ++ and changes the sign if the degree of this polynomial is odd.
      ++ This is a form of degree k
    rightRankPolynomial : () -> SparseUnivariatePolynomial FPR
      ++ rightRankPolynomial() returns the right minimimal polynomial
      ++ of the generic element
    genericRightMinimalPolynomial : % -> SparseUnivariatePolynomial FPR
      ++ genericRightMinimalPolynomial(a) substitutes the coefficients
      ++ of \spad{a} for the generic coefficients in
      ++ \spadfun{rightRankPolynomial}
    genericRightTrace : % -> FPR
      ++ genericRightTrace(a) substitutes the coefficients
      ++ of \spad{a} for the generic coefficients into the
```

```
      ++ coefficient of the second highest term in
      ++ \spadfun{rightRankPolynomial} and changes the sign
    genericRightNorm : % -> FPR
      ++ genericRightNorm(a) substitutes the coefficients
      ++ of \spad{a} for the generic coefficients into the
      ++ coefficient of the constant term in \spadfun{rightRankPolynomial}
      ++ and changes the sign if the degree of this polynomial is odd
    genericLeftTraceForm : (%,%) -> FPR
      ++ genericLeftTraceForm (a,b) is defined to be
      ++ \spad{genericLeftTrace (a*b)}, this defines
      ++ a symmetric bilinear form on the algebra
    genericLeftDiscriminant: () -> FPR
      ++ genericLeftDiscriminant() is the determinant of the
      ++ generic left trace forms of all products of basis element,
      ++ if the generic left trace form is associative, an algebra
      ++ is separable if the generic left discriminant is invertible,
      ++ if it is non-zero, there is some ring extension which
      ++ makes the algebra separable
    genericRightTraceForm : (%,%) -> FPR
      ++ genericRightTraceForm (a,b) is defined to be
      ++ \spadfun{genericRightTrace (a*b)}, this defines
      ++ a symmetric bilinear form on the algebra
    genericRightDiscriminant: () -> FPR
      ++ genericRightDiscriminant() is the determinant of the
      ++ generic left trace forms of all products of basis element,
      ++ if the generic left trace form is associative, an algebra
      ++ is separable if the generic left discriminant is invertible,
      ++ if it is non-zero, there is some ring extension which
      ++ makes the algebra separable
    conditionsForIdempotents: Vector % -> List Polynomial R
      ++ conditionsForIdempotents([v1,...,vn]) determines a complete list
      ++ of polynomial equations for the coefficients of idempotents
      ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}
    conditionsForIdempotents: () -> List Polynomial R
      ++ conditionsForIdempotents() determines a complete list
      ++ of polynomial equations for the coefficients of idempotents
      ++ with respect to the fixed \spad{R}-module basis

 private ==> AlgebraGivenByStructuralConstants(FPR,n,ls,_
        coerce(gamma)$CoerceVectorMatrixPackage(R) ) add

   listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
   symbolsForCoef : V Symbol :=
       [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
   genericElement : % :=
     v : Vector PR :=
       [monomial(1$PR, [symbolsForCoef.i],[1]) for i in 1..n]
     convert map(coerce,v)$VectorFunctions2(PR,FPR)

   eval : (FPR, %) -> FPR
```

```
eval(rf,a) ==
  -- for the moment we only substitute the numerators
  -- of the coefficients
  coefOfa : List PR :=
    map(numer, entries coordinates a)$ListFunctions2(FPR,PR)
  ls : List PR :=[monomial(1$PR, [s],[1]) for s in entries symbolsForCoef]
  lEq : List Equation PR := []
  for i in 1..maxIndex ls repeat
    lEq := cons(equation(ls.i,coefOfa.i)$Equation(PR) , lEq)
  top : PR := eval(numer(rf),lEq)$PR
  bot : PR := eval(numer(rf),lEq)$PR
  top/bot


if R has IntegralDomain then

  genericLeftTraceForm(a,b) == genericLeftTrace(a*b)
  genericLeftDiscriminant() ==
    listBasis : List % := entries basis()$%
    m : Matrix FPR := matrix
      [[genericLeftTraceForm(a,b) for a in listBasis] for b in listBasis]
    determinant m

  genericRightTraceForm(a,b) == genericRightTrace(a*b)
  genericRightDiscriminant() ==
    listBasis : List % := entries basis()$%
    m : Matrix FPR := matrix
      [[genericRightTraceForm(a,b) for a in listBasis] for b in listBasis]
    determinant m



  leftRankPoly : SparseUnivariatePolynomial FPR := 0
  initLeft? : Boolean :=true

  initializeLeft: () -> Void
  initializeLeft() ==
    -- reset initialize flag
    initLeft?:=false
    leftRankPoly := leftMinimalPolynomial genericElement
    void()$Void

  rightRankPoly : SparseUnivariatePolynomial FPR := 0
  initRight? : Boolean :=true

  initializeRight: () -> Void
  initializeRight() ==
    -- reset initialize flag
    initRight?:=false
    rightRankPoly := rightMinimalPolynomial genericElement
```

```
    void()$Void

  leftRankPolynomial() ==
    if initLeft? then initializeLeft()
    leftRankPoly

  rightRankPolynomial() ==
    if initRight? then initializeRight()
    rightRankPoly

  genericLeftMinimalPolynomial a ==
    if initLeft? then initializeLeft()
    map(x+->eval(x,a),leftRankPoly)$SUP(FPR)

  genericRightMinimalPolynomial a ==
    if initRight? then initializeRight()
    map(x+->eval(x,a),rightRankPoly)$SUP(FPR)

  genericLeftTrace a ==
    if initLeft? then initializeLeft()
    d1 : NNI := (degree leftRankPoly - 1) :: NNI
    rf : FPR := coefficient(leftRankPoly, d1)
    rf := eval(rf,a)
    - rf

  genericRightTrace a ==
    if initRight? then initializeRight()
    d1 : NNI := (degree rightRankPoly - 1) :: NNI
    rf : FPR := coefficient(rightRankPoly, d1)
    rf := eval(rf,a)
    - rf

  genericLeftNorm a ==
    if initLeft? then initializeLeft()
    rf : FPR := coefficient(leftRankPoly, 1)
    if odd? degree leftRankPoly then rf := - rf
    rf

  genericRightNorm a ==
    if initRight? then initializeRight()
    rf : FPR := coefficient(rightRankPoly, 1)
    if odd? degree rightRankPoly then rf := - rf
    rf

conditionsForIdempotents(b: V %) : List Polynomial R ==
  x : % := generic(b)
  map(numer,entries coordinates(x*x-x,b))$ListFunctions2(FPR,PR)

conditionsForIdempotents(): List Polynomial R ==
  x : % := genericElement
```

```
      map(numer,entries coordinates(x*x-x))$ListFunctions2(FPR,PR)

    generic() ==  genericElement

    generic(vs:V S, ve: V %): % ==
      maxIndex v > maxIndex ve =>
        error "generic: too little symbols"
      v : Vector PR :=
        [monomial(1$PR, [vs.i],[1]) for i in 1..maxIndex ve]
      represents(map(coerce,v)$VectorFunctions2(PR,FPR),ve)

    generic(s: S, ve: V %): % ==
      lON : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..maxIndex ve]
      sFC : Vector Symbol :=
        [concat(s pretend String, i)::Symbol  for i in lON]
      generic(sFC, ve)

    generic(ve : V %) ==
      lON : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..maxIndex ve]
      sFC : Vector Symbol :=
        [concat("%", concat("x", i))::Symbol  for i in lON]
      v : Vector PR :=
        [monomial(1$PR, [sFC.i],[1]) for i in 1..maxIndex ve]
      represents(map(coerce,v)$VectorFunctions2(PR,FPR),ve)

    generic(vs:V S): % == generic(vs, basis()$%)

    generic(s: S): % == generic(s, basis()$%)

    -- variations on eval
    --coefOfa : List FPR := entries coordinates a
    --ls : List Symbol := entries symbolsForCoef
    -- a very dangerous sequential implementation for  the moment,
    -- because the compiler doesn't manage the parallel code
    -- also doesn't run:
    -- not known that (Fraction (Polynomial R)) has (has (Polynomial R)
    --  (Evalable (Fraction (Polynomial R))))
    --res : FPR := rf
    --for eq in lEq repeat res := eval(res,eq)$FPR
    --res
    --rf
    --eval(rf, le)$FPR
    --eval(rf, entries symbolsForCoef, coefOfa)$FPR
    --eval(rf, ls, coefOfa)$FPR
    --le : List Equation PR := [equation(lh,rh) for lh in ls for rh in coefOfa]
```

———

— GCNAALG.dotabb —

```
"GCNAALG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GCNAALG"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"GCNAALG" -> "FRNAALG"
```

8.4 domain GPOLSET GeneralPolynomialSet

— GeneralPolynomialSet.input —

```
)set break resume
)sys rm -f GeneralPolynomialSet.output
)spool GeneralPolynomialSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralPolynomialSet
--R GeneralPolynomialSet(R: Ring,E: OrderedAbelianMonoidSup,VarSet: OrderedSet,P: RecursivePo
--R Abbreviation for GeneralPolynomialSet is GPOLSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GPOLSET
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> List P
--R coerce : % -> OutputForm              collect : (%,VarSet) -> %
--R collectUnder : (%,VarSet) -> %        collectUpper : (%,VarSet) -> %
--R construct : List P -> %               convert : List P -> %
--R copy : % -> %                         empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             latex : % -> String
--R mainVariables : % -> List VarSet      map : ((P -> P),%) -> %
--R mvar : % -> VarSet                    retract : List P -> %
--R sample : () -> %                      trivialIdeal? : % -> Boolean
--R variables : % -> List VarSet          ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if $ has finiteAggregate and P has SETCAT
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
```

```
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R mainVariable? : (VarSet,%) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if $ has finiteAggregate and P has SETCAT
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if $ has finiteAggregate and P has SETCAT
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if $ has finiteAggregate and P has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and P has SETCAT
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,VarSet) -> Record(under: %,floor: %,upper: %)
--R triangular? : % -> Boolean if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

———

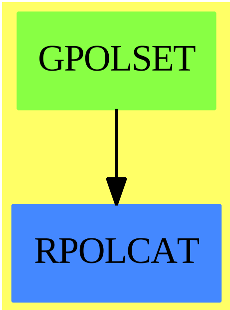
— GeneralPolynomialSet.help —

```
====================================================================
GeneralPolynomialSet examples
====================================================================

See Also:
o )show GeneralPolynomialSet
```

———

8.4.1 GeneralPolynomialSet (GPOLSET)



Exports:

any?	coerce
collect	collectUnder
collectUpper	construct
convert	copy
count	empty
empty?	eq?
eval	every?
find	hash
headRemainder	latex
less?	mainVariables
mainVariable?	map
map!	member?
members	more?
mvar	parts
reduce	remainder
remove	removeDuplicates
retract	retractIfCan
rewriteIdealWithHeadRemainder	rewriteIdealWithRemainder
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
triangular?	trivialIdeal?
variables	#?
?=?	?~=?

— **domain GPOLSET GeneralPolynomialSet** —

```
)abbrev domain GPOLSET GeneralPolynomialSet
++ Author: Marc Moreno Maza
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
```

```
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References:
++ Description:
++ A domain for polynomial sets.

GeneralPolynomialSet(R,E,VarSet,P) : Exports == Implementation where

  R:Ring
  VarSet:OrderedSet
  E:OrderedAbelianMonoidSup
  P:RecursivePolynomialCategory(R,E,VarSet)
  LP ==> List P
  PtoP ==> P -> P

  Exports ==  PolynomialSetCategory(R,E,VarSet,P)  with

     convert : LP -> $
       ++ \axiom{convert(lp)} returns the polynomial set whose members
       ++ are the polynomials of \axiom{lp}.

     finiteAggregate
     shallowlyMutable

  Implementation == add

     Rep := List P

     construct lp ==
       (removeDuplicates(lp)$List(P))::$

     copy ps ==
       construct(copy(members(ps)$$)$LP)$$

     empty() ==
       []

     parts ps ==
       ps pretend LP

     map (f : PtoP, ps : $) : $ ==
       construct(map(f,members(ps))$LP)$$

     map! (f : PtoP, ps : $) : $  ==
       construct(map!(f,members(ps))$LP)$$

     member? (p,ps) ==
```

```
      member?(p,members(ps))$LP

    ps1 = ps2 ==
      {p for p in parts(ps1)} =$(Set P) {p for p in parts(ps2)}

    coerce(ps:$) : OutputForm ==
      lp : List(P) := sort(infRittWu?,members(ps))$(List P)
      brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm

    mvar ps ==
      empty? ps => error"Error from GPOLSET in mvar : #1 is empty"
      lv : List VarSet := variables(ps)
      empty? lv =>
       error "Error from GPOLSET in mvar : every polynomial in #1 is constant"
      reduce(max,lv)$(List VarSet)

    retractIfCan(lp) ==
      (construct(lp))::Union($,"failed")

    coerce(ps:$) : (List P) ==
      ps pretend (List P)

    convert(lp:LP) : $ ==
      construct lp
```

— GPOLSET.dotabb —

```
"GPOLSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GPOLSET"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"GPOLSET" -> "RPOLCAT"
```

8.5 domain GSTBL GeneralSparseTable

— GeneralSparseTable.input —

```
)set break resume
)sys rm -f GeneralSparseTable.output
)spool GeneralSparseTable.output
)set message test on
)set message auto off
```

```
)set break resume
)clear all
--S 1 of 7
patrons: GeneralSparseTable(String, Integer, KeyedAccessFile(Integer), 0) := table() ;
--E 1

--S 2 of 7
patrons."Smith" := 10500
--E 2

--S 3 of 7
patrons."Jones" := 22000
--E 3

--S 4 of 7
patrons."Jones"
--E 4

--S 5 of 7
patrons."Stingy"
--E 5

--S 6 of 7
reduce(+, entries patrons)
--E 6

--S 7 of 7
)system rm -r kaf*.sdata
--E 7
)spool
)lisp (bye)
```

— **GeneralSparseTable.help** —

```
====================================================================
GeneralSparseTable
====================================================================

Sometimes when working with tables there is a natural value to use as
the entry in all but a few cases.  The GeneralSparseTable constructor
can be used to provide any table type with a default value for
entries.

Suppose we launched a fund-raising campaign to raise fifty thousand
dollars.  To record the contributions, we want a table with strings as
keys (for the names) and integer entries (for the amount).  In a data
base of cash contributions, unless someone has been explicitly
```

```
entered, it is reasonable to assume they have made a zero dollar
contribution.

This creates a keyed access file with default entry 0.

  patrons: GeneralSparseTable(String, Integer, KeyedAccessFile(Integer), 0) := table() ;

Now patrons can be used just as any other table.  Here we record two gifts.

  patrons."Smith" := 10500

  patrons."Jones" := 22000

Now let us look up the size of the contributions from Jones and Stingy.

  patrons."Jones"

  patrons."Stingy"

Have we met our seventy thousand dollar goal?

  reduce(+, entries patrons)

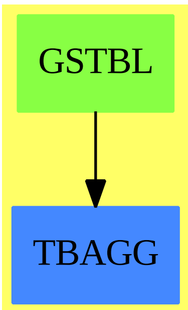
So the project is cancelled and we can delete the data base:

  )system rm -r kaf*.sdata

See Also:
o )show GeneralSparseTable
```

———–

8.5.1 GeneralSparseTable (GSTBL)



See

⇒ "HashTable" (HASHTBL) 9.1.1 on page 1085
⇒ "InnerTable" (INTABL) 10.27.1 on page 1299
⇒ "Table" (TABLE) 21.1.1 on page 2621
⇒ "EqTable" (EQTBL) 6.2.1 on page 667
⇒ "StringTable" (STRTBL) 20.32.1 on page 2569
⇒ "SparseTable" (STBL) 20.16.1 on page 2409

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?~=?	?.?

— domain GSTBL GeneralSparseTable —

```
)abbrev domain GSTBL GeneralSparseTable
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ A sparse table has a default entry, which is returned if no other
++ value has been explicitly stored for a key.

GeneralSparseTable(Key, Entry, Tbl, dent): TableAggregate(Key, Entry) == Impl
  where
    Key, Entry: SetCategory
    Tbl:  TableAggregate(Key, Entry)
    dent: Entry

    Impl ==> Tbl add
        Rep := Tbl

        elt(t:%, k:Key) ==
            (u := search(k, t)$Rep) case "failed" => dent
```

```
            u::Entry

        setelt(t:%, k:Key, e:Entry) ==
            e = dent => (remove_!(k, t); e)
            setelt(t, k, e)$Rep

        search(k:Key, t:%) ==
            (u := search(k, t)$Rep) case "failed" => dent
            u
```

— GSTBL.dotabb —

```
"GSTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GSTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"GSTBL" -> "TBAGG"
```

8.6 domain GTSET GeneralTriangularSet

— GeneralTriangularSet.input —

```
)set break resume
)sys rm -f GeneralTriangularSet.output
)spool GeneralTriangularSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralTriangularSet
--R GeneralTriangularSet(R: IntegralDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: Recurs
--R Abbreviation for GeneralTriangularSet is GTSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GTSET
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List V      coerce : % -> List P
--R coerce : % -> OutputForm              collect : (%,V) -> %
--R collectQuasiMonic : % -> %            collectUnder : (%,V) -> %
--R collectUpper : (%,V) -> %             construct : List P -> %
```

```
--R copy : % -> %                        degree : % -> NonNegativeInteger
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               extend : (%,P) -> %
--R first : % -> Union(P,"failed")       hash : % -> SingleInteger
--R headReduce : (P,%) -> P              headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean      infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P         initiallyReduced? : % -> Boolean
--R initials : % -> List P               last : % -> Union(P,"failed")
--R latex : % -> String                  mainVariable? : (V,%) -> Boolean
--R mainVariables : % -> List V          map : ((P -> P),%) -> %
--R mvar : % -> V                        normalized? : % -> Boolean
--R normalized? : (P,%) -> Boolean       reduceByQuasiMonic : (P,%) -> P
--R removeZero : (P,%) -> P              rest : % -> Union(%,"failed")
--R retract : List P -> %                sample : () -> %
--R stronglyReduce : (P,%) -> P          stronglyReduced? : % -> Boolean
--R trivialIdeal? : % -> Boolean         variables : % -> List V
--R zeroSetSplit : List P -> List %      ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
--R basicSet : (List P,(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"failed")
--R basicSet : (List P,((P,P) -> Boolean)) -> Union(Record(bas: %,top: List P),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if $ has finiteAggregate and P has SETCAT
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if $ has finiteAggregate and P has SETCAT
--R members : % -> List P if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R quasiComponent : % -> Record(close: List P,open: List P)
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if $ has finiteAggregate and P has SETCAT
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if $ has finiteAggregate and P has SETCAT
```

```
--R removeDuplicates : % -> % if $ has finiteAggregate and P has SETCAT
--R retractIfCan : List P -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,V) -> Union(P,"failed")
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R stronglyReduced? : (P,%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %,open: List P)
--R
--E 1

)spool
)lisp (bye)
```

———

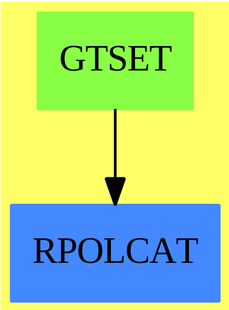
— **GeneralTriangularSet.help** —

```
====================================================================
GeneralTriangularSet examples
====================================================================

See Also:
o )show GeneralTriangularSet
```

———

8.6.1 GeneralTriangularSet (GTSET)



See

⇒ "WuWenTsunTriangularSet" (WUTSET) 24.2.1 on page 2884

Exports:

algebraic?	algebraicVariables
any?	autoReduced?
basicSet	coerce
collect	collectQuasiMonic
collectUnder	collectUpper
coHeight	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headReduced?	headRemainder
infRittWu?	initiallyReduce
initiallyReduced?	initials
last	latex
less?	mainVariable?
mainVariables	map
map!	member?
members	more?
mvar	normalized?
normalized?	parts
quasiComponent	reduce
reduceByQuasiMonic	reduced?
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
stronglyReduce	stronglyReduced?
triangular?	trivialIdeal?
variables	zeroSetSplit
zeroSetSplitIntoTriangularSystems	#?
?=?	?~=?

— domain GTSET GeneralTriangularSet —

```
)abbrev domain GTSET GeneralTriangularSet
++ Author: Marc Moreno Maza (marc@nag.co.uk)
```

```
++ Date Created: 10/06/1995
++ Date Last Updated: 06/12/1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++  [1] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++ Description:
++ A domain constructor of the category \axiomType{TriangularSetCategory}.
++ The only requirement for a list of polynomials to be a member of such
++ a domain is the following: no polynomial is constant and two distinct
++ polynomials have distinct main variables. Such a triangular set may
++ not be auto-reduced or consistent. Triangular sets are stored
++ as sorted lists w.r.t. the main variables of their members but they
++ are displayed in reverse order.

GeneralTriangularSet(R,E,V,P) : Exports == Implementation where

  R : IntegralDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)
  N ==> NonNegativeInteger
  Z ==> Integer
  B ==> Boolean
  LP ==> List P
  PtoP ==> P -> P

  Exports ==  TriangularSetCategory(R,E,V,P)

  Implementation == add

     Rep ==> LP

     rep(s:$):Rep == s pretend Rep
     per(l:Rep):$ == l pretend $

     copy ts ==
       per(copy(rep(ts))$LP)
     empty() ==
       per([])
     empty?(ts:$) ==
       empty?(rep(ts))
     parts ts ==
       rep(ts)
     members ts ==
       rep(ts)
```

```
map (f : PtoP, ts : $) : $ ==
  construct(map(f,rep(ts))$LP)$$
map! (f : PtoP, ts : $) : $  ==
  construct(map!(f,rep(ts))$LP)$$
member? (p,ts) ==
  member?(p,rep(ts))$LP

unitIdealIfCan() ==
  "failed"::Union($,"failed")
roughUnitIdeal? ts ==
  false

-- the following assume that rep(ts) is decreasingly sorted
-- w.r.t. the main variavles of the polynomials in rep(ts)
coerce(ts:$) : OutputForm ==
  lp : List(P) := reverse(rep(ts))
  brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm
mvar ts ==
  empty? ts => error"failed in mvar : $ -> V from GTSET"
  mvar(first(rep(ts)))$P
first ts ==
  empty? ts => "failed"::Union(P,"failed")
  first(rep(ts))::Union(P,"failed")
last ts ==
  empty? ts => "failed"::Union(P,"failed")
  last(rep(ts))::Union(P,"failed")
rest ts ==
  empty? ts => "failed"::Union($,"failed")
  per(rest(rep(ts)))::Union($,"failed")
coerce(ts:$) : (List P) ==
  rep(ts)
collectUpper (ts,v) ==
  empty? ts => ts
  lp := rep(ts)
  newlp : Rep := []
  while (not empty? lp) and (mvar(first(lp)) > v) repeat
    newlp := cons(first(lp),newlp)
    lp := rest lp
  per(reverse(newlp))
collectUnder (ts,v) ==
  empty? ts => ts
  lp := rep(ts)
  while (not empty? lp) and (mvar(first(lp)) >= v) repeat
    lp := rest lp
  per(lp)

-- for another domain of TSETCAT build on this domain GTSET
-- the following operations must be redefined
extendIfCan(ts:$,p:P) ==
  ground? p => "failed"::Union($,"failed")
```

```
      empty? ts => (per([unitCanonical(p)]$LP))::Union($,"failed")
      not (mvar(ts) < mvar(p)) => "failed"::Union($,"failed")
      (per(cons(p,rep(ts))))::Union($,"failed")
```

———-

— GTSET.dotabb —

```
"GTSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GTSET"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"GTSET" -> "RPOLCAT"
```

———-

8.7 domain GSERIES GeneralUnivariatePowerSeries

— GeneralUnivariatePowerSeries.input —

```
)set break resume
)sys rm -f GeneralUnivariatePowerSeries.output
)spool GeneralUnivariatePowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralUnivariatePowerSeries
--R GeneralUnivariatePowerSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for GeneralUnivariatePowerSeries is GSERIES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GSERIES
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       center : % -> Coef
--R coerce : Variable var -> %           coerce : Integer -> %
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> Fraction Integer       hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> Coef
```

```
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : % -> Fraction Integer        pole? : % -> Boolean
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R sample : () -> %                     variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING S
--R D : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDR
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef an
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) 
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Co
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : UnivariatePuiseuxSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING S
--R differentiate : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDR
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef an
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef)
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
```

```
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———-

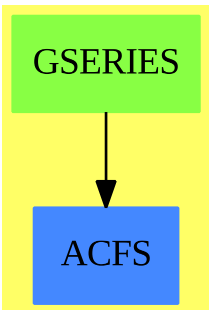
— GeneralUnivariatePowerSeries.help —

```
====================================================================
GeneralUnivariatePowerSeries examples
====================================================================

See Also:
o )show GeneralUnivariatePowerSeries
```

———-

8.7.1 GeneralUnivariatePowerSeries (GSERIES)



Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	lcm	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiEuclidean	multiplyExponents	nthRoot
one?	order	pi	pole?
prime?	principalIdeal	recip	reductum
sample	sec	sech	series
sin	sinh	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	tan
tanh	terms	truncate	unit?
unitCanonical	unitNormal	variable	variables
zero?	?+?	?-?	-?
?=?	?^?	?~=?	?*?
?**?	?/?	?.?	
?quo?	?rem?		

— domain GSERIES GeneralUnivariatePowerSeries —

```
)abbrev domain GSERIES GeneralUnivariatePowerSeries
++ Author: Clifton J. Williamson
++ Date Created: 22 September 1993
++ Date Last Updated: 23 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux
++ Examples:
++ References:
++ Description:
++ This is a category of univariate Puiseux series constructed
++ from univariate Laurent series.  A Puiseux series is represented
++ by a pair \spad{[r,f(x)]}, where r is a positive rational number and
++ \spad{f(x)} is a Laurent series.  This pair represents the Puiseux
++ series \spad{f(x\^r)}.

GeneralUnivariatePowerSeries(Coef,var,cen): Exports == Implementation where
```

```
Coef : Ring
var  : Symbol
cen  : Coef
I      ==> Integer
UTS    ==> UnivariateTaylorSeries
ULS    ==> UnivariateLaurentSeries
UPXS   ==> UnivariatePuiseuxSeries
EFULS  ==> ElementaryFunctionsUnivariateLaurentSeries
EFUPXS ==> ElementaryFunctionsUnivariatePuiseuxSeries
FS2UPS ==> FunctionSpaceToUnivariatePowerSeries

Exports ==> UnivariatePuiseuxSeriesCategory Coef with
  coerce: Variable(var) -> %
    ++ coerce(var) converts the series variable \spad{var} into a
    ++ Puiseux series.
  coerce: UPXS(Coef,var,cen) -> %
    ++ coerce(f) converts a Puiseux series to a general power series.
  differentiate: (%,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} returns the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  if Coef has Algebra Fraction Integer then
    integrate: (%,Variable(var)) -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.

Implementation ==> UnivariatePuiseuxSeries(Coef,var,cen) add

  coerce(upxs:UPXS(Coef,var,cen)) == upxs pretend %

  puiseux: % -> UPXS(Coef,var,cen)
  puiseux f == f pretend UPXS(Coef,var,cen)

  if Coef has Algebra Fraction Integer then

    differentiate f ==
      str1 : String := "'differentiate' unavailable on this domain;  "
      str2 : String := "use 'approximate' first"
      error concat(str1,str2)

    differentiate(f:%,v:Variable(var)) == differentiate f

    if Coef has PartialDifferentialRing(Symbol) then
      differentiate(f:%,s:Symbol) ==
        (s = variable(f)) =>
          str1 : String := "'differentiate' unavailable on this domain;  "
          str2 : String := "use 'approximate' first"
          error concat(str1,str2)
        dcds := differentiate(center f,s)
```

```
        deriv := differentiate(puiseux f) :: %
        map(x+->differentiate(x,s),f) - dcds * deriv

    integrate f ==
      str1 : String := "'integrate' unavailable on this domain;  "
      str2 : String := "use 'approximate' first"
      error concat(str1,str2)

    integrate(f:%,v:Variable(var)) == integrate f

    if Coef has integrate: (Coef,Symbol) -> Coef and _
       Coef has variables: Coef -> List Symbol then

      integrate(f:%,s:Symbol) ==
        (s = variable(f)) =>
          str1 : String := "'integrate' unavailable on this domain;  "
          str2 : String := "use 'approximate' first"
          error concat(str1,str2)
        not entry?(s,variables center f) => map(x+->integrate(x,s),f)
        error "integrate: center is a function of variable of integration"

    if Coef has TranscendentalFunctionCategory and _
       Coef has PrimitiveFunctionCategory and _
       Coef has AlgebraicallyClosedFunctionSpace Integer then

      integrateWithOneAnswer: (Coef,Symbol) -> Coef
      integrateWithOneAnswer(f,s) ==
        res := integrate(f,s)$FunctionSpaceIntegration(Integer,Coef)
        res case Coef => res :: Coef
        first(res :: List Coef)

      integrate(f:%,s:Symbol) ==
        (s = variable(f)) =>
          str1 : String := "'integrate' unavailable on this domain;  "
          str2 : String := "use 'approximate' first"
          error concat(str1,str2)
        not entry?(s,variables center f) =>
          map(x+->integrateWithOneAnswer(x,s),f)
        error "integrate: center is a function of variable of integration"
```

— GSERIES.dotabb —

```
"GSERIES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GSERIES"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"GSERIES" -> "ACFS"
```

8.8 domain GRIMAGE GraphImage

— GraphImage.input —

```
)set break resume
)sys rm -f GraphImage.output
)spool GraphImage.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GraphImage
--R GraphImage  is a domain constructor
--R Abbreviation for GraphImage is GRIMAGE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GRIMAGE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R graphImage : () -> %                  hash : % -> SingleInteger
--R key : % -> Integer                    latex : % -> String
--R makeGraphImage : % -> %               ranges : % -> List Segment Float
--R units : % -> List Float               ?~=? : (%,%) -> Boolean
--R appendPoint : (%,Point DoubleFloat) -> Void
--R coerce : List List Point DoubleFloat -> %
--R component : (%,Point DoubleFloat,Palette,Palette,PositiveInteger) -> Void
--R component : (%,Point DoubleFloat) -> Void
--R component : (%,List Point DoubleFloat,Palette,Palette,PositiveInteger) -> Void
--R figureUnits : List List Point DoubleFloat -> List DoubleFloat
--R makeGraphImage : (List List Point DoubleFloat,List Palette,List Palette,List PositiveInte
--R makeGraphImage : (List List Point DoubleFloat,List Palette,List Palette,List PositiveInte
--R makeGraphImage : List List Point DoubleFloat -> %
--R point : (%,Point DoubleFloat,Palette) -> Void
--R pointLists : % -> List List Point DoubleFloat
--R putColorInfo : (List List Point DoubleFloat,List Palette) -> List List Point DoubleFloat
--R ranges : (%,List Segment Float) -> List Segment Float
--R units : (%,List Float) -> List Float
--R
--E 1

)spool
)lisp (bye)
```

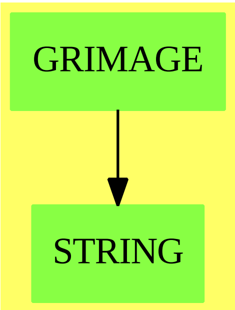
— GraphImage.help —

```
====================================================================
GraphImage examples
====================================================================

See Also:
o )show GraphImage
```

———-

8.8.1 GraphImage (GRIMAGE)



Exports:

appendPoint	coerce	component	figureUnits	graphImage
hash	key	latex	makeGraphImage	point
pointLists	putColorInfo	ranges	units	?~=?
?=?				

— domain GRIMAGE GraphImage —

```
)abbrev domain GRIMAGE GraphImage
++ Author: Jim Wen
++ Date Created: 27 April 1989
++ Date Last Updated: 1995 September 20, Mike Richardson (MGR)
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ TwoDimensionalGraph creates virtual two dimensional graphs
++ (to be displayed on TwoDimensionalViewports).
```

```
GraphImage (): Exports == Implementation where

  VIEW    ==> VIEWPORTSERVER$Lisp
  sendI   ==> SOCK_-SEND_-INT
  sendSF  ==> SOCK_-SEND_-FLOAT
  sendSTR ==> SOCK_-SEND_-STRING
  getI    ==> SOCK_-GET_-INT
  getSF   ==> SOCK_-GET_-FLOAT

  typeGRAPH  ==> 2
  typeVIEW2D ==> 3

  makeGRAPH  ==> (-1)$SingleInteger
  makeVIEW2D ==> (-1)$SingleInteger

  I   ==> Integer
  PI  ==> PositiveInteger
  NNI ==> NonNegativeInteger
  SF  ==> DoubleFloat
  F   ==> Float
  L   ==> List
  P   ==> Point(SF)
  V   ==> Vector
  SEG ==> Segment
  RANGESF   ==> L SEG SF
  RANGEF    ==> L SEG F
  UNITSF   ==> L SF
  UNITF    ==> L F
  PAL ==> Palette
  E   ==> OutputForm
  DROP ==> DrawOption
  PP ==> PointPackage(SF)
  COORDSYS ==> CoordinateSystems(SF)

  Exports ==> SetCategory with
    graphImage      :  ()                                     -> $
      ++ graphImage() returns an empty graph with 0 point lists
      ++ of the domain \spadtype{GraphImage}.  A graph image contains
      ++ the graph data component of a two dimensional viewport.
    makeGraphImage  :  $                                      -> $
      ++ makeGraphImage(gi) takes the given graph, \spad{gi} of the
      ++ domain \spadtype{GraphImage}, and sends it's data to the
      ++ viewport manager where it waits to be included in a two-dimensional
      ++ viewport window.  \spad{gi} cannot be an empty graph, and it's
      ++ elements must have been created using the \spadfun{point} or
      ++ \spadfun{component} functions, not by a previous
      ++ \spadfun{makeGraphImage}.
    makeGraphImage  :  (L L P)                                -> $
      ++ makeGraphImage(llp) returns a graph of the domain
```

```
    ++ \spadtype{GraphImage} which is composed of the points and
    ++ lines from the list of lists of points, \spad{llp}, with
    ++ default point size and default point and line colours. The graph
    ++ data is then sent to the viewport manager where it waits to be
    ++ included in a two-dimensional viewport window.
  makeGraphImage  :  (L L P,L PAL,L PAL,L PI)            -> $
    ++ makeGraphImage(llp,lpal1,lpal2,lp) returns a graph of the
    ++ domain \spadtype{GraphImage} which is composed of the points
    ++ and lines from the list of lists of points, \spad{llp}, whose
    ++ point colors are indicated by the list of palette colors,
    ++ \spad{lpal1}, and whose lines are colored according to the list
    ++ of palette colors, \spad{lpal2}.  The paramater lp is a list of
    ++ integers which denote the size of the data points.  The graph
    ++ data is then sent to the viewport manager where it waits to be
    ++ included in a two-dimensional viewport window.
  makeGraphImage  :  (L L P,L PAL,L PAL,L PI,L DROP)     -> $
    ++ makeGraphImage(llp,lpal1,lpal2,lp,lopt) returns a graph of
    ++ the domain \spadtype{GraphImage} which is composed of the
    ++ points and lines from the list of lists of points, \spad{llp},
    ++ whose point colors are indicated by the list of palette colors,
    ++ \spad{lpal1}, and whose lines are colored according to the list
    ++ of palette colors, \spad{lpal2}.  The paramater lp is a list of
    ++ integers which denote the size of the data points, and \spad{lopt}
    ++ is the list of draw command options.  The graph data is then sent
    ++ to the viewport manager where it waits to be included in a
    ++ two-dimensional viewport window.
  pointLists      :  $                                   -> L L P
    ++ pointLists(gi) returns the list of lists of points which compose
    ++ the given graph, \spad{gi}, of the domain \spadtype{GraphImage}.
  key             :  $                                   -> I
    ++ key(gi) returns the process ID of the given graph, \spad{gi},
    ++ of the domain \spadtype{GraphImage}.
  ranges          :  $                                   -> RANGEF
    ++ ranges(gi) returns the list of ranges of the point components from
    ++ the indicated graph, \spad{gi}, of the domain \spadtype{GraphImage}.
  ranges          :  ($,RANGEF)                          -> RANGEF
    ++ ranges(gi,lr) modifies the list of ranges for the given graph,
    ++ \spad{gi} of the domain \spadtype{GraphImage}, to be that of the
    ++ list of range segments, \spad{lr}, and returns the new range list
    ++ for \spad{gi}.
  units           :  $                                   -> UNITF
    ++ units(gi) returns the list of unit increments for the x and y
    ++ axes of the indicated graph, \spad{gi}, of the domain
    ++ \spadtype{GraphImage}.
  units           :  ($,UNITF)                           -> UNITF
    ++ units(gi,lu) modifies the list of unit increments for the x and y
    ++ axes of the given graph, \spad{gi} of the domain
    ++ \spadtype{GraphImage}, to be that of the list of unit increments,
    ++ \spad{lu}, and returns the new list of units for \spad{gi}.
  component       :  ($,L P,PAL,PAL,PI)                  -> Void
```

```
    ++ component(gi,lp,pal1,pal2,p) sets the components of the
    ++ graph, \spad{gi} of the domain \spadtype{GraphImage}, to the
    ++ values given.  The point list for \spad{gi} is set to the list
    ++ \spad{lp}, the color of the points in \spad{lp} is set to
    ++ the palette color \spad{pal1}, the color of the lines which
    ++ connect the points \spad{lp} is set to the palette color
    ++ \spad{pal2}, and the size of the points in \spad{lp} is given
    ++ by the integer p.
  component       :  ($,P)                                  -> Void
    ++ component(gi,pt) modifies the graph \spad{gi} of the domain
    ++ \spadtype{GraphImage} to contain one point component, \spad{pt}
    ++ whose point color, line color and point size are determined by
    ++ the default functions \spadfun{pointColorDefault},
    ++ \spadfun{lineColorDefault}, and \spadfun{pointSizeDefault}.
  component       :  ($,P,PAL,PAL,PI)                       -> Void
    ++ component(gi,pt,pal1,pal2,ps) modifies the graph \spad{gi} of
    ++ the domain \spadtype{GraphImage} to contain one point component,
    ++ \spad{pt} whose point color is set to the palette color \spad{pal1},
    ++ line color is set to the palette color \spad{pal2}, and point
    ++ size is set to the positive integer \spad{ps}.
  appendPoint     :  ($,P)                                  -> Void
    ++ appendPoint(gi,pt) appends the point \spad{pt} to the end
    ++ of the list of points component for the graph, \spad{gi}, which is
    ++ of the domain \spadtype{GraphImage}.
  point           :  ($,P,PAL)                              -> Void
    ++ point(gi,pt,pal) modifies the graph \spad{gi} of the domain
    ++ \spadtype{GraphImage} to contain one point component, \spad{pt}
    ++ whose point color is set to be the palette color \spad{pal}, and
    ++ whose line color and point size are determined by the default
    ++ functions \spadfun{lineColorDefault} and \spadfun{pointSizeDefault}.
  coerce          :  L L P                                  -> $
    ++ coerce(llp)
    ++ component(gi,pt) creates and returns a graph of the domain
    ++ \spadtype{GraphImage} which is composed of the list of list
    ++ of points given by \spad{llp}, and whose point colors, line colors
    ++ and point sizes are determined by the default functions
    ++ \spadfun{pointColorDefault}, \spadfun{lineColorDefault}, and
    ++ \spadfun{pointSizeDefault}.  The graph data is then sent to the
    ++ viewport manager where it waits to be included in a two-dimensional
    ++ viewport window.
  coerce          :  $                                      -> E
    ++ coerce(gi) returns the indicated graph, \spad{gi}, of domain
    ++ \spadtype{GraphImage} as output of the domain \spadtype{OutputForm}.
  putColorInfo    : (L L P,L PAL)                           -> L L P
    ++ putColorInfo(llp,lpal) takes a list of list of points, \spad{llp},
    ++ and returns the points with their hue and shade components
    ++ set according to the list of palette colors, \spad{lpal}.
  figureUnits : L L P                      -> UNITSF

 Implementation ==> add
```

```
    import Color()
    import Palette()
    import ViewDefaultsPackage()
    import PlotTools()
    import DrawOptionFunctions0
    import P
    import PP
    import COORDSYS

    Rep := Record(key: I, rangesField: RANGESF, unitsField: UNITSF, _
       llPoints: L L P, pointColors: L PAL, lineColors: L PAL, pointSizes: L PI, _
       optionsField: L DROP)

--%Internal Functions

    graph       : RANGEF                          -> $
    scaleStep   : SF                          -> SF
    makeGraph   :  $                          -> $


    numberCheck(nums:Point SF):Void ==
      for i in minIndex(nums)..maxIndex(nums) repeat
        COMPLEXP(nums.(i::PositiveInteger))$Lisp =>
          error "An unexpected complex number was encountered in the calculations."


    doOptions(g:Rep):Void ==
      lr : RANGEF := ranges(g.optionsField,ranges g)
      if (#lr > 1$I) then
        g.rangesField := [segment(convert(lo(lr.1))@SF,convert(hi(lr.1))@SF)$(Segment(SF)),
                          segment(convert(lo(lr.2))@SF,convert(hi(lr.2))@SF)$(Segment(SF))]
      else
        g.rangesField := []
      lu : UNITF := units(g.optionsField,units g)
      if (#lu > 1$I) then
        g.unitsField := [convert(lu.1)@SF,convert(lu.2)@SF]
      else
        g.unitsField := []
    -- etc - graphimage specific stuff...

    putColorInfo(llp,listOfPalettes) ==
      llp2 : L L P := []
      for lp in llp for pal in listOfPalettes repeat
        lp2 : L P := []
        daHue   := (hue(hue pal))::SF
        daShade := (shade pal)::SF
        for p in lp repeat
          if (d := dimension p) < 3 then
            p := extend(p,[daHue,daShade])
          else
```

```
        p.3 := daHue
        d < 4 => p := extend(p,[daShade])
        p.4 := daShade
      lp2 := cons(p,lp2)
    llp2 := cons(reverse_! lp2,llp2)
  reverse_! llp2

graph demRanges ==
  null demRanges =>  [ 0, [], [], [], [], [], [], [] ]
  demRangesSF : RANGESF := _
    [ segment(convert(lo demRanges.1)@SF,convert(hi demRanges.1)@SF)$(Segment(SF)), _
      segment(convert(lo demRanges.1)@SF,convert(hi demRanges.1)@SF)$(Segment(SF)) ]
  [ 0, demRangesSF, [], [], [], [], [], [] ]

scaleStep(range) ==                        -- MGR

  adjust:NNI
  tryStep:SF
  scaleDown:SF
  numerals:String
  adjust := 0
  while range < 100.0::SF repeat
    adjust := adjust + 1
    range := range * 10.0::SF -- might as well take big steps
  tryStep := range/10.0::SF
  numerals := string(((retract(ceiling(tryStep)$SF)$SF)@I))$String
  scaleDown := (10@I **$I (((#(numerals)@I) - 1$I) pretend PI))::SF
  scaleDown*ceiling(tryStep/scaleDown - 0.5::SF)/((10 **$I adjust)::SF)

figureUnits(listOfListsOfPoints) ==
    -- figure out the min/max and divide by 10 for unit markers
  xMin := xMax := xCoord first first listOfListsOfPoints
  yMin := yMax := yCoord first first listOfListsOfPoints
  if xMin ~= xMin then xMin:=max()
  if xMax ~= xMax then xMax:=min()
  if yMin ~= yMin then yMin:=max()
  if yMax ~= yMax then yMax:=min()
  for pL in listOfListsOfPoints repeat
    for p in pL repeat
      if ((px := (xCoord p)) < xMin) then
        xMin := px
      if px > xMax then
        xMax := px
      if ((py := (yCoord p)) < yMin) then
        yMin := py
      if py > yMax then
        yMax := py
  if xMin = xMax then
    xMin := xMin - convert(0.5)$Float
    xMax := xMax + convert(0.5)$Float
```

```
    if yMin = yMax then
      yMin := yMin - convert(0.5)$Float
      yMax := yMax + convert(0.5)$Float
    [scaleStep(xMax-xMin),scaleStep(yMax-yMin)]

  plotLists(graf:Rep,listOfListsOfPoints:L L P,listOfPointColors:L PAL,listOfLineColors:L PAL,listOfPo
    givenLen := #listOfListsOfPoints
      -- take out point lists that are actually empty
    listOfListsOfPoints := [ l for l in listOfListsOfPoints | ^null l ]
    if (null listOfListsOfPoints) then
      error "GraphImage was given a list that contained no valid point lists"
    if ((len := #listOfListsOfPoints) ^= givenLen) then
      sayBrightly(["   Warning: Ignoring pointless point list"::E]$List(E))$Lisp
    graf.llPoints := listOfListsOfPoints
      -- do point colors
    if ((givenLen := #listOfPointColors) > len) then
       -- pad or discard elements if given list has length different from the point list
      graf.pointColors := concat(listOfPointColors,
          new((len - givenLen)::NonNegativeInteger + 1, pointColorDefault()))
    else graf.pointColors := first(listOfPointColors, len)
      -- do line colors
    if ((givenLen := #listOfLineColors) > len) then
      graf.lineColors := concat(listOfLineColors,
           new((len - givenLen)::NonNegativeInteger + 1, lineColorDefault()))
    else graf.lineColors := first(listOfLineColors, len)
      -- do point sizes
    if ((givenLen := #listOfPointSizes) > len) then
      graf.pointSizes := concat(listOfPointSizes,
           new((len - givenLen)::NonNegativeInteger + 1, pointSizeDefault()))
    else graf.pointSizes := first(listOfPointSizes, len)
    graf

  makeGraph graf ==
    doOptions(graf)
    (s := #(graf.llPoints)) = 0 =>
      error "You are trying to make a graph with no points"
    key graf ^= 0 =>
      error "You are trying to draw over an existing graph"
    transform := coord(graf.optionsField,cartesian$COORDSYS)$DrawOptionFunctions0
    graf.llPoints:= putColorInfo(graf.llPoints,graf.pointColors)
    if null(ranges graf) then  -- figure out best ranges for points
      graf.rangesField := calcRanges(graf.llPoints)  --::V SEG SF
    if null(units graf) then  -- figure out best ranges for points
      graf.unitsField := figureUnits(graf.llPoints)  --::V SEG SF
    sayBrightly(["   Graph data being transmitted to the viewport manager..."::E]$List(E))$Lisp
    sendI(VIEW,typeGRAPH)$Lisp
    sendI(VIEW,makeGRAPH)$Lisp
    tonto := (graf.rangesField)::RANGESF
    sendSF(VIEW,lo(first tonto))$Lisp
    sendSF(VIEW,hi(first tonto))$Lisp
```

```
      sendSF(VIEW,lo(second tonto))$Lisp
      sendSF(VIEW,hi(second tonto))$Lisp
      sendSF(VIEW,first (graf.unitsField))$Lisp
      sendSF(VIEW,second (graf.unitsField))$Lisp
      sendI(VIEW,s)$Lisp     -- how many lists of points are being sent
      for aList in graf.llPoints for pColor in graf.pointColors for lColor in graf.lineColors
        sendI(VIEW,#aList)$Lisp  -- how many points in this list
        for p in aList repeat
          aPoint := transform p
          sendSF(VIEW,xCoord aPoint)$Lisp
          sendSF(VIEW,yCoord aPoint)$Lisp
          sendSF(VIEW,hue(p)$PP)$Lisp  -- ?use aPoint as well...?
          sendSF(VIEW,shade(p)$PP)$Lisp
        hueShade := hue hue pColor + shade pColor * numberOfHues()
        sendI(VIEW,hueShade)$Lisp
        hueShade := (hue hue lColor -1)*5 + shade lColor
        sendI(VIEW,hueShade)$Lisp
        sendI(VIEW,s)$Lisp
      graf.key := getI(VIEW)$Lisp
      graf


--%Exported Functions
    makeGraphImage(graf:$)    == makeGraph graf
    key graf                  == graf.key
    pointLists graf           == graf.llPoints
    ranges graf                ==
      null graf.rangesField => []
      [segment(convert(lo graf.rangesField.1)@F,convert(hi graf.rangesField.1)@F), _
       segment(convert(lo graf.rangesField.2)@F,convert(hi graf.rangesField.2)@F)]
    ranges(graf,rangesList)     ==
      graf.rangesField :=
        [segment(convert(lo rangesList.1)@SF,convert(hi rangesList.1)@SF), _
         segment(convert(lo rangesList.2)@SF,convert(hi rangesList.2)@SF)]
      rangesList
    units graf                ==
      null(graf.unitsField) => []
      [convert(graf.unitsField.1)@F,convert(graf.unitsField.2)@F]
    units (graf,unitsToBe)    ==
      graf.unitsField := [convert(unitsToBe.1)@SF,convert(unitsToBe.2)@SF]
      unitsToBe
    graphImage                == graph []

    makeGraphImage(llp) ==
      makeGraphImage(llp,
        [pointColorDefault() for i in 1..(l:=#llp)],
         [lineColorDefault() for i in 1..l],
          [pointSizeDefault() for i in 1..l])

    makeGraphImage(llp,lpc,llc,lps) ==
```

```
      makeGraphImage(llp,lpc,llc,lps,[])

    makeGraphImage(llp,lpc,llc,lps,opts) ==
      graf := graph(ranges(opts,[]))
      graf.optionsField := opts
      graf := plotLists(graf,llp,lpc,llc,lps)
      transform := coord(graf.optionsField,cartesian$COORDSYS)$DrawOptionFunctions0
      for aList in graf.llPoints repeat
        for p in aList repeat
          aPoint := transform p
          numberCheck aPoint
      makeGraph graf

    component (graf:$,ListOfPoints:L P,PointColor:PAL,LineColor:PAL,PointSize:PI) ==
      graf.llPoints    := append(graf.llPoints,[ListOfPoints])
      graf.pointColors := append(graf.pointColors,[PointColor])
      graf.lineColors  := append(graf.lineColors,[LineColor])
      graf.pointSizes  := append(graf.pointSizes,[PointSize])

    component (graf,aPoint) ==
      component(graf,aPoint,pointColorDefault(),lineColorDefault(),pointSizeDefault())

    component (graf:$,aPoint:P,PointColor:PAL,LineColor:PAL,PointSize:PI) ==
      component (graf,[aPoint],PointColor,LineColor,PointSize)

    appendPoint (graf,aPoint) ==
      num : I  := #(graf.llPoints) - 1
      num < 0 => error "No point lists to append to!"
      (graf.llPoints.num) := append((graf.llPoints.num),[aPoint])

    point (graf,aPoint,PointColor) ==
      component(graf,aPoint,PointColor,lineColorDefault(),pointSizeDefault())

    coerce (llp : L L P) : $ ==
      makeGraphImage(llp,
          [pointColorDefault() for i in 1..(l:=#llp)],
           [lineColorDefault() for i in 1..l],
            [pointSizeDefault() for i in 1..l])

    coerce (graf : $) : E ==
      hconcat( ["Graph with " :: E,(p := # pointLists graf) :: E,
         (p=1 => " point list"; " point lists") :: E])
```

— GRIMAGE.dotabb —

```
"GRIMAGE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GRIMAGE"]
```

```
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"GRIMAGE" -> "STRING"
```

———-

8.9 domain GOPT GuessOption

— GuessOption.input —

```
)set break resume
)sys rm -f GuessOption.output
)spool GuessOption.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GuessOption
--R GuessOption  is a domain constructor
--R Abbreviation for GuessOption is GOPT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GOPT
--R
--R------------------------------ Operations --------------------------------
--R ?=? : (%,%) -> Boolean                allDegrees : Boolean -> %
--R checkExtraValues : Boolean -> %       coerce : % -> OutputForm
--R debug : Boolean -> %                  displayKind : Symbol -> %
--R functionName : Symbol -> %            functionNames : List Symbol -> %
--R hash : % -> SingleInteger             indexName : Symbol -> %
--R latex : % -> String                   one : Boolean -> %
--R safety : NonNegativeInteger -> %      variableName : Symbol -> %
--R ?~=? : (%,%) -> Boolean
--R Somos : Union(PositiveInteger,Boolean) -> %
--R check : Union(skip,MonteCarlo,deterministic) -> %
--R homogeneous : Union(PositiveInteger,Boolean) -> %
--R maxDegree : Union(NonNegativeInteger,arbitrary) -> %
--R maxDerivative : Union(NonNegativeInteger,arbitrary) -> %
--R maxLevel : Union(NonNegativeInteger,arbitrary) -> %
--R maxMixedDegree : NonNegativeInteger -> %
--R maxPower : Union(PositiveInteger,arbitrary) -> %
--R maxShift : Union(NonNegativeInteger,arbitrary) -> %
--R maxSubst : Union(PositiveInteger,arbitrary) -> %
--R option : (List %,Symbol) -> Union(Any,"failed")
--R
--E 1
```

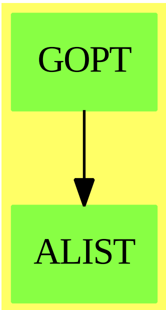
```
)spool
)lisp (bye)
```

— **GuessOption.help** —

```
====================================================================
GuessOption examples
====================================================================

See Also:
o )show GuessOption
```

8.9.1 GuessOption (GOPT)



Exports:

?=?	?~=?	Somos	allDegrees	check
checkExtraValues	coerce	debug	displayKind	functionName
functionNames	hash	homogeneous	indexName	latex
maxDegree	maxDerivative	maxLevel	maxMixedDegree	maxPower
maxShift	maxSubst	one	option	safety variableName

— **domain GOPT GuessOption** —

```
)abbrev domain GOPT GuessOption
++ Author: Martin Rubey
++ Description: GuessOption is a domain whose elements are various options used
++ by Guess.
GuessOption(): Exports == Implementation where

  Exports == SetCategory with
```

```
maxDerivative: Union(NonNegativeInteger, "arbitrary") -> %
  ++ maxDerivative(d) specifies the maximum derivative in an algebraic
  ++ differential equation.  This option is expressed in the form
  ++ \spad{maxDerivative == d}.

maxShift: Union(NonNegativeInteger, "arbitrary") -> %
  ++ maxShift(d) specifies the maximum shift in a recurrence
  ++ equation.  This option is expressed in the form \spad{maxShift == d}.

maxSubst: Union(PositiveInteger, "arbitrary") -> %
  ++ maxSubst(d) specifies the maximum degree of the monomial substituted
  ++ into the function we are looking for.  That is, if \spad{maxSubst ==
  ++ d}, we look for polynomials such that $p(f(x), f(x^2), ...,
  ++ f(x^d))=0$.  equation.  This option is expressed in the form
  ++ \spad{maxSubst == d}.

maxPower: Union(PositiveInteger, "arbitrary") -> %
  ++ maxPower(d) specifies the maximum degree in an algebraic differential
  ++ equation. For example, the degree of (f'')^3 f' is 4. maxPower(-1)
  ++ specifies that the maximum exponent can be arbitrary. This option is
  ++ expressed in the form \spad{maxPower == d}.

homogeneous: Union(PositiveInteger, Boolean) -> %
  ++ homogeneous(d) specifies whether we allow only homogeneous algebraic
  ++ differential equations.  This option is expressed in the form
  ++ \spad{homogeneous == d}.  If true, then maxPower must be
  ++ set, too, and ADEs with constant total degree are allowed.
  ++ If a PositiveInteger is given, only ADE's with this total degree are
  ++ allowed.

Somos: Union(PositiveInteger, Boolean) -> %
  ++ Somos(d) specifies whether we want that the total degree of the
  ++ differential operators is constant, and equal to d, or maxDerivative
  ++ if true. If true, maxDerivative must be set, too.

maxLevel: Union(NonNegativeInteger, "arbitrary") -> %
  ++ maxLevel(d) specifies the maximum number of recursion levels operators
  ++ guessProduct and guessSum will be applied. This option is expressed in
  ++ the form spad{maxLevel == d}.

maxDegree: Union(NonNegativeInteger, "arbitrary") -> %
  ++ maxDegree(d) specifies the maximum degree of the coefficient
  ++ polynomials in an algebraic differential equation or a recursion with
  ++ polynomial coefficients. For rational functions with an exponential
  ++ term, \spad{maxDegree} bounds the degree of the denominator
  ++ polynomial.
  ++ This option is expressed in the form \spad{maxDegree == d}.

maxMixedDegree: NonNegativeInteger -> %
```

```
    ++ maxMixedDegree(d) specifies the maximum q-degree of the coefficient
    ++ polynomials in a recurrence with polynomial coefficients, in the case
    ++ of mixed shifts.  Although slightly inconsistent, maxMixedDegree(0)
    ++ specifies that no mixed shifts are allowed. This option is expressed
    ++ in the form \spad{maxMixedDegree == d}.

  allDegrees: Boolean -> %
    ++ allDegrees(d) specifies whether all possibilities of the degree vector
    ++ - taking into account maxDegree - should be tried. This is mainly
    ++ interesting for rational interpolation. This option is expressed in
    ++ the form \spad{allDegrees == d}.

  safety: NonNegativeInteger -> %
    ++ safety(d) specifies the number of values reserved for testing any
    ++ solutions found. This option is expressed in the form \spad{safety ==
    ++ d}.

  check: Union("skip", "MonteCarlo", "deterministic") -> %
    ++ check(d) specifies how we want to check the solution.  If
    ++ the value is "skip", we return the solutions found by the
    ++ interpolation routine without checking.  If the value is
    ++ "MonteCarlo", we use a probabilistic check.  This option is
    ++ expressed in the form \spad{check == d}

  checkExtraValues: Boolean -> %
    ++ checkExtraValues(d) specifies whether we want to check the
    ++ solution beyond the order given by the degree bounds. This
    ++ option is expressed in the form \spad{checkExtraValues == d}

  one: Boolean -> %
    ++ one(d) specifies whether we are happy with one solution. This option
    ++ is expressed in the form \spad{one == d}.

  debug: Boolean -> %
    ++ debug(d) specifies whether we want additional output on the
    ++ progress. This option is expressed in the form \spad{debug == d}.

  functionName: Symbol -> %
    ++ functionName(d) specifies the name of the function given by the
    ++ algebraic differential equation or recurrence. This option is
    ++ expressed in the form \spad{functionName == d}.

  functionNames: List(Symbol) -> %
    ++ functionNames(d) specifies the names for the function in
    ++ algebraic dependence. This option is
    ++ expressed in the form \spad{functionNames == d}.

  variableName: Symbol -> %
    ++ variableName(d) specifies the variable used in by the algebraic
    ++ differential equation. This option is expressed in the form
```

```
      ++ \spad{variableName == d}.

    indexName: Symbol -> %
      ++ indexName(d) specifies the index variable used for the formulas. This
      ++ option is expressed in the form \spad{indexName == d}.

    displayKind: Symbol -> %
      ++ displayKind(d) specifies kind of the result: generating function,
      ++ recurrence or equation. This option should not be set by the
      ++ user, but rather by the HP-specification.

    option : (List %, Symbol) -> Union(Any, "failed")
      ++ option(l, option) returns which options are given.

  Implementation ==> add
    import AnyFunctions1(Boolean)
    import AnyFunctions1(Symbol)
    import AnyFunctions1(NonNegativeInteger)
    import AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))
    import AnyFunctions1(Union(PositiveInteger, "arbitrary"))
    import AnyFunctions1(Union(PositiveInteger, Boolean))
    import AnyFunctions1(Union("skip", "MonteCarlo", "deterministic"))

    Rep := Record(keyword: Symbol, value: Any)

    maxLevel d        == ['maxLevel,       d::Any]
    maxDerivative d   == ['maxDerivative,  d::Any]
    maxShift d        == maxDerivative d
    maxSubst d        ==
        if d case PositiveInteger
        then maxDerivative((d::Integer-1)::NonNegativeInteger)
        else maxDerivative d
    maxDegree d        == ['maxDegree,        d::Any]
    maxMixedDegree d   == ['maxMixedDegree,   d::Any]
    allDegrees d       == ['allDegrees,       d::Any]
    maxPower d         == ['maxPower,         d::Any]
    safety d           == ['safety,           d::Any]
    homogeneous d      == ['homogeneous,      d::Any]
    Somos d            == ['Somos,            d::Any]
    debug d            == ['debug,            d::Any]
    check d            == ['check,            d::Any]
    checkExtraValues d == ['checkExtraValues, d::Any]
    one d              == ['one,              d::Any]
    functionName d     == ['functionName,     d::Any]
    functionNames d ==
        ['functionNames, coerce(d)$AnyFunctions1(List(Symbol))]
    variableName d     == ['variableName,     d::Any]
    indexName d        == ['indexName,        d::Any]
    displayKind d      == ['displayKind,      d::Any]
```

```
    coerce(x:%):OutputForm == x.keyword::OutputForm = x.value::OutputForm
    x:% = y:%              == x.keyword = y.keyword and x.value = y.value

    option(l, s) ==
      for x in l repeat
        x.keyword = s => return(x.value)
      "failed"
```

———-

— GOPT.dotabb —

```
"GOPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GOPT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"GOPT" -> "ALIST"
```

———-

8.10 domain GOPT0 GuessOptionFunctions0

— GuessOptionFunctions0.input —

```
)set break resume
)sys rm -f GuessOptionFunctions0.output
)spool GuessOptionFunctions0.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GuessOptionFunctions0
--R GuessOptionFunctions0  is a domain constructor
--R Abbreviation for GuessOptionFunctions0 is GOPT0
--R This constructor is not exposed in this frame.
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one : List(GuessOption) -> Boolean    ?~=? : (%,%) -> Boolean
--R MonteCarlo : List(GuessOption) -> Boolean
--R Somos : List(GuessOption) -> Union(PositiveInteger,Boolean)
--R allDegrees : List(GuessOption) -> Boolean
--R check : List(GuessOption) -> Boolean
--R checkOptions : List(GuessOption) -> Void
--R debug : List(GuessOption) -> Boolean
```

```
--R displayAsGF : List(GuessOption) -> Boolean
--R functionName : List(GuessOption) -> Symbol
--R homogeneous : List(GuessOption) -> Union(PositiveInteger,Boolean)
--R indexName : List(GuessOption) -> Symbol
--R maxDegree : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxDerivative : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxLevel : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxMixedDegree : List(GuessOption) -> NonNegativeInteger
--R maxPower : List(GuessOption) -> Union(PositiveInteger,arbitrary)
--R maxShift : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxSubst : List(GuessOption) -> Union(PositiveInteger,arbitrary)
--R safety : List(GuessOption) -> NonNegativeInteger
--R variableName : List(GuessOption) -> Symbol
--R
--E 1

)spool
)lisp (bye)
```

———-

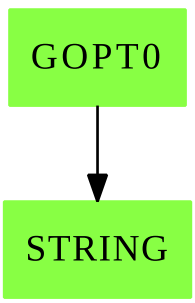
— **GuessOptionFunctions0.help** —

```
====================================================================
GuessOptionFunctions0 examples
====================================================================

See Also:
o )show GuessOptionFunctions0
```

———-

8.10.1 GuessOptionFunctions0 (GOPT0)



Exports:

?=?	?~=?	MonteCarlo	Somos	allDegrees
check	checkOptions	coerce	debug	displayAsGF
functionName	hash	homogeneous	indexName	latex
maxDegree	maxDerivative	maxLevel	maxMixedDegree	maxPower
maxShift	maxSubst	one	safety	variableName

— domain GOPT0 GuessOptionFunctions0 —

```
)abbrev domain GOPT0 GuessOptionFunctions0
++ Author: Martin Rubey
++ Description:
++ GuessOptionFunctions0 provides operations that extract the
++ values of options for Guess.
GuessOptionFunctions0(): Exports == Implementation where

  LGOPT ==> List GuessOption

  Exports == SetCategory with

    maxDerivative: LGOPT -> Union(NonNegativeInteger, "arbitrary")
      ++ maxDerivative returns the specified maxDerivative.

    maxShift: LGOPT -> Union(NonNegativeInteger, "arbitrary")
      ++ maxShift returns the specified maxShift.

    maxSubst: LGOPT -> Union(PositiveInteger, "arbitrary")
      ++ maxSubst returns the specified maxSubst.

    maxPower: LGOPT -> Union(PositiveInteger, "arbitrary")
      ++ maxPower returns the specified maxPower.

    homogeneous: LGOPT -> Union(PositiveInteger, Boolean)
      ++ homogeneous returns whether we allow only homogeneous algebraic
      ++ differential equations, default being false

    Somos: LGOPT -> Union(PositiveInteger, Boolean)
      ++ Somos returns whether we allow only Somos-like operators, default
      ++ being false

    maxLevel: LGOPT -> Union(NonNegativeInteger, "arbitrary")
      ++ maxLevel returns the specified maxLevel.

    maxDegree: LGOPT -> Union(NonNegativeInteger, "arbitrary")
      ++ maxDegree returns the specified maxDegree.

    maxMixedDegree: LGOPT -> NonNegativeInteger
      ++ maxMixedDegree returns the specified maxMixedDegree.

    allDegrees: LGOPT -> Boolean
```

```
    ++ allDegrees returns whether all possibilities of the degree vector
    ++ should be tried, the default being false.

  safety: LGOPT -> NonNegativeInteger
    ++ safety returns the specified safety or 1 as default.

  check: LGOPT -> Union("skip", "MonteCarlo", "deterministic")
    ++ check(d) specifies how we want to check the solution.  If
    ++ the value is "skip", we return the solutions found by the
    ++ interpolation routine without checking.  If the value is
    ++ "MonteCarlo", we use a probabilistic check.  The default is
    ++ "deterministic".

  checkExtraValues: LGOPT -> Boolean
    ++ checkExtraValues(d) specifies whether we want to check the
    ++ solution beyond the order given by the degree bounds.  The
    ++ default is true.

  one: LGOPT -> Boolean
    ++ one returns whether we need only one solution, default being true.

  functionName: LGOPT -> Symbol
    ++ functionName returns the name of the function given by the algebraic
    ++ differential equation, default being f

  variableName: LGOPT -> Symbol
    ++ variableName returns the name of the variable used in by the
    ++ algebraic differential equation, default being x

  indexName: LGOPT -> Symbol
    ++ indexName returns the name of the index variable used for the
    ++ formulas, default being n

  displayAsGF: LGOPT -> Boolean
    ++ displayAsGF specifies whether the result is a generating function
    ++ or a recurrence. This option should not be set by the user, but rather
    ++ by the HP-specification, therefore, there is no default.

  debug: LGOPT -> Boolean
    ++ debug returns whether we want additional output on the progress,
    ++ default being false

  checkOptions: LGOPT -> Void
    ++ checkOptions checks whether the given options are consistent, and
    ++ yields an error otherwise

Implementation == add

  maxLevel l ==
    if (opt := option(l, 'maxLevel)) case "failed" then
```

```
      "arbitrary"
    else
      retract(opt::Any)$AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))

  maxDerivative l ==
    if (opt := option(l, 'maxDerivative)) case "failed" then
      "arbitrary"
    else
      retract(opt::Any)$AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))

  maxShift l == maxDerivative l

  maxSubst l ==
      d := maxDerivative l
      if d case NonNegativeInteger
      then (d+1)::PositiveInteger
      else d

  maxDegree l ==
    if (opt := option(l, 'maxDegree)) case "failed" then
      "arbitrary"
    else
      retract(opt::Any)$AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))

  maxMixedDegree l ==
    if (opt := option(l, 'maxMixedDegree)) case "failed" then
      0
    else
      retract(opt :: Any)$AnyFunctions1(NonNegativeInteger)

  allDegrees l ==
    if (opt := option(l, 'allDegrees)) case "failed" then
      false
    else
      retract(opt :: Any)$AnyFunctions1(Boolean)

  maxPower l ==
    if (opt := option(l, 'maxPower)) case "failed" then
      "arbitrary"
    else
      retract(opt :: Any)$AnyFunctions1(Union(PositiveInteger, "arbitrary"))

  safety l ==
    if (opt := option(l, 'safety)) case "failed" then
      1$NonNegativeInteger
    else
      retract(opt :: Any)$AnyFunctions1(NonNegativeInteger)

  check l ==
     if (opt := option(l, 'check)) case "failed" then
```

```
            "deterministic"
        else
            retract(opt::Any)$AnyFunctions1(_
                          Union("skip", "MonteCarlo", "deterministic"))

    checkExtraValues l ==
        if (opt := option(l, 'checkExtraValues)) case "failed" then
            true
        else
            retract(opt :: Any)$AnyFunctions1(Boolean)

    one l ==
      if (opt := option(l, 'one)) case "failed" then
        true
      else
        retract(opt :: Any)$AnyFunctions1(Boolean)

    debug l ==
      if (opt := option(l, 'debug)) case "failed" then
        false
      else
        retract(opt :: Any)$AnyFunctions1(Boolean)

    homogeneous l ==
      if (opt := option(l, 'homogeneous)) case "failed" then
        false
      else
        retract(opt :: Any)$AnyFunctions1(Union(PositiveInteger, Boolean))

    Somos l ==
      if (opt := option(l, 'Somos)) case "failed" then
        false
      else
        retract(opt :: Any)$AnyFunctions1(Union(PositiveInteger, Boolean))

    variableName l ==
      if (opt := option(l, 'variableName)) case "failed" then
        'x
      else
        retract(opt :: Any)$AnyFunctions1(Symbol)

    functionName l ==
      if (opt := option(l, 'functionName)) case "failed" then
        'f
      else
        retract(opt :: Any)$AnyFunctions1(Symbol)

    indexName l ==
      if (opt := option(l, 'indexName)) case "failed" then
        'n
```

```
    else
      retract(opt :: Any)$AnyFunctions1(Symbol)

  displayAsGF l ==
    if (opt := option(l, 'displayAsGF)) case "failed" then
      error "GuessOption: displayAsGF not set"
    else
      retract(opt :: Any)$AnyFunctions1(Boolean)

  NNI ==> NonNegativeInteger
  PI  ==> PositiveInteger

  checkOptions l ==
    maxD := maxDerivative l
    maxP := maxPower l
    homo := homogeneous l
    Somo := Somos l

    if Somo case PI then
        if one? Somo then
            error "Guess: Somos must be Boolean or at least two"

        if maxP case PI and one? maxP then
            error "Guess: Somos requires that maxPower is at least two"

        if maxD case NNI and maxD > Somo then
            err:String:=concat [_
              "Guess: if Somos is an integer, it should be larger than ",_
              "maxDerivative/maxShift or at least as big as maxSubst" ]
            error err
    else
        if Somo then
            if maxP case PI and one? maxP then
                error "Guess: Somos requires that maxPower is at least two"

            if not (maxD case NNI) or zero? maxD or one? maxD then
                err:String:= concat [_
                  "Guess: Somos==true requires that maxDerivative/maxShift",_
                  " is an integer, at least two, or maxSubst is an ",_
                  "integer, at least three" ]
                error err

            if not (maxP case PI) and homo case Boolean and not homo then
                err:String:= concat [_
                  "Guess: Somos requires that maxPower is set or ", _
                  "homogeneous is not false" ]
                error err

    if homo case PI then
        if maxP case PI and maxP ~= homo then
```

```
              err:String:= _
                "Guess: only one of homogeneous and maxPower may be an integer"
              error err

          if maxD case NNI and zero? maxD then
              err:String:= concat [_
                "Guess: homogeneous requires that maxShift/maxDerivative ",_
                "is at least one or maxSubst is at least two" ]
              error err
      else
          if homo then
              if not maxP case PI then
                  err:String:= concat [_
                    "Guess: homogeneous==true requires that maxPower is ", _
                    "an integer" ]
                  error err

              if maxD case NNI and zero? maxD then
                  err:String:= concat [_
                    "Guess: homogeneous requires that maxShift/maxDerivative",_
                    " is at least one or maxSubst is at least two" ]
                  error err
```

— GOPT0.dotabb —

```
"GOPT0" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GOPT0"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"GOPT0" -> "STRING"
```

Chapter 9

Chapter H

9.1 domain HASHTBL HashTable

— HashTable.input —

```
)set break resume
)sys rm -f HashTable.output
)spool HashTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HashTable
--R HashTable(Key: SetCategory,Entry: SetCategory,hashfn: String)  is a domain constructor
--R Abbreviation for HashTable is HASHTBL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HASHTBL
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         dictionary : () -> %
--R elt : (%,Key,Entry) -> Entry          ?.? : (%,Key) -> Entry
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Entry             eq? : (%,%) -> Boolean
--R index? : (Key,%) -> Boolean           indices : % -> List Key
--R key? : (Key,%) -> Boolean             keys : % -> List Key
--R map : ((Entry -> Entry),%) -> %       qelt : (%,Key) -> Entry
--R sample : () -> %                      setelt : (%,Key,Entry) -> Entry
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Key,entry: Entry) -> %
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if $ has finiteAggregate and Entry has SETCAT
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has fi
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(ke
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) 
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: En
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggreg
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: En
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ ha
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if $ has finiteAggregate and Reco
--R members : % -> List Entry if $ has finiteAggregate
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Entry if $ has finiteAggregate
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: K
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: K
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: K
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate and Record(key:
--R remove! : (Key,%) -> Union(Entry,"failed")
```

```
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
--R table : List Record(key: Key,entry: Entry) -> %
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

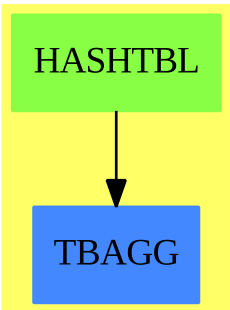
— **HashTable.help** —

```
====================================================================
HashTable examples
====================================================================

See Also:
o )show HashTable
```

———-

9.1.1 HashTable (HASHTBL)



See

⇒ “InnerTable” (INTABL) 10.27.1 on page 1299
⇒ “Table” (TABLE) 21.1.1 on page 2621

⇒ "EqTable" (EQTBL) 6.2.1 on page 667
⇒ "StringTable" (STRTBL) 20.32.1 on page 2569
⇒ "GeneralSparseTable" (GSTBL) 8.5.1 on page 1044
⇒ "SparseTable" (STBL) 20.16.1 on page 2409

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	entry?	elt
empty	empty?	entries	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?~=?	?.?

— domain HASHTBL HashTable —

```
)abbrev domain HASHTBL HashTable
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table, EqTable, StringTable
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides access to the underlying Lisp hash tables.
++ By varying the hashfn parameter, tables suited for different
++ purposes can be obtained.

HashTable(Key, Entry, hashfn): Exports == Implementation where
    Key, Entry: SetCategory
    hashfn: String --  Union("EQ", "UEQUAL", "CVEC", "ID")

    Exports ==> TableAggregate(Key, Entry) with
                     finiteAggregate

    Implementation ==> add
        Pair ==> Record(key: Key, entry: Entry)
        Ex   ==> OutputForm
        failMsg := GENSYM()$Lisp

        t1 = t2              == EQ(t1, t2)$Lisp
```

```
        keys t               == HKEYS(t)$Lisp
        # t                  == HASH_-TABLE_-COUNT(t)$Lisp
        setelt(t, k, e)      == HPUT(t,k,e)$Lisp
        remove_!(k:Key, t:%) ==
          r := HGET(t,k,failMsg)$Lisp
          not EQ(r,failMsg)$Lisp =>
            HREM(t, k)$Lisp
            r pretend Entry
          "failed"

        empty() ==
            MAKE_-HASHTABLE(INTERN(hashfn)$Lisp,
                            INTERN("STRONG")$Lisp)$Lisp

        search(k:Key, t:%)  ==
            r := HGET(t, k, failMsg)$Lisp
            not EQ(r, failMsg)$Lisp => r pretend Entry
            "failed"
```

— HASHTBL.dotabb —

```
"HASHTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HASHTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"HASHTBL" -> "TBAGG"
```

9.2 domain HEAP Heap

— Heap.input —

```
)set break resume
)sys rm -f Heap.output
)spool Heap.output
)set message test on
)set message auto off
)clear all

--S 1 of 42
a:Heap INT:= heap [1,2,3,4,5]
--R
--R
```

```
--R   (1)  [5,4,2,1,3]
--R                                                     Type: Heap Integer
--E 1

--S 2 of 42
bag([1,2,3,4,5])$Heap(INT)
--R
--R
--R   (2)  [5,4,3,1,2]
--R                                                     Type: Heap Integer
--E 2

--S 3 of 42
c:=copy a
--R
--R
--R   (3)  [5,4,2,1,3]
--R                                                     Type: Heap Integer
--E 3

--S 4 of 42
empty? a
--R
--R
--R   (4)  false
--R                                                          Type: Boolean
--E 4

--S 5 of 42
b:=empty()$(Heap INT)
--R
--R
--R   (5)  []
--R                                                     Type: Heap Integer
--E 5

--S 6 of 42
empty? b
--R
--R
--R   (6)  true
--R                                                          Type: Boolean
--E 6

--S 7 of 42
eq?(a,c)
--R
--R
--R   (7)  false
--R                                                          Type: Boolean
```

```
--E 7

--S 8 of 42
extract! a
--R
--R
--R   (8)  5
--R                                                    Type: PositiveInteger
--E 8

--S 8 of 42
h:=heap [17,-4,9,-11,2,7,-7]
--R
--R
--R   (9)  [17,2,9,- 11,- 4,7,- 7]
--R                                                      Type: Heap Integer
--E 8

--S 9 of 42
[extract!(h) while not empty?(h)]
--R
--R
--R   (10)  [17,9,7,2,- 4,- 7,- 11]
--R                                                      Type: List Integer
--E 9

--S 10 of 42
heapsort(x) == (empty? x => []; cons(extract!(x),heapsort x))
--R
--R                                                           Type: Void
--E 10

--S 11 of 42
h1 := heapsort heap [17,-4,9,-11,2,7,-7]
--R
--R   Compiling function heapsort with type Heap Integer -> List Integer
--R
--R   (12)  [17,9,7,2,- 4,- 7,- 11]
--R                                                      Type: List Integer
--E 11

--S 12 of 42
(a=c)@Boolean
--R
--R
--R   (13)  false
--R                                                         Type: Boolean
--E 12

--S 13 of 42
```

```
(a~=c)
--R
--R
--R   (14)  true
--R                                                            Type: Boolean
--E 13

--S 14 of 42
a
--R
--R
--R   (15)  [4,3,2,1]
--R                                                       Type: Heap Integer
--E 14

--S 15 of 42
inspect a
--R
--R
--R   (16)  4
--R                                                    Type: PositiveInteger
--E 15

--S 16 of 42
insert!(9,a)
--R
--R
--R   (17)  [9,4,2,1,3]
--R                                                       Type: Heap Integer
--E 16

--S 17 of 42
map(x+->x+10,a)
--R
--R
--R   (18)  [19,14,12,11,13]
--R                                                       Type: Heap Integer
--E 17

--S 18 of 42
a
--R
--R
--R   (19)  [9,4,2,1,3]
--R                                                       Type: Heap Integer
--E 18

--S 19 of 42
map!(x+->x+10,a)
--R
```

```
--R
--R   (20)  [19,14,12,11,13]
--R                                                     Type: Heap Integer
--E 19

--S 20 of 42
a
--R
--R
--R   (21)  [19,14,12,11,13]
--R                                                     Type: Heap Integer
--E 20

--S 21 of 42
max a
--R
--R
--R   (22)  19
--R                                                  Type: PositiveInteger
--E 21

--S 22 of 42
merge(a,c)
--R
--R
--R   (23)  [19,14,12,11,13,5,4,2,1,3]
--R                                                     Type: Heap Integer
--E 22

--S 23 of 42
a
--R
--R
--R   (24)  [19,14,12,11,13]
--R                                                     Type: Heap Integer
--E 23

--S 24 of 42
merge!(a,c)
--R
--R
--R   (25)  [19,14,12,11,13,5,4,2,1,3]
--R                                                     Type: Heap Integer
--E 24

--S 25 of 42
a
--R
--R
--R   (26)  [19,14,12,11,13,5,4,2,1,3]
```

```
--R                                                   Type: Heap Integer
--E 25

--S 26 of 42
c
--R
--R
--R   (27)  [5,4,2,1,3]
--R                                                   Type: Heap Integer
--E 26

--S 27 of 42
sample()$Heap(INT)
--R
--R
--R   (28)  []
--R                                                   Type: Heap Integer
--E 27

--S 28 of 42
#a
--R
--R
--R   (29)  10
--R                                                Type: PositiveInteger
--E 28

--S 29 of 42
any?(x+->(x=14),a)
--R
--R
--R   (30)  true
--R                                                        Type: Boolean
--E 29

--S 30 of 42
every?(x+->(x=11),a)
--R
--R
--R   (31)  false
--R                                                        Type: Boolean
--E 30

--S 31 of 42
parts a
--R
--R
--R   (32)  [19,14,12,11,13,5,4,2,1,3]
--R                                                   Type: List Integer
--E 31
```

```
--S 32 of 42
size?(a,9)
--R
--R
--R   (33)  false
--R                                                            Type: Boolean
--E 32

--S 33 of 42
more?(a,9)
--R
--R
--R   (34)  true
--R                                                            Type: Boolean
--E 33

--S 34 of 42
less?(a,9)
--R
--R
--R   (35)  false
--R                                                            Type: Boolean
--E 34

--S 35 of 42
members a
--R
--R
--R   (36)  [19,14,12,11,13,5,4,2,1,3]
--R                                                       Type: List Integer
--E 35

--S 36 of 42
member?(14,a)
--R
--R
--R   (37)  true
--R                                                            Type: Boolean
--E 36

--S 37 of 42
latex a
--R
--R
--R   (38)  "\mbox{\bf Unimplemented}"
--R                                                             Type: String
--E 37

--S 38 of 42
```

```
hash a
--R
--R
--I   (39)  36647017
--R                                                       Type: SingleInteger
--E 38

--S 39 of 42
count(14,a)
--R
--R
--R   (40)  1
--R                                                     Type: PositiveInteger
--E 39

--S 40 of 42
count(x+->(x>13),a)
--R
--R
--R   (41)  2
--R                                                     Type: PositiveInteger
--E 40

--S 41 of 42
coerce a
--R
--R
--R   (42)  [19,14,12,11,13,5,4,2,1,3]
--R                                                          Type: OutputForm
--E 41

--S 42 of 42
)show Heap
--R
--R Heap S: OrderedSet  is a domain constructor
--R Abbreviation for Heap is HEAP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HEAP
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                    copy : % -> %
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               extract! : % -> S
--R heap : List S -> %                   insert! : (S,%) -> %
--R inspect : % -> S                     map : ((S -> S),%) -> %
--R max : % -> S                         merge : (%,%) -> %
--R merge! : (%,%) -> %                  sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 42

)spool
)lisp (bye)
```

— **Heap.help** —

```
====================================================================
Heap examples
====================================================================

The domain Heap(S) implements a priority queue of objects of type S
such that the operation extract! removes and returns the maximum
element.  The implementation represents heaps as flexible arrays The
representation and algorithms give complexity of O(log(n)) for
insertion and extractions, and O(n) for construction.

Create a heap of five elements:

   a:Heap INT:= heap [1,2,3,4,5]
        [5,4,2,1,3]

Use bag to convert a Bag into a Heap:

   bag([1,2,3,4,5])$Heap(INT)
        [5,4,3,1,2]

The operation copy can be used to copy a Heap:
```

```
   c:=copy a
        [5,4,2,1,3]
```

Use empty? to check if the heap is empty:

```
   empty? a
        false
```

Use empty to create a new, empty heap:

```
   b:=empty()$(Heap INT)
        []
```

and we can see that the newly created heap is empty:

```
   empty? b
        true
```

The eq? function compares the reference of one heap to another:

```
   eq?(a,c)
        false
```

The extract! function removes largest element of the heap:

```
   extract! a
        5
```

Now extract! elements repeatedly until none are left, collecting the elements in a list.

```
  [extract!(h) while not empty?(h)]
    [9,7,3,2,- 4,- 7]
                      Type: List Integer
```

Another way to produce the same result is by defining a heapsort function.

```
  heapsort(x) == (empty? x => []; cons(extract!(x),heapsort x))
                      Type: Void
```

Create another sample heap.

```
  h1 := heap [17,-4,9,-11,2,7,-7]
    [17,2,9,- 11,- 4,7,- 7]
                      Type: Heap Integer
```

Apply heapsort to present elements in order.

```
  heapsort h1
```

```
 [17,9,7,2,- 4,- 7,- 11]
                    Type: List Integer
```

Heaps can be compared with =

```
(a=c)@Boolean
     false
```

and ~=

```
(a~=c)
    true
```

The inspect function shows the largest element in the heap:

```
inspect a
    4
```

The insert! function adds an element to the heap:

```
insert!(9,a)
    [9,4,2,1,3]
```

The map function applies a function to every element of the heap and returns a new heap:

```
map(x+->x+10,a)
    [19,14,12,11,13]
```

The original heap is unchanged:

```
a
    [9,4,2,1,3]
```

The map! function applies a function to every element of the heap and returns the original heap with modifications:

```
map!(x+->x+10,a)
    [19,14,12,11,13]
```

The original heap has been modified:

```
a
    [19,14,12,11,13]
```

The max function returns the largest element in the heap:

```
max a
    19
```

The merge function takes two heaps and creates a new heap with all of the elements:

```
   merge(a,c)
       [19,14,12,11,13,5,4,2,1,3]
```

Notice that the original heap is unchanged:

```
   a
       [19,14,12,11,13]
```

The merge! function takes two heaps and modifies the first heap argument to contain all of the elements:

```
   merge!(a,c)
       [19,14,12,11,13,5,4,2,1,3]
```

Notice that the first argument was modified:

```
   a
       [19,14,12,11,13,5,4,2,1,3]
```

but the second argument was not:

```
   c
       [5,4,2,1,3]
```

A new, empty heap can be created with sample:

```
   sample()$Heap(INT)
       []
```

The # function gives the size of the heap:

```
   #a
       10
```

The any? function tests each element against a predicate function and returns true if any pass:

```
   any?(x+->(x=14),a)
       true
```

The every? function tests each element against a predicate function and returns true if they all pass:

```
   every?(x+->(x=11),a)
       false
```

The parts function returns a list of the elements in the heap:

```
   parts a
       [19,14,12,11,13,5,4,2,1,3]
```

The size? predicate compares the size of the heap to a value:

```
   size?(a,9)
       false
```

The more? predicate asks if the heap size is larger than a value:

```
   more?(a,9)
       true
```

The less? predicate asks if the heap size is smaller than a value:

```
   less?(a,9)
       false
```

The members function returns a list of the elements of the heap:

```
   members a
       [19,14,12,11,13,5,4,2,1,3]
```

The member? predicate asks if an element is in the heap:

```
   member?(14,a)
       true
```

The count function has two forms, one of which counts the number
of copies of an element in the heap:

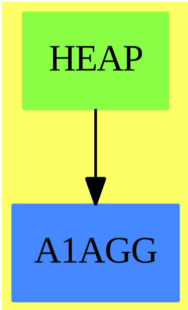
```
   count(14,a)
       1
```

The second form of the count function accepts a predicate to test
against each member of the heap and counts the number of true results:

```
   count(x+->(x>13),a)
       2
```

See Also:
o)show Stack
o)show ArrayStack
o)show Queue
o)show Dequeue
o)show Heap
o)show BagAggregate

9.2.1 Heap (HEAP)



See

⇒ "Stack" (STACK) 20.28.1 on page 2521
⇒ "ArrayStack" (ASTACK) 2.10.1 on page 65
⇒ "Queue" (QUEUE) 18.5.1 on page 2143
⇒ "Dequeue" (DEQUEUE) 5.5.1 on page 497

Exports:

any?	bag	coerce	copy	count
empty	empty?	eq?	eval	every?
extract!	hash	heap	insert!	inspect
latex	less?	map	map!	max
member?	members	merge	merge!	more?
parts	sample	size?	#?	?=?
?~=?				

— **domain HEAP Heap** —

```
)abbrev domain HEAP Heap
++ Author: Michael Monagan and Stephen Watt
++ Date Created:June 86 and July 87
++ Date Last Updated:Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Heap implemented in a flexible array to allow for insertions
++ Complexity: O(log n) insertion, extraction and O(n) construction
```

```
--% Dequeue and Heap data types

Heap(S:OrderedSet): Exports == Implementation where
  Exports == PriorityQueueAggregate S with
    heap : List S -> %
      ++ heap(ls) creates a heap of elements consisting of the
      ++ elements of ls.
      ++
      ++E i:Heap INT := heap [1,6,3,7,5,2,4]

 -- Inherited Signatures repeated for examples documentation

    bag : List S -> %
      ++
      ++X bag([1,2,3,4,5])$Heap(INT)
    copy : % -> %
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X copy a
    empty? : % -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X empty? a
    empty : () -> %
      ++
      ++X b:=empty()$(Heap INT)
    eq? : (%,%) -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X b:=copy a
      ++X eq?(a,b)
    extract_! : % -> S
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X extract! a
      ++X a
    insert_! : (S,%) -> %
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X insert!(8,a)
      ++X a
    inspect : % -> S
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X inspect a
    map :  ((S -> S),%) -> %
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X map(x+->x+10,a)
      ++X a
```

```
max : % -> S
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X max a
merge : (%,%) -> %
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X b:Heap INT:= heap [6,7,8,9,10]
  ++X merge(a,b)
merge! : (%,%) -> %
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X b:Heap INT:= heap [6,7,8,9,10]
  ++X merge!(a,b)
  ++X a
  ++X b
sample : () -> %
  ++
  ++X sample()$Heap(INT)
less? : (%,NonNegativeInteger) -> Boolean
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X less?(a,9)
more? : (%,NonNegativeInteger) -> Boolean
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X more?(a,9)
size? : (%,NonNegativeInteger) -> Boolean
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X size?(a,5)
if $ has shallowlyMutable then
  map! :  ((S -> S),%) -> %
    ++
    ++X a:Heap INT:= heap [1,2,3,4,5]
    ++X map!(x+->x+10,a)
    ++X a
if S has SetCategory then
  latex : % -> String
    ++
    ++X a:Heap INT:= heap [1,2,3,4,5]
    ++X latex a
  hash : % -> SingleInteger
    ++
    ++X a:Heap INT:= heap [1,2,3,4,5]
    ++X hash a
  coerce : % -> OutputForm
    ++
    ++X a:Heap INT:= heap [1,2,3,4,5]
    ++X coerce a
```

```
    "=": (%,%) -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X b:Heap INT:= heap [1,2,3,4,5]
      ++X (a=b)@Boolean
    "~=" : (%,%) -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X b:=copy a
      ++X (a~=b)
  if % has finiteAggregate then
    every? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X every?(x+->(x=4),a)
    any? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X any?(x+->(x=4),a)
    count :  ((S -> Boolean),%) -> NonNegativeInteger
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X count(x+->(x>2),a)
    _# : % -> NonNegativeInteger
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X #a
    parts : % -> List S
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X parts a
    members : % -> List S
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X members a
  if % has finiteAggregate and S has SetCategory then
    member? : (S,%) -> Boolean
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X member?(3,a)
    count : (S,%) -> NonNegativeInteger
      ++
      ++X a:Heap INT:= heap [1,2,3,4,5]
      ++X count(4,a)

Implementation == IndexedFlexibleArray(S,0) add
  Rep := IndexedFlexibleArray( S,0)
  empty() == empty()$Rep
  heap l ==
    n := #l
```

```
  h := empty()
  n = 0 => h
  for x in l repeat insert_!(x,h)
  h
siftUp: (%,Integer,Integer) -> Void
siftUp(r,i,n) ==
   -- assertion 0 <= i < n
   t := r.i
   while (j := 2*i+1) < n repeat
      if (k := j+1) < n and r.j < r.k then j := k
      if t < r.j then (r.i := r.j; r.j := t; i := j) else leave

extract_! r ==
   -- extract the maximum from the heap O(log n)
   n := #r :: Integer
   n = 0 => error "empty heap"
   t := r(0)
   r(0) := r(n-1)
   delete_!(r,n-1)
   n = 1 => t
   siftUp(r,0,n-1)
   t

insert_!(x,r) ==
   -- Williams' insertion algorithm O(log n)
   j := (#r) :: Integer
   r:=concat_!(r,concat(x,empty()$Rep))
   while j > 0 repeat
      i := (j-1) quo 2
      if r(i) >= x then leave
      r(j) := r(i)
      j := i
   r(j):=x
   r

max r == if #r = 0 then error "empty heap" else r.0
inspect r == max r

makeHeap(r:%):% ==
   -- Floyd's heap construction algorithm O(n)
   n := #r
   for k in n quo 2 -1 .. 0 by -1 repeat siftUp(r,k,n)
   r
bag l == makeHeap construct(l)$Rep
merge(a,b) == makeHeap concat(a,b)
merge_!(a,b) == makeHeap concat_!(a,b)
```

———-

— **HEAP.dotabb** —

```
"HEAP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HEAP"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"HEAP" -> "A1AGG"
```

————

9.3 domain HEXADEC HexadecimalExpansion

— **HexadecimalExpansion.input** —

```
)set break resume
)sys rm -f HexadecimalExpansion.output
)spool HexadecimalExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
r := hex(22/7)
--R
--R
--R           ---
--R   (1)  3.249
--R                                                   Type: HexadecimalExpansion
--E 1

--S 2 of 7
r + hex(6/7)
--R
--R
--R   (2)  4
--R                                                   Type: HexadecimalExpansion
--E 2

--S 3 of 7
[hex(1/i) for i in 350..354]
--R
--R
--R   (3)
--R       ---------------    ---------      -----    ----------------------
--R   [0.00BB3EE721A54D88, 0.00BAB6561, 0.00BA2E8, 0.00B9A7862A0FF465879D5F,
--R       -----------------------------
--R    0.00B92143FA36F5E02E4850FE8DBD78]
--R                                              Type: List HexadecimalExpansion
```

```
--E 3

--S 4 of 7
hex(1/1007)
--R
--R
--R   (4)
--R   0.
--R     OVERBAR
--R        0041149783F0BF2C7D13933192AF6980619EE345E91EC2BB9D5CCA5C071E40926E54E8D
--R          DAE24196C0B2F8A0AAD60DBA57F5D4C8536262210C74F1
--R                                                     Type: HexadecimalExpansion
--E 4

--S 5 of 7
p := hex(1/4)*x**2 + hex(2/3)*x + hex(4/9)
--R
--R
--R              2      _      ___
--R   (5)  0.4x  + 0.Ax + 0.71C
--R                                          Type: Polynomial HexadecimalExpansion
--E 5

--S 6 of 7
q := D(p, x)
--R
--R
--R                      _
--R   (6)  0.8x + 0.A
--R                                          Type: Polynomial HexadecimalExpansion
--E 6

--S 7 of 7
g := gcd(p, q)
--R
--R
--R                  _
--R   (7)  x + 1.5
--R                                          Type: Polynomial HexadecimalExpansion
--E 7
)spool
)lisp (bye)
```

— **HexadecimalExpansion.help** —

```
====================================================================
HexadecimalExpansion
```

```
====================================================================

All rationals have repeating hexadecimal expansions.  The operation
hex returns these expansions of type HexadecimalExpansion.  Operations
to access the individual numerals of a hexadecimal expansion can be
obtained by converting the value to RadixExpansion(16).  More examples
of expansions are available in the DecimalExpansion, BinaryExpansion,
and RadixExpansion.

This is a hexadecimal expansion of a rational number.

  r := hex(22/7)
      ---
    3.249
                      Type: HexadecimalExpansion

Arithmetic is exact.

  r + hex(6/7)
    4
                      Type: HexadecimalExpansion

The period of the expansion can be short or long ...

  [hex(1/i) for i in 350..354]
        ---------------    ---------      -----    ----------------------
   [0.00BB3EE721A54D88, 0.00BAB6561, 0.00BA2E8, 0.00B9A7862A0FF465879D5F,
        -----------------------------
    0.00B92143FA36F5E02E4850FE8DBD78]
                      Type: List HexadecimalExpansion

or very long!

  hex(1/1007)
      ----------------------------------------------------------------------
   0.0041149783F0BF2C7D13933192AF6980619EE345E91EC2BB9D5CCA5C071E40926E54E8D
     ----------------------------------------------
     DAE24196C0B2F8A0AAD60DBA57F5D4C8536262210C74F1
                      Type: HexadecimalExpansion

These numbers are bona fide algebraic objects.

  p := hex(1/4)*x**2 + hex(2/3)*x + hex(4/9)
        2      -      ---
    0.4x  + 0.Ax + 0.71C
                      Type: Polynomial HexadecimalExpansion

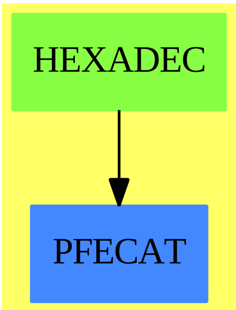
  q := D(p, x)
               -
    0.8x + 0.A
```

```
                      Type: Polynomial HexadecimalExpansion

  g := gcd(p, q)
             _
    x + 1.5
                      Type: Polynomial HexadecimalExpansion

See Also:
o )help RadixExpansion
o )help BinaryExpansion
o )help DecimalExpansion
o )show HexadecimalExpansion
```

9.3.1 HexadecimalExpansion (HEXADEC)



See

⇒ "RadixExpansion" (RADIX) 19.2.1 on page 2165
⇒ "BinaryExpansion" (BINARY) 3.7.1 on page 274
⇒ "DecimalExpansion" (DECIMAL) 5.3.1 on page 451

Exports:

0	1
abs	associates?
ceiling	characteristic
charthRoot	coerce
conditionP	convert
D	denom
denominator	differentiate
divide	euclideanSize
eval	expressIdealMember
exquo	extendedEuclidean
factor	factorPolynomial
factorSquareFreePolynomial	floor
fractionPart	gcd
gcdPolynomial	hash
hex	init
inv	latex
lcm	map
max	min
multiEuclidean	negative?
nextItem	numer
numerator	one?
patternMatch	positive?
prime?	principalIdeal
random	recip
reducedSystem	retract
retractIfCan	sample
sign	sizeLess?
solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial
subtractIfCan	unit?
unitCanonical	unitNormal
wholePart	zero?
?*?	?**?
?+?	?-?
-?	?/?
?=?	?^?
?~=?	?<?
?<=?	?>?
?>=?	?.?
?quo?	?rem?

— domain HEXADEC HexadecimalExpansion —

```
)abbrev domain HEXADEC HexadecimalExpansion
++ Author: Clifton J. Williamson
```

```
++ Date Created: April 26, 1990
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains: RadixExpansion
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, hexadecimal
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ hexadecimal expansions.

HexadecimalExpansion(): Exports == Implementation where
  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
      ++ coerce(h) converts a hexadecimal expansion to a rational number.
    coerce: % -> RadixExpansion(16)
      ++ coerce(h) converts a hexadecimal expansion to a radix expansion
      ++ with base 16.
    fractionPart: % -> Fraction Integer
      ++ fractionPart(h) returns the fractional part of a hexadecimal expansion.
    hex: Fraction Integer -> %
      ++ hex(r) converts a rational number to a hexadecimal expansion.

  Implementation ==> RadixExpansion(16) add
    hex r == r :: %
    coerce(x:%): RadixExpansion(16) == x pretend RadixExpansion(16)
```

— **HEXADEC.dotabb** —

```
"HEXADEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HEXADEC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"HEXADEC" -> "PFECAT"
```

9.4 package HTMLFORM HTMLFormat

Here I have put some information about 'how to use' and 'the benefits of' this HTML formatter. Also some information for programmers if they want to extend this package.

If you want information about creating output formatters in general then, rather than duplicating content here I refer you to mathml.spad.pamphlet containing the MathMLFormat domain by Arthur C. Ralfs. This contains useful information for writers of output formatters.

9.4.1 Overview

This package allows users to cut and paste output from the Axiom command line to a HTML page. This output is enabled by typing:

```
)set output html on
```

After this the command line will output html (in addition to other formats that are enabled) and this html code can then be copied and pasted into a HTML document.

The HTML produced is well formed XML, that is, all tags have equivalent closing tags.

9.4.2 Why output to HTML?

In some ways HTMLFormat is a compromise between the standard text output and specialised formats like MathMLFormat. The potential quality is never going to be as good as output to a specialised maths renderer but on the other hand it is a lot better than the clunky fixed width font text output. The quality is not the only issue though, the direct output in any format is unlikely to be exactly what the user wants, so possibly more important than quality is the ability to edit the output.

HTMLFormat has advantages that the other output formats don't, for instance,

- It works with any browser without the need for plugins (as far as I know most computers should have the required fonts)
- Users can easily annotate and add comments using colour, bold, underline and so on.
- Annotations can easily be done with whatever html editor or text editor you are familiar with.
- Edits to the output will cause the width of columns and so on to be automatically adjusted, no need to try to insert spaces to get the superscripts to line up again!
- It is very easy to customise output so, for instance, we can fit a lot of information in a compact space on the page.

9.5 Using the formatter

We can cause the command line interpreter to output in html by typing the following:

```
)set output html on
```

After this the command line will output html (in addition to other formats that are enabled) and this html code can then be copied and pasted into an existing HTML document.

If you do not already have an html page to copy the output to then you can create one with a text editor and entering the following:

```
<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.1//EN"
 "http://www.w3.org/TR/xhtml11/DTD/xhtml11.dtd">
<html xmlns="http://www.w3.org/1999/xhtml" >
 <head>
  <title>Enter Your Title Here</title>
 </head>
 <body>
  Copy and paste the output from command line here.
 </body>
</html>
```

Or using any program that will export to html such as OpenOffice.org writer.

9.6 Form of the output

```
HTMLFormat does not try to interpret syntax, for instance in an example like:
(1) -> integral(x^x,x)
it just takes what OutputForm provides and does not try to replace
%A with the bound variable x.
```

9.7 Matrix Formatting

A big requirement for me is to fit big matrices on ordinary web pages.

At the moment the default output for a matrix is a grid, however it easy to modify this for a single matrix, or a whole page or whole site by using css (cascading style sheets). For instance we can get a more conventional looking matrix by adding the following style to the top of the page after the ¡head¿ tag:

```
<style type="text/css">
#matl {border-left-style:solid}
#matr {border-right-style:solid}
#matlt {border-left-style:solid;border-top-style:solid}
#matrt {border-right-style:solid;border-top-style:solid}
#matlb {border-left-style:solid;border-bottom-style:solid}
#matrb {border-right-style:solid;border-bottom-style:solid}
</style>
```

There are many other possibilities, for instance we can generate a matrix with bars either side to indicate a determinant. All we have to do is change the css for the site, page or individual element.

9.8 Programmers Guide

This package converts from OutputForm, which is a hierarchical tree structure, to html which uses tags arranged in a hierarchical tree structure. So the package converts from one tree (graph) structure to another.

This conversion is done in two stages using an intermediate Tree String structure. This Tree String structure represents HTML where:

- leafs represents unstructured text
- string in leafs contains the text
- non-leafs represents xml elements
- string in non-leafs represents xml attributes

This is created by traversing OutputForm while building up the Tree String structure.

The second stage is to convert the Tree Structure to text. All text output is done using:

```
sayTeX$Lisp
```

I have not produced and output to String as I don't know a way to append to a long string efficiently and I don't know how to insert carriage- returns into a String.

9.8.1 Future Developments

There would be some benefits in creating a XMLFormat category which would contain common elements for all xml formatted outputs such as HTMLFormat, MathMLFormat, SVGFormat and X3DFormat. However programming effort might be better spent creating a version of OutputForm which has better syntax information.

— HTMLFormat.input —

```
)set break resume
)sys rm -f HTMLFormat.output
)spool HTMLFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 9
)show HTMLFormat
--R HTMLFormat  is a domain constructor
--R Abbreviation for HTMLFormat is HTMLFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HTMLFORM
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : OutputForm -> String
--R coerce : % -> OutputForm             coerceL : OutputForm -> String
--R coerceS : OutputForm -> String       display : String -> Void
--R exprex : OutputForm -> String        hash : % -> SingleInteger
--R latex : % -> String                  ?~=? : (%,%) -> Boolean
--R
--E 1

--S 2 of 9
coerce("3+4"::OutputForm)$HTMLFORM
--R
--R"3+4"
--R
--R   (1)  " "
--R                                                            Type: String
--E 2

--S 3 of 9
coerce("sqrt(3+4)"::OutputForm)$HTMLFORM
--R
--R"sqrt(3+4)"
--R
--R   (2)  " "
--R                                                            Type: String
--E 3

--S 4 of 9
coerce(sqrt(3+4)::OutputForm)$HTMLFORM
--R
--R&radic;7
--R
--R   (3)  " "
--R                                                            Type: String
--E 4

--S 5 of 9
coerce(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
```

```
--R
--R   (4)  " "
--R                                                                Type: String
--E 5

--S 6 of 9
coerceS(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
--R
--R   (5)  " "
--R                                                                Type: String
--E 6

--S 7 of 9
coerceL(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
--R
--R   (6)  " "
--R                                                                Type: String
--E 7

--S 8 of 9
exprex(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R
--R   (7)  "{{ROOT}{{+}{x}{3}}}"
--R                                                                Type: String
--E 8

--S 9 of 9
```

```
display(coerce(sqrt(3+x)::OutputForm)$HTMLFORM)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
--R
--R                                                            Type: Void
--E 9
)spool
)lisp (bye)
```

— HTMLFormat.help —

```
====================================================================
HTMLFormat examples
====================================================================

coerce("3+4"::OutputForm)$HTMLFORM
  "3+4"

coerce("sqrt(3+4)"::OutputForm)$HTMLFORM
  "sqrt(3+4)"

coerce(sqrt(3+4)::OutputForm)$HTMLFORM
  &radic;7

coerce(sqrt(3+x)::OutputForm)$HTMLFORM
  <table border='0' id='root'>
  <tr id='root'>
  <td id='root'>
  &radic;
  </td>
  <td id='root' style='border-top-style:solid'>
  x+3
  </td>
  </tr>
  </table>

coerceS(sqrt(3+x)::OutputForm)$HTMLFORM
  <table border='0' id='root'>
```

```
  <tr id='root'>
  <td id='root'>
  &radic;
  </td>
  <td id='root' style='border-top-style:solid'>
  x+3
  </td>
  </tr>
  </table>

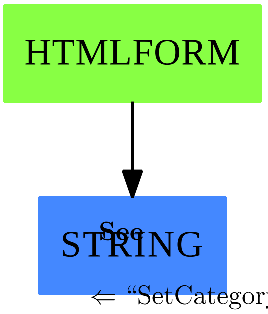
coerceL(sqrt(3+x)::OutputForm)$HTMLFORM
  <table border='0' id='root'>
  <tr id='root'>
  <td id='root'>
  &radic;
  </td>
  <td id='root' style='border-top-style:solid'>
  x+3
  </td>
  </tr>
  </table>

exprex(sqrt(3+x)::OutputForm)$HTMLFORM
  "{{ROOT}{{+}{x}{3}}}"

display(coerce(sqrt(3+x)::OutputForm)$HTMLFORM)$HTMLFORM
  <table border='0' id='root'>
  <tr id='root'>
  <td id='root'>
  &radic;
  </td>
  <td id='root' style='border-top-style:solid'>
  x+3
  </td>
  </tr>
  </table>

See Also:
o )show HTMLFormat
```

9.8.2 HTMLFormat (HTMLFORM)



See
⇐ “SetCategory” (SETCAT) ?? on page ??

Exports:

?=?	? =?	coerce	coerceL	coerceS
display	exprex	hash	latex	

— domain HTMLFORM HTMLFormat —

```
)abbrev domain HTMLFORM HTMLFormat
++ Author: Martin J Baker, Arthur C. Ralfs, Robert S. Sutor
++ Date: January 2010
++ Description:
++ HtmlFormat provides a coercion from OutputForm to html.
HTMLFormat(): public == private where
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String

  public == SetCategory with
    coerce:    E -> S
      ++ coerce(o) changes o in the standard output format to html format.
      ++
      ++X coerce(sqrt(3+x)::OutputForm)$HTMLFORM
    coerceS:   E -> S
      ++ coerceS(o) changes o in the standard output format to html
      ++ format and displays formatted result.
      ++
      ++X coerceS(sqrt(3+x)::OutputForm)$HTMLFORM
    coerceL:   E -> S
      ++ coerceL(o) changes o in the standard output format to html
      ++ format and displays result as one long string.
      ++
      ++X coerceL(sqrt(3+x)::OutputForm)$HTMLFORM
    exprex:    E -> S
```

```
      ++ exprex(o) coverts \spadtype{OutputForm} to \spadtype{String}
      ++
      ++X exprex(sqrt(3+x)::OutputForm)$HTMLFORM
    display:   S -> Void
      ++ display(o) prints the string returned by coerce.
      ++
      ++X display(coerce(sqrt(3+x)::OutputForm)$HTMLFORM)$HTMLFORM

  private == add
    import OutputForm
    import Character
    import Integer
    import List OutputForm
    import List String

    expr: E
    prec,opPrec: I
    str:  S
    blank         : S := " \  "

    maxPrec       : I   := 1000000
    minPrec       : I   := 0

    unaryOps      : L S := ["-"]$(L S)
    unaryPrecs    : L I := [700]$(L I)

    -- the precedence of / in the following is relatively low because
    -- the bar obviates the need for parentheses.
    binaryOps     : L S := ["+->","|","^","/","<",">","=","OVER"]$(L S)
    binaryPrecs   : L I := [0,0,900,700,400,400,400,700]$(L I)
    naryOps       : L S := ["-","+","*",blank,",",";"," ","ROW","",
       " \cr ","&","/\","\/"]$(L S)
    naryPrecs     : L I := [700,700,800,800,110,110,0,0,0,0,0,600,600]$(L I)
    naryNGOps     : L S := ["ROW","&"]$(L S)
    plexOps       : L S := ["SIGMA","SIGMA2","PI","PI2","INTSIGN",_
                            "INDEFINTEGRAL"]$(L S)
    plexPrecs     : L I := [700,800,700,800,700,700]$(L I)
    specialOps    : L S := ["MATRIX","BRACKET","BRACE","CONCATB","VCONCAT",_
                            "AGGLST","CONCAT","OVERBAR","ROOT","SUB","TAG",_
                            "SUPERSUB","ZAG","AGGSET","SC","PAREN",_
                            "SEGMENT","QUOTE","theMap", "SLASH"]

    -- the next two lists provide translations for some strings for
    -- which HTML has some special character codes.
    specialStrings : L S :=
      ["cos", "cot", "csc", "log", "sec", "sin", "tan", _
       "cosh", "coth", "csch", "sech", "sinh", "tanh", _
       "acos","asin","atan","erf","...","$","infinity","Gamma", _
       "%pi","%e","%i"]
    specialStringsInHTML : L S :=
```

```
  ["cos","cot","csc","log","sec","sin","tan", _
   "cosh","coth","csch","sech","sinh","tanh", _
   "arccos","arcsin","arctan","erf","&#x2026;","$","&#x221E;",_
   "&#x0413;","&#x003C0;","&#x02147;","&#x02148;"]

debug := false

atomize:E -> L E

formatBinary:(S,L E, I) -> Tree S

formatFunction:(Tree S,L E, I) -> Tree S

formatMatrix:L E -> Tree S

formatNary:(S,L E, I) -> Tree S

formatNaryNoGroup:(S,L E, I) -> Tree S

formatNullary:S -> Tree S

formatPlex:(S,L E, I) -> Tree S

formatSpecial:(S,L E, I) -> Tree S

formatUnary:(S,  E, I) -> Tree S

formatHtml:(E,I) -> Tree S

precondition:E -> E
  -- this function is applied to the OutputForm expression before
  -- doing anything else.

outputTree:Tree S -> Void
  -- This function traverses the tree and linierises it into a string.
  -- To get the formatting we use a nested set of tables. It also checks
  -- for +- and removes the +. it may also need to remove the outer
  -- set of brackets.

stringify:E -> S

coerce(expr : E): S ==
  outputTree formatHtml(precondition expr, minPrec)
  " "

coerceS(expr : E): S ==
  outputTree formatHtml(precondition expr, minPrec)
  " "

coerceL(expr : E): S ==
```

```
    outputTree formatHtml(precondition expr, minPrec)
    " "

  display(html : S): Void ==
    sayTeX$Lisp html
    void()$Void

  newNode(tag:S,node: Tree S): (Tree S) ==
    t := tree(S,[node])
    setvalue!(t,tag)
    t

  newNodes(tag:S,nodes: L Tree S): (Tree S) ==
    t := tree(S,nodes)
    setvalue!(t,tag)
    t

  -- returns true if this can be represented without a table
  notTable?(node: Tree S): Boolean ==
    empty?(node) => true
    leaf?(node) => true
    prefix?("table",value(node))$String => false
    c := children(node)
    for a in c repeat
      if not notTable?(a) then return false
    true

  -- this retuns a string representation of OutputForm arguments
  -- it is used when debug is true to trace the calling of functions
  -- in this package
  argsToString(args : L E): S ==
    sop : S := exprex first args
    args := rest args
    s : S := concat ["{",sop]
    for a in args repeat
        s1 : S := exprex a
        s := concat [s,s1]
    s := concat [s,"}"]

  exprex(expr : E): S ==
    -- This breaks down an expression into atoms and returns it as
    -- a string.  It's for developmental purposes to help understand
    -- the expressions.
    a : E
    expr := precondition expr
    (ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
      concat ["{",stringify expr,"}"]
    le : L E := (expr pretend L E)
    op := first le
    sop : S := exprex op
```

```
  args : L E := rest le
  nargs : I := #args
  s : S := concat ["{",sop]
  if nargs > 0  then
    for a in args repeat
      s1 : S := exprex a
      s := concat [s,s1]
  s := concat [s,"}"]

atomize(expr : E): L E ==
  -- This breaks down an expression into a flat list of atomic
  -- expressions.
  -- expr should be preconditioned.
  le : L E := nil()
  a : E
  letmp : L E
  (ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
    le := append(le,list(expr))
  letmp := expr pretend L E
  for a in letmp repeat
    le := append(le,atomize a)
  le

-- output html test using tables and
-- remove unnecessary '+' at end of first string
-- when second string starts with '-'
outputTree(t: Tree S): Void ==
  endWithPlus:Boolean := false -- if the last string ends with '+'
  -- and the next string starts with '-' then the '+' needs to be
  -- removed
  if empty?(t) then
    --if debug then sayTeX$Lisp "outputTree empty"
    return void()$Void
  if leaf?(t) then
    --if debug then sayTeX$Lisp concat("outputTree leaf:",value(t))
    sayTeX$Lisp value(t)
    return void()$Void
  tagName := copy value(t)
  tagPos := position(char(" "),tagName,1)$String
  if tagPos > 1 then
    tagName := split(tagName,char(" ")).1
    --sayTeX$Lisp "outputTree: tagPos="string(tagPos)" "tagName
  if value(t) ~= "" then sayTeX$Lisp concat ["<",value(t),">"]
  c := children(t)
  enableGrid:Boolean := (#c > 1) and not notTable?(t)
  if enableGrid then
    if tagName = "table" then enableGrid := false
    if tagName = "tr" then enableGrid := false
  b:List Boolean := [leaf?(c1) for c1 in c]
  -- if all children are strings then no need to wrap in table
```

```
  allString: Boolean := true
  for c1 in c repeat if not leaf?(c1) then allString := false
  if allString then
    s:String := ""
    for c1 in c repeat s := concat(s,value(c1))
    sayTeX$Lisp s
    if value(t) ~= "" then sayTeX$Lisp concat ["</",tagName,">"]
    return void()$Void
  if enableGrid then
    sayTeX$Lisp "<table border='0'>"
    sayTeX$Lisp "<tr>"
  for c1 in c repeat
    if enableGrid then sayTeX$Lisp "<td>"
    outputTree(c1)
    if enableGrid then sayTeX$Lisp "</td>"
  if enableGrid then
    sayTeX$Lisp "</tr>"
    sayTeX$Lisp "</table>"
  if value(t) ~= "" then sayTeX$Lisp concat ["</",tagName,">"]
  void()$Void

stringify expr == (mathObject2String$Lisp expr)@S

precondition expr ==
  outputTran$Lisp expr

-- I dont know what SC is so put it in a table for now
formatSC(args : L E, prec : I)  : Tree S ==
  if debug then sayTeX$Lisp "formatSC: "concat [" args=",_
    argsToString(args)," prec=",string(prec)$S]
  null args => tree("")
  cells:L Tree S := [_
    newNode("td id='sc' style='border-bottom-style:solid'",_
    formatHtml(a,prec)) for a in args]
  row:Tree S := newNodes("tr id='sc'",cells)
  newNode("table border='0' id='sc'",row)

-- to build an overbar we put it in a single column,
-- single row table and set the top border to solid
buildOverbar(content : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildOverbar"
  cell:Tree S := _
    newNode("td id='overbar' style='border-top-style:solid'",content)
  row:Tree S := newNode("tr id='overbar'",cell)
  newNode("table border='0' id='overbar'",row)

-- to build an square root we put it in a double column,
-- single row table and set the top border of the second column to
-- solid
buildRoot(content : Tree S) : Tree S ==
```

```
  if debug then sayTeX$Lisp "buildRoot"
  if leaf?(content) then
    -- root of a single term so no need for overbar
    return newNodes("",[tree("&radic;"),content])
  cell1:Tree S := newNode("td id='root'",tree("&radic;"))
  cell2:Tree S := _
    newNode("td id='root' style='border-top-style:solid'",content)
  row:Tree S := newNodes("tr id='root'",[cell1,cell2])
  newNode("table border='0' id='root'",row)

-- to build an 'n'th root we put it in a double column,
-- single row table and set the top border of the second column to
-- solid
buildNRoot(content : Tree S,nth: Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildNRoot"
  power:Tree S := newNode("sup",nth)
  if leaf?(content) then
    -- root of a single term so no need for overbar
    return newNodes("",[power,tree("&radic;"),content])
  cell1:Tree S := newNodes("td id='nroot'",[power,tree("&radic;")])
  cell2:Tree S := _
    newNode("td id='nroot' style='border-top-style:solid'",content)
  row:Tree S := newNodes("tr id='nroot'",[cell1,cell2])
  newNode("table border='0' id='nroot'",row)

-- formatSpecial handles "theMap","AGGLST","AGGSET","TAG","SLASH",
-- "VCONCAT", "CONCATB","CONCAT","QUOTE","BRACKET","BRACE","PAREN",
-- "OVERBAR","ROOT", "SEGMENT","SC","MATRIX","ZAG"
-- note "SUB" and "SUPERSUB" are handled directly by formatHtml
formatSpecial(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp _
    "formatSpecial: " concat ["op=",op," args=",argsToString(args),_
      " prec=",string(prec)$S]
  arg : E
  prescript : Boolean := false
  op = "theMap" => tree("theMap(...)")
  op = "AGGLST" =>
    formatNary(",",args,prec)
  op = "AGGSET" =>
    formatNary(";",args,prec)
  op = "TAG" =>
    newNodes("",[formatHtml(first args,prec),tree("&#x02192;"),_
      formatHtml(second args,prec)])
    --RightArrow
  op = "SLASH" =>
    newNodes("",[formatHtml(first args, prec),tree("/"),_
      formatHtml(second args,prec)])
  op = "VCONCAT" =>
    newNodes("table",[newNode("td",formatHtml(u, minPrec))_
       for u in args]::L Tree S)
```

```
    op = "CONCATB" =>
      formatNary(" ",args,prec)
    op = "CONCAT" =>
      formatNary("",args,minPrec)
    op = "QUOTE" =>
      newNodes("",[tree("'"),formatHtml(first args, minPrec)])
    op = "BRACKET" =>
      newNodes("",[tree("["),formatHtml(first args, minPrec),tree("]")])
    op = "BRACE" =>
      newNodes("",[tree("{"),formatHtml(first args, minPrec),tree("}")])
    op = "PAREN" =>
      newNodes("",[tree("("),formatHtml(first args, minPrec),tree(")")])
    op = "OVERBAR" =>
      null args => tree("")
      buildOverbar(formatHtml(first args,minPrec))
    op = "ROOT" and #args < 1 => tree("")
    op = "ROOT" and #args = 1 => _
      buildRoot(formatHtml(first args, minPrec))
    op = "ROOT" and #args > 1 => _
      buildNRoot(formatHtml(first args, minPrec),_
        formatHtml(second args, minPrec))
    op = "SEGMENT" =>
      -- '..' indicates a range in a list for example
      tmp : Tree S := newNodes("",[formatHtml(first args, minPrec),_
        tree("..")])
      null rest args =>  tmp
      newNodes("",[tmp,formatHtml(first rest args, minPrec)])
    op = "SC" => formatSC(args,minPrec)
    op = "MATRIX" => formatMatrix rest args
    op = "ZAG" =>
      -- {{+}{3}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}_
      --      {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
      -- to format continued fraction traditionally need to intercept
      -- it at the formatNary of the "+"
      newNodes("",[tree(" \zag{"),formatHtml(first args, minPrec),
        tree("}{"),
        formatHtml(first rest args,minPrec),tree("}")])
    tree("formatSpecial not implemented:"op)

  formatSuperSub(expr : E, args : L E, opPrec : I) : Tree S ==
    -- This one produces ordinary derivatives with differential notation,
    -- it needs a little more work yet.
    -- first have to divine the semantics, add cases as needed
    if debug then sayTeX$Lisp _
      "formatSuperSub: " concat ["expr=",stringify expr," args=",_
        argsToString(args)," prec=",string(opPrec)$S]
    atomE : L E := atomize(expr)
    op : S := stringify first atomE
    op ~= "SUPERSUB" => tree("Mistake in formatSuperSub: no SUPERSUB")
    #args ~= 1 => tree("Mistake in SuperSub: #args <> 1")
```

```
  var : E := first args
  -- should be looking at something like {{SUPERSUB}{var}{ }{,,...,}}
  -- for example here's the second derivative of y w.r.t. x
  -- {{{SUPERSUB}{y}{ }{,,}}{x}}, expr is the first {} and args is the
  -- {x}
  funcS : S := stringify first rest atomE
  bvarS : S := stringify first args
  -- count the number of commas
  commaS : S := stringify first rest rest rest atomE
  commaTest : S := ","
  ndiffs : I := 0
  while position(commaTest,commaS,1) > 0 repeat
    ndiffs := ndiffs+1
    commaTest := commaTest","
  res:Tree S := newNodes("",_
    [tree("&#x02146;"string(ndiffs)""funcS"&#x02146;"),_
      formatHtml(first args,minPrec),tree(""string(ndiffs)"&#x02061;"),_
        formatHtml(first args,minPrec),tree(")")])
  res

-- build structure such as integral as a table
buildPlex3(main:Tree S,supsc:Tree S,op:Tree S,subsc:Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildPlex"
  ssup:Tree S := newNode("td id='plex'",supsc)
  sop:Tree S := newNode("td id='plex'",op)
  ssub:Tree S := newNode("td id='plex'",subsc)
  m:Tree S := newNode("td rowspan='3' id='plex'",main)
  rows:(List Tree S) := [newNodes("tr id='plex'",[ssup,m]),_
    newNode("tr id='plex'",sop),newNode("tr id='plex'",ssub)]
  newNodes("table border='0' id='plex'",rows)

-- build structure such as integral as a table
buildPlex2(main : Tree S,supsc : Tree S,op : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildPlex"
  ssup:Tree S := newNode("td id='plex'",supsc)
  sop:Tree S := newNode("td id='plex'",op)
  m:Tree S := newNode("td rowspan='2' id='plex'",main)
  rows:(List Tree S) := [newNodes("tr id='plex'",[sop,m]),_
    newNode("tr id='plex'",ssup)]
  newNodes("table border='0' id='plex'",rows)

-- format an integral
-- args.1 = "NOTHING"
-- args.2 = bound variable
-- args.3 = body, thing being integrated
--
-- axiom replaces the bound variable with somthing like
-- %A and puts the original variable used
-- in the input command as a superscript on the integral sign.
formatIntSign(args : L E, opPrec : I) : Tree S ==
```

```
  -- the original OutputForm expression looks something like this:
  -- {{INTSIGN}{NOTHING or lower limit?}
  -- {bvar or upper limit?}{{*}{integrand}{{CONCAT}{d}{axiom var}}}}
  -- the args list passed here consists of the rest of this list, i.e.
  -- starting at the NOTHING or ...
  if debug then sayTeX$Lisp "formatIntSign: " concat [" args=",_
    argsToString(args)," prec=",string(opPrec)$S]
  (stringify first args) = "NOTHING" =>
    buildPlex2(formatHtml(args.3,opPrec),tree("&int;"),_
      formatHtml(args.2,opPrec)) -- could use &#x0222B; or &int;
  buildPlex3(formatHtml(first args,opPrec),formatHtml(args.3,opPrec),_
    tree("&int;"),formatHtml(args.2,opPrec))

-- plex ops are "SIGMA","SIGMA2","PI","PI2","INTSIGN","INDEFINTEGRAL"
-- expects 2 or 3 args
formatPlex(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatPlex: " concat ["op=",op," args=",_
    argsToString(args)," prec=",string(prec)$S]
  checkarg:Boolean := false
  hold : S
  p : I := position(op,plexOps)
  p < 1 => error "unknown plex op"
  op = "INTSIGN" => formatIntSign(args,minPrec)
  opPrec := plexPrecs.p
  n : I := #args
  (n ~= 2) and (n ~= 3) => error "wrong number of arguments for plex"
  s : Tree S :=
    op = "SIGMA"   =>
      checkarg := true
      tree("&#x02211;")
    -- Sum
    op = "SIGMA2"   =>
      checkarg := true
      tree("&#x02211;")
    -- Sum
    op = "PI"      =>
      checkarg := true
      tree("&#x0220F;")
    -- Product
    op = "PI2"     =>
      checkarg := true
      tree("&#x0220F;")
    -- Product
    op = "INTSIGN" => tree("&#x0222B;")
    -- Integral, int
    op = "INDEFINTEGRAL" => tree("&#x0222B;")
    -- Integral, int
    tree("formatPlex: unexpected op:"op)
  -- if opPrec < prec then perhaps we should parenthesize?
  -- but we need to be careful we don't get loads of unnecessary
```

```
  -- brackets
  if n=2 then return buildPlex2(formatHtml(first args,minPrec),_
    formatHtml(args.2,minPrec),s)
  buildPlex3(formatHtml(first args,minPrec),formatHtml(args.2,minPrec),_
    s,formatHtml(args.3,minPrec))

-- an example is: op=ROW arg={{ROW}{1}{2}}
formatMatrixRow(op : S, arg : E, prec : I,y:I,h:I)  : L Tree S ==
  if debug then sayTeX$Lisp "formatMatrixRow: " concat ["op=",op,_
    " args=",stringify arg," prec=",string(prec)$S]
  ATOM(arg)$Lisp@Boolean => [_
    tree("formatMatrixRow does not contain row")]
  l : L E := (arg pretend L E)
  op : S := stringify first l
  args : L E := rest l
  --sayTeX$Lisp "formatMatrixRow op="op" args="argsToString(args)
  w:I := #args
  cells:(List Tree S) := empty()
  for x in 1..w repeat
    --sayTeX$Lisp "formatMatrixRow: x="string(x)$S" width="string(w)$S
    attrib:S := "td id='mat'"
    if x=1 then attrib := "td id='matl'"
    if x=w then attrib := "td id='matr'"
    if y=1 then attrib := "td id='matt'"
    if y=h then attrib := "td id='matb'"
    if x=1 and y=1 then attrib := "td id='matlt'"
    if x=1 and y=h then attrib := "td id='matlb'"
    if x=w and y=1  then attrib := "td id='matrt'"
    if x=w and y=h  then attrib := "td id='matrb'"
    cells := append(cells,[newNode(attrib,formatHtml(args.(x),prec))])
  cells

-- an example is: op=MATRIX args={{ROW}{1}{2}}{{ROW}{3}{4}}
formatMatrixContent(op : S, args : L E, prec : I)  : L Tree S ==
  if debug then sayTeX$Lisp "formatMatrixContent: " concat ["op=",op,_
    " args=",argsToString(args)," prec=",string(prec)$S]
  y:I := 0
  rows:(List Tree S) := [newNodes("tr id='mat'",_
    formatMatrixRow("ROW",e,prec,y:=y+1,#args)) for e in args]
  rows

formatMatrix(args : L E) : Tree S ==
  -- format for args is [[ROW ...],[ROW ...],[ROW ...]]
  -- generate string for formatting columns (centered)
  if debug then sayTeX$Lisp "formatMatrix: " concat ["args=",_
    argsToString(args)]
  newNodes("table border='1' id='mat'",_
    formatMatrixContent("MATRIX",args,minPrec))

-- output arguments in column table
```

```
buildColumnTable(elements : List Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildColumnTable"
  cells:(List Tree S) := [newNode("td id='col'",j) for j in elements]
  rows:(List Tree S) := [newNode("tr id='col'",i) for i in cells]
  newNodes("table border='0' id='col'",rows)

-- build superscript structure as either sup tag or
-- if it contains anything that won't go into a
-- sup tag then build it as a table
buildSuperscript(main : Tree S,super : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildSuperscript"
  notTable?(super) => newNodes("",[main,newNode("sup",super)])
  m:Tree S := newNode("td rowspan='2' id='sup'",main)
  su:Tree S := newNode("td id='sup'",super)
  e:Tree S := newNode("td id='sup'",tree(" "))
  rows:(List Tree S) := [newNodes("tr id='sup'",[m,su]),_
    newNode("tr id='sup'",e)]
  newNodes("table border='0' id='sup'",rows)

-- build subscript structure as either sub tag or
-- if it contains anything that won't go into a
-- sub tag then build it as a table
buildSubscript(main : Tree S,subsc : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildSubscript"
  notTable?(subsc) => newNodes("",[main,newNode("sub",subsc)])
  m:Tree S := newNode("td rowspan='2' id='sub'",main)
  su:Tree S := newNode("td id='sub'",subsc)
  e:Tree S := newNode("td id='sub'",tree(" "))
  rows:(List Tree S) := [newNodes("tr id='sub'",[m,e]),_
    newNode("tr id='sub'",su)]
  newNodes("table border='0' id='sub'",rows)

formatSub(expr : E, args : L E, opPrec : I) : Tree S ==
  -- format subscript
  -- this function expects expr to start with SUB
  -- it expects first args to be the operator or value that
  -- the subscript is applied to
  -- and the rest args to be the subscript
  if debug then sayTeX$Lisp "formatSub: " concat ["expr=",_
    stringify expr," args=",argsToString(args)," prec=",_
      string(opPrec)$S]
  atomE : L E := atomize(expr)
  if empty?(atomE) then
    if debug then sayTeX$Lisp "formatSub: expr=empty"
    return tree("formatSub: expr=empty")
  op : S := stringify first atomE
  op ~= "SUB" =>
    if debug then sayTeX$Lisp "formatSub: expr~=SUB"
    tree("formatSub: expr~=SUB")
  -- assume args.1 is the expression and args.2 is its subscript
```

```
  if #args < 2 then
    if debug then sayTeX$Lisp concat("formatSub: num args=",_
      string(#args)$String)$String
    return tree(concat("formatSub: num args=",_
      string(#args)$String)$String)
  if #args > 2 then
    if debug then sayTeX$Lisp concat("formatSub: num args=",_
      string(#args)$String)$String
    return buildSubscript(formatHtml(first args,opPrec),_
      newNodes("",[formatHtml(e,opPrec) for e in rest args]))
  buildSubscript(formatHtml(first args,opPrec),_
    formatHtml(args.2,opPrec))

formatFunction(op : Tree S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatFunction: " concat ["args=",_
    argsToString(args)," prec=",string(prec)$S]
  newNodes("",[op,tree("("),formatNary(",",args,minPrec),tree(")")])

formatNullary(op : S) : Tree S ==
  if debug then sayTeX$Lisp "formatNullary: " concat ["op=",op]
  op = "NOTHING" => empty()$Tree(S)
  tree(op"()")

-- implement operation with single argument
-- an example is minus '-'
-- prec is precidence of operator, used to force brackets where
-- more tightly bound operation is next to less tightly bound operation
formatUnary(op : S, arg : E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatUnary: " concat ["op=",op," arg=",_
    stringify arg," prec=",string(prec)$S]
  p : I := position(op,unaryOps)
  p < 1 => error "unknown unary op"
  opPrec := unaryPrecs.p
  s : Tree S := newNodes("",[tree(op),formatHtml(arg,opPrec)])
  opPrec < prec => newNodes("",[tree("("),s,tree(")")])
  s

-- output division with numerator above the denominator
-- implemented as a table
buildOver(top : Tree S,bottom : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildOver"
  topCell:Tree S := newNode("td",top)
  bottomCell:Tree S := newNode("td style='border-top-style:solid'",_
    bottom)
  rows:(List Tree S) := [newNode("tr id='col'",topCell),_
    newNode("tr id='col'",bottomCell)]
  newNodes("table border='0' id='col'",rows)

-- op may be: "|","^","/","OVER","+->"
-- note: "+" and "*" are n-ary ops
```

```
formatBinary(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatBinary: " concat ["op=",op,_
    " args=",argsToString(args)," prec=",string(prec)$S]
  p : I := position(op,binaryOps)
  p < 1 => error "unknown binary op"
  opPrec := binaryPrecs.p
  -- if base op is product or sum need to add parentheses
  if ATOM(first args)$Lisp@Boolean then
    opa:S := stringify first args
  else
    la : L E := (first args pretend L E)
    opa : S := stringify first la
  if (opa = "SIGMA" or opa = "SIGMA2" or opa = "PI" or opa = "PI2")_
    and op = "^" then
      s1 : Tree S := newNodes("",[tree("("),formatHtml(first args,_
        opPrec),tree(")")])
  else
    s1 : Tree S := formatHtml(first args, opPrec)
  s2 : Tree S := formatHtml(first rest args, opPrec)
  op = "|" => newNodes("",[s1,tree(op),s2])
  op = "^" => buildSuperscript(s1,s2)
  op = "/" => newNodes("",[s1,tree(op),s2])
  op = "OVER" => buildOver(s1,s2)
  op = "+->" => newNodes("",[s1,tree("|&mdash;&rsaquo;"),s2])
  newNodes("",[s1,tree(op),s2])

-- build a zag from a table with a right part and a
-- upper and lower left part
buildZag(top:Tree S,lowerLeft:Tree S,lowerRight:Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildZag"
  cellTop:Tree S := _
    newNode("td colspan='2' id='zag' style='border-bottom-style:solid'",_
     top)
  cellLowerLeft:Tree S := newNodes("td id='zag'",[lowerLeft,tree("+")])
  cellLowerRight:Tree S := newNode("td id='zag'",lowerRight)
  row1:Tree S := newNodes("tr id='zag'",[cellTop])
  row2:Tree S := newNodes("tr id='zag'",[cellLowerLeft,cellLowerRight])
  newNodes("table border='0' id='zag'",[row1,row2])

formatZag(args : L E,nestLevel:I)  : Tree S ==
  -- args will be a list of things like this {{ZAG}{1}{7}}, the ZAG
  -- must be there, the '1' and '7' could conceivably be more complex
  -- expressions
  --
  -- ex 1. continuedFraction(314159/100000)
  -- {{+}{3}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}
  -- {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
  -- this is the preconditioned output form
  -- including "op", the args list would be the rest of this
  -- i.e op = '+' and args = {{3}{{ZAG}{1}{7}}{{ZAG}{1}{15}}
```

```
  -- {{ZAG}{1}{1}}{{ZAG}{1}{25}}{{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
  --
  -- ex 2. continuedFraction(14159/100000)
  -- this one doesn't have the leading integer
  -- {{+}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}
  -- {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
  --
  -- ex 3. continuedFraction(3,repeating [1], repeating [3,6])
  -- {{+}{3}{{ZAG}{1}{3}}{{ZAG}{1}{6}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}
  -- {{ZAG}{1}{3}}{{ZAG}{1}{6}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}
  -- {{ZAG}{1}{3}}{{ZAG}{1}{6}}{...}}
  --
  -- In each of these examples the args list consists of the terms
  -- following the '+' op
  -- so the first arg could be a "ZAG" or something
  -- else, but the second arg looks like it has to be "ZAG", so maybe
  -- test for #args > 1 and args.2 contains "ZAG".
  -- Note that since the resulting tables are nested we need
  -- to handle the whole continued fraction at once, i.e. we can't
  -- just look for, e.g., {{ZAG}{1}{6}}
  --
  -- we will assume that the font starts at 16px and reduce it by 4
  -- <span style='font-size:16px'>outer zag</span>
  -- <span style='font-size:14px'>next zag</span>
  -- <span style='font-size:12px'>next zag</span>
  -- <span style='font-size:10px'>next zag</span>
  -- <span style='font-size:9px'>lowest zag</span>
  if debug then sayTeX$Lisp "formatZag: " concat ["args=",_
    argsToString(args)]
  tmpZag : L E := first args pretend L E
  fontAttrib : S :=
    nestLevel < 2 => "span style='font-size:16px'"
    nestLevel = 2 => "span style='font-size:14px'"
    nestLevel = 3 => "span style='font-size:12px'"
    nestLevel = 4 => "span style='font-size:10px'"
    "span style='font-size:9px'"
  -- may want to test that tmpZag contains 'ZAG'
  #args > 1 =>
    newNode(fontAttrib,buildZag(formatHtml(first rest tmpZag,minPrec),_
      formatHtml(first rest rest tmpZag,minPrec),_
        formatZag(rest args,nestLevel+1)))
  (first args = "...":: E)@Boolean => tree("&#x2026;")
  op:S := stringify first args
  position("ZAG",op,1) > 0 =>
    newNode(fontAttrib,buildOver(formatHtml(first rest tmpZag,minPrec),_
      formatHtml(first rest rest tmpZag,minPrec)))
  tree("formatZag: Last argument in ZAG construct unknown operator: "op)

-- returns true if this term starts with a minus '-' sign
-- this is used so that we can suppress any plus '+' in front
```

```
-- of the - so we dont get terms like +-
neg?(arg : E) : Boolean ==
  if debug then sayTeX$Lisp "neg?: " concat ["arg=",argsToString([arg])]
  ATOM(arg)$Lisp@Boolean => false
  l : L E := (arg pretend L E)
  op : S := stringify first l
  op = "-" => true
  false

formatNary(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatNary: " concat ["op=",op," args=",_
    argsToString(args)," prec=",string(prec)$S]
  formatNaryNoGroup(op, args, prec)

-- possible op values are:
-- ",",";","*"," ","ROW","+","-"
-- an example is content of matrix which gives:
-- {{ROW}{1}{2}}{{ROW}{3}{4}}
-- or AGGLST which gives op=, args={{1}{2}}
--
-- need to:
-- format ZAG
-- check for +-
-- add brackets for sigma or pi or root ("SIGMA","SIGMA2","PI","PI2")
formatNaryNoGroup(op : S, args : L E, prec : I)  : Tree S ==
  if debug then sayTeX$Lisp "formatNaryNoGroup: " concat ["op=",op,_
    " args=",argsToString(args)," prec=",string(prec)$S]
  checkargs:Boolean := false
  null args => empty()$Tree(S)
  p : I := position(op,naryOps)
  p < 1 => error "unknown nary op"
  -- need to test for "ZAG" case and divert it here
  (#args > 1) and (position("ZAG",stringify first rest args,1) > 0) =>
       tmpS : S := stringify first args
       position("ZAG",tmpS,1) > 0 => formatZag(args,1)
       newNodes("",[formatHtml(first args,minPrec),tree("+"),_
        formatZag(rest args,1)])
  -- At least for the ops "*","+","-" we need to test to see if a
  -- sigma or pi is one of their arguments because we might need
  -- parentheses as indicated
  -- by the problem with summation(operator(f)(i),i=1..n)+1 versus
  -- summation(operator(f)(i)+1,i=1..n) having identical displays as of
  -- 2007-12-21
  l := empty()$Tree(S)
  opPrec := naryPrecs.p
  -- if checkargs is true check each arg except last one to see if it's
  -- a sigma or pi and if so add parentheses. Other op's may have to be
  -- checked for in future
  count:I := 1
  tags : (L Tree S)
```

```
  if opPrec < prec then tags := [tree("("),formatHtml(args.1,opPrec)]
  if opPrec >= prec then tags := [formatHtml(args.1,opPrec)]
  for a in rest args repeat
    if op ~= "+" or not neg?(a) then tags := append(tags,[tree(op)])
    tags := append(tags,[formatHtml(a,opPrec)])
  if opPrec < prec then tags := append(tags,[tree(")")])
  newNodes("",tags)

-- expr is a tree structure
-- prec is the precision of integers
-- formatHtml returns a string for this node in the tree structure
-- and calls recursivly to evaluate sub expressions
formatHtml(arg : E,prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatHtml: " concat ["arg=",_
    argsToString([arg])," prec=",string(prec)$S]
  i,len : Integer
  intSplitLen : Integer := 20
  ATOM(arg)$Lisp@Boolean =>
    if debug then sayTeX$Lisp "formatHtml atom: " concat ["expr=",_
      stringify arg," prec=",string(prec)$S]
    str := stringify arg
    (i := position(str,specialStrings)) > 0 =>
      tree(specialStringsInHTML.i)
    tree(str)
  l : L E := (arg pretend L E)
  null l => tree(blank)
  op : S := stringify first l
  args : L E := rest l
  nargs : I := #args
  -- need to test here in case first l is SUPERSUB case and then
  -- pass first l and args to formatSuperSub.
  position("SUPERSUB",op,1) > 0 =>
    formatSuperSub(first l,args,minPrec)
  -- now test for SUB
  position("SUB",op,1) > 0 =>
    formatSub(first l,args,minPrec)
  -- special cases
  -- specialOps are:
  -- MATRIX, BRACKET, BRACE, CONCATB, VCONCAT
  -- AGGLST, CONCAT, OVERBAR, ROOT, SUB, TAG
  -- SUPERSUB, ZAG, AGGSET, SC, PAREN
  -- SEGMENT, QUOTE, theMap, SLASH
  member?(op, specialOps) => formatSpecial(op,args,prec)
  -- specialOps are:
  -- SIGMA, SIGMA2, PI, PI2, INTSIGN, INDEFINTEGRAL
  member?(op, plexOps)    => formatPlex(op,args,prec)
  -- nullary case: function with no aguments
  0 = nargs => formatNullary op
  -- unary case: function with one agument such as '-'
  (1 = nargs) and member?(op, unaryOps) =>
```

```
        formatUnary(op, first args, prec)
      -- binary case
      -- binary ops include special processing for | ^ / OVER and +->
      (2 = nargs) and member?(op, binaryOps) =>
        formatBinary(op, args, prec)
      -- nary case: including '+' and '*'
      member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
      member?(op,naryOps) => formatNary(op,args, prec)

      op1 := formatHtml(first l,minPrec)
      formatFunction(op1,args,prec)
```

—— HTMLFORM.dotabb ——

```
"HTMLFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HTMLFORM"]
"STRING" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRING"]
"HTMLFORM" -> "STRING"
```

9.9 domain HDP HomogeneousDirectProduct

—— HomogeneousDirectProduct.input ——

```
)set break resume
)sys rm -f HomogeneousDirectProduct.output
)spool HomogeneousDirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HomogeneousDirectProduct
--R HomogeneousDirectProduct(dim: NonNegativeInteger,S: OrderedAbelianMonoidSup)  is a domain constructo
--R Abbreviation for HomogeneousDirectProduct is HDP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HDP
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if S has RING            1 : () -> % if S has MONOID
--R 0 : () -> % if S has CABMON          coerce : % -> Vector S
--R copy : % -> %                        directProduct : Vector S -> %
```

```
--R ?.? : (%,Integer) -> S                elt : (%,Integer,S) -> S
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List S                 eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R map : ((S -> S),%) -> %               qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (PositiveInteger,%) -> % if S has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if S has CABMON
--R ?*? : (S,%) -> % if S has RING
--R ?*? : (%,S) -> % if S has RING
--R ?*? : (%,%) -> % if S has MONOID
--R ?*? : (Integer,%) -> % if S has RING
--R ?**? : (%,PositiveInteger) -> % if S has MONOID
--R ?**? : (%,NonNegativeInteger) -> % if S has MONOID
--R ?+? : (%,%) -> % if S has ABELSG
--R ?-? : (%,%) -> % if S has RING
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : % -> % if S has DIFRING and S has RING
--R ?^? : (%,PositiveInteger) -> % if S has MONOID
--R ?^? : (%,NonNegativeInteger) -> % if S has MONOID
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : S -> % if S has SETCAT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : % -> % if S has DIFRING and S has RING
```

```
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R index : PositiveInteger -> % if S has FINITE
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has MONOID
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(%,"failed") if S has MONOID
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R retract : % -> S if S has SETCAT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S,"failed") if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT and S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed") if S has CABMON
--R sup : (%,%) -> % if S has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R zero? : % -> Boolean if S has CABMON
--R ?~=? : (%,%) -> Boolean if S has SETCAT
```

```
--R
--E 1

)spool
)lisp (bye)
```

————-

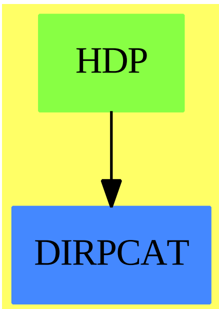
— **HomogeneousDirectProduct.help** —

```
====================================================================
HomogeneousDirectProduct examples
====================================================================

See Also:
o )show HomogeneousDirectProduct
```

————-

9.9.1 HomogeneousDirectProduct (HDP)



See

⇒ "OrderedDirectProduct" (ODP) 16.13.1 on page 1778
⇒ "SplitHomogeneousDirectProduct" (SHDP) 20.23.1 on page 2467

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
#?	?*?	?**?	?+?	?-?
?/?	?<?	?<=?	?=?	?>?
?>=?	?^?	?~=?	-?	?.?

— domain HDP HomogeneousDirectProduct —

```
)abbrev domain HDP HomogeneousDirectProduct
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See: OrderedDirectProduct, SplitHomogeneousDirectproduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
++ underlying ordered component type. The vectors are ordered first
++ by the sum of their components, and then refined using a reverse
++ lexicographic ordering. This type is a suitable third argument for
++ \spadtype{GeneralDistributedMultivariatePolynomial}.

HomogeneousDirectProduct(dim,S) : T == C where
   dim : NonNegativeInteger
   S         : OrderedAbelianMonoidSup

   T == DirectProductCategory(dim,S)
   C == DirectProduct(dim,S) add
        Rep:=Vector(S)
        v1:% < v2:% ==
 -- reverse lexicographical ordering
          n1:S:=0
          n2:S:=0
          for i in 1..dim repeat
            n1:= n1+qelt(v1,i)
```

```
      n2:=n2+qelt(v2,i)
    n1<n2 => true
    n2<n1 => false
    for i in reverse(1..dim) repeat
      if qelt(v2,i) < qelt(v1,i) then return true
      if qelt(v1,i) < qelt(v2,i) then return false
    false
```

— HDP.dotabb —

```
"HDP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HDP"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"HDP" -> "DIRPCAT"
```

9.10 domain HDMP HomogeneousDistributedMultivariatePolynomial

— HomogeneousDistributedMultivariatePolynomial.input —

```
)set break resume
)sys rm -f HomogeneousDistributedMultivariatePolynomial.output
)spool HomogeneousDistributedMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
--R
--R                                                                   Type: Void
--E 1

--S 2 of 10
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
--R
--R
--R                 2       2
--R   (2)  - 4z + 4y x + 16x  + 1
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 2
```

```
--S 3 of 10
d2 := 2*z*y**2 + 4*x + 1
--R
--R
--R            2
--R   (3)  2z y  + 4x + 1
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 3

--S 4 of 10
d3 := 2*z*x**2 - 2*y**2 - x
--R
--R
--R            2     2
--R   (4)  2z x  - 2y  - x
--R            Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 4

--S 5 of 10
groebner [d1,d2,d3]
--R
--R
--R   (5)
--R        1568  6   1264  5    6   4   182  3   2047  2    103      2857
--R   [z - ---- x  - ---- x  + --- x  + --- x  - ---- x  - ---- x - -----,
--R        2745       305      305      549       610      2745     10980
--R     2    112  6    84  5   1264  4    13  3    84  2   1772       2
--R    y  + ---- x  - --- x  - ---- x  - --- x  + --- x  + ---- x + ----,
--R         2745      305       305      549      305      2745     2745
--R     7   29  6   17  4   11  3    1  2   15     1
--R    x  + -- x  - -- x  - -- x  + -- x  + -- x + -]
--R          4      16       8      32      16     4
--R       Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 5

--S 6 of 10
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
--R
--R                                                                Type: Void
--E 6

--S 7 of 10
n1 := d1
--R
--R
--R          2       2
--R   (7)  4y x + 16x  - 4z + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 7
```

```
--S 8 of 10
n2 := d2
--R
--R
--R            2
--R   (8)  2z y  + 4x + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 8

--S 9 of 10
n3 := d3
--R
--R
--R            2     2
--R   (9)  2z x  - 2y  - x
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 9

--S 10 of 10
groebner [n1,n2,n3]
--R
--R
--R   (10)
--R     4     3   3  2   1     1   4   29  3   1  2   7        9     1
--R   [y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
--R               2      2     8        4      8      4       16     4
--R       2        1   2      2       1     2    2   1
--R    z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
--R                2                  4             2
--R     2     2     2   1     3
--R    z  - 4y  + 2x  - - z - - x]
--R                     4     2
--RType: List HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 10
)spool
)lisp (bye)
```

— **HomogeneousDistributedMultivariatePolynomial.help** —

```
====================================================================
MultivariatePolynomial
DistributedMultivariatePolynomial
HomogeneousDistributedMultivariatePolynomial
GeneralDistributedMultivariatePolynomial
====================================================================
```

```
DistributedMultivariatePolynomial which is abbreviated as DMP and
HomogeneousDistributedMultivariatePolynomial, which is abbreviated
as HDMP, are very similar to MultivariatePolynomial except that
they are represented and displayed in a non-recursive manner.

  (d1,d2,d3) : DMP([z,y,x],FRAC INT)
                      Type: Void

The constructor DMP orders its monomials lexicographically while
HDMP orders them by total order refined by reverse lexicographic
order.

  d1 := -4*z + 4*y**2*x + 16*x**2 + 1
            2       2
   - 4z + 4y x + 16x  + 1
           Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  d2 := 2*z*y**2 + 4*x + 1
       2
   2z y  + 4x + 1
           Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  d3 := 2*z*x**2 - 2*y**2 - x
       2     2
   2z x  - 2y  - x
           Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

These constructors are mostly used in Groebner basis calculations.

  groebner [d1,d2,d3]
        1568  6   1264  5    6   4   182  3   2047  2    103      2857
   [z - ---- x  - ---- x  + --- x  + --- x  - ---- x  - ---- x - -----,
        2745       305      305      549       610      2745     10980
     2    112  6    84  5   1264  4    13  3    84  2   1772       2
    y  + ---- x  - --- x  - ---- x  - --- x  + --- x  + ---- x + ----,
         2745      305      305      549      305      2745     2745
     7   29  6   17  4   11  3    1  2   15     1
    x  + -- x  - -- x  - -- x  + -- x  + -- x + -]
          4      16       8      32      16     4
       Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  (n1,n2,n3) : HDMP([z,y,x],FRAC INT)
                      Type: Void

  n1 := d1
     2       2
   4y x + 16x  - 4z + 1
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  n2 := d2
```

```
      2
   2z y  + 4x + 1
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

  n3 := d3
      2     2
   2z x  - 2y  - x
 Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

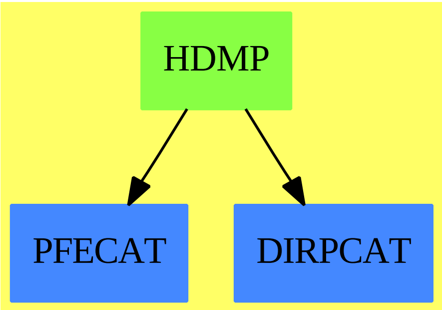
Note that we get a different Groebner basis when we use the HDMP polynomials, as expected.

```
  groebner [n1,n2,n3]
     4     3   3  2   1     1   4   29  3   1  2   7        9     1
   [y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
               2      2     8       4      8      4       16     4
      2        1   2      2       1     2    2   1
    z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
               2                  4              2
     2     2     2   1     3
    z  - 4y  + 2x  - - z - - x]
                     4     2
      Type: List HomogeneousDistributedMultivariatePolynomial([z,y,x],
                                                          Fraction Integer)
```

GeneralDistributedMultivariatePolynomial is somewhat more flexible in the sense that as well as accepting a list of variables to specify the variable ordering, it also takes a predicate on exponent vectors to specify the term ordering. With this polynomial type the user can experiment with the effect of using completely arbitrary term orderings. This flexibility is mostly important for algorithms such as Groebner basis calculations which can be very sensitive to term ordering.

See Also:
o)help Polynomial
o)help UnivariatePolynomial
o)help MultivariatePolynomial
o)help DistributedMultivariatePolynomial
o)help GeneralDistributedMultivariatePolynomial
o)show HomogeneousDistributedMultivariatePolynomial

9.10.1 HomogeneousDistributedMultivariatePolynomial (HDMP)



See

⇒ "GeneralDistributedMultivariatePolynomial" (GDMP) 8.1.1 on page 1018
⇒ "DistributedMultivariatePolynomial" (DMP) 5.13.1 on page 557

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	isExpt
isPlus	isTimes	latex
lcm	leadingCoefficient	leadingMonomial
mainVariable	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multivariate	numberOfMonomials
one?	patternMatch	pomopo!
prime?	primitiveMonomials	primitivePart
recip	reducedSystem	reductum
reorder	resultant	retract
retractIfCan	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	totalDegree	unit?
unitCanonical	unitNormal	univariate
variables	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?/?	?<?
?<=?	?>?	?>=?
?^?		

— domain HDMP HomogeneousDistributedMultivariatePolynomial —

```
)abbrev domain HDMP HomogeneousDistributedMultivariatePolynomial
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd, leadingCoefficient
++ Related Constructors: DistributedMultivariatePolynomial,
++ GeneralDistributedMultivariatePolynomial
++ Also See: Polynomial
++ AMS Classifications:
++ Keywords: polynomial, multivariate, distributed
++ References:
++ Description:
```

```
++ This type supports distributed multivariate polynomials
++ whose variables are from a user specified list of symbols.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.
++ The term ordering is total degree ordering refined by reverse
++ lexicographic ordering with respect to the position that the variables
++ appear in the list of variables parameter.

HomogeneousDistributedMultivariatePolynomial(vl,R): public == private where
  vl : List Symbol
  R  : Ring
  E   ==> HomogeneousDirectProduct(#vl,NonNegativeInteger)
  OV  ==> OrderedVariableList(vl)
  public == PolynomialCategory(R,E,OV) with
      reorder: (%,List Integer) -> %
        ++ reorder(p, perm) applies the permutation perm to the variables
        ++ in a polynomial and returns the new correctly ordered polynomial
  private ==
    GeneralDistributedMultivariatePolynomial(vl,R,E)
```

——

— **HDMP.dotabb** —

```
"HDMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HDMP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"HDMP" -> "PFECAT"
"HDMP" -> "DIRPCAT"
```

——

9.11 domain HELLFDIV HyperellipticFiniteDivisor

— **HyperellipticFiniteDivisor.input** —

```
)set break resume
)sys rm -f HyperellipticFiniteDivisor.output
)spool HyperellipticFiniteDivisor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show HyperellipticFiniteDivisor
--R HyperellipticFiniteDivisor(F: Field,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePo
--R Abbreviation for HyperellipticFiniteDivisor is HELLFDIV
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HELLFDIV
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R divisor : (R,UP,UP,UP,F) -> %         divisor : (F,F,Integer) -> %
--R divisor : (F,F) -> %                  divisor : R -> %
--R hash : % -> SingleInteger             latex : % -> String
--R principal? : % -> Boolean             reduce : % -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R decompose : % -> Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
--R divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
--R generator : % -> Union(R,"failed")
--R ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

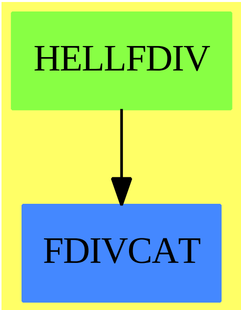
— HyperellipticFiniteDivisor.help —

```
====================================================================
HyperellipticFiniteDivisor examples
====================================================================

See Also:
o )show HyperellipticFiniteDivisor
```

———

9.11.1 HyperellipticFiniteDivisor (HELLFDIV)



See

⇒ "FractionalIdeal" (FRIDEAL) 7.25.1 on page 961
⇒ "FramedModule" (FRMOD) 7.26.1 on page 967
⇒ "FiniteDivisor" (FDIV) 7.4.1 on page 781

Exports:

0	coerce	decompose	divisor	hash
ideal	generator	latex	principal?	reduce
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?=?	

— domain HELLFDIV HyperellipticFiniteDivisor —

```
)abbrev domain HELLFDIV HyperellipticFiniteDivisor
++ Author: Manuel Bronstein
++ Date Created: 19 May 1993
++ Date Last Updated: 20 July 1998
++ Keywords: divisor, algebraic, curve.
++ Description:
++ This domains implements finite rational divisors on an hyperelliptic curve,
++ that is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.
++ The equation of the curve must be  y^2 = f(x) and f must have odd degree.

HyperellipticFiniteDivisor(F, UP, UPUP, R): Exports == Implementation where
  F   : Field
  UP  : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP
  R   : FunctionFieldCategory(F, UP, UPUP)

  O   ==> OutputForm
  Z   ==> Integer
  RF  ==> Fraction UP
  ID  ==> FractionalIdeal(UP, RF, UPUP, R)
  ERR ==> error "divisor: incomplete implementation for hyperelliptic curves"
```

```
  Exports ==> FiniteDivisorCategory(F, UP, UPUP, R)

  Implementation ==> add
    if (uhyper:Union(UP, "failed") := hyperelliptic()$R) case "failed" then
              error "HyperellipticFiniteDivisor: curve must be hyperelliptic"

-- we use the semi-reduced representation from D.Cantor, "Computing in the
-- Jacobian of a HyperellipticCurve", Mathematics of Computation, vol 48,
-- no.177, January 1987, 95-101.
-- The representation [a,b,f] for D means D = [a,b] + div(f)
-- and [a,b] is a semi-reduced representative on the Jacobian
    Rep := Record(center:UP, polyPart:UP, principalPart:R, reduced?:Boolean)

    hyper:UP := uhyper::UP
    gen:Z    := ((degree(hyper)::Z - 1) exquo 2)::Z     -- genus of the curve
    dvd:O    := "div"::Symbol::O
    zer:O    := 0::Z::O

    makeDivisor  : (UP, UP, R) -> %
    intReduc     : (R, UP) -> R
    princ?       : % -> Boolean
    polyIfCan    : R -> Union(UP, "failed")
    redpolyIfCan : (R, UP) -> Union(UP, "failed")
    intReduce    : (R, UP) -> R
    mkIdeal      : (UP, UP) -> ID
    reducedTimes : (Z, UP, UP) -> %
    reducedDouble: (UP, UP) -> %

    0                    == divisor(1$R)
    divisor(g:R)         == [1, 0, g, true]
    makeDivisor(a, b, g) == [a, b, g, false]
--    princ? d             == one?(d.center) and zero?(d.polyPart)
    princ? d             == (d.center = 1) and zero?(d.polyPart)
    ideal d     == ideal([d.principalPart]) * mkIdeal(d.center, d.polyPart)
    decompose d == [ideal makeDivisor(d.center, d.polyPart, 1),d.principalPart]
    mkIdeal(a, b) == ideal [a::RF::R, reduce(monomial(1, 1)$UPUP-b::RF::UPUP)]

-- keep the sum reduced if d1 and d2 are both reduced at the start
    d1 + d2 ==
      a1  := d1.center;   a2 := d2.center
      b1  := d1.polyPart; b2 := d2.polyPart
      rec := principalIdeal [a1, a2, b1 + b2]
      d   := rec.generator
      h   := rec.coef              -- d = h1 a1 + h2 a2 + h3(b1 + b2)
      a   := ((a1 * a2) exquo d**2)::UP
      b:UP:= first(h) * a1 * b2
      b   := b + second(h) * a2 * b1
      b   := b + third(h) * (b1*b2 + hyper)
      b   := (b exquo d)::UP rem a
      dd  := makeDivisor(a, b, d::RF * d1.principalPart * d2.principalPart)
```

```
      d1.reduced? and d2.reduced? => reduce dd
      dd

-- if is cheaper to keep on reducing as we exponentiate if d is already reduced
    n:Z * d:% ==
      zero? n => 0
      n < 0 => (-n) * (-d)
      divisor(d.principalPart ** n) + divisor(mkIdeal(d.center,d.polyPart)**n)

    divisor(i:ID) ==
--      one?(n := #(v := basis minimize i)) => divisor v minIndex v
      (n := #(v := basis minimize i)) = 1 => divisor v minIndex v
      n ^= 2 => ERR
      a := v minIndex v
      h := v maxIndex v
      (u := polyIfCan a) case UP =>
        (w := redpolyIfCan(h, u::UP)) case UP => makeDivisor(u::UP, w::UP, 1)
        ERR
      (u := polyIfCan h) case UP =>
        (w := redpolyIfCan(a, u::UP)) case UP => makeDivisor(u::UP, w::UP, 1)
        ERR
      ERR

    polyIfCan a ==
      (u := retractIfCan(a)@Union(RF, "failed")) case "failed" => "failed"
      (v := retractIfCan(u::RF)@Union(UP, "failed")) case "failed" => "failed"
      v::UP

    redpolyIfCan(h, a) ==
      degree(p := lift h) ^= 1 => "failed"
      q := - coefficient(p, 0) / coefficient(p, 1)
      rec := extendedEuclidean(denom q, a)
      not ground?(rec.generator) => "failed"
      ((numer(q) * rec.coef1) exquo rec.generator)::UP rem a

    coerce(d:%):O ==
      r := bracket [d.center::O, d.polyPart::O]
      g := prefix(dvd, [d.principalPart::O])
--      z := one?(d.principalPart)
      z := (d.principalPart = 1)
      princ? d => (z => zer; g)
      z => r
      r + g

    reduce d ==
      d.reduced? => d
      degree(a := d.center) <= gen => (d.reduced? := true; d)
      b  := d.polyPart
      a0 := ((hyper - b**2) exquo a)::UP
      b0 := (-b) rem a0
```

```
      g  := d.principalPart * reduce(b::RF::UPUP-monomial(1,1)$UPUP)/a0::RF::R
      reduce makeDivisor(a0, b0, g)

    generator d ==
      d := reduce d
      princ? d => d.principalPart
      "failed"

    - d ==
      a := d.center
      makeDivisor(a, - d.polyPart, inv(a::RF * d.principalPart))

    d1 = d2 ==
      d1 := reduce d1
      d2 := reduce d2
      d1.center = d2.center and d1.polyPart = d2.polyPart
        and d1.principalPart = d2.principalPart

    divisor(a, b) ==
      x := monomial(1, 1)$UP
      not ground? gcd(d := x - a::UP, retract(discriminant())@UP) =>
                                error "divisor: point is singular"
      makeDivisor(d, b::UP, 1)

    intReduce(h, b) ==
      v := integralCoordinates(h).num
      integralRepresents(
                [qelt(v, i) rem b for i in minIndex v .. maxIndex v], 1)

-- with hyperelliptic curves, it is cheaper to keep divisors in reduced form
    divisor(h, a, dp, g, r) ==
      h  := h - (r * dp)::RF::R
      a  := gcd(a, retract(norm h)@UP)
      h  := intReduce(h, a)
      if not ground? gcd(g, a) then h := intReduce(h ** rank(), a)
      hh := lift h
      b  := - coefficient(hh, 0) / coefficient(hh, 1)
      rec := extendedEuclidean(denom b, a)
      not ground?(rec.generator) => ERR
      bb := ((numer(b) * rec.coef1) exquo rec.generator)::UP rem a
      reduce makeDivisor(a, bb, 1)
```

———-

— HELLFDIV.dotabb —

```
"HELLFDIV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HELLFDIV"]
"FDIVCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FDIVCAT"]
```

```
"HELLFDIV" -> "FDIVCAT"
```

———

Chapter 10

Chapter I

10.1 domain ICP InfClsPt

— InfClsPt.input —

```
)set break resume
)sys rm -f InfClsPt.output
)spool InfClsPt.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfClsPt
--R InfClsPt(K: Field,symb: List Symbol,BLMET: BlowUpMethodCategory)  is a domain constructor
--R Abbreviation for InfClsPt is ICP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ICP
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                actualExtensionV : % -> K
--R chartV : % -> BLMET                   coerce : % -> OutputForm
--R degree : % -> PositiveInteger         excpDivV : % -> Divisor Places K
--R fullOut : % -> OutputForm             fullOutput : () -> Boolean
--R fullOutput : Boolean -> Boolean       hash : % -> SingleInteger
--R latex : % -> String                   localPointV : % -> AffinePlane K
--R multV : % -> NonNegativeInteger       pointV : % -> ProjectivePlane K
--R setchart! : (%,BLMET) -> BLMET        symbNameV : % -> Symbol
--R ?~=? : (%,%) -> Boolean
--R create : (ProjectivePlane K,DistributedMultivariatePolynomial(symb,K)) -> %
--R create : (ProjectivePlane K,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),AffinePla
--R curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
```

```
--R localParamV : % -> List NeitherSparseOrDensePowerSeries K
--R setcurve! : (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) -> Distri
--R setexcpDiv! : (%,Divisor Places K) -> Divisor Places K
--R setlocalParam! : (%,List NeitherSparseOrDensePowerSeries K) -> List NeitherSparseOrDense
--R setlocalPoint! : (%,AffinePlane K) -> AffinePlane K
--R setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--R setpoint! : (%,ProjectivePlane K) -> ProjectivePlane K
--R setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--R setsymbName! : (%,Symbol) -> Symbol
--R subMultV : % -> NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

———-

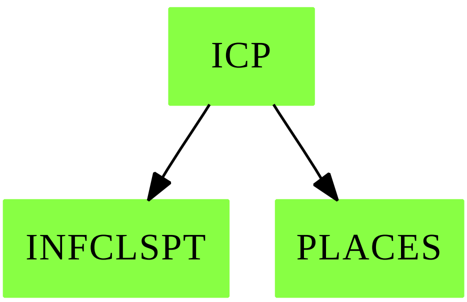
— **InfClsPt.help** —

```
====================================================================
InfClsPt examples
====================================================================

See Also:
o )show InfClsPt
```

———-

10.1.1 InfClsPt (ICP)



Exports:

?=?	?~=?	actualExtensionV
chartV	coerce	create
curveV	degree	excpDivV
fullOut	fullOutput	hash
latex	localParamV	localPointV
multV	pointV	setchart!
setcurve!	setexcpDiv!	setlocalParam!
setlocalPoint!	setmult!	setpoint!
setsubmult!	setsymbName!	subMultV
symbNameV		

— domain ICP InfClsPt —

```
)abbrev domain ICP InfClsPt
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
InfClsPt(K,symb,BLMET):Exports == Implementation where
  K:Field
  symb: List Symbol
  BLMET : BlowUpMethodCategory

  E         ==> DirectProduct(#symb,NonNegativeInteger)
  PolyRing  ==> DistributedMultivariatePolynomial(symb,K)
  ProjPt    ==> ProjectivePlane(K)
  PCS       ==> NeitherSparseOrDensePowerSeries(K)
  Plc       ==> Places(K)
  DIVISOR   ==> Divisor(Plc)

  Exports == InfinitlyClosePointCategory(K,symb,PolyRing,E,ProjPt,_
                                         PCS,Plc,DIVISOR,BLMET) with
    fullOut: % -> OutputForm
      ++ fullOut(tr) yields a full output of tr (see function fullOutput).

    fullOutput: Boolean -> Boolean
      ++ fullOutput(b) sets a flag such that when true, a coerce to
      ++ OutputForm yields the full output of tr, otherwise encode(tr) is
      ++ output (see encode function). The default is false.

    fullOutput: () -> Boolean
      ++ fullOutput returns the value of the flag set by fullOutput(b).

  Implementation == InfinitlyClosePoint(K,symb,PolyRing,E,ProjPt,_
                                        PCS,Plc,DIVISOR,BLMET)
```

— ICP.dotabb —

```
"ICP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ICP"]
"INFCLSPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFCLSPT"]
"PLACES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLACES"]
"ICP" -> "INFCLSPT"
"ICP" -> "PLACES"
```

10.2 domain ICARD IndexCard

— IndexCard.input —

```
)set break resume
)sys rm -f IndexCard.output
)spool IndexCard.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexCard
--R IndexCard  is a domain constructor
--R Abbreviation for IndexCard is ICARD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ICARD
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : String -> %
--R coerce : % -> OutputForm              display : % -> Void
--R ?.? : (%,Symbol) -> String            fullDisplay : % -> Void
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

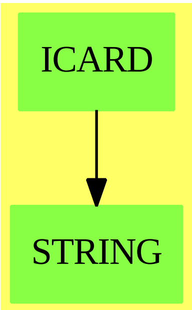
— **IndexCard.help** —

```
====================================================================
IndexCard examples
====================================================================

See Also:
o )show IndexCard
```

———-

10.2.1 IndexCard (ICARD)



See

⇒ "DataList" (DLIST) 5.2.1 on page 445
⇒ "Database" (DBASE) 5.1.1 on page 440
⇒ "QueryEquation" (QEQUAT) 18.4.1 on page 2129

Exports:

coerce	display	fullDisplay	hash	latex
max	min	?~=?	?<?	?<=?
?=?	?>?	?>=?	?.?	

— **domain ICARD IndexCard** —

```
)abbrev domain ICARD IndexCard
++ Author: Mark Botch
++ Description:
++ This domain implements a container of information about the AXIOM library

IndexCard() : Exports == Implementation where
  Exports == OrderedSet with
    elt: (%,Symbol) -> String
      ++ elt(ic,s) selects a particular field from \axiom{ic}.  Valid fields
```

```
      ++ are \axiom{name, nargs, exposed, type, abbreviation, kind, origin,
      ++ params, condition, doc}.
    display: % -> Void
      ++ display(ic) prints a summary of information contained in \axiom{ic}.
    fullDisplay: % -> Void
      ++ fullDisplay(ic) prints all of the information contained in \axiom{ic}.
    coerce: String -> %
      ++ coerce(s) converts \axiom{s} into an \axiom{IndexCard}.  Warning: if
      ++ \axiom{s} is not of the right format then an error will occur
  Implementation == add
    x<y==(x pretend String) < (y pretend String)
    x=y==(x pretend String) = (y pretend String)
    display(x) ==
      name : OutputForm := dbName(x)$Lisp
      type : OutputForm := dbPart(x,4,1$Lisp)$Lisp
      output(hconcat(name,hconcat(" : ",type)))$OutputPackage
    fullDisplay(x) ==
      name : OutputForm := dbName(x)$Lisp
      type : OutputForm := dbPart(x,4,1$Lisp)$Lisp
      origin:OutputForm :=
          hconcat(alqlGetOrigin(x$Lisp)$Lisp,alqlGetParams(x$Lisp)$Lisp)
      fromPart : OutputForm := hconcat(" from ",origin)
      condition : String := dbPart(x,6,1$Lisp)$Lisp
      ifPart : OutputForm :=
        condition = "" => empty()
        hconcat(" if ",condition::OutputForm)
      exposed? : String := SUBSTRING(dbPart(x,3,1)$Lisp,0,1)$Lisp
      exposedPart : OutputForm :=
        exposed? = "n" => " (unexposed)"
        empty()
      firstPart := hconcat(name,hconcat(" : ",type))
      secondPart := hconcat(fromPart,hconcat(ifPart,exposedPart))
      output(hconcat(firstPart,secondPart))$OutputPackage
    coerce(s:String): % == (s pretend %)
    coerce(x): OutputForm == (x pretend String)::OutputForm
    elt(x,sel) ==
      s := PNAME(sel)$Lisp pretend String
      s = "name" => dbName(x)$Lisp
      s = "nargs" => dbPart(x,2,1$Lisp)$Lisp
      s = "exposed" => SUBSTRING(dbPart(x,3,1)$Lisp,0,1)$Lisp
      s = "type" => dbPart(x,4,1$Lisp)$Lisp
      s = "abbreviation" => dbPart(x,5,1$Lisp)$Lisp
      s = "kind" => alqlGetKindString(x)$Lisp
      s = "origin" => alqlGetOrigin(x)$Lisp
      s = "params" => alqlGetParams(x)$Lisp
      s = "condition" => dbPart(x,6,1$Lisp)$Lisp
      s = "doc" => dbComments(x)$Lisp
      error "unknown selector"
```

———-

— ICARD.dotabb —

```
"ICARD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ICARD"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"ICARD" -> "STRING"
```

———-

10.3 domain IBITS IndexedBits

— IndexedBits.input —

```
)set break resume
)sys rm -f IndexedBits.output
)spool IndexedBits.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
a:IBITS(32):=new(32,false)
--R
--R
--R   (1)  "00000000000000000000000000000000"
--R                                                        Type: IndexedBits 32
--E 1

--S 2 of 13
b:IBITS(32):=new(32,true)
--R
--R
--R   (2)  "11111111111111111111111111111111"
--R                                                        Type: IndexedBits 32
--E 2

--S 3 of 13
elt(a,3)
--R
--R
--R   (3)  false
--R                                                               Type: Boolean
--E 3

--S 4 of 13
```

```
setelt(a,3,true)
--R
--R
--R   (4)  true
--R                                                            Type: Boolean
--E 4

--S 5 of 13
a
--R
--R
--R   (5)  "00000000000000000000000000000100"
--R                                                     Type: IndexedBits 32
--E 5

--S 6 of 13
#a
--R
--R
--R   (6)  32
--R                                                    Type: PositiveInteger
--E 6

--S 7 of 13
(a=a)$IBITS(32)
--R
--R
--R   (7)  true
--R                                                            Type: Boolean
--E 7

--S 8 of 13
(a=b)$IBITS(32)
--R
--R
--R   (8)  false
--R                                                            Type: Boolean
--E 8

--S 9 of 13
(a ~= b)
--R
--R
--R   (9)  true
--R                                                            Type: Boolean
--E 9

--S 10 of 13
Or(a,b)
--R
```

```
--R
--R   (10)  "11111111111111111111111111111111"
--R                                                     Type: IndexedBits 32
--E 10

--S 11 of 13
And(a,b)
--R
--R
--R   (11)  "00000000000000000000000000000100"
--R                                                     Type: IndexedBits 32
--E 11

--S 12 of 13
Not(a)
--R
--R
--R   (12)  "11111111111111111111111111111011"
--R                                                     Type: IndexedBits 32
--E 12

--S 13 of 13
c:=copy a
--R
--R
--R   (13)  "00000000000000000000000000000100"
--R                                                     Type: IndexedBits 32
--E 13
)spool
)lisp (bye)
```

— **IndexedBits.help** —

```
====================================================================
IndexedBits
====================================================================

a:IBITS(32):=new(32,false)
  "00000000000000000000000000000000"

b:IBITS(32):=new(32,true)
  "11111111111111111111111111111111"

elt(a,3)
  false

setelt(a,3,true)
```

```
  true

a
  "00000000000000000000000000000100"

#a
  32

(a=a)$IBITS(32)
  true

(a=b)$IBITS(32)
  false

(a ~= b)
  true

Or(a,b)
  "11111111111111111111111111111111"

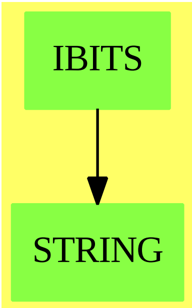
And(a,b)
  "00000000000000000000000000000100"

Not(a)
  "11111111111111111111111111111011"

c:=copy a
  "00000000000000000000000000000100"

See Also:
o )show IndexedBits
```

10.3.1 IndexedBits (IBITS)



See

⇒ "Reference" (REF) 19.5.1 on page 2209
⇒ "Boolean" (BOOLEAN) 3.15.1 on page 304
⇒ "Bits" (BITS) 3.12.1 on page 297

Exports:

And	any?	coerce	concat	construct
convert	copy	copyInto!	count	count
delete	elt	empty	empty?	entries
entry?	eq?	eval	every?	fill!
find	first	hash	index?	indices
insert	latex	less?	map	map!
max	maxIndex	member?	members	merge
min	minIndex	more?	nand	new
nor	Not	not?	Or	parts
position	qelt	qsetelt!	reduce	removeDuplicates
reverse	reverse!	sample	select	size?
sort	sort!	sorted?	swap!	xor
#?	?.?	?/\?	?<?	?<=?
?=?	?>?	?>=?	?\/?	^?
?.?	~?	?~=?	?or?	?and?

— domain IBITS IndexedBits —

```
)abbrev domain IBITS IndexedBits
++ Author: Stephen Watt and Michael Monagan
++ Date Created: July 86
++ Change History:  Oct 87
++ Basic Operations: range
++ Related Constructors:
++ Keywords: indexed bits
++ Description:
++ \spadtype{IndexedBits} is a domain to compactly represent
++ large quantities of Boolean data.
```

```
IndexedBits(mn:Integer): BitAggregate() with
        -- temporaries until parser gets better
        Not: % -> %
            ++ Not(n) returns the bit-by-bit logical Not of n.
        Or : (%, %) -> %
            ++ Or(n,m)  returns the bit-by-bit logical Or of
            ++ n and m.
        And: (%, %) -> %
            ++ And(n,m)  returns the bit-by-bit logical And of
            ++ n and m.
    == add

        range: (%, Integer) -> Integer
          --++ range(j,i) returnes the range i of the boolean j.

        minIndex u  == mn

        range(v, i) ==
          i >= 0 and i < #v => i
          error "Index out of range"

        coerce(v):OutputForm ==
            t:Character := char "1"
            f:Character := char "0"
            s := new(#v, space()$Character)$String
            for i in minIndex(s)..maxIndex(s) for j in mn.. repeat
              s.i := if v.j then t else f
            s::OutputForm

        new(n, b)       == BVEC_-MAKE_-FULL(n,TRUTH_-TO_-BIT(b)$Lisp)$Lisp
        empty()         == BVEC_-MAKE_-FULL(0,0)$Lisp
        copy v          == BVEC_-COPY(v)$Lisp
        #v              == BVEC_-SIZE(v)$Lisp
        v = u           == BVEC_-EQUAL(v, u)$Lisp
        v < u           == BVEC_-GREATER(u, v)$Lisp
        _and(u, v)      == (#v=#u => BVEC_-AND(v,u)$Lisp; map("and",v,u))
        _or(u, v)       == (#v=#u => BVEC_-OR(v, u)$Lisp; map("or", v,u))
        xor(v,u)        == (#v=#u => BVEC_-XOR(v,u)$Lisp; map("xor",v,u))
        setelt(v:%, i:Integer, f:Boolean) ==
          BVEC_-SETELT(v, range(v, abs(i-mn)), TRUTH_-TO_-BIT(f)$Lisp)$Lisp
        elt(v:%, i:Integer) ==
          BIT_-TO_-TRUTH(BVEC_-ELT(v, range(v, abs(i-mn)))$Lisp)$Lisp

        Not v           == BVEC_-NOT(v)$Lisp
        And(u, v)       == (#v=#u => BVEC_-AND(v,u)$Lisp; map("and",v,u))
        Or(u, v)        == (#v=#u => BVEC_-OR(v, u)$Lisp; map("or", v,u))
```

————–

— **IBITS.dotabb** —

```
"IBITS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IBITS"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"IBITS" -> "STRING"
```

10.4 domain IDPAG IndexedDirectProductAbelianGroup

— **IndexedDirectProductAbelianGroup.input** —

```
)set break resume
)sys rm -f IndexedDirectProductAbelianGroup.output
)spool IndexedDirectProductAbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductAbelianGroup
--R IndexedDirectProductAbelianGroup(A: AbelianGroup,S: OrderedSet)  is a domain constructor
--R Abbreviation for IndexedDirectProductAbelianGroup is IDPAG
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPAG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 0 : () -> %                          coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R leadingCoefficient : % -> A          leadingSupport : % -> S
--R map : ((A -> A),%) -> %              monomial : (A,S) -> %
--R reductum : % -> %                    sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

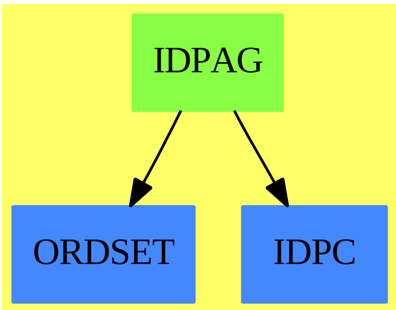
— **IndexedDirectProductAbelianGroup.help** —

```
====================================================================
IndexedDirectProductAbelianGroup examples
====================================================================

See Also:
o )show IndexedDirectProductAbelianGroup
```

———-

10.4.1 IndexedDirectProductAbelianGroup (IDPAG)



See

⇒ "IndexedDirectProductObject" (IDPO) 10.6.1 on page 1175
⇒ "IndexedDirectProductAbelianMonoid" (IDPAM) 10.5.1 on page 1171
⇒ "IndexedDirectProductOrderedAbelianMonoid" (IDPOAM) 10.7.1 on page 1178
⇒ "IndexedDirectProductOrderedAbelianMonoidSup" (IDPOAMS) 10.8.1 on page 1180

Exports:

0	coerce	hash	latex	leadingCoefficient
leadingSupport	map	monomial	reductum	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

— **domain IDPAG IndexedDirectProductAbelianGroup** —

```
)abbrev domain IDPAG IndexedDirectProductAbelianGroup
++ Author: Mark Botch
++ Description:
++ Indexed direct products of abelian groups over an abelian group \spad{A} of
++ generators indexed by the ordered set S.
++ All items have finite support: only non-zero terms are stored.
```

```
IndexedDirectProductAbelianGroup(A:AbelianGroup,S:OrderedSet):
    Join(AbelianGroup,IndexedDirectProductCategory(A,S))
 ==  IndexedDirectProductAbelianMonoid(A,S) add
    --representations
       Term:=  Record(k:S,c:A)
       Rep:=  List Term
       x,y: %
       r: A
       n: Integer
       f: A -> A
       s: S
       -x == [[u.k,-u.c] for u in x]
       n * x  ==
             n = 0 => 0
             n = 1 => x
             [[u.k,a] for u in x | (a:=n*u.c) ^= 0$A]

       qsetrest!: (Rep, Rep) -> Rep
       qsetrest!(l: Rep, e: Rep): Rep == RPLACD(l, e)$Lisp

       x - y ==
                null x => -y
                null y => x
                endcell: Rep := empty()
                res:  Rep := empty()
                while not empty? x and not empty? y repeat
                        newcell := empty()
                        if x.first.k = y.first.k then
                                r:= x.first.c - y.first.c
                                if not zero? r then
                                        newcell := cons([x.first.k, r], empty())
                                x := rest x
                                y := rest y
                        else if x.first.k > y.first.k then
                                newcell := cons(x.first, empty())
                                x := rest x
                        else
                                newcell := cons([y.first.k,-y.first.c], empty())
                                y := rest y
                        if not empty? newcell then
                                if not empty? endcell then
                                        qsetrest!(endcell, newcell)
                                        endcell := newcell
                                else
                                        res     := newcell;
                                        endcell := res
                if empty? x then end := - y
                else end := x
                if empty? res then res := end
                else qsetrest!(endcell, end)
```

```
            res

--      x - y  ==
--         empty? x => - y
--         empty? y => x
--         y.first.k > x.first.k => cons([y.first.k,-y.first.c],(x - y.rest))
--         x.first.k > y.first.k => cons(x.first,(x.rest - y))
--         r:= x.first.c - y.first.c
--         r = 0 => x.rest - y.rest
--         cons([x.first.k,r],(x.rest - y.rest))
```

— IDPAG.dotabb —

```
"IDPAG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPAG"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"IDPC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=IDPC"]
"IDPAG" -> "IDPC"
"IDPAG" -> "ORDSET"
```

10.5 domain IDPAM IndexedDirectProductAbelianMonoid

— IndexedDirectProductAbelianMonoid.input —

```
)set break resume
)sys rm -f IndexedDirectProductAbelianMonoid.output
)spool IndexedDirectProductAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductAbelianMonoid
--R IndexedDirectProductAbelianMonoid(A: AbelianMonoid,S: OrderedSet)  is a domain constructo
--R Abbreviation for IndexedDirectProductAbelianMonoid is IDPAM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPAM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
```

```
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> A
--R leadingSupport : % -> S              map : ((A -> A),%) -> %
--R monomial : (A,S) -> %                reductum : % -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R
--E 1

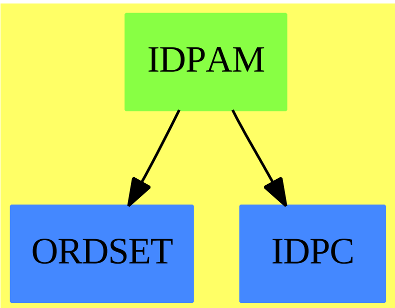
)spool
)lisp (bye)
```

—— IndexedDirectProductAbelianMonoid.help ——

```
====================================================================
IndexedDirectProductAbelianMonoid examples
====================================================================

See Also:
o )show IndexedDirectProductAbelianMonoid
```

10.5.1 IndexedDirectProductAbelianMonoid (IDPAM)



See

⇒ "IndexedDirectProductObject" (IDPO) 10.6.1 on page 1175
⇒ "IndexedDirectProductOrderedAbelianMonoid" (IDPOAM) 10.7.1 on page 1178
⇒ "IndexedDirectProductOrderedAbelianMonoidSup" (IDPOAMS) 10.8.1 on page 1180
⇒ "IndexedDirectProductAbelianGroup" (IDPAG) 10.4.1 on page 1168

Exports:

0	coerce	hash	latex	leadingCoefficient
leadingSupport	map	monomial	reductum	sample
zero?	?~=?	?*?	?+?	?=?

— domain IDPAM IndexedDirectProductAbelianMonoid —

```
)abbrev domain IDPAM IndexedDirectProductAbelianMonoid
++ Author: Mark Botch
++ Description:
++ Indexed direct products of abelian monoids over an abelian monoid
++ \spad{A} of generators indexed by the ordered set S. All items have
++ finite support. Only non-zero terms are stored.

IndexedDirectProductAbelianMonoid(A:AbelianMonoid,S:OrderedSet):
    Join(AbelianMonoid,IndexedDirectProductCategory(A,S))
 ==  IndexedDirectProductObject(A,S) add
    --representations
       Term:=  Record(k:S,c:A)
       Rep:=  List Term
       x,y: %
       r: A
       n: NonNegativeInteger
       f: A -> A
       s: S
       0  == []
       zero? x ==  null x

        -- PERFORMANCE CRITICAL; Should build list up
        --  by merging 2 sorted lists.   Doing this will
        -- avoid the recursive calls (very useful if there is a
        -- large number of vars in a polynomial.
--       x + y  ==
--          null x => y
--          null y => x
--          y.first.k > x.first.k => cons(y.first,(x + y.rest))
--          x.first.k > y.first.k => cons(x.first,(x.rest + y))
--          r:= x.first.c + y.first.c
--          r = 0 => x.rest + y.rest
--          cons([x.first.k,r],(x.rest + y.rest))
       qsetrest!: (Rep, Rep) -> Rep
       qsetrest!(l: Rep, e: Rep): Rep == RPLACD(l, e)$Lisp

       x + y ==
        null x => y
        null y => x
        endcell: Rep := empty()
        res:  Rep := empty()
        while not empty? x and not empty? y repeat
            newcell := empty()
```

```
        if x.first.k = y.first.k then
            r:= x.first.c + y.first.c
            if not zero? r then
                newcell := cons([x.first.k, r], empty())
            x := rest x
            y := rest y
        else if x.first.k > y.first.k then
            newcell := cons(x.first, empty())
            x := rest x
        else
            newcell := cons(y.first, empty())
            y := rest y
        if not empty? newcell then
            if not empty? endcell then
                qsetrest!(endcell, newcell)
                endcell := newcell
            else
                res     := newcell;
                endcell := res
    if empty? x then end := y
    else end := x
    if empty? res then res := end
    else qsetrest!(endcell, end)
    res

   n * x  ==
         n = 0 => 0
         n = 1 => x
         [[u.k,a] for u in x | (a:=n*u.c) ^= 0$A]

   monomial(r,s) == (r = 0 => 0; [[s,r]])
   map(f,x) == [[tm.k,a] for tm in x | (a:=f(tm.c)) ^= 0$A]

   reductum x     == (null x => 0; rest x)
   leadingCoefficient x  == (null x => 0; x.first.c)
```

———-

— IDPAM.dotabb —

```
"IDPAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPAM"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"IDPC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=IDPC"]
"IDPAM" -> "IDPC"
"IDPAM" -> "ORDSET"
```

———-

10.6 domain IDPO IndexedDirectProductObject

— IndexedDirectProductObject.input —

```
)set break resume
)sys rm -f IndexedDirectProductObject.output
)spool IndexedDirectProductObject.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductObject
--R IndexedDirectProductObject(A: SetCategory,S: OrderedSet)  is a domain constructor
--R Abbreviation for IndexedDirectProductObject is IDPO
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPO
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> A           leadingSupport : % -> S
--R map : ((A -> A),%) -> %               monomial : (A,S) -> %
--R reductum : % -> %                     ?~=? : (%,%) -> Boolean
--R
--E 1

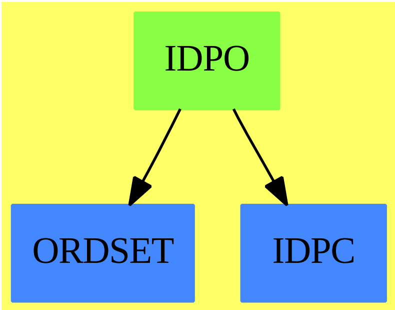
)spool
)lisp (bye)
```

— IndexedDirectProductObject.help —

```
====================================================================
IndexedDirectProductObject examples
====================================================================

See Also:
o )show IndexedDirectProductObject
```

10.6.1 IndexedDirectProductObject (IDPO)



See

⇒ "IndexedDirectProductAbelianMonoid" (IDPAM) 10.5.1 on page 1171
⇒ "IndexedDirectProductOrderedAbelianMonoid" (IDPOAM) 10.7.1 on page 1178
⇒ "IndexedDirectProductOrderedAbelianMonoidSup" (IDPOAMS) 10.8.1 on page 1180
⇒ "IndexedDirectProductAbelianGroup" (IDPAG) 10.4.1 on page 1168

Exports:

coerce	hash	latex	leadingCoefficient	leadingSupport
map	monomial	reductum	?=?	?~=?

— domain IDPO IndexedDirectProductObject —

```
)abbrev domain IDPO IndexedDirectProductObject
++ Author: Mark Botch
++ Description:
++ Indexed direct products of objects over a set \spad{A}
++ of generators indexed by an ordered set S. All items have finite support.

IndexedDirectProductObject(A:SetCategory,S:OrderedSet): _
  IndexedDirectProductCategory(A,S)
 == add
    --representations
       Term:=  Record(k:S,c:A)
       Rep:=  List Term
    --declarations
       x,y: %
       f: A -> A
       s: S
    --define
       x = y ==
         while not null x and _^ null y repeat
           x.first.k ^= y.first.k => return false
           x.first.c ^= y.first.c => return false
           x:=x.rest
           y:=y.rest
         null x and null y
```

```
   coerce(x:%):OutputForm ==
      bracket [rarrow(t.k :: OutputForm, t.c :: OutputForm) for t in x]

   -- sample():% == [[sample()$S,sample()$A]$Term]$Rep

   monomial(r,s) == [[s,r]]
   map(f,x) == [[tm.k,f(tm.c)] for tm in x]

   reductum x     ==
      rest x
   leadingCoefficient x  ==
      null x => error "Can't take leadingCoefficient of empty product element"
      x.first.c
   leadingSupport x  ==
      null x => error "Can't take leadingCoefficient of empty product element"
      x.first.k
```

— **IDPO.dotabb** —

```
"IDPO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPO"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"IDPC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=IDPC"]
"IDPO" -> "IDPC"
"IDPO" -> "ORDSET"
```

10.7 domain IDPOAM IndexedDirectProductOrderedAbelianMonoid

— **IndexedDirectProductOrderedAbelianMonoid.input** —

```
)set break resume
)sys rm -f IndexedDirectProductOrderedAbelianMonoid.output
)spool IndexedDirectProductOrderedAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show IndexedDirectProductOrderedAbelianMonoid
--R IndexedDirectProductOrderedAbelianMonoid(A: OrderedAbelianMonoid,S: OrderedSet)  is a domain constru
--R Abbreviation for IndexedDirectProductOrderedAbelianMonoid is IDPOAM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPOAM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              0 : () -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> A
--R leadingSupport : % -> S              map : ((A -> A),%) -> %
--R max : (%,%) -> %                     min : (%,%) -> %
--R monomial : (A,S) -> %                reductum : % -> %
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

—————

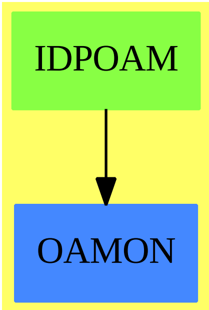
— **IndexedDirectProductOrderedAbelianMonoid.help** —

```
====================================================================
IndexedDirectProductOrderedAbelianMonoid examples
====================================================================

See Also:
o )show IndexedDirectProductOrderedAbelianMonoid
```

—————

10.7.1 IndexedDirectProductOrderedAbelianMonoid (IDPOAM)



See

⇒ "IndexedDirectProductObject" (IDPO) 10.6.1 on page 1175
⇒ "IndexedDirectProductAbelianMonoid" (IDPAM) 10.5.1 on page 1171
⇒ "IndexedDirectProductOrderedAbelianMonoidSup" (IDPOAMS) 10.8.1 on page 1180
⇒ "IndexedDirectProductAbelianGroup" (IDPAG) 10.4.1 on page 1168

Exports:

0	coerce	hash	latex	leadingCoefficient
leadingSupport	map	max	min	monomial
reductum	sample	zero?	?~=?	?*?
?+?	?<?	?<=?	?=?	?>?
?>=?				

— domain IDPOAM IndexedDirectProductOrderedAbelianMonoid —

```
)abbrev domain IDPOAM IndexedDirectProductOrderedAbelianMonoid
++ Author: Mark Botch
++ Description:
++ Indexed direct products of ordered abelian monoids \spad{A} of
++ generators indexed by the ordered set S.
++ The inherited order is lexicographical.
++ All items have finite support: only non-zero terms are stored.

IndexedDirectProductOrderedAbelianMonoid(A:OrderedAbelianMonoid,S:OrderedSet):
    Join(OrderedAbelianMonoid,IndexedDirectProductCategory(A,S))
 ==  IndexedDirectProductAbelianMonoid(A,S) add
    --representations
       Term:=  Record(k:S,c:A)
       Rep:=  List Term
       x,y: %
       x<y ==
         empty? y => false
         empty? x => true   -- note careful order of these two lines
         y.first.k > x.first.k => true
```

```
        y.first.k < x.first.k => false
        y.first.c > x.first.c => true
        y.first.c < x.first.c => false
        x.rest < y.rest
```

———-

— IDPOAM.dotabb —

```
"IDPOAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPOAM"]
"OAMON" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMON"]
"IDPOAM" -> "OAMON"
```

———-

10.8 domain IDPOAMS IndexedDirectProductOrderedAbelianMonoidSup

— IndexedDirectProductOrderedAbelianMonoidSup.input —

```
)set break resume
)sys rm -f IndexedDirectProductOrderedAbelianMonoidSup.output
)spool IndexedDirectProductOrderedAbelianMonoidSup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductOrderedAbelianMonoidSup
--R IndexedDirectProductOrderedAbelianMonoidSup(A: OrderedAbelianMonoidSup,S: OrderedSet)  is a domain c
--R Abbreviation for IndexedDirectProductOrderedAbelianMonoidSup is IDPOAMS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPOAMS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              0 : () -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> A
--R leadingSupport : % -> S              map : ((A -> A),%) -> %
--R max : (%,%) -> %                     min : (%,%) -> %
```

```
--R monomial : (A,S) -> %                 reductum : % -> %
--R sample : () -> %                      sup : (%,%) -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

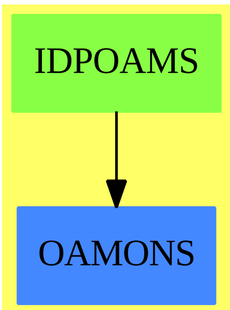
— **IndexedDirectProductOrderedAbelianMonoidSup.help** —

```
====================================================================
IndexedDirectProductOrderedAbelianMonoidSup examples
====================================================================

See Also:
o )show IndexedDirectProductOrderedAbelianMonoidSup
```

———-

10.8.1 IndexedDirectProductOrderedAbelianMonoidSup (IDPOAMS)



See

⇒ "IndexedDirectProductObject" (IDPO) 10.6.1 on page 1175
⇒ "IndexedDirectProductAbelianMonoid" (IDPAM) 10.5.1 on page 1171
⇒ "IndexedDirectProductOrderedAbelianMonoid" (IDPOAM) 10.7.1 on page 1178
⇒ "IndexedDirectProductAbelianGroup" (IDPAG) 10.4.1 on page 1168

Exports:

0	coerce	hash	latex	leadingCoefficient
leadingSupport	map	max	min	monomial
reductum	sample	subtractIfCan	sup	zero?
?~=?	?*?	?+?	?<?	?<=?
?=?	?>?	?>=?		

— domain IDPOAMS IndexedDirectProductOrderedAbelianMonoid-Sup —

```
)abbrev domain IDPOAMS IndexedDirectProductOrderedAbelianMonoidSup
++ Author: Mark Botch
++ Description:
++ Indexed direct products of ordered abelian monoid sups \spad{A},
++ generators indexed by the ordered set S.
++ All items have finite support: only non-zero terms are stored.

IndexedDirectProductOrderedAbelianMonoidSup(A:OrderedAbelianMonoidSup,S:OrderedSet):
    Join(OrderedAbelianMonoidSup,IndexedDirectProductCategory(A,S))
 ==  IndexedDirectProductOrderedAbelianMonoid(A,S) add
    --representations
       Term:=  Record(k:S,c:A)
       Rep:=  List Term
       x,y: %
       r: A
       s: S

       subtractIfCan(x,y) ==
         empty? y => x
         empty? x => "failed"
         x.first.k < y.first.k => "failed"
         x.first.k > y.first.k =>
             t:= subtractIfCan(x.rest, y)
             t case "failed" => "failed"
             cons( x.first, t)
         u:=subtractIfCan(x.first.c, y.first.c)
         u case "failed" => "failed"
         zero? u => subtractIfCan(x.rest, y.rest)
         t:= subtractIfCan(x.rest, y.rest)
         t case "failed" => "failed"
         cons([x.first.k,u],t)

       sup(x,y) ==
         empty? y => x
         empty? x => y
         x.first.k < y.first.k => cons(y.first,sup(x,y.rest))
         x.first.k > y.first.k => cons(x.first,sup(x.rest,y))
         u:=sup(x.first.c, y.first.c)
         cons([x.first.k,u],sup(x.rest,y.rest))
```

— IDPOAMS.dotabb —

```
"IDPOAMS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPOAMS"]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
"IDPOAMS" -> "OAMONS"
```

10.9 domain INDE IndexedExponents

— IndexedExponents.input —

```
)set break resume
)sys rm -f IndexedExponents.output
)spool IndexedExponents.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedExponents
--R IndexedExponents Varset: OrderedSet  is a domain constructor
--R Abbreviation for IndexedExponents is INDE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INDE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              0 : () -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  leadingSupport : % -> Varset
--R max : (%,%) -> %                     min : (%,%) -> %
--R reductum : % -> %                    sample : () -> %
--R sup : (%,%) -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R leadingCoefficient : % -> NonNegativeInteger
--R map : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R monomial : (NonNegativeInteger,Varset) -> %
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

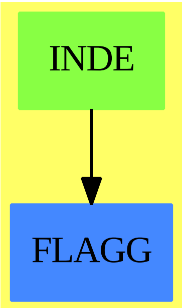
— IndexedExponents.help —

```
====================================================================
IndexedExponents examples
====================================================================

See Also:
o )show IndexedExponents
```

———-

10.9.1 IndexedExponents (INDE)



See

⇒ "Polynomial" (POLY) 17.25.1 on page 2037
⇒ "MultivariatePolynomial" (MPOLY) 14.16.1 on page 1645
⇒ "SparseMultivariatePolynomial" (SMP) 20.14.1 on page 2381

Exports:

0	coerce	hash	latex	leadingCoefficient
leadingSupport	map	max	min	monomial
reductum	sample	subtractIfCan	sup	zero?
?~=?	?*?	?+?	?<?	?<=?
?=?	?>?	?>=?		

— domain INDE IndexedExponents —

```
)abbrev domain INDE IndexedExponents
++ Author: James Davenport
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ IndexedExponents of an ordered set of variables gives a representation
++ for the degree of polynomials in commuting variables. It gives an ordered
++ pairing of non negative integer exponents with variables

IndexedExponents(Varset:OrderedSet): C == T where
  C == Join(OrderedAbelianMonoidSup,
            IndexedDirectProductCategory(NonNegativeInteger,Varset))
  T == IndexedDirectProductOrderedAbelianMonoidSup(NonNegativeInteger,Varset) add
      Term:=  Record(k:Varset,c:NonNegativeInteger)
      Rep:=  List Term
      x:%
      t:Term
      coerceOF(t):OutputForm ==     --++ converts term to OutputForm
         t.c = 1 => (t.k)::OutputForm
         (t.k)::OutputForm ** (t.c)::OutputForm
      coerce(x):OutputForm == ++ converts entire exponents to OutputForm
         null x => 1::Integer::OutputForm
         null rest x => coerceOF(first x)
         reduce("*",[coerceOF t for t in x])
```

— INDE.dotabb —

```
"INDE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INDE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"INDE" -> "FLAGG"
```

10.10 domain IFARRAY IndexedFlexibleArray

— IndexedFlexibleArray.input —

```
)set break resume
)sys rm -f IndexedFlexibleArray.output
)spool IndexedFlexibleArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedFlexibleArray
--R IndexedFlexibleArray(S: Type,mn: Integer)  is a domain constructor
--R Abbreviation for IndexedFlexibleArray is IFARRAY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IFARRAY
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R concat : (S,%) -> %                   concat : (%,S) -> %
--R concat! : (%,S) -> %                  concat! : (%,%) -> %
--R construct : List S -> %               copy : % -> %
--R delete : (%,Integer) -> %             delete! : (%,Integer) -> %
--R ?.? : (%,Integer) -> S                elt : (%,Integer,S) -> S
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List S                 eq? : (%,%) -> Boolean
--R flexibleArray : List S -> %           index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %           insert! : (S,%,Integer) -> %
--R insert! : (%,%,Integer) -> %          map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %               new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S               reverse : % -> %
--R sample : () -> %                      shrinkable : Boolean -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
```

```
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R merge! : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R physicalLength : % -> NonNegativeInteger
--R physicalLength! : (%,Integer) -> %
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,%) -> % if S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates! : % -> % if S has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

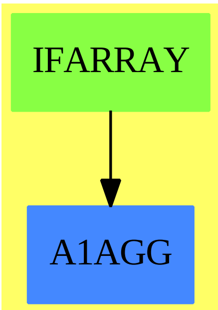
— **IndexedFlexibleArray.help** —

```
====================================================================
IndexedFlexibleArray examples
====================================================================

See Also:
o )show IndexedFlexibleArray
```

———-

10.10.1 IndexedFlexibleArray (IFARRAY)



See

⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208
⇒ “OneDimensionalArray” (ARRAY1) 16.3.1 on page 1736

Exports:

concat	concat!	construct	copy
delete	delete!	elt	empty
empty?	entries	eq?	flexibleArray
index?	indices	insert	insert!
map	new	qelt	reverse
sample	shrinkable	any?	coerce
convert	copyInto!	count	delete
delete!	entry?	eval	every?
fill!	find	first	hash
latex	less?	map!	max
maxIndex	member?	members	merge
merge!	min	minIndex	more?
parts	physicalLength	physicalLength!	position
qsetelt!	reduce	remove	remove!
removeDuplicates	removeDuplicates!	reverse!	select
select!	setelt	size?	sort
sort!	sorted?	swap!	#?
?<?	?<=?	?=?	?>?
?>=?	?~=?	?.?	

— domain IFARRAY IndexedFlexibleArray —

```
)abbrev domain IFARRAY IndexedFlexibleArray
++ Author: Michael Monagan July/87, modified SMW June/91
++ Description:
++ A FlexibleArray is the notion of an array intended to allow for growth
++ at the end only.  Hence the following efficient operations\br
++ \spad{append(x,a)} meaning append item x at the end of the array \spad{a}\br
++ \spad{delete(a,n)} meaning delete the last item from the array \spad{a}\br
++ Flexible arrays support the other operations inherited from
++ \spadtype{ExtensibleLinearAggregate}. However, these are not efficient.
++ Flexible arrays combine the \spad{O(1)} access time property of arrays
++ with growing and shrinking at the end in \spad{O(1)} (average) time.
++ This is done by using an ordinary array which may have zero or more
++ empty slots at the end.  When the array becomes full it is copied
++ into a new larger (50% larger) array.  Conversely, when the array
++ becomes less than 1/2 full, it is copied into a smaller array.
++ Flexible arrays provide for an efficient implementation of many
++ data structures in particular heaps, stacks and sets.

IndexedFlexibleArray(S:Type, mn: Integer): Exports == Implementation where
  A ==> PrimitiveArray S
  I ==> Integer
  N ==> NonNegativeInteger
  U ==> UniversalSegment Integer
  Exports ==
   Join(OneDimensionalArrayAggregate S,ExtensibleLinearAggregate S) with
```

```
    flexibleArray : List S -> %
     ++ flexibleArray(l) creates a flexible array from the list of elements l
     ++
     ++X T1:=IndexedFlexibleArray(Integer,20)
     ++X flexibleArray([i for i in 1..10])$T1

    physicalLength : % -> NonNegativeInteger
     ++ physicalLength(x) returns the number of elements x can
     ++ accomodate before growing
     ++
     ++X T1:=IndexedFlexibleArray(Integer,20)
     ++X t2:=flexibleArray([i for i in 1..10])$T1
     ++X physicalLength t2

    physicalLength_!: (%, I) -> %
     ++ physicalLength!(x,n) changes the physical length of x to be n and
     ++ returns the new array.
     ++
     ++X T1:=IndexedFlexibleArray(Integer,20)
     ++X t2:=flexibleArray([i for i in 1..10])$T1
     ++X physicalLength!(t2,15)

    shrinkable: Boolean -> Boolean
     ++ shrinkable(b) sets the shrinkable attribute of flexible arrays to b
     ++ and returns the previous value
     ++
     ++X T1:=IndexedFlexibleArray(Integer,20)
     ++X shrinkable(false)$T1

  Implementation == add
    Rep := Record(physLen:I, logLen:I, f:A)
    shrinkable? : Boolean := true
    growAndFill : (%, I, S) -> %
    growWith    : (%, I, S) -> %
    growAdding  : (%, I, %) -> %
    shrink: (%, I)    -> %
    newa  : (N, A) -> A

    physicalLength(r) == (r.physLen) pretend NonNegativeInteger
    physicalLength_!(r, n) ==
       r.physLen = 0  => error "flexible array must be non-empty"
       growWith(r, n, r.f.0)

    empty()      == [0, 0, empty()]
    #r           == (r.logLen)::N
    fill_!(r, x) == (fill_!(r.f, x); r)
    maxIndex r   == r.logLen - 1 + mn
    minIndex r   == mn
    new(n, a)    == [n, n, new(n, a)]
```

```
shrinkable(b) ==
  oldval := shrinkable?
  shrinkable? := b
  oldval

flexibleArray l ==
   n := #l
   n = 0 => empty()
   x := l.1
   a := new(n,x)
   for i in mn + 1..mn + n-1 for y in rest l repeat a.i := y
   a

-- local utility operations
newa(n, a) ==
   zero? n => empty()
   new(n, a.0)

growAdding(r, b, s) ==
   b = 0 => r
   #r > 0 => growAndFill(r, b, (r.f).0)
   #s > 0 => growAndFill(r, b, (s.f).0)
   error "no default filler element"

growAndFill(r, b, x) ==
   (r.logLen := r.logLen + b) <= r.physLen => r
   -- enlarge by 50% + b
   n := r.physLen + r.physLen quo 2 + 1
   if r.logLen > n then n := r.logLen
   growWith(r, n, x)

growWith(r, n, x) ==
   y := new(n::N, x)$PrimitiveArray(S)
   a := r.f
   for k in 0 .. r.physLen-1 repeat y.k := a.k
   r.physLen := n
   r.f := y
   r

shrink(r, i) ==
   r.logLen := r.logLen - i
   negative?(n := r.logLen) => error "internal bug in flexible array"
   2*n+2 > r.physLen => r
   not shrinkable? => r
   if n < r.logLen
     then error "cannot shrink flexible array to indicated size"
   n = 0 => empty()
   r.physLen := n
   y := newa(n::N, a := r.f)
   for k in 0 .. n-1 repeat y.k := a.k
```

```
      r.f := y
      r

   copy r ==
      n := #r
      a := r.f
      v := newa(n, a := r.f)
      for k in 0..n-1 repeat v.k := a.k
      [n, n, v]


   elt(r:%, i:I) ==
      i < mn or i >= r.logLen + mn =>
          error "index out of range"
      r.f.(i-mn)

   setelt(r:%, i:I, x:S) ==
      i < mn or i >= r.logLen + mn =>
          error "index out of range"
      r.f.(i-mn) := x

   -- operations inherited from extensible aggregate
   merge(g, a, b)   == merge_!(g, copy a, b)
   concat(x:S, r:%) == insert_!(x, r, mn)

   concat_!(r:%, x:S) ==
      growAndFill(r, 1, x)
      r.f.(r.logLen-1) := x
      r

   concat_!(a:%, b:%) ==
      if eq?(a, b) then b := copy b
      n := #a
      growAdding(a, #b, b)
      copyInto_!(a, b, n + mn)

   remove_!(g:(S->Boolean), a:%) ==
      k:I := 0
      for i in 0..maxIndex a - mn repeat
         if not g(a.i) then (a.k := a.i; k := k+1)
      shrink(a, #a - k)

   delete_!(r:%, i1:I) ==
      i := i1 - mn
      i < 0 or i > r.logLen => error "index out of range"
      for k in i..r.logLen-2 repeat r.f.k := r.f.(k+1)
      shrink(r, 1)

   delete_!(r:%, i:U) ==
      l := lo i - mn; m := maxIndex r - mn
```

```
   h := (hasHi i => hi i - mn; m)
   l < 0 or h > m => error "index out of range"
   for j in l.. for k in h+1..m repeat r.f.j := r.f.k
   shrink(r, max(0,h-l+1))

insert_!(x:S, r:%, i1:I):% ==
   i := i1 - mn
   n := r.logLen
   i < 0 or i > n => error "index out of range"
   growAndFill(r, 1, x)
   for k in n-1 .. i by -1 repeat r.f.(k+1) := r.f.k
   r.f.i := x
   r

insert_!(a:%, b:%, i1:I):% ==
   i := i1 - mn
   if eq?(a, b) then b := copy b
   m := #a; n := #b
   i < 0 or i > n => error "index out of range"
   growAdding(b, m, a)
   for k in n-1 .. i by -1 repeat b.f.(m+k) := b.f.k
   for k in m-1 .. 0 by -1 repeat b.f.(i+k) := a.f.k
   b

merge_!(g, a, b) ==
   m := #a; n := #b; growAdding(a, n, b)
   for i in m-1..0 by -1 for j in m+n-1.. by -1 repeat a.f.j := a.f.i
   i := n; j := 0
   for k in 0.. while i < n+m and j < n repeat
      if g(a.f.i,b.f.j) then (a.f.k := a.f.i; i := i+1)
      else (a.f.k := b.f.j; j := j+1)
   for k in k.. for j in j..n-1 repeat a.f.k := b.f.j
   a

select_!(g:(S->Boolean), a:%) ==
   k:I := 0
   for i in 0..maxIndex a - mn repeat_
      if g(a.f.i) then (a.f.k := a.f.i;k := k+1)
   shrink(a, #a - k)

if S has SetCategory then
  removeDuplicates_! a ==
     ct := #a
     ct < 2 => a

     i     := mn
     nlim  := mn + ct
     nlim0 := nlim
     while i < nlim repeat
        j := i+1
```

```
            for k in j..nlim-1 | a.k ^= a.i repeat
                a.j := a.k
                j := j+1
            nlim := j
            i := i+1
         nlim ^= nlim0 => delete_!(a, i..)
         a
```

———-

— IFARRAY.dotabb —

```
"IFARRAY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IFARRAY"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"IFARRAY" -> "A1AGG"
```

———-

10.11 domain ILIST IndexedList

— IndexedList.input —

```
)set break resume
)sys rm -f IndexedList.output
)spool IndexedList.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedList
--R IndexedList(S: Type,mn: Integer)  is a domain constructor
--R Abbreviation for IndexedList is ILIST
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ILIST
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %               concat : (%,S) -> %
--R concat : List % -> %                 concat : (S,%) -> %
--R concat : (%,%) -> %                  concat! : (%,S) -> %
--R concat! : (%,%) -> %                 construct : List S -> %
--R copy : % -> %                        cycleEntry : % -> %
--R cycleTail : % -> %                   cyclic? : % -> Boolean
--R delete : (%,Integer) -> %            delete! : (%,Integer) -> %
```

```
--R distance : (%,%) -> Integer          elt : (%,Integer,S) -> S
--R ?.? : (%,Integer) -> S               ?.last : (%,last) -> S
--R ?.rest : (%,rest) -> %               ?.first : (%,first) -> S
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List S
--R eq? : (%,%) -> Boolean               explicitlyFinite? : % -> Boolean
--R first : % -> S                       index? : (Integer,%) -> Boolean
--R indices : % -> List Integer          insert : (S,%,Integer) -> %
--R insert : (%,%,Integer) -> %          insert! : (S,%,Integer) -> %
--R insert! : (%,%,Integer) -> %         last : % -> S
--R leaf? : % -> Boolean                 leaves : % -> List S
--R list : S -> %                        map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %              new : (NonNegativeInteger,S) -> %
--R nodes : % -> List %                  possiblyInfinite? : % -> Boolean
--R qelt : (%,Integer) -> S              rest : % -> %
--R reverse : % -> %                     sample : () -> %
--R second : % -> S                      tail : % -> %
--R third : % -> S                       value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
```

```
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R merge! : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R position : ((S -> Boolean),%) -> Integer
--R position : (S,%) -> Integer if S has SETCAT
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,%) -> % if S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates! : % -> % if S has SETCAT
--R rest : (%,NonNegativeInteger) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((S,S) -> Boolean),%) -> %
--R sort : % -> % if S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if S has ORDSET
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
```

```
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

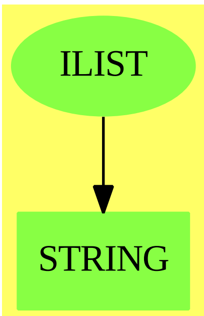
— IndexedList.help —

```
====================================================================
IndexedList examples
====================================================================

See Also:
o )show IndexedList
```

———-

10.11.1 IndexedList (ILIST)



See

⇒ "List" (LIST) 13.9.1 on page 1468
⇒ "AssociationList" (ALIST) 2.42.1 on page 218

Exports:

any?	child?	children	coerce
concat	convert	concat!	copyInto!
construct	copy	count	cycleEntry
cycleLength	cycleSplit!	cycleTail	cyclic?
delete	delete!	distance	elt
empty	empty?	entries	entry?
eq?	eval	every?	explicitlyFinite?
fill!	find	first	hash
index?	indices	insert	insert!
last	latex	leaf?	leaves
less?	list	map	map!
max	maxIndex	member?	members
merge	merge!	min	minIndex
more?	new	node?	nodes
parts	position	possiblyInfinite?	qelt
qsetelt!	reduce	remove	remove!
removeDuplicates	removeDuplicates!	rest	reverse
reverse!	sample	second	select
select!	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?
sort	sort!	sorted?	split!
swap!	tail	third	value
#?	?<?	?<=?	?=?
?>?	?>=?	?~=?	?.?
?.last	?.rest	?.first	?.value

— domain ILIST IndexedList —

```
)abbrev domain ILIST IndexedList
++ Author: Michael Monagan
++ Date Created: Sep 1987
++ Change History:
++ Basic Operations:
++   \#, concat, concat!, construct, copy, elt, elt, empty,
++   empty?, eq?, first, member?, merge!, mergeSort, minIndex,
++   parts, removeDuplicates!, rest, rest, reverse, reverse!,
++   setelt, setfirst!, setrest!, sort!, split!
++ Related Constructors: List
++ Also See:
++ AMS Classification:
++ Keywords: list, aggregate, index
++ Description:
++ \spadtype{IndexedList} is a basic implementation of the functions
++ in \spadtype{ListAggregate}, often using functions in the underlying
++ LISP system. The second parameter to the constructor (\spad{mn})
++ is the beginning index of the list. That is, if \spad{l} is a
++ list, then \spad{elt(l,mn)} is the first value. This constructor
```

```
++ is probably best viewed as the implementation of singly-linked
++ lists that are addressable by index rather than as a mere wrapper
++ for LISP lists.

IndexedList(S:Type, mn:Integer): Exports == Implementation where
 cycleMax ==> 1000        -- value used in checking for cycles

-- The following seems to be a bit out of date, but is kept in case
-- a knowledgeable person wants to update it:
--   The following LISP dependencies are divided into two groups
--   Those that are required
--   CONS, EQ, NIL, NULL, QCAR, QCDR, RPLACA, RPLACD
--   Those that are included for efficiency only
--   NEQ, LIST, CAR, CDR, NCONC2, NREVERSE, LENGTH
--   Also REVERSE, since it's called in Polynomial Ring

 Qfirst  ==> QCAR$Lisp
 Qrest   ==> QCDR$Lisp
 Qnull   ==> NULL$Lisp
 Qeq     ==> EQ$Lisp
 Qneq    ==> NEQ$Lisp
 Qcons   ==> CONS$Lisp
 Qpush   ==> PUSH$Lisp

 Exports ==> ListAggregate S
 Implementation ==>
  add
   #x                 == LENGTH(x)$Lisp
   concat(s:S,x:%)    == CONS(s,x)$Lisp
   eq?(x,y)           == EQ(x,y)$Lisp
   first x            == SPADfirst(x)$Lisp
   elt(x,"first")     == SPADfirst(x)$Lisp
   empty()            == NIL$Lisp
   empty? x           == NULL(x)$Lisp
   rest x             == CDR(x)$Lisp
   elt(x,"rest")      == CDR(x)$Lisp
   setfirst_!(x,s)    ==
      empty? x => error "Cannot update an empty list"
      Qfirst RPLACA(x,s)$Lisp
   setelt(x,"first",s) ==
      empty? x => error "Cannot update an empty list"
      Qfirst RPLACA(x,s)$Lisp
   setrest_!(x,y)     ==
      empty? x => error "Cannot update an empty list"
      Qrest RPLACD(x,y)$Lisp
   setelt(x,"rest",y)  ==
      empty? x => error "Cannot update an empty list"
      Qrest RPLACD(x,y)$Lisp
   construct l        == l pretend %
   parts s            == s pretend List S
```

```
reverse_! x          == NREVERSE(x)$Lisp
reverse x            == REVERSE(x)$Lisp
minIndex x           == mn

rest(x, n) ==
   for i in 1..n repeat
      if Qnull x then error "index out of range"
      x := Qrest x
   x

copy x ==
   y := empty()
   for i in 0.. while not Qnull x repeat
      if Qeq(i,cycleMax) and cyclic? x then error "cyclic list"
      y := Qcons(Qfirst x,y)
      x := Qrest x
   (NREVERSE(y)$Lisp)@%

if S has SetCategory then
  coerce(x):OutputForm ==
     -- displays cycle with overbar over the cycle
     y := empty()$List(OutputForm)
     s := cycleEntry x
     while Qneq(x, s) repeat
       y := concat((first x)::OutputForm, y)
       x := rest x
     y := reverse_! y
     empty? s => bracket y
     -- cyclic case: z is cylic part
     z := list((first x)::OutputForm)
     while Qneq(s, rest x) repeat
        x := rest x
        z := concat((first x)::OutputForm, z)
     bracket concat_!(y, overbar commaSeparate reverse_! z)

  x = y ==
    Qeq(x,y) => true
    while not Qnull x and not Qnull y repeat
       Qfirst x ^=$S Qfirst y => return false
       x := Qrest x
       y := Qrest y
    Qnull x and Qnull y

  latex(x : %): String ==
    s : String := "\left["
    while not Qnull x repeat
      s := concat(s, latex(Qfirst x)$S)$String
      x := Qrest x
      if not Qnull x then s := concat(s, ", ")$String
    concat(s, " \right]")$String
```

```
     member?(s,x) ==
        while not Qnull x repeat
           if s = Qfirst x then return true else x := Qrest x
        false

   -- Lots of code from parts of AGGCAT, repeated here to
   -- get faster compilation
   concat_!(x:%,y:%) ==
      Qnull x =>
        Qnull y => x
        Qpush(first y,x)
        QRPLACD(x,rest y)$Lisp
        x
      z:=x
      while not Qnull Qrest z repeat
        z:=Qrest z
      QRPLACD(z,y)$Lisp
      x

   -- Then a quicky:
   if S has SetCategory then
     removeDuplicates_! l ==
       p := l
       while not Qnull p repeat
--       p := setrest_!(p, remove_!(#1 = Qfirst p, Qrest p))
-- far too expensive - builds closures etc.
         pp:=p
         f:S:=Qfirst p
         p:=Qrest p
         while not Qnull (pr:=Qrest pp) repeat
           if (Qfirst pr)@S = f then QRPLACD(pp,Qrest pr)$Lisp
           else pp:=pr
       l

   -- then sorting
   mergeSort: ((S, S) -> Boolean, %, Integer) -> %

   sort_!(f, l)       == mergeSort(f, l, #l)

   merge_!(f, p, q) ==
     Qnull p => q
     Qnull q => p
     Qeq(p, q) => error "cannot merge a list into itself"
     if f(Qfirst p, Qfirst q)
       then (r := t := p; p := Qrest p)
       else (r := t := q; q := Qrest q)
     while not Qnull p and not Qnull q repeat
       if f(Qfirst p, Qfirst q)
         then (QRPLACD(t, p)$Lisp; t := p; p := Qrest p)
```

```
        else (QRPLACD(t, q)$Lisp; t := q; q := Qrest q)
    QRPLACD(t, if Qnull p then q else p)$Lisp
    r

  split_!(p, n) ==
     n < 1 => error "index out of range"
     p := rest(p, (n - 1)::NonNegativeInteger)
     q := Qrest p
     QRPLACD(p, NIL$Lisp)$Lisp
     q

  mergeSort(f, p, n) ==
    if n = 2 and f(first rest p, first p) then p := reverse_! p
    n < 3 => p
    l := (n quo 2)::NonNegativeInteger
    q := split_!(p, l)
    p := mergeSort(f, p, l)
    q := mergeSort(f, q, n - l)
    merge_!(f, p, q)
```

— **ILIST.dotabb** —

```
"ILIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ILIST",
          shape=ellipse]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"ILIST" -> "STRING"
```

10.12 domain IMATRIX IndexedMatrix

— **IndexedMatrix.input** —

```
)set break resume
)sys rm -f IndexedMatrix.output
)spool IndexedMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedMatrix
```

```
--R IndexedMatrix(R: Ring,mnRow: Integer,mnCol: Integer)  is a domain constructor
--R Abbreviation for IndexedMatrix is IMATRIX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IMATRIX
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %              ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                    ?*? : (%,%) -> %
--R ?+? : (%,%) -> %                    -? : % -> %
--R ?-? : (%,%) -> %                    antisymmetric? : % -> Boolean
--R copy : % -> %                       diagonal? : % -> Boolean
--R diagonalMatrix : List % -> %        diagonalMatrix : List R -> %
--R elt : (%,Integer,Integer,R) -> R    elt : (%,Integer,Integer) -> R
--R empty : () -> %                     empty? : % -> Boolean
--R eq? : (%,%) -> Boolean              fill! : (%,R) -> %
--R horizConcat : (%,%) -> %            listOfLists : % -> List List R
--R map : (((R,R) -> R),%,%,R) -> %     map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %             map! : ((R -> R),%) -> %
--R matrix : List List R -> %           maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer          minColIndex : % -> Integer
--R minRowIndex : % -> Integer          ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger     parts : % -> List R
--R qelt : (%,Integer,Integer) -> R     sample : () -> %
--R square? : % -> Boolean              squareTop : % -> %
--R symmetric? : % -> Boolean           transpose : % -> %
--R vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (IndexedVector(R,mnCol),%) -> IndexedVector(R,mnCol)
--R ?*? : (%,IndexedVector(R,mnRow)) -> IndexedVector(R,mnRow)
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : IndexedVector(R,mnRow) -> %
--R coerce : % -> OutputForm if R has SETCAT
--R column : (%,Integer) -> IndexedVector(R,mnRow)
--R columnSpace : % -> List IndexedVector(R,mnRow) if R has EUCDOM
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R elt : (%,List Integer,List Integer) -> %
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R hash : % -> SingleInteger if R has SETCAT
--R inverse : % -> Union(%,"failed") if R has FIELD
```

```
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R nullSpace : % -> List IndexedVector(R,mnRow) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R pfaffian : % -> R if R has COMRING
--R qsetelt! : (%,Integer,Integer,R) -> R
--R rank : % -> NonNegativeInteger if R has INTDOM
--R row : (%,Integer) -> IndexedVector(R,mnCol)
--R rowEchelon : % -> % if R has EUCDOM
--R scalarMatrix : (NonNegativeInteger,R) -> %
--R setColumn! : (%,Integer,IndexedVector(R,mnRow)) -> %
--R setRow! : (%,Integer,IndexedVector(R,mnCol)) -> %
--R setelt : (%,List Integer,List Integer,%) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R setsubMatrix! : (%,Integer,Integer,%) -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R subMatrix : (%,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (%,Integer,Integer) -> %
--R swapRows! : (%,Integer,Integer) -> %
--R transpose : IndexedVector(R,mnCol) -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

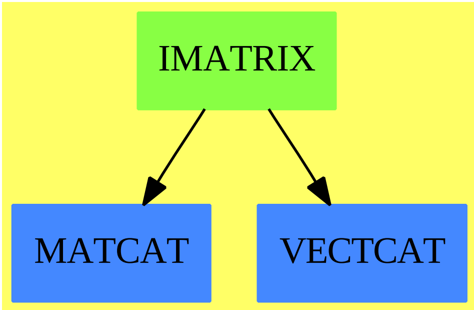
— IndexedMatrix.help —

```
====================================================================
IndexedMatrix examples
====================================================================

See Also:
o )show IndexedMatrix
```

———-

10.12.1 IndexedMatrix (IMATRIX)



See

⇒ "Matrix" (MATRIX) 14.7.1 on page 1586
⇒ "RectangularMatrix" (RMATRIX) 19.4.1 on page 2205
⇒ "SquareMatrix" (SQMATRIX) 20.27.1 on page 2505

Exports:

any?	antisymmetric?	coerce	column	copy
count	determinant	diagonal?	diagonalMatrix	elt
empty	empty?	eq?	eval	every?
exquo	fill!	hash	horizConcat	inverse
latex	less?	listOfLists	map	map!
matrix	maxColIndex	maxRowIndex	member?	members
minColIndex	minordet	minRowIndex	more?	ncols
new	nrows	nullSpace	nullity	parts
qelt	qsetelt!	rank	row	rowEchelon
sample	scalarMatrix	setColumn!	setRow!	setelt
setsubMatrix!	size?	square?	squareTop	subMatrix
swapColumns!	swapRows!	symmetric?	transpose	vertConcat
zero	#?	?*?	?**?	?/?
?=?	?~=?	?+?	-?	?-?

— domain IMATRIX IndexedMatrix —

```
)abbrev domain IMATRIX IndexedMatrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: Matrix, RectangularMatrix, SquareMatrix,
++   StorageEfficientMatrixOperations
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
```

```
++ Description:
++ An \spad{IndexedMatrix} is a matrix where the minimal row and column
++ indices are parameters of the type.  The domains Row and Col
++ are both IndexedVectors.
++ The index of the 'first' row may be obtained by calling the
++ function \spadfun{minRowIndex}.  The index of the 'first' column may
++ be obtained by calling the function \spadfun{minColIndex}.  The index of
++ the first element of a 'Row' is the same as the index of the
++ first column in a matrix and vice versa.

IndexedMatrix(R,mnRow,mnCol): Exports == Implementation where
  R : Ring
  mnRow, mnCol : Integer
  Row ==> IndexedVector(R,mnCol)
  Col ==> IndexedVector(R,mnRow)
  MATLIN ==> MatrixLinearAlgebraFunctions(R,Row,Col,$)

  Exports ==> MatrixCategory(R,Row,Col)

  Implementation ==>
    InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col) add

      swapRows_!(x,i1,i2) ==
        (i1 < minRowIndex(x)) or (i1 > maxRowIndex(x)) or _
           (i2 < minRowIndex(x)) or (i2 > maxRowIndex(x)) =>
             error "swapRows!: index out of range"
        i1 = i2 => x
        minRow := minRowIndex x
        xx := x pretend PrimitiveArray(PrimitiveArray(R))
        n1 := i1 - minRow; n2 := i2 - minRow
        row1 := qelt(xx,n1)
        qsetelt_!(xx,n1,qelt(xx,n2))
        qsetelt_!(xx,n2,row1)
        xx pretend $

      if R has commutative("*") then

        determinant x == determinant(x)$MATLIN
        minordet    x == minordet(x)$MATLIN

      if R has EuclideanDomain then

        rowEchelon  x == rowEchelon(x)$MATLIN

      if R has IntegralDomain then

        rank        x == rank(x)$MATLIN
        nullity     x == nullity(x)$MATLIN
        nullSpace   x == nullSpace(x)$MATLIN
```

```
    if R has Field then

      inverse     x == inverse(x)$MATLIN
```

———-

— IMATRIX.dotabb —

```
"IMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IMATRIX"]
"MATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=MATCAT"]
"VECTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=VECTCAT"]
"IMATRIX" -> "MATCAT"
"IMATRIX" -> "VECTCAT"
```

———-

10.13 domain IARRAY1 IndexedOneDimensionalArray

— IndexedOneDimensionalArray.input —

```
)set break resume
)sys rm -f IndexedOneDimensionalArray.output
)spool IndexedOneDimensionalArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedOneDimensionalArray
--R IndexedOneDimensionalArray(S: Type,mn: Integer)  is a domain constructor
--R Abbreviation for IndexedOneDimensionalArray is IARRAY1
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IARRAY1
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R concat : (S,%) -> %                   concat : (%,S) -> %
--R construct : List S -> %               copy : % -> %
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List S
--R eq? : (%,%) -> Boolean                index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %           map : (((S,S) -> S),%,%) -> %
```

```
--R map : ((S -> S),%) -> %               new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S               reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
```

```
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

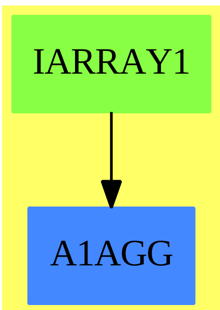
— IndexedOneDimensionalArray.help —

```
====================================================================
IndexedOneDimensionalArray examples
====================================================================

See Also:
o )show IndexedOneDimensionalArray
```

———-

10.13.1 IndexedOneDimensionalArray (IARRAY1)



See

⇒ "PrimitiveArray" (PRIMARR) 17.30.1 on page 2069
⇒ "Tuple" (TUPLE) 21.12.1 on page 2711
⇒ "IndexedFlexibleArray" (IFARRAY) 10.10.1 on page 1187
⇒ "FlexibleArray" (FARRAY) 7.14.1 on page 853
⇒ "OneDimensionalArray" (ARRAY1) 16.3.1 on page 1736

Exports:

concat	construct	copy	delete	elt
empty	empty?	entries	eq?	index?
indices	insert	insert	map	map
new	qelt	reverse	sample	any?
coerce	convert	copyInto!	count	count
delete	entry?	eval	eval	eval
eval	every?	fill!	find	first
hash	latex	less?	map!	max
maxIndex	member?	members	merge	merge
min	minIndex	more?	parts	position
position	position	qsetelt!	reduce	reduce
reduce	remove	remove	removeDuplicates	reverse!
select	setelt	setelt	size?	sort
sort	sort!	sort!	sorted?	sorted?
swap!	#?	?<?	?<=?	?=?
?>?	?>=?	?~=?	?.?	

— domain IARRAY1 IndexedOneDimensionalArray —

```
)abbrev domain IARRAY1 IndexedOneDimensionalArray
++ Author Micheal Monagan Aug/87
++ Description:
++ This is the basic one dimensional array data type.

IndexedOneDimensionalArray(S:Type, mn:Integer):
 OneDimensionalArrayAggregate S == add
   Qmax ==> QVMAXINDEX$Lisp
   Qsize ==> QVSIZE$Lisp
--   Qelt ==> QVELT$Lisp
--   Qsetelt ==> QSETVELT$Lisp
   Qelt ==> ELT$Lisp
   Qsetelt ==> SETELT$Lisp
--   Qelt1 ==> QVELT_-1$Lisp
--   Qsetelt1 ==> QSETVELT_-1$Lisp
   Qnew ==> MAKE_-ARRAY$Lisp
   I ==> Integer

   #x              == Qsize x
   fill_!(x, s)    == (for i in 0..Qmax x repeat Qsetelt(x, i, s); x)
   minIndex x      == mn

   empty()         == Qnew(0$Lisp)
```

```
   new(n, s)        == fill_!(Qnew n,s)

   map_!(f, s1)  ==
      n:Integer := Qmax(s1)
      n < 0 => s1
      for i in 0..n repeat Qsetelt(s1, i, f(Qelt(s1,i)))
      s1

   map(f, s1)       ==
      n:Integer := Qmax(s1)
      n < 0 => s1
      ss2:% := Qnew(n+1)
      for i in 0..n repeat Qsetelt(ss2, i, f(Qelt(s1,i)))
      ss2

   map(f, a, b)   ==
      maxind:Integer := min(Qmax a, Qmax b)
      maxind < 0 => empty()
      c:% := Qnew(maxind+1)
      for i in 0..maxind repeat
        Qsetelt(c, i, f(Qelt(a,i),Qelt(b,i)))
      c

   if zero? mn then
     qelt(x, i)       == Qelt(x, i)
     qsetelt_!(x, i, s) == Qsetelt(x, i, s)

     elt(x:%, i:I) ==
       negative? i or i > maxIndex(x) => error "index out of range"
       qelt(x, i)

     setelt(x:%, i:I, s:S) ==
       negative? i or i > maxIndex(x) => error "index out of range"
       qsetelt_!(x, i, s)

--   else if one? mn then
   else if (mn = 1) then
     maxIndex x       == Qsize x
     qelt(x, i)       == Qelt(x, i-1)
     qsetelt_!(x, i, s) == Qsetelt(x, i-1, s)

     elt(x:%, i:I) ==
       QSLESSP(i,1$Lisp)$Lisp or QSLESSP(Qsize x,i)$Lisp =>
         error "index out of range"
       Qelt(x, i-1)

     setelt(x:%, i:I, s:S) ==
       QSLESSP(i,1$Lisp)$Lisp or QSLESSP(Qsize x,i)$Lisp =>
         error "index out of range"
       Qsetelt(x, i-1, s)
```

```
    else
       qelt(x, i)       == Qelt(x, i - mn)
       qsetelt_!(x, i, s) == Qsetelt(x, i - mn, s)

       elt(x:%, i:I) ==
         i < mn or i > maxIndex(x) => error "index out of range"
         qelt(x, i)

       setelt(x:%, i:I, s:S) ==
         i < mn or i > maxIndex(x) => error "index out of range"
         qsetelt_!(x, i, s)
```

— **IARRAY1.dotabb** —

```
"IARRAY1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IARRAY1"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"IARRAY1" -> "A1AGG"
```

10.14 domain ISTRING IndexedString

— **IndexedString.input** —

```
)set break resume
)sys rm -f IndexedString.output
)spool IndexedString.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedString
--R IndexedString mn: Integer  is a domain constructor
--R Abbreviation for IndexedString is ISTRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ISTRING
--R
--R------------------------------- Operations --------------------------------
--R coerce : Character -> %              concat : List % -> %
--R concat : (%,%) -> %                  concat : (Character,%) -> %
```

```
--R concat : (%,Character) -> %          construct : List Character -> %
--R copy : % -> %                        delete : (%,Integer) -> %
--R ?.? : (%,%) -> %                     ?.? : (%,Integer) -> Character
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List Character        eq? : (%,%) -> Boolean
--R hash : % -> Integer                   index? : (Integer,%) -> Boolean
--R indices : % -> List Integer          insert : (%,%,Integer) -> %
--R leftTrim : (%,Character) -> %        lowerCase : % -> %
--R lowerCase! : % -> %                  prefix? : (%,%) -> Boolean
--R qelt : (%,Integer) -> Character      reverse : % -> %
--R rightTrim : (%,Character) -> %       sample : () -> %
--R split : (%,Character) -> List %      suffix? : (%,%) -> Boolean
--R trim : (%,CharacterClass) -> %       trim : (%,Character) -> %
--R upperCase : % -> %                   upperCase! : % -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Character has ORDSET
--R ?<=? : (%,%) -> Boolean if Character has ORDSET
--R ?=? : (%,%) -> Boolean if Character has SETCAT
--R ?>? : (%,%) -> Boolean if Character has ORDSET
--R ?>=? : (%,%) -> Boolean if Character has ORDSET
--R any? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Character has SETCAT
--R convert : % -> InputForm if Character has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Character,%) -> NonNegativeInteger if $ has finiteAggregate and Character has SE
--R count : ((Character -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Character) -> Character
--R entry? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R eval : (%,List Character,List Character) -> % if Character has EVALAB CHAR and Character
--R eval : (%,Character,Character) -> % if Character has EVALAB CHAR and Character has SETCAT
--R eval : (%,Equation Character) -> % if Character has EVALAB CHAR and Character has SETCAT
--R eval : (%,List Equation Character) -> % if Character has EVALAB CHAR and Character has SE
--R every? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Character) -> % if $ has shallowlyMutable
--R find : ((Character -> Boolean),%) -> Union(Character,"failed")
--R first : % -> Character if Integer has ORDSET
--R hash : % -> SingleInteger if Character has SETCAT
--R insert : (Character,%,Integer) -> %
--R latex : % -> String if Character has SETCAT
--R leftTrim : (%,CharacterClass) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Character,Character) -> Character),%,%) -> %
--R map : ((Character -> Character),%) -> %
--R map! : ((Character -> Character),%) -> % if $ has shallowlyMutable
--R match : (%,%,Character) -> NonNegativeInteger
--R match? : (%,%,Character) -> Boolean
--R max : (%,%) -> % if Character has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
```

```
--R member? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R members : % -> List Character if $ has finiteAggregate
--R merge : (%,%) -> % if Character has ORDSET
--R merge : (((Character,Character) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Character has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Character) -> %
--R parts : % -> List Character if $ has finiteAggregate
--R position : (CharacterClass,%,Integer) -> Integer
--R position : (%,%,Integer) -> Integer
--R position : (Character,%,Integer) -> Integer if Character has SETCAT
--R position : (Character,%) -> Integer if Character has SETCAT
--R position : ((Character -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R reduce : (((Character,Character) -> Character),%) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character,Character) -> Character if $ has finiteAg
--R remove : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Character,%) -> % if $ has finiteAggregate and Character has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and Character has SETCAT
--R replace : (%,UniversalSegment Integer,%) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R rightTrim : (%,CharacterClass) -> %
--R select : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Character) -> Character if $ has shallowlyMutable
--R setelt : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Character has ORDSET
--R sort : (((Character,Character) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and Character has ORDSET
--R sort! : (((Character,Character) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Character has ORDSET
--R sorted? : (((Character,Character) -> Boolean),%) -> Boolean
--R split : (%,CharacterClass) -> List %
--R substring? : (%,%,Integer) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Character has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———

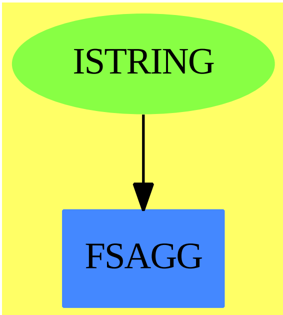
— **IndexedString.help** —

```
====================================================================
```

```
IndexedString examples
====================================================================

See Also:
o )show IndexedString
```

10.14.1 IndexedString (ISTRING)



See

⇒ “Character” (CHAR) 4.3.1 on page 357
⇒ “CharacterClass” (CCLASS) 4.4.1 on page 365
⇒ “String” (STRING) 20.31.1 on page 2565

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	prefix?	position
qelt	qsetelt!	reduce	remove	removeDuplicates
replace	reverse	reverse!	rightTrim	sample
select	setelt	size?	sort	sort!
sorted?	split	suffix?	substring?	swap!
trim	upperCase	upperCase!	#?	?<?
?<=?	?=?	?>?	?>=?	?~=?
?.?				

— domain ISTRING IndexedString —

```
)abbrev domain ISTRING IndexedString
++ Authors: Stephen Watt, Michael Monagan, Manuel Bronstein 1986 .. 1991
-- The following Lisp dependencies are divided into two groups
-- Those that are required
-- QENUM QESET QCSIZE MAKE-FULL-CVEC EQ QSLESSP QSGREATERP
-- Those that can are included for efficiency only
-- COPY STRCONC SUBSTRING STRPOS RPLACSTR DOWNCASE UPCASE CGREATERP
++ Description:
++ This domain implements low-level strings

IndexedString(mn:Integer): Export == Implementation where
  B ==> Boolean
  C ==> Character
  I ==> Integer
  N ==> NonNegativeInteger
  U ==> UniversalSegment Integer

  Export ==> StringAggregate() with
      hash: % -> I
        ++ hash(x) provides a hashing function for strings

  Implementation ==> add
    -- These assume Character's Rep is Small I
    Qelt    ==> QENUM$Lisp
    Qequal  ==> EQUAL$Lisp
    Qsetelt ==> QESET$Lisp
    Qsize   ==> QCSIZE$Lisp
    Cheq    ==> EQL$Lisp
    Chlt    ==> QSLESSP$Lisp
    Chgt    ==> QSGREATERP$Lisp

    c: Character
    cc: CharacterClass

--  new n                  == MAKE_-FULL_-CVEC(n, space$C)$Lisp
    new(n, c)              == MAKE_-FULL_-CVEC(n, c)$Lisp
    empty()                == MAKE_-FULL_-CVEC(0$Lisp)$Lisp
    empty?(s)              == Qsize(s) = 0
    #s                     == Qsize(s)
    s = t                  == Qequal(s, t)
    s < t                  == CGREATERP(t,s)$Lisp
    concat(s:%,t:%)        == STRCONC(s,t)$Lisp
    copy s                 == COPY_-SEQ(s)$Lisp
    insert(s:%, t:%, i:I)  == concat(concat(s(mn..i-1), t), s(i..))
    coerce(s:%):OutputForm == outputForm(s pretend String)
    minIndex s             == mn
    upperCase_! s          == map_!(upperCase, s)
    lowerCase_! s          == map_!(lowerCase, s)

    latex s             == concat("\mbox{``", concat(s pretend String, "''}"))
```

```
replace(s, sg, t) ==
  l := lo(sg) - mn
  m := #s
  n := #t
  h:I := if hasHi sg then hi(sg) - mn else maxIndex s - mn
  l < 0 or h >= m or h < l-1 => error "index out of range"
  r := new((m-(h-l+1)+n)::N, space$C)
  for k in 0.. for i in 0..l-1 repeat Qsetelt(r, k, Qelt(s, i))
  for k in k.. for i in 0..n-1 repeat Qsetelt(r, k, Qelt(t, i))
  for k in k.. for i in h+1..m-1 repeat Qsetelt(r, k, Qelt(s, i))
  r

setelt(s:%, i:I, c:C) ==
  i < mn or i > maxIndex(s) => error "index out of range"
  Qsetelt(s, i - mn, c)
  c

substring?(part, whole, startpos) ==
  np:I := Qsize part
  nw:I := Qsize whole
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  np > nw - startpos => false
  for ip in 0..np-1 for iw in startpos.. repeat
      not Cheq(Qelt(part, ip), Qelt(whole, iw)) => return false
  true

position(s:%, t:%, startpos:I) ==
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  startpos >= Qsize t => mn - 1
  r:I := STRPOS(s, t, startpos, NIL$Lisp)$Lisp
  EQ(r, NIL$Lisp)$Lisp => mn - 1
  r + mn

position(c: Character, t: %, startpos: I) ==
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  startpos >= Qsize t => mn - 1
  for r in startpos..Qsize t - 1 repeat
      if Cheq(Qelt(t, r), c) then return r + mn
  mn - 1

position(cc: CharacterClass, t: %, startpos: I) ==
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  startpos >= Qsize t => mn - 1
  for r in startpos..Qsize t - 1 repeat
      if member?(Qelt(t,r), cc) then return r + mn
  mn - 1

suffix?(s, t) ==
  (m := maxIndex s) > (n := maxIndex t) => false
```

```
    substring?(s, t, mn + n - m)

  split(s, c) ==
    n := maxIndex s
    for i in mn..n while s.i = c repeat 0
    l := empty()$List(%)
    j:Integer -- j is conditionally intialized
    while i <= n and (j := position(c, s, i)) >= mn repeat
        l := concat(s(i..j-1), l)
        for i in j..n while s.i = c repeat 0
    if i <= n then l := concat(s(i..n), l)
    reverse_! l

  split(s, cc) ==
    n := maxIndex s
    for i in mn..n while member?(s.i,cc) repeat 0
    l := empty()$List(%)
    j:Integer -- j is conditionally intialized
    while i <= n and (j := position(cc, s, i)) >= mn repeat
        l := concat(s(i..j-1), l)
        for i in j..n while member?(s.i,cc) repeat 0
    if i <= n then l := concat(s(i..n), l)
    reverse_! l

  leftTrim(s, c) ==
    n := maxIndex s
    for i in mn .. n while s.i = c repeat 0
    s(i..n)

  leftTrim(s, cc) ==
    n := maxIndex s
    for i in mn .. n while member?(s.i,cc) repeat 0
    s(i..n)

  rightTrim(s, c) ==
    for j in maxIndex s .. mn by -1 while s.j = c repeat 0
    s(minIndex(s)..j)

  rightTrim(s, cc) ==
    for j in maxIndex s .. mn by -1 while member?(s.j, cc) repeat 0
    s(minIndex(s)..j)

  concat l ==
    t := new(+/[#s for s in l], space$C)
    i := mn
    for s in l repeat
        copyInto_!(t, s, i)
        i := i + #s
    t
```

```
    copyInto_!(y, x, s) ==
      m := #x
      n := #y
      s := s - mn
      s < 0 or s+m > n => error "index out of range"
      RPLACSTR(y, s, m, x, 0, m)$Lisp
      y

    elt(s:%, i:I) ==
      i < mn or i > maxIndex(s) => error "index out of range"
      Qelt(s, i - mn)

    elt(s:%, sg:U) ==
      l := lo(sg) - mn
      h := if hasHi sg then hi(sg) - mn else maxIndex s - mn
      l < 0 or h >= #s => error "index out of bound"
      SUBSTRING(s, l, max(0, h-l+1))$Lisp

    hash(s:$):Integer ==
      n:I := Qsize s
      zero? n => 0
--      one? n => ord(s.mn)
      (n = 1) => ord(s.mn)
      ord(s.mn) * ord s(mn+n-1) * ord s(mn + n quo 2)

    match(pattern,target,wildcard) ==
      stringMatch(pattern,target,CHARACTER(wildcard)$Lisp)$Lisp

    match?(pattern, target, dontcare) ==
      n := maxIndex pattern
      p := position(dontcare, pattern, m := minIndex pattern)::N
      p = m-1 => pattern = target
      (p ^= m) and not prefix?(pattern(m..p-1), target) => false
      i := p      -- index into target
      q := position(dontcare, pattern, p + 1)::N
      while q ^= m-1 repeat
         s := pattern(p+1..q-1)
         i := position(s, target, i)::N
         i = m-1 => return false
         i := i + #s
         p := q
         q := position(dontcare, pattern, q + 1)::N
      (p ^= n) and not suffix?(pattern(p+1..n), target) => false
      true
```

———-

— **ISTRING.dotabb** —

```
"ISTRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ISTRING",
          shape=ellipse]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"ISTRING" -> "FSAGG"
```

10.15 domain IARRAY2 IndexedTwoDimensionalArray

An IndexedTwoDimensionalArray is a 2-dimensional array where the minimal row and column indices are parameters of the type. Rows and columns are returned as IndexedOneDimensionalArray's with minimal indices matching those of the IndexedTwoDimensionalArray. The index of the 'first' row may be obtained by calling the function 'minRowIndex'. The index of the 'first' column may be obtained by calling the function 'minColIndex'. The index of the first element of a 'Row' is the same as the index of the first column in an array and vice versa.

— IndexedTwoDimensionalArray.input —

```
)set break resume
)sys rm -f IndexedTwoDimensionalArray.output
)spool IndexedTwoDimensionalArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedTwoDimensionalArray
--R IndexedTwoDimensionalArray(R: Type,mnRow: Integer,mnCol: Integer)  is a domain constructor
--R Abbreviation for IndexedTwoDimensionalArray is IARRAY2
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IARRAY2
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         elt : (%,Integer,Integer,R) -> R
--R elt : (%,Integer,Integer) -> R        empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R fill! : (%,R) -> %                    map : (((R,R) -> R),%,%,R) -> %
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R map! : ((R -> R),%) -> %              maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       parts : % -> List R
--R qelt : (%,Integer,Integer) -> R       sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if R has SETCAT
```

```
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R column : (%,Integer) -> IndexedOneDimensionalArray(R,mnRow)
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R qsetelt! : (%,Integer,Integer,R) -> R
--R row : (%,Integer) -> IndexedOneDimensionalArray(R,mnCol)
--R setColumn! : (%,Integer,IndexedOneDimensionalArray(R,mnRow)) -> %
--R setRow! : (%,Integer,IndexedOneDimensionalArray(R,mnCol)) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

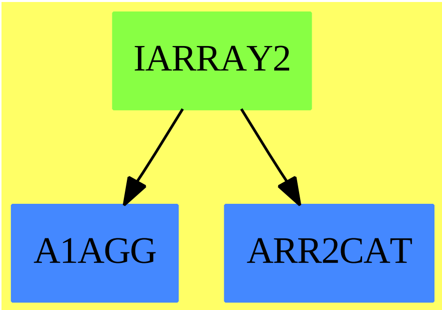
— **IndexedTwoDimensionalArray.help** —

```
====================================================================
IndexedTwoDimensionalArray examples
====================================================================

See Also:
o )show IndexedTwoDimensionalArray
```

———-

10.15.1 IndexedTwoDimensionalArray (IARRAY2)



See

⇒ "InnerIndexedTwoDimensionalArray" (IIARRAY2) 10.23.1 on page 1254
⇒ "TwoDimensionalArray" (ARRAY2) 21.13.1 on page 2722

Exports:

any?	coerce	column	copy	count
count	elt	empty	empty?	eq?
eval	every?	fill!	hash	latex
less?	maxColIndex	maxRowIndex	map	map!
member?	members	minColIndex	minRowIndex	more?
ncols	new	nrows	parts	qelt
qsetelt!	row	sample	setColumn!	setRow!
setelt	size?	#?	?=?	?~=?

— **domain IARRAY2 IndexedTwoDimensionalArray** —

```
)abbrev domain IARRAY2 IndexedTwoDimensionalArray
++ Author: Mark Botch
++ Description:
++ This domain implements two dimensional arrays

IndexedTwoDimensionalArray(R,mnRow,mnCol):Exports == Implementation where
  R : Type
  mnRow, mnCol : Integer
  Row ==> IndexedOneDimensionalArray(R,mnCol)
  Col ==> IndexedOneDimensionalArray(R,mnRow)

  Exports ==> TwoDimensionalArrayCategory(R,Row,Col)

  Implementation ==>
    InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col)
```

———–

— IARRAY2.dotabb —

```
"IARRAY2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IARRAY2"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ARR2CAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ARR2CAT"]
"IARRAY2" -> "ARR2CAT"
"IARRAY2" -> "A1AGG"
```

10.16 domain IVECTOR IndexedVector

— IndexedVector.input —

```
)set break resume
)sys rm -f IndexedVector.output
)spool IndexedVector.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedVector
--R IndexedVector(R: Type,mn: Integer)  is a domain constructor
--R Abbreviation for IndexedVector is IVECTOR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IVECTOR
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has ABELGRP          concat : List % -> %
--R concat : (%,%) -> %                   concat : (R,%) -> %
--R concat : (%,R) -> %                   construct : List R -> %
--R copy : % -> %                         delete : (%,Integer) -> %
--R ?.? : (%,Integer) -> R                elt : (%,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List R                 eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           insert : (R,%,Integer) -> %
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R new : (NonNegativeInteger,R) -> %     qelt : (%,Integer) -> R
--R reverse : % -> %                      sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,R) -> % if R has MONOID
--R ?*? : (R,%) -> % if R has MONOID
--R ?*? : (Integer,%) -> % if R has ABELGRP
```

```
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (%,%) -> % if R has RING
--R delete : (%,UniversalSegment Integer) -> %
--R dot : (%,%) -> R if R has RING
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R length : % -> R if R has RADCAT and R has RING
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RADCAT and R has RING
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R min : (%,%) -> % if R has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix R if R has RING
--R parts : % -> List R if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if $ has finiteAggregate and R has SETCAT
```

```
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if $ has finiteAggregate and R has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and R has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and R has ORDSET
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

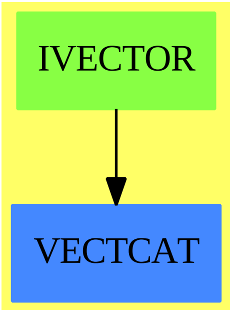
— **IndexedVector.help** —

```
====================================================================
IndexedVector examples
====================================================================

See Also:
o )show IndexedVector
```

———-

10.16.1 IndexedVector (IVECTOR)



Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	cross	delete
dot	elt	empty	empty?	entries
entry?	eq?	eval	every?	fill!
find	first	hash	index?	indices
insert	latex	length	less?	magnitude
map!	max	maxIndex	member?	members
merge	min	minIndex	more?	new
outerProduct	parts	position	qelt	qsetelt!
reduce	remove	removeDuplicates	reverse	reverse!
sample	select	setelt	size?	sort
sort!	sorted?	swap!	zero	#?
?*?	?+?	?-?	?<?	?<=?
?=?	?>?	?>=?	?~=?	-?
?.?				

— domain IVECTOR IndexedVector —

```
)abbrev domain IVECTOR IndexedVector
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents vector like objects with varying lengths
++ and a user-specified initial index.

IndexedVector(R:Type, mn:Integer):
```

```
  VectorCategory R == IndexedOneDimensionalArray(R, mn)
```

———-

— IVECTOR.dotabb —

```
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"VECTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=VECTCAT"]
"IVECTOR" -> "VECTCAT"
```

———-

10.17 domain ITUPLE InfiniteTuple

— InfiniteTuple.input —

```
)set break resume
)sys rm -f InfiniteTuple.output
)spool InfiniteTuple.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfiniteTuple
--R InfiniteTuple S: Type  is a domain constructor
--R Abbreviation for InfiniteTuple is ITUPLE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ITUPLE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              construct : % -> Stream S
--R generate : ((S -> S),S) -> %          map : ((S -> S),%) -> %
--R select : ((S -> Boolean),%) -> %
--R filterUntil : ((S -> Boolean),%) -> %
--R filterWhile : ((S -> Boolean),%) -> %
--R
--E 1

)spool
)lisp (bye)
```

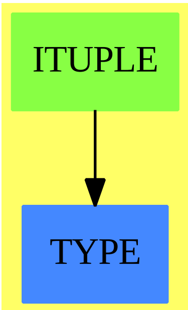
———-

— InfiniteTuple.help —

```
====================================================================
InfiniteTuple examples
====================================================================

See Also:
o )show InfiniteTuple
```

10.17.1 InfiniteTuple (ITUPLE)



Exports:
coerce construct filterUntil filterWhile generate map select

— domain ITUPLE InfiniteTuple —

```
)abbrev domain ITUPLE InfiniteTuple
++ Author: Clifton J. Williamson
++ Date Created: 16 February 1990
++ Date Last Updated: 16 February 1990
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This package implements 'infinite tuples' for the interpreter.
++ The representation is a stream.

InfiniteTuple(S:Type): Exports == Implementation where

  Exports ==> CoercibleTo OutputForm with
    map: (S -> S, %) -> %
```

```
      ++ map(f,t) replaces the tuple t
      ++ by \spad{[f(x) for x in t]}.
    filterWhile: (S -> Boolean, %) -> %
      ++ filterWhile(p,t) returns \spad{[x for x in t while p(x)]}.
    filterUntil: (S -> Boolean, %) -> %
      ++ filterUntil(p,t) returns \spad{[x for x in t while not p(x)]}.
    select: (S -> Boolean, %) -> %
      ++ select(p,t) returns \spad{[x for x in t | p(x)]}.
    generate: (S -> S,S) -> %
      ++ generate(f,s) returns \spad{[s,f(s),f(f(s)),...]}.
    construct: % -> Stream S
      ++ construct(t) converts an infinite tuple to a stream.

  Implementation ==> Stream S add
    generate(f,x) == generate(f,x)$Stream(S) pretend %
    filterWhile(f, x) == filterWhile(f,x pretend Stream(S))$Stream(S) pretend %
    filterUntil(f, x) == filterUntil(f,x pretend Stream(S))$Stream(S) pretend %
    select(f, x) == select(f,x pretend Stream(S))$Stream(S) pretend %
    construct x == x pretend Stream(S)
--    coerce x ==
--      coerce(x)$Stream(S)
```

———-

— ITUPLE.dotabb —

```
"ITUPLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ITUPLE"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"ITUPLE" -> "TYPE"
```

———-

10.18 domain INFCLSPT InfinitlyClosePoint

— InfinitlyClosePoint.input —

```
)set break resume
)sys rm -f InfinitlyClosePoint.output
)spool InfinitlyClosePoint.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show InfinitlyClosePoint
--R InfinitlyClosePoint(K: Field,symb: List Symbol,PolyRing: PolynomialCategory(K,E,OrderedVariableList
--R Abbreviation for InfinitlyClosePoint is INFCLSPT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INFCLSPT
--R
--R------------------------------ Operations --------------------------------
--R ?=? : (%,%) -> Boolean                actualExtensionV : % -> K
--R chartV : % -> BLMET                   coerce : % -> OutputForm
--R create : (ProjPt,PolyRing) -> %       degree : % -> PositiveInteger
--R excpDivV : % -> DIVISOR               fullOut : % -> OutputForm
--R fullOutput : () -> Boolean            fullOutput : Boolean -> Boolean
--R hash : % -> SingleInteger             latex : % -> String
--R localParamV : % -> List PCS           localPointV : % -> AffinePlane K
--R multV : % -> NonNegativeInteger       pointV : % -> ProjPt
--R setchart! : (%,BLMET) -> BLMET        setpoint! : (%,ProjPt) -> ProjPt
--R symbNameV : % -> Symbol               ?~=? : (%,%) -> Boolean
--R create : (ProjPt,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),AffinePlane K,NonNeg
--R curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
--R setcurve! : (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) -> DistributedMultiva
--R setexcpDiv! : (%,DIVISOR) -> DIVISOR
--R setlocalParam! : (%,List PCS) -> List PCS
--R setlocalPoint! : (%,AffinePlane K) -> AffinePlane K
--R setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--R setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--R setsymbName! : (%,Symbol) -> Symbol
--R subMultV : % -> NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

———-

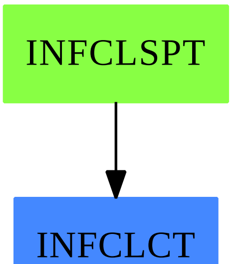
— InfinitlyClosePoint.help —

```
====================================================================
InfinitlyClosePoint examples
====================================================================

See Also:
o )show InfinitlyClosePoint
```

———-

10.18.1 InfinitlyClosePoint (INFCLSPT)



Exports:

?=?	?~=?	actualExtensionV
chartV	coerce	create
curveV	degree	excpDivV
fullOut	fullOutput	fullOutput
hash	latex	localParamV
localPointV	multV	pointV
setchart!	setcurve!	setexcpDiv!
setlocalParam!	setlocalPoint!	setmult!
setpoint!	setsubmult!	setsymbName!
subMultV	symbNameV	

— domain INFCLSPT InfinitlyClosePoint —

```
)abbrev domain INFCLSPT InfinitlyClosePoint
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
InfinitlyClosePoint(K,symb,PolyRing,E,ProjPt,PCS,Plc,DIVISOR,BLMET):Exports == Implementatio
  K:Field
  symb: List Symbol
  E:DirectProductCategory(#symb,NonNegativeInteger)
  OV ==> OrderedVariableList(symb)
  PolyRing:  PolynomialCategory(K,E,OV)

  bls      ==> ['X,'Y]
  BlUpRing ==> DistributedMultivariatePolynomial(bls , K)
  E2       ==> DirectProduct( #bls , NonNegativeInteger )
  outRec   ==> Record(name:Symbol,mult:NonNegativeInteger)
  AFP      ==> AffinePlane(K)
  OV2      ==> OrderedVariableList( bls )

  PCS: LocalPowerSeriesCategory(K)
```

```
ProjPt:ProjectiveSpaceCategory(K)
Plc: PlacesCategory(K,PCS)
DIVISOR: DivisorCategory(Plc)
BLMET : BlowUpMethodCategory

bigoutRecBLQT ==> Record(dominate:ProjPt,_
                     name:Symbol,_
                     mult:NonNegativeInteger,_
                     defCurve:BlUpRing,_
                     localPoint:AFP,_
                     chart:BLMET,_
                     expD:DIVISOR)

bigoutRecHN  ==> Record(dominate:ProjPt,_
                     name:Symbol,_
                     mult:NonNegativeInteger,_
                     defCurve:BlUpRing,_
                     localPoint:AFP,_
                     chart:BLMET,_
                     subMultip: NonNegativeInteger,_
                     expD:DIVISOR)


representation   ==>  Record(point:ProjPt,_
                             curve:BlUpRing,_
                             localPoint:AFP,_
                             mult:NonNegativeInteger,_
                             chrt:BLMET,_
                             subMultiplicity:NonNegativeInteger,_
                             excpDiv:DIVISOR,_
                             localParam:List(PCS),_
                             actualExtension:K,_
                             symbName:Symbol)


Exports == InfinitlyClosePointCategory(K,symb,PolyRing,E,ProjPt,PCS,Plc,DIVISOR,BLMET) with

  fullOut: % -> OutputForm
    ++ fullOut(tr) yields a full output of tr (see function fullOutput).

  fullOutput: Boolean -> Boolean
    ++ fullOutput(b) sets a flag such that when true, a coerce to
    ++ OutputForm yields the full output of tr, otherwise encode(tr) is
    ++ output (see encode function). The default is false.

  fullOutput: () -> Boolean
    ++ fullOutput returns the value of the flag set by fullOutput(b).

Implementation == representation add
  Rep := representation
```

```
polyRing2BiRing: (PolyRing, Integer) -> BlUpRing
polyRing2BiRing(pol,nV)==
  zero? pol => 0$BlUpRing
  d:= degree pol
  lc:= leadingCoefficient pol
  dd: List NonNegativeInteger := entries d
  ddr:=vector([dd.i for i in 1..#dd | ^(i=nV)])$Vector(NonNegativeInteger)
  ddre:E2 := directProduct( ddr )$E2
  monomial(lc,ddre)$BlUpRing  + polyRing2BiRing( reductum pol , nV )

projPt2affPt: (ProjPt, Integer) -> AFP
projPt2affPt(pt,nV)==
  ll:= pt :: List(K)
  l:= [ ll.i for i in 1..#ll | ^(i = nV )]
  affinePoint( l)

fullOut(a)==
  oo: bigoutRecBLQT
  oo2: bigoutRecHN
  BLMET has BlowUpWithQuadTrans =>
    oo:=  [ pointV(a), symbNameV(a), multV(a), curveV(a), _
          localPointV(a), chartV(a),  excpDivV(a) ]$bigoutRecBLQT
    oo :: OutputForm
  BLMET has BlowUpWithHamburgerNoether =>
    oo2:=  [ pointV(a), symbNameV(a), multV(a), curveV(a), _
          localPointV(a), chartV(a), subMultV(a), excpDivV(a) ]$bigoutRecHN
    oo2 :: OutputForm

fullOutputFlag:Boolean:=false()

fullOutput(f)== fullOutputFlag:=f

fullOutput == fullOutputFlag

coerce(a:%):OutputForm==
  fullOutput() => fullOut(a)
  oo:outRec:= [ symbNameV(a) , multV(a) ]$outRec
  oo :: OutputForm

degree(a)==
  K has PseudoAlgebraicClosureOfPerfectFieldCategory  => extDegree actualExtensionV a
  1

create(pointA,curveA,localPointA,multA,chartA,subM,excpDivA,atcL,aName)==  -- CHH
  ([pointA,curveA,localPointA,multA,chartA,subM,excpDivA,empty()$List(PCS),atcL,aName]$Re

create(pointA,curveA)==
  nV := lastNonNul pointA
  localPointA := projPt2affPt(pointA,nV)
```

```
      multA:NonNegativeInteger:=0$NonNegativeInteger
      chartA:BLMET
      if BLMET has QuadraticTransform then chartA:=( [0,0, nV] :: List Integer ) :: BLMET   -- CHH
      if BLMET has HamburgerNoether then
        chartA := createHN( 0,0,nV,0,0,true,"right")   -- A changer le "right"
      excpDivA:DIVISOR:= 0$DIVISOR
      actL:K:=definingField pointA
      aName:Symbol:=new(P)$Symbol
      affCurvA : BlUpRing := polyRing2BiRing(curveA,nV)
      create(pointA,affCurvA,localPointA,multA,chartA,0$NonNegativeInteger,excpDivA,actL,aName)

    subMultV(a:%)== (a:Rep)(subMultiplicity)

    setsubmult_!(a:%,sm:NonNegativeInteger)== (a:Rep)(subMultiplicity) := sm

    pointV(a:%)     ==(a:Rep)(point)

    symbNameV(a:%)     ==(a:Rep)(symbName)

    curveV(a:%)  ==(a:Rep)(curve)

    localPointV(a:%)   ==(a:Rep)(localPoint)

    multV(a:%)    ==(a:Rep)(mult)

    chartV(a:%)   ==(a:Rep)(chrt)  -- CHH

    excpDivV(a:%) ==(a:Rep)(excpDiv)

    localParamV(a:%) ==(a:Rep)(localParam)

    actualExtensionV(a:%) == (a:Rep)(actualExtension)

    setpoint_!(a:%,n:ProjPt)       ==(a:Rep)(point):=n

    setcurve_!(a:%,n:BlUpRing)   ==(a:Rep)(curve):=n

    setlocalPoint_!(a:%,n:AFP)  ==(a:Rep)(localPoint):=n

    setmult_!(a:%,n:NonNegativeInteger)         ==(a:Rep)(mult):=n

    setchart_!(a:%,n:BLMET)  ==(a:Rep)(chrt):=n  -- CHH

    setlocalParam_!(a:%,n:List(PCS)) ==(a:Rep)(localParam):=n

    setexcpDiv_!(a:%,n:DIVISOR) ==(a:Rep)(excpDiv):=n

    setsymbName_!(a:%,n:Symbol) ==(a:Rep)(symbName):=n
```

——

— INFCLSPT.dotabb —

```
"INFCLSPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFCLSPT"]
"INFCLCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=INFCLCT"]
"INFCLSPT" -> "INFCLCT"
```

——

10.19 domain INFCLSPS InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField

— InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField.output
)spool InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField
--R InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField(K: FiniteFieldCategory,symb: L
--R Abbreviation for InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField is INFCLSPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INFCLSPS
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                chartV : % -> BLMET
--R coerce : % -> OutputForm              degree : % -> PositiveInteger
--R fullOut : % -> OutputForm             fullOutput : () -> Boolean
--R fullOutput : Boolean -> Boolean       hash : % -> SingleInteger
--R latex : % -> String                   multV : % -> NonNegativeInteger
--R setchart! : (%,BLMET) -> BLMET        symbNameV : % -> Symbol
--R ?~=? : (%,%) -> Boolean
--R actualExtensionV : % -> PseudoAlgebraicClosureOfFiniteField K
--R create : (ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K,DistributedMultivariat
--R create : (ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K,DistributedMultivariat
--R curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],PseudoAlgebraic
--R excpDivV : % -> Divisor PlacesOverPseudoAlgebraicClosureOfFiniteField K
--R localParamV : % -> List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteFie
--R localPointV : % -> AffinePlane PseudoAlgebraicClosureOfFiniteField K
```

```
--R pointV : % -> ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K
--R setcurve! : (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],PseudoAlgebraicClosureOfF
--R setexcpDiv! : (%,Divisor PlacesOverPseudoAlgebraicClosureOfFiniteField K) -> Divisor PlacesOverPseud
--R setlocalParam! : (%,List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteField K) -> L
--R setlocalPoint! : (%,AffinePlane PseudoAlgebraicClosureOfFiniteField K) -> AffinePlane PseudoAlgebrai
--R setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--R setpoint! : (%,ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K) -> ProjectivePlaneOverPseud
--R setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--R setsymbName! : (%,Symbol) -> Symbol
--R subMultV : % -> NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

———-

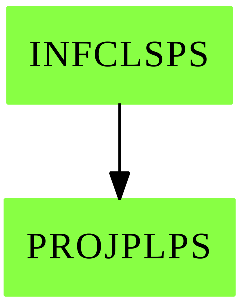
— InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField.help —

```
====================================================================
InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField examples
====================================================================

See Also:
o )show InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField
```

———-

10.19.1 InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField (INFCLSPS)



Exports:

?=?	?~=?	actualExtensionV
chartV	coerce	create
curveV	degree	excpDivV
fullOut	fullOutput	hash
latex	localParamV	localPointV
multV	pointV	setchart!
setcurve!	setexcpDiv!	setlocalParam!
setlocalPoint!	setmult!	setpoint!
setsubmult!	setsymbName!	subMultV
symbNameV		

— domain INFCLSPS InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField —

```
)abbrev domain INFCLSPS InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField(K,symb,BLMET):_
 Exports == Implementation where

  K:FiniteFieldCategory
  symb: List Symbol
  BLMET : BlowUpMethodCategory

  E           ==> DirectProduct(#symb,NonNegativeInteger)
  KK          ==> PseudoAlgebraicClosureOfFiniteField(K)
  PolyRing    ==> DistributedMultivariatePolynomial(symb,KK)
  ProjPt      ==> ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField(K)
  PCS         ==> NeitherSparseOrDensePowerSeries(KK)
  Plc         ==> PlacesOverPseudoAlgebraicClosureOfFiniteField(K)
  DIVISOR     ==> Divisor(Plc)

  Exports == InfinitlyClosePointCategory(KK,symb,PolyRing,E,ProjPt,_
                                        PCS,Plc,DIVISOR,BLMET) with

    fullOut: % -> OutputForm
      ++ fullOut(tr) yields a full output of tr (see function fullOutput).

    fullOutput: Boolean -> Boolean

      ++ fullOutput(b) sets a flag such that when true, a coerce to OutputForm
      ++  yields the full output of tr, otherwise encode(tr) is output
      ++ (see encode function). The default is false.

    fullOutput: () -> Boolean
```

```
    ++ fullOutput returns the value of the flag set by fullOutput(b).

  Implementation == InfinitlyClosePoint(KK,symb,PolyRing,E,ProjPt,_
                                        PCS,Plc,DIVISOR,BLMET)
```

———-

— INFCLSPS.dotabb —

```
"INFCLSPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFCLSPS"]
"PROJPLPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJPLPS"]
"INFCLSPS" -> "PROJPLPS"
```

———-

10.20 domain IAN InnerAlgebraicNumber

— InnerAlgebraicNumber.input —

```
)set break resume
)sys rm -f InnerAlgebraicNumber.output
)spool InnerAlgebraicNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerAlgebraicNumber
--R InnerAlgebraicNumber  is a domain constructor
--R Abbreviation for InnerAlgebraicNumber is IAN
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IAN
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?*? : (Integer,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (%,Fraction Integer) -> %
--R ?*? : (Fraction Integer,%) -> %      ?**? : (%,PositiveInteger) -> %
--R ?**? : (%,Integer) -> %              ?**? : (%,Fraction Integer) -> %
--R ?+? : (%,%) -> %                     -? : % -> %
--R ?-? : (%,%) -> %                     ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              D : % -> %
--R D : (%,NonNegativeInteger) -> %      1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
```

```
--R ?^? : (%,Integer) -> %               associates? : (%,%) -> Boolean
--R belong? : BasicOperator -> Boolean   box : List % -> %
--R box : % -> %                         coerce : Integer -> %
--R coerce : % -> %                      coerce : Fraction Integer -> %
--R coerce : Kernel % -> %               coerce : % -> OutputForm
--R convert : % -> Complex Float         convert : % -> DoubleFloat
--R convert : % -> Float                 differentiate : % -> %
--R distribute : (%,%) -> %              distribute : % -> %
--R elt : (BasicOperator,%,%) -> %       elt : (BasicOperator,%) -> %
--R eval : (%,List %,List %) -> %        eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %           eval : (%,List Equation %) -> %
--R eval : (%,Kernel %,%) -> %           factor : % -> Factored %
--R freeOf? : (%,Symbol) -> Boolean      freeOf? : (%,%) -> Boolean
--R gcd : (%,%) -> %                     gcd : List % -> %
--R hash : % -> SingleInteger            height : % -> NonNegativeInteger
--R inv : % -> %                         is? : (%,Symbol) -> Boolean
--R kernel : (BasicOperator,%) -> %      kernels : % -> List Kernel %
--R latex : % -> String                  lcm : (%,%) -> %
--R lcm : List % -> %                    map : ((% -> %),Kernel %) -> %
--R max : (%,%) -> %                     min : (%,%) -> %
--R norm : (%,List Kernel %) -> %        norm : (%,Kernel %) -> %
--R nthRoot : (%,Integer) -> %           one? : % -> Boolean
--R paren : List % -> %                  paren : % -> %
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       reduce : % -> %
--R ?rem? : (%,%) -> %                   retract : % -> Fraction Integer
--R retract : % -> Integer               retract : % -> Kernel %
--R rootOf : Polynomial % -> %           rootsOf : Polynomial % -> List %
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R sqrt : % -> %                        squareFree : % -> Factored %
--R squareFreePart : % -> %              subst : (%,Equation %) -> %
--R tower : % -> List Kernel %           trueEqual : (%,%) -> Boolean
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 zeroOf : Polynomial % -> %
--R zerosOf : Polynomial % -> List %     ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coerce : SparseMultivariatePolynomial(Integer,Kernel %) -> %
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(Integer,Kernel %)
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
```

```
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
--R eval : (%,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R is? : (%,BasicOperator) -> Boolean
--R kernel : (BasicOperator,List %) -> %
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R norm : (SparseUnivariatePolynomial %,List Kernel %) -> SparseUnivariatePolynomial %
--R norm : (SparseUnivariatePolynomial %,Kernel %) -> SparseUnivariatePolynomial %
--R numer : % -> SparseMultivariatePolynomial(Integer,Kernel %)
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reducedSystem : Matrix % -> Matrix Fraction Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction Integer,vec: Vector Fraction Inte
--R reducedSystem : Matrix % -> Matrix Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R rootOf : SparseUnivariatePolynomial % -> %
--R rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R rootsOf : SparseUnivariatePolynomial % -> List %
--R rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R zeroOf : SparseUnivariatePolynomial % -> %
--R zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R zerosOf : SparseUnivariatePolynomial % -> List %
--R zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R
--E 1

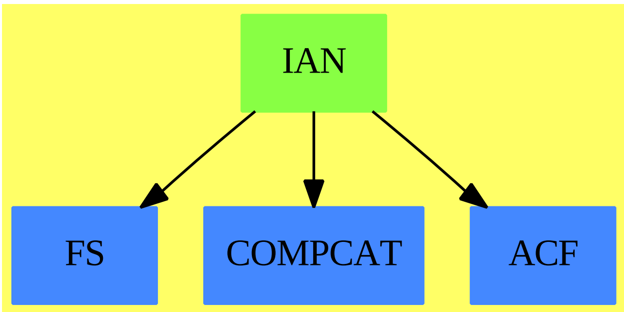
)spool
)lisp (bye)
```

— InnerAlgebraicNumber.help —

```
====================================================================
InnerAlgebraicNumber examples
====================================================================

See Also:
o )show InnerAlgebraicNumber
```

10.20.1 InnerAlgebraicNumber (IAN)



See

⇒ "AlgebraicNumber" (AN) 2.6.1 on page 35

Exports:

0	1	associates?	belong?
box	characteristic	coerce	convert
D	definingPolynomial	denom	differentiate
distribute	divide	elt	euclideanSize
eval	even?	expressIdealMember	exquo
extendedEuclidean	factor	freeOf?	gcd
gcdPolynomial	hash	height	inv
is?	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	norm
nthRoot	numer	odd?	one?
operator	operators	paren	prime?
principalIdeal	recip	reduce	reducedSystem
retract	retractIfCan	rootOf	rootsOf
sample	sizeLess?	sqrt	squareFree
squareFreePart	subst	subtractIfCan	tower
trueEqual	unit?	unitCanonical	unitNormal
zero?	zeroOf	zerosOf	?*?
?**?	?+?	-?	?-?
?/?	?<?	?<=?	?=?
?>?	?>=?	?^?	?~=?
?*?	?**?	?quo?	?rem?

— domain IAN InnerAlgebraicNumber —

```
)abbrev domain IAN InnerAlgebraicNumber
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 4 October 1995 (JHD)
++ Keywords: algebraic, number.
++ Description:
++ Algebraic closure of the rational numbers.

InnerAlgebraicNumber(): Exports == Implementation where
  Z   ==> Integer
  FE  ==> Expression Z
  K   ==> Kernel %
  P   ==> SparseMultivariatePolynomial(Z, K)
  ALGOP ==> "%alg"
  SUP ==>  SparseUnivariatePolynomial

  Exports ==> Join(ExpressionSpace, AlgebraicallyClosedField,
                   RetractableTo Z, RetractableTo Fraction Z,
                   LinearlyExplicitRingOver Z, RealConstant,
                   LinearlyExplicitRingOver Fraction Z,
                   CharacteristicZero,
                   ConvertibleTo Complex Float, DifferentialRing) with
```

```
    coerce : P -> %
      ++ coerce(p) returns p viewed as an algebraic number.
    numer  : % -> P
      ++ numer(f) returns the numerator of f viewed as a
      ++ polynomial in the kernels over Z.
    denom  : % -> P
      ++ denom(f) returns the denominator of f viewed as a
      ++ polynomial in the kernels over Z.
    reduce : % -> %
      ++ reduce(f) simplifies all the unreduced algebraic numbers
      ++ present in f by applying their defining relations.
    trueEqual : (%,%) -> Boolean
      ++ trueEqual(x,y) tries to determine if the two numbers are equal
    norm : (SUP(%),Kernel %) -> SUP(%)
      ++ norm(p,k) computes the norm of the polynomial p
      ++ with respect to the extension generated by kernel k
    norm : (SUP(%),List Kernel %) -> SUP(%)
      ++ norm(p,l) computes the norm of the polynomial p
      ++ with respect to the extension generated by kernels l
    norm : (%,Kernel %) -> %
      ++ norm(f,k) computes the norm of the algebraic number f
      ++ with respect to the extension generated by kernel k
    norm : (%,List Kernel %) -> %
      ++ norm(f,l) computes the norm of the algebraic number f
      ++ with respect to the extension generated by kernels l
  Implementation ==> FE add

    Rep := FE

    -- private
    mainRatDenom(f:%):% ==
       ratDenom(f::Rep::FE)$AlgebraicManipulations(Integer, FE)::Rep::%
--        mv:= mainVariable denom f
--        mv case "failed" => f
--        algv:=mv::K
--       q:=univariate(f, algv, minPoly(algv))_
--       $PolynomialCategoryQuotientFunctions(IndexedExponents K,K,Integer,P,%)
--       q(algv::%)

    findDenominator(z:SUP %):Record(num:SUP %,den:%) ==
       zz:=z
       while not(zz=0) repeat
          dd:=(denom leadingCoefficient zz)::%
          not(dd=1) =>
             rec:=findDenominator(dd*z)
             return [rec.num,rec.den*dd]
          zz:=reductum zz
       [z,1]
    makeUnivariate(p:P,k:Kernel %):SUP % ==
      map(x+->x::%,univariate(p,k))$SparseUnivariatePolynomialFunctions2(P,%)
```

```
    -- public
    a,b:%
    differentiate(x:%):% == 0
    zero? a == zero? numer a
--    one? a == one? numer a and one? denom a
    one? a == (numer a = 1) and (denom a = 1)
    x:% / y:%        == mainRatDenom(x /$Rep y)
    x:% ** n:Integer ==
      n < 0 => mainRatDenom (x **$Rep n)
      x **$Rep n
    trueEqual(a,b) ==
       -- if two algebraic numbers have the same norm (after deleting repeated
       -- roots, then they are certainly conjugates. Note that we start with a
       -- monic polynomial, so don't have to check for constant factors.
       -- this will be fooled by sqrt(2) and -sqrt(2), but the = in
       -- AlgebraicNumber knows what to do about this.
       ka:=reverse tower a
       kb:=reverse tower b
       empty? ka and empty? kb => retract(a)@Fraction Z = retract(b)@Fraction Z
       pa,pb:SparseUnivariatePolynomial %
       pa:=monomial(1,1)-monomial(a,0)
       pb:=monomial(1,1)-monomial(b,0)
       na:=map(retract,norm(pa,ka))_
         $SparseUnivariatePolynomialFunctions2(%,Fraction Z)
       nb:=map(retract,norm(pb,kb))_
         $SparseUnivariatePolynomialFunctions2(%,Fraction Z)
       (sa:=squareFreePart(na)) = (sb:=squareFreePart(nb)) => true
       g:=gcd(sa,sb)
       (dg:=degree g) = 0 => false
       -- of course, if these have a factor in common, then the
       -- answer is really ambiguous, so we ought to be using Duval-type
       -- technology
       dg = degree sa or dg = degree sb => true
       false
    norm(z:%,k:Kernel %): % ==
       p:=minPoly k
       n:=makeUnivariate(numer z,k)
       d:=makeUnivariate(denom z,k)
       resultant(n,p)/resultant(d,p)
    norm(z:%,l:List Kernel %): % ==
       for k in l repeat
           z:=norm(z,k)
       z
    norm(z:SUP %,k:Kernel %):SUP % ==
       p:=map(x +-> x::SUP %,minPoly k)_
         $SparseUnivariatePolynomialFunctions2(%,SUP %)
       f:=findDenominator z
       zz:=map(x +-> makeUnivariate(numer x,k),f.num)_
         $SparseUnivariatePolynomialFunctions2( %,SUP %)
       zz:=swap(zz)$CommuteUnivariatePolynomialCategory(%,SUP %,SUP SUP %)
```

```
      resultant(p,zz)/norm(f.den,k)
   norm(z:SUP %,l:List Kernel %): SUP % ==
      for k in l repeat
          z:=norm(z,k)
      z
   belong? op          == belong?(op)$ExpressionSpace_&(%) or has?(op, ALGOP)

   convert(x:%):Float ==
     retract map(y +-> y::Float, x pretend FE)$ExpressionFunctions2(Z,Float)

   convert(x:%):DoubleFloat ==
     retract map(y +-> y::DoubleFloat,x pretend FE)_
       $ExpressionFunctions2(Z, DoubleFloat)

   convert(x:%):Complex(Float) ==
     retract map(y +-> y::Complex(Float),x pretend FE)_
       $ExpressionFunctions2(Z, Complex Float)
```

— IAN.dotabb —

```
"IAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IAN"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"IAN" -> "ACF"
"IAN" -> "FS"
"IAN" -> "COMPCAT"
```

10.21 domain IFF InnerFiniteField

— InnerFiniteField.input —

```
)set break resume
)sys rm -f InnerFiniteField.output
)spool InnerFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show InnerFiniteField
--R InnerFiniteField(p: PositiveInteger,n: PositiveInteger)  is a domain constructor
--R Abbreviation for InnerFiniteField is IFF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IFF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (InnerPrimeField p,%) -> %      ?*? : (%,InnerPrimeField p) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,InnerPrimeField p) -> %      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        basis : () -> Vector %
--R coerce : InnerPrimeField p -> %       coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R norm : % -> InnerPrimeField p         one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> InnerPrimeField p      sample : () -> %
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               trace : % -> InnerPrimeField p
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if InnerPrimeField p has FINITE
--R D : % -> % if InnerPrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if InnerPrimeField p has FINITE
--R Frobenius : % -> % if InnerPrimeField p has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if InnerPrimeField p has CHARNZ or InnerPrimeField p has FINITE
--R charthRoot : % -> % if InnerPrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %,"failed") if InnerPrimeField p has FINITE
--R coordinates : Vector % -> Matrix InnerPrimeField p
--R coordinates : % -> Vector InnerPrimeField p
--R createNormalElement : () -> % if InnerPrimeField p has FINITE
```

```
--R createPrimitiveElement : () -> % if InnerPrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial InnerPrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if InnerPrimeField p has FINITE
--R differentiate : % -> % if InnerPrimeField p has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if InnerPrimeField p has CHARNZ
--R discreteLog : % -> NonNegativeInteger if InnerPrimeField p has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if InnerP
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R generator : () -> % if InnerPrimeField p has FINITE
--R index : PositiveInteger -> % if InnerPrimeField p has FINITE
--R init : () -> % if InnerPrimeField p has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial InnerPrimeField p) -> % if InnerPrime
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial InnerPrimeField p,"failed
--R linearAssociatedLog : % -> SparseUnivariatePolynomial InnerPrimeField p if InnerPrimeFiel
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial InnerPrimeField p if InnerPrimeFi
--R lookup : % -> PositiveInteger if InnerPrimeField p has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if InnerPrimeFiel
--R minimalPolynomial : % -> SparseUnivariatePolynomial InnerPrimeField p
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if InnerPrimeField p has FINITE
--R norm : (%,PositiveInteger) -> % if InnerPrimeField p has FINITE
--R normal? : % -> Boolean if InnerPrimeField p has FINITE
--R normalElement : () -> % if InnerPrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if InnerPrimeField p has CHARNZ or InnerP
--R order : % -> PositiveInteger if InnerPrimeField p has FINITE
--R primeFrobenius : % -> % if InnerPrimeField p has CHARNZ or InnerPrimeField p has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if InnerPrimeField p has CHARNZ or InnerPrim
--R primitive? : % -> Boolean if InnerPrimeField p has FINITE
--R primitiveElement : () -> % if InnerPrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if InnerPrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if InnerPrimeField p h
--R represents : Vector InnerPrimeField p -> %
--R retractIfCan : % -> Union(InnerPrimeField p,"failed")
--R size : () -> NonNegativeInteger if InnerPrimeField p has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Inner
--R trace : (%,PositiveInteger) -> % if InnerPrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

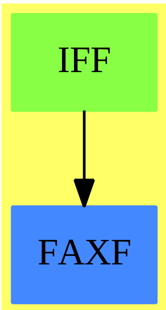
— InnerFiniteField.help —

```
====================================================================
InnerFiniteField examples
====================================================================

See Also:
o )show InnerFiniteField
```

———-

10.21.1 InnerFiniteField (IFF)



See

⇒ "FiniteFieldExtensionByPolynomial" (FFP) 7.10.1 on page 818
⇒ "FiniteFieldExtension" (FFX) 7.9.1 on page 813
⇒ "FiniteField" (FF) 7.5.1 on page 787

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
dimension	differentiate	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	index	inGroundField?
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain IFF InnerFiniteField —

```
)abbrev domain IFF InnerFiniteField
++ Author: Mark Botch
++ Date Created: ???
++ Date Last Updated: 29 May 1990
++ Basic Operations:
++ Related Constructors: FiniteFieldExtensionByPolynomial,
++  FiniteFieldPolynomialPackage
++ Also See: FiniteFieldCyclicGroup, FiniteFieldNormalBasis
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++   finite extension, finite field, Galois field
++ Reference:
++  R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
++   Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
```

```
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++   AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ InnerFiniteField(p,n) implements finite fields with \spad{p**n} elements
++ where p is assumed prime but does not check.
++ For a version which checks that p is prime, see \spadtype{FiniteField}.

InnerFiniteField(p:PositiveInteger, n:PositiveInteger) ==
     FiniteFieldExtension(InnerPrimeField p, n)
```

——————

— **IFF.dotabb** —

```
"IFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IFF"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"IFF" -> "FAXF"
```

——————

10.22 domain IFAMON InnerFreeAbelianMonoid

— **InnerFreeAbelianMonoid.input** —

```
)set break resume
)sys rm -f InnerFreeAbelianMonoid.output
)spool InnerFreeAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerFreeAbelianMonoid
--R InnerFreeAbelianMonoid(S: SetCategory,E: CancellationAbelianMonoid,un: E)  is a domain constructor
--R Abbreviation for InnerFreeAbelianMonoid is IFAMON
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IFAMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (E,S) -> %                      ?*? : (PositiveInteger,%) -> %
--R ?+? : (S,%) -> %                      ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coefficient : (S,%) -> E              coerce : S -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
```

```
--R latex : % -> String                   mapCoef : ((E -> E),%) -> %
--R mapGen : ((S -> S),%) -> %            nthCoef : (%,Integer) -> E
--R nthFactor : (%,Integer) -> S          retract : % -> S
--R sample : () -> %                      size : % -> NonNegativeInteger
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R highCommonTerms : (%,%) -> % if E has OAMON
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(gen: S,exp: E)
--R
--E 1

)spool
)lisp (bye)
```

———-

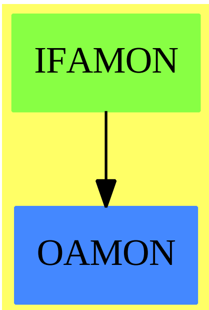
— **InnerFreeAbelianMonoid.help** —

```
====================================================================
InnerFreeAbelianMonoid examples
====================================================================

See Also:
o )show InnerFreeAbelianMonoid
```

———-

10.22.1 InnerFreeAbelianMonoid (IFAMON)



See

⇒ "ListMonoidOps" (LMOPS) 13.10.1 on page 1473
⇒ "FreeMonoid" (FMONOID) 7.32.1 on page 987

⇒ "FreeGroup" (FGROUP) 7.29.1 on page 976
⇒ "FreeAbelianMonoid" (FAMONOID) 7.28.1 on page 974
⇒ "FreeAbelianGroup" (FAGROUP) 7.27.1 on page 971

Exports:

0	coefficient	coerce	hash	highCommonTerms
latex	mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size	subtractIfCan
terms	zero?	?~=?	?*?	?+?
?=?				

— domain IFAMON InnerFreeAbelianMonoid —

```
)abbrev domain IFAMON InnerFreeAbelianMonoid
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ Internal implementation of a free abelian monoid on any set of generators

InnerFreeAbelianMonoid(S: SetCategory, E:CancellationAbelianMonoid, un:E):
  FreeAbelianMonoidCategory(S, E) == ListMonoidOps(S, E, un) add
        Rep := ListMonoidOps(S, E, un)

        0                          == makeUnit()
        zero? f                    == empty? listOfMonoms f
        terms f                    == copy listOfMonoms f
        nthCoef(f, i)              == nthExpon(f, i)
        nthFactor(f, i)            == nthFactor(f, i)$Rep
        s:S + f:$                  == plus(s, un, f)
        f:$ + g:$                  == plus(f, g)
        (f:$ = g:$):Boolean        == commutativeEquality(f,g)
        n:E * s:S                  == makeTerm(s, n)
        n:NonNegativeInteger * f:$ == mapExpon(x +-> n*x, f)
        coerce(f:$):OutputForm     == outputForm(f, "+", (x,y) +-> y*x, 0)
        mapCoef(f, x)              == mapExpon(f, x)
        mapGen(f, x)               == mapGen(f, x)$Rep

        coefficient(s, f) ==
          for x in terms f repeat
            x.gen = s => return(x.exp)
          0

        if E has OrderedAbelianMonoid then
          highCommonTerms(f, g) ==
            makeMulti [[x.gen, min(x.exp, n)] for x in listOfMonoms f |
                                 (n := coefficient(x.gen, g)) > 0]
```

————

— IFAMON.dotabb —

```
"IFAMON" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IFAMON"]
"OAMON" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMON"]
"IFAMON" -> "OAMON"
```

10.23 domain IIARRAY2 InnerIndexedTwoDimensionalArray

This is an internal type which provides an implementation of 2-dimensional arrays as PrimitiveArray's of PrimitiveArray's.

— InnerIndexedTwoDimensionalArray.input —

```
)set break resume
)sys rm -f InnerIndexedTwoDimensionalArray.output
)spool InnerIndexedTwoDimensionalArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerIndexedTwoDimensionalArray
--R InnerIndexedTwoDimensionalArray(R: Type,mnRow: Integer,mnCol: Integer,Row: FiniteLinearAg
--R Abbreviation for InnerIndexedTwoDimensionalArray is IIARRAY2
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IIARRAY2
--R
--R------------------------------- Operations --------------------------------
--R column : (%,Integer) -> Col          copy : % -> %
--R elt : (%,Integer,Integer,R) -> R     elt : (%,Integer,Integer) -> R
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               fill! : (%,R) -> %
--R map : (((R,R) -> R),%,%,R) -> %      map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %              map! : ((R -> R),%) -> %
--R maxColIndex : % -> Integer           maxRowIndex : % -> Integer
--R minColIndex : % -> Integer           minRowIndex : % -> Integer
--R ncols : % -> NonNegativeInteger      nrows : % -> NonNegativeInteger
--R parts : % -> List R                  qelt : (%,Integer,Integer) -> R
--R row : (%,Integer) -> Row             sample : () -> %
--R setRow! : (%,Integer,Row) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if R has SETCAT
```

```
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R qsetelt! : (%,Integer,Integer,R) -> R
--R setColumn! : (%,Integer,Col) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

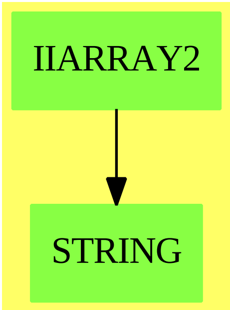
— InnerIndexedTwoDimensionalArray.help —

```
====================================================================
InnerIndexedTwoDimensionalArray examples
====================================================================

See Also:
o )show InnerIndexedTwoDimensionalArray
```

———-

10.23.1 InnerIndexedTwoDimensionalArray (IIARRAY2)



See

⇒ "IndexedTwoDimensionalArray" (IARRAY2) 10.15.1 on page 1221
⇒ "TwoDimensionalArray" (ARRAY2) 21.13.1 on page 2722

Exports:

any?	coerce	column	copy	count
elt	empty	empty?	eq?	eval
every?	fill!	hash	latex	less?
map	map!	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
new	nrows	parts	qelt	qsetelt!
row	sample	setColumn!	setelt	setRow!
size?	#?	?=?	?~=?	

— **domain IIARRAY2 InnerIndexedTwoDimensionalArray** —

```
)abbrev domain IIARRAY2 InnerIndexedTwoDimensionalArray
++ Author: Mark Botch
++ Description:
++ There is no description for this domain

InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col):_
       Exports == Implementation where
  R : Type
  mnRow, mnCol : Integer
  Row : FiniteLinearAggregate R
  Col : FiniteLinearAggregate R

  Exports ==> TwoDimensionalArrayCategory(R,Row,Col)

  Implementation ==> add

    Rep := PrimitiveArray PrimitiveArray R

--% Predicates
```

```
    empty? m == empty?(m)$Rep

--% Primitive array creation

    empty() == empty()$Rep

    new(rows,cols,a) ==
      rows = 0 =>
        error "new: arrays with zero rows are not supported"
--      cols = 0 =>
--        error "new: arrays with zero columns are not supported"
      arr : PrimitiveArray PrimitiveArray R := new(rows,empty())
      for i in minIndex(arr)..maxIndex(arr) repeat
        qsetelt_!(arr,i,new(cols,a))
      arr

--% Size inquiries

    minRowIndex m == mnRow
    minColIndex m == mnCol
    maxRowIndex m == nrows m + mnRow - 1
    maxColIndex m == ncols m + mnCol - 1

    nrows m == (# m)$Rep

    ncols m ==
      empty? m => 0
      # m(minIndex(m)$Rep)

--% Part selection/assignment

    qelt(m,i,j) ==
      qelt(qelt(m,i - minRowIndex m)$Rep,j - minColIndex m)

    elt(m:%,i:Integer,j:Integer) ==
      i < minRowIndex(m) or i > maxRowIndex(m) =>
        error "elt: index out of range"
      j < minColIndex(m) or j > maxColIndex(m) =>
        error "elt: index out of range"
      qelt(m,i,j)

    qsetelt_!(m,i,j,r) ==
      setelt(qelt(m,i - minRowIndex m)$Rep,j - minColIndex m,r)

    setelt(m:%,i:Integer,j:Integer,r:R) ==
      i < minRowIndex(m) or i > maxRowIndex(m) =>
        error "setelt: index out of range"
      j < minColIndex(m) or j > maxColIndex(m) =>
        error "setelt: index out of range"
```

```
      qsetelt_!(m,i,j,r)

    if R has SetCategory then
        latex(m : %) : String ==
          s : String := "\left[ \begin{array}{"
          j : Integer
          for j in minColIndex(m)..maxColIndex(m) repeat
            s := concat(s,"c")$String
          s := concat(s,"} ")$String
          i : Integer
          for i in minRowIndex(m)..maxRowIndex(m) repeat
            for j in minColIndex(m)..maxColIndex(m) repeat
              s := concat(s, latex(qelt(m,i,j))$R)$String
              if j < maxColIndex(m) then s := concat(s, " & ")$String
            if i < maxRowIndex(m) then s := concat(s, " \\ ")$String
          concat(s, "\end{array} \right]")$String
```

————

— IIARRAY2.dotabb —

```
"IIARRAY2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IIARRAY2"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"IIARRAY2" -> "STRING"
```

————

10.24 domain IPADIC InnerPAdicInteger

— InnerPAdicInteger.input —

```
)set break resume
)sys rm -f InnerPAdicInteger.output
)spool InnerPAdicInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerPAdicInteger
--R InnerPAdicInteger(p: Integer,unBalanced?: Boolean)  is a domain constructor
--R Abbreviation for InnerPAdicInteger is IPADIC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IPADIC
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %         ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,PositiveInteger) -> %         associates? : (%,%) -> Boolean
--R coerce : % -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm               complete : % -> %
--R digits : % -> Stream Integer           extend : (%,Integer) -> %
--R gcd : List % -> %                      gcd : (%,%) -> %
--R hash : % -> SingleInteger              latex : % -> String
--R lcm : List % -> %                      lcm : (%,%) -> %
--R moduloP : % -> Integer                 modulus : () -> Integer
--R one? : % -> Boolean                    order : % -> NonNegativeInteger
--R ?quo? : (%,%) -> %                     quotientByP : % -> %
--R recip : % -> Union(%,"failed")         ?rem? : (%,%) -> %
--R sample : () -> %                       sizeLess? : (%,%) -> Boolean
--R sqrt : (%,Integer) -> %                unit? : % -> Boolean
--R unitCanonical : % -> %                 zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R root : (SparseUnivariatePolynomial Integer,Integer) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

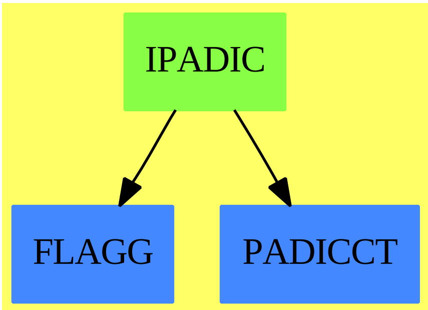
— **InnerPAdicInteger.help** —

```
====================================================================
InnerPAdicInteger examples
====================================================================

See Also:
o )show InnerPAdicInteger
```

———-

10.24.1 InnerPAdicInteger (IPADIC)



See

⇒ "PAdicInteger" (PADIC) 17.1.1 on page 1841
⇒ "BalancedPAdicInteger" (BPADIC) 3.2.1 on page 240
⇒ "PAdicRationalConstructor" (PADICRC) 17.3.1 on page 1850
⇒ "PAdicRational" (PADICRAT) 17.2.1 on page 1845
⇒ "BalancedPAdicRational" (BPADICRT) 3.3.1 on page 244

Exports:

0	1	approximate	associates?
characteristic	coerce	complete	digits
divide	euclideanSize	expressIdealMember	exquo
extend	extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm	multiEuclidean
moduloP	modulus	one?	order
principalIdeal	quotientByP	recip	root
sample	sizeLess?	sqrt	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?
?quo?	?rem?		

— domain IPADIC InnerPAdicInteger —

```
)abbrev domain IPADIC InnerPAdicInteger
```

```
++ Author: Clifton J. Williamson
++ Date Created: 20 August 1989
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ This domain implements Zp, the p-adic completion of the integers.
++ This is an internal domain.

InnerPAdicInteger(p,unBalanced?): Exports == Implementation where
  p           : Integer
  unBalanced? : Boolean
  I   ==> Integer
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  L   ==> List
  ST  ==> Stream
  SUP ==> SparseUnivariatePolynomial

  Exports ==> PAdicIntegerCategory p

  Implementation ==> add

    PEXPR := p :: OUT

    Rep := ST I

    characteristic() == 0
    euclideanSize(x) == order(x)

    stream(x:%):ST I == x pretend ST(I)
    padic(x:ST I):% == x pretend %
    digits x == stream x

    extend(x,n) == extend(x,n + 1)$Rep
    complete x == complete(x)$Rep

--     notBalanced?:() -> Boolean
--     notBalanced?() == unBalanced?

    modP:I -> I
    modP n ==
      unBalanced? or (p = 2) => positiveRemainder(n,p)
      symmetricRemainder(n,p)
```

```
modPInfo:I -> Record(digit:I,carry:I)
modPInfo n ==
  dv := divide(n,p)
  r0 := dv.remainder; q := dv.quotient
  if (r := modP r0) ^= r0 then q := q + ((r0 - r) quo p)
  [r,q]

invModP: I -> I
invModP n == invmod(n,p)

modulus()     == p
moduloP x     == (empty? x => 0; frst x)
quotientByP x == (empty? x => x; rst x)

approximate(x,n) ==
  n <= 0 or empty? x => 0
  frst x + p * approximate(rst x,n - 1)

x = y ==
  st : ST I := stream(x - y)
  n : I := _$streamCount$Lisp
  for i in 0..n repeat
    empty? st => return true
    frst st ^= 0 => return false
    st := rst st
  empty? st

order x ==
  st := stream x
  for i in 0..1000 repeat
    empty? st => return 0
    frst st ^= 0 => return i
    st := rst st
  error "order: series has more than 1000 leading zero coefs"

0 == padic concat(0$I,empty())
1 == padic concat(1$I,empty())

intToPAdic: I -> ST I
intToPAdic n == delay
  n = 0 => empty()
  modp := modPInfo n
  concat(modp.digit,intToPAdic modp.carry)

intPlusPAdic: (I,ST I) -> ST I
intPlusPAdic(n,x) == delay
  empty? x => intToPAdic n
  modp := modPInfo(n + frst x)
  concat(modp.digit,intPlusPAdic(modp.carry,rst x))
```

```
intMinusPAdic: (I,ST I) -> ST I
intMinusPAdic(n,x) == delay
  empty? x => intToPAdic n
  modp := modPInfo(n - frst x)
  concat(modp.digit,intMinusPAdic(modp.carry,rst x))

plusAux: (I,ST I,ST I) -> ST I
plusAux(n,x,y) == delay
  empty? x and empty? y => intToPAdic n
  empty? x => intPlusPAdic(n,y)
  empty? y => intPlusPAdic(n,x)
  modp := modPInfo(n + frst x + frst y)
  concat(modp.digit,plusAux(modp.carry,rst x,rst y))

minusAux: (I,ST I,ST I) -> ST I
minusAux(n,x,y) == delay
  empty? x and empty? y => intToPAdic n
  empty? x => intMinusPAdic(n,y)
  empty? y => intPlusPAdic(n,x)
  modp := modPInfo(n + frst x - frst y)
  concat(modp.digit,minusAux(modp.carry,rst x,rst y))

x + y == padic plusAux(0,stream x,stream y)
x - y == padic minusAux(0,stream x,stream y)
- y   == padic intMinusPAdic(0,stream y)
coerce(n:I) == padic intToPAdic n

intMult:(I,ST I) -> ST I
intMult(n,x) == delay
  empty? x => empty()
  modp := modPInfo(n * frst x)
  concat(modp.digit,intPlusPAdic(modp.carry,intMult(n,rst x)))

(n:I) * (x:%) ==
  padic intMult(n,stream x)

timesAux:(ST I,ST I) -> ST I
timesAux(x,y) == delay
  empty? x or empty? y => empty()
  modp := modPInfo(frst x * frst y)
  car := modp.digit
  cdr : ST I --!!
  cdr := plusAux(modp.carry,intMult(frst x,rst y),timesAux(rst x,y))
  concat(car,cdr)

(x:%) * (y:%) == padic timesAux(stream x,stream y)

quotientAux:(ST I,ST I) -> ST I
quotientAux(x,y) == delay
```

```
    empty? x => error "quotientAux: first argument"
    empty? y => empty()
    modP frst x = 0 =>
      modP frst y = 0 => quotientAux(rst x,rst y)
      error "quotient: quotient not integral"
    z0 := modP(invModP frst x * frst y)
    yy : ST I --!!
    yy := rest minusAux(0,y,intMult(z0,x))
    concat(z0,quotientAux(x,yy))

  recip x ==
    empty? x or modP frst x = 0 => "failed"
    padic quotientAux(stream x,concat(1,empty()))

  iExquo: (%,%,I) -> Union(%,"failed")
  iExquo(xx,yy,n) ==
    n > 1000 =>
      error "exquo: quotient by series with many leading zero coefs"
    empty? yy => "failed"
    empty? xx => 0
    zero? frst yy =>
      zero? frst xx => iExquo(rst xx,rst yy,n + 1)
      "failed"
    (rec := recip yy) case "failed" => "failed"
    xx * (rec :: %)

  x exquo y == iExquo(stream x,stream y,0)

  divide(x,y) ==
    (z:=x exquo y) case "failed" => [0,x]
    [z, 0]

  iSqrt: (I,I,I,%) -> %
  iSqrt(pn,an,bn,bSt) == delay
    bn1 := (empty? bSt => bn; bn + pn * frst(bSt))
    c := (bn1 - an*an) quo pn
    aa := modP(c * invmod(2*an,p))
    nSt := (empty? bSt => bSt; rst bSt)
    concat(aa,iSqrt(pn*p,an + pn*aa,bn1,nSt))

  sqrt(b,a) ==
    p = 2 =>
      error "sqrt: no square roots in Z2 yet"
    not zero? modP(a*a - (bb := moduloP b)) =>
      error "sqrt: not a square root (mod p)"
    b := (empty? b => b; rst b)
    a := modP a
    concat(a,iSqrt(p,a,bb,b))

  iRoot: (SUP I,I,I,I) -> ST I
```

```
iRoot(f,alpha,invFpx0,pPow) == delay
  num := -((f(alpha) exquo pPow) :: I)
  digit := modP(num * invFpx0)
  concat(digit,iRoot(f,alpha + digit * pPow,invFpx0,p * pPow))

root(f,x0) ==
  x0 := modP x0
  not zero? modP f(x0) =>
    error "root: not a root (mod p)"
  fpx0 := modP (differentiate f)(x0)
  zero? fpx0 =>
    error "root: approximate root must be a simple root (mod p)"
  invFpx0 := modP invModP fpx0
  padic concat(x0,iRoot(f,x0,invFpx0,p))

termOutput:(I,I) -> OUT
termOutput(k,c) ==
  k = 0 => c :: OUT
  mon := (k = 1 => PEXPR; PEXPR ** (k :: OUT))
  c = 1 => mon
  c = -1 => -mon
  (c :: OUT) * mon

showAll?:() -> Boolean
-- check a global Lisp variable
showAll?() == true

coerce(x:%):OUT ==
  empty?(st := stream x) => 0 :: OUT
  n : NNI ; count : NNI := _$streamCount$Lisp
  l : L OUT := empty()
  for n in 0..count while not empty? st repeat
    if frst(st) ^= 0 then
      l := concat(termOutput(n :: I,frst st),l)
    st := rst st
  if showAll?() then
    for n in (count + 1).. while explicitEntries? st and _
           not eq?(st,rst st) repeat
      if frst(st) ^= 0 then
        l := concat(termOutput(n pretend I,frst st),l)
      st := rst st
  l :=
    explicitlyEmpty? st => l
    eq?(st,rst st) and frst st = 0 => l
    concat(prefix("O" :: OUT,[PEXPR ** (n :: OUT)]),l)
  empty? l => 0 :: OUT
  reduce("+",reverse_! l)
```

———

— IPADIC.dotabb —

```
"IPADIC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IPADIC"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"PADICCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PADICCT"]
"IPADIC" -> "PADICCT"
"IPADIC" -> "FLAGG"
```

10.25 domain IPF InnerPrimeField

— InnerPrimeField.input —

```
)set break resume
)sys rm -f InnerPrimeField.output
)spool InnerPrimeField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerPrimeField
--R InnerPrimeField p: PositiveInteger  is a domain constructor
--R Abbreviation for InnerPrimeField is IPF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IPF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R D : % -> %                           D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean            associates? : (%,%) -> Boolean
--R basis : () -> Vector %               charthRoot : % -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R convert : % -> Integer               coordinates : % -> Vector %
--R createPrimitiveElement : () -> %     degree : % -> PositiveInteger
--R differentiate : % -> %               dimension : () -> CardinalNumber
```

```
--R factor : % -> Factored %            gcd : List % -> %
--R gcd : (%,%) -> %                    hash : % -> SingleInteger
--R inGroundField? : % -> Boolean       index : PositiveInteger -> %
--R init : () -> %                      inv : % -> %
--R latex : % -> String                 lcm : List % -> %
--R lcm : (%,%) -> %                    lookup : % -> PositiveInteger
--R norm : % -> %                       one? : % -> Boolean
--R order : % -> PositiveInteger        prime? : % -> Boolean
--R primeFrobenius : % -> %             primitive? : % -> Boolean
--R primitiveElement : () -> %          ?quo? : (%,%) -> %
--R random : () -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                  represents : Vector % -> %
--R retract : % -> %                    sample : () -> %
--R size : () -> NonNegativeInteger     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %        squareFreePart : % -> %
--R trace : % -> %                      transcendent? : % -> Boolean
--R unit? : % -> Boolean                unitCanonical : % -> %
--R zero? : % -> Boolean                ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if $ has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if $ has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix % -> Union(Vector %,"failed")
--R coordinates : Vector % -> Matrix %
--R createNormalElement : () -> % if $ has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R extensionDegree : () -> PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if $ has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial %) -> % if $ has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial % if $ has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial %,"failed") if $ has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial % if $ has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial %
```

```
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if $ has FINITE
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R norm : (%,PositiveInteger) -> % if $ has FINITE
--R normal? : % -> Boolean if $ has FINITE
--R normalElement : () -> % if $ has FINITE
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R retractIfCan : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R trace : (%,PositiveInteger) -> % if $ has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

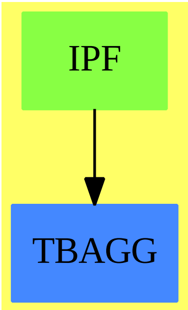
— InnerPrimeField.help —

```
====================================================================
InnerPrimeField examples
====================================================================

See Also:
o )show InnerPrimeField
```

———-

10.25.1 InnerPrimeField (IPF)



See

⇒ "PrimeField" (PF) 17.29.1 on page 2064

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
convert	coordinates	createPrimitiveElement
createNormalElement	D	definingPolynomial
degree	differentiate	dimension
discreteLog	divide	euclideanSize
expressIdealMember	exquo	extendedEuclidean
extensionDegree	factor	factorsOfCyclicGroupSize
Frobenius	gcd	gcdPolynomial
generator	hash	inGroundField?
index	init	inv
latex	lcm	linearAssociatedExp
linearAssociatedLog	linearAssociatedOrder	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	normal?	normalElement
one?	order	prime?
primeFrobenius	primitive?	primitiveElement
principalIdeal	random	recip
representationType	represents	retract
retractIfCan	sample	size
sizeLess?	squareFree	squareFreePart
subtractIfCan	tableForDiscreteLogarithm	trace
transcendenceDegree	transcendent?	unit?
unitCanonical	unitNormal	zero?
?*?	?**?	?+?
?-?	-?	?/?
?=?	?^?	?~=?
?quo?	?rem?	

— domain IPF InnerPrimeField —

```
)abbrev domain IPF InnerPrimeField
++ Authors: N.N., J.Grabmeier, A.Scheerhorn
++ Date Created: ?, November 1990, 26.03.1991
++ Date Last Updated: 12 April 1991
++ Basic Operations:
++ Related Constructors: PrimeField
++ Also See:
++ AMS Classifications:
++ Keywords: prime characteristic, prime field, finite field
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++  AXIOM Technical Report Series, to appear.
++ Description:
++ InnerPrimeField(p) implements the field with p elements.
++ Note: argument p MUST be a prime (this domain does not check).
++ See \spadtype{PrimeField} for a domain that does check.

InnerPrimeField(p:PositiveInteger): Exports == Implementation where

  I   ==> Integer
  NNI ==> NonNegativeInteger
  PI  ==> PositiveInteger
  TBL ==> Table(PI,NNI)
  R   ==> Record(key:PI,entry:NNI)
  SUP ==> SparseUnivariatePolynomial
  OUT ==> OutputForm

  Exports ==> Join(FiniteFieldCategory,FiniteAlgebraicExtensionField($),_
              ConvertibleTo(Integer))

  Implementation ==> IntegerMod p add

    initializeElt:() -> Void
    initializeLog:() -> Void

-- global variables ====================================================

    primitiveElt:PI:=1
    -- for the lookup the primitive Element computed by createPrimitiveElement()

    sizeCG  :=(p-1) pretend NonNegativeInteger
    -- the size of the cyclic group

    facOfGroupSize := nil()$(List Record(factor:Integer,exponent:Integer))
    -- the factorization of the cyclic group size
```

```
    initlog?:Boolean:=true
    -- gets false after initialization of the logarithm table

    initelt?:Boolean:=true
    -- gets false after initialization of the primitive Element


    discLogTable:Table(PI,TBL):=table()$Table(PI,TBL)
    -- tables indexed by the factors of the size q of the cyclic group
    -- discLogTable.factor is a table of with keys
    -- primitiveElement() ** (i * (q quo factor)) and entries i for
    -- i in 0..n-1, n computed in initialize() in order to use
    -- the minimal size limit 'limit' optimal.

-- functions ===========================================================

    generator() == 1

    -- This uses x**(p-1)=1 (mod p), so x**(q(p-1)+r) = x**r (mod p)
    x:$ ** n:Integer ==
      zero?(n) => 1
      zero?(x) => 0
      r := positiveRemainder(n,p-1)::NNI
      ((x pretend IntegerMod p) **$IntegerMod(p) r) pretend $

    if p <= convert(max()$SingleInteger)@Integer then
      q := p::SingleInteger

      recip x ==
        zero?(y := convert(x)@Integer :: SingleInteger) => "failed"
        invmod(y, q)::Integer::$
    else
      recip x ==
        zero?(y := convert(x)@Integer) => "failed"
        invmod(y, p)::$

    convert(x:$) == x pretend I

    normalElement() == 1

    createNormalElement() == 1

    characteristic() == p

    factorsOfCyclicGroupSize() ==
      p=2 => facOfGroupSize -- this fixes an infinite loop of functions
                            -- calls, problem was that factors factor(1)
                            -- is the empty list
      if empty? facOfGroupSize then initializeElt()
      facOfGroupSize
```

```
   representationType() == "prime"

   tableForDiscreteLogarithm(fac) ==
     if initlog? then initializeLog()
     tbl:=search(fac::PI,discLogTable)$Table(PI,TBL)
     tbl case "failed" =>
       error "tableForDiscreteLogarithm: argument must be prime divisor_
 of the order of the multiplicative group"
     tbl pretend TBL

   primitiveElement() ==
     if initelt? then initializeElt()
     index(primitiveElt)

   initializeElt() ==
     facOfGroupSize:=factors(factor(sizeCG)$I)$(Factored I)
     -- get a primitive element
     primitiveElt:=lookup(createPrimitiveElement())
     -- set initialization flag
     initelt? := false
     void$Void

   initializeLog() ==
     if initelt? then initializeElt()
     -- set up tables for discrete logarithm
     limit:Integer:=30
     -- the minimum size for the discrete logarithm table
     for f in facOfGroupSize repeat
       fac:=f.factor
       base:$:=primitiveElement() ** (sizeCG quo fac)
       l:Integer:=length(fac)$Integer
       n:Integer:=0
       if odd?(l)$Integer then n:=shift(fac,-(l quo 2))
                          else n:=shift(1,(l quo 2))
       if n < limit then
         d:=(fac-1) quo limit + 1
         n:=(fac-1) quo d + 1
       tbl:TBL:=table()$TBL
       a:$:=1
       for i in (0::NNI)..(n-1)::NNI repeat
         insert_!([lookup(a),i::NNI]$R,tbl)$TBL
         a:=a*base
       insert_!([fac::PI,copy(tbl)$TBL]_
              $Record(key:PI,entry:TBL),discLogTable)$Table(PI,TBL)
     -- tell user about initialization
     --    print("discrete logarithm table initialized"::OUT)
     -- set initialization flag
     initlog? := false
     void$Void
```

```
    degree(x):PI == 1::PositiveInteger
    extensionDegree():PI == 1::PositiveInteger

--    sizeOfGroundField() == p::NonNegativeInteger

    inGroundField?(x)  == true

    coordinates(x) == new(1,x)$(Vector $)

    represents(v)  == v.1

    retract(x) == x

    retractIfCan(x) == x

    basis() == new(1,1::$)$(Vector $)
    basis(n:PI) ==
      n = 1 => basis()
      error("basis: argument must divide extension degree")

    definingPolynomial() ==
      monomial(1,1)$(SUP $) - monomial(1,0)$(SUP $)


    minimalPolynomial(x) ==
      monomial(1,1)$(SUP $) - monomial(x,0)$(SUP $)

    charthRoot x == x
```

———-

— IPF.dotabb —

```
"IPF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IPF"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"IPF" -> "TBAGG"
```

———-

10.26 domain ISUPS InnerSparseUnivariatePowerSeries

— InnerSparseUnivariatePowerSeries.input —

```
)set break resume
```

```
)sys rm -f InnerSparseUnivariatePowerSeries.output
)spool InnerSparseUnivariatePowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerSparseUnivariatePowerSeries
--R InnerSparseUnivariatePowerSeries Coef: Ring  is a domain constructor
--R Abbreviation for InnerSparseUnivariatePowerSeries is ISUPS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ISUPS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       center : % -> Coef
--R coefficient : (%,Integer) -> Coef    coerce : Integer -> %
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> Integer                ?.? : (%,Integer) -> Coef
--R extend : (%,Integer) -> %            hash : % -> SingleInteger
--R iCompose : (%,%) -> %                latex : % -> String
--R leadingCoefficient : % -> Coef       leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %        monomial : (Coef,Integer) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : (%,Integer) -> Integer       order : % -> Integer
--R pole? : % -> Boolean                 recip : % -> Union(%,"failed")
--R reductum : % -> %                    sample : () -> %
--R taylorQuoByVar : % -> %              truncate : (%,Integer) -> %
--R variable : % -> Symbol               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef an
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has co
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R cAcos : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R cAcosh : % -> % if Coef has ALGEBRA FRAC INT
--R cAcot : % -> % if Coef has ALGEBRA FRAC INT
--R cAcoth : % -> % if Coef has ALGEBRA FRAC INT
--R cAcsc : % -> % if Coef has ALGEBRA FRAC INT
--R cAcsch : % -> % if Coef has ALGEBRA FRAC INT
--R cAsec : % -> % if Coef has ALGEBRA FRAC INT
--R cAsech : % -> % if Coef has ALGEBRA FRAC INT
--R cAsin : % -> % if Coef has ALGEBRA FRAC INT
--R cAsinh : % -> % if Coef has ALGEBRA FRAC INT
--R cAtan : % -> % if Coef has ALGEBRA FRAC INT
--R cAtanh : % -> % if Coef has ALGEBRA FRAC INT
--R cCos : % -> % if Coef has ALGEBRA FRAC INT
--R cCosh : % -> % if Coef has ALGEBRA FRAC INT
--R cCot : % -> % if Coef has ALGEBRA FRAC INT
--R cCoth : % -> % if Coef has ALGEBRA FRAC INT
--R cCsc : % -> % if Coef has ALGEBRA FRAC INT
--R cCsch : % -> % if Coef has ALGEBRA FRAC INT
--R cExp : % -> % if Coef has ALGEBRA FRAC INT
--R cLog : % -> % if Coef has ALGEBRA FRAC INT
--R cPower : (%,Coef) -> % if Coef has ALGEBRA FRAC INT
--R cRationalPower : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R cSec : % -> % if Coef has ALGEBRA FRAC INT
--R cSech : % -> % if Coef has ALGEBRA FRAC INT
--R cSin : % -> % if Coef has ALGEBRA FRAC INT
--R cSinh : % -> % if Coef has ALGEBRA FRAC INT
--R cTan : % -> % if Coef has ALGEBRA FRAC INT
--R cTanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBO
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef and Coef ha
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef a
--R ?.? : (%,%) -> % if Integer has SGROUP
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R getRef : % -> Reference OrderedCompletion Integer
--R getStream : % -> Stream Record(k: Integer,c: Coef)
--R iExquo : (%,%,Boolean) -> Union(%,"failed")
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R makeSeries : (Reference OrderedCompletion Integer,Stream Record(k: Integer,c: Coef)) -> %
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
```

```
--R series : Stream Record(k: Integer,c: Coef) -> %
--R seriesToOutputForm : (Stream Record(k: Integer,c: Coef),Reference OrderedCompletion Integ
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

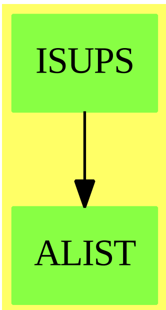
)spool
)lisp (bye)
```

— **InnerSparseUnivariatePowerSeries.help** —

```
====================================================================
InnerSparseUnivariatePowerSeries examples
====================================================================

See Also:
o )show InnerSparseUnivariatePowerSeries
```

10.26.1 InnerSparseUnivariatePowerSeries (ISUPS)



Exports:

0	1	approximate	associates?
cAcos	cAcosh	cAcot	cAcoth
cAcsc	cAcsch	cAsec	cAsech
cAsin	cAsinh	cAtan	cAtanh
cCos	cCosh	cCot	cCoth
cCsc	cCsch	center	cExp
cLog	coefficient	cPower	cRationalPower
cSec	cSech	cSin	cSinh
cTan	cTanh	characteristic	charthRoot
coerce	complete	D	differentiate
degree	eval	exquo	extend
getRef	getStream	hash	iCompose
iExquo	integrate	latex	leadingCoefficient
leadingMonomial	makeSeries	map	monomial
monomial?	multiplyCoefficients	multiplyExponents	one?
order	pole?	recip	reductum
sample	series	seriesToOutputForm	subtractIfCan
taylorQuoByVar	terms	truncate	unit?
unitCanonical	unitNormal	variable	variables
zero?	?**?	?**?	?+?
?-?	-?	?=?	?^?
?.?	?~=?	?/?	?^?
?.?			

— domain ISUPS InnerSparseUnivariatePowerSeries —

```
)abbrev domain ISUPS InnerSparseUnivariatePowerSeries
++ Author: Clifton J. Williamson
++ Date Created: 28 October 1994
++ Date Last Updated: 9 March 1995
++ Basic Operations:
++ Related Domains: SparseUnivariateTaylorSeries, SparseUnivariateLaurentSeries
++   SparseUnivariatePuiseuxSeries
++ Also See:
++ AMS Classifications:
++ Keywords: sparse, series
++ Examples:
++ References:
++ Description:
++ InnerSparseUnivariatePowerSeries is an internal domain
++ used for creating sparse Taylor and Laurent series.

InnerSparseUnivariatePowerSeries(Coef): Exports == Implementation where
  Coef  : Ring
  B    ==> Boolean
  COM  ==> OrderedCompletion Integer
  I    ==> Integer
```

```
L    ==> List
NNI  ==> NonNegativeInteger
OUT  ==> OutputForm
PI   ==> PositiveInteger
REF  ==> Reference OrderedCompletion Integer
RN   ==> Fraction Integer
Term ==> Record(k:Integer,c:Coef)
SG   ==> String
ST   ==> Stream Term

Exports ==> UnivariatePowerSeriesCategory(Coef,Integer) with
  makeSeries: (REF,ST) -> %
    ++ makeSeries(refer,str) creates a power series from the reference
    ++ \spad{refer} and the stream \spad{str}.
  getRef: % -> REF
    ++ getRef(f) returns a reference containing the order to which the
    ++ terms of f have been computed.
  getStream: % -> ST
    ++ getStream(f) returns the stream of terms representing the series f.
  series: ST -> %
    ++ series(st) creates a series from a stream of non-zero terms,
    ++ where a term is an exponent-coefficient pair.  The terms in the
    ++ stream should be ordered by increasing order of exponents.
  monomial?: % -> B
    ++ monomial?(f) tests if f is a single monomial.
  multiplyCoefficients: (I -> Coef,%) -> %
    ++ multiplyCoefficients(fn,f) returns the series
    ++ \spad{sum(fn(n) * an * x^n,n = n0..)},
    ++ where f is the series \spad{sum(an * x^n,n = n0..)}.
  iExquo: (%,%,B) -> Union(%,"failed")
    ++ iExquo(f,g,taylor?) is the quotient of the power series f and g.
    ++ If \spad{taylor?} is \spad{true}, then we must have
    ++ \spad{order(f) >= order(g)}.
  taylorQuoByVar: % -> %
    ++ taylorQuoByVar(a0 + a1 x + a2 x**2 + ...)
    ++ returns \spad{a1 + a2 x + a3 x**2 + ...}
  iCompose: (%,%) -> %
    ++ iCompose(f,g) returns \spad{f(g(x))}.  This is an internal function
    ++ which should only be called for Taylor series \spad{f(x)} and
    ++ \spad{g(x)} such that the constant coefficient of \spad{g(x)} is zero.
  seriesToOutputForm: (ST,REF,Symbol,Coef,RN) -> OutputForm
    ++ seriesToOutputForm(st,refer,var,cen,r) prints the series
    ++ \spad{f((var - cen)^r)}.
  if Coef has Algebra Fraction Integer then
    integrate: % -> %
      ++ integrate(f(x)) returns an anti-derivative of the power series
      ++ \spad{f(x)} with constant coefficient 0.
      ++ Warning: function does not check for a term of degree -1.
    cPower: (%,Coef) -> %
      ++ cPower(f,r) computes \spad{f^r}, where f has constant coefficient 1.
```

```
    ++ For use when the coefficient ring is commutative.
  cRationalPower: (%,RN) -> %
    ++ cRationalPower(f,r) computes \spad{f^r}.
    ++ For use when the coefficient ring is commutative.
  cExp: % -> %
    ++ cExp(f) computes the exponential of the power series f.
    ++ For use when the coefficient ring is commutative.
  cLog: % -> %
    ++ cLog(f) computes the logarithm of the power series f.
    ++ For use when the coefficient ring is commutative.
  cSin: % -> %
    ++ cSin(f) computes the sine of the power series f.
    ++ For use when the coefficient ring is commutative.
  cCos: % -> %
    ++ cCos(f) computes the cosine of the power series f.
    ++ For use when the coefficient ring is commutative.
  cTan: % -> %
    ++ cTan(f) computes the tangent of the power series f.
    ++ For use when the coefficient ring is commutative.
  cCot: % -> %
    ++ cCot(f) computes the cotangent of the power series f.
    ++ For use when the coefficient ring is commutative.
  cSec: % -> %
    ++ cSec(f) computes the secant of the power series f.
    ++ For use when the coefficient ring is commutative.
  cCsc: % -> %
    ++ cCsc(f) computes the cosecant of the power series f.
    ++ For use when the coefficient ring is commutative.
  cAsin: % -> %
    ++ cAsin(f) computes the arcsine of the power series f.
    ++ For use when the coefficient ring is commutative.
  cAcos: % -> %
    ++ cAcos(f) computes the arccosine of the power series f.
    ++ For use when the coefficient ring is commutative.
  cAtan: % -> %
    ++ cAtan(f) computes the arctangent of the power series f.
    ++ For use when the coefficient ring is commutative.
  cAcot: % -> %
    ++ cAcot(f) computes the arccotangent of the power series f.
    ++ For use when the coefficient ring is commutative.
  cAsec: % -> %
    ++ cAsec(f) computes the arcsecant of the power series f.
    ++ For use when the coefficient ring is commutative.
  cAcsc: % -> %
    ++ cAcsc(f) computes the arccosecant of the power series f.
    ++ For use when the coefficient ring is commutative.
  cSinh: % -> %
    ++ cSinh(f) computes the hyperbolic sine of the power series f.
    ++ For use when the coefficient ring is commutative.
  cCosh: % -> %
```

```
        ++ cCosh(f) computes the hyperbolic cosine of the power series f.
        ++ For use when the coefficient ring is commutative.
      cTanh: % -> %
        ++ cTanh(f) computes the hyperbolic tangent of the power series f.
        ++ For use when the coefficient ring is commutative.
      cCoth: % -> %
        ++ cCoth(f) computes the hyperbolic cotangent of the power series f.
        ++ For use when the coefficient ring is commutative.
      cSech: % -> %
        ++ cSech(f) computes the hyperbolic secant of the power series f.
        ++ For use when the coefficient ring is commutative.
      cCsch: % -> %
        ++ cCsch(f) computes the hyperbolic cosecant of the power series f.
        ++ For use when the coefficient ring is commutative.
      cAsinh: % -> %
        ++ cAsinh(f) computes the inverse hyperbolic sine of the power
        ++ series f.  For use when the coefficient ring is commutative.
      cAcosh: % -> %
        ++ cAcosh(f) computes the inverse hyperbolic cosine of the power
        ++ series f.  For use when the coefficient ring is commutative.
      cAtanh: % -> %
        ++ cAtanh(f) computes the inverse hyperbolic tangent of the power
        ++ series f.  For use when the coefficient ring is commutative.
      cAcoth: % -> %
        ++ cAcoth(f) computes the inverse hyperbolic cotangent of the power
        ++ series f.  For use when the coefficient ring is commutative.
      cAsech: % -> %
        ++ cAsech(f) computes the inverse hyperbolic secant of the power
        ++ series f.  For use when the coefficient ring is commutative.
      cAcsch: % -> %
        ++ cAcsch(f) computes the inverse hyperbolic cosecant of the power
        ++ series f.  For use when the coefficient ring is commutative.

  Implementation ==> add
    import REF

    Rep := Record(%ord: REF,%str: Stream Term)
    -- when the value of 'ord' is n, this indicates that all non-zero
    -- terms of order up to and including n have been computed;
    -- when 'ord' is plusInfinity, all terms have been computed;
    -- lazy evaluation of 'str' has the side-effect of modifying the value
    -- of 'ord'

--% Local functions

    makeTerm:        (Integer,Coef) -> Term
    getCoef:         Term -> Coef
    getExpon:        Term -> Integer
    iSeries:         (ST,REF) -> ST
    iExtend:         (ST,COM,REF) -> ST
```

```
    iTruncate0:      (ST,REF,REF,COM,I,I) -> ST
    iTruncate:       (%,COM,I) -> %
    iCoefficient:    (ST,Integer) -> Coef
    iOrder:          (ST,COM,REF) -> I
    iMap1:           ((Coef,I) -> Coef,I -> I,B,ST,REF,REF,Integer) -> ST
    iMap2:           ((Coef,I) -> Coef,I -> I,B,%) -> %
    iPlus1:          ((Coef,Coef) -> Coef,ST,REF,ST,REF,REF,I) -> ST
    iPlus2:          ((Coef,Coef) -> Coef,%,%) -> %
    productByTerm:   (Coef,I,ST,REF,REF,I) -> ST
    productLazyEval: (ST,REF,ST,REF,COM) -> Void
    iTimes:          (ST,REF,ST,REF,REF,I) -> ST
    iDivide:         (ST,REF,ST,REF,Coef,I,REF,I) -> ST
    divide:          (%,I,%,I,Coef) -> %
    compose0:        (ST,REF,ST,REF,I,%,%,I,REF,I) -> ST
    factorials?:     () -> Boolean
    termOutput:      (RN,Coef,OUT) -> OUT
    showAll?:        () -> Boolean

--% macros

    makeTerm(exp,coef) == [exp,coef]
    getCoef term == term.c
    getExpon term == term.k

    makeSeries(refer,x) == [refer,x]
    getRef ups == ups.%ord
    getStream ups == ups.%str

--% creation and destruction of series

    monomial(coef,expon) ==
      nix : ST := empty()
      st :=
        zero? coef => nix
        concat(makeTerm(expon,coef),nix)
      makeSeries(ref plusInfinity(),st)

    monomial? ups == (not empty? getStream ups) and (empty? rst getStream ups)

    coerce(n:I)    == n :: Coef :: %
    coerce(r:Coef) == monomial(r,0)

    iSeries(x,refer) ==
      empty? x => (setelt(refer,plusInfinity()); empty())
      setelt(refer,(getExpon frst x) :: COM)
      concat(frst x,iSeries(rst x,refer))

    series(x:ST) ==
      empty? x => 0
      n := getExpon frst x; refer := ref(n :: COM)
```

```
      makeSeries(refer,iSeries(x,refer))

--% values

    characteristic() == characteristic()$Coef

    0 == monomial(0,0)
    1 == monomial(1,0)

    iExtend(st,n,refer) ==
      (elt refer) < n =>
        explicitlyEmpty? st => (setelt(refer,plusInfinity()); st)
        explicitEntries? st => iExtend(rst st,n,refer)
        iExtend(lazyEvaluate st,n,refer)
      st

    extend(x,n) == (iExtend(getStream x,n :: COM,getRef x); x)
    complete x  == (iExtend(getStream x,plusInfinity(),getRef x); x)

    iTruncate0(x,xRefer,refer,minExp,maxExp,n) == delay
      explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
      nn := n :: COM
      while (elt xRefer) < nn repeat lazyEvaluate x
      explicitEntries? x =>
        (nx := getExpon(xTerm := frst x)) > maxExp =>
          (setelt(refer,plusInfinity()); empty())
        setelt(refer,nx :: COM)
        (nx :: COM) >= minExp =>
          concat(makeTerm(nx,getCoef xTerm),_
                 iTruncate0(rst x,xRefer,refer,minExp,maxExp,nx + 1))
        iTruncate0(rst x,xRefer,refer,minExp,maxExp,nx + 1)
      -- can't have elt(xRefer) = infty unless all terms have been computed
      degr := retract(elt xRefer)@I
      setelt(refer,degr :: COM)
      iTruncate0(x,xRefer,refer,minExp,maxExp,degr + 1)

    iTruncate(ups,minExp,maxExp) ==
      x := getStream ups; xRefer := getRef ups
      explicitlyEmpty? x => 0
      explicitEntries? x =>
        deg := getExpon frst x
        refer := ref((deg - 1) :: COM)
        makeSeries(refer,iTruncate0(x,xRefer,refer,minExp,maxExp,deg))
      -- can't have elt(xRefer) = infty unless all terms have been computed
      degr := retract(elt xRefer)@I
      refer := ref(degr :: COM)
      makeSeries(refer,iTruncate0(x,xRefer,refer,minExp,maxExp,degr + 1))

    truncate(ups,n) == iTruncate(ups,minusInfinity(),n)
    truncate(ups,n1,n2) ==
```

```
      if n1 > n2 then (n1,n2) := (n2,n1)
      iTruncate(ups,n1 :: COM,n2)

    iCoefficient(st,n) ==
      explicitEntries? st =>
        term := frst st
        (expon := getExpon term) > n => 0
        expon = n => getCoef term
        iCoefficient(rst st,n)
      0

    coefficient(x,n)   == (extend(x,n); iCoefficient(getStream x,n))
    elt(x:%,n:Integer) == coefficient(x,n)

    iOrder(st,n,refer) ==
      explicitlyEmpty? st => error "order: series has infinite order"
      explicitEntries? st =>
        ((r := getExpon frst st) :: COM) >= n => retract(n)@Integer
        r
      -- can't have elt(xRefer) = infty unless all terms have been computed
      degr := retract(elt refer)@I
      (degr :: COM) >= n => retract(n)@Integer
      iOrder(lazyEvaluate st,n,refer)

    order x    == iOrder(getStream x,plusInfinity(),getRef x)
    order(x,n) == iOrder(getStream x,n :: COM,getRef x)

    terms x    == getStream x

--% predicates

    zero? ups ==
      x := getStream ups; ref := getRef ups
      whatInfinity(n := elt ref) = 1 => explicitlyEmpty? x
      count : NNI := _$streamCount$Lisp
      for i in 1..count repeat
        explicitlyEmpty? x => return true
        explicitEntries? x => return false
        lazyEvaluate x
      false

    ups1 = ups2 == zero?(ups1 - ups2)

--% arithmetic

    iMap1(cFcn,eFcn,check?,x,xRefer,refer,n) == delay
      -- when this function is called, all terms in 'x' of order < n have been
      -- computed and we compute the eFcn(n)th order coefficient of the result
      explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
      -- if terms in 'x' up to order n have not been computed,
```

```
    -- apply lazy evaluation
    nn := n :: COM
    while (elt xRefer) < nn repeat lazyEvaluate x
    -- 'x' may now be empty: retest
    explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
    -- must have nx >= n
    explicitEntries? x =>
      xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
      newCoef := cFcn(xCoef,nx); m := eFcn nx
      setelt(refer,m :: COM)
      not check? =>
        concat(makeTerm(m,newCoef),_
               iMap1(cFcn,eFcn,check?,rst x,xRefer,refer,nx + 1))
      zero? newCoef => iMap1(cFcn,eFcn,check?,rst x,xRefer,refer,nx + 1)
      concat(makeTerm(m,newCoef),_
             iMap1(cFcn,eFcn,check?,rst x,xRefer,refer,nx + 1))
    -- can't have elt(xRefer) = infty unless all terms have been computed
    degr := retract(elt xRefer)@I
    setelt(refer,eFcn(degr) :: COM)
    iMap1(cFcn,eFcn,check?,x,xRefer,refer,degr + 1)

  iMap2(cFcn,eFcn,check?,ups) ==
    -- 'eFcn' must be a strictly increasing function,
    -- i.e. i < j => eFcn(i) < eFcn(j)
    xRefer := getRef ups; x := getStream ups
    explicitlyEmpty? x => 0
    explicitEntries? x =>
      deg := getExpon frst x
      refer := ref(eFcn(deg - 1) :: COM)
      makeSeries(refer,iMap1(cFcn,eFcn,check?,x,xRefer,refer,deg))
    -- can't have elt(xRefer) = infty unless all terms have been computed
    degr := retract(elt xRefer)@I
    refer := ref(eFcn(degr) :: COM)
    makeSeries(refer,iMap1(cFcn,eFcn,check?,x,xRefer,refer,degr + 1))

  map(fcn,x)                == iMap2((y,n) +-> fcn(y), z +->z,      true, x)
  differentiate x           == iMap2((y,n) +-> n*y,    z +-> z - 1, true, x)
  multiplyCoefficients(f,x) == iMap2((y,n) +-> f(n)*y, z +-> z,     true, x)
  multiplyExponents(x,n)    == iMap2((y,m) +-> y,      z +-> n*z,   false, x)

  iPlus1(op,x,xRefer,y,yRefer,refer,n) == delay
    -- when this function is called, all terms in 'x' and 'y' of order < n
    -- have been computed and we are computing the nth order coefficient of
    -- the result; note the 'op' is either '+' or '-'
    explicitlyEmpty? x =>
      iMap1((x1,m) +-> op(0,x1), z +-> z, false, y, yRefer, refer, n)
    explicitlyEmpty? y =>
      iMap1((x1,m) +-> op(x1,0), z +-> z, false, x, xRefer, refer, n)
    -- if terms up to order n have not been computed,
    -- apply lazy evaluation
```

```
nn := n :: COM
while (elt xRefer) < nn repeat lazyEvaluate x
while (elt yRefer) < nn repeat lazyEvaluate y
-- 'x' or 'y' may now be empty: retest
explicitlyEmpty? x =>
  iMap1((x1,m) +-> op(0,x1), z +-> z, false, y, yRefer, refer, n)
explicitlyEmpty? y =>
  iMap1((x1,m) +-> op(x1,0), z +-> z, false, x, xRefer, refer, n)
-- must have nx >= n, ny >= n
-- both x and y have explicit terms
explicitEntries?(x) and explicitEntries?(y) =>
  xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
  yCoef := getCoef(yTerm := frst y); ny := getExpon yTerm
  nx = ny =>
    setelt(refer,nx :: COM)
    zero? (coef := op(xCoef,yCoef)) =>
      iPlus1(op,rst x,xRefer,rst y,yRefer,refer,nx + 1)
    concat(makeTerm(nx,coef),_
           iPlus1(op,rst x,xRefer,rst y,yRefer,refer,nx + 1))
  nx < ny =>
    setelt(refer,nx :: COM)
    concat(makeTerm(nx,op(xCoef,0)),_
           iPlus1(op,rst x,xRefer,y,yRefer,refer,nx + 1))
  setelt(refer,ny :: COM)
  concat(makeTerm(ny,op(0,yCoef)),_
         iPlus1(op,x,xRefer,rst y,yRefer,refer,ny + 1))
-- y has no term of degree n
explicitEntries? x =>
  xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
  -- can't have elt(yRefer) = infty unless all terms have been computed
  (degr := retract(elt yRefer)@I) < nx =>
    setelt(refer,elt yRefer)
    iPlus1(op,x,xRefer,y,yRefer,refer,degr + 1)
  setelt(refer,nx :: COM)
  concat(makeTerm(nx,op(xCoef,0)),_
         iPlus1(op,rst x,xRefer,y,yRefer,refer,nx + 1))
-- x has no term of degree n
explicitEntries? y =>
  yCoef := getCoef(yTerm := frst y); ny := getExpon yTerm
  -- can't have elt(xRefer) = infty unless all terms have been computed
  (degr := retract(elt xRefer)@I) < ny =>
    setelt(refer,elt xRefer)
    iPlus1(op,x,xRefer,y,yRefer,refer,degr + 1)
  setelt(refer,ny :: COM)
  concat(makeTerm(ny,op(0,yCoef)),_
         iPlus1(op,x,xRefer,rst y,yRefer,refer,ny + 1))
-- neither x nor y has a term of degree n
setelt(refer,xyRef := min(elt xRefer,elt yRefer))
-- can't have xyRef = infty unless all terms have been computed
iPlus1(op,x,xRefer,y,yRefer,refer,retract(xyRef)@I + 1)
```

```
iPlus2(op,ups1,ups2) ==
  xRefer := getRef ups1; x := getStream ups1
  xDeg :=
    explicitlyEmpty? x => return map(z +-> op(0$Coef,z),ups2)
    explicitEntries? x => (getExpon frst x) - 1
    -- can't have elt(xRefer) = infty unless all terms have been computed
    retract(elt xRefer)@I
  yRefer := getRef ups2; y := getStream ups2
  yDeg :=
    explicitlyEmpty? y => return map(z +-> op(z,0$Coef),ups1)
    explicitEntries? y => (getExpon frst y) - 1
    -- can't have elt(yRefer) = infty unless all terms have been computed
    retract(elt yRefer)@I
  deg := min(xDeg,yDeg); refer := ref(deg :: COM)
  makeSeries(refer,iPlus1(op,x,xRefer,y,yRefer,refer,deg + 1))

x + y == iPlus2((xi,yi) +-> xi + yi, x, y)
x - y == iPlus2((xi,yi) +-> xi - yi, x, y)
- y   == iMap2((x,n) +-> -x, z +-> z, false, y)

-- gives correct defaults for I, NNI and PI
n:I   * x:% == (zero? n => 0; map(z +-> n*z, x))
n:NNI * x:% == (zero? n => 0; map(z +-> n*z, x))
n:PI  * x:% == (zero? n => 0; map(z +-> n*z, x))

productByTerm(coef,expon,x,xRefer,refer,n) ==
  iMap1((y,m) +-> coef*y, z +-> z+expon, true, x, xRefer, refer, n)

productLazyEval(x,xRefer,y,yRefer,nn) ==
  explicitlyEmpty?(x) or explicitlyEmpty?(y) => void()
  explicitEntries? x =>
    explicitEntries? y => void()
    xDeg := (getExpon frst x) :: COM
    while (xDeg + elt(yRefer)) < nn repeat lazyEvaluate y
    void()
  explicitEntries? y =>
    yDeg := (getExpon frst y) :: COM
    while (yDeg + elt(xRefer)) < nn repeat lazyEvaluate x
    void()
  lazyEvaluate x
  -- if x = y, then y may now have explicit entries
  if lazy? y then lazyEvaluate y
  productLazyEval(x,xRefer,y,yRefer,nn)

iTimes(x,xRefer,y,yRefer,refer,n) == delay
  -- when this function is called, we are computing the nth order
  -- coefficient of the product
  productLazyEval(x,xRefer,y,yRefer,n :: COM)
  explicitlyEmpty?(x) or explicitlyEmpty?(y) =>
```

```
    (setelt(refer,plusInfinity()); empty())
  -- must have nx + ny >= n
  explicitEntries?(x) and explicitEntries?(y) =>
    xCoef := getCoef(xTerm := frst x); xExpon := getExpon xTerm
    yCoef := getCoef(yTerm := frst y); yExpon := getExpon yTerm
    expon := xExpon + yExpon
    setelt(refer,expon :: COM)
    scRefer := ref(expon :: COM)
    scMult := productByTerm(xCoef,xExpon,rst y,yRefer,scRefer,yExpon + 1)
    prRefer := ref(expon :: COM)
    pr := iTimes(rst x,xRefer,y,yRefer,prRefer,expon + 1)
    sm := iPlus1((a,b) +-> a+b,scMult,scRefer,pr,prRefer,refer,expon + 1)
    zero?(coef := xCoef * yCoef) => sm
    concat(makeTerm(expon,coef),sm)
  explicitEntries? x =>
    xExpon := getExpon frst x
    -- can't have elt(yRefer) = infty unless all terms have been computed
    degr := retract(elt yRefer)@I
    setelt(refer,(xExpon + degr) :: COM)
    iTimes(x,xRefer,y,yRefer,refer,xExpon + degr + 1)
  explicitEntries? y =>
    yExpon := getExpon frst y
    -- can't have elt(xRefer) = infty unless all terms have been computed
    degr := retract(elt xRefer)@I
    setelt(refer,(yExpon + degr) :: COM)
    iTimes(x,xRefer,y,yRefer,refer,yExpon + degr + 1)
  -- can't have elt(xRefer) = infty unless all terms have been computed
  xDegr := retract(elt xRefer)@I
  yDegr := retract(elt yRefer)@I
  setelt(refer,(xDegr + yDegr) :: COM)
  iTimes(x,xRefer,y,yRefer,refer,xDegr + yDegr + 1)

ups1:% * ups2:% ==
  xRefer := getRef ups1; x := getStream ups1
  xDeg :=
    explicitlyEmpty? x => return 0
    explicitEntries? x => (getExpon frst x) - 1
    -- can't have elt(xRefer) = infty unless all terms have been computed
    retract(elt xRefer)@I
  yRefer := getRef ups2; y := getStream ups2
  yDeg :=
    explicitlyEmpty? y => return 0
    explicitEntries? y => (getExpon frst y) - 1
    -- can't have elt(yRefer) = infty unless all terms have been computed
    retract(elt yRefer)@I
  deg := xDeg + yDeg + 1; refer := ref(deg :: COM)
  makeSeries(refer,iTimes(x,xRefer,y,yRefer,refer,deg + 1))

iDivide(x,xRefer,y,yRefer,rym,m,refer,n) == delay
  -- when this function is called, we are computing the nth order
```

```
  -- coefficient of the result
  explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
  -- if terms up to order n - m have not been computed,
  -- apply lazy evaluation
  nm := (n + m) :: COM
  while (elt xRefer) < nm repeat lazyEvaluate x
  -- 'x' may now be empty: retest
  explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
  -- must have nx >= n + m
  explicitEntries? x =>
    newCoef := getCoef(xTerm := frst x) * rym; nx := getExpon xTerm
    prodRefer := ref(nx :: COM)
    prod := productByTerm(-newCoef,nx - m,rst y,yRefer,prodRefer,1)
    sumRefer := ref(nx :: COM)
    sum := iPlus1((a,b)+->a+b,rst x,xRefer,prod,prodRefer,sumRefer,nx + 1)
    setelt(refer,(nx - m) :: COM); term := makeTerm(nx - m,newCoef)
    concat(term,iDivide(sum,sumRefer,y,yRefer,rym,m,refer,nx - m + 1))
  -- can't have elt(xRefer) = infty unless all terms have been computed
  degr := retract(elt xRefer)@I
  setelt(refer,(degr - m) :: COM)
  iDivide(x,xRefer,y,yRefer,rym,m,refer,degr - m + 1)

divide(ups1,deg1,ups2,deg2,r) ==
  xRefer := getRef ups1; x := getStream ups1
  yRefer := getRef ups2; y := getStream ups2
  refer := ref((deg1 - deg2) :: COM)
  makeSeries(refer,iDivide(x,xRefer,y,yRefer,r,deg2,refer,deg1 - deg2 + 1))

iExquo(ups1,ups2,taylor?) ==
  xRefer := getRef ups1; x := getStream ups1
  yRefer := getRef ups2; y := getStream ups2
  n : I := 0
  -- try to find first non-zero term in y
  -- give up after 1000 lazy evaluations
  while not explicitEntries? y repeat
    explicitlyEmpty? y => return "failed"
    lazyEvaluate y
    (n := n + 1) > 1000 => return "failed"
  yCoef := getCoef(yTerm := frst y); ny := getExpon yTerm
  (ry := recip yCoef) case "failed" => "failed"
  nn := ny :: COM
  if taylor? then
    while (elt(xRefer) < nn) repeat
      explicitlyEmpty? x => return 0
      explicitEntries? x => return "failed"
      lazyEvaluate x
  -- check if ups2 is a monomial
  empty? rst y => iMap2((y1,m) +-> y1*(ry::Coef),z +->z-ny, false, ups1)
  explicitlyEmpty? x => 0
  nx :=
```

```
      explicitEntries? x =>
        ((deg := getExpon frst x) < ny) and taylor? => return "failed"
        deg - 1
      -- can't have elt(xRefer) = infty unless all terms have been computed
      retract(elt xRefer)@I
    divide(ups1,nx,ups2,ny,ry :: Coef)

  taylorQuoByVar ups ==
    iMap2((y,n) +-> y, z +-> z-1,false,ups - monomial(coefficient(ups,0),0))

  compose0(x,xRefer,y,yRefer,yOrd,y1,yn0,n0,refer,n) == delay
    -- when this function is called, we are computing the nth order
    -- coefficient of the composite
    explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
    -- if terms in 'x' up to order n have not been computed,
    -- apply lazy evaluation
    nn := n :: COM; yyOrd := yOrd :: COM
    while (yyOrd * elt(xRefer)) < nn repeat lazyEvaluate x
    explicitEntries? x =>
      xCoef := getCoef(xTerm := frst x); n1 := getExpon xTerm
      zero? n1 =>
        setelt(refer,n1 :: COM)
        concat(makeTerm(n1,xCoef),_
               compose0(rst x,xRefer,y,yRefer,yOrd,y1,yn0,n0,refer,n1 + 1))
      yn1 := yn0 * y1 ** ((n1 - n0) :: NNI)
      z := getStream yn1; zRefer := getRef yn1
      degr := yOrd * n1; prodRefer := ref((degr - 1) :: COM)
      prod := iMap1((s,k)+->xCoef*s,m+->m,true,z,zRefer,prodRefer,degr)
      coRefer := ref((degr + yOrd - 1) :: COM)
      co := compose0(rst x,xRefer,y,yRefer,yOrd,y1,yn1,n1,coRefer,degr+yOrd)
      setelt(refer,(degr - 1) :: COM)
      iPlus1((a,b)+->a+b,prod,prodRefer,co,coRefer,refer,degr)
    -- can't have elt(xRefer) = infty unless all terms have been computed
    degr := yOrd * (retract(elt xRefer)@I + 1)
    setelt(refer,(degr - 1) :: COM)
    compose0(x,xRefer,y,yRefer,yOrd,y1,yn0,n0,refer,degr)

  iCompose(ups1,ups2) ==
    x := getStream ups1; xRefer := getRef ups1
    y := getStream ups2; yRefer := getRef ups2
    -- try to compute the order of 'ups2'
    n : I := _$streamCount$Lisp
    for i in 1..n while not explicitEntries? y repeat
      explicitlyEmpty? y => coefficient(ups1,0) :: %
      lazyEvaluate y
    explicitlyEmpty? y => coefficient(ups1,0) :: %
    yOrd : I :=
      explicitEntries? y => getExpon frst y
      retract(elt yRefer)@I
    compRefer := ref((-1) :: COM)
```

```
      makeSeries(compRefer,_
                 compose0(x,xRefer,y,yRefer,yOrd,ups2,1,0,compRefer,0))

    if Coef has Algebra Fraction Integer then

      integrate x == iMap2((y,n) +-> 1/(n+1)*y, z +-> z+1, true, x)

--% Fixed point computations

      Ys ==> Y$ParadoxicalCombinatorsForStreams(Term)

      integ0: (ST,REF,REF,I) -> ST
      integ0(x,intRef,ansRef,n) == delay
        nLess1 := (n - 1) :: COM
        while (elt intRef) < nLess1 repeat lazyEvaluate x
        explicitlyEmpty? x => (setelt(ansRef,plusInfinity()); empty())
        explicitEntries? x =>
          xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
          setelt(ansRef,(n1 := (nx + 1)) :: COM)
          concat(makeTerm(n1,inv(n1 :: RN) * xCoef),_
                 integ0(rst x,intRef,ansRef,n1))
        -- can't have elt(intRef) = infty unless all terms have been computed
        degr := retract(elt intRef)@I; setelt(ansRef,(degr + 1) :: COM)
        integ0(x,intRef,ansRef,degr + 2)

      integ1: (ST,REF,REF) -> ST
      integ1(x,intRef,ansRef) == integ0(x,intRef,ansRef,1)

      lazyInteg: (Coef,() -> ST,REF,REF) -> ST
      lazyInteg(a,xf,intRef,ansRef) ==
        ansStr : ST := integ1(delay xf,intRef,ansRef)
        concat(makeTerm(0,a),ansStr)

      cPower(f,r) ==
        -- computes f^r.  f should have constant coefficient 1.
        fp := differentiate f
        fInv := iExquo(1,f,false) :: %; y := r * fp * fInv
        yRef := getRef y; yStr := getStream y
        intRef := ref((-1) :: COM); ansRef := ref(0 :: COM)
        ansStr :=
         Ys(s+->lazyInteg(1,iTimes(s,ansRef,yStr,yRef,intRef,0),intRef,ansRef))
        makeSeries(ansRef,ansStr)

      iExp: (%,Coef) -> %
      iExp(f,cc) ==
        -- computes exp(f).  cc = exp coefficient(f,0)
        fp := differentiate f
        fpRef := getRef fp; fpStr := getStream fp
        intRef := ref((-1) :: COM); ansRef := ref(0 :: COM)
        ansStr :=
```

```
         Ys(s+->lazyInteg(cc,
                  iTimes(s,ansRef,fpStr,fpRef,intRef,0),intRef,ansRef))
        makeSeries(ansRef,ansStr)

      sincos0: (Coef,Coef,L ST,REF,REF,ST,REF,ST,REF) -> L ST
      sincos0(sinc,cosc,list,sinRef,cosRef,fpStr,fpRef,fpStr2,fpRef2) ==
        sinStr := first list; cosStr := second list
        prodRef1 := ref((-1) :: COM); prodRef2 := ref((-1) :: COM)
        prodStr1 := iTimes(cosStr,cosRef,fpStr,fpRef,prodRef1,0)
        prodStr2 := iTimes(sinStr,sinRef,fpStr2,fpRef2,prodRef2,0)
        [lazyInteg(sinc,prodStr1,prodRef1,sinRef),_
         lazyInteg(cosc,prodStr2,prodRef2,cosRef)]

      iSincos: (%,Coef,Coef,I) -> Record(%sin: %, %cos: %)
      iSincos(f,sinc,cosc,sign) ==
        fp := differentiate f
        fpRef := getRef fp; fpStr := getStream fp
--        fp2 := (one? sign => fp; -fp)
        fp2 := ((sign = 1) => fp; -fp)
        fpRef2 := getRef fp2; fpStr2 := getStream fp2
        sinRef := ref(0 :: COM); cosRef := ref(0 :: COM)
        sincos :=
         Ys(s+->sincos0(sinc,cosc,s,sinRef,cosRef,fpStr,fpRef,fpStr2,fpRef2),2)
        sinStr := (zero? sinc => rst first sincos; first sincos)
        cosStr := (zero? cosc => rst second sincos; second sincos)
        [makeSeries(sinRef,sinStr),makeSeries(cosRef,cosStr)]

      tan0: (Coef,ST,REF,ST,REF,I) -> ST
      tan0(cc,ansStr,ansRef,fpStr,fpRef,sign) ==
        sqRef := ref((-1) :: COM)
        sqStr := iTimes(ansStr,ansRef,ansStr,ansRef,sqRef,0)
        one : % := 1; oneStr := getStream one; oneRef := getRef one
        yRef := ref((-1) :: COM)
        yStr : ST :=
--          one? sign => iPlus1(#1 + #2,oneStr,oneRef,sqStr,sqRef,yRef,0)
          (sign = 1) => iPlus1((a,b)+->a+b,oneStr,oneRef,sqStr,sqRef,yRef,0)
          iPlus1((a,b)+->a-b,oneStr,oneRef,sqStr,sqRef,yRef,0)
        intRef := ref((-1) :: COM)
        lazyInteg(cc,iTimes(yStr,yRef,fpStr,fpRef,intRef,0),intRef,ansRef)

      iTan: (%,%,Coef,I) -> %
      iTan(f,fp,cc,sign) ==
        -- computes the tangent (and related functions) of f.
        fpRef := getRef fp; fpStr := getStream fp
        ansRef := ref(0 :: COM)
        ansStr := Ys(s+->tan0(cc,s,ansRef,fpStr,fpRef,sign))
        zero? cc => makeSeries(ansRef,rst ansStr)
        makeSeries(ansRef,ansStr)

--% Error Reporting
```

```
      TRCONST : SG := "series expansion involves transcendental constants"
      NPOWERS : SG := "series expansion has terms of negative degree"
      FPOWERS : SG := "series expansion has terms of fractional degree"
      MAYFPOW : SG := "series expansion may have terms of fractional degree"
      LOGS : SG := "series expansion has logarithmic term"
      NPOWLOG : SG :=
         "series expansion has terms of negative degree or logarithmic term"
      NOTINV : SG := "leading coefficient not invertible"

--% Rational powers and transcendental functions

      orderOrFailed : % -> Union(I,"failed")
      orderOrFailed uts ==
      -- returns the order of x or "failed"
      -- if -1 is returned, the series is identically zero
        x := getStream uts
        for n in 0..1000 repeat
          explicitlyEmpty? x => return -1
          explicitEntries? x => return getExpon frst x
          lazyEvaluate x
        "failed"

      RATPOWERS : Boolean := Coef has "**": (Coef,RN) -> Coef
      TRANSFCN  : Boolean := Coef has TranscendentalFunctionCategory

      cRationalPower(uts,r) ==
        (ord0 := orderOrFailed uts) case "failed" =>
          error "**: series with many leading zero coefficients"
        order := ord0 :: I
        (n := order exquo denom(r)) case "failed" =>
          error "**: rational power does not exist"
        cc := coefficient(uts,order)
        (ccInv := recip cc) case "failed" => error concat("**: ",NOTINV)
        ccPow :=
--          one? cc => cc
          (cc = 1) => cc
--          one? denom r =>
          (denom r) = 1 =>
            not negative?(num := numer r) => cc ** (num :: NNI)
            (ccInv :: Coef) ** ((-num) :: NNI)
          RATPOWERS => cc ** r
          error "** rational power of coefficient undefined"
        uts1 := (ccInv :: Coef) * uts
        uts2 := uts1 * monomial(1,-order)
        monomial(ccPow,(n :: I) * numer(r)) * cPower(uts2,r :: Coef)

      cExp uts ==
        zero?(cc := coefficient(uts,0)) => iExp(uts,1)
        TRANSFCN => iExp(uts,exp cc)
```

```
        error concat("exp: ",TRCONST)

      cLog uts ==
        zero?(cc := coefficient(uts,0)) =>
          error "log: constant coefficient should not be 0"
--        one? cc => integrate(differentiate(uts) * (iExquo(1,uts,true) :: %))
        (cc = 1) => integrate(differentiate(uts) * (iExquo(1,uts,true) :: %))
        TRANSFCN =>
          y := iExquo(1,uts,true) :: %
          (log(cc) :: %) + integrate(y * differentiate(uts))
        error concat("log: ",TRCONST)

      sincos: % -> Record(%sin: %, %cos: %)
      sincos uts ==
        zero?(cc := coefficient(uts,0)) => iSincos(uts,0,1,-1)
        TRANSFCN => iSincos(uts,sin cc,cos cc,-1)
        error concat("sincos: ",TRCONST)

      cSin uts == sincos(uts).%sin
      cCos uts == sincos(uts).%cos

      cTan uts ==
        zero?(cc := coefficient(uts,0)) => iTan(uts,differentiate uts,0,1)
        TRANSFCN => iTan(uts,differentiate uts,tan cc,1)
        error concat("tan: ",TRCONST)

      cCot uts ==
        zero? uts => error "cot: cot(0) is undefined"
        zero?(cc := coefficient(uts,0)) => error error concat("cot: ",NPOWERS)
        TRANSFCN => iTan(uts,-differentiate uts,cot cc,1)
        error concat("cot: ",TRCONST)

      cSec uts ==
        zero?(cc := coefficient(uts,0)) => iExquo(1,cCos uts,true) :: %
        TRANSFCN =>
          cosUts := cCos uts
          zero? coefficient(cosUts,0) => error concat("sec: ",NPOWERS)
          iExquo(1,cosUts,true) :: %
        error concat("sec: ",TRCONST)

      cCsc uts ==
        zero? uts => error "csc: csc(0) is undefined"
        TRANSFCN =>
          sinUts := cSin uts
          zero? coefficient(sinUts,0) => error concat("csc: ",NPOWERS)
          iExquo(1,sinUts,true) :: %
        error concat("csc: ",TRCONST)

      cAsin uts ==
        zero?(cc := coefficient(uts,0)) =>
```

```
    integrate(cRationalPower(1 - uts*uts,-1/2) * differentiate(uts))
  TRANSFCN =>
    x := 1 - uts * uts
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("asin: ",MAYFPOW)
      (order := ord :: I) = -1 => return asin(cc) :: %
      odd? order => error concat("asin: ",FPOWERS)
      c0 := asin(cc) :: %
      c0 + integrate(cRationalPower(x,-1/2) * differentiate(uts))
    c0 := asin(cc) :: %
    c0 + integrate(cRationalPower(x,-1/2) * differentiate(uts))
  error concat("asin: ",TRCONST)

cAcos uts ==
  zero? uts =>
    TRANSFCN => acos(0)$Coef :: %
    error concat("acos: ",TRCONST)
  TRANSFCN =>
    x := 1 - uts * uts
    cc := coefficient(uts,0)
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("acos: ",MAYFPOW)
      (order := ord :: I) = -1 => return acos(cc) :: %
      odd? order => error concat("acos: ",FPOWERS)
      c0 := acos(cc) :: %
      c0 + integrate(-cRationalPower(x,-1/2) * differentiate(uts))
    c0 := acos(cc) :: %
    c0 + integrate(-cRationalPower(x,-1/2) * differentiate(uts))
  error concat("acos: ",TRCONST)

cAtan uts ==
  zero?(cc := coefficient(uts,0)) =>
    y := iExquo(1,(1 :: %) + uts*uts,true) :: %
    integrate(y * (differentiate uts))
  TRANSFCN =>
    (y := iExquo(1,(1 :: %) + uts*uts,true)) case "failed" =>
      error concat("atan: ",LOGS)
    (atan(cc) :: %) + integrate((y :: %) * (differentiate uts))
  error concat("atan: ",TRCONST)

cAcot uts ==
  TRANSFCN =>
    (y := iExquo(1,(1 :: %) + uts*uts,true)) case "failed" =>
      error concat("acot: ",LOGS)
    cc := coefficient(uts,0)
    (acot(cc) :: %) + integrate(-(y :: %) * (differentiate uts))
```

```
    error concat("acot: ",TRCONST)

  cAsec uts ==
    zero?(cc := coefficient(uts,0)) =>
      error "asec: constant coefficient should not be 0"
    TRANSFCN =>
      x := uts * uts - 1
      y :=
        cc = 1 or cc = -1 =>
          -- compute order of 'x'
          (ord := orderOrFailed x) case "failed" =>
            error concat("asec: ",MAYFPOW)
          (order := ord :: I) = -1 => return asec(cc) :: %
          odd? order => error concat("asec: ",FPOWERS)
          cRationalPower(x,-1/2) * differentiate(uts)
        cRationalPower(x,-1/2) * differentiate(uts)
      (z := iExquo(y,uts,true)) case "failed" =>
        error concat("asec: ",NOTINV)
      (asec(cc) :: %) + integrate(z :: %)
    error concat("asec: ",TRCONST)

  cAcsc uts ==
    zero?(cc := coefficient(uts,0)) =>
      error "acsc: constant coefficient should not be 0"
    TRANSFCN =>
      x := uts * uts - 1
      y :=
        cc = 1 or cc = -1 =>
          -- compute order of 'x'
          (ord := orderOrFailed x) case "failed" =>
            error concat("acsc: ",MAYFPOW)
          (order := ord :: I) = -1 => return acsc(cc) :: %
          odd? order => error concat("acsc: ",FPOWERS)
          -cRationalPower(x,-1/2) * differentiate(uts)
        -cRationalPower(x,-1/2) * differentiate(uts)
      (z := iExquo(y,uts,true)) case "failed" =>
        error concat("asec: ",NOTINV)
      (acsc(cc) :: %) + integrate(z :: %)
    error concat("acsc: ",TRCONST)

  sinhcosh: % -> Record(%sinh: %, %cosh: %)
  sinhcosh uts ==
    zero?(cc := coefficient(uts,0)) =>
      tmp := iSincos(uts,0,1,1)
      [tmp.%sin,tmp.%cos]
    TRANSFCN =>
      tmp := iSincos(uts,sinh cc,cosh cc,1)
      [tmp.%sin,tmp.%cos]
    error concat("sinhcosh: ",TRCONST)
```

```
cSinh uts == sinhcosh(uts).%sinh
cCosh uts == sinhcosh(uts).%cosh

cTanh uts ==
  zero?(cc := coefficient(uts,0)) => iTan(uts,differentiate uts,0,-1)
  TRANSFCN => iTan(uts,differentiate uts,tanh cc,-1)
  error concat("tanh: ",TRCONST)

cCoth uts ==
  tanhUts := cTanh uts
  zero? tanhUts => error "coth: coth(0) is undefined"
  zero? coefficient(tanhUts,0) => error concat("coth: ",NPOWERS)
  iExquo(1,tanhUts,true) :: %

cSech uts ==
  coshUts := cCosh uts
  zero? coefficient(coshUts,0) => error concat("sech: ",NPOWERS)
  iExquo(1,coshUts,true) :: %

cCsch uts ==
  sinhUts := cSinh uts
  zero? coefficient(sinhUts,0) => error concat("csch: ",NPOWERS)
  iExquo(1,sinhUts,true) :: %

cAsinh uts ==
  x := 1 + uts * uts
  zero?(cc := coefficient(uts,0)) => cLog(uts + cRationalPower(x,1/2))
  TRANSFCN =>
    (ord := orderOrFailed x) case "failed" =>
      error concat("asinh: ",MAYFPOW)
    (order := ord :: I) = -1 => return asinh(cc) :: %
    odd? order => error concat("asinh: ",FPOWERS)
    -- the argument to 'log' must have a non-zero constant term
    cLog(uts + cRationalPower(x,1/2))
  error concat("asinh: ",TRCONST)

cAcosh uts ==
  zero? uts =>
    TRANSFCN => acosh(0)$Coef :: %
    error concat("acosh: ",TRCONST)
  TRANSFCN =>
    cc := coefficient(uts,0); x := uts*uts - 1
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("acosh: ",MAYFPOW)
      (order := ord :: I) = -1 => return acosh(cc) :: %
      odd? order => error concat("acosh: ",FPOWERS)
      -- the argument to 'log' must have a non-zero constant term
      cLog(uts + cRationalPower(x,1/2))
```

```
      cLog(uts + cRationalPower(x,1/2))
    error concat("acosh: ",TRCONST)

  cAtanh uts ==
    half := inv(2 :: RN) :: Coef
    zero?(cc := coefficient(uts,0)) =>
      half * (cLog(1 + uts) - cLog(1 - uts))
    TRANSFCN =>
      cc = 1 or cc = -1 => error concat("atanh: ",LOGS)
      half * (cLog(1 + uts) - cLog(1 - uts))
    error concat("atanh: ",TRCONST)

  cAcoth uts ==
    zero? uts =>
      TRANSFCN => acoth(0)$Coef :: %
      error concat("acoth: ",TRCONST)
    TRANSFCN =>
      cc := coefficient(uts,0); half := inv(2 :: RN) :: Coef
      cc = 1 or cc = -1 => error concat("acoth: ",LOGS)
      half * (cLog(uts + 1) - cLog(uts - 1))
    error concat("acoth: ",TRCONST)

  cAsech uts ==
    zero? uts => error "asech: asech(0) is undefined"
    TRANSFCN =>
      zero?(cc := coefficient(uts,0)) =>
        error concat("asech: ",NPOWLOG)
      x := 1 - uts * uts
      cc = 1 or cc = -1 =>
        -- compute order of 'x'
        (ord := orderOrFailed x) case "failed" =>
          error concat("asech: ",MAYFPOW)
        (order := ord :: I) = -1 => return asech(cc) :: %
        odd? order => error concat("asech: ",FPOWERS)
        (utsInv := iExquo(1,uts,true)) case "failed" =>
          error concat("asech: ",NOTINV)
        cLog((1 + cRationalPower(x,1/2)) * (utsInv :: %))
      (utsInv := iExquo(1,uts,true)) case "failed" =>
        error concat("asech: ",NOTINV)
      cLog((1 + cRationalPower(x,1/2)) * (utsInv :: %))
    error concat("asech: ",TRCONST)

  cAcsch uts ==
    zero? uts => error "acsch: acsch(0) is undefined"
    TRANSFCN =>
      zero?(cc := coefficient(uts,0)) => error concat("acsch: ",NPOWLOG)
      x := uts * uts + 1
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("acsc: ",MAYFPOW)
```

```
          (order := ord :: I) = -1 => return acsch(cc) :: %
          odd? order => error concat("acsch: ",FPOWERS)
          (utsInv := iExquo(1,uts,true)) case "failed" =>
            error concat("acsch: ",NOTINV)
          cLog((1 + cRationalPower(x,1/2)) * (utsInv :: %))
        error concat("acsch: ",TRCONST)

--% Output forms

    -- check a global Lisp variable
    factorials?() == false

    termOutput(k,c,vv) ==
    -- creates a term c * vv ** k
      k = 0 => c :: OUT
      mon := (k = 1 => vv; vv ** (k :: OUT))
--       if factorials?() and k > 1 then
--         c := factorial(k)$IntegerCombinatoricFunctions * c
--         mon := mon / hconcat(k :: OUT,"!" :: OUT)
      c = 1 => mon
      c = -1 => -mon
      (c :: OUT) * mon

    -- check a global Lisp variable
    showAll?() == true

    seriesToOutputForm(st,refer,var,cen,r) ==
      vv :=
        zero? cen => var :: OUT
        paren(var :: OUT - cen :: OUT)
      l : L OUT := empty()
      while explicitEntries? st repeat
        term := frst st
        l := concat(termOutput(getExpon(term) * r,getCoef term,vv),l)
        st := rst st
      l :=
        explicitlyEmpty? st => l
        (deg := retractIfCan(elt refer)@Union(I,"failed")) case I =>
          concat(prefix("O" :: OUT,[vv ** ((((deg :: I) + 1) * r) :: OUT)]),l)
        l
      empty? l => (0$Coef) :: OUT
      reduce("+",reverse_! l)
```

— ISUPS.dotabb —

```
"ISUPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ISUPS"]
```

```
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ISUPS" -> "ALIST"
```

———-

10.27 domain INTABL InnerTable

— InnerTable.input —

```
)set break resume
)sys rm -f InnerTable.output
)spool InnerTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerTable
--R InnerTable(Key: SetCategory,Entry: SetCategory,addDom) where
--R   addDom: TableAggregate(Key,Entry) with
--R        finiteAggregate  is a domain constructor
--R Abbreviation for InnerTable is INTABL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INTABL
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         dictionary : () -> %
--R elt : (%,Key,Entry) -> Entry          ?.? : (%,Key) -> Entry
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Entry             eq? : (%,%) -> Boolean
--R index? : (Key,%) -> Boolean           indices : % -> List Key
--R key? : (Key,%) -> Boolean             keys : % -> List Key
--R map : ((Entry -> Entry),%) -> %       qelt : (%,Key) -> Entry
--R sample : () -> %                      setelt : (%,Key,Entry) -> Entry
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Key,entry: Entry) -> %
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if $ has finiteAggregate and Entry has SETCAT
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if $ has finiteAggregate and Record(
```

```
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has fi
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(ke
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: En
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggreg
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: En
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCA
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ ha
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if $ has finiteAggregate and Reco
--R members : % -> List Entry if $ has finiteAggregate
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Entry if $ has finiteAggregate
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key:
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key:
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key:
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate and Record(key:
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Key,entry: Entry) has
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
```

```
--R table : List Record(key: Key,entry: Entry) -> %
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

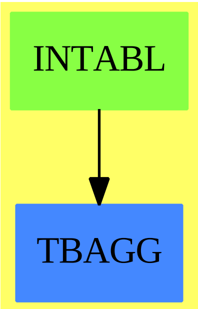
— **InnerTable.help** —

```
====================================================================
InnerTable examples
====================================================================

See Also:
o )show InnerTable
```

———-

10.27.1 InnerTable (INTABL)



See

⇒ "HashTable" (HASHTBL) 9.1.1 on page 1085
⇒ "Table" (TABLE) 21.1.1 on page 2621
⇒ "EqTable" (EQTBL) 6.2.1 on page 667
⇒ "StringTable" (STRTBL) 20.32.1 on page 2569
⇒ "GeneralSparseTable" (GSTBL) 8.5.1 on page 1044
⇒ "SparseTable" (STBL) 20.16.1 on page 2409

Exports:

any?	any?	bag	coerce	construct
convert	copy	count	dictionary	entry?
elt	empty	empty?	entries	eq?
eval	every?	extract!	fill!	find
first	hash	index?	indices	insert!
inspect	key?	keys	latex	less?
map	map!	maxIndex	member?	members
minIndex	more?	parts	sample	qelt
qsetelt!	reduce	remove	remove!	removeDuplicates
search	select	select!	setelt	size?
swap!	table	?.?	#?	?=?
?~=?				

— **domain INTABL InnerTable** —

```
)abbrev domain INTABL InnerTable
++ Author: Barry Trager
++ Date Created: 1992
++ Date Last Updated: Sept 15, 1992
++ Basic Operations:
++ Related Domains: HashTable, AssociationList, Table
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain is used to provide a conditional "add" domain
++ for the implementation of \spadtype{Table}.

InnerTable(Key: SetCategory, Entry: SetCategory, addDom):Exports == Implementation where
    addDom : TableAggregate(Key, Entry) with
                     finiteAggregate
    Exports ==> TableAggregate(Key, Entry) with
                     finiteAggregate
    Implementation ==> addDom
```

———

— **INTABL.dotabb** —

```
"INTABL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INTABL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"INTABL" -> "TBAGG"
```

———

10.28 domain ITAYLOR InnerTaylorSeries

— **InnerTaylorSeries.input** —

```
)set break resume
)sys rm -f InnerTaylorSeries.output
)spool InnerTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerTaylorSeries
--R InnerTaylorSeries Coef: Ring  is a domain constructor
--R Abbreviation for InnerTaylorSeries is ITAYLOR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ITAYLOR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Integer) -> %               ?*? : (%,Coef) -> %
--R ?*? : (Coef,%) -> %                  ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> Stream Coef      coerce : Integer -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  one? : % -> Boolean
--R order : % -> NonNegativeInteger      pole? : % -> Boolean
--R recip : % -> Union(%,"failed")       sample : () -> %
--R series : Stream Coef -> %            zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R characteristic : () -> NonNegativeInteger
--R coerce : % -> % if Coef has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R
--E 1
```

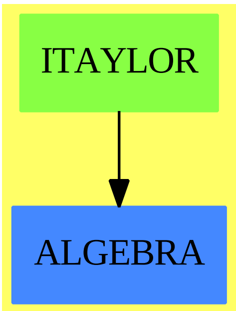
```
)spool
)lisp (bye)
```

— InnerTaylorSeries.help —

```
====================================================================
InnerTaylorSeries examples
====================================================================

See Also:
o )show InnerTaylorSeries
```

10.28.1 InnerTaylorSeries (ITAYLOR)



See

⇒ “UnivariateTaylorSeries” (UTS) 22.9.1 on page 2834

Exports:

0	1	associates?	characteristic	coefficients
coerce	exquo	hash	latex	one?
order	pole?	recip	sample	series
subtractIfCan	unit?	unitCanonical	unitNormal	zero?
?~=?	?*?	?**?	?^?	?+?
?-?	-?	?=?		

— domain ITAYLOR InnerTaylorSeries —

```
)abbrev domain ITAYLOR InnerTaylorSeries
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 25 February 1989
```

```
++ Basic Operations:
++ Related Domains: UnivariateTaylorSeries(Coef,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: stream, dense Taylor series
++ Examples:
++ References:
++ Description:
++ Internal package for dense Taylor series.
++ This is an internal Taylor series type in which Taylor series
++ are represented by a \spadtype{Stream} of \spadtype{Ring} elements.
++ For univariate series, the \spad{Stream} elements are the Taylor
++ coefficients. For multivariate series, the \spad{n}th Stream element
++ is a form of degree n in the power series variables.

InnerTaylorSeries(Coef): Exports == Implementation where
  Coef  : Ring
  I   ==> Integer
  NNI ==> NonNegativeInteger
  ST  ==> Stream Coef
  STT ==> StreamTaylorSeriesOperations Coef

  Exports ==> Ring with
    coefficients: % -> Stream Coef
      ++\spad{coefficients(x)} returns a stream of ring elements.
      ++ When x is a univariate series, this is a stream of Taylor
      ++ coefficients. When x is a multivariate series, the
      ++ \spad{n}th element of the stream is a form of
      ++ degree n in the power series variables.
    series: Stream Coef -> %
      ++\spad{series(s)} creates a power series from a stream of
      ++ ring elements.
      ++ For univariate series types, the stream s should be a stream
      ++ of Taylor coefficients. For multivariate series types, the
      ++ stream s should be a stream of forms the \spad{n}th element
      ++ of which is a
      ++ form of degree n in the power series variables.
    pole?: % -> Boolean
      ++\spad{pole?(x)} tests if the series x has a pole.
      ++ Note: this is false when x is a Taylor series.
    order: % -> NNI
      ++\spad{order(x)} returns the order of a power series x,
      ++ i.e. the degree of the first non-zero term of the series.
    order: (%,NNI) -> NNI
      ++\spad{order(x,n)} returns the minimum of n and the order of x.
    "*" : (Coef,%)->%
      ++\spad{c*x} returns the product of c and the series x.
    "*" : (%,Coef)->%
      ++\spad{x*c} returns the product of c and the series x.
    "*" : (%,Integer)->%
```

```
      ++\spad{x*i} returns the product of integer i and the series x.
    if Coef has IntegralDomain then IntegralDomain
      --++ An IntegralDomain provides 'exquo'

  Implementation ==> add

    Rep := Stream Coef

--% declarations
    x,y: %

--% definitions

    -- In what follows, we will be calling operations on Streams
    -- which are NOT defined in the package Stream.  Thus, it is
    -- necessary to explicitly pass back and forth between Rep and %.
    -- This will be done using the functions 'stream' and 'series'.

    stream : % -> Stream Coef
    stream x  == x pretend Stream(Coef)
    series st == st pretend %

    0 == coerce(0)$STT
    1 == coerce(1)$STT

    x = y ==
      -- tests if two power series are equal
      -- difference must be a finite stream of zeroes of length <= n + 1,
      -- where n = $streamCount$Lisp
      st : ST := stream(x - y)
      n : I := _$streamCount$Lisp
      for i in 0..n repeat
        empty? st => return true
        frst st ^= 0 => return false
        st := rst st
      empty? st

    coefficients x == stream x

    x + y            == stream(x) +$STT stream(y)
    x - y            == stream(x) -$STT stream(y)
    (x:%) * (y:%)    == stream(x) *$STT stream(y)
    - x              == -$STT (stream x)
    (i:I) * (x:%)    == (i::Coef) *$STT stream x
    (x:%) * (i:I)    == stream(x) *$STT (i::Coef)
    (c:Coef) * (x:%) == c *$STT stream x
    (x:%) * (c:Coef) == stream(x) *$STT c

    recip x ==
      (rec := recip$STT stream x) case "failed" => "failed"
```

```
      series(rec :: ST)

    if Coef has IntegralDomain then

      x exquo y ==
        (quot := stream(x) exquo$STT stream(y)) case "failed" => "failed"
        series(quot :: ST)

    x:% ** n:NNI ==
      n = 0 => 1
      expt(x,n :: PositiveInteger)$RepeatedSquaring(%)

    characteristic() == characteristic()$Coef
    pole? x == false

    iOrder: (ST,NNI,NNI) -> NNI
    iOrder(st,n,n0) ==
      (n = n0) or (empty? st) => n0
      zero? frst st => iOrder(rst st,n + 1,n0)
      n

    order(x,n) == iOrder(stream x,0,n)

    iOrder2: (ST,NNI) -> NNI
    iOrder2(st,n) ==
      empty? st => error "order: series has infinite order"
      zero? frst st => iOrder2(rst st,n + 1)
      n

    order x == iOrder2(stream x,0)
```

— **ITAYLOR.dotabb** —

```
"ITAYLOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ITAYLOR"]
"ALGEBRA" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ALGEBRA"]
"ITAYLOR" -> "ALGEBRA"
```

10.29 domain INFORM InputForm

— **InputForm.input** —

```
)set break resume
)sys rm -f InputForm.output
)spool InputForm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InputForm
--R InputForm  is a domain constructor
--R Abbreviation for InputForm is INFORM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INFORM
--R
--R------------------------------- Operations --------------------------------
--R #? : % -> Integer                     ?*? : (%,%) -> %
--R ?**? : (%,Integer) -> %               ?+? : (%,%) -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R atom? : % -> Boolean                  binary : (%,List %) -> %
--R car : % -> %                          cdr : % -> %
--R coerce : % -> OutputForm              convert : SExpression -> %
--R convert : % -> SExpression            convert : OutputForm -> %
--R convert : DoubleFloat -> %            convert : Integer -> %
--R convert : Symbol -> %                 convert : String -> %
--R convert : List % -> %                 declare : List % -> Symbol
--R destruct : % -> List %                ?.? : (%,List Integer) -> %
--R ?.? : (%,Integer) -> %                eq : (%,%) -> Boolean
--R expr : % -> OutputForm                flatten : % -> %
--R float : % -> DoubleFloat              float? : % -> Boolean
--R hash : % -> SingleInteger             integer : % -> Integer
--R integer? : % -> Boolean               interpret : % -> Any
--R lambda : (%,List Symbol) -> %         latex : % -> String
--R list? : % -> Boolean                  null? : % -> Boolean
--R pair? : % -> Boolean                  parse : String -> %
--R string : % -> String                  string? : % -> Boolean
--R symbol : % -> Symbol                  symbol? : % -> Boolean
--R unparse : % -> String                 ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R compile : (Symbol,List %) -> Symbol
--R function : (%,List Symbol,Symbol) -> %
--R
--E 1

)spool
)lisp (bye)
```

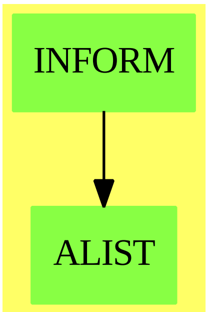
— **InputForm.help** —

```
====================================================================
InputForm examples
====================================================================

See Also:
o )show InputForm
```

———–

10.29.1 InputForm (INFORM)



Exports:

0	1	atom?	binary	car
cdr	coerce	compile	convert	declare
destruct	eq	expr	flatten	float
float?	function	hash	integer	integer?
interpret	lambda	latex	list?	null?
pair?	parse	string	string?	symbol
symbol?	unparse	#?	?~=?	?**?
?*?	?+?	?/?	?=?	?.?

— **domain INFORM InputForm** —

```
)abbrev domain INFORM InputForm
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 19 April 1991
++ Description:
++ Domain of parsed forms which can be passed to the interpreter.
++ This is also the interface between algebra code and facilities
++ in the interpreter.
```

```
--)boot $noSubsumption := true

InputForm():
  Join(SExpressionCategory(String,Symbol,Integer,DoubleFloat,OutputForm),
       ConvertibleTo SExpression) with
    interpret: % -> Any
      ++ interpret(f) passes f to the interpreter.
    convert  : SExpression -> %
      ++ convert(s) makes s into an input form.
    binary   : (%, List %) -> %
      ++ \spad{binary(op, [a1,...,an])} returns the input form
      ++ corresponding to  \spad{a1 op a2 op ... op an}.
      ++
      ++X a:=[1,2,3]::List(InputForm)
      ++X binary(_+::InputForm,a)

    function : (%, List Symbol, Symbol) -> %
      ++ \spad{function(code, [x1,...,xn], f)} returns the input form
      ++ corresponding to \spad{f(x1,...,xn) == code}.
    lambda   : (%, List Symbol) -> %
      ++ \spad{lambda(code, [x1,...,xn])} returns the input form
      ++ corresponding to \spad{(x1,...,xn) +-> code} if \spad{n > 1},
      ++ or to \spad{x1 +-> code} if \spad{n = 1}.
    "+"      : (%, %) -> %
      ++ \spad{a + b} returns the input form corresponding to \spad{a + b}.
    "*"      : (%, %) -> %
      ++ \spad{a * b} returns the input form corresponding to \spad{a * b}.
    "/"      : (%, %) -> %
      ++ \spad{a / b} returns the input form corresponding to \spad{a / b}.
    "**"     : (%, NonNegativeInteger) -> %
      ++ \spad{a ** b} returns the input form corresponding to \spad{a ** b}.
    "**"     : (%, Integer) -> %
      ++ \spad{a ** b} returns the input form corresponding to \spad{a ** b}.
    0        : constant -> %
      ++ \spad{0} returns the input form corresponding to 0.
    1        : constant -> %
      ++ \spad{1} returns the input form corresponding to 1.
    flatten  : % -> %
      ++ flatten(s) returns an input form corresponding to s with
      ++ all the nested operations flattened to triples using new
      ++ local variables.
      ++ If s is a piece of code, this speeds up
      ++ the compilation tremendously later on.
    unparse  : % -> String
      ++ unparse(f) returns a string s such that the parser
      ++ would transform s to f.
      ++ Error: if f is not the parsed form of a string.
    parse : String -> %
      ++ parse is the inverse of unparse. It parses a string to InputForm.
```

```
    declare  : List %   -> Symbol
      ++ declare(t) returns a name f such that f has been
      ++ declared to the interpreter to be of type t, but has
      ++ not been assigned a value yet.
      ++ Note: t should be created as \spad{devaluate(T)$Lisp} where T is the
      ++ actual type of f (this hack is required for the case where
      ++ T is a mapping type).
    compile  : (Symbol, List %) -> Symbol
      ++ \spad{compile(f, [t1,...,tn])} forces the interpreter to compile
      ++ the function f with signature \spad{(t1,...,tn) -> ?}.
      ++ returns the symbol f if successful.
      ++ Error: if f was not defined beforehand in the interpreter,
      ++ or if the ti's are not valid types, or if the compiler fails.
 == SExpression add
    Rep := SExpression

    mkProperOp: Symbol -> %
    strsym    : % -> String
    tuplify   : List Symbol -> %
    flatten0  : (%, Symbol, NonNegativeInteger) ->
                                          Record(lst: List %, symb:%)

    0                         == convert(0::Integer)
    1                         == convert(1::Integer)
    convert(x:%):SExpression == x pretend SExpression
    convert(x:SExpression):% == x

    conv(ll : List %): % ==
      convert(ll pretend List SExpression)$SExpression pretend %

    lambda(f,l) == conv([convert("+->"::Symbol),tuplify l,f]$List(%))

    interpret x ==
      v := interpret(x)$Lisp
      mkObj(unwrap(objVal(v)$Lisp)$Lisp, objMode(v)$Lisp)$Lisp

    convert(x:DoubleFloat):% ==
      zero? x => 0
--      one? x => 1
      (x = 1) => 1
      convert(x)$Rep

    flatten s ==
      -- will not compile if I use 'or'
      atom? s => s
      every?(atom?,destruct s)$List(%) => s
      sy := new()$Symbol
      n:NonNegativeInteger := 0
      l2 := [flatten0(x, sy, n := n + 1) for x in rest(l := destruct s)]
      conv(concat(convert("SEQ"::Symbol)@%,
```

```
    concat(concat [u.lst for u in l2], conv(
       [convert("exit"::Symbol)@%, 1$%, conv(concat(first l,
           [u.symb for u in l2]))@%]$List(%))@%)))@%

flatten0(s, sy, n) ==
  atom? s => [nil(), s]
  a := convert(concat(string sy, convert(n)@String)::Symbol)@%
  l2 := [flatten0(x, sy, n := n+1) for x in rest(l := destruct s)]
  [concat(concat [u.lst for u in l2], conv([convert(
    "LET"::Symbol)@%, a, conv(concat(first l,
         [u.symb for u in l2]))@%]$List(%))@%), a]

strsym s ==
  string? s => string s
  symbol? s => string symbol s
  error "strsym: form is neither a string or symbol"

unparse x ==
  atom?(s:% := form2String(x)$Lisp) => strsym s
  concat [strsym a for a in destruct s]

parse(s:String):% ==
  ncParseFromString(s)$Lisp

declare signature ==
  declare(name := new()$Symbol, signature)$Lisp
  name

compile(name, types) ==
  symbol car cdr car
    selectLocalMms(mkProperOp name, convert(name)@%,
      types, nil$List(%))$Lisp

mkProperOp name ==
  op := mkAtree(nme := convert(name)@%)$Lisp
  transferPropsToNode(nme, op)$Lisp
  convert op

binary(op, args) ==
  (n := #args) < 2 => error "Need at least 2 arguments"
  n = 2 => convert([op, first args, last args]$List(%))
  convert([op, first args, binary(op, rest args)]$List(%))

tuplify l ==
  empty? rest l => convert first l
  conv
    concat(convert("Tuple"::Symbol), [convert x for x in l]$List(%))

function(f, l, name) ==
  nn := convert(new(1 + #l, convert(nil()$List(%)))$List(%))@%
```

```
      conv([convert("DEF"::Symbol), conv(cons(convert(name)@%,
                      [convert(x)@% for x in l])), nn, nn, f]$List(%))

    s1 + s2 ==
      s1 = 0 => s2
      s2 = 0 => s1
      conv [convert("+"::Symbol), s1, s2]$List(%)

    s1 * s2 ==
      s1 = 0 or s2 = 0 => 0
      s1 = 1 => s2
      s2 = 1 => s1
      conv [convert("*"::Symbol), s1, s2]$List(%)

    s1:% ** n:Integer ==
      s1 = 0 and n > 0 => 0
      s1 = 1 or zero? n => 1
--      one? n => s1
      (n = 1) => s1
      conv [convert("**"::Symbol), s1, convert n]$List(%)

    s1:% ** n:NonNegativeInteger == s1 ** (n::Integer)

    s1 / s2 ==
      s2 = 1 => s1
      conv [convert("/"::Symbol), s1, s2]$List(%)
```

—— INFORM.dotabb ——

```
"INFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFORM"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"INFORM" -> "ALIST"
```

10.30 domain INT Integer

The function **one?** has been rewritten back to its original form. The NAG version called a lisp primitive that exists only in Codemist Common Lisp and is not defined in Common Lisp.

—— **Integer.input** ——

```
)set break resume
)sys rm -f Integer.output
)spool Integer.output
)set message test on
)set message auto off
)clear all
--S 1 of 42
2**(5678 - 4856 + 2 * 17)
--R
--R
--R   (1)
--R  480481077043500814718154092512592439123952613987168226347385561008808420007 6_
--R   308293086342527091412083743074572278211496076276922026433435687527334980249_
--R   539302425425230458177649495442143929053063884787051467457680738771416988598_
--R   15495632935288783334250628775936
--R                                                      Type: PositiveInteger
--E 1

--S 2 of 42
x := -101
--R
--R
--R   (2)  - 101
--R                                                              Type: Integer
--E 2

--S 3 of 42
abs(x)
--R
--R
--R   (3)  101
--R                                                      Type: PositiveInteger
--E 3

--S 4 of 42
sign(x)
--R
--R
--R   (4)  - 1
--R                                                              Type: Integer
--E 4

--S 5 of 42
x < 0
--R
--R
--R   (5)  true
--R                                                              Type: Boolean
--E 5
```

```
--S 6 of 42
x <= -1
--R
--R
--R   (6)  true
--R                                                           Type: Boolean
--E 6

--S 7 of 42
negative?(x)
--R
--R
--R   (7)  true
--R                                                           Type: Boolean
--E 7

--S 8 of 42
x > 0
--R
--R
--R   (8)  false
--R                                                           Type: Boolean
--E 8

--S 9 of 42
x >= 1
--R
--R
--R   (9)  false
--R                                                           Type: Boolean
--E 9

--S 10 of 42
positive?(x)
--R
--R
--R   (10)  false
--R                                                           Type: Boolean
--E 10

--S 11 of 42
zero?(x)
--R
--R
--R   (11)  false
--R                                                           Type: Boolean
--E 11

--S 12 of 42
one?(x)
```

```
--R
--R
--R   (12)  false
--R                                                                Type: Boolean
--E 12

--S 13 of 42
(x = -101)@Boolean
--R
--R
--R   (13)  true
--R                                                                Type: Boolean
--E 13

--S 14 of 42
odd?(x)
--R
--R
--R   (14)  true
--R                                                                Type: Boolean
--E 14

--S 15 of 42
even?(x)
--R
--R
--R   (15)  false
--R                                                                Type: Boolean
--E 15

--S 16 of 42
gcd(56788,43688)
--R
--R
--R   (16)  4
--R                                                        Type: PositiveInteger
--E 16

--S 17 of 42
lcm(56788,43688)
--R
--R
--R   (17)  620238536
--R                                                        Type: PositiveInteger
--E 17

--S 18 of 42
max(678,567)
--R
--R
```

```
--R   (18)  678
--R                                                    Type: PositiveInteger
--E 18

--S 19 of 42
min(678,567)
--R
--R
--R   (19)  567
--R                                                    Type: PositiveInteger
--E 19

--S 20 of 42
reduce(max,[2,45,-89,78,100,-45])
--R
--R
--R   (20)  100
--R                                                    Type: PositiveInteger
--E 20

--S 21 of 42
reduce(min,[2,45,-89,78,100,-45])
--R
--R
--R   (21)  - 89
--R                                                            Type: Integer
--E 21

--S 22 of 42
reduce(gcd,[2,45,-89,78,100,-45])
--R
--R
--R   (22)  1
--R                                                    Type: PositiveInteger
--E 22

--S 23 of 42
reduce(lcm,[2,45,-89,78,100,-45])
--R
--R
--R   (23)  1041300
--R                                                    Type: PositiveInteger
--E 23

--S 24 of 42
13 / 4
--R
--R
--R         13
--R   (24)  --
```

```
--R          4
--R                                                       Type: Fraction Integer
--E 24

--S 25 of 42
13 quo 4
--R
--R
--R   (25)  3
--R                                                        Type: PositiveInteger
--E 25

--S 26 of 42
13 rem 4
--R
--R
--R   (26)  1
--R                                                        Type: PositiveInteger
--E 26

--S 27 of 42
zero?(167604736446952 rem 2003644)
--R
--R
--R   (27)  true
--R                                                                Type: Boolean
--E 27

--S 28 of 42
d := divide(13,4)
--R
--R
--R   (28)  [quotient= 3,remainder= 1]
--R                              Type: Record(quotient: Integer,remainder: Integer)
--E 28

--S 29 of 42
d.quotient
--R
--R
--R   (29)  3
--R                                                        Type: PositiveInteger
--E 29

--S 30 of 42
d.remainder
--R
--R
--R   (30)  1
--R                                                        Type: PositiveInteger
```

```
--E 30

--S 31 of 42
factor 102400
--R
--R
--R          12 2
--R   (31)  2  5
--R                                                     Type: Factored Integer
--E 31

--S 32 of 42
prime? 7
--R
--R
--R   (32)  true
--R                                                              Type: Boolean
--E 32

--S 33 of 42
prime? 8
--R
--R
--R   (33)  false
--R                                                              Type: Boolean
--E 33

--S 34 of 42
nextPrime 100
--R
--R
--R   (34)  101
--R                                                      Type: PositiveInteger
--E 34

--S 35 of 42
prevPrime 100
--R
--R
--R   (35)  97
--R                                                      Type: PositiveInteger
--E 35

--S 36 of 42
primes(100,175)
--R
--R
--R   (36)  [173,167,163,157,151,149,139,137,131,127,113,109,107,103,101]
--R                                                         Type: List Integer
--E 36
```

```
--S 37 of 42
factor(2 :: Complex Integer)
--R
--R
--R                        2
--R   (37)  - %i (1 + %i)
--R                                                  Type: Factored Complex Integer
--E 37

--S 38 of 42
[fibonacci(k) for k in 0..]
--R
--R
--R   (38)  [0,1,1,2,3,5,8,13,21,34,...]
--R                                                            Type: Stream Integer
--E 38

--S 39 of 42
[legendre(i,11) for i in 0..10]
--R
--R
--R   (39)  [0,1,- 1,1,1,1,- 1,- 1,- 1,1,- 1]
--R                                                              Type: List Integer
--E 39

--S 40 of 42
[jacobi(i,15) for i in 0..9]
--R
--R
--R   (40)  [0,1,1,0,1,0,0,- 1,1,0]
--R                                                              Type: List Integer
--E 40

--S 41 of 42
[eulerPhi i for i in 1..]
--R
--R
--R   (41)  [1,1,2,2,4,2,6,4,6,4,...]
--R                                                            Type: Stream Integer
--E 41

--S 42 of 42
[moebiusMu i for i in 1..]
--R
--R
--R   (42)  [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R                                                            Type: Stream Integer
--E 42
)spool
```

```
)lisp (bye)
```

———-

— Integer.help —

```
====================================================================
Integer examples
====================================================================

Axiom provides many operations for manipulating arbitrary precision
integers.  In this section we will show some of those that come from
Integer itself plus some that are implemented in other packages.

\subsection{Basic Functions}

The size of an integer in Axiom is only limited by the amount of
computer storage you have available.  The usual arithmetic operations
are available.

  2**(5678 - 4856 + 2 * 17)
  480481077043500814718154092512592439123952613987168226347385561008808420007б_
   308293086342527091412083743074572278211496076276922026433435687527334980249_
   539302425425230458177649495442143929053063884787051467457680738771416988598_
   15495632935288783334250628775936
                        Type: PositiveInteger

There are a number of ways of working with the sign of an integer.
Let's use this x as an example.

  x := -101
    - 101
                        Type: Integer

First of all, there is the absolute value function.

  abs(x)
    101
                        Type: PositiveInteger

The sign operation returns -1 if its argument is negative, 0 if zero
and 1 if positive.

  sign(x)
    - 1
                        Type: Integer

You can determine if an integer is negative in several other ways.
```

```
  x < 0
    true
                         Type: Boolean

  x <= -1
    true
                         Type: Boolean

  negative?(x)
    true
                         Type: Boolean
```

Similarly, you can find out if it is positive.

```
  x > 0
    false
                         Type: Boolean

  x >= 1
    false
                         Type: Boolean

  positive?(x)
    false
                         Type: Boolean
```

This is the recommended way of determining whether an integer is zero.

```
  zero?(x)
    false
                         Type: Boolean
```

Use the zero? operation whenever you are testing any mathematical object for equality with zero. This is usually more efficient that using = (think of matrices: it is easier to tell if a matrix is zero by just checking term by term than constructing another "zero" matrix and comparing the two matrices term by term) and also avoids the problem that = is usually used for creating equations.

This is the recommended way of determining whether an integer is equal to one.

```
  one?(x)
    false
                        Type: Boolean
```

This syntax is used to test equality using =. It says that you want a Boolean (true or false) answer rather than an equation.

```
  (x = -101)@Boolean
```

```
    true
                        Type: Boolean
```

The operations odd? and even? determine whether an integer is odd or even, respectively. They each return a Boolean object.

```
  odd?(x)
    true
                        Type: Boolean

  even?(x)
    false
                        Type: Boolean
```

The operation gcd computes the greatest common divisor of two integers.

```
  gcd(56788,43688)
    4
                        Type: PositiveInteger
```

The operation lcm computes their least common multiple.

```
  lcm(56788,43688)
    620238536
                        Type: PositiveInteger
```

To determine the maximum of two integers, use max.

```
  max(678,567)
    678
                        Type: PositiveInteger
```

To determine the minimum, use min.

```
  min(678,567)
    567
                        Type: PositiveInteger
```

The reduce operation is used to extend binary operations to more than two arguments. For example, you can use reduce to find the maximum integer in a list or compute the least common multiple of all integers in the list.

```
  reduce(max,[2,45,-89,78,100,-45])
    100
                        Type: PositiveInteger

  reduce(min,[2,45,-89,78,100,-45])
    - 89
                        Type: Integer
```

```
  reduce(gcd,[2,45,-89,78,100,-45])
    1
                       Type: PositiveInteger

  reduce(lcm,[2,45,-89,78,100,-45])
    1041300
                       Type: PositiveInteger
```

The infix operator "/" is not used to compute the quotient of integers.
Rather, it is used to create rational numbers as described in Fraction.

```
  13 / 4
     13
     --
      4
                       Type: Fraction Integer
```

The infix operation quo computes the integer quotient.

```
  13 quo 4
    3
                       Type: PositiveInteger
```

The infix operation rem computes the integer remainder.

```
  13 rem 4
    1
                       Type: PositiveInteger
```

One integer is evenly divisible by another if the remainder is zero.
The operation exquo can also be used.

```
  zero?(167604736446952 rem 2003644)
    true
                       Type: Boolean
```

The operation divide returns a record of the quotient and remainder
and thus is more efficient when both are needed.

```
  d := divide(13,4)
    [quotient= 3,remainder= 1]
                       Type: Record(quotient: Integer,remainder: Integer)

  d.quotient
    3
                       Type: PositiveInteger
```

See help on Records for details on Records.

```
  d.remainder
    1
                       Type: PositiveInteger

====================================================================
Primes and Factorization
====================================================================

Use the operation factor to factor integers.  It returns an object of
type Factored Integer.

  factor 102400
     12 2
    2  5
                       Type: Factored Integer

The operation prime? returns true or false depending on whether its
argument is a prime.

  prime? 7
    true
                       Type: Boolean

  prime? 8
    false
                       Type: Boolean

The operation nextPrime returns the least prime number greater than
its argument.

  nextPrime 100
    101
                       Type: PositiveInteger

The operation prevPrime returns the greatest prime number less than
its argument.

  prevPrime 100
    97
                       Type: PositiveInteger

To compute all primes between two integers (inclusively), use the
operation primes.

  primes(100,175)
    [173,167,163,157,151,149,139,137,131,127,113,109,107,103,101]
                       Type: List Integer

You might sometimes want to see the factorization of an integer
when it is considered a Gaussian integer.
```

```
  factor(2 :: Complex Integer)
                 2
    - %i (1 + %i)
                        Type: Factored Complex Integer

====================================================================
Some Number Theoretic Functions
====================================================================

Axiom provides several number theoretic operations for integers.

The operation fibonacci computes the Fibonacci numbers.  The algorithm
has running time O(log^3n) for argument n.

  [fibonacci(k) for k in 0..]
    [0,1,1,2,3,5,8,13,21,34,...]
                       Type: Stream Integer

The operation legendre computes the Legendre symbol for its two integer
arguments where the second one is prime.  If you know the second argument
to be prime, use jacobi instead where no check is made.

  [legendre(i,11) for i in 0..10]
    [0,1,- 1,1,1,1,- 1,- 1,- 1,1,- 1]
                       Type: List Integer

The operation jacobi computes the Jacobi symbol for its two integer
arguments.  By convention, 0 is returned if the greatest common divisor
of the numerator and denominator is not 1.

  [jacobi(i,15) for i in 0..9]
    [0,1,1,0,1,0,0,- 1,1,0]
                       Type: List Integer

The operation eulerPhi computes the values of Euler's \phi-function
where \phi(n) equals the number of positive integers less than or equal
to n that are relatively prime to the positive integer n.

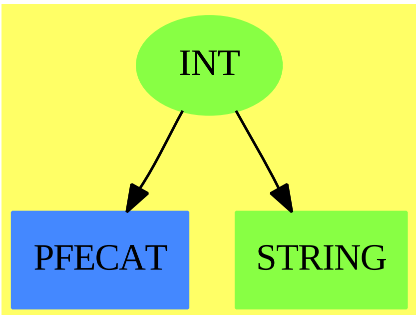
  [eulerPhi i for i in 1..]
    [1,1,2,2,4,2,6,4,6,4,...]
                       Type: Stream Integer

The operation moebiusMu computes the Moebius mu function.

  [moebiusMu i for i in 1..]
    [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
                       Type: Stream Integer
```

```
See Also:
o )help Complex
o )help Factored
o )help Records
o )help Fraction
o )help RadixExpansion
o )help HexadecimalExpansion
o )help BinaryExpansion
o )help DecimalExpansion
o )help IntegerNumberTheoryFunctions
o )help RomanNumeral
o )show Integer
```

10.30.1 Integer (INT)



See

⇒ "NonNegativeInteger" (NNI) 15.5.1 on page 1702
⇒ "PositiveInteger" (PI) 17.28.1 on page 2060
⇒ "RomanNumeral" (ROMAN) 19.12.1 on page 2286

Exports:

0	1	abs	addmod
associates?	base	binomial	bit?
characteristic	coerce	convert	copy
D	dec	differentiate	divide
euclideanSize	even?	expressIdealMember	exquo
extendedEuclidean	extendedEuclidean	factor	factorial
gcd	gcdPolynomial	hash	inc
init	invmod	latex	lcm
length	mask	max	min
mulmod	multiEuclidean	negative?	nextItem
odd?	OMwrite	one?	patternMatch
permutation	positive?	positiveRemainder	powmod
prime?	principalIdeal	random	rational
rational?	rationalIfCan	recip	reducedSystem
retract	retractIfCan	sample	shift
sign	sizeLess?	squareFree	squareFreePart
submod	subtractIfCan	symmetricRemainder	unit?
unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?
?<?	?<=?	?=?	?>?
?>=?	?^?	?~=?	?quo?
?rem?			

— domain INT Integer —

```
)abbrev domain INT Integer
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations:
++ Related Constructors:
++ Keywords: integer
++ Description:
++ \spadtype{Integer} provides the domain of arbitrary precision integers.

Integer: Join(IntegerNumberSystem, ConvertibleTo String, OpenMath) with
    random   : % -> %
      ++ random(n) returns a random integer from 0 to \spad{n-1}.
    canonical
      ++ mathematical equality is data structure equality.
    canonicalsClosed
      ++ two positives multiply to give positive.
    noetherian
      ++ ascending chain condition on ideals.
    infinite
      ++ nextItem never returns "failed".
 == add
```

```
ZP ==> SparseUnivariatePolynomial %
ZZP ==> SparseUnivariatePolynomial Integer
x,y: %
n: NonNegativeInteger

writeOMInt(dev: OpenMathDevice, x: %): Void ==
  if x < 0 then
    OMputApp(dev)
    OMputSymbol(dev, "arith1", "unary__minus")
    OMputInteger(dev, (-x) pretend Integer)
    OMputEndApp(dev)
  else
    OMputInteger(dev, x pretend Integer)

OMwrite(x: %): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  OMputObject(dev)
  writeOMInt(dev, x)
  OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(x: %, wholeObj: Boolean): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  if wholeObj then
    OMputObject(dev)
  writeOMInt(dev, x)
  if wholeObj then
    OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  writeOMInt(dev, x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
  writeOMInt(dev, x)
  if wholeObj then
    OMputEndObject(dev)
```

```
      zero? x == ZEROP(x)$Lisp
--      one? x == ONEP(x)$Lisp
      one? x == x = 1
      0 == 0$Lisp
      1 == 1$Lisp
      base()  == 2$Lisp
      copy x  == x
      inc  x  == x + 1
      dec  x  == x - 1
      hash x == SXHASH(x)$Lisp
      negative? x == MINUSP(x)$Lisp
      coerce(x):OutputForm == outputForm(x pretend Integer)
      coerce(m:Integer):% == m pretend %
      convert(x:%):Integer == x pretend Integer
      length a == INTEGER_-LENGTH(a)$Lisp
      addmod(a, b, p) ==
         (c:=a + b) >= p => c - p
         c
      submod(a, b, p) ==
         (c:=a - b) < 0 => c + p
         c
      mulmod(a, b, p) == (a * b) rem p
      convert(x:%):Float       == coerce(x pretend Integer)$Float
      convert(x:%):DoubleFloat  == coerce(x pretend Integer)$DoubleFloat
      convert(x:%):InputForm   == convert(x pretend Integer)$InputForm
      convert(x:%):String      == string(x pretend Integer)$String

      latex(x:%):String ==
        s : String := string(x pretend Integer)$String
        (-1 < (x pretend Integer)) and ((x  pretend Integer) < 10) => s
        concat("{", concat(s, "}")$String)$String

      positiveRemainder(a, b) ==
        negative?(r := a rem b) =>
           negative? b => r - b
           r + b
        r

      reducedSystem(m:Matrix %):Matrix(Integer) ==
        m pretend Matrix(Integer)

      reducedSystem(m:Matrix %, v:Vector %):
       Record(mat:Matrix(Integer), vec:Vector(Integer)) ==
        [m pretend Matrix(Integer), vec pretend Vector(Integer)]

      abs(x) == ABS(x)$Lisp
      random() == random()$Lisp
      random(x) == RANDOM(x)$Lisp
      x = y == EQL(x,y)$Lisp
      x < y == (x<y)$Lisp
```

```
      - x == (-x)$Lisp
      x + y == (x+y)$Lisp
      x - y == (x-y)$Lisp
      x * y == (x*y)$Lisp
      (m:Integer) * (y:%) == (m*y)$Lisp -- for subsumption problem
      x ** n == EXPT(x,n)$Lisp
      odd? x == ODDP(x)$Lisp
      max(x,y) == MAX(x,y)$Lisp
      min(x,y) == MIN(x,y)$Lisp
      divide(x,y) == DIVIDE2(x,y)$Lisp
      x quo y == QUOTIENT2(x,y)$Lisp
      x rem y == REMAINDER2(x,y)$Lisp
      shift(x, y) == ASH(x,y)$Lisp
      x exquo y ==
         zero? y => "failed"
         zero?(x rem y) => x quo y
         "failed"
--     recip(x) == if one? x or x=-1 then x else "failed"
      recip(x) == if (x = 1) or x=-1 then x else "failed"
      gcd(x,y) == GCD(x,y)$Lisp
      UCA ==> Record(unit:%,canonical:%,associate:%)
      unitNormal x ==
         x < 0 => [-1,-x,-1]$UCA
         [1,x,1]$UCA
      unitCanonical x == abs x
      solveLinearPolynomialEquation(lp:List ZP,p:ZP):Union(List ZP,"failed") ==
         solveLinearPolynomialEquation(lp pretend List ZZP,
               p pretend ZZP)$IntegerSolveLinearPolynomialEquation pretend
                     Union(List ZP,"failed")
      squareFreePolynomial(p:ZP):Factored ZP ==
        squareFree(p)$UnivariatePolynomialSquareFree(%,ZP)
      factorPolynomial(p:ZP):Factored ZP ==
         -- GaloisGroupFactorizer doesn't factor the content
         -- so we have to do this by hand
         pp:=primitivePart p
         leadingCoefficient pp = leadingCoefficient p =>
             factor(p)$GaloisGroupFactorizer(ZP)
         mergeFactors(factor(pp)$GaloisGroupFactorizer(ZP),
                      map((x1:%):ZP+->x1::ZP,
                          factor((leadingCoefficient p exquo
                                  leadingCoefficient pp)
                                 ::%))$FactoredFunctions2(%,ZP)
                                   )$FactoredFunctionUtilities(ZP)
      factorSquareFreePolynomial(p:ZP):Factored ZP ==
           factorSquareFree(p)$GaloisGroupFactorizer(ZP)
      gcdPolynomial(p:ZP, q:ZP):ZP ==
         zero? p => unitCanonical q
         zero? q => unitCanonical p
         gcd([p,q])$HeuGcd(ZP)
--    myNextPrime: (%,NonNegativeInteger) -> %
```

```
--    myNextPrime(x,n) ==
--       nextPrime(x)$IntegerPrimesPackage(%)
--    TT:=InnerModularGcd(%,ZP,67108859 pretend %,myNextPrime)
--    gcdPolynomial(p,q) == modularGcd(p,q)$TT
```

———-

— INT.dotabb —

```
"INT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INT",
          shape=ellipse]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"INT" -> "STRING"
"INT" -> "PFECAT"
```

———-

10.31 domain ZMOD IntegerMod

— IntegerMod.input —

```
)set break resume
)sys rm -f IntegerMod.output
)spool IntegerMod.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegerMod
--R IntegerMod p: PositiveInteger  is a domain constructor
--R Abbreviation for IntegerMod is ZMOD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ZMOD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Integer -> %
--R coerce : % -> OutputForm              convert : % -> Integer
```

```
--R hash : % -> SingleInteger            index : PositiveInteger -> %
--R init : () -> %                       latex : % -> String
--R lookup : % -> PositiveInteger        one? : % -> Boolean
--R random : () -> %                     recip : % -> Union(%,"failed")
--R sample : () -> %                     size : () -> NonNegativeInteger
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R nextItem : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

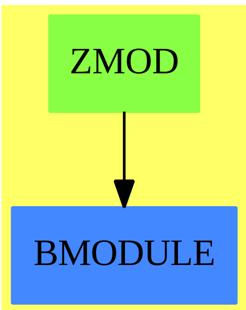
— IntegerMod.help —

```
====================================================================
IntegerMod examples
====================================================================

See Also:
o )show IntegerMod
```

———-

10.31.1 IntegerMod (ZMOD)



Exports:

0	1	characteristic	coerce	convert
hash	index	init	latex	lookup
nextItem	one?	random	recip	sample
size	subtractIfCan	zero?	?~=?	?*?
?**?	?^?	?+?	?-?	-?
?=?				

— domain ZMOD IntegerMod —

```
)abbrev domain ZMOD IntegerMod
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ IntegerMod(n) creates the ring of integers reduced modulo the integer n.

IntegerMod(p:PositiveInteger):
 Join(CommutativeRing, Finite, ConvertibleTo Integer, StepThrough) == add
  size()           == p
  characteristic() == p
  lookup x == (zero? x => p; (convert(x)@Integer) :: PositiveInteger)

-- Code is duplicated for the optimizer to kick in.
  if p <= convert(max()$SingleInteger)@Integer then
    Rep:= SingleInteger
    q := p::SingleInteger

    bloodyCompiler: Integer -> %
    bloodyCompiler n == positiveRemainder(n, p)$Integer :: Rep

    convert(x:%):Integer == convert(x)$Rep
    coerce(x):OutputForm == coerce(x)$Rep
    coerce(n:Integer):%  == bloodyCompiler n
    0                    == 0$Rep
    1                    == 1$Rep
    init                 == 0$Rep
    nextItem(n)          ==
                              m:=n+1
                              m=0 => "failed"
                              m
    x = y                == x =$Rep y
    x:% * y:%            == mulmod(x, y, q)
```

```
    n:Integer * x:%      == mulmod(bloodyCompiler n, x, q)
    x + y                == addmod(x, y, q)
    x - y                == submod(x, y, q)
    random()             == random(q)$Rep
    index a              == positiveRemainder(a::%, q)
    - x                  == (zero? x => 0; q -$Rep x)

    x:% ** n:NonNegativeInteger ==
      n < p => powmod(x, n::Rep, q)
      powmod(convert(x)@Integer, n, p)$Integer :: Rep

    recip x ==
       (c1, c2, g) := extendedEuclidean(x, q)$Rep
--       not one? g => "failed"
       not (g = 1) => "failed"
       positiveRemainder(c1, q)

  else
    Rep:= Integer

    convert(x:%):Integer == convert(x)$Rep
    coerce(n:Integer):%  == positiveRemainder(n::Rep, p)
    coerce(x):OutputForm == coerce(x)$Rep
    0                    == 0$Rep
    1                    == 1$Rep
    init                 == 0$Rep
    nextItem(n)          ==
                              m:=n+1
                              m=0 => "failed"
                              m
    x = y                == x =$Rep y
    x:% * y:%            == mulmod(x, y, p)
    n:Integer * x:%      == mulmod(positiveRemainder(n::Rep, p), x, p)
    x + y                == addmod(x, y, p)
    x - y                == submod(x, y, p)
    random()             == random(p)$Rep
    index a              == positiveRemainder(a::Rep, p)
    - x                  == (zero? x => 0; p -$Rep x)
    x:% ** n:NonNegativeInteger == powmod(x, n::Rep, p)

    recip x ==
       (c1, c2, g) := extendedEuclidean(x, p)$Rep
--       not one? g => "failed"
       not (g = 1) => "failed"
       positiveRemainder(c1, p)
```

— **ZMOD.dotabb** —

```
"ZMOD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ZMOD"]
"BMODULE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BMODULE"]
"ZMOD" -> "BMODULE"
```

———-

10.32 domain INTFTBL IntegrationFunctionsTable

— **IntegrationFunctionsTable.input** —

```
)set break resume
)sys rm -f IntegrationFunctionsTable.output
)spool IntegrationFunctionsTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegrationFunctionsTable
--R IntegrationFunctionsTable  is a domain constructor
--R Abbreviation for IntegrationFunctionsTable is INTFTBL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INTFTBL
--R
--R------------------------------- Operations --------------------------------
--R clearTheFTable : () -> Void           showTheFTable : () -> %
--R entries : % -> List Record(key: Record(var: Symbol,fn: Expression DoubleFloat,range: Segm
--R entry : Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion Do
--R fTable : List Record(key: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R insert! : Record(key: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ordere
--R keys : % -> List Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComp
--R showAttributes : Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComp
--R
--E 1

)spool
)lisp (bye)
```

———-

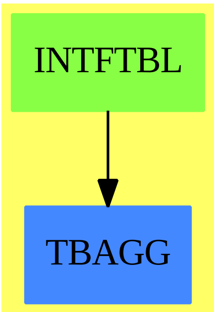
— **IntegrationFunctionsTable.help** —

```
====================================================================
IntegrationFunctionsTable examples
====================================================================

See Also:
o )show IntegrationFunctionsTable
```

———-

10.32.1 IntegrationFunctionsTable (INTFTBL)



Exports:
clearTheFTable entries entry fTable insert! keys showAttributes showTheFTable

— domain INTFTBL IntegrationFunctionsTable —

```
)abbrev domain INTFTBL IntegrationFunctionsTable
++ Author: Brian Dupee
++ Date Created: March 1995
++ Date Last Updated: June 1995
++ Description:
++ There is no description for this domain

IntegrationFunctionsTable(): E == I where
  EF2  ==> ExpressionFunctions2
  EFI  ==> Expression Fraction Integer
  FI  ==> Fraction Integer
  LEDF  ==> List Expression DoubleFloat
  KEDF  ==> Kernel Expression DoubleFloat
  EEDF  ==> Equation Expression DoubleFloat
  EDF  ==> Expression DoubleFloat
  PDF  ==> Polynomial DoubleFloat
  LDF  ==> List DoubleFloat
  SDF  ==> Stream DoubleFloat
```

```
DF  ==> DoubleFloat
F  ==> Float
ST  ==> String
LST  ==> List String
SI  ==> SingleInteger
SOCDF  ==> Segment OrderedCompletion DoubleFloat
OCDF  ==> OrderedCompletion DoubleFloat
OCEDF  ==> OrderedCompletion Expression DoubleFloat
EOCEFI  ==> Equation OrderedCompletion Expression Fraction Integer
OCEFI   ==> OrderedCompletion Expression Fraction Integer
OCFI    ==> OrderedCompletion Fraction Integer
NIA  ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
INT  ==> Integer
CTYPE  ==> Union(continuous: "Continuous at the end points",
           lowerSingular: "There is a singularity at the lower end point",
            upperSingular: "There is a singularity at the upper end point",
             bothSingular: "There are singularities at both end points",
              notEvaluated: "End point continuity not yet evaluated")
RTYPE  ==> Union(finite: "The range is finite",
            lowerInfinite: "The bottom of range is infinite",
              upperInfinite: "The top of range is infinite",
                bothInfinite: "Both top and bottom points are infinite",
                  notEvaluated: "Range not yet evaluated")
STYPE  ==> Union(str:SDF,
                 notEvaluated:"Internal singularities not yet evaluated")
ATT  ==> Record(endPointContinuity:CTYPE,
                  singularitiesStream:STYPE,range:RTYPE)
ROA  ==> Record(key:NIA,entry:ATT)

E ==> with

  showTheFTable:() -> $
    ++ showTheFTable() returns the current table of functions.
  clearTheFTable : () -> Void
    ++ clearTheFTable() clears the current table of functions.
  keys : $ -> List(NIA)
    ++ keys(f) returns the list of keys of f
  fTable: List Record(key:NIA,entry:ATT) -> $
    ++ fTable(l) creates a functions table from the elements of l.
  insert!:Record(key:NIA,entry:ATT) -> $
    ++ insert!(r) inserts an entry r into theIFTable
  showAttributes:NIA -> Union(ATT,"failed")
    ++ showAttributes(x) is not documented
  entries : $ -> List Record(key:NIA,entry:ATT)
    ++ entries(x) is not documented
  entry:NIA -> ATT
    ++ entry(n) is not documented
I ==> add

  Rep := Table(NIA,ATT)
```

```
    import Rep

    theFTable:$ := empty()$Rep

    showTheFTable():$ ==
      theFTable

    clearTheFTable():Void ==
      theFTable := empty()$Rep
      void()$Void

    fTable(l:List Record(key:NIA,entry:ATT)):$ ==
      theFTable := table(l)$Rep

    insert!(r:Record(key:NIA,entry:ATT)):$ ==
      insert!(r,theFTable)$Rep

    keys(t:$):List NIA ==
      keys(t)$Rep

    showAttributes(k:NIA):Union(ATT,"failed") ==
      search(k,theFTable)$Rep

    entries(t:$):List Record(key:NIA,entry:ATT) ==
      members(t)$Rep

    entry(k:NIA):ATT ==
      qelt(theFTable,k)$Rep
```

— INTFTBL.dotabb —

```
"INTFTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INTFTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"INTFTBL" -> "TBAGG"
```

10.33 domain IR IntegrationResult

— IntegrationResult.input —

```
)set break resume
```

```
)sys rm -f IntegrationResult.output
)spool IntegrationResult.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegrationResult
--R IntegrationResult F: Field  is a domain constructor
--R Abbreviation for IntegrationResult is IR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction Integer) -> %       ?*? : (Fraction Integer,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : F -> %
--R coerce : % -> OutputForm              differentiate : (%,(F -> F)) -> F
--R elem? : % -> Boolean                  hash : % -> SingleInteger
--R integral : (F,F) -> %                 latex : % -> String
--R ratpart : % -> F                      retract : % -> F
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R differentiate : (%,Symbol) -> F if F has PDRING SYMBOL
--R integral : (F,Symbol) -> % if F has RETRACT SYMBOL
--R logpart : % -> List Record(scalar: Fraction Integer,coeff: SparseUnivariatePolynomial F,l
--R mkAnswer : (F,List Record(scalar: Fraction Integer,coeff: SparseUnivariatePolynomial F,lo
--R notelem : % -> List Record(integrand: F,intvar: F)
--R retractIfCan : % -> Union(F,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

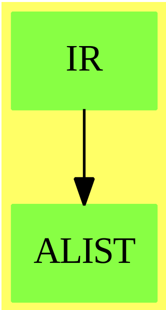
— IntegrationResult.help —

```
====================================================================
IntegrationResult examples
====================================================================

See Also:
o )show IntegrationResult
```

———-

10.33.1 IntegrationResult (IR)



Exports:

0	coerce	differentiate	elem?	hash
integral	latex	logpart	mkAnswer	notelem
ratpart	retract	retractIfCan	subtractIfCan	sample
zero?	?~=?	?*?	?+?	?-?
-?	?=?			

— **domain IR IntegrationResult** —

```
)abbrev domain IR IntegrationResult
++ Author: Barry Trager, Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 12 August 1992
++ Keywords: integration.
++ Description:
++ The result of a transcendental integration.
++ If a function f has an elementary integral g, then g can be written
++ in the form \spad{g = h + c1 log(u1) + c2 log(u2) + ... + cn log(un)}
++ where h, which is in the same field than f, is called the rational
++ part of the integral, and \spad{c1 log(u1) + ... cn log(un)} is called the
++ logarithmic part of the integral. This domain manipulates integrals
++ represented in that form, by keeping both parts separately. The logs
++ are not explicitly computed.

IntegrationResult(F:Field): Exports == Implementation where
  O   ==> OutputForm
  B   ==> Boolean
  Z   ==> Integer
  Q   ==> Fraction Integer
  SE  ==> Symbol
```

```
UP  ==> SparseUnivariatePolynomial F
LOG ==> Record(scalar:Q, coeff:UP, logand:UP)
NE  ==> Record(integrand:F, intvar:F)

Exports ==> (Module Q, RetractableTo F) with
  mkAnswer: (F, List LOG, List NE) -> %
    ++ mkAnswer(r,l,ne) creates an integration result from
    ++ a rational part r, a logarithmic part l, and a non-elementary part ne.
  ratpart : % -> F
    ++ ratpart(ir) returns the rational part of an integration result
  logpart : % -> List LOG
    ++ logpart(ir) returns the logarithmic part of an integration result
  notelem : % -> List NE
    ++ notelem(ir) returns the non-elementary part of an integration result
  elem?   : % -> B
    ++ elem?(ir) tests if an integration result is elementary over F?
  integral: (F, F) -> %
    ++ integral(f,x) returns the formal integral of f with respect to x
  differentiate: (%, F -> F) -> F
    ++ differentiate(ir,D) differentiates ir with respect to the derivation D.
  if F has PartialDifferentialRing(SE) then
    differentiate: (%, Symbol) -> F
      ++ differentiate(ir,x) differentiates ir with respect to x
  if F has RetractableTo Symbol then
    integral: (F, Symbol) -> %
      ++ integral(f,x) returns the formal integral of f with respect to x

Implementation ==> add
  Rep := Record(ratp: F, logp: List LOG, nelem: List NE)

  timelog : (Q, LOG) -> LOG
  timene  : (Q, NE)  -> NE
  LOG2O   : LOG      -> O
  NE2O    : NE       -> O
  Q2F     : Q        -> F
  nesimp  : List NE  -> List NE
  neselect: (List NE, F) -> F
  pLogDeriv: (LOG, F -> F) -> F
  pNeDeriv : (NE,  F -> F) -> F


  alpha:O := new()$Symbol :: O

  - u               == (-1$Z) * u
  0                 == mkAnswer(0, empty(), empty())
  coerce(x:F):%     == mkAnswer(x, empty(), empty())
  ratpart u         == u.ratp
  logpart u         == u.logp
  notelem u         == u.nelem
  elem? u           == empty? notelem u
```

```
    mkAnswer(x, l, n) == [x, l, nesimp n]
    timelog(r, lg)    == [r * lg.scalar, lg.coeff, lg.logand]
    integral(f:F,x:F) == (zero? f => 0; mkAnswer(0, empty(), [[f, x]]))
    timene(r, ne)     == [Q2F(r) * ne.integrand, ne.intvar]
    n:Z * u:%         == (n::Q) * u
    Q2F r             == numer(r)::F / denom(r)::F
    neselect(l, x)    == _+/[ne.integrand for ne in l | ne.intvar = x]

    if F has RetractableTo Symbol then
      integral(f:F, x:Symbol):% == integral(f, x::F)

    LOG2O rec ==
--      one? degree rec.coeff =>
      (degree rec.coeff) = 1 =>
        -- deg 1 minimal poly doesn't get sigma
        lastc := - coefficient(rec.coeff, 0) / coefficient(rec.coeff, 1)
        lg    := (rec.logand) lastc
        logandp := prefix("log"::Symbol::O, [lg::O])
        (cc := Q2F(rec.scalar) * lastc) = 1 => logandp
        cc = -1 => - logandp
        cc::O * logandp
      coeffp:O := (outputForm(rec.coeff, alpha) = 0::Z::O)@O
      logandp :=
           alpha * prefix("log"::Symbol::O, [outputForm(rec.logand, alpha)])
      if (cc := Q2F(rec.scalar)) ^= 1 then
        logandp := cc::O * logandp
      sum(logandp, coeffp)

    nesimp l ==
      [[u,x] for x in removeDuplicates_!([ne.intvar for ne in l]$List(F))
                                       | (u := neselect(l, x)) ^= 0]

    if (F has LiouvillianFunctionCategory) and (F has RetractableTo Symbol) then
      retractIfCan u ==
        empty? logpart u =>
          ratpart u +
             _+/[integral(ne.integrand, retract(ne.intvar)@Symbol)$F
                for ne in notelem u]
        "failed"

    else
      retractIfCan u ==
        elem? u and empty? logpart u => ratpart u
        "failed"

    r:Q * u:% ==
      r = 0 => 0
      mkAnswer(Q2F(r) * ratpart u, map(x1+->timelog(r, x1), logpart u),
                                   map(x2+->timene(r, x2), notelem u))
```

```
    -- Initial attempt, quick and dirty, no simplification
    u + v ==
      mkAnswer(ratpart u + ratpart v, concat(logpart u, logpart v),
                                 nesimp concat(notelem u, notelem v))

    if F has PartialDifferentialRing(Symbol) then
      differentiate(u:%, x:Symbol):F ==
        differentiate(u, x1+->differentiate(x1, x))

    differentiate(u:%, derivation:F -> F):F ==
      derivation ratpart u +
          _+/[pLogDeriv(log, derivation) for log in logpart u]
               + _+/[pNeDeriv(ne, derivation) for ne in notelem u]

    pNeDeriv(ne, derivation) ==
--      one? derivation(ne.intvar) => ne.integrand
      (derivation(ne.intvar) = 1) => ne.integrand
      zero? derivation(ne.integrand) => 0
      error "pNeDeriv: cannot differentiate not elementary part into F"

    pLogDeriv(log, derivation) ==
      map(derivation, log.coeff) ^= 0 =>
        error "pLogDeriv: can only handle logs with constant coefficients"
--      one?(n := degree(log.coeff)) =>
      ((n := degree(log.coeff)) = 1) =>
        c   := - (leadingCoefficient reductum log.coeff)
                                     / (leadingCoefficient log.coeff)
        ans := (log.logand) c
        Q2F(log.scalar) * c * derivation(ans) / ans
      numlog := map(derivation, log.logand)
      diflog := extendedEuclidean(log.logand, log.coeff,
                                 numlog)::Record(coef1:UP, coef2:UP)
      algans := diflog.coef1
      ans:F := 0
      for i in 0..(n-1) repeat
        algans := algans * monomial(1, 1) rem log.coeff
        ans := ans + coefficient(algans, i)
      Q2F(log.scalar) * ans

    coerce(u:%):O ==
      (r := retractIfCan u) case F => r::F::O
      l := reverse_! [LOG2O f for f in logpart u]$List(O)
      if ratpart u ^= 0 then l := concat(ratpart(u)::O, l)
      if not elem? u then l := concat([NE2O f for f in notelem u], l)
      null l => 0::O
      reduce("+", l)

    NE2O ne ==
      int((ne.integrand)::O * hconcat ["d"::Symbol::O, (ne.intvar)::O])
```

———-

— IR.dotabb —

```
"IR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"IR" -> "ALIST"
```

———-

10.34 domain INTRVL Interval

— Interval.input —

```
)set break resume
)sys rm -f Interval.output
)spool Interval.output
)set message test on
)set message auto off
)clear all

--S 1 of 13
t1:=0::Interval(Float)
--R
--R
--R   (1)  [0.0,0.0]
--R                                                    Type: Interval Float
--E 1

--S 2 of 13
t2:=1::Interval(Float)
--R
--R
--R   (2)  [1.0,1.0]
--R                                                    Type: Interval Float
--E 2

--S 3 of 13
t3:=3*t1
--R
--R
--R   (3)  [- 0.3 E -20,0.3 E -20]
--R                                                    Type: Interval Float
--E 3
```

```
--S 4 of 13
t4:=1/4*t2
--R
--R
--R        [1.0,1.0]
--R   (4)  ---------
--R        [4.0,4.0]
--R                                            Type: Fraction Interval Float
--E 4

--S 5 of 13
acos(t3)
--R
--R
--R   (5)  [1.5707963267 948966192,1.5707963267 948966192]
--R                                                     Type: Interval Float
--E 5

--S 6 of 13
t6:=t4*t4
--R
--R
--R         [1.0,1.0]
--R   (6)  -----------
--R        [16.0,16.0]
--R                                            Type: Fraction Interval Float
--E 6

--S 7 of 13
sup(t6)
--R
--R
--R   (7)  0.0625000000 0000000000 1
--R                                                              Type: Float
--E 7

--S 8 of 13
inf(t6)
--R
--R
--R   (8)  0.0624999999 9999999999 9
--R                                                              Type: Float
--E 8

--S 9 of 13
width(t6)
--R
--R
--R   (9)  0.1 E -20
--R                                                              Type: Float
```

```
--E 9

--S 10 of 13
positive? t3
--R
--R
--R   (10)  false
--R                                                            Type: Boolean
--E 10

--S 11 of 13
negative? t3
--R
--R
--R   (11)  false
--R                                                            Type: Boolean
--E 11

--S 12 of 13
contains?(t3,0.3)
--R
--R
--R   (12)  false
--R                                                            Type: Boolean
--E 12

--S 13 of 13
)show Interval
--R
--R Interval R: Join(FloatingPointSystem,TranscendentalFunctionCategory)  is a domain constructor
--R Abbreviation for Interval is INTRVL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INTRVL
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Fraction Integer) -> %
--R ?**? : (%,%) -> %                     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        acos : % -> %
--R acosh : % -> %                        acot : % -> %
--R acoth : % -> %                        acsc : % -> %
--R acsch : % -> %                        asec : % -> %
--R asech : % -> %                        asin : % -> %
--R asinh : % -> %                        associates? : (%,%) -> Boolean
--R atan : % -> %                         atanh : % -> %
```

```
--R coerce : Integer -> %               coerce : % -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R contains? : (%,R) -> Boolean        cos : % -> %
--R cosh : % -> %                       cot : % -> %
--R coth : % -> %                       csc : % -> %
--R csch : % -> %                       exp : % -> %
--R gcd : List % -> %                   gcd : (%,%) -> %
--R hash : % -> SingleInteger           inf : % -> R
--R interval : Fraction Integer -> %    interval : R -> %
--R interval : (R,R) -> %               latex : % -> String
--R lcm : List % -> %                   lcm : (%,%) -> %
--R log : % -> %                        max : (%,%) -> %
--R min : (%,%) -> %                    negative? : % -> Boolean
--R nthRoot : (%,Integer) -> %          one? : % -> Boolean
--R pi : () -> %                        positive? : % -> Boolean
--R qinterval : (R,R) -> %              recip : % -> Union(%,"failed")
--R retract : % -> Integer              sample : () -> %
--R sec : % -> %                        sech : % -> %
--R sin : % -> %                        sinh : % -> %
--R sqrt : % -> %                       sup : % -> R
--R tan : % -> %                        tanh : % -> %
--R unit? : % -> Boolean                unitCanonical : % -> %
--R width : % -> R                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 13

)spool
)lisp (bye)
```

———-

— Interval.help —

```
====================================================================
Interval examples
====================================================================

t1:=0::Interval(Float)
```

```
   [0.0,0.0]

t2:=1::Interval(Float)
   [1.0,1.0]

t3:=3*t1
   [- 0.3 E -20,0.3 E -20]

t4:=1/4*t2
   [1.0,1.0]
   ---------
   [4.0,4.0]

acos(t3)
   [1.5707963267 948966192,1.5707963267 948966192]

t6:=t4*t4
   [1.0,1.0]
   -----------
   [16.0,16.0]

sup(t6)
   0.0625000000 0000000000 1

inf(t6)
   0.0624999999 9999999999 9

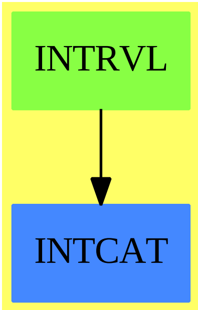
width(t6)
   0.1 E -20

positive? t3
   false

negative? t3
   false

contains?(t3,0.3)
   false
```

10.34.1 Interval (INTRVL)



Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	coerce	contains?	cos	cosh
cot	coth	csc	csch	exp
exquo	gcd	gcdPolynomial	hash	inf
interval	latex	lcm	log	max
min	negative?	nthRoot	one?	pi
positive?	qinterval	recip	retract	retractIfCan
sample	sec	sech	sin	sinh
sqrt	subtractIfCan	sup	tan	tanh
unit?	unitCanonical	unitNormal	width	zero?
?*?	?**?	?+?	?-?	-?
?<?	?<=?	?=?	?>?	?>=?
?^?	?~=?			

— domain INTRVL Interval —

```
)abbrev domain INTRVL Interval
++ Author: Mike Dewar
++ Date Created: November 1996
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain is an implementation of interval arithmetic and transcendental
++ functions over intervals.

Interval(R:Join(FloatingPointSystem,TranscendentalFunctionCategory)): IntervalCategory(R) ==
```

```
  import Integer
--  import from R

  Rep := Record(Inf:R, Sup:R)

  roundDown(u:R):R ==
    if zero?(u) then float(-1,-(bits() pretend Integer))
                else float(mantissa(u) - 1,exponent(u))

  roundUp(u:R):R   ==
    if zero?(u) then float(1, -(bits()) pretend Integer)
                else float(mantissa(u) + 1,exponent(u))

  -- Sometimes the float representation does not use all the bits (e.g. when
  -- representing an integer in software using arbitrary-length Integers as
  -- your mantissa it is convenient to keep them exact).  This function
  -- normalises things so that rounding etc. works as expected.  It is only
  -- called when creating new intervals.
  normaliseFloat(u:R):R ==
    zero? u => u
    m : Integer := mantissa u
    b : Integer := bits() pretend Integer
    l : Integer := length(m)
    if l < b then
      BASE : Integer := base()$R pretend Integer
      float(m*BASE**((b-l) pretend PositiveInteger),exponent(u)-b+l)
    else
      u

  interval(i:R,s:R):% ==
    i > s =>  [roundDown normaliseFloat s,roundUp normaliseFloat i]
    [roundDown normaliseFloat i,roundUp normaliseFloat s]

  interval(f:R):% ==
    zero?(f) => 0
    one?(f)  => 1
    -- This next part is necessary to allow e.g. mapping between Expressions:
    -- AXIOM assumes that Integers stay as Integers!
--    import from Union(value1:Integer,failed:"failed")
    fnew : R := normaliseFloat f
    retractIfCan(f)@Union(Integer,"failed") case "failed" =>
      [roundDown fnew, roundUp fnew]
    [fnew,fnew]

  qinterval(i:R,s:R):% ==
    [roundDown normaliseFloat i,roundUp normaliseFloat s]

  exactInterval(i:R,s:R):% == [i,s]
  exactSupInterval(i:R,s:R):% == [roundDown i,s]
```

```
exactInfInterval(i:R,s:R):% == [i,roundUp s]

inf(u:%):R == u.Inf
sup(u:%):R == u.Sup
width(u:%):R == u.Sup - u.Inf

contains?(u:%,f:R):Boolean == (f > inf(u)) and (f < sup(u))

positive?(u:%):Boolean == inf(u) > 0
negative?(u:%):Boolean == sup(u) < 0

_< (a:%,b:%):Boolean ==
  if inf(a) < inf(b) then
    true
  else if inf(a) > inf(b) then
    false
  else
    sup(a) < sup(b)

_+ (a:%,b:%):% ==
  -- A couple of blatent hacks to preserve the Ring Axioms!
  if zero?(a) then return(b) else if zero?(b) then return(a)
  if a = b then return qinterval(2*inf(a),2*sup(a))
  qinterval(inf(a) + inf(b), sup(a) + sup(b))


_- (a:%,b:%):% ==
  if zero?(a) then return(-b) else if zero?(b) then return(a)
  if a = b then 0 else qinterval(inf(a) - sup(b), sup(a) - inf(b))


_* (a:%,b:%):% ==
  -- A couple of blatent hacks to preserve the Ring Axioms!
  if one?(a) then return(b) else if one?(b) then return(a)
  if zero?(a) then return(0) else if zero?(b) then return(0)
  prods : List R :=  sort [inf(a)*inf(b),sup(a)*sup(b),
                           inf(a)*sup(b),sup(a)*inf(b)]
  qinterval(first prods, last prods)


_* (a:Integer,b:%):% ==
  if (a > 0) then
    qinterval(a*inf(b),a*sup(b))
  else if (a < 0) then
    qinterval(a*sup(b),a*inf(b))
  else
    0

_* (a:PositiveInteger,b:%):% == qinterval(a*inf(b),a*sup(b))
```

```
  _*_* (a:%,n:PositiveInteger):% ==
    contains?(a,0) and zero?((n pretend Integer) rem 2) =>
      interval(0,max(inf(a)**n,sup(a)**n))
    interval(inf(a)**n,sup(a)**n)


  _^ (a:%,n:PositiveInteger):% ==
    contains?(a,0) and zero?((n pretend Integer) rem 2) =>
      interval(0,max(inf(a)**n,sup(a)**n))
    interval(inf(a)**n,sup(a)**n)

  _- (a:%):% == exactInterval(-sup(a),-inf(a))

  _= (a:%,b:%):Boolean == (inf(a)=inf(b)) and (sup(a)=sup(b))
  _~_= (a:%,b:%):Boolean == (inf(a)~=inf(b)) or (sup(a)~=sup(b))

  1 ==
    one : R := normaliseFloat 1
    [one,one]

  0 == [0,0]

  recip(u:%):Union(%,"failed") ==
   contains?(u,0) => "failed"
   vals:List R := sort [1/inf(u),1/sup(u)]$List(R)
   qinterval(first vals, last vals)


  unit?(u:%):Boolean == contains?(u,0)

  _exquo(u:%,v:%):Union(%,"failed") ==
   contains?(v,0) => "failed"
   one?(v) => u
   u=v => 1
   u=-v => -1
   vals:List R := sort [inf(u)/inf(v),inf(u)/sup(v),sup(u)/inf(v),sup(u)/sup(v)]$List(R)
   qinterval(first vals, last vals)


  gcd(u:%,v:%):% == 1

  coerce(u:Integer):% ==
    ur := normaliseFloat(u::R)
    exactInterval(ur,ur)


  interval(u:Fraction Integer):% ==
--    import   log2 : % -> %
--             coerce : Integer -> %
--             retractIfCan : % -> Union(value1:Integer,failed:"failed")
```

```
--    from Float
    flt := u::R

    -- Test if the representation in R is exact
    --den := denom(u)::Float
    bin : Union(Integer,"failed") := retractIfCan(log2(denom(u)::Float))
    bin case Integer and length(numer u)$Integer < (bits() pretend Integer) =>
      flt := normaliseFloat flt
      exactInterval(flt,flt)

    qinterval(flt,flt)


  retractIfCan(u:%):Union(Integer,"failed") ==
    not zero? width(u) => "failed"
    retractIfCan inf u


  retract(u:%):Integer ==
    not zero? width(u) =>
      error "attempt to retract a non-Integer interval to an Integer"
    retract inf u


  coerce(u:%):OutputForm ==
    bracket([coerce inf(u), coerce sup(u)]$List(OutputForm))

  characteristic():NonNegativeInteger == 0


  -- Explicit export from TranscendentalFunctionCategory
  pi():% == qinterval(pi(),pi())

  -- From ElementaryFunctionCategory
  log(u:%):% ==
    positive?(u) => qinterval(log inf u, log sup u)
    error "negative logs in interval"


  exp(u:%):% == qinterval(exp inf u, exp sup u)

  _*_* (u:%,v:%):% ==
    zero?(v) => if zero?(u) then error "0**0 is undefined" else 1
    one?(u)  => 1
    expts : List R :=  sort [inf(u)**inf(v),sup(u)**sup(v),
                             inf(u)**sup(v),sup(u)**inf(v)]
    qinterval(first expts, last expts)

  -- From TrigonometricFunctionCategory
```

```
-- This function checks whether an interval contains a value of the form
-- `offset + 2 n pi'.
hasTwoPiMultiple(offset:R,ipi:R,i:%):Boolean ==
  next : Integer := retract ceiling( (inf(i) - offset)/(2*ipi) )
  contains?(i,offset+2*next*ipi)


-- This function checks whether an interval contains a value of the form
-- `offset + n pi'.
hasPiMultiple(offset:R,ipi:R,i:%):Boolean ==
  next : Integer := retract ceiling( (inf(i) - offset)/ipi )
  contains?(i,offset+next*ipi)


sin(u:%):% ==
  ipi : R := pi()$R
  hasOne? : Boolean := hasTwoPiMultiple(ipi/(2::R),ipi,u)
  hasMinusOne? : Boolean := hasTwoPiMultiple(3*ipi/(2::R),ipi,u)

  if hasOne? and hasMinusOne? then
    exactInterval(-1,1)
  else
    vals : List R := sort [sin inf u, sin sup u]
    if hasOne? then
      exactSupInterval(first vals, 1)
    else if hasMinusOne? then
      exactInfInterval(-1,last vals)
    else
      qinterval(first vals, last vals)



cos(u:%):% ==
  ipi : R := pi()
  hasOne? : Boolean := hasTwoPiMultiple(0,ipi,u)
  hasMinusOne? : Boolean := hasTwoPiMultiple(ipi,ipi,u)

  if hasOne? and hasMinusOne? then
    exactInterval(-1,1)
  else
    vals : List R := sort [cos inf u, cos sup u]
    if hasOne? then
      exactSupInterval(first vals, 1)
    else if hasMinusOne? then
      exactInfInterval(-1,last vals)
    else
      qinterval(first vals, last vals)
```

```
  tan(u:%):% ==
    ipi : R := pi()
    if width(u) > ipi then
      error "Interval contains a singularity"
    else
      -- Since we know the interval is less than pi wide, monotonicity implies
      -- that there is no singularity.  If there is a singularity on a endpoint
      -- of the interval the user will see the error generated by R.
      lo : R := tan inf u
      hi : R := tan sup u

      lo > hi => error "Interval contains a singularity"
      qinterval(lo,hi)



  csc(u:%):% ==
    ipi : R := pi()
    if width(u) > ipi then
      error "Interval contains a singularity"
    else
--      import from Integer
      -- singularities are at multiples of Pi
      if hasPiMultiple(0,ipi,u) then error "Interval contains a singularity"
      vals : List R := sort [csc inf u, csc sup u]
      if hasTwoPiMultiple(ipi/(2::R),ipi,u) then
        exactInfInterval(1,last vals)
      else if hasTwoPiMultiple(3*ipi/(2::R),ipi,u) then
        exactSupInterval(first vals,-1)
      else
        qinterval(first vals, last vals)



  sec(u:%):% ==
    ipi : R := pi()
    if width(u) > ipi then
      error "Interval contains a singularity"
    else
--      import from Integer
      -- singularities are at Pi/2 + n Pi
      if hasPiMultiple(ipi/(2::R),ipi,u) then
        error "Interval contains a singularity"
      vals : List R := sort [sec inf u, sec sup u]
      if hasTwoPiMultiple(0,ipi,u) then
        exactInfInterval(1,last vals)
      else if hasTwoPiMultiple(ipi,ipi,u) then
        exactSupInterval(first vals,-1)
      else
        qinterval(first vals, last vals)
```

```
cot(u:%):% ==
  ipi : R := pi()
  if width(u) > ipi then
    error "Interval contains a singularity"
  else
    -- Since we know the interval is less than pi wide, monotonicity implies
    -- that there is no singularity.  If there is a singularity on a endpoint
    -- of the interval the user will see the error generated by R.
    hi : R := cot inf u
    lo : R := cot sup u

    lo > hi => error "Interval contains a singularity"
    qinterval(lo,hi)



-- From ArcTrigonometricFunctionCategory

asin(u:%):% ==
  lo : R := inf(u)
  hi : R := sup(u)
  if (lo < -1) or (hi > 1) then error "asin only defined on the region -1..1"
  qinterval(asin lo,asin hi)


acos(u:%):% ==
  lo : R := inf(u)
  hi : R := sup(u)
  if (lo < -1) or (hi > 1) then error "acos only defined on the region -1..1"
  qinterval(acos hi,acos lo)


atan(u:%):% == qinterval(atan inf u, atan sup u)

acot(u:%):% == qinterval(acot sup u, acot inf u)

acsc(u:%):% ==
  lo : R := inf(u)
  hi : R := sup(u)
  if ((lo <= -1) and (hi >= -1)) or ((lo <= 1) and (hi >= 1)) then
    error "acsc not defined on the region -1..1"
  qinterval(acsc hi, acsc lo)


asec(u:%):% ==
  lo : R := inf(u)
```

```
  hi : R := sup(u)
  if ((lo < -1) and (hi > -1)) or ((lo < 1) and (hi > 1)) then
    error "asec not defined on the region -1..1"
  qinterval(asec lo, asec hi)


-- From HyperbolicFunctionCategory

tanh(u:%):% == qinterval(tanh inf u, tanh sup u)

sinh(u:%):% == qinterval(sinh inf u, sinh sup u)

sech(u:%):% ==
  negative? u => qinterval(sech inf u, sech sup u)
  positive? u => qinterval(sech sup u, sech inf u)
  vals : List R := sort [sech inf u, sech sup u]
  exactSupInterval(first vals,1)


cosh(u:%):% ==
  negative? u => qinterval(cosh sup u, cosh inf u)
  positive? u => qinterval(cosh inf u, cosh sup u)
  vals : List R := sort [cosh inf u, cosh sup u]
  exactInfInterval(1,last vals)


csch(u:%):% ==
  contains?(u,0) => error "csch: singularity at zero"
  qinterval(csch sup u, csch inf u)


coth(u:%):% ==
  contains?(u,0) => error "coth: singularity at zero"
  qinterval(coth sup u, coth inf u)


-- From ArcHyperbolicFunctionCategory

acosh(u:%):% ==
  inf(u)<1 => error "invalid argument: acosh only defined on the region 1.."
  qinterval(acosh inf u, acosh sup u)


acoth(u:%):% ==
  lo : R := inf(u)
  hi : R := sup(u)
  if ((lo <= -1) and (hi >= -1)) or ((lo <= 1) and (hi >= 1)) then
    error "acoth not defined on the region -1..1"
  qinterval(acoth hi, acoth lo)
```

```
  acsch(u:%):% ==
    contains?(u,0) => error "acsch: singularity at zero"
    qinterval(acsch sup u, acsch inf u)


  asech(u:%):% ==
    lo : R := inf(u)
    hi : R := sup(u)
    if  (lo <= 0) or (hi > 1) then
      error "asech only defined on the region 0 < x <= 1"
    qinterval(asech hi, asech lo)


  asinh(u:%):% == qinterval(asinh inf u, asinh sup u)

  atanh(u:%):% ==
    lo : R := inf(u)
    hi : R := sup(u)
    if  (lo <= -1) or (hi >= 1) then
      error "atanh only defined on the region -1 < x < 1"
    qinterval(atanh lo, atanh hi)


  -- From RadicalCategory
  _*_* (u:%,n:Fraction Integer):% == interval(inf(u)**n,sup(u)**n)
```

— INTRVL.dotabb —

```
"INTRVL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INTRVL"]
"INTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=INTCAT"]
"INTRVL" -> "INTCAT"
```

Chapter 11

Chapter J

Chapter 12

Chapter K

12.1 domain KERNEL Kernel

— Kernel.input —

```
)set break resume
)sys rm -f Kernel.output
)spool Kernel.output
)set message test on
)set message auto off
)clear all
--S 1 of 19
x :: Expression Integer
--R
--R
--R   (1)  x
--R                                                 Type: Expression Integer
--E 1

--S 2 of 19
kernel x
--R
--R
--R   (2)  x
--R                                          Type: Kernel Expression Integer
--E 2

--S 3 of 19
sin(x) + cos(x)
--R
--R
--R   (3)  sin(x) + cos(x)
```

```
--R                                                        Type: Expression Integer
--E 3

--S 4 of 19
kernels %
--R
--R
--R   (4)  [sin(x),cos(x)]
--R                                             Type: List Kernel Expression Integer
--E 4

--S 5 of 19
sin(x)**2 + sin(x) + cos(x)
--R
--R
--R              2
--R   (5)  sin(x)  + sin(x) + cos(x)
--R                                                        Type: Expression Integer
--E 5

--S 6 of 19
kernels %
--R
--R
--R   (6)  [sin(x),cos(x)]
--R                                             Type: List Kernel Expression Integer
--E 6

--S 7 of 19
kernels(1 :: Expression Integer)
--R
--R
--R   (7)  []
--R                                             Type: List Kernel Expression Integer
--E 7

--S 8 of 19
mainKernel(cos(x) + tan(x))
--R
--R
--R   (8)  tan(x)
--R                                    Type: Union(Kernel Expression Integer,...)
--E 8

--S 9 of 19
height kernel x
--R
--R
--R   (9)  1
--R                                                           Type: PositiveInteger
```

```
--E 9

--S 10 of 19
height mainKernel(sin x)
--R
--R
--R   (10)  2
--R                                                    Type: PositiveInteger
--E 10

--S 11 of 19
height mainKernel(sin cos x)
--R
--R
--R   (11)  3
--R                                                    Type: PositiveInteger
--E 11

--S 12 of 19
height mainKernel(sin cos (tan x + sin x))
--R
--R
--R   (12)  4
--R                                                    Type: PositiveInteger
--E 12

--S 13 of 19
operator mainKernel(sin cos (tan x + sin x))
--R
--R
--R   (13)  sin
--R                                                      Type: BasicOperator
--E 13

--S 14 of 19
name mainKernel(sin cos (tan x + sin x))
--R
--R
--R   (14)  sin
--R                                                             Type: Symbol
--E 14

--S 15 of 19
f := operator 'f
--R
--R
--R   (15)  f
--R                                                      Type: BasicOperator
--E 15
```

```
--S 16 of 19
e := f(x, y, 10)
--R
--R
--R   (16)  f(x,y,10)
--R                                                  Type: Expression Integer
--E 16

--S 17 of 19
is?(e, f)
--R
--R
--R   (17)  true
--R                                                             Type: Boolean
--E 17

--S 18 of 19
is?(e, 'f)
--R
--R
--R   (18)  true
--R                                                             Type: Boolean
--E 18

--S 19 of 19
argument mainKernel e
--R
--R
--R   (19)  [x,y,10]
--R                                             Type: List Expression Integer
--E 19
)spool
)lisp (bye)
```

— **Kernel.help** —

```
====================================================================
Kernel examples
====================================================================

A kernel is a symbolic function application (such as sin(x+ y))
or a symbol (such as x).  More precisely, a non-symbol
kernel over a set S is an operator applied to a given list of
arguments from S.  The operator has type BasicOperator
and the kernel object is usually part of an Expression object.

Kernels are created implicitly for you when you create expressions.
```

```
x :: Expression Integer
  x
                         Type: Expression Integer
```

You can directly create a "symbol" kernel by using the kernel operation.

```
kernel x
  x
                         Type: Kernel Expression Integer
```

This expression has two different kernels.

```
sin(x) + cos(x)
  sin(x) + cos(x)
                         Type: Expression Integer
```

The operator kernels returns a list of the kernels in an object of type Expression.

```
kernels %
  [sin(x),cos(x)]
                         Type: List Kernel Expression Integer
```

This expression also has two different kernels.

```
sin(x)**2 + sin(x) + cos(x)
        2
  sin(x)  + sin(x) + cos(x)
                         Type: Expression Integer
```

The sin(x) kernel is used twice.

```
kernels %
  [sin(x),cos(x)]
                         Type: List Kernel Expression Integer
```

An expression need not contain any kernels.

```
kernels(1 :: Expression Integer)
  []
                         Type: List Kernel Expression Integer
```

If one or more kernels are present, one of them is designated the main kernel.

```
mainKernel(cos(x) + tan(x))
  tan(x)
                         Type: Union(Kernel Expression Integer,...)
```

Kernels can be nested. Use height to determine the nesting depth.

```
height kernel x
  1
                         Type: PositiveInteger
```

This has height 2 because the x has height 1 and then we apply an operator to that.

```
height mainKernel(sin x)
  2
                         Type: PositiveInteger

height mainKernel(sin cos x)
  3
                         Type: PositiveInteger

height mainKernel(sin cos (tan x + sin x))
  4
                         Type: PositiveInteger
```

Use the operator operation to extract the operator component of the kernel. The operator has type BasicOperator.

```
operator mainKernel(sin cos (tan x + sin x))
  sin
                         Type: BasicOperator
```

Use the name operation to extract the name of the operator component of the kernel. The name has type Symbol. This is really just a shortcut for a two-step process of extracting the operator and then calling name on the operator.

```
name mainKernel(sin cos (tan x + sin x))
  sin
                         Type: Symbol
```

Axiom knows about functions such as sin, cos and so on and can make kernels and then expressions using them. To create a kernel and expression using an arbitrary operator, use operator.

Now f can be used to create symbolic function applications.

```
f := operator 'f
  f
                        Type: BasicOperator

e := f(x, y, 10)
  f(x,y,10)
                        Type: Expression Integer
```

```
Use the is? operation to learn if the operator component of a kernel
is equal to a given operator.

  is?(e, f)
    true
                        Type: Boolean

You can also use a symbol or a string as the second argument to is?.

  is?(e, 'f)
    true
                        Type: Boolean

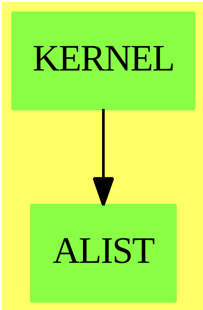
Use the argument operation to get a list containing the argument
component of a kernel.

  argument mainKernel e
    [x,y,10]
                        Type: List Expression Integer

Conceptually, an object of type Expression can be thought of a
quotient of multivariate polynomials, where the "variables" are
kernels.  The arguments of the kernels are again expressions and so
the structure recurses.  See Expression for examples of using kernels
to take apart expression objects.

See Also:
o )help Expression
o )help BasicOperator
o )show Kernel
```

12.1.1 Kernel (KERNEL)



See

⇒ "MakeCachableSet" (MKCHSET) 14.5.1 on page 1534

Exports:

argument	coerce	convert	hash	height
is?	kernel	latex	max	min
name	operator	position	setPosition	symbolIfCan
?~=?	?<?	?<=?	?=?	?>?
?>=?				

— domain KERNEL Kernel —

```
)abbrev domain KERNEL Kernel
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 10 August 1994
++ Description:
++ A kernel over a set S is an operator applied to a given list
++ of arguments from S.

Kernel(S:OrderedSet): Exports == Implementation where
  O  ==> OutputForm
  N  ==> NonNegativeInteger
  OP ==> BasicOperator

  SYMBOL  ==> "%symbol"
  PMPRED  ==> "%pmpredicate"
  PMOPT   ==> "%pmoptional"
  PMMULT  ==> "%pmmultiple"
  PMCONST ==> "%pmconstant"
  SPECIALDISP  ==> "%specialDisp"
  SPECIALEQUAL ==> "%specialEqual"
  SPECIALINPUT ==> "%specialInput"

  Exports ==> Join(CachableSet, Patternable S) with
    name    : % -> Symbol
```

```
    ++ name(op(a1,...,an)) returns the name of op.
  operator: % -> OP
    ++ operator(op(a1,...,an)) returns the operator op.
  argument: % -> List S
    ++ argument(op(a1,...,an)) returns \spad{[a1,...,an]}.
  height  : % -> N
    ++ height(k) returns the nesting level of k.
  kernel  : (OP, List S, N) -> %
    ++ kernel(op, [a1,...,an], m) returns the kernel \spad{op(a1,...,an)}
    ++ of nesting level m.
    ++ Error: if op is k-ary for some k not equal to m.
  kernel  : Symbol -> %
    ++ kernel(x) returns x viewed as a kernel.
  symbolIfCan: % -> Union(Symbol, "failed")
    ++ symbolIfCan(k) returns k viewed as a symbol if k is a symbol, and
    ++ "failed" otherwise.
  is?     : (%, OP) -> Boolean
    ++ is?(op(a1,...,an), f) tests if op = f.
  is?     : (%, Symbol) -> Boolean
    ++ is?(op(a1,...,an), s) tests if the name of op is s.
  if S has ConvertibleTo InputForm then ConvertibleTo InputForm

Implementation ==> add
  import SortedCache(%)

  Rep := Record(op:OP, arg:List S, nest:N, posit:N)

  clearCache()

  B2Z   : Boolean -> Integer
  triage: (%, %)  -> Integer
  preds : OP      -> List Any

  is?(k:%, s:Symbol) == is?(operator k, s)
  is?(k:%, o:OP)     == (operator k) = o
  name k             == name operator k
  height k           == k.nest
  operator k         == k.op
  argument k         == k.arg
  position k         == k.posit
  setPosition(k, n)  == k.posit := n
  B2Z flag           == (flag => -1; 1)
  kernel s           == kernel(assert(operator(s,0),SYMBOL), nil(), 1)

  preds o ==
    (u := property(o, PMPRED)) case "failed" => nil()
    (u::None) pretend List(Any)

  symbolIfCan k ==
    has?(operator k, SYMBOL) => name operator k
```

```
    "failed"

  k1 = k2 ==
    if k1.posit = 0 then enterInCache(k1, triage)
    if k2.posit = 0 then enterInCache(k2, triage)
    k1.posit = k2.posit

  k1 < k2 ==
    if k1.posit = 0 then enterInCache(k1, triage)
    if k2.posit = 0 then enterInCache(k2, triage)
    k1.posit < k2.posit

  kernel(fn, x, n) ==
    ((u := arity fn) case N) and (#x ^= u::N)
                              => error "Wrong number of arguments"
    enterInCache([fn, x, n, 0]$Rep, triage)

  -- SPECIALDISP contains a map List S -> OutputForm
  -- it is used when the converting the arguments first is not good,
  -- for instance with formal derivatives.
  coerce(k:%):OutputForm ==
    (v := symbolIfCan k) case Symbol => v::Symbol::OutputForm
    (f := property(o := operator k, SPECIALDISP)) case None =>
      ((f::None) pretend (List S -> OutputForm)) (argument k)
    l := [x::OutputForm for x in argument k]$List(OutputForm)
    (u := display o) case "failed" => prefix(name(o)::OutputForm, l)
    (u::(List OutputForm -> OutputForm)) l

  triage(k1, k2) ==
    k1.nest   ^= k2.nest   => B2Z(k1.nest   < k2.nest)
    k1.op ^= k2.op => B2Z(k1.op < k2.op)
    (n1 := #(argument k1)) ^= (n2 := #(argument k2)) => B2Z(n1 < n2)
    ((func := property(operator k1, SPECIALEQUAL)) case None) and
      (((func::None) pretend ((%, %) -> Boolean)) (k1, k2)) => 0
    for x1 in argument(k1) for x2 in argument(k2) repeat
      x1 ^= x2 => return B2Z(x1 < x2)
    0

  if S has ConvertibleTo InputForm then
    convert(k:%):InputForm ==
      (v := symbolIfCan k) case Symbol => convert(v::Symbol)@InputForm
      (f := property(o := operator k, SPECIALINPUT)) case None =>
        ((f::None) pretend (List S -> InputForm)) (argument k)
      l := [convert x for x in argument k]$List(InputForm)
      (u := input operator k) case "failed" =>
        convert concat(convert name operator k, l)
      (u::(List InputForm -> InputForm)) l

  if S has ConvertibleTo Pattern Integer then
    convert(k:%):Pattern(Integer) ==
```

```
        o := operator k
        (v := symbolIfCan k) case Symbol =>
          s  := patternVariable(v::Symbol,
                    has?(o, PMCONST), has?(o, PMOPT), has?(o, PMMULT))
          empty?(l := preds o) => s
          setPredicates(s, l)
        o [convert x for x in k.arg]$List(Pattern Integer)

    if S has ConvertibleTo Pattern Float then
      convert(k:%):Pattern(Float) ==
        o := operator k
        (v := symbolIfCan k) case Symbol =>
          s  := patternVariable(v::Symbol,
                    has?(o, PMCONST), has?(o, PMOPT), has?(o, PMMULT))
          empty?(l := preds o) => s
          setPredicates(s, l)
        o [convert x for x in k.arg]$List(Pattern Float)
```

— **KERNEL.dotabb** —

```
"KERNEL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=KERNEL"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"KERNEL" -> "ALIST"
```

12.2 domain KAFILE KeyedAccessFile

— **KeyedAccessFile.input** —

```
)set break resume
)sys rm -f KeyedAccessFile.output
)spool KeyedAccessFile.output
)set message test on
)set message auto off
)clear all

--S 1 of 20
ey: KeyedAccessFile(Integer) := open("editor.year", "output")
--R
--R
--R   (1)  "editor.year"
```

```
--R                                                Type: KeyedAccessFile Integer
--E 1

--S 2 of 20
ey."Char":= 1986
--R
--R
--R   (2)  1986
--R                                                        Type: PositiveInteger
--E 2

--S 3 of 20
ey."Caviness" := 1985
--R
--R
--R   (3)  1985
--R                                                        Type: PositiveInteger
--E 3

--S 4 of 20
ey."Fitch"    := 1984
--R
--R
--R   (4)  1984
--R                                                        Type: PositiveInteger
--E 4

--S 5 of 20
ey."Char"
--R
--R
--R   (5)  1986
--R                                                        Type: PositiveInteger
--E 5

--S 6 of 20
ey("Char")
--R
--R
--R   (6)  1986
--R                                                        Type: PositiveInteger
--E 6

--S 7 of 20
ey "Char"
--R
--R
--R   (7)  1986
--R                                                        Type: PositiveInteger
--E 7
```

```
--S 8 of 20
search("Char", ey)
--R
--R
--R   (8)  1986
--R                                                  Type: Union(Integer,...)
--E 8

--S 9 of 20
search("Smith", ey)
--R
--R
--R   (9)  "failed"
--R                                                 Type: Union("failed",...)
--E 9

--S 10 of 20
remove!("Char", ey)
--R
--R
--R   (10)  1986
--R                                                  Type: Union(Integer,...)
--E 10

--S 11 of 20
keys ey
--R
--R
--R   (11)  ["Fitch","Caviness"]
--R                                                         Type: List String
--E 11

--S 12 of 20
#ey
--R
--R
--R   (12)  2
--R                                                    Type: PositiveInteger
--E 12

--S 13 of 20
KE := Record(key: String, entry: Integer)
--R
--R
--R   (13)  Record(key: String,entry: Integer)
--R                                                              Type: Domain
--E 13

--S 14 of 20
```

```
reopen!(ey, "output")
--R
--R
--R   (14)  "editor.year"
--R                                              Type: KeyedAccessFile Integer
--E 14

--S 15 of 20
write!(ey, ["van Hulzen", 1983]$KE)
--R
--R
--R   (15)  [key= "van Hulzen",entry= 1983]
--R                                    Type: Record(key: String,entry: Integer)
--E 15

--S 16 of 20
write!(ey, ["Calmet", 1982]$KE)
--R
--R
--R   (16)  [key= "Calmet",entry= 1982]
--R                                    Type: Record(key: String,entry: Integer)
--E 16

--S 17 of 20
write!(ey, ["Wang", 1981]$KE)
--R
--R
--R   (17)  [key= "Wang",entry= 1981]
--R                                    Type: Record(key: String,entry: Integer)
--E 17

--S 18 of 20
close! ey
--R
--R
--R   (18)  "editor.year"
--R                                              Type: KeyedAccessFile Integer
--E 18

--S 19 of 20
keys ey
--R
--R
--R   (19)  ["Wang","Calmet","van Hulzen","Fitch","Caviness"]
--R                                                          Type: List String
--E 19

--S 20 of 20
members ey
--R
```

```
--R
--R   (20)  [1981,1982,1983,1984,1985]
--R                                                  Type: List Integer
--E 20

)system rm -r editor.year

)spool
)lisp (bye)
```

— KeyedAccessFile.help —

```
====================================================================
KeyedAccessFile examples
====================================================================

The domain KeyedAccessFile(S) provides files which can be used
as associative tables.  Data values are stored in these files and can
be retrieved according to their keys.  The keys must be strings so
this type behaves very much like the StringTable(S) domain.  The
difference is that keyed access files reside in secondary storage
while string tables are kept in memory.

Before a keyed access file can be used, it must first be opened.
A new file can be created by opening it for output.

  ey: KeyedAccessFile(Integer) := open("editor.year", "output")

Just as for vectors, tables or lists, values are saved in a keyed
access file by setting elements.

  ey."Char":= 1986

  ey."Caviness" := 1985

  ey."Fitch"    := 1984

Values are retrieved using application, in any of its syntactic forms.

  ey."Char"

  ey("Char")

  ey "Char"

Attempting to retrieve a non-existent element in this way causes an
error.  If it is not known whether a key exists, you should use the
```

```
search operation.

  search("Char", ey)

  search("Smith", ey)

When an entry is no longer needed, it can be removed from the file.

  remove!("Char", ey)

The keys operation returns a list of all the keys for a given file.

  keys ey

The # operation gives the number of entries.

  #ey

The table view of keyed access files provides safe operations.  That
is, if the Axiom program is terminated between file operations, the
file is left in a consistent, current state.  This means, however,
that the operations are somewhat costly.  For example, after each
update the file is closed.

Here we add several more items to the file, then check its contents.

  KE := Record(key: String, entry: Integer)

  reopen!(ey, "output")

If many items are to be added to a file at the same time, then it is
more efficient to use the write operation.

  write!(ey, ["van Hulzen", 1983]$KE)

  write!(ey, ["Calmet", 1982]$KE)

  write!(ey, ["Wang", 1981]$KE)

  close! ey

The read operation is also available from the file view, but it
returns elements in a random order.  It is generally clearer and more
efficient to use the keys operation and to extract elements by key.

  keys ey

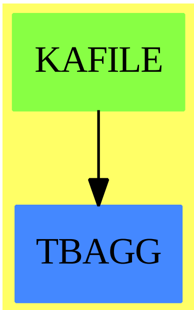
  members ey

  )system rm -r editor.year
```

```
See Also:
o )help Table
o )help StringTable
o )help File
o )help TextFile
o )help Library
o )show KeyedAccessFile
```

———–

12.2.1 KeyedAccessFile (KAFILE)



See

⇒ "File" (FILE) 7.2.1 on page 770
⇒ "TextFile" (TEXTFILE) 21.5.1 on page 2651
⇒ "BinaryFile" (BINFILE) 3.8.1 on page 277
⇒ "Library" (LIB) 13.2.1 on page 1392

Exports:

any?	any?	bag	close!	coerce
construct	convert	copy	count	count
dictionary	elt	empty	empty?	entries
entry?	eq?	eval	every?	every?
extract!	fill!	find	first	hash
index?	indices	insert!	inspect	iomode
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	name	open	pack!	parts
parts	qelt	qsetelt!	read!	reduce
remove	remove!	removeDuplicates	reopen!	sample
search	select	select!	setelt	size?
swap!	table	write!	#?	?~=?
?=?	?.?			

— domain KAFILE KeyedAccessFile —

```
)abbrev domain KAFILE KeyedAccessFile
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain allows a random access file to be viewed both as a table
++ and as a file object. The KeyedAccessFile format is a directory
++ containing a single file called ``index.kaf''. This file is a random
++ access file. The first thing in the file is an integer which is the
++ byte offset of an association list (the dictionary) at the end of
++ the file. The association list is of the form
++ ((key . byteoffset) (key . byteoffset)...)
++ where the byte offset is the number of bytes from the beginning of
++ the file. This offset contains an s-expression for the value of the key.

KeyedAccessFile(Entry): KAFcategory == KAFcapsule where
    Name  ==> FileName
    Key   ==> String
    Entry :   SetCategory

    KAFcategory ==
        Join(FileCategory(Name, Record(key: Key, entry: Entry)),
             TableAggregate(Key, Entry)) with
                 finiteAggregate
```

```
            pack_!: % -> %
                ++ pack!(f) reorganizes the file f on disk to recover
                ++ unused space.

    KAFcapsule == add

        CLASS     ==> 131   -- an arbitrary no. greater than 127
        FileState ==> SExpression
        IOMode    ==> String


        Cons:= Record(car: SExpression, cdr: SExpression)
        Rep := Record(fileName:    Name,      _
                     fileState:   FileState,   _
                     fileIOmode:  IOMode)

        defstream(fn: Name, mode: IOMode): FileState ==
            kafstring:=concat(fn::String,"/index.kaf")::FileName
            mode = "input"  =>
              not readable? kafstring => error ["File is not readable", fn]
              RDEFINSTREAM(fn)$Lisp
            mode = "output" =>
              not writable? fn => error ["File is not writable", fn]
              RDEFOUTSTREAM(fn)$Lisp
            error ["IO mode must be input or output", mode]

        ---- From Set ----
        f1 = f2 ==
            f1.fileName = f2.fileName
        coerce(f: %): OutputForm ==
            f.fileName::OutputForm

        ---- From FileCategory ----
        open fname ==
            open(fname, "either")
        open(fname, mode) ==
            mode = "either" =>
                exists? fname =>
                    open(fname, "input")
                writable? fname =>
                    reopen_!(open(fname, "output"), "input")
                error "File does not exist and cannot be created"
            [fname, defstream(fname, mode), mode]
        reopen_!(f, mode) ==
            close_! f
            if mode ^= "closed" then
                f.fileState := defstream(f.fileName, mode)
                f.fileIOmode  := mode
            f
        close_! f  ==
```

```
    if f.fileIOmode ^= "closed" then
        RSHUT(f.fileState)$Lisp
        f.fileIOmode  := "closed"
    f
read_! f ==
    f.fileIOmode ^= "input" => error ["File not in read state",f]
    ks: List Symbol := RKEYIDS(f.fileName)$Lisp
    null ks => error ["Attempt to read empty file", f]
    ix := random()$Integer rem #ks
    k: String := PNAME(ks.ix)$Lisp
    [k, SPADRREAD(k, f.fileState)$Lisp]
write_!(f, pr) ==
    f.fileIOmode ^= "output" => error ["File not in write state",f]
    SPADRWRITE(pr.key, pr.entry, f.fileState)$Lisp
    pr
name f ==
    f.fileName
iomode f ==
    f.fileIOmode

---- From TableAggregate ----
empty() ==
    fn := new("", "kaf", "sdata")$Name
    open fn
keys f ==
    close_! f
    l: List SExpression := RKEYIDS(f.fileName)$Lisp
    [PNAME(n)$Lisp for n in l]
# f ==
    # keys f
elt(f,k) ==
    reopen_!(f, "input")
    SPADRREAD(k, f.fileState)$Lisp
setelt(f,k,e) ==
    -- Leaves f in a safe, closed state.  For speed use "write".
    reopen_!(f, "output")
    UNWIND_-PROTECT(write_!(f, [k,e]), close_! f)$Lisp
    close_! f
    e
search(k,f) ==
    not member?(k, keys f) => "failed"   -- can't trap RREAD error
    reopen_!(f, "input")
    (SPADRREAD(k, f.fileState)$Lisp)@Entry
remove_!(k:String,f:%)  ==
    result := search(k,f)
    result case "failed" => result
    close_! f
    RDROPITEMS(NAMESTRING(f.fileName)$Lisp, LIST(k)$Lisp)$Lisp
    result
pack_! f ==
```

```
close_! f
RPACKFILE(f.fileName)$Lisp
f
```

———-

— KAFILE.dotabb —

```
"KAFILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=KAFILE"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"KAFILE" -> "TBAGG"
```

———-

Chapter 13

Chapter L

13.1 domain LAUPOL LaurentPolynomial

— **LaurentPolynomial.input** —

```
)set break resume
)sys rm -f LaurentPolynomial.output
)spool LaurentPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LaurentPolynomial
--R LaurentPolynomial(R: IntegralDomain,UP: UnivariatePolynomialCategory R)  is a domain constructor
--R Abbreviation for LaurentPolynomial is LAUPOL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LAUPOL
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : % -> % if UP has DIFRING          D : (%,(UP -> UP)) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coefficient : (%,Integer) -> R        coerce : UP -> %
--R coerce : R -> %                       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R convert : % -> Fraction UP            degree : % -> Integer
--R hash : % -> SingleInteger             latex : % -> String
```

```
--R leadingCoefficient : % -> R          monomial : (R,Integer) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : % -> Integer                 recip : % -> Union(%,"failed")
--R reductum : % -> %                    retract : % -> UP
--R retract : % -> R                     sample : () -> %
--R trailingCoefficient : % -> R         unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if UP has DIFRING
--R D : (%,Symbol) -> % if UP has PDRING SYMBOL
--R D : (%,List Symbol) -> % if UP has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R D : (%,(UP -> UP),NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R differentiate : % -> % if UP has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if UP has DIFRING
--R differentiate : (%,Symbol) -> % if UP has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if UP has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R differentiate : (%,(UP -> UP),NonNegativeInteger) -> %
--R differentiate : (%,(UP -> UP)) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R gcd : (%,%) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R lcm : (%,%) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R ?quo? : (%,%) -> % if R has FIELD
--R ?rem? : (%,%) -> % if R has FIELD
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(UP,"failed")
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R separate : Fraction UP -> Record(polyPart: %,fracPart: Fraction UP) if R has FIELD
```

```
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

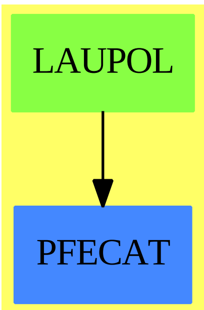
)spool
)lisp (bye)
```

— **LaurentPolynomial.help** —

```
====================================================================
LaurentPolynomial examples
====================================================================

See Also:
o )show LaurentPolynomial
```

13.1.1 LaurentPolynomial (LAUPOL)



Exports:

0	1	associates?
characteristic	charthRoot	coefficient
coerce	convert	D
degree	differentiate	divide
euclideanSize	expressIdealMember	exquo
extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm
leadingCoefficient	monomial	monomial?
multiEuclidean	one?	order
principalIdeal	recip	reductum
retract	retractIfCan	sample
separate	sizeLess?	subtractIfCan
trailingCoefficient	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?quo?	?rem?

— domain LAUPOL LaurentPolynomial —

```
)abbrev domain LAUPOL LaurentPolynomial
++ Author: Manuel Bronstein
++ Date Created: May 1988
++ Date Last Updated: 26 Apr 1990
++ Description:
++ Univariate polynomials with negative and positive exponents.

LaurentPolynomial(R, UP): Exports == Implementation where
  R : IntegralDomain
  UP: UnivariatePolynomialCategory R

  O   ==> OutputForm
  B   ==> Boolean
  N   ==> NonNegativeInteger
  Z   ==> Integer
  RF  ==> Fraction UP

  Exports ==> Join(DifferentialExtension UP, IntegralDomain,
          ConvertibleTo RF, FullyRetractableTo R, RetractableTo UP) with
    monomial?          : %  -> B
      ++ monomial?(x) is not documented
    degree             : %  -> Z
      ++ degree(x) is not documented
    order              : %  -> Z
      ++ order(x) is not documented
    reductum           : %  -> %
      ++ reductum(x) is not documented
```

```
    leadingCoefficient : %  -> R
      ++ leadingCoefficient is not documented
    trailingCoefficient: %  -> R
      ++ trailingCoefficient is not documented
    coefficient        : (%, Z) -> R
      ++ coefficient(x,n) is not documented
    monomial           : (R, Z) -> %
      ++ monomial(x,n) is not documented
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has Field then
      EuclideanDomain
      separate: RF -> Record(polyPart:%, fracPart:RF)
        ++ separate(x) is not documented

  Implementation ==> add
    Rep := Record(polypart: UP, order0: Z)

    poly   : %  -> UP
    check0 : (Z, UP) -> %
    mkgpol : (Z, UP) -> %
    gpol   : (UP, Z) -> %
    toutput: (R, Z, O) -> O
    monTerm: (R, Z, O) -> O

    0                 == [0, 0]
    1                 == [1, 0]
    p = q             == p.order0 = q.order0 and p.polypart = q.polypart
    poly p            == p.polypart
    order p           == p.order0
    gpol(p, n)        == [p, n]
    monomial(r, n)    == check0(n, r::UP)
    coerce(p:UP):%    == mkgpol(0, p)
    reductum p        == check0(order p, reductum poly p)
    n:Z * p:%         == check0(order p, n * poly p)
    characteristic() == characteristic()$R
    coerce(n:Z):%     == n::R::%
    degree p          == degree(poly p)::Z + order p
    monomial? p       == monomial? poly p
    coerce(r:R):%     == gpol(r::UP, 0)
    convert(p:%):RF  == poly(p) * (monomial(1, 1)$UP)::RF ** order p
    p:% * q:%         == check0(order p + order q, poly p * poly q)
    - p               == gpol(- poly p, order p)
    check0(n, p)      == (zero? p => 0; gpol(p, n))
    trailingCoefficient p   == coefficient(poly p, 0)
    leadingCoefficient p    == leadingCoefficient poly p

    coerce(p:%):O ==
      zero? p => 0::Z::O
      l := nil()$List(O)
```

```
      v := monomial(1, 1)$UP :: O
      while p ^= 0 repeat
        l := concat(l, toutput(leadingCoefficient p, degree p, v))
        p := reductum p
      reduce("+", l)

    coefficient(p, n) ==
      (m := n - order p) < 0 => 0
      coefficient(poly p, m::N)

    differentiate(p:%, derivation:UP -> UP) ==
      t := monomial(1, 1)$UP
      mkgpol(order(p) - 1,
              derivation(poly p) * t + order(p) * poly(p) * derivation t)

    monTerm(r, n, v) ==
      zero? n => r::O
--      one? n => v
      (n = 1) => v
      v ** (n::O)

    toutput(r, n, v) ==
      mon := monTerm(r, n, v)
--      zero? n or one? r => mon
      zero? n or (r = 1) => mon
      r = -1 => - mon
      r::O * mon

    recip p ==
      (q := recip poly p) case "failed" => "failed"
      gpol(q::UP, - order p)

    p + q ==
      zero? q => p
      zero? p => q
      (d := order p - order q) > 0 =>
                    gpol(poly(p) * monomial(1, d::N) + poly q, order q)
      d < 0 => gpol(poly(p) + poly(q) * monomial(1, (-d)::N), order p)
      mkgpol(order p, poly(p) + poly q)

    mkgpol(n, p) ==
      zero? p => 0
      d := order(p, monomial(1, 1)$UP)
      gpol((p exquo monomial(1, d))::UP, n + d::Z)

    p exquo q ==
      (r := poly(p) exquo poly q) case "failed" => "failed"
      check0(order p - order q, r::UP)

    retractIfCan(p:%):Union(UP, "failed") ==
```

```
      order(p) < 0 => error "Not retractable"
      poly(p) * monomial(1, order(p)::N)$UP

    retractIfCan(p:%):Union(R, "failed") ==
      order(p) ^= 0 => "failed"
      retractIfCan poly p

    if R has Field then
      gcd(p, q) == gcd(poly p, poly q)::%

      separate f ==
        n  := order(q := denom f, monomial(1, 1))
        q  := (q exquo (tn := monomial(1, n)$UP))::UP
        bc := extendedEuclidean(tn,q,numer f)::Record(coef1:UP,coef2:UP)
        qr := divide(bc.coef1, q)
        [mkgpol(-n, bc.coef2 + tn * qr.quotient), qr.remainder / q]

-- returns (z, r) s.t. p = q z + r,
-- and degree(r) < degree(q), order(r) >= min(order(p), order(q))
      divide(p, q) ==
        c  := min(order p, order q)
        qr := divide(poly(p) * monomial(1, (order p - c)::N)$UP, poly q)
        [mkgpol(c - order q, qr.quotient), mkgpol(c, qr.remainder)]

      euclideanSize p == degree poly p

      extendedEuclidean(a, b, c) ==
        (bc := extendedEuclidean(poly a, poly b, poly c)) case "failed"
          => "failed"
        [mkgpol(order c - order a, bc.coef1),
                                 mkgpol(order c - order b, bc.coef2)]
```

———

— LAUPOL.dotabb —

```
"LAUPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LAUPOL"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"LAUPOL" -> "PFECAT"
```

———

13.2 domain LIB Library

— Library.input —

```
)set break resume
)sys rm -f Library.output
)spool Library.output
)set message test on
)set message auto off
)clear all

--S 1 of 7
stuff := library "Neat.stuff"
--R
--R
--R   (1)  "Neat.stuff"
--R                                                        Type: Library
--E 1

--S 2 of 7
stuff.int := 32**2
--R
--R
--R   (2)  1024
--R                                                Type: PositiveInteger
--E 2

--S 3 of 7
stuff."poly" := x**2 + 1
--R
--R
--R         2
--R   (3)  x  + 1
--R                                             Type: Polynomial Integer
--E 3

--S 4 of 7
stuff.str := "Hello"
--R
--R
--R   (4)  "Hello"
--R                                                         Type: String
--E 4

--S 5 of 7
keys stuff
--R
--R
--R   (5)  ["str","poly","int"]
--R                                                    Type: List String
--E 5

--S 6 of 7
stuff.poly
```

```
--R
--R
--R         2
--R   (6)  x  + 1
--R                                                       Type: Polynomial Integer
--E 6

--S 7 of 7
stuff("poly")
--R
--R
--R         2
--R   (7)  x  + 1
--R                                                       Type: Polynomial Integer
--E 7

)system rm -rf Neat.stuff
)spool
)lisp (bye)
```

——

— **Library.help** —

```
====================================================================
Library examples
====================================================================

The Library domain provides a simple way to store Axiom values
in a file.  This domain is similar to KeyedAccessFile but fewer
declarations are needed and items of different types can be saved
together in the same file.

To create a library, you supply a file name.

  stuff := library "Neat.stuff"

Now values can be saved by key in the file.  The keys should be
mnemonic, just as the field names are for records.  They can be given
either as strings or symbols.

  stuff.int := 32**2

  stuff."poly" := x**2 + 1

  stuff.str := "Hello"

You obtain the set of available keys using the keys operation.
```

```
  keys stuff

You extract values by giving the desired key in this way.

  stuff.poly

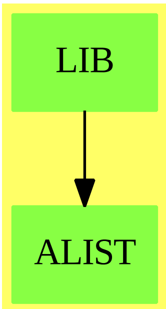
  stuff("poly")

When the file is no longer needed, you should remove it from the
file system.

  )system rm -rf Neat.stuff

See Also:
o )help File
o )help TextFile
o )help KeyedAccessFile
o )show Library
```

13.2.1 Library (LIB)



See

⇒ "File" (FILE) 7.2.1 on page 770
⇒ "TextFile" (TEXTFILE) 21.5.1 on page 2651
⇒ "BinaryFile" (BINFILE) 3.8.1 on page 277
⇒ "KeyedAccessFile" (KAFILE) 12.2.1 on page 1377

Exports:

any?	any?	bag	close!	coerce
copy	construct	convert	count	dictionary
elt	empty	empty?	entries	entry?
eq?	eval	every?	every?	extract!
fill!	find	first	hash	index?
indices	insert!	inspect	key?	keys
latex	less?	library	map	map!
maxIndex	member?	members	minIndex	more?
pack!	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?~=?	?.?

— domain LIB Library —

```
)abbrev domain LIB Library
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: KeyedAccessFile
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides a simple way to save values in files.

Library(): TableAggregate(String, Any) with
         library:  FileName -> %
             ++ library(ln) creates a new library file.
         pack_!: % -> %
             ++ pack!(f) reorganizes the file f on disk to recover
             ++ unused space.

         elt : (%, Symbol) -> Any
             ++ elt(lib,k) or lib.k  extracts the value corresponding to
             ++ the key \spad{k} from the library \spad{lib}.

         setelt : (%, Symbol, Any) -> Any
             ++ \spad{lib.k := v} saves the value \spad{v} in the library
             ++ \spad{lib}.  It can later be extracted using the key \spad{k}.

         close_!: % -> %
          ++ close!(f) returns the library f closed to input and output.

    == KeyedAccessFile(Any) add
```

```
Rep := KeyedAccessFile(Any)
library f == open f
elt(f:%,v:Symbol) == elt(f, string v)
setelt(f:%, v:Symbol, val:Any) == setelt(f, string v, val)
```

— LIB.dotabb —

```
"LIB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LIB"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LIB" -> "ALIST"
```

13.3 domain LEXP LieExponentials

— LieExponentials.input —

```
)set break resume
)sys rm -f LieExponentials.output
)spool LieExponentials.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
a: Symbol := 'a
--R
--R
--R   (1)  a
--R                                                            Type: Symbol
--E 1

--S 2 of 13
b: Symbol := 'b
--R
--R
--R   (2)  b
--R                                                            Type: Symbol
--E 2

--S 3 of 13
coef := Fraction(Integer)
--R
```

```
--R
--R   (3)  Fraction Integer
--R                                                            Type: Domain
--E 3

--S 4 of 13
group := LieExponentials(Symbol, coef, 3)
--R
--R
--R   (4)  LieExponentials(Symbol,Fraction Integer,3)
--R                                                            Type: Domain
--E 4

--S 5 of 13
lpoly := LiePolynomial(Symbol, coef)
--R
--R
--R   (5)  LiePolynomial(Symbol,Fraction Integer)
--R                                                            Type: Domain
--E 5

--S 6 of 13
poly := XPBWPolynomial(Symbol, coef)
--R
--R
--R   (6)  XPBWPolynomial(Symbol,Fraction Integer)
--R                                                            Type: Domain
--E 6

--S 7 of 13
ea := exp(a::lpoly)$group
--R
--R
--R         [a]
--R   (7)  e
--R                               Type: LieExponentials(Symbol,Fraction Integer,3)
--E 7

--S 8 of 13
eb := exp(b::lpoly)$group
--R
--R
--R         [b]
--R   (8)  e
--R                               Type: LieExponentials(Symbol,Fraction Integer,3)
--E 8

--S 9 of 13
g: group := ea*eb
--R
```

```
--R
--R              1     2         1   2
--R              - [a b ]        - [a b]
--R         [b]  2         [a b] 2        [a]
--R   (9)  e   e          e     e        e
--R                               Type: LieExponentials(Symbol,Fraction Integer,3)
--E 9

--S 10 of 13
g :: poly
--R
--R
--R   (10)
--R                        1                            1          1
--R     1 + [a] + [b] + - [a][a] + [a b] + [b][a] + - [b][b] + - [a][a][a]
--R                        2                            2          6
--R   +
--R     1   2                 1     2    1                         1
--R     - [a b] + [a b][a] + - [a b ] + - [b][a][a] + [b][a b] + - [b][b][a]
--R     2                     2          2                         2
--R   +
--R     1
--R     - [b][b][b]
--R     6
--R                                  Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 10

--S 11 of 13
log(g)$group
--R
--R
--R                        1          1   2      1     2
--R   (11)  [a] + [b] + - [a b] + -- [a b] + -- [a b ]
--R                        2         12         12
--R                                   Type: LiePolynomial(Symbol,Fraction Integer)
--E 11

--S 12 of 13
g1: group := inv(g)
--R
--R
--R          - [b] - [a]
--R   (12)  e     e
--R                               Type: LieExponentials(Symbol,Fraction Integer,3)
--E 12

--S 13 of 13
g*g1
--R
--R
```

```
--R   (13)  1
--R                            Type: LieExponentials(Symbol,Fraction Integer,3)
--E 13
)spool
)lisp (bye)
```

— LieExponentials.help —

```
====================================================================
LieExponentials examples
====================================================================

  a: Symbol := 'a
    a
                        Type: Symbol

  b: Symbol := 'b
    b
                        Type: Symbol

====================================================================
Declarations of domains
====================================================================

  coef := Fraction(Integer)
    Fraction Integer
                        Type: Domain

  group := LieExponentials(Symbol, coef, 3)
    LieExponentials(Symbol,Fraction Integer,3)
                        Type: Domain

  lpoly := LiePolynomial(Symbol, coef)
    LiePolynomial(Symbol,Fraction Integer)
                        Type: Domain

  poly := XPBWPolynomial(Symbol, coef)
    XPBWPolynomial(Symbol,Fraction Integer)
                        Type: Domain

====================================================================
Calculations
====================================================================

  ea := exp(a::lpoly)$group
     [a]
    e
```

```
                        Type: LieExponentials(Symbol,Fraction Integer,3)

eb := exp(b::lpoly)$group
   [b]
  e
                        Type: LieExponentials(Symbol,Fraction Integer,3)

g: group := ea*eb
       1     2        1   2
       - [a b ]       - [a b]
   [b] 2        [a b] 2       [a]
  e   e        e     e       e
                        Type: LieExponentials(Symbol,Fraction Integer,3)

g :: poly
                  1                           1          1
   1 + [a] + [b] + - [a][a] + [a b] + [b][a] + - [b][b] + - [a][a][a]
                  2                           2          6
 +
   1   2                 1     2    1                       1
   - [a b] + [a b][a] + - [a b ] + - [b][a][a] + [b][a b] + - [b][b][a]
   2                     2          2                       2
 +
   1
   - [b][b][b]
   6
                       Type: XPBWPolynomial(Symbol,Fraction Integer)

log(g)$group
             1          1   2      1     2
  [a] + [b] + - [a b] + -- [a b] + -- [a b ]
             2          12         12
                       Type: LiePolynomial(Symbol,Fraction Integer)

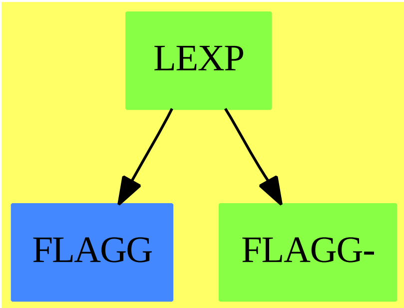
g1: group := inv(g)
   - [b] - [a]
  e     e
                       Type: LieExponentials(Symbol,Fraction Integer,3)

g*g1
  1
                       Type: LieExponentials(Symbol,Fraction Integer,3)
```

See Also:
o)show LieExponentials

———–

13.3.1 LieExponentials (LEXP)



Exports:

1	coerce	commutator	conjugate
exp	hash	identification	inv
latex	log	listOfTerms	LyndonBasis
LyndonCoordinates	mirror	one?	recip
sample	varList	?~=?	?^?
?*?	?**?	?/?	?=?

— **domain LEXP LieExponentials** —

```
)abbrev domain LEXP LieExponentials
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Management of the Lie Group associated with a
++ free nilpotent Lie algebra. Every Lie bracket with
++ length greater than \axiom{Order} are assumed to be null.
++ The implementation inherits from the \spadtype{XPBWPolynomial}
++ domain constructor: Lyndon coordinates are exponential coordinates
++ of the second kind.

LieExponentials(VarSet, R, Order): XDPcat == XDPdef where

  EX      ==> OutputForm
  PI      ==> PositiveInteger
  NNI     ==> NonNegativeInteger
  I       ==> Integer
```

```
RN     ==> Fraction(I)
R      : Join(CommutativeRing, Module RN)
Order  : PI
VarSet : OrderedSet
LWORD  ==> LyndonWord(VarSet)
LWORDS ==> List LWORD
BASIS  ==> PoincareBirkhoffWittLyndonBasis(VarSet)
TERM   ==> Record(k:BASIS, c:R)
LTERMS ==> List(TERM)
LPOLY  ==> LiePolynomial(VarSet,R)
XDPOLY ==> XDistributedPolynomial(VarSet,R)
PBWPOLY==> XPBWPolynomial(VarSet, R)
TERM1  ==> Record(k:LWORD, c:R)
EQ     ==> Equation(R)

XDPcat == Group with
  exp         : LPOLY -> $
    ++ \axiom{exp(p)} returns the exponential of \axiom{p}.
  log         : $ -> LPOLY
    ++ \axiom{log(p)} returns the logarithm of \axiom{p}.
  listOfTerms : $ -> LTERMS
    ++ \axiom{listOfTerms(p)} returns the internal representation of \axiom{p}.
  coerce      : $ -> XDPOLY
    ++ \axiom{coerce(g)} returns the internal representation of \axiom{g}.
  coerce      : $ -> PBWPOLY
    ++ \axiom{coerce(g)} returns the internal representation of \axiom{g}.
  mirror      : $ -> $
    ++ \axiom{mirror(g)} is the mirror of the internal representation of \axiom{g}.
  varList     : $ -> List VarSet
    ++ \axiom{varList(g)} returns the list of variables of \axiom{g}.
  LyndonBasis : List VarSet -> List LPOLY
    ++ \axiom{LyndonBasis(lv)} returns the Lyndon basis of the nilpotent free
    ++ Lie algebra.
  LyndonCoordinates: $ -> List TERM1
    ++ \axiom{LyndonCoordinates(g)} returns the exponential coordinates of \axiom{g}.
  identification: ($,$) -> List EQ
    ++ \axiom{identification(g,h)} returns the list of equations \axiom{g_i = h_i},
    ++ where \axiom{g_i} (resp. \axiom{h_i}) are exponential coordinates
    ++ of \axiom{g} (resp. \axiom{h}).

XDPdef == PBWPOLY add

  -- Representation
     Rep := PBWPOLY

  -- local functions
     compareTerm1s: (TERM1, TERM1) -> Boolean
     out: TERM1 -> EX
     ident: (List TERM1, List TERM1) -> List EQ
```

```
-- functions locales
   ident(l1, l2) ==
     import(TERM1)
     null l1 => [equation(0$R,t.c)$EQ for t in l2]
     null l2 => [equation(t.c, 0$R)$EQ for t in l1]
     u1 : LWORD := l1.first.k; c1 :R := l1.first.c
     u2 : LWORD := l2.first.k; c2 :R := l2.first.c
     u1 = u2 =>
        r: R := c1 - c2
        r = 0 => ident(rest l1, rest l2)
        cons(equation(c1,c2)$EQ , ident(rest l1, rest l2))
     lexico(u1, u2)$LWORD =>
        cons(equation(0$R,c2)$EQ , ident(l1, rest l2))
     cons(equation(c1,0$R)$EQ , ident(rest l1, l2))

   -- ordre lexico decroissant
   compareTerm1s(u:TERM1, v:TERM1):Boolean == lexico(v.k, u.k)$LWORD

   out(t:TERM1):EX ==
     t.c =$R 1 => char("e")$Character :: EX ** t.k ::EX
     char("e")$Character :: EX ** (t.c::EX * t.k::EX)

-- definitions
   identification(x,y) ==
      l1: List TERM1 := LyndonCoordinates x
      l2: List TERM1 := LyndonCoordinates y
      ident(l1, l2)

   LyndonCoordinates x ==
     lt: List TERM1 := [[l::LWORD, t.c]$TERM1 for t in listOfTerms x | _
                        (l := retractIfCan(t.k)$BASIS) case LWORD ]
     lt := sort(compareTerm1s,lt)

   x:$ * y:$ == product(x::Rep, y::Rep, Order::I::NNI)$Rep

   exp p == exp(p::Rep , Order::I::NNI)$Rep

   log p == LiePolyIfCan(log(p,Order::I::NNI))$Rep :: LPOLY

   coerce(p:$):EX ==
      p = 1$$ => 1$R :: EX
      lt : List TERM1 := LyndonCoordinates p
      reduce(_*, [out t for t in lt])$List(EX)


   LyndonBasis(lv) ==
     [LiePoly(l)$LPOLY for l in LyndonWordsList(lv,Order)$LWORD]

   coerce(p:$):PBWPOLY == p::Rep
```

```
    inv x ==
      x = 1 => 1
      lt:LTERMS := listOfTerms mirror x
      lt:= [[t.k, (odd? length(t.k)$BASIS => - t.c; t.c)]$TERM for t in lt ]
      lt pretend $
```

———-

— LEXP.dotabb —

```
"LEXP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LEXP"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"LEXP" -> "FLAGG-"
"LEXP" -> "FLAGG"
```

———-

13.4 domain LPOLY LiePolynomial

— LiePolynomial.input —

```
)set break resume
)sys rm -f LiePolynomial.output
)spool LiePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 28
RN := Fraction Integer
--R
--R
--R   (1)  Fraction Integer
--R                                                                 Type: Domain
--E 1

--S 2 of 28
Lpoly := LiePolynomial(Symbol,RN)
--R
--R
--R   (2)  LiePolynomial(Symbol,Fraction Integer)
--R                                                                 Type: Domain
--E 2
```

```
--S 3 of 28
Dpoly := XDPOLY(Symbol,RN)
--R
--R
--R   (3)  XDistributedPolynomial(Symbol,Fraction Integer)
--R                                                           Type: Domain
--E 3

--S 4 of 28
Lword := LyndonWord Symbol
--R
--R
--R   (4)  LyndonWord Symbol
--R                                                           Type: Domain
--E 4

--S 5 of 28
a:Symbol := 'a
--R
--R
--R   (5)  a
--R                                                           Type: Symbol
--E 5

--S 6 of 28
b:Symbol := 'b
--R
--R
--R   (6)  b
--R                                                           Type: Symbol
--E 6

--S 7 of 28
c:Symbol := 'c
--R
--R
--R   (7)  c
--R                                                           Type: Symbol
--E 7

--S 8 of 28
aa: Lpoly := a
--R
--R
--R   (8)  [a]
--R                               Type: LiePolynomial(Symbol,Fraction Integer)
--E 8

--S 9 of 28
bb: Lpoly := b
```

```
--R
--R
--R   (9)  [b]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 9

--S 10 of 28
cc: Lpoly := c
--R
--R
--R   (10)  [c]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 10

--S 11 of 28
p : Lpoly := [aa,bb]
--R
--R
--R   (11)  [a b]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 11

--S 12 of 28
q : Lpoly := [p,bb]
--R
--R
--R             2
--R   (12)  [a b ]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 12

--S 13 of 28
liste : List Lword := LyndonWordsList([a,b], 4)
--R
--R
--R                             2       2    3     2 2      3
--R   (13)  [[a],[b],[a b],[a b],[a b ],[a b],[a b ],[a b ]]
--R                                                 Type: List LyndonWord Symbol
--E 13

--S 14 of 28
r: Lpoly := p + q + 3*LiePoly(liste.4)$Lpoly
--R
--R
--R                       2         2
--R   (14)  [a b] + 3[a b] + [a b ]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 14

--S 15 of 28
```

```
s:Lpoly := [p,r]
--R
--R
--R                2                 2
--R   (15)  - 3[a b a b] + [a b a b ]
--R                                 Type: LiePolynomial(Symbol,Fraction Integer)
--E 15

--S 16 of 28
t:Lpoly  := s  + 2*LiePoly(liste.3) - 5*LiePoly(liste.5)
--R
--R
--R                         2       2                 2
--R   (16)  2[a b] - 5[a b ] - 3[a b a b] + [a b a b ]
--R                                 Type: LiePolynomial(Symbol,Fraction Integer)
--E 16

--S 17 of 28
degree t
--R
--R
--R   (17)  5
--R                                                        Type: PositiveInteger
--E 17

--S 18 of 28
mirror t
--R
--R
--R                           2       2                 2
--R   (18)  - 2[a b] - 5[a b ] - 3[a b a b] + [a b a b ]
--R                                 Type: LiePolynomial(Symbol,Fraction Integer)
--E 18

--S 19 of 28
Jacobi(p: Lpoly, q: Lpoly, r: Lpoly): Lpoly == _
   [ [p,q]$Lpoly, r] + [ [q,r]$Lpoly, p] + [ [r,p]$Lpoly, q]
--R
--R   Function declaration Jacobi : (LiePolynomial(Symbol,Fraction Integer
--R      ),LiePolynomial(Symbol,Fraction Integer),LiePolynomial(Symbol,
--R      Fraction Integer)) -> LiePolynomial(Symbol,Fraction Integer) has
--R      been added to workspace.
--R                                                                   Type: Void
--E 19

--S 20 of 28
test: Lpoly := Jacobi(a,b,b)
--R
--R   Compiling function Jacobi with type (LiePolynomial(Symbol,Fraction
--R      Integer),LiePolynomial(Symbol,Fraction Integer),LiePolynomial(
```

```
--R      Symbol,Fraction Integer)) -> LiePolynomial(Symbol,Fraction
--R      Integer)
--R
--R   (20)  0
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 20

--S 21 of 28
test: Lpoly := Jacobi(p,q,r)
--R
--R
--R   (21)  0
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 21

--S 22 of 28
test: Lpoly := Jacobi(r,s,t)
--R
--R
--R   (22)  0
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 22

--S 23 of 28
eval(p, a, p)$Lpoly
--R
--R
--R             2
--R   (23)  [a b ]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 23

--S 24 of 28
eval(p, [a,b], [2*bb, 3*aa])$Lpoly
--R
--R
--R   (24)  - 6[a b]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 24

--S 25 of 28
r: Lpoly := [p,c]
--R
--R
--R   (25)  [a b c] + [a c b]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 25

--S 26 of 28
r1: Lpoly := eval(r, [a,b,c], [bb, cc, aa])$Lpoly
```

```
--R
--R
--R   (26)  - [a b c]
--R                                  Type: LiePolynomial(Symbol,Fraction Integer)
--E 26

--S 27 of 28
r2: Lpoly := eval(r, [a,b,c], [cc, aa, bb])$Lpoly
--R
--R
--R   (27)  - [a c b]
--R                                  Type: LiePolynomial(Symbol,Fraction Integer)
--E 27

--S 28 of 28
r + r1 + r2
--R
--R
--R   (28)  0
--R                                  Type: LiePolynomial(Symbol,Fraction Integer)
--E 28
)spool

)lisp (bye)
```

— LiePolynomial.help —

```
====================================================================
LiePolynomial examples
====================================================================

====================================================================
Declaration of domains
====================================================================

  RN := Fraction Integer
    Fraction Integer
                     Type: Domain

  Lpoly := LiePolynomial(Symbol,RN)
    LiePolynomial(Symbol,Fraction Integer)
                     Type: Domain

  Dpoly := XDPOLY(Symbol,RN)
    XDistributedPolynomial(Symbol,Fraction Integer)
                     Type: Domain
```

```
  Lword := LyndonWord Symbol
    LyndonWord Symbol
                     Type: Domain

====================================================================
Initialisation
====================================================================

  a:Symbol := 'a
    a
                     Type: Symbol

  b:Symbol := 'b
    b
                     Type: Symbol

  c:Symbol := 'c
    c
                     Type: Symbol

  aa: Lpoly := a
    [a]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  bb: Lpoly := b
    [b]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  cc: Lpoly := c
    [c]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  p : Lpoly := [aa,bb]
    [a b]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  q : Lpoly := [p,bb]
        2
    [a b ]
                     Type: LiePolynomial(Symbol,Fraction Integer)

All the Lyndon words of order 4

  liste : List Lword := LyndonWordsList([a,b], 4)
                     2       2    3     2 2      3
    [[a],[b],[a b],[a b],[a b ],[a b],[a b ],[a b ]]
                     Type: List LyndonWord Symbol

  r: Lpoly := p + q + 3*LiePoly(liste.4)$Lpoly
              2         2
```

```
    [a b] + 3[a b] + [a b ]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  s:Lpoly := [p,r]
         2                  2
    - 3[a b a b] + [a b a b ]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  t:Lpoly  := s  + 2*LiePoly(liste.3) - 5*LiePoly(liste.5)
                  2       2                  2
    2[a b] - 5[a b ] - 3[a b a b] + [a b a b ]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  degree t
    5
                     Type: PositiveInteger

  mirror t
                    2       2                  2
    - 2[a b] - 5[a b ] - 3[a b a b] + [a b a b ]
                     Type: LiePolynomial(Symbol,Fraction Integer)

====================================================================
Jacobi Relation
====================================================================

 Jacobi(p: Lpoly, q: Lpoly, r: Lpoly): Lpoly == _
   [ [p,q]\$Lpoly, r] + [ [q,r]\$Lpoly, p] + [ [r,p]\$Lpoly, q]
                    Type: Void

====================================================================
Tests
====================================================================

  test: Lpoly := Jacobi(a,b,b)
    0
                    Type: LiePolynomial(Symbol,Fraction Integer)

  test: Lpoly := Jacobi(p,q,r)
    0
                    Type: LiePolynomial(Symbol,Fraction Integer)

  test: Lpoly := Jacobi(r,s,t)
    0
                    Type: LiePolynomial(Symbol,Fraction Integer)

====================================================================
Evaluation
====================================================================
```

```
  eval(p, a, p)$Lpoly
        2
    [a b ]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  eval(p, [a,b], [2*bb, 3*aa])$Lpoly
    - 6[a b]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  r: Lpoly := [p,c]
    [a b c] + [a c b]
                     Type: LiePolynomial(Symbol,Fraction Integer)

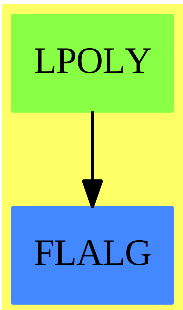
  r1: Lpoly := eval(r, [a,b,c], [bb, cc, aa])$Lpoly
    - [a b c]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  r2: Lpoly := eval(r, [a,b,c], [cc, aa, bb])$Lpoly
    - [a c b]
                     Type: LiePolynomial(Symbol,Fraction Integer)

  r + r1 + r2
    0
                     Type: LiePolynomial(Symbol,Fraction Integer)

See Also:
o )help LyndonWord
o )help XDistributedPolynomial
o )show LiePolynomial
```

13.4.1 LiePolynomial (LPOLY)



Exports:

0	coef	coefficient	coefficients
coerce	construct	degree	eval
hash	latex	leadingCoefficient	leadingMonomial
leadingTerm	LiePoly	LiePolyIfCan	listOfTerms
lquo	map	mirror	monom
monomial?	monomials	numberOfMonomials	reductum
retract	retractIfCan	rquo	sample
subtractIfCan	trunc	varList	zero?
?~=?	?*?	?/?	?+?
?-?	-?	?=?	

— domain LPOLY LiePolynomial —

```
)abbrev domain LPOLY LiePolynomial
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Free Lie Algebras by C. Reutenauer (Oxford science publications).
++ Description:
++ This type supports Lie polynomials in Lyndon basis
++ see Free Lie Algebras by C. Reutenauer
++ (Oxford science publications).

LiePolynomial(VarSet:OrderedSet, R:CommutativeRing) : Public == Private where
   MAGMA   ==> Magma(VarSet)
   LWORD   ==> LyndonWord(VarSet)
   WORD    ==> OrderedFreeMonoid(VarSet)
   XDPOLY  ==> XDistributedPolynomial(VarSet,R)
   XRPOLY  ==> XRecursivePolynomial(VarSet,R)
   NNI     ==> NonNegativeInteger
   RN      ==> Fraction Integer
   EX      ==> OutputForm
   TERM    ==> Record(k: LWORD, c: R)

   Public == Join(FreeLieAlgebra(VarSet,R), FreeModuleCat(R,LWORD)) with
      LiePolyIfCan: XDPOLY -> Union($, "failed")
        ++ \axiom{LiePolyIfCan(p)} returns \axiom{p} in Lyndon basis
        ++ if \axiom{p} is a Lie polynomial, otherwise \axiom{"failed"}
        ++ is returned.
      construct: (LWORD, LWORD) -> $
        ++ \axiom{construct(x,y)} returns the Lie bracket \axiom{[x,y]}.
```

```
   construct: (LWORD, $) -> $
     ++ \axiom{construct(x,y)} returns the Lie bracket \axiom{[x,y]}.
   construct: ($, LWORD) -> $
     ++ \axiom{construct(x,y)} returns the Lie bracket \axiom{[x,y]}.

Private ==  FreeModule1(R, LWORD) add
     import(TERM)

   --representation
     Rep :=  List TERM

   -- fonctions locales
     cr1 : (LWORD, $    ) -> $
     cr2 : ($, LWORD    ) -> $
     crw : (LWORD, LWORD) -> $     -- crochet de 2 mots de Lyndon
     DPoly: LWORD -> XDPOLY
     lquo1: (XRPOLY , LWORD) -> XRPOLY
     lyndon: (LWORD, LWORD) -> $
     makeLyndon: (LWORD, LWORD) -> LWORD
     rquo1: (XRPOLY , LWORD) -> XRPOLY
     RPoly: LWORD -> XRPOLY
     eval1: (LWORD, VarSet, $) -> $                    -- 08/03/98
     eval2: (LWORD, List VarSet, List $) -> $          -- 08/03/98


   -- Evaluation
     eval1(lw,v,nv) ==                                 -- 08/03/98
       not member?(v, varList(lw)$LWORD) => LiePoly lw
       (s := retractIfCan(lw)$LWORD) case VarSet =>
          if (s::VarSet) = v then nv else LiePoly lw
       l: LWORD := left lw
       r: LWORD := right lw
       construct(eval1(l,v,nv), eval1(r,v,nv))

     eval2(lw,lv,lnv) ==                               -- 08/03/98
       p: Integer
       (s := retractIfCan(lw)$LWORD) case VarSet =>
          p := position(s::VarSet, lv)$List(VarSet)
          if p=0 then lw::$ else elt(lnv,p)$List($)
       l: LWORD := left lw
       r: LWORD := right lw
       construct(eval2(l,lv,lnv), eval2(r,lv,lnv))

     eval(p:$, v: VarSet, nv: $): $ ==                 -- 08/03/98
       +/ [t.c * eval1(t.k, v, nv) for t in p]

     eval(p:$, lv: List(VarSet), lnv: List($)): $ ==   -- 08/03/98
       +/ [t.c * eval2(t.k, lv, lnv) for t in p]

     lquo1(p,lw) ==
```

```
      constant? p => 0$XRPOLY
      retractable? lw => lquo(p, retract lw)$XRPOLY
      lquo1(lquo1(p, left lw),right lw) - lquo1(lquo1(p, right lw),left lw)
    rquo1(p,lw) ==
      constant? p => 0$XRPOLY
      retractable? lw => rquo(p, retract lw)$XRPOLY
      rquo1(rquo1(p, left lw),right lw) - rquo1(rquo1(p, right lw),left lw)

    coef(p, lp) == coef(p, lp::XRPOLY)$XRPOLY

    lquo(p, lp) ==
      lp = 0 => 0$XRPOLY
      +/ [t.c * lquo1(p,t.k) for t in lp]

    rquo(p, lp) ==
      lp = 0 => 0$XRPOLY
      +/ [t.c * rquo1(p,t.k) for t in lp]

    LiePolyIfCan p ==         -- inefficace a cause de la rep. de XDPOLY
       not quasiRegular? p => "failed"
       p1: XDPOLY := p ; r:$ := 0
       while p1 ^= 0 repeat
         t: Record(k:WORD, c:R) := mindegTerm p1
         w: WORD := t.k; coef:R := t.c
         (l := lyndonIfCan(w)$LWORD) case "failed" => return "failed"
         lp:$ := coef * LiePoly(l::LWORD)
         r := r + lp
         p1 := p1 - lp::XDPOLY
       r

  --definitions locales
    makeLyndon(u,v) == (u::MAGMA * v::MAGMA) pretend LWORD

    crw(u,v) ==               -- u et v sont des mots de Lyndon
      u = v => 0
      lexico(u,v) => lyndon(u,v)
      - lyndon (v,u)

    lyndon(u,v) ==            -- u et v sont des mots de Lyndon tq u < v
      retractable? u => monom(makeLyndon(u,v),1)
      u1: LWORD := left u
      u2: LWORD := right u
      lexico(u2,v) => cr1(u1, lyndon(u2,v)) + cr2(lyndon(u1,v), u2)
      monom(makeLyndon(u,v),1)

    cr1 (l, p) ==
        +/[t.c * crw(l, t.k) for t in p]

    cr2 (p, l) ==
        +/[t.c * crw(t.k, l) for t in p]
```

```
      DPoly w ==
         retractable? w => retract(w) :: XDPOLY
         l:XDPOLY := DPoly left w
         r:XDPOLY := DPoly right w
         l*r - r*l

      RPoly w ==
         retractable? w => retract(w) :: XRPOLY
         l:XRPOLY := RPoly left w
         r:XRPOLY := RPoly right w
         l*r - r*l

    -- definitions

      coerce(v:VarSet) == monom(v::LWORD , 1)

      construct(x:$ , y:$):$ ==
          +/[t.c * cr1(t.k, y) for t in x]

      construct(l:LWORD , p:$):$ == cr1(l,p)
      construct(p:$ , l:LWORD):$ == cr2(p,l)
      construct(u:LWORD , v:LWORD):$ == crw(u,v)

      coerce(p:$):XDPOLY ==
          +/ [t.c * DPoly(t.k) for t in p]

      coerce(p:$):XRPOLY ==
          +/ [t.c * RPoly(t.k) for t in p]

      LiePoly(l) == monom(l,1)

      varList p ==
        le : List VarSet := "setUnion"/[varList(t.k)$LWORD for t in p]
        sort(le)$List(VarSet)

      mirror p ==
        [[t.k, (odd? length t.k => t.c; -t.c)]$TERM for t in p]

      trunc(p, n) ==
        degree(p) > n => trunc( reductum p , n)
        p

      degree p ==
        null p => 0
        length( p.first.k)$LWORD

    --  listOfTerms p == p pretend List TERM

--        coerce(x) : EX ==
```

```
--          null x => (0$R) :: EX
--          le : List EX := nil
--          for rec in x repeat
--            rec.c = 1$R => le := cons(rec.k :: EX, le)
--            le := cons(mkBinary("*"::EX,  rec.c :: EX, rec.k :: EX), le)
--          1 = #le => first le
--          mkNary("+" :: EX,le)
```

— **LPOLY.dotabb** —

```
"LPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LPOLY"]
"FLALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLALG"]
"LPOLY" -> "FLALG"
```

13.5 domain LSQM LieSquareMatrix

— **LieSquareMatrix.input** —

```
)set break resume
)sys rm -f LieSquareMatrix.output
)spool LieSquareMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LieSquareMatrix
--R LieSquareMatrix(n: PositiveInteger,R: CommutativeRing)  is a domain constructor
--R Abbreviation for LieSquareMatrix is LSQM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LSQM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : % -> % if R has DIFRING           D : (%,(R -> R)) -> %
--R 1 : () -> %                           0 : () -> %
```

```
--R ?^? : (%,PositiveInteger) -> %        alternative? : () -> Boolean
--R antiAssociative? : () -> Boolean      antiCommutative? : () -> Boolean
--R antiCommutator : (%,%) -> %           antisymmetric? : % -> Boolean
--R apply : (Matrix R,%) -> %             associative? : () -> Boolean
--R associator : (%,%,%) -> %             basis : () -> Vector %
--R coerce : % -> Matrix R                coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R commutative? : () -> Boolean          commutator : (%,%) -> %
--R convert : % -> Vector R               convert : Vector R -> %
--R coordinates : % -> Vector R           copy : % -> %
--R diagonal? : % -> Boolean              diagonalMatrix : List R -> %
--R diagonalProduct : % -> R              ?.? : (%,Integer) -> R
--R elt : (%,Integer,Integer) -> R        elt : (%,Integer,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                flexible? : () -> Boolean
--R hash : % -> SingleInteger             jacobiIdentity? : () -> Boolean
--R jordanAdmissible? : () -> Boolean     jordanAlgebra? : () -> Boolean
--R latex : % -> String                   leftAlternative? : () -> Boolean
--R leftDiscriminant : Vector % -> R      leftDiscriminant : () -> R
--R leftNorm : % -> R                     leftTrace : % -> R
--R leftTraceMatrix : () -> Matrix R      lieAdmissible? : () -> Boolean
--R lieAlgebra? : () -> Boolean           listOfLists : % -> List List R
--R map : ((R -> R),%) -> %               map : (((R,R) -> R),%,%) -> %
--R matrix : List List R -> %             maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       one? : % -> Boolean
--R powerAssociative? : () -> Boolean     qelt : (%,Integer,Integer) -> R
--R rank : () -> PositiveInteger          recip : % -> Union(%,"failed")
--R represents : Vector R -> %            retract : % -> R
--R rightAlternative? : () -> Boolean     rightDiscriminant : Vector % -> R
--R rightDiscriminant : () -> R           rightNorm : % -> R
--R rightTrace : % -> R                   rightTraceMatrix : () -> Matrix R
--R sample : () -> %                      scalarMatrix : R -> %
--R someBasis : () -> Vector %            square? : % -> Boolean
--R symmetric? : % -> Boolean             trace : % -> R
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (DirectProduct(n,R),%) -> DirectProduct(n,R)
--R ?*? : (%,DirectProduct(n,R)) -> DirectProduct(n,R)
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,(R -> R),NonNegativeInteger) -> %
```

```
--R ?^? : (%,NonNegativeInteger) -> %
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R associatorDependence : () -> List Vector R if R has INTDOM
--R characteristic : () -> NonNegativeInteger
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R column : (%,Integer) -> DirectProduct(n,R)
--R conditionsForIdempotents : Vector % -> List Polynomial R
--R conditionsForIdempotents : () -> List Polynomial R
--R coordinates : (%,Vector %) -> Vector R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : Vector % -> Matrix R
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R diagonal : % -> DirectProduct(n,R)
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,(R -> R)) -> %
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R inverse : % -> Union(%,"failed") if R has FIELD
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
--R leftRegularRepresentation : (%,Vector %) -> Matrix R
--R leftRegularRepresentation : % -> Matrix R
--R leftTraceMatrix : Vector % -> Matrix R
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R noncommutativeJordanAlgebra? : () -> Boolean
--R nullSpace : % -> List DirectProduct(n,R) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List R if $ has finiteAggregate
```

```
--R plenaryPower : (%,PositiveInteger) -> %
--R rank : % -> NonNegativeInteger if R has INTDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R represents : (Vector R,Vector %) -> %
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : (%,Vector %) -> Matrix R
--R rightRegularRepresentation : % -> Matrix R
--R rightTraceMatrix : Vector % -> Matrix R
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R row : (%,Integer) -> DirectProduct(n,R)
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%,NonNegativeInteger) -> Boolean
--R structuralConstants : Vector % -> Vector Matrix R
--R structuralConstants : () -> Vector Matrix R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

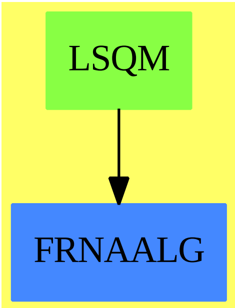
)spool
)lisp (bye)
```

— **LieSquareMatrix.help** —

```
====================================================================
LieSquareMatrix examples
====================================================================

See Also:
o )show LieSquareMatrix
```

13.5.1 LieSquareMatrix (LSQM)



See

⇒ "AssociatedLieAlgebra" (LIE) 2.41.1 on page 211
⇒ "AssociatedJordanAlgebra" (JORDAN) 2.40.1 on page 206

Exports:

0	1	alternative?
antiAssociative?	antiCommutator	antisymmetric?
any?	apply	associative?
associator	associatorDependence	basis
characteristic	coerce	column
commutative?	commutator	conditionsForIdempotents
convert	coordinates	copy
count	D	determinant
diagonal	diagonal?	diagonalMatrix
diagonalProduct	differentiate	elt
empty	empty?	eq?
eval	every?	exquo
flexible?	hash	inverse
jacobiIdentity?	jordanAdmissible?	jordanAlgebra?
latex	leftAlternative?	leftCharacteristicPolynomial
leftDiscriminant	leftDiscriminant	leftMinimalPolynomial
leftNorm	leftPower	leftRankPolynomial
leftRecip	leftRegularRepresentation	leftRegularRepresentation
leftTrace	leftTraceMatrix	leftUnit
leftUnits	less?	lieAdmissible?
lieAlgebra?	listOfLists	map
map!	matrix	maxColIndex
maxRowIndex	member?	members
minColIndex	minordet	minRowIndex
more?	ncols	noncommutativeJordanAlgebra?
nrows	nullSpace	nullity
one?	parts	plenaryPower
powerAssociative?	qelt	rank
recip	reducedSystem	represents
retract	retractIfCan	rightAlternative?
rightCharacteristicPolynomial	rightDiscriminant	rightMinimalPolynomial
rightNorm	rightPower	rightRankPolynomial
rightRecip	rightRegularRepresentation	rightTrace
rightTraceMatrix	rightUnit	rightUnits
row	rowEchelon	sample
scalarMatrix	size?	someBasis
square?	structuralConstants	structuralConstants
subtractIfCan	symmetric?	trace
unit	zero?	#?
?~=?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?.?		

— domain LSQM LieSquareMatrix —

```
)abbrev domain LSQM LieSquareMatrix
++ Author: J. Grabmeier
++ Date Created: 07 March 1991
++ Date Last Updated: 08 March 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ LieSquareMatrix(n,R) implements the Lie algebra of the n by n
++ matrices over the commutative ring R.
++ The Lie bracket (commutator) of the algebra is given by\br
++ \spad{a*b := (a *$SQMATRIX(n,R) b - b *$SQMATRIX(n,R) a)},\br
++ where \spadfun{*$SQMATRIX(n,R)} is the usual matrix multiplication.

LieSquareMatrix(n,R): Exports == Implementation where

  n    : PositiveInteger
  R    : CommutativeRing

  Row ==> DirectProduct(n,R)
  Col ==> DirectProduct(n,R)

  Exports ==> Join(SquareMatrixCategory(n,R,Row,Col), CoercibleTo Matrix R,_
      FramedNonAssociativeAlgebra R) --with

  Implementation ==> AssociatedLieAlgebra (R,SquareMatrix(n, R)) add

    Rep :=  AssociatedLieAlgebra (R,SquareMatrix(n, R))
      -- local functions
    n2 : PositiveInteger := n*n

    convDM : DirectProduct(n2,R) -> %
    conv : DirectProduct(n2,R) ->  SquareMatrix(n,R)
      --++ converts n2-vector to (n,n)-matrix row by row
    conv v  ==
      cond : Matrix(R) := new(n,n,0$R)$Matrix(R)
      z : Integer := 0
      for i in 1..n repeat
        for j in 1..n  repeat
          z := z+1
          setelt(cond,i,j,v.z)
      squareMatrix(cond)$SquareMatrix(n, R)


    coordinates(a:%,b:Vector(%)):Vector(R) ==
      -- only valid for b canonicalBasis
      res : Vector R := new(n2,0$R)
```

```
      z : Integer := 0
      for i in 1..n repeat
        for j in 1..n repeat
          z := z+1
          res.z := elt(a,i,j)$%
      res


    convDM v ==
      sq := conv v
      coerce(sq)$Rep :: %

    basis() ==
      n2 : PositiveInteger := n*n
      ldp : List DirectProduct(n2,R) :=
        [unitVector(i::PositiveInteger)$DirectProduct(n2,R) for i in 1..n2]
      res:Vector % := vector map(convDM,_
        ldp)$ListFunctions2(DirectProduct(n2,R), %)

    someBasis() == basis()
    rank() == n*n


--    transpose: % -> %
--      ++ computes the transpose of a matrix
--    squareMatrix: Matrix R -> %
--      ++ converts a Matrix to a LieSquareMatrix
--    coerce: % -> Matrix R
--      ++ converts a LieSquareMatrix to a Matrix
--    symdecomp : % -> Record(sym:%,antisym:%)
--    if R has commutative("*") then
--      minorsVect: -> Vector(Union(R,"uncomputed")) --range: 1..2**n-1
--    if R has commutative("*") then central
--    if R has commutative("*") and R has unitsKnown then unitsKnown
```

———

— LSQM.dotabb —

```
"LSQM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LSQM"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"LSQM" -> "FRNAALG"
```

———

13.6 domain LODO LinearOrdinaryDifferentialOperator

— LinearOrdinaryDifferentialOperator.input —

```
)set break resume
)sys rm -f LinearOrdinaryDifferentialOperator.output
)spool LinearOrdinaryDifferentialOperator.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
Dx: LODO(EXPR INT, f +-> D(f, x))
--R
--R                                                                   Type: Void
--E 1

--S 2 of 16
Dx := D()
--R
--R
--R   (2)  D
--IType: LinearOrdinaryDifferentialOperator(Expression Integer,theMap LAMBDA-CLOSURE(NIL,NIL,NIL,G1404 e
--E 2

--S 3 of 16
Dop:= Dx^3 + G/x^2*Dx + H/x^3 - 1
--R
--R
--R                          3
--R         3    G     - x  + H
--R   (3)  D  + -- D + --------
--R              2        3
--R             x        x
--IType: LinearOrdinaryDifferentialOperator(Expression Integer,theMap LAMBDA-CLOSURE(NIL,NIL,NIL,G1404 e
--E 3

--S 4 of 16
n == 3
--R
--R                                                                   Type: Void
--E 4

--S 5 of 16
phi == reduce(+,[subscript(s,[i])*exp(x)/x^i for i in 0..n])
--R
--R                                                                   Type: Void
--E 5
```

```
--S 6 of 16
phi1 ==  Dop(phi) / exp x
--R
--R                                                                   Type: Void
--E 6

--S 7 of 16
phi2 == phi1 *x**(n+3)
--R
--R                                                                   Type: Void
--E 7

--S 8 of 16
phi3 == retract(phi2)@(POLY INT)
--R
--R                                                                   Type: Void
--E 8

--S 9 of 16
pans == phi3 ::UP(x,POLY INT)
--R
--R                                                                   Type: Void
--E 9

--S 10 of 16
pans1 == [coefficient(pans, (n+3-i) :: NNI) for i in 2..n+1]
--R
--R                                                                   Type: Void
--E 10

--S 11 of 16
leq == solve(pans1,[subscript(s,[i]) for i in 1..n])
--R
--R                                                                   Type: Void
--E 11

--S 12 of 16
leq
--R
--R   Compiling body of rule n to compute value of type PositiveInteger
--R   Compiling body of rule phi to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi1 to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi2 to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi3 to compute value of type Polynomial
--R      Integer
--R   Compiling body of rule pans to compute value of type
--R      UnivariatePolynomial(x,Polynomial Integer)
```

```
--R   Compiling body of rule pans1 to compute value of type List
--R      Polynomial Integer
--R   Compiling body of rule leq to compute value of type List List
--R      Equation Fraction Polynomial Integer
--I   Compiling function G3349 with type Integer -> Boolean
--R
--R   (12)
--R                          2                                3        2
--R         s G     3s H + s G  + 6s G     (9s G + 54s )H + s G  + 18s G  + 72s G
--R          0        0     0      0         0       0      0        0        0
--R   [[s = ---,s = ------------------,s = -------------------------------------]]
--R      1   3   2          18         3                    162
--R                            Type: List List Equation Fraction Polynomial Integer
--E 12

--S 13 of 16
n==4
--R
--R   Compiled code for n has been cleared.
--R   Compiled code for leq has been cleared.
--R   Compiled code for pans1 has been cleared.
--R   Compiled code for phi2 has been cleared.
--R   Compiled code for phi has been cleared.
--R   Compiled code for phi3 has been cleared.
--R   Compiled code for phi1 has been cleared.
--R   Compiled code for pans has been cleared.
--R   1 old definition(s) deleted for function or rule n
--R                                                                    Type: Void
--E 13

--S 14 of 16
leq
--R
--R   Compiling body of rule n to compute value of type PositiveInteger
--R   Compiling body of rule phi to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi1 to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi2 to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi3 to compute value of type Polynomial
--R      Integer
--R   Compiling body of rule pans to compute value of type
--R      UnivariatePolynomial(x,Polynomial Integer)
--R   Compiling body of rule pans1 to compute value of type List
--R      Polynomial Integer
--R   Compiling body of rule leq to compute value of type List List
--R      Equation Fraction Polynomial Integer
--R
--R   (14)
```

```
--R   [
--R                              2
--R          s G      3s H + s G  + 6s G
--R           0         0     0       0
--R     [s = ---, s = ------------------,
--R       1   3    2          18
--R                                3        2
--R          (9s G + 54s )H + s G  + 18s G  + 72s G
--R             0       0      0        0        0
--R      s = -------------------------------------,
--R       3                   162
--R
--R       s  =
--R        4
--R                 2         2                          4        3         2
--R            27s H  + (18s G  + 378s G + 1296s )H + s G  + 36s G  + 396s G
--R               0         0         0         0      0        0         0
--R          +
--R            1296s G
--R                 0
--R        /
--R          1944
--R       ]
--R     ]
--R                         Type: List List Equation Fraction Polynomial Integer
--E 14

--S 15 of 16
n==7
--R
--R   Compiled code for n has been cleared.
--R   Compiled code for leq has been cleared.
--R   Compiled code for pans1 has been cleared.
--R   Compiled code for phi2 has been cleared.
--R   Compiled code for phi has been cleared.
--R   Compiled code for phi3 has been cleared.
--R   Compiled code for phi1 has been cleared.
--R   Compiled code for pans has been cleared.
--R   1 old definition(s) deleted for function or rule n
--R                                                                    Type: Void
--E 15

--S 16 of 16
leq
--R
--R   Compiling body of rule n to compute value of type PositiveInteger
--R   Compiling body of rule phi to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi1 to compute value of type Expression
--R      Integer
```

```
--R   Compiling body of rule phi2 to compute value of type Expression
--R      Integer
--R   Compiling body of rule phi3 to compute value of type Polynomial
--R      Integer
--R   Compiling body of rule pans to compute value of type
--R      UnivariatePolynomial(x,Polynomial Integer)
--R   Compiling body of rule pans1 to compute value of type List
--R      Polynomial Integer
--R   Compiling body of rule leq to compute value of type List List
--R      Equation Fraction Polynomial Integer
--R
--R   (16)
--R   [
--R                               2
--R         s G      3s H + s G  + 6s G
--R          0         0     0       0
--R     [s = ---, s = ------------------,
--R       1   3    2          18
--R                                3        2
--R         (9s G + 54s )H + s G  + 18s G  + 72s G
--R            0       0      0        0        0
--R      s = -------------------------------------,
--R       3                    162
--R
--R       s  =
--R        4
--R                 2         2                          4        3         2
--R            27s H  + (18s G  + 378s G + 1296s )H + s G  + 36s G  + 396s G
--R               0         0         0         0      0        0         0
--R          +
--R            1296s G
--R                 0
--R        /
--R          1944
--R       ,
--R
--R       s  =
--R        5
--R                                 2         3          2
--R            (135s G + 2268s )H  + (30s G  + 1350s G  + 16416s G + 38880s )H
--R                 0         0          0          0           0          0
--R          +
--R               5        4          3          2
--R            s G  + 60s G  + 1188s G  + 9504s G  + 25920s G
--R             0        0          0          0           0
--R        /
--R          29160
--R       ,
--R
--R       s  =
```

```
--R        6
--R                   3          2                       2
--R             405s H  + (405s G  + 18468s G + 174960s )H
--R                 0          0           0           0
--R           +
--R                   4          3           2                                6
--R             (45s G  + 3510s G  + 88776s G  + 777600s G + 1166400s )H + s G
--R                 0          0           0            0            0      0
--R           +
--R                  5          4           3           2
--R             90s G  + 2628s G  + 27864s G  + 90720s G
--R                0          0           0           0
--R        /
--R           524880
--R       ,
--R
--R       s  =
--R        7
--R                                    3
--R             (2835s G + 91854s )H
--R                   0          0
--R           +
--R                    3           2                            2
--R             (945s G  + 81648s G  + 2082996s G + 14171760s )H
--R                  0           0             0             0
--R           +
--R                   5          4            3             2
--R             (63s G  + 7560s G  + 317520s G  + 5554008s G  + 34058880s G)H
--R                 0          0            0             0              0
--R           +
--R                7         6          5           4             3              2
--R             s G  + 126s G  + 4788s G  + 25272s G  - 1744416s G  - 26827200s G
--R              0         0          0           0             0              0
--R           +
--R             - 97977600s G
--R                          0
--R        /
--R           11022480
--R       ]
--R     ]
--R                         Type: List List Equation Fraction Polynomial Integer
--E 16
)spool
```

———-

— LinearOrdinaryDifferentialOperator.help —

```
====================================================================
LinearOrdinaryDifferentialOperator examples
====================================================================

LinearOrdinaryDifferentialOperator(A, diff) is the domain of linear
ordinary differential operators with coefficients in a ring A with a
given derivation.

====================================================================
Differential Operators with Series Coefficients
====================================================================

Problem:
  Find the first few coefficients of exp(x)/x^i of Dop phi where

    Dop := D^3 + G/x^2 * D + H/x^3 - 1
    phi := sum(s[i]*exp(x)/x^i, i = 0..)

Solution:

Define the differential.

  Dx: LODO(EXPR INT, f +-> D(f, x))
                       Type: Void

  Dx := D()
   D
     Type: LinearOrdinaryDifferentialOperator(Expression Integer,
      theMap LAMBDA-CLOSURE(NIL,NIL,NIL,G1404 envArg,
              SPADCALL(G1404,QUOTE x,
               ELT(*1;anonymousFunction;0;frame0;internal;MV,0))))

Now define the differential operator Dop.

  Dop:= Dx^3 + G/x^2*Dx + H/x^3 - 1
                       3
     3    G     - x  + H
    D  + -- D + --------
          2        3
         x        x
     Type: LinearOrdinaryDifferentialOperator(Expression Integer,
      theMap LAMBDA-CLOSURE(NIL,NIL,NIL,G1404 envArg,
              SPADCALL(G1404,QUOTE x,
               ELT(*1;anonymousFunction;0;frame0;internal;MV,0))))

  n == 3
                        Type: Void

  phi == reduce(+,[subscript(s,[i])*exp(x)/x^i for i in 0..n])
                        Type: Void
```

```
  phi1 ==  Dop(phi) / exp x
                         Type: Void

  phi2 == phi1 *x**(n+3)
                         Type: Void

  phi3 == retract(phi2)@(POLY INT)
                         Type: Void

  pans == phi3 ::UP(x,POLY INT)
                         Type: Void

  pans1 == [coefficient(pans, (n+3-i) :: NNI) for i in 2..n+1]
                         Type: Void

  leq == solve(pans1,[subscript(s,[i]) for i in 1..n])
                         Type: Void
```

Evaluate this for several values of n.

```
  leq
                        2                                 3        2
      s G     3s H + s G  + 6s G     (9s G + 54s )H + s G  + 18s G  + 72s G
       0        0     0       0         0       0      0        0        0
 [[s = ---,s = ------------------,s = -------------------------------------]]
    1   3   2          18          3                    162
                        Type: List List Equation Fraction Polynomial Integer

  n==4
                         Type: Void

  leq
   [
                           2
        s G      3s H + s G  + 6s G
         0          0     0       0
     [s = ---, s = ------------------,
       1   3    2           18
                            3        2
        (9s G + 54s )H + s G  + 18s G  + 72s G
           0       0      0        0        0
      s = -------------------------------------,
       3                    162

       s  =
        4
              2          2                          4        3         2
           27s H  + (18s G  + 378s G + 1296s )H + s G  + 36s G  + 396s G
              0          0          0         0      0        0         0
```

```
         +
           1296s G
                0
      /
         1944
     ]
   ]
                       Type: List List Equation Fraction Polynomial Integer
```

```
n==7
                        Type: Void
```

```
leq
 [
                           2
        s G      3s H + s G  + 6s G
         0          0     0       0
   [s = ---, s = -----------------,
     1   3    2           18
                            3         2
        (9s G + 54s )H + s G  + 18s G  + 72s G
           0       0      0        0        0
    s = -------------------------------------,
     3                    162

     s  =
      4
               2          2                          4        3         2
           27s H  + (18s G  + 378s G + 1296s )H + s G  + 36s G  + 396s G
              0          0          0         0      0        0         0
         +
           1296s G
                0
      /
         1944
     ,

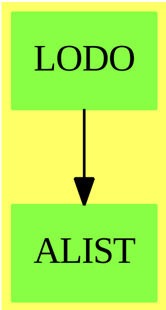
     s  =
      5
                          2         3          2
           (135s G + 2268s )H  + (30s G  + 1350s G  + 16416s G + 38880s )H
                0         0           0          0           0          0
         +
              5        4          3          2
           s G  + 60s G  + 1188s G  + 9504s G  + 25920s G
            0        0          0          0           0
      /
         29160
     ,
```

```
     s  =
      6
              3          2                         2
         405s H  + (405s G  + 18468s G + 174960s )H
             0          0           0           0
       +
              4          3           2                             6
         (45s G  + 3510s G  + 88776s G  + 777600s G + 1166400s )H + s G
             0          0           0            0            0      0
       +
             5          4           3           2
         90s G  + 2628s G  + 27864s G  + 90720s G
            0          0           0           0
      /
        524880
     ,

     s  =
      7
                               3
         (2835s G + 91854s )H
               0          0
       +
               3           2                              2
         (945s G  + 81648s G  + 2082996s G + 14171760s )H
              0           0             0             0
       +
              5          4            3             2
         (63s G  + 7560s G  + 317520s G  + 5554008s G  + 34058880s G)H
             0          0            0             0               0
       +
            7         6           5           4             3              2
         s G  + 126s G  + 4788s G  + 25272s G  - 1744416s G  - 26827200s G
          0         0           0           0             0              0
       +
         - 97977600s G
                    0
      /
        11022480
     ]
   ]
                      Type: List List Equation Fraction Polynomial Integer
```

See Also:
o)show LinearOrdinaryDifferentialOperator

13.6.1 LinearOrdinaryDifferentialOperator (LODO)



See

⇒ "LinearOrdinaryDifferentialOperator1" (LODO1) 13.7.1 on page 1443
⇒ "LinearOrdinaryDifferentialOperator2" (LODO2) 13.8.1 on page 1455

Exports:

0	1	adjoint
apply	characteristic	coefficient
coefficients	coerce	content
D	degree	directSum
exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient
leftExtendedGcd	leftGcd	leftLcm
leftQuotient	leftRemainder	minimumDegree
monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip
reductum	retract	retractIfCan
rightDivide	rightExactQuotient	rightExtendedGcd
rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan
symmetricPower	symmetricProduct	symmetricSquare
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?.?
?~=?		

— domain LODO LinearOrdinaryDifferentialOperator —

```
)abbrev domain LODO LinearOrdinaryDifferentialOperator
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 15 April 1994
++ Keywords: differential operator
++ Description:
++ \spad{LinearOrdinaryDifferentialOperator} defines a ring of
```

```
++ differential operators with coefficients in a ring A with a given
++ derivation.
++ Multiplication of operators corresponds to functional composition:\br
++ \spad{(L1 * L2).(f) = L1 L2 f}

LinearOrdinaryDifferentialOperator(A:Ring, diff: A -> A):
    LinearOrdinaryDifferentialOperatorCategory A
      == SparseUnivariateSkewPolynomial(A, 1, diff) add
        Rep := SparseUnivariateSkewPolynomial(A, 1, diff)

        outputD := "D"@String :: Symbol :: OutputForm

        coerce(l:%):OutputForm == outputForm(l, outputD)
        elt(p:%, a:A):A        == apply(p, 0, a)

        if A has Field then
            import LinearOrdinaryDifferentialOperatorsOps(A, %)

            symmetricProduct(a, b) == symmetricProduct(a, b, diff)
            symmetricPower(a, n)   == symmetricPower(a, n, diff)
            directSum(a, b)        == directSum(a, b, diff)
```

———-

— **LODO.dotabb** —

```
"LODO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LODO"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LODO" -> "ALIST"
```

———-

13.7 domain LODO1 LinearOrdinaryDifferentialOperator1

— **LinearOrdinaryDifferentialOperator1.input** —

```
)set break resume
)sys rm -f LinearOrdinaryDifferentialOperator1.output
)spool LinearOrdinaryDifferentialOperator1.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 20
RFZ := Fraction UnivariatePolynomial('x, Integer)
--R
--R
--R   (1)  Fraction UnivariatePolynomial(x,Integer)
--R                                                                 Type: Domain
--E 1

--S 2 of 20
x : RFZ := 'x
--R
--R
--R   (2)  x
--R                                  Type: Fraction UnivariatePolynomial(x,Integer)
--E 2

--S 3 of 20
Dx : LODO1 RFZ := D()
--R
--R
--R   (3)  D
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 3

--S 4 of 20
b : LODO1 RFZ := 3*x**2*Dx**2 + 2*Dx + 1/x
--R
--R
--R          2 2        1
--R   (4)  3x D  + 2D + -
--R                     x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 4

--S 5 of 20
a : LODO1 RFZ := b*(5*x*Dx + 7)
--R
--R
--R           3 3       2        2         7
--R   (5)  15x D  + (51x  + 10x)D  + 29D + -
--R                                        x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 5

--S 6 of 20
p := x**2 + 1/x**2
--R
--R
--R         4
--R        x  + 1
```

```
--R   (6)  ------
--R          2
--R         x
--R                                  Type: Fraction UnivariatePolynomial(x,Integer)
--E 6

--S 7 of 20
(a*b - b*a) p
--R
--R
--R            4
--R       - 75x  + 540x - 75
--R   (7)  ------------------
--R                4
--R               x
--R                                  Type: Fraction UnivariatePolynomial(x,Integer)
--E 7

--S 8 of 20
ld := leftDivide(a,b)
--R
--R
--R   (8)  [quotient= 5x D + 7,remainder= 0]
--RType: Record(quotient: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x
--E 8

--S 9 of 20
a = b * ld.quotient + ld.remainder
--R
--R
--R           3 3       2        2         7     3 3       2        2         7
--R   (9)  15x D  + (51x  + 10x)D  + 29D + -= 15x D  + (51x  + 10x)D  + 29D + -
--R                                        x                                  x
--RType: Equation LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer
--E 9

--S 10 of 20
rd := rightDivide(a,b)
--R
--R
--R                                              5
--R   (10)  [quotient= 5x D + 7,remainder= 10D + -]
--R                                              x
--RType: Record(quotient: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x
--E 10

--S 11 of 20
a = rd.quotient * b + rd.remainder
--R
--R
```

```
--R            3 3       2        2         7     3 3       2        2         7
--R   (11)  15x D  + (51x  + 10x)D  + 29D + -= 15x D  + (51x  + 10x)D  + 29D + -
--R                                         x                                  x
--RType: Equation LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 11

--S 12 of 20
rightQuotient(a,b)
--R
--R
--R   (12)  5x D + 7
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 12

--S 13 of 20
rightRemainder(a,b)
--R
--R
--R               5
--R   (13)  10D + -
--R               x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 13

--S 14 of 20
leftExactQuotient(a,b)
--R
--R
--R   (14)  5x D + 7
--RType: Union(LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer),...)
--E 14

--S 15 of 20
e := leftGcd(a,b)
--R
--R
--R           2 2        1
--R   (15)  3x D  + 2D + -
--R                      x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 15

--S 16 of 20
leftRemainder(a, e)
--R
--R
--R   (16)  0
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 16
```

```
--S 17 of 20
rightRemainder(a, e)
--R
--R
--R                5
--R   (17)  10D + -
--R                x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 17

--S 18 of 20
f := rightLcm(a,b)
--R
--R
--R             3 3       2        2         7
--R   (18)  15x D  + (51x  + 10x)D  + 29D + -
--R                                          x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 18

--S 19 of 20
rightRemainder(f, b)
--R
--R
--R                5
--R   (19)  10D + -
--R                x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 19

--S 20 of 20
leftRemainder(f, b)
--R
--R
--R   (20)  0
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 20
)spool
)lisp (bye)
```

— LinearOrdinaryDifferentialOperator1.help —

```
====================================================================
LinearOrdinaryDifferentialOperator1 example
====================================================================

LinearOrdinaryDifferentialOperator1(A) is the domain of linear
```

```
ordinary differential operators with coefficients in the differential
ring A.

====================================================================
Differential Operators with Rational Function Coefficients
====================================================================

This example shows differential operators with rational function
coefficients.  In this case operator multiplication is non-commutative and,
since the coefficients form a field, an operator division algorithm exists.

We begin by defining RFZ to be the rational functions in x with
integer coefficients and Dx to be the differential operator for d/dx.

  RFZ := Fraction UnivariatePolynomial('x, Integer)
    Fraction UnivariatePolynomial(x,Integer)
                         Type: Domain

  x : RFZ := 'x
    x
                         Type: Fraction UnivariatePolynomial(x,Integer)

  Dx : LODO1 RFZ := D()
    D
                         Type: LinearOrdinaryDifferentialOperator1
                                 Fraction UnivariatePolynomial(x,Integer)

Operators are created using the usual arithmetic operations.

  b : LODO1 RFZ := 3*x**2*Dx**2 + 2*Dx + 1/x
      2 2        1
    3x D  + 2D + -
                 x
                         Type: LinearOrdinaryDifferentialOperator1
                                Fraction UnivariatePolynomial(x,Integer)

  a : LODO1 RFZ := b*(5*x*Dx + 7)
       3 3       2        2        7
    15x D  + (51x  + 10x)D  + 29D + -
                                   x
                        Type: LinearOrdinaryDifferentialOperator1
                                Fraction UnivariatePolynomial(x,Integer)

Operator multiplication corresponds to functional composition.

  p := x**2 + 1/x**2
     4
    x  + 1
    ------
       2
```

```
     x
                         Type: Fraction UnivariatePolynomial(x,Integer)
```

Since operator coefficients depend on x, the multiplication is not commutative.

```
  (a*b - b*a) p
         4
    - 75x  + 540x - 75
    ------------------
             4
            x
                         Type: Fraction UnivariatePolynomial(x,Integer)
```

When the coefficients of operator polynomials come from a field, as in this case, it is possible to define operator division. Division on the left and division on the right yield different results when the multiplication is non-commutative.

The results of leftDivide and rightDivide are quotient-remainder pairs satisfying:

```
  leftDivide(a,b) = [q, r] such that  a = b*q + r
  rightDivide(a,b) = [q, r] such that a = q*b + r
```

In both cases, the degree of the remainder, r, is less than the degree of b.

```
  ld := leftDivide(a,b)
    [quotient= 5x D + 7,remainder= 0]
      Type: Record(quotient: LinearOrdinaryDifferentialOperator1
                               Fraction UnivariatePolynomial(x,Integer),
                   remainder: LinearOrdinaryDifferentialOperator1
                               Fraction UnivariatePolynomial(x,Integer))

  a = b * ld.quotient + ld.remainder
       3 3       2        2         7     3 3       2        2         7
    15x D  + (51x  + 10x)D  + 29D + -= 15x D  + (51x  + 10x)D  + 29D + -
      Type: Equation LinearOrdinaryDifferentialOperator1
                       Fraction UnivariatePolynomial(x,Integer)
```

The operations of left and right division are so-called because the quotient is obtained by dividing a on that side by b.

```
  rd := rightDivide(a,b)
    [quotient= 5x D + 7,remainder= 10D + -]
      Type: Record(quotient: LinearOrdinaryDifferentialOperator1
                               Fraction UnivariatePolynomial(x,Integer),
                   remainder: LinearOrdinaryDifferentialOperator1
                               Fraction UnivariatePolynomial(x,Integer))
```

```
  a = rd.quotient * b + rd.remainder
       3 3       2        2         7     3 3       2        2         7
    15x D  + (51x  + 10x)D  + 29D + -= 15x D  + (51x  + 10x)D  + 29D + -
     Type: Equation LinearOrdinaryDifferentialOperator1
                     Fraction UnivariatePolynomial(x,Integer)
```

Operations rightQuotient and rightRemainder are available if only one of the quotient or remainder are of interest to you. This is the quotient from right division.

```
  rightQuotient(a,b)
    5x D + 7
     Type: LinearOrdinaryDifferentialOperator1
             Fraction UnivariatePolynomial(x,Integer)
```

This is the remainder from right division. The corresponding "left" functions, leftQuotient and leftRemainder are also available.

```
  rightRemainder(a,b)
          5
    10D + -
          x
      Type: LinearOrdinaryDifferentialOperator1
              Fraction UnivariatePolynomial(x,Integer)
```

For exact division, operations leftExactQuotient and rightExactQuotient are supplied. These return the quotient but only if the remainder is zero. The call rightExactQuotient(a,b) would yield an error.

```
  leftExactQuotient(a,b)
    5x D + 7
     Type: Union(LinearOrdinaryDifferentialOperator1
                  Fraction UnivariatePolynomial(x,Integer),...)
```

The division operations allow the computation of left and right greatest common divisors, leftGcd and rightGcd via remainder sequences, and consequently the computation of left and right least common multiples, rightLcm and leftLcm.

```
  e := leftGcd(a,b)
      2 2        1
    3x D  + 2D + -
                 x
     Type: LinearOrdinaryDifferentialOperator1
             Fraction UnivariatePolynomial(x,Integer)
```

Note that a greatest common divisor doesn't necessarily divide a and b on both sides. Here the left greatest common divisor does not divide a on the right.

```
  leftRemainder(a, e)
    0
      Type: LinearOrdinaryDifferentialOperator1
              Fraction UnivariatePolynomial(x,Integer)

  rightRemainder(a, e)
          5
    10D + -
          x
      Type: LinearOrdinaryDifferentialOperator1
              Fraction UnivariatePolynomial(x,Integer)
```

Similarly, a least common multiple is not necessarily divisible from both sides.

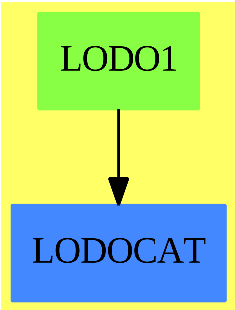
```
  f := rightLcm(a,b)
       3 3       2        2         7
    15x D  + (51x  + 10x)D  + 29D + -
      Type: LinearOrdinaryDifferentialOperator1
              Fraction UnivariatePolynomial(x,Integer)

  rightRemainder(f, b)
          5
    10D + -
          x
      Type: LinearOrdinaryDifferentialOperator1
              Fraction UnivariatePolynomial(x,Integer)

  leftRemainder(f, b)
    0
      Type: LinearOrdinaryDifferentialOperator1
              Fraction UnivariatePolynomial(x,Integer)
```

See Also:
o)show LinearOrdinaryDifferentialOperator1

13.7.1 LinearOrdinaryDifferentialOperator1 (LODO1)



See

⇒ "LinearOrdinaryDifferentialOperator" (LODO) 13.6.1 on page 1433
⇒ "LinearOrdinaryDifferentialOperator2" (LODO2) 13.8.1 on page 1455

Exports:

0	1	adjoint	apply
characteristic	coefficient	coefficients	coerce
content	D	degree	directSum
exquo	hash	latex	leadingCoefficient
leftDivide	leftExactQuotient	leftExtendedGcd	leftGcd
leftLcm	leftQuotient	leftRemainder	minimumDegree
monicLeftDivide	monicRightDivide	monomial	one?
primitivePart	recip	reductum	retract
retractIfCan	rightDivide	rightExactQuotient	rightExtendedGcd
rightGcd	rightLcm	rightQuotient	rightRemainder
sample	subtractIfCan	symmetricPower	symmetricProduct
symmetricSquare	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?.?	?~=?	

— **domain LODO1 LinearOrdinaryDifferentialOperator1** —

```
)abbrev domain LODO1 LinearOrdinaryDifferentialOperator1
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 31 January 1994
++ Keywords: differential operator
++ Description:
++ \spad{LinearOrdinaryDifferentialOperator1} defines a ring of
++ differential operators with coefficients in a differential ring A.
++ Multiplication of operators corresponds to functional composition:\br
++ \spad{(L1 * L2).(f) = L1 L2 f}

LinearOrdinaryDifferentialOperator1(A:DifferentialRing) ==
  LinearOrdinaryDifferentialOperator(A, differentiate$A)
```

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— LODO1.dotabb —

```
"LODO1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LODO1"]
"LODOCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=LODOCAT"]
"LODO1" -> "LODOCAT"
```

———-

13.8 domain LODO2 LinearOrdinaryDifferentialOperator2

— LinearOrdinaryDifferentialOperator2.input —

```
)set break resume
)sys rm -f LinearOrdinaryDifferentialOperator2.output
)spool LinearOrdinaryDifferentialOperator2.output
)set message test on
)set message auto off
)clear all
--S 1 of 26
Q  := Fraction Integer
--R
--R
--R   (1)  Fraction Integer
--R                                                                    Type: Domain
--E 1

--S 2 of 26
PQ := UnivariatePolynomial('x, Q)
--R
--R
--R   (2)  UnivariatePolynomial(x,Fraction Integer)
--R                                                                    Type: Domain
--E 2

--S 3 of 26
x: PQ := 'x
--R
--R
--R   (3)  x
```

```
--R                                   Type: UnivariatePolynomial(x,Fraction Integer)
--E 3

--S 4 of 26
Dx: LODO2(Q, PQ) := D()
--R
--R
--R   (4)  D
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 4

--S 5 of 26
a := Dx  + 1
--R
--R
--R   (5)  D + 1
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 5

--S 6 of 26
b := a + 1/2*Dx**2 - 1/2
--R
--R
--R        1  2       1
--R   (6)  - D  + D + -
--R        2          2
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 6

--S 7 of 26
p := 4*x**2 + 2/3
--R
--R
--R          2   2
--R   (7)  4x  + -
--R              3
--R                                   Type: UnivariatePolynomial(x,Fraction Integer)
--E 7

--S 8 of 26
a p
--R
--R
--R          2       2
--R   (8)  4x  + 8x + -
--R                   3
--R                                   Type: UnivariatePolynomial(x,Fraction Integer)
--E 8

--S 9 of 26
```

```
(a * b) p = a b p
--R
--R
--R          2          37    2          37
--R   (9)  2x  + 12x + --= 2x  + 12x + --
--R                     3                3
--R                      Type: Equation UnivariatePolynomial(x,Fraction Integer)
--E 9

--S 10 of 26
c := (1/9)*b*(a + b)^2
--R
--R
--R          1  6    5  5   13  4   19  3   79  2    7     1
--R   (10)  -- D  + -- D  + -- D  + -- D  + -- D  + -- D + -
--R         72      36      24      18      72      12     8
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction
--E 10

--S 11 of 26
(a**2 - 3/4*b + c) (p + 1)
--R
--R
--R           2   44     541
--R   (11)  3x  + -- x + ---
--R                3      36
--R                              Type: UnivariatePolynomial(x,Fraction Integer)
--E 11
)clear all
--S 12 of 26
PZ := UnivariatePolynomial(x,Integer)
--R
--R
--R   (1)  UnivariatePolynomial(x,Integer)
--R                                                            Type: Domain
--E 12

--S 13 of 26
x:PZ := 'x
--R
--R
--R   (2)  x
--R                                       Type: UnivariatePolynomial(x,Integer)
--E 13

--S 14 of 26
Mat  := SquareMatrix(3,PZ)
--R
--R
--R   (3)  SquareMatrix(3,UnivariatePolynomial(x,Integer))
```

```
--R                                                           Type: Domain
--E 14

--S 15 of 26
Vect := DPMM(3, PZ, Mat, PZ)
--R
--R
--R   (4)
--R  DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,Un
--R  ivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer))
--R                                                           Type: Domain
--E 15

--S 16 of 26
Modo := LODO2(Mat, Vect)
--R
--R
--R   (5)
--R  LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Int
--R  eger)),DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatr
--R  ix(3,UnivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer)))
--R                                                           Type: Domain
--E 16

--S 17 of 26
m:Mat := matrix [ [x^2,1,0],[1,x^4,0],[0,0,4*x^2] ]
--R
--R
--R        + 2         +
--R        |x   1    0 |
--R        |           |
--R   (6)  |     4     |
--R        |1   x    0 |
--R        |           |
--R        |          2|
--R        +0   0   4x +
--R                        Type: SquareMatrix(3,UnivariatePolynomial(x,Integer))
--E 17

--S 18 of 26
p:Vect := directProduct [3*x^2+1,2*x,7*x^3+2*x]
--R
--R
--R           2          3
--R   (7)  [3x  + 1,2x,7x  + 2x]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 18

--S 19 of 26
q: Vect := m * p
```

```
--R
--R
--R            4    2        5     2        5     3
--R   (8)  [3x  + x  + 2x,2x  + 3x  + 1,28x  + 8x ]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,Univariat
--E 19

--S 20 of 26
Dx : Modo := D()
--R
--R
--R   (9)  D
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer))
--E 20

--S 21 of 26
a : Modo := Dx  + m
--R
--R
--R             + 2         +
--R             |x   1    0 |
--R             |           |
--R   (10)  D + |     4     |
--R             |1   x    0 |
--R             |           |
--R             |          2|
--R             +0   0   4x +
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer))
--E 21

--S 22 of 26
b : Modo := m*Dx  + 1
--R
--R
--R         + 2         +
--R         |x   1    0 |    +1  0  0+
--R         |           |    |       |
--R   (11)  |     4     |D + |0  1  0|
--R         |1   x    0 |    |       |
--R         |           |    +0  0  1+
--R         |          2|
--R         +0   0   4x +
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer))
--E 22

--S 23 of 26
c := a*b
--R
--R
--R   (12)
```

```
--R   + 2         +     + 4             4    2                 +    + 2         +
--R   |x   1    0 |     |x  + 2x + 2    x  + x           0     |    |x   1    0 |
--R   |           | 2   |                                      |    |           |
--R   |     4     |D  + |   4    2     8     3                 |D + |     4     |
--R   |1   x    0 |     |  x  + x     x  + 4x  + 2       0     |    |1   x    0 |
--R   |           |     |                                      |    |           |
--R   |          2|     |                             4        |    |          2|
--R   +0   0   4x +     +     0            0       16x  + 8x + 1+    +0   0   4x +
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer)),DirectProdu
--E 23

--S 24 of 26
a p
--R
--R
--R           4    2        5     2        5     3      2
--R   (13)  [3x  + x  + 8x,2x  + 3x  + 3,28x  + 8x  + 21x  + 2]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 24

--S 25 of 26
b p
--R
--R
--R           3     2       4         4     3     2
--R   (14)  [6x  + 3x  + 3,2x  + 8x,84x  + 7x  + 8x  + 2x]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 25

--S 26 of 26
(a + b + c) (p + q)
--R
--R
--R   (15)
--R       8      7      6      5      4      3      2
--R   [10x  + 12x  + 16x  + 30x  + 85x  + 94x  + 40x  + 40x + 17,
--R       12      9      8      7     6      5      4      3      2
--R    10x   + 10x  + 12x  + 92x  + 6x  + 32x  + 72x  + 28x  + 49x  + 32x + 19,
--R         8       7        6        5       4       3      2
--R    2240x  + 224x  + 1280x  + 3508x  + 492x  + 751x  + 98x  + 18x + 4]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 26
)spool
)lisp (bye)
```

— **LinearOrdinaryDifferentialOperator2.help** —

```
====================================================================
LinearOrdinaryDifferentialOperator2
====================================================================

LinearOrdinaryDifferentialOperator2(A, M) is the domain of linear
ordinary differential operators with coefficients in the differential
ring A and operating on M, an A-module.  This includes the cases of
operators which are polynomials in D acting upon scalar or vector
expressions of a single variable.  The coefficients of the operator
polynomials can be integers, rational functions, matrices or elements
of other domains.

====================================================================
Differential Operators with Constant Coefficients
====================================================================

This example shows differential operators with rational number
coefficients operating on univariate polynomials.

We begin by making type assignments so we can conveniently refer
to univariate polynomials in x over the rationals.

  Q  := Fraction Integer
    Fraction Integer
                         Type: Domain

  PQ := UnivariatePolynomial('x, Q)
    UnivariatePolynomial(x,Fraction Integer)
                         Type: Domain

  x: PQ := 'x
    x
                         Type: UnivariatePolynomial(x,Fraction Integer)

Now we assign Dx to be the differential operator D corresponding to d/dx.

  Dx: LODO2(Q, PQ) := D()
    D
     Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
             UnivariatePolynomial(x,Fraction Integer))

New operators are created as polynomials in D().

  a := Dx  + 1
    D + 1
     Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
             UnivariatePolynomial(x,Fraction Integer))

  b := a + 1/2*Dx**2 - 1/2
    1  2       1
```

```
    - D  + D + -
    2         2
      Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
              UnivariatePolynomial(x,Fraction Integer))
```

To apply the operator a to the value p the usual function call syntax
is used.

```
  p := 4*x**2 + 2/3
      2   2
    4x  + -
          3
                     Type: UnivariatePolynomial(x,Fraction Integer)

  a p
      2         2
    4x  + 8x + -
               3
                     Type: UnivariatePolynomial(x,Fraction Integer)
```

Operator multiplication is defined by the identity (a*b)p = a(b(p))

```
  (a * b) p = a b p
      2         37    2         37
    2x  + 12x + --= 2x  + 12x + --
                 3               3
                     Type: Equation UnivariatePolynomial(x,Fraction Integer)
```

Exponentiation follows from multiplication.

```
  c := (1/9)*b*(a + b)^2
      1  6    5  5   13  4   19  3   79  2    7     1
     -- D  + -- D  + -- D  + -- D  + -- D  + -- D + -
     72      36      24      18      72      12     8
      Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
              UnivariatePolynomial(x,Fraction Integer))
```

Finally, note that operator expressions may be applied directly.

```
  (a**2 - 3/4*b + c) (p + 1)
       2   44     541
     3x  + -- x + ---
            3      36
                     Type: UnivariatePolynomial(x,Fraction Integer)
```

==
Differential Operators with Matrix Coefficients Operating on Vectors}
==

This is another example of linear ordinary differential operators with non-commutative multiplication. Unlike the rational function case, the differential ring of square matrices (of a given dimension) with univariate polynomial entries does not form a field. Thus the number of operations available is more limited.

In this section, the operators have three by three matrix coefficients with polynomial entries.

```
PZ := UnivariatePolynomial(x,Integer)
  UnivariatePolynomial(x,Integer)
                       Type: Domain

x:PZ := 'x
  x
                       Type: UnivariatePolynomial(x,Integer)

Mat  := SquareMatrix(3,PZ)
  SquareMatrix(3,UnivariatePolynomial(x,Integer))
                       Type: Domain
```

The operators act on the vectors considered as a Mat-module.

```
Vect := DPMM(3, PZ, Mat, PZ)
DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,Un
ivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer))
                       Type: Domain

Modo := LODO2(Mat, Vect)
LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Int
eger)),DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatr
ix(3,UnivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer)))
                       Type: Domain
```

The matrix m is used as a coefficient and the vectors p and q are operated upon.

```
m:Mat := matrix [ [x^2,1,0],[1,x^4,0],[0,0,4*x^2] ]
      + 2         +
      |x   1    0 |
      |           |
      |     4     |
      |1   x    0 |
      |           |
      |          2|
      +0   0   4x +
                Type: SquareMatrix(3,UnivariatePolynomial(x,Integer))

p:Vect := directProduct [3*x^2+1,2*x,7*x^3+2*x]
     2          3
  [3x  + 1,2x,7x  + 2x]
```

```
    Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
                UnivariatePolynomial(x,Integer))

  q: Vect := m * p
      4    2        5     2        5     3
    [3x  + x  + 2x,2x  + 3x  + 1,28x  + 8x ]
      Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
         SquareMatrix(3,UnivariatePolynomial(x,Integer)),
            UnivariatePolynomial(x,Integer))
```

Now form a few operators.

```
  Dx : Modo := D()
    D
      Type: LinearOrdinaryDifferentialOperator2(
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             UnivariatePolynomial(x,Integer)))

  a : Modo := Dx  + m
             + 2         +
             |x   1    0 |
             |           |
          D + |     4     |
             |1   x    0 |
             |           |
             |          2|
             +0   0   4x +
    Type: LinearOrdinaryDifferentialOperator2(
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             UnivariatePolynomial(x,Integer)))

  b : Modo := m*Dx  + 1
         + 2         +
         |x   1    0 |    +1  0  0+
         |           |    |       |
         |     4     |D + |0  1  0|
         |1   x    0 |    |       |
         |           |    +0  0  1+
         |          2|
         +0   0   4x +
     Type: LinearOrdinaryDifferentialOperator2(
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             UnivariatePolynomial(x,Integer)))
```

```
c := a*b
+ 2        +     + 4              4    2                +    + 2        +
|x   1    0 |     |x   + 2x + 2    x   + x           0      |    |x   1    0 |
|           | 2   |                                         |    |            |
|     4     |D  + |   4    2     8     3                    |D + |     4      |
|1    x    0 |     |  x   + x     x   + 4x  + 2       0      |    |1    x    0 |
|           |     |                                         |    |            |
|          2|     |                              4          |    |          2|
+0    0   4x +     +     0           0        16x  + 8x + 1+    +0    0   4x +
  Type: LinearOrdinaryDifferentialOperator2(
          SquareMatrix(3,UnivariatePolynomial(x,Integer)),
          DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
          SquareMatrix(3,UnivariatePolynomial(x,Integer)),
          UnivariatePolynomial(x,Integer)))
```

These operators can be applied to vector values.

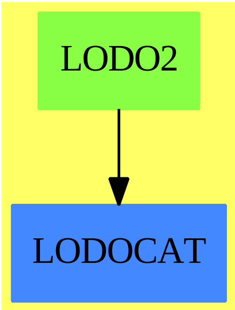
```
a p
      4    2        5     2        5     3      2
   [3x  + x  + 8x,2x  + 3x  + 3,28x  + 8x  + 21x  + 2]
     Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
             SquareMatrix(3,UnivariatePolynomial(x,Integer)),
                UnivariatePolynomial(x,Integer))
```

```
b p
      3     2       4         4     3     2
   [6x  + 3x  + 3,2x  + 8x,84x  + 7x  + 8x  + 2x]
      Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
              SquareMatrix(3,UnivariatePolynomial(x,Integer)),
                UnivariatePolynomial(x,Integer))
```

```
(a + b + c) (p + q)
     8      7      6      5      4      3      2
 [10x  + 12x  + 16x  + 30x  + 85x  + 94x  + 40x  + 40x + 17,
     12      9      8      7      6      5      4      3      2
  10x   + 10x  + 12x  + 92x  + 6x  + 32x  + 72x  + 28x  + 49x  + 32x + 19,
       8       7        6        5       4       3      2
  2240x  + 224x  + 1280x  + 3508x  + 492x  + 751x  + 98x  + 18x + 4]
   Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
           SquareMatrix(3,UnivariatePolynomial(x,Integer)),
             UnivariatePolynomial(x,Integer))
```

See Also:
o)show LinearOrdinaryDifferentialOperator2

13.8.1 LinearOrdinaryDifferentialOperator2 (LODO2)



See

⇒ "LinearOrdinaryDifferentialOperator" (LODO) 13.6.1 on page 1433
⇒ "LinearOrdinaryDifferentialOperator1" (LODO1) 13.7.1 on page 1443

Exports:

0	1	adjoint	apply
characteristic	coefficient	coefficients	coerce
content	D	degree	directSum
exquo	hash	latex	leadingCoefficient
leftDivide	leftExactQuotient	leftExtendedGcd	leftGcd
leftLcm	leftQuotient	leftRemainder	minimumDegree
monicLeftDivide	monicRightDivide	monomial	one?
primitivePart	recip	reductum	retract
retractIfCan	rightDivide	rightExactQuotient	rightExtendedGcd
rightGcd	rightLcm	rightQuotient	rightRemainder
sample	subtractIfCan	symmetricPower	symmetricProduct
symmetricSquare	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?.?	?~=?	

— **domain LODO2 LinearOrdinaryDifferentialOperator2** —

```
)abbrev domain LODO2 LinearOrdinaryDifferentialOperator2
++ Author: Stephen M. Watt, Manuel Bronstein
++ Date Created: 1986
++ Date Last Updated: 1 February 1994
++ Keywords: differential operator
++ Description:
++ \spad{LinearOrdinaryDifferentialOperator2} defines a ring of
++ differential operators with coefficients in a differential ring A
++ and acting on an A-module M.
++ Multiplication of operators corresponds to functional composition:\br
++ \spad{(L1 * L2).(f) = L1 L2 f}

LinearOrdinaryDifferentialOperator2(A, M): Exports == Implementation where
```

```
  A: DifferentialRing
  M: LeftModule A with
      differentiate: $ -> $
        ++ differentiate(x) returns the derivative of x

  Exports ==> Join(LinearOrdinaryDifferentialOperatorCategory A, Eltable(M, M))

  Implementation ==> LinearOrdinaryDifferentialOperator(A, differentiate$A) add
      elt(p:%, m:M):M ==
        apply(p, differentiate, m)$ApplyUnivariateSkewPolynomial(A, M, %)
```

— LODO2.dotabb —

```
"LODO2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LODO2"]
"LODOCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=LODOCAT"]
"LODO2" -> "LODOCAT"
```

13.9 domain LIST List

— List.input —

```
)set break resume
)sys rm -f List.output
)spool List.output
)set message test on
)set message auto off
)clear all
--S 1 of 34
[2, 4, 5, 6]
--R
--R
--R   (1)  [2,4,5,6]
--R                                                  Type: List PositiveInteger
--E 1

--S 2 of 34
[1]
--R
--R
--R   (2)  [1]
```

```
--R                                                   Type: List PositiveInteger
--E 2

--S 3 of 34
list(1)
--R
--R
--R   (3)  [1]
--R                                                   Type: List PositiveInteger
--E 3

--S 4 of 34
append([1,2,3],[5,6,7])
--R
--R
--R   (4)  [1,2,3,5,6,7]
--R                                                   Type: List PositiveInteger
--E 4

--S 5 of 34
cons(10,[9,8,7])
--R
--R
--R   (5)  [10,9,8,7]
--R                                                   Type: List PositiveInteger
--E 5

--S 6 of 34
empty? [x+1]
--R
--R
--R   (6)  false
--R                                                             Type: Boolean
--E 6

--S 7 of 34
([] = nil)@Boolean
--R
--R
--R   (7)  true
--R                                                             Type: Boolean
--E 7

--S 8 of 34
k := [4,3,7,3,8,5,9,2]
--R
--R
--R   (8)  [4,3,7,3,8,5,9,2]
--R                                                   Type: List PositiveInteger
--E 8
```

```
--S 9 of 34
first k
--R
--R
--R   (9)  4
--R                                                      Type: PositiveInteger
--E 9

--S 10 of 34
k.first
--R
--R
--R   (10)  4
--R                                                      Type: PositiveInteger
--E 10

--S 11 of 34
k.1
--R
--R
--R   (11)  4
--R                                                      Type: PositiveInteger
--E 11

--S 12 of 34
k(1)
--R
--R
--R   (12)  4
--R                                                      Type: PositiveInteger
--E 12

--S 13 of 34
n := #k
--R
--R
--R   (13)  8
--R                                                      Type: PositiveInteger
--E 13

--S 14 of 34
last k
--R
--R
--R   (14)  2
--R                                                      Type: PositiveInteger
--E 14

--S 15 of 34
```

```
k.last
--R
--R
--R   (15)  2
--R                                                   Type: PositiveInteger
--E 15

--S 16 of 34
k.(#k)
--R
--R
--R   (16)  2
--R                                                   Type: PositiveInteger
--E 16

--S 17 of 34
k := [4,3,7,3,8,5,9,2]
--R
--R
--R   (17)  [4,3,7,3,8,5,9,2]
--R                                              Type: List PositiveInteger
--E 17

--S 18 of 34
k.1 := 999
--R
--R
--R   (18)  999
--R                                                   Type: PositiveInteger
--E 18

--S 19 of 34
k
--R
--R
--R   (19)  [999,3,7,3,8,5,9,2]
--R                                              Type: List PositiveInteger
--E 19

--S 20 of 34
k := [1,2]
--R
--R
--R   (20)  [1,2]
--R                                              Type: List PositiveInteger
--E 20

--S 21 of 34
m := cons(0,k)
--R
```

```
--R
--R   (21)  [0,1,2]
--R                                                      Type: List Integer
--E 21

--S 22 of 34
m.2 := 99
--R
--R
--R   (22)  99
--R                                                    Type: PositiveInteger
--E 22

--S 23 of 34
m
--R
--R
--R   (23)  [0,99,2]
--R                                                      Type: List Integer
--E 23

--S 24 of 34
k
--R
--R
--R   (24)  [99,2]
--R                                               Type: List PositiveInteger
--E 24

--S 25 of 34
k := [1,2,3]
--R
--R
--R   (25)  [1,2,3]
--R                                               Type: List PositiveInteger
--E 25

--S 26 of 34
rest k
--R
--R
--R   (26)  [2,3]
--R                                               Type: List PositiveInteger
--E 26

--S 27 of 34
removeDuplicates [4,3,4,3,5,3,4]
--R
--R
--R   (27)  [4,3,5]
```

```
--R                                                  Type: List PositiveInteger
--E 27

--S 28 of 34
reverse [1,2,3,4,5,6]
--R
--R
--R   (28)  [6,5,4,3,2,1]
--R                                                  Type: List PositiveInteger
--E 28

--S 29 of 34
member?(1/2,[3/4,5/6,1/2])
--R
--R
--R   (29)  true
--R                                                            Type: Boolean
--E 29

--S 30 of 34
member?(1/12,[3/4,5/6,1/2])
--R
--R
--R   (30)  false
--R                                                            Type: Boolean
--E 30

--S 31 of 34
reverse(rest(reverse(k)))
--R
--R
--R   (31)  [1,2]
--R                                                  Type: List PositiveInteger
--E 31

--S 32 of 34
[1..3,10,20..23]
--R
--R
--R   (32)  [1..3,10..10,20..23]
--R                                          Type: List Segment PositiveInteger
--E 32

--S 33 of 34
expand [1..3,10,20..23]
--R
--R
--R   (33)  [1,2,3,10,20,21,22,23]
--R                                                       Type: List Integer
--E 33
```

```
--S 34 of 34
expand [1..]
--R
--R
--R   (34)  [1,2,3,4,5,6,7,8,9,10,...]
--R                                                      Type: Stream Integer
--E 34
)spool
)lisp (bye)
```

— List.help —

```
====================================================================
List examples
====================================================================

A list is a finite collection of elements in a specified order that
can contain duplicates.  A list is a convenient structure to work with
because it is easy to add or remove elements and the length need not
be constant.  There are many different kinds of lists in Axiom, but
the default types (and those used most often) are created by the List
constructor.  For example, there are objects of type List Integer,
List Float and List Polynomial Fraction Integer.  Indeed, you can even
have List List List Boolean (that is, lists of lists of lists of
Boolean values).  You can have lists of any type of Axiom object.

====================================================================
Creating Lists
====================================================================

The easiest way to create a list with, for example, the elements
2, 4, 5, 6 is to enclose the elements with square brackets and
separate the elements with commas.

The spaces after the commas are optional, but they do improve the
readability.

  [2, 4, 5, 6]
    [2,4,5,6]
                             Type: List PositiveInteger

To create a list with the single element 1, you can use either [1] or
the operation list.

  [1]
    [1]
```

```
                          Type: List PositiveInteger

  list(1)
    [1]
                          Type: List PositiveInteger
```

Once created, two lists k and m can be concatenated by issuing append(k,m). append does not physically join the lists, but rather produces a new list with the elements coming from the two arguments.

```
  append([1,2,3],[5,6,7])
    [1,2,3,5,6,7]
                           Type: List PositiveInteger
```

Use cons to append an element onto the front of a list.

```
  cons(10,[9,8,7])
    [10,9,8,7]
                           Type: List PositiveInteger
```

==
Accessing List Elements
==

To determine whether a list has any elements, use the operation empty?.

```
  empty? [x+1]
    false
                           Type: Boolean
```

Alternatively, equality with the list constant nil can be tested.

```
  ([] = nil)@Boolean
    true
                           Type: Boolean
```

We'll use this in some of the following examples.

```
  k := [4,3,7,3,8,5,9,2]
    [4,3,7,3,8,5,9,2]
                           Type: List PositiveInteger
```

Each of the next four expressions extracts the first element of k.

```
  first k
    4
                           Type: PositiveInteger

  k.first
    4
```

```
                           Type: PositiveInteger

  k.1
    4
                           Type: PositiveInteger

  k(1)
    4
                           Type: PositiveInteger
```

The last two forms generalize to k.i and k(i), respectively, where
1 <= i <= n and n equals the length of k.

This length is calculated by #.

```
  n := #k
    8
                           Type: PositiveInteger
```

Performing an operation such as k.i is sometimes referred to as
indexing into k or elting into k. The latter phrase comes about
because the name of the operation that extracts elements is called
elt. That is, k.3 is just alternative syntax for elt(k,3). It is
important to remember that list indices begin with 1. If we issue
k := [1,3,2,9,5] then k.4 returns 9. It is an error to use an index
that is not in the range from 1 to the length of the list.

The last element of a list is extracted by any of the following three
expressions.

```
  last k
    2
                              Type: PositiveInteger

  k.last
    2
                              Type: PositiveInteger
```

This form computes the index of the last element and then extracts the
element from the list.

```
  k.(#k)
    2
                              Type: PositiveInteger
```

==
Changing List Elements
==

We'll use this in some of the following examples.

```
  k := [4,3,7,3,8,5,9,2]
    [4,3,7,3,8,5,9,2]
                              Type: List PositiveInteger
```

List elements are reset by using the k.i form on the left-hand side of an assignment. This expression resets the first element of k to 999.

```
  k.1 := 999
    999
                              Type: PositiveInteger
```

As with indexing into a list, it is an error to use an index that is not within the proper bounds. Here you see that k was modified.

```
  k
    [999,3,7,3,8,5,9,2]
                              Type: List PositiveInteger
```

The operation that performs the assignment of an element to a particular position in a list is called setelt. This operation is destructive in that it changes the list. In the above example, the assignment returned the value 999 and k was modified. For this reason, lists are called mutable objects: it is possible to change part of a list (mutate it) rather than always returning a new list reflecting the intended modifications.

Moreover, since lists can share structure, changes to one list can sometimes affect others.

```
  k := [1,2]
    [1,2]
                               Type: List PositiveInteger

  m := cons(0,k)
    [0,1,2]
                               Type: List Integer
```

Change the second element of m.

```
  m.2 := 99
    99
                               Type: PositiveInteger
```

See, m was altered.

```
  m
    [0,99,2]
                               Type: List Integer
```

```
But what about k?  It changed too!

  k
    [99,2]
                               Type: List PositiveInteger

====================================================================
Other Functions
====================================================================

An operation that is used frequently in list processing is that
which returns all elements in a list after the first element.

  k := [1,2,3]
    [1,2,3]
                              Type: List PositiveInteger

Use the rest operation to do this.

  rest k
    [2,3]
                              Type: List PositiveInteger

To remove duplicate elements in a list k, use removeDuplicates.

  removeDuplicates [4,3,4,3,5,3,4]
    [4,3,5]
                              Type: List PositiveInteger

To get a list with elements in the order opposite to those in a list
k, use reverse.

  reverse [1,2,3,4,5,6]
    [6,5,4,3,2,1]
                              Type: List PositiveInteger

To test whether an element is in a list, use member?: member?(a,k)
returns true or false depending on whether a is in k or not.

  member?(1/2,[3/4,5/6,1/2])
    true
                              Type: Boolean

  member?(1/12,[3/4,5/6,1/2])
    false
                              Type: Boolean

We can get a list containing all but the last of the elements in a
given non-empty list k.
```

```
  reverse(rest(reverse(k)))
    [1,2]
                                    Type: List PositiveInteger

====================================================================
Dot, Dot
====================================================================

Certain lists are used so often that Axiom provides an easy way of
constructing them.  If n and m are integers, then expand [n..m]
creates a list containing n, n+1, ... m.  If n > m then the list is
empty.  It is actually permissible to leave off the m in the dot-dot
construction (see below).

The dot-dot notation can be used more than once in a list construction
and with specific elements being given.  Items separated by dots are
called segments.

  [1..3,10,20..23]
    [1..3,10..10,20..23]
                                   Type: List Segment PositiveInteger

Segments can be expanded into the range of items between the endpoints
by using expand.

  expand [1..3,10,20..23]
    [1,2,3,10,20,21,22,23]
                                   Type: List Integer

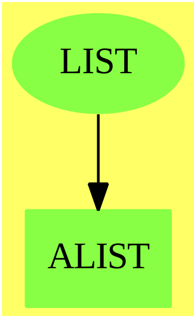
What happens if we leave off a number on the right-hand side of ..?

  expand [1..]
    [1,2,3,4,5,6,7,8,9,10,...]
                                   Type: Stream Integer

What is created in this case is a Stream which is a generalization of
a list.

See Also:
o )help Stream
o )show List
```

13.9.1 List (LIST)



See

⇒ "IndexedList" (ILIST) 10.11.1 on page 1196
⇒ "AssociationList" (ALIST) 2.42.1 on page 218

Exports:

any?	append	child?	children	coerce
concat	concat!	cons	construct	convert
copy	copyInto!	count	cycleEntry	cycleLength
cycleSplit!	cycleTail	cyclic?	delete	delete!
distance	elt	empty	empty?	entries
entry?	eq?	eval	every?	explicitlyFinite?
fill!	find	first	hash	index?
indices	insert	insert!	last	latex
leaf?	leaves	less?	list	map
map!	max	maxIndex	member?	members
merge	merge!	min	minIndex	more?
node?	new	nil	nodes	null
OMwrite	parts	position	possiblyInfinite?	qelt
qsetelt!	reduce	remove	remove!	removeDuplicates
removeDuplicates!	rest	reverse	reverse!	sample
second	select	select!	setDifference	setIntersection
setUnion	setchildren!	setelt	setfirst!	setlast!
setrest!	setvalue!	size?	sort	sort!
sorted?	split!	swap!	tail	third
value	#?	?<?	?<=?	?=?
?>?	?>=?	?.?	?~=?	?.?
?.last	?.rest	?.first	?.value	

— domain LIST List —

```
)abbrev domain LIST List
++ Author: Michael Monagan
++ Date Created: Sep 1987
++ Change History:
```

```
++ Related Constructors: ListFunctions2, ListFunctions3, ListToMap
++ Also See: IndexList, ListAggregate
++ AMS Classification:
++ Keywords: list, index, aggregate, lisp
++ Description:
++ \spadtype{List} implements singly-linked lists that are
++ addressable by indices; the index of the first element
++ is 1. In addition to the operations provided by
++ \spadtype{IndexedList}, this constructor provides some
++ LISP-like functions such as \spadfun{null} and \spadfun{cons}.

List(S:Type): Exports == Implementation where
 LISTMININDEX ==> 1       -- this is the minimum list index

 Exports ==> ListAggregate S with
  nil             : ()     -> %
    ++ nil() returns the empty list.
  null            : %      -> Boolean
    ++ null(u) tests if list \spad{u} is the
    ++ empty list.
  cons            : (S, %) -> %
    ++ cons(element,u) appends \spad{element} onto the front
    ++ of list \spad{u} and returns the new list. This new list
    ++ and the old one will share some structure.
  append          : (%, %) -> %
    ++ append(u1,u2) appends the elements of list \spad{u1}
    ++ onto the front of list \spad{u2}. This new list
    ++ and \spad{u2} will share some structure.
  if S has SetCategory then
    setUnion        : (%, %) -> %
      ++ setUnion(u1,u2) appends the two lists u1 and u2, then
      ++ removes all duplicates. The order of elements in the
      ++ resulting list is unspecified.
    setIntersection : (%, %) -> %
      ++ setIntersection(u1,u2) returns a list of the elements
      ++ that lists \spad{u1} and \spad{u2} have in common.
      ++ The order of elements in the resulting list is unspecified.
    setDifference   : (%, %) -> %
      ++ setDifference(u1,u2) returns a list of the elements
      ++ of \spad{u1} that are not also in \spad{u2}.
      ++ The order of elements in the resulting list is unspecified.
  if S has OpenMath then OpenMath

 Implementation ==>
   IndexedList(S, LISTMININDEX) add
      nil()                   == NIL$Lisp
      null l                  == NULL(l)$Lisp
      cons(s, l)              == CONS(s, l)$Lisp
      append(l:%, t:%)        == APPEND(l, t)$Lisp
```

```
if S has OpenMath then
  writeOMList(dev: OpenMathDevice, x: %): Void ==
    OMputApp(dev)
    OMputSymbol(dev, "list1", "list")
    -- The following didn't compile because the compiler isn't
    -- convinced that 'xval' is a S.  Duhhh! MCD.
    --for xval in x repeat
    --  OMwrite(dev, xval, false)
    while not null x repeat
      OMwrite(dev,first x,false)
      x := rest x
    OMputEndApp(dev)

  OMwrite(x: %): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    OMputObject(dev)
    writeOMList(dev, x)
    OMputEndObject(dev)
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

  OMwrite(x: %, wholeObj: Boolean): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    if wholeObj then
      OMputObject(dev)
    writeOMList(dev, x)
    if wholeObj then
      OMputEndObject(dev)
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

  OMwrite(dev: OpenMathDevice, x: %): Void ==
    OMputObject(dev)
    writeOMList(dev, x)
    OMputEndObject(dev)

  OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
    if wholeObj then
      OMputObject(dev)
    writeOMList(dev, x)
    if wholeObj then
      OMputEndObject(dev)

if S has SetCategory then
```

```
    setUnion(l1:%,l2:%)      == removeDuplicates concat(l1,l2)

    setIntersection(l1:%,l2:%) ==
      u :% := empty()
      l1 := removeDuplicates l1
      while not empty? l1 repeat
        if member?(first l1,l2) then u := cons(first l1,u)
        l1 := rest l1
      u

    setDifference(l1:%,l2:%) ==
      l1 := removeDuplicates l1
      lu:% := empty()
      while not empty? l1 repeat
        l11:=l1.1
        if not member?(l11,l2) then lu := concat(l11,lu)
        l1 := rest l1
      lu

  if S has ConvertibleTo InputForm then
    convert(x:%):InputForm ==
      convert concat(convert("construct"::Symbol)@InputForm,
            [convert a for a in (x pretend List S)]$List(InputForm))
```

— LIST.dotabb —

```
"LIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LIST",
          shape=ellipse]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LIST" -> "ALIST"
```

13.10 domain LMOPS ListMonoidOps

— ListMonoidOps.input —

```
)set break resume
)sys rm -f ListMonoidOps.output
)spool ListMonoidOps.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show ListMonoidOps
--R ListMonoidOps(S: SetCategory,E: AbelianMonoid,un: E)  is a domain constructor
--R Abbreviation for ListMonoidOps is LMOPS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LMOPS
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : S -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   leftMult : (S,%) -> %
--R makeTerm : (S,E) -> %                 makeUnit : () -> %
--R mapExpon : ((E -> E),%) -> %          mapGen : ((S -> S),%) -> %
--R nthExpon : (%,Integer) -> E           nthFactor : (%,Integer) -> S
--R plus : (%,%) -> %                     plus : (S,E,%) -> %
--R retract : % -> S                      reverse : % -> %
--R reverse! : % -> %                     rightMult : (%,S) -> %
--R size : % -> NonNegativeInteger        ?~=? : (%,%) -> Boolean
--R commutativeEquality : (%,%) -> Boolean
--R listOfMonoms : % -> List Record(gen: S,exp: E)
--R makeMulti : List Record(gen: S,exp: E) -> %
--R outputForm : (%,((OutputForm,OutputForm) -> OutputForm),((OutputForm,OutputForm) -> Outpu
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

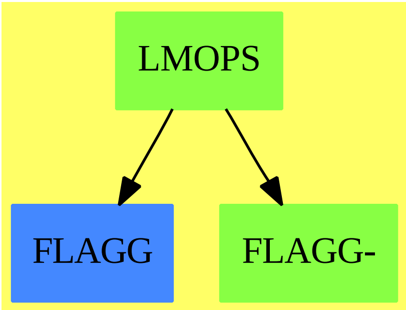
)spool
)lisp (bye)
```

— **ListMonoidOps.help** —

```
====================================================================
ListMonoidOps examples
====================================================================

See Also:
o )show ListMonoidOps
```

13.10.1 ListMonoidOps (LMOPS)



See

⇒ "FreeMonoid" (FMONOID) 7.32.1 on page 987
⇒ "FreeGroup" (FGROUP) 7.29.1 on page 976
⇒ "InnerFreeAbelianMonoid" (IFAMON) 10.22.1 on page 1250
⇒ "FreeAbelianMonoid" (FAMONOID) 7.28.1 on page 974
⇒ "FreeAbelianGroup" (FAGROUP) 7.27.1 on page 971

Exports:

coerce	commutativeEquality	hash	latex	leftMult
listOfMonoms	makeTerm	makeUnit	mapExpon	mapGen
makeMulti	nthExpon	nthFactor	outputForm	plus
retract	retractIfCan	reverse	reverse!	rightMult
size	?=?	?~=?		

— domain LMOPS ListMonoidOps —

```
)abbrev domain LMOPS ListMonoidOps
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ This internal package represents monoid (abelian or not, with or
++ without inverses) as lists and provides some common operations
++ to the various flavors of monoids.

ListMonoidOps(S, E, un): Exports == Implementation where
  S : SetCategory
  E : AbelianMonoid
  un: E

  REC ==> Record(gen:S, exp: E)
  O   ==> OutputForm

  Exports ==> Join(SetCategory, RetractableTo S) with
    outputForm    : ($, (O, O) -> O, (O, O) -> O, Integer) -> O
```

```
    ++ outputForm(l, fop, fexp, unit) converts the monoid element
    ++ represented by l to an \spadtype{OutputForm}.
    ++ Argument unit is the output form
    ++ for the \spadignore{unit} of the monoid (e.g. 0 or 1),
    ++ \spad{fop(a, b)} is the
    ++ output form for the monoid operation applied to \spad{a} and b
    ++ (e.g. \spad{a + b}, \spad{a * b}, \spad{ab}),
    ++ and \spad{fexp(a, n)} is the output form
    ++ for the exponentiation operation applied to \spad{a} and n
    ++ (e.g. \spad{n a}, \spad{n * a}, \spad{a ** n}, \spad{a\^n}).
  listOfMonoms  : $ -> List REC
    ++ listOfMonoms(l) returns the list of the monomials forming l.
  makeTerm      : (S, E) -> $
    ++ makeTerm(s, e) returns the monomial s exponentiated by e
    ++ (e.g. s^e or e * s).
  makeMulti     : List REC -> $
    ++ makeMulti(l) returns the element whose list of monomials is l.
  nthExpon      : ($, Integer) -> E
    ++ nthExpon(l, n) returns the exponent of the n^th monomial of l.
  nthFactor     : ($, Integer) -> S
    ++ nthFactor(l, n) returns the factor of the n^th monomial of l.
  reverse       : $ -> $
    ++ reverse(l) reverses the list of monomials forming l. This
    ++ has some effect if the monoid is non-abelian, i.e.
    ++ \spad{reverse(a1\^e1 ... an\^en) = an\^en ... a1\^e1} which is different.
  reverse_!     : $ -> $
    ++ reverse!(l) reverses the list of monomials forming l, destroying
    ++ the element l.
  size          : $ -> NonNegativeInteger
    ++ size(l) returns the number of monomials forming l.
  makeUnit      : () -> $
    ++ makeUnit() returns the unit element of the monomial.
  rightMult     : ($, S) -> $
    ++ rightMult(a, s) returns \spad{a * s} where \spad{*}
    ++ is the monoid operation,
    ++ which is assumed non-commutative.
  leftMult      : (S, $) -> $
    ++ leftMult(s, a) returns \spad{s * a} where
    ++ \spad{*} is the monoid operation,
    ++ which is assumed non-commutative.
  plus          : (S, E, $) -> $
    ++ plus(s, e, x) returns \spad{e * s + x} where \spad{+}
    ++ is the monoid operation,
    ++ which is assumed commutative.
  plus          : ($, $) -> $
    ++ plus(x, y) returns \spad{x + y} where \spad{+}
    ++ is the monoid operation,
    ++ which is assumed commutative.
  commutativeEquality: ($, $) -> Boolean
    ++ commutativeEquality(x,y) returns true if x and y are equal
```

```
    ++ assuming commutativity
  mapExpon      : (E -> E, $) -> $
    ++ mapExpon(f, a1\^e1 ... an\^en) returns \spad{a1\^f(e1) ... an\^f(en)}.
  mapGen        : (S -> S, $) -> $
    ++ mapGen(f, a1\^e1 ... an\^en) returns \spad{f(a1)\^e1 ... f(an)\^en}.

 Implementation ==> add
  Rep := List REC

  localplus: ($, $) -> $

  makeUnit()       == empty()$Rep
  size l           == # listOfMonoms l
  coerce(s:S):$    == [[s, un]]
  coerce(l:$):O    == coerce(l)$Rep
  makeTerm(s, e)   == (zero? e => makeUnit(); [[s, e]])
  makeMulti l      == l
  f = g            == f =$Rep g
  listOfMonoms l   == l pretend List(REC)
  nthExpon(f, i)   == f.(i-1+minIndex f).exp
  nthFactor(f, i)  == f.(i-1+minIndex f).gen
  reverse l        == reverse(l)$Rep
  reverse_! l      == reverse_!(l)$Rep
  mapGen(f, l)     == [[f(x.gen), x.exp] for x in l]

  mapExpon(f, l) ==
    ans:List(REC) := empty()
    for x in l repeat
      if (a := f(x.exp)) ^= 0 then ans := concat([x.gen, a], ans)
    reverse_! ans

  outputForm(l, op, opexp, id) ==
    empty? l => id::OutputForm
    l:List(O) :=
       [(p.exp = un => p.gen::O; opexp(p.gen::O, p.exp::O)) for p in l]
    reduce(op, l)

  retractIfCan(l:$):Union(S, "failed") ==
    not empty? l and empty? rest l and l.first.exp = un => l.first.gen
    "failed"

  rightMult(f, s) ==
    empty? f => s::$
    s = f.last.gen => (setlast_!(h := copy f, [s, f.last.exp + un]); h)
    concat(f, [s, un])

  leftMult(s, f) ==
    empty? f => s::$
    s = f.first.gen => concat([s, f.first.exp + un], rest f)
    concat([s, un], f)
```

```
    commutativeEquality(s1:$, s2:$):Boolean ==
      #s1 ^= #s2 => false
      for t1 in s1 repeat
          if not member?(t1,s2) then return false
      true

    plus_!(s:S, n:E, f:$):$ ==
      h := g := concat([s, n], f)
      h1 := rest h
      while not empty? h1 repeat
        s = h1.first.gen =>
          l :=
            zero?(m := n + h1.first.exp) => rest h1
            concat([s, m], rest h1)
          setrest_!(h, l)
          return rest g
        h := h1
        h1 := rest h1
      g

    plus(s, n, f) == plus_!(s,n,copy f)

    plus(f, g) ==
      #f < #g => localplus(f, g)
      localplus(g, f)

    localplus(f, g) ==
      g := copy g
      for x in f repeat
        g := plus(x.gen, x.exp, g)
      g
```

— **LMOPS.dotabb** —

```
"LMOPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LMOPS"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"LMOPS" -> "FLAGG"
"LMOPS" -> "FLAGG-"
```

13.11 domain LMDICT ListMultiDictionary

— ListMultiDictionary.input —

```
)set break resume
)sys rm -f ListMultiDictionary.output
)spool ListMultiDictionary.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ListMultiDictionary
--R ListMultiDictionary S: SetCategory  is a domain constructor
--R Abbreviation for ListMultiDictionary is LMDICT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LMDICT
--R
--R------------------------------- Operations --------------------------------
--R bag : List S -> %                     construct : List S -> %
--R copy : % -> %                         dictionary : List S -> %
--R dictionary : () -> %                  duplicates? : % -> Boolean
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R insert! : (S,%) -> %                  inspect : % -> S
--R map : ((S -> S),%) -> %               removeDuplicates! : % -> %
--R sample : () -> %                      substitute : (S,S,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List Record(entry: S,count: NonNegativeInteger)
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R insert! : (S,%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
```

```
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

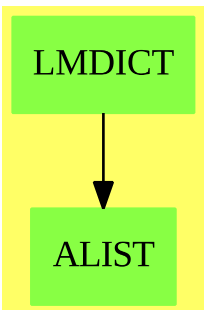
)spool
)lisp (bye)
```

— **ListMultiDictionary.help** —

```
====================================================================
ListMultiDictionary examples
====================================================================

See Also:
o )show ListMultiDictionary
```

13.11.1 ListMultiDictionary (LMDICT)



Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	dictionary	duplicates
duplicates?	empty	empty?	eq?	eval
extract!	every?	find	hash	insert!
inspect	latex	less?	map	map!
member?	members	more?	parts	reduce
remove	remove!	removeDuplicates	removeDuplicates!	sample
select	select!	size?	substitute	#?
?~=?	?=?			

— domain LMDICT ListMultiDictionary —

```
)abbrev domain LMDICT ListMultiDictionary
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated: 13 June 1994 Frederic Lehobey
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The \spadtype{ListMultiDictionary} domain implements a
++ dictionary with duplicates
++ allowed.  The representation is a list with duplicates represented
++ explicitly.  Hence most operations will be relatively inefficient
++ when the number of entries in the dictionary becomes large.
++ If the objects in the dictionary belong to an ordered set,
++ the entries are maintained in ascending order.

ListMultiDictionary(S:SetCategory): EE == II where

 NNI ==> NonNegativeInteger
 D ==> Record(entry:S, count:NonNegativeInteger)

 EE ==> MultiDictionary(S) with
   finiteAggregate
   duplicates?: % -> Boolean
     ++ duplicates?(d) tests if dictionary d has duplicate entries.
   substitute : (S, S, %) -> %
     ++ substitute(x,y,d) replace x's with y's in dictionary d.
 II ==> add
   Rep := Reference List S

   sub: (S, S, S) -> S

   coerce(s:%):OutputForm ==
```

```
    prefix("dictionary"::OutputForm, [x::OutputForm for x in parts s])

  #s                 == # parts s
  copy s             == dictionary copy parts s
  empty? s           == empty? parts s
  bag l              == dictionary l
  dictionary()       == dictionary empty()

  empty():% == ref empty()

  dictionary(ls:List S):% ==
    empty? ls => empty()
    lmd := empty()
    for x in ls repeat insert_!(x,lmd)
    lmd

  if S has ConvertibleTo InputForm then
    convert(lmd:%):InputForm ==
      convert [convert("dictionary"::Symbol)@InputForm,
       convert(parts lmd)@InputForm]

  map(f, s)          == dictionary map(f, parts s)
  map_!(f, s)        == dictionary map_!(f, parts s)
  parts s            == deref s
  sub(x, y, z)       == (z = x => y; z)
  insert_!(x, s, n)  == (for i in 1..n repeat insert_!(x, s); s)
  substitute(x, y, s) == dictionary map(z1 +-> sub(x, y, z1), parts s)
  removeDuplicates_! s == dictionary removeDuplicates_! parts s

  inspect s ==
    empty? s => error "empty dictionary"
    first parts s

  extract_! s ==
    empty? s => error "empty dictionary"
    x := first(p := parts s)
    setref(s, rest p)
    x

  duplicates? s ==
    empty?(p := parts s) => false
    q := rest p
    while not empty? q repeat
      first p = first q => return true
      p := q
      q := rest q
    false

  remove_!(p: S->Boolean, lmd:%):% ==
    for x in removeDuplicates parts lmd | p(x) repeat remove_!(x,lmd)
```

```
    lmd

  select_!(p: S->Boolean, lmd:%):% == remove_!((z:S):Boolean+->not p(z), lmd)

  duplicates(lmd:%):List D ==
    ld: List D := empty()
    for x in removeDuplicates parts lmd | (n := count(x, lmd)) >
     1$NonNegativeInteger repeat
      ld := cons([x, n], ld)
    ld

  if S has OrderedSet then
     s = t == parts s = parts t

     remove_!(x:S, s:%) ==
        p := deref s
        while not empty? p and x = first p repeat p := rest p
        setref(s, p)
        empty? p => s
        q := rest p
        while not empty? q and x > first q repeat (p := q; q := rest q)
        while not empty? q and x = first q repeat q := rest q
        p.rest := q
        s

     insert_!(x, s) ==
        p := deref s
        empty? p or x < first p =>
           setref(s, concat(x, p))
           s
        q := rest p
        while not empty? q and x > first q repeat (p := q; q := rest q)
        p.rest := concat(x, q)
        s

  else
     remove_!(x:S, s:%) == (setref(s, remove_!(x, parts s)); s)

     s = t ==
        a := copy s
        while not empty? a repeat
           x := inspect a
           count(x, s) ^= count(x, t) => return false
           remove_!(x, a)
        true

     insert_!(x, s) ==
        p := deref s
        while not empty? p repeat
           x = first p =>
```

```
            p.rest := concat(x, rest p)
            s
         p := rest p
      setref(s, concat(x, deref s))
      s
```

———-

— LMDICT.dotabb —

```
"LMDICT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LMDICT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LMDICT" -> "ALIST"
```

———-

13.12 domain LA LocalAlgebra

— LocalAlgebra.input —

```
)set break resume
)sys rm -f LocalAlgebra.output
)spool LocalAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LocalAlgebra
--R LocalAlgebra(A: Algebra R,R: CommutativeRing,S: SubsetCategory(Monoid,R))  is a domain co
--R Abbreviation for LocalAlgebra is LA
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LA
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (A,S) -> %
--R ?/? : (%,S) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
```

```
--R denom : % -> S                         hash : % -> SingleInteger
--R latex : % -> String                    numer : % -> A
--R one? : % -> Boolean                    recip : % -> Union(%,"failed")
--R sample : () -> %                       zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<? : (%,%) -> Boolean if A has ORDRING
--R ?<=? : (%,%) -> Boolean if A has ORDRING
--R ?>? : (%,%) -> Boolean if A has ORDRING
--R ?>=? : (%,%) -> Boolean if A has ORDRING
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if A has ORDRING
--R characteristic : () -> NonNegativeInteger
--R max : (%,%) -> % if A has ORDRING
--R min : (%,%) -> % if A has ORDRING
--R negative? : % -> Boolean if A has ORDRING
--R positive? : % -> Boolean if A has ORDRING
--R sign : % -> Integer if A has ORDRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

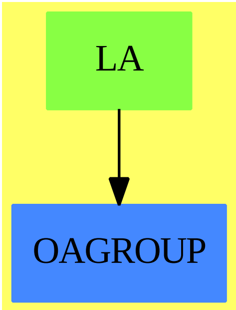
— LocalAlgebra.help —

```
====================================================================
LocalAlgebra examples
====================================================================

See Also:
o )show LocalAlgebra
```

———-

13.12.1 LocalAlgebra (LA)



See

⇒ “Localize” (LO) 13.13.1 on page 1486
⇒ “Fraction” (FRAC) 7.24.1 on page 952

Exports:

0	1	abs	characteristic	coerce
denom	hash	latex	max	min
negative?	numer	one?	positive?	recip
sample	sign	subtractIfCan	zero?	?~=?
?*?	?**?	?<?	?<=?	?>?
?>=?	?^?	?+?	?-?	-?
?/?	?=?			

— domain LA LocalAlgebra —

```
)abbrev domain LA LocalAlgebra
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ LocalAlgebra produces the localization of an algebra, i.e.
++ fractions whose numerators come from some R algebra.

LocalAlgebra(A: Algebra R,
             R: CommutativeRing,
             S: SubsetCategory(Monoid, R)): Algebra R with
          if A has OrderedRing then OrderedRing
          _/ : (%,S) -> %
            ++ x / d divides the element x by d.
          _/ : (A,S) -> %
```

```
            ++ a / d divides the element \spad{a} by d.
          numer: % -> A
            ++ numer x returns the numerator of x.
          denom: % -> S
            ++ denom x returns the denominator of x.
 == Localize(A, R, S) add
        1 == 1$A / 1$S
        x:% * y:% == (numer(x) * numer(y)) / (denom(x) * denom(y))
        characteristic() == characteristic()$A
```

— LA.dotabb —

```
"LA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LA"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"LA" -> "OAGROUP"
```

13.13 domain LO Localize

— Localize.input —

```
)set break resume
)sys rm -f Localize.output
)spool Localize.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Localize
--R Localize(M: Module R,R: CommutativeRing,S: SubsetCategory(Monoid,R))  is a domain constructor
--R Abbreviation for Localize is LO
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LO
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (M,S) -> %
--R ?/? : (%,S) -> %                      ?=? : (%,%) -> Boolean
```

```
--R 0 : () -> %                          coerce : % -> OutputForm
--R denom : % -> S                       hash : % -> SingleInteger
--R latex : % -> String                  numer : % -> M
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?<? : (%,%) -> Boolean if M has OAGROUP
--R ?<=? : (%,%) -> Boolean if M has OAGROUP
--R ?>? : (%,%) -> Boolean if M has OAGROUP
--R ?>=? : (%,%) -> Boolean if M has OAGROUP
--R max : (%,%) -> % if M has OAGROUP
--R min : (%,%) -> % if M has OAGROUP
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

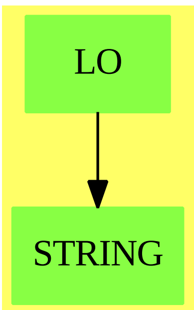
)spool
)lisp (bye)
```

— **Localize.help** —

```
====================================================================
Localize examples
====================================================================

See Also:
o )show Localize
```

13.13.1 Localize (LO)



See

⇒ "LocalAlgebra" (LA) 13.12.1 on page 1484
⇒ "Fraction" (FRAC) 7.24.1 on page 952

Exports:

0	coerce	denom	hash	latex
max	min	numer	sample	subtractIfCan
zero?	?~=?	?*?	?<?	?<=?
?>?	?>=?	?+?	?-?	-?
?/?	?=?			

— domain LO Localize —

```
)abbrev domain LO Localize
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: + - / numer denom
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: localization
++ References:
++ Description:
++ Localize(M,R,S) produces fractions with numerators
++ from an R module M and denominators from some multiplicative subset D of R.

Localize(M:Module R,
         R:CommutativeRing,
         S:SubsetCategory(Monoid, R)): Module R with
      if M has OrderedAbelianGroup then OrderedAbelianGroup
      _/ :(%,S) -> %
         ++ x / d divides the element x by d.
      _/ :(M,S) -> %
         ++ m / d divides the element m by d.
      numer: % -> M
         ++ numer x returns the numerator of x.
      denom: % -> S
         ++ denom x returns the denominator of x.
 ==
  add
    --representation
      Rep:= Record(num:M,den:S)
    --declarations
      x,y: %
      n: Integer
      m: M
      r: R
      d: S
    --definitions
      0 == [0,1]
```

```
      zero? x == zero? (x.num)
      -x== [-x.num,x.den]
      x=y == y.den*x.num = x.den*y.num
      numer x == x.num
      denom x == x.den
      if M has OrderedAbelianGroup then
        x < y ==
--             if y.den::R < 0 then (x,y):=(y,x)
--             if x.den::R < 0 then (x,y):=(y,x)
             y.den*x.num < x.den*y.num
      x+y == [y.den*x.num+x.den*y.num, x.den*y.den]
      n*x == [n*x.num,x.den]
      r*x == if r=x.den then [x.num,1] else [r*x.num,x.den]
      x/d ==
        zero?(u:S:=d*x.den) => error "division by zero"
        [x.num,u]
      m/d == if zero? d then error "division by zero" else [m,d]
      coerce(x:%):OutputForm ==
--        one?(xd:=x.den) => (x.num)::OutputForm
        ((xd:=x.den) = 1) => (x.num)::OutputForm
        (x.num)::OutputForm / (xd::OutputForm)
      latex(x:%): String ==
--        one?(xd:=x.den) => latex(x.num)
        ((xd:=x.den) = 1) => latex(x.num)
        nl : String := concat("{", concat(latex(x.num), "}")$String)$String
        dl : String := concat("{", concat(latex(x.den), "}")$String)$String
        concat("{ ", concat(nl, concat(" \over ", concat(dl, " }")$String)$String)$String)$St
```

———-

— LO.dotabb —

```
"LO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LO"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"LO" -> "STRING"
```

———-

13.14 domain LWORD LyndonWord

— LyndonWord.input —

```
)set break resume
)sys rm -f LyndonWord.output
```

```
)spool LyndonWord.output
)set message test on
)set message auto off
)clear all
--S 1 of 22
a:Symbol :='a
--R
--R
--R   (1)  a
--R                                                              Type: Symbol
--E 1

--S 2 of 22
b:Symbol :='b
--R
--R
--R   (2)  b
--R                                                              Type: Symbol
--E 2

--S 3 of 22
c:Symbol :='c
--R
--R
--R   (3)  c
--R                                                              Type: Symbol
--E 3

--S 4 of 22
lword:= LyndonWord(Symbol)
--R
--R
--R   (4)  LyndonWord Symbol
--R                                                              Type: Domain
--E 4

--S 5 of 22
magma := Magma(Symbol)
--R
--R
--R   (5)  Magma Symbol
--R                                                              Type: Domain
--E 5

--S 6 of 22
word := OrderedFreeMonoid(Symbol)
--R
--R
--R   (6)  OrderedFreeMonoid Symbol
--R                                                              Type: Domain
```

```
--E 6

--S 7 of 22
LyndonWordsList1([a,b,c],3)$lword
--R
--R
--R   (7)
--R   [[[a],[b],[c]], [[a b],[a c],[b c]],
--R       2     2       2                      2    2       2
--R    [[a b],[a c],[a b ],[a b c],[a c b],[a c ],[b c],[b c ]]]
--R                              Type: OneDimensionalArray List LyndonWord Symbol
--E 7

--S 8 of 22
LyndonWordsList([a,b,c],3)$lword
--R
--R
--R   (8)
--R                                            2      2        2
--R   [[a], [b], [c], [a b], [a c], [b c], [a b], [a c], [a b ], [a b c], [a c b],
--R        2     2        2
--R    [a c ], [b c], [b c ]]
--R                                                  Type: List LyndonWord Symbol
--E 8

--S 9 of 22
lw := LyndonWordsList([a,b],5)$lword
--R
--R
--R   (9)
--R                        2        2     3      2 2       3     4      3 2
--R   [[a], [b], [a b], [a b], [a b ], [a b], [a b ], [a b ], [a b], [a b ],
--R      2          2 3          2       4
--R    [a b a b], [a b ], [a b a b ], [a b ]]
--R                                                  Type: List LyndonWord Symbol
--E 9

--S 10 of 22
w1 : word := lw.4 :: word
--R
--R
--R         2
--R   (10)  a b
--R                                                Type: OrderedFreeMonoid Symbol
--E 10

--S 11 of 22
w2 : word := lw.5 :: word
--R
--R
```

```
--R            2
--R   (11)  a b
--R                                              Type: OrderedFreeMonoid Symbol
--E 11

--S 12 of 22
factor(a::word)$lword
--R
--R
--R   (12)  [[a]]
--R                                                Type: List LyndonWord Symbol
--E 12

--S 13 of 22
factor(w1*w2)$lword
--R
--R
--R            2     2
--R   (13)  [[a b a b ]]
--R                                                Type: List LyndonWord Symbol
--E 13

--S 14 of 22
factor(w2*w2)$lword
--R
--R
--R              2      2
--R   (14)  [[a b ],[a b ]]
--R                                                Type: List LyndonWord Symbol
--E 14

--S 15 of 22
factor(w2*w1)$lword
--R
--R
--R              2    2
--R   (15)  [[a b ],[a b]]
--R                                                Type: List LyndonWord Symbol
--E 15

--S 16 of 22
lyndon?(w1)$lword
--R
--R
--R   (16)  true
--R                                                             Type: Boolean
--E 16

--S 17 of 22
lyndon?(w1*w2)$lword
```

```
--R
--R
--R   (17)  true
--R                                                                Type: Boolean
--E 17

--S 18 of 22
lyndon?(w2*w1)$lword
--R
--R
--R   (18)  false
--R                                                                Type: Boolean
--E 18

--S 19 of 22
lyndonIfCan(w1)$lword
--R
--R
--R           2
--R   (19)  [a b]
--R                                           Type: Union(LyndonWord Symbol,...)
--E 19

--S 20 of 22
lyndonIfCan(w2*w1)$lword
--R
--R
--R   (20)  "failed"
--R                                                   Type: Union("failed",...)
--E 20

--S 21 of 22
lyndon(w1)$lword
--R
--R
--R           2
--R   (21)  [a b]
--R                                                  Type: LyndonWord Symbol
--E 21

--S 22 of 22
lyndon(w1*w2)$lword
--R
--R
--R           2     2
--R   (22)  [a b a b ]
--R                                                  Type: LyndonWord Symbol
--E 22
)spool
)lisp (bye)
```

— LyndonWord.help —

```
====================================================================
LyndonWord examples
====================================================================

A function f in [0,1] is called acyclic if C(F) consists of n
different objects. The canonical representative of the orbit of an
acyclic function is usually called a Lyndon Word. If f is acyclic,
then all elements in the orbit C(f) are acyclic as well, and we call
C(f) an acyclic orbit.

====================================================================
Initialisations
====================================================================

  a:Symbol :='a
    a
                          Type: Symbol

  b:Symbol :='b
    b
                          Type: Symbol

  c:Symbol :='c
    c
                          Type: Symbol

  lword:= LyndonWord(Symbol)
    LyndonWord Symbol
                          Type: Domain

  magma := Magma(Symbol)
    Magma Symbol
                          Type: Domain

  word := OrderedFreeMonoid(Symbol)
    OrderedFreeMonoid Symbol
                          Type: Domain

All Lyndon words of with a, b, c to order 3

  LyndonWordsList1([a,b,c],3)$lword
   [[[a],[b],[c]], [[a b],[a c],[b c]],
       2     2       2                       2    2       2
    [[a b],[a c],[a b ],[a b c],[a c b],[a c ],[b c],[b c ]]]
                          Type: OneDimensionalArray List LyndonWord Symbol
```

All Lyndon words of with a, b, c to order 3 in flat list

```
  LyndonWordsList([a,b,c],3)$lword
                                            2      2        2
   [[a], [b], [c], [a b], [a c], [b c], [a b], [a c], [a b ], [a b c], [a c b],
       2     2        2
    [a c ], [b c], [b c ]]
                            Type: List LyndonWord Symbol
```

All Lyndon words of with a, b to order 5

```
  lw := LyndonWordsList([a,b],5)$lword
                        2        2     3      2 2       3     4      3 2
   [[a], [b], [a b], [a b], [a b ], [a b], [a b ], [a b ], [a b], [a b ],
     2          2 3           2       4
    [a b a b], [a b ], [a b a b ], [a b ]]
                            Type: List LyndonWord Symbol
```

```
  w1 : word := lw.4 :: word
     2
    a b
                             Type: OrderedFreeMonoid Symbol
```

```
  w2 : word := lw.5 :: word
       2
    a b
                             Type: OrderedFreeMonoid Symbol
```

Let's try factoring

```
  factor(a::word)$lword
    [[a]]
                             Type: List LyndonWord Symbol
```

```
  factor(w1*w2)$lword
       2     2
    [[a b a b ]]
                             Type: List LyndonWord Symbol
```

```
  factor(w2*w2)$lword
         2      2
    [[a b ],[a b ]]
                             Type: List LyndonWord Symbol
```

```
  factor(w2*w1)$lword
         2    2
    [[a b ],[a b]]
                             Type: List LyndonWord Symbol
```

```
====================================================================
Checks and coercions
====================================================================

  lyndon?(w1)$lword
    true
                             Type: Boolean

  lyndon?(w1*w2)$lword
    true
                             Type: Boolean

  lyndon?(w2*w1)$lword
    false
                             Type: Boolean

  lyndonIfCan(w1)$lword
      2
    [a b]
                             Type: Union(LyndonWord Symbol,...)

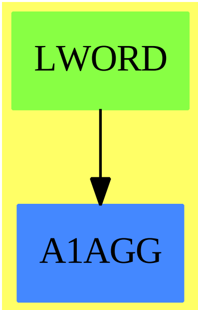
  lyndonIfCan(w2*w1)$lword
    "failed"
                             Type: Union("failed",...)

  lyndon(w1)$lword
      2
    [a b]
                             Type: LyndonWord Symbol

  lyndon(w1*w2)$lword
      2     2
    [a b a b ]
                             Type: LyndonWord Symbol

See Also:
o )show LyndonWord
```

13.14.1 LyndonWord (LWORD)



Exports:

coerce	factor	hash	latex	left
length	lexico	lyndon	lyndon?	lyndonIfCan
LyndonWordsList	LyndonWordsList1	max	min	retract
retractIfCan	retractable?	right	varList	?<?
?<=?	?=?	?>?	?>=?	?~=?

— domain LWORD LyndonWord —

```
)abbrev domain LWORD LyndonWord
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Free Lie Algebras by C. Reutenauer (Oxford science publications).
++ Description:
++ Lyndon words over arbitrary (ordered) symbols:
++ see Free Lie Algebras by C. Reutenauer (Oxford science publications).
++ A Lyndon word is a word which is smaller than any of its right factors
++ w.r.t. the pure lexicographical ordering.
++ If \axiom{a} and \axiom{b} are two Lyndon words such that \axiom{a < b}
++ holds w.r.t lexicographical ordering then \axiom{a*b} is a Lyndon word.
++ Parenthesized Lyndon words can be generated from symbols by using the
++ following rule:\br
++ \axiom{[[a,b],c]} is a Lyndon word iff \axiom{a*b < c <= b} holds.\br
++ Lyndon words are internally represented by binary trees using the
++ \spadtype{Magma} domain constructor.
++ Two ordering are provided: lexicographic and
++ length-lexicographic.
```

```
LyndonWord(VarSet:OrderedSet):Public == Private where
   OFMON ==> OrderedFreeMonoid(VarSet)
   PI    ==> PositiveInteger
   NNI   ==> NonNegativeInteger
   I     ==> Integer
   OF    ==> OutputForm
   ARRAY1==> OneDimensionalArray

   Public == Join(OrderedSet,RetractableTo VarSet) with
      retractable?  : $ -> Boolean
        ++ \axiom{retractable?(x)} tests if \axiom{x} is a tree
        ++ with only one entry.
      left          : $ -> $
        ++ \axiom{left(x)} returns left subtree of \axiom{x} or
        ++ error if retractable?(x) is true.
      right  :  $ -> $
        ++ \axiom{right(x)} returns right subtree of \axiom{x} or
        ++ error if retractable?(x) is true.
      length :  $ -> PI
        ++ \axiom{length(x)} returns the number of entries in \axiom{x}.
      lexico :  ($,$) -> Boolean
        ++ \axiom{lexico(x,y)} returns \axiom{true} iff  \axiom{x} is smaller than
        ++ \axiom{y} w.r.t. the lexicographical ordering induced by \axiom{VarSet}.
      coerce :  $ -> OFMON
        ++ \axiom{coerce(x)} returns the element of \axiomType{OrderedFreeMonoid}(VarSet)
        ++ corresponding to \axiom{x}.
      coerce :  $ -> Magma VarSet
        ++ \axiom{coerce(x)} returns the element of \axiomType{Magma}(VarSet)
        ++ corresponding to \axiom{x}.
      factor :  OFMON -> List $
        ++ \axiom{factor(x)} returns the decreasing factorization into Lyndon words.
      lyndon?:  OFMON -> Boolean
        ++ \axiom{lyndon?(w)} test if \axiom{w} is a Lyndon word.
      lyndon :  OFMON -> $
        ++ \axiom{lyndon(w)} convert \axiom{w} into a Lyndon word,
        ++ error if \axiom{w} is not a Lyndon word.
      lyndonIfCan : OFMON -> Union($, "failed")
        ++ \axiom{lyndonIfCan(w)} convert \axiom{w} into a Lyndon word.
      varList     : $ -> List VarSet
        ++ \axiom{varList(x)} returns the list of distinct entries of \axiom{x}.
      LyndonWordsList1: (List VarSet, PI)  -> ARRAY1 List $
        ++ \axiom{LyndonWordsList1(vl, n)} returns an array of lists of Lyndon
        ++ words over the alphabet \axiom{vl}, up to order \axiom{n}.
      LyndonWordsList : (List VarSet, PI)  -> List $
        ++ \axiom{LyndonWordsList(vl, n)} returns the list of Lyndon
        ++ words over the alphabet \axiom{vl}, up to order \axiom{n}.

   Private == Magma(VarSet) add
     -- Representation
```

```
  Rep:= Magma(VarSet)

-- Fonctions locales
  LetterList : OFMON -> List VarSet
  factor1    : (List $, $, List $) -> List $

-- Definitions
  lyndon? w ==
    w = 1$OFMON => false
    f: OFMON := rest w
    while f ^= 1$OFMON repeat
      not lexico(w,f) => return false
      f := rest f
    true

  lyndonIfCan w ==
    l: List $ := factor w
    # l = 1 => first l
    "failed"

  lyndon w ==
    l: List $ := factor w
    # l = 1 => first l
    error "This word is not a Lyndon word"

  LetterList w ==
    w = 1 => []
    cons(first w , LetterList rest w)

  factor1 (gauche, x, droite) ==
    g: List $ := gauche; d: List $ := droite
    while not null g repeat            ++ (l in g or l=x) et u in d
      lexico(  g.first , x ) =>        ++  => right(l) >= u
         x  := g.first *$Rep x         -- crochetage
         null(d) => g := rest g
         g := cons( x, rest g)         -- mouvement a droite
         x  := first d
         d := rest d
      d := cons( x , d)                -- mouvement a gauche
      x  := first g
      g := rest g
    return cons(x, d)

  factor w ==
    w = 1 => []
    l : List $ := reverse [ u::$ for u in LetterList w]
    factor1( rest l, first l , [] )

  x < y ==                     -- lexicographique par longueur
    lx,ly: PI
```

```
      lx:= length x ; ly:= length y
      lx = ly => lexico(x,y)
      lx < ly

    coerce(x:$):OF == bracket(x::OFMON::OF)
    coerce(x:$):Magma VarSet == x::Rep

    LyndonWordsList1 (vl,n) ==    -- a ameliorer !!!!!!!!!!!
         null vl => error "empty list"
         base: ARRAY1 List $ := new(n::I::NNI ,[])

        -- mots de longueur 1
         lbase1:List $ := [w::$ for w in sort(vl)]
         base.1 := lbase1

        -- calcul des mots de longueur ll
         for ll in 2..n:I  repeat
            lbase1 := []
            for a in base(1) repeat              -- lettre + mot
               for b in base(ll-1) repeat
                  if lexico(a,b) then lbase1:=cons(a*b,lbase1)

            for i in 2..ll-1 repeat              -- mot + mot
              for a in base(i) repeat
                for b in base(ll-i) repeat
                  if lexico(a,b) and (lexico(b,right a) or b = right a )
                  then lbase1:=cons(a*b,lbase1)

            base(ll):= sort_!(lexico, lbase1)
         return base

    LyndonWordsList (vl,n) ==
        v:ARRAY1 List $ := LyndonWordsList1(vl,n)
        "append"/ [v.i for i in 1..n]
```

———

— LWORD.dotabb —

```
"LWORD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LWORD"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"LWORD" -> "A1AGG"
```

———

Chapter 14

Chapter M

14.1 domain MCMPLX MachineComplex

— MachineComplex.input —

```
)set break resume
)sys rm -f MachineComplex.output
)spool MachineComplex.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MachineComplex
--R MachineComplex  is a domain constructor
--R Abbreviation for MachineComplex is MCMPLX
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MCMPLX
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,MachineFloat) -> %          ?*? : (MachineFloat,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean              ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean               ?>=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       associates? : (%,%) -> Boolean
--R basis : () -> Vector %               coerce : % -> Complex Float
--R coerce : Complex Integer -> %        coerce : Complex Float -> %
--R coerce : MachineFloat -> %           coerce : Integer -> %
```

```
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              conjugate : % -> %
--R discriminant : () -> MachineFloat     generator : () -> %
--R hash : % -> SingleInteger             imag : % -> MachineFloat
--R imaginary : () -> %                   latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R norm : % -> MachineFloat              one? : % -> Boolean
--R rank : () -> PositiveInteger          real : % -> MachineFloat
--R recip : % -> Union(%,"failed")        retract : % -> MachineFloat
--R retract : % -> Integer                sample : () -> %
--R trace : % -> MachineFloat             unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if MachineFloat has FIELD
--R ?*? : (Fraction Integer,%) -> % if MachineFloat has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if MachineFloat has FIELD
--R ?**? : (%,Fraction Integer) -> % if MachineFloat has RADCAT and MachineFloat has TRANFUN
--R ?**? : (%,%) -> % if MachineFloat has TRANFUN
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if MachineFloat has FIELD
--R D : % -> % if MachineFloat has DIFRING
--R D : (%,NonNegativeInteger) -> % if MachineFloat has DIFRING
--R D : (%,Symbol) -> % if MachineFloat has PDRING SYMBOL
--R D : (%,List Symbol) -> % if MachineFloat has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R D : (%,(MachineFloat -> MachineFloat),NonNegativeInteger) -> %
--R D : (%,(MachineFloat -> MachineFloat)) -> %
--R ?^? : (%,Integer) -> % if MachineFloat has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if MachineFloat has RNS
--R acos : % -> % if MachineFloat has TRANFUN
--R acosh : % -> % if MachineFloat has TRANFUN
--R acot : % -> % if MachineFloat has TRANFUN
--R acoth : % -> % if MachineFloat has TRANFUN
--R acsc : % -> % if MachineFloat has TRANFUN
--R acsch : % -> % if MachineFloat has TRANFUN
--R argument : % -> MachineFloat if MachineFloat has TRANFUN
--R asec : % -> % if MachineFloat has TRANFUN
--R asech : % -> % if MachineFloat has TRANFUN
--R asin : % -> % if MachineFloat has TRANFUN
--R asinh : % -> % if MachineFloat has TRANFUN
--R atan : % -> % if MachineFloat has TRANFUN
--R atanh : % -> % if MachineFloat has TRANFUN
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> SparseUnivariatePolynomial MachineFloat
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and MachineFloat has EUCDOM and Machi
--R charthRoot : % -> % if MachineFloat has FFIELDC
--R coerce : Fraction Integer -> % if MachineFloat has FIELD or MachineFloat has RETRACT FRAC
```

```
--R coerce : Complex MachineInteger -> %
--R coerce : Complex MachineFloat -> %
--R complex : (MachineFloat,MachineFloat) -> %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and MachineFloat has EUCDOM and Ma
--R convert : % -> Vector MachineFloat
--R convert : Vector MachineFloat -> %
--R convert : % -> SparseUnivariatePolynomial MachineFloat
--R convert : SparseUnivariatePolynomial MachineFloat -> %
--R convert : % -> Pattern Integer if MachineFloat has KONVERT PATTERN INT
--R convert : % -> Pattern Float if MachineFloat has KONVERT PATTERN FLOAT
--R convert : % -> Complex Float if MachineFloat has REAL
--R convert : % -> Complex DoubleFloat if MachineFloat has REAL
--R convert : % -> InputForm if MachineFloat has KONVERT INFORM
--R coordinates : (%,Vector %) -> Vector MachineFloat
--R coordinates : (Vector %,Vector %) -> Matrix MachineFloat
--R coordinates : % -> Vector MachineFloat
--R coordinates : Vector % -> Matrix MachineFloat
--R cos : % -> % if MachineFloat has TRANFUN
--R cosh : % -> % if MachineFloat has TRANFUN
--R cot : % -> % if MachineFloat has TRANFUN
--R coth : % -> % if MachineFloat has TRANFUN
--R createPrimitiveElement : () -> % if MachineFloat has FFIELDC
--R csc : % -> % if MachineFloat has TRANFUN
--R csch : % -> % if MachineFloat has TRANFUN
--R definingPolynomial : () -> SparseUnivariatePolynomial MachineFloat
--R derivationCoordinates : (Vector %,(MachineFloat -> MachineFloat)) -> Matrix MachineFloat if MachineF
--R differentiate : % -> % if MachineFloat has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if MachineFloat has DIFRING
--R differentiate : (%,Symbol) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (%,(MachineFloat -> MachineFloat),NonNegativeInteger) -> %
--R differentiate : (%,(MachineFloat -> MachineFloat)) -> %
--R discreteLog : % -> NonNegativeInteger if MachineFloat has FFIELDC
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if MachineFloat has FFIELDC
--R discriminant : Vector % -> MachineFloat
--R divide : (%,%) -> Record(quotient: %,remainder: %) if MachineFloat has EUCDOM
--R ?.? : (%,MachineFloat) -> % if MachineFloat has ELTAB(MFLOAT,MFLOAT)
--R euclideanSize : % -> NonNegativeInteger if MachineFloat has EUCDOM
--R eval : (%,List MachineFloat,List MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (%,MachineFloat,MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (%,Equation MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (%,List Equation MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (%,List Symbol,List MachineFloat) -> % if MachineFloat has IEVALAB(SYMBOL,MFLOAT)
--R eval : (%,Symbol,MachineFloat) -> % if MachineFloat has IEVALAB(SYMBOL,MFLOAT)
--R exp : % -> % if MachineFloat has TRANFUN
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if MachineFloat has EUCDOM
--R exquo : (%,MachineFloat) -> Union(%,"failed") if MachineFloat has INTDOM
--R exquo : (%,%) -> Union(%,"failed")
```

```
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if MachineFloat has E
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if MachineFloat
--R factor : % -> Factored % if MachineFloat has EUCDOM and MachineFloat has PFECAT or Machin
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if Machin
--R gcd : (%,%) -> % if MachineFloat has EUCDOM
--R gcd : List % -> % if MachineFloat has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R index : PositiveInteger -> % if MachineFloat has FINITE
--R init : () -> % if MachineFloat has FFIELDC
--R inv : % -> % if MachineFloat has FIELD
--R lcm : (%,%) -> % if MachineFloat has EUCDOM
--R lcm : List % -> % if MachineFloat has EUCDOM
--R lift : % -> SparseUnivariatePolynomial MachineFloat
--R log : % -> % if MachineFloat has TRANFUN
--R lookup : % -> PositiveInteger if MachineFloat has FINITE
--R map : ((MachineFloat -> MachineFloat),%) -> %
--R minimalPolynomial : % -> SparseUnivariatePolynomial MachineFloat if MachineFloat has FIEL
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if MachineFloat has EUCDOM
--R nextItem : % -> Union(%,"failed") if MachineFloat has FFIELDC
--R nthRoot : (%,Integer) -> % if MachineFloat has RADCAT and MachineFloat has TRANFUN
--R order : % -> PositiveInteger if MachineFloat has FFIELDC
--R order : % -> OnePointCompletion PositiveInteger if MachineFloat has FFIELDC
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pi : () -> % if MachineFloat has TRANFUN
--R polarCoordinates : % -> Record(r: MachineFloat,phi: MachineFloat) if MachineFloat has RNS
--R prime? : % -> Boolean if MachineFloat has EUCDOM and MachineFloat has PFECAT or MachineFl
--R primeFrobenius : (%,NonNegativeInteger) -> % if MachineFloat has FFIELDC
--R primeFrobenius : % -> % if MachineFloat has FFIELDC
--R primitive? : % -> Boolean if MachineFloat has FFIELDC
--R primitiveElement : () -> % if MachineFloat has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if MachineFloat has EUCDOM
--R ?quo? : (%,%) -> % if MachineFloat has EUCDOM
--R random : () -> % if MachineFloat has FINITE
--R rational : % -> Fraction Integer if MachineFloat has INS
--R rational? : % -> Boolean if MachineFloat has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if MachineFloat has INS
--R reduce : SparseUnivariatePolynomial MachineFloat -> %
--R reduce : Fraction SparseUnivariatePolynomial MachineFloat -> Union(%,"failed") if Machine
--R reducedSystem : Matrix % -> Matrix Integer if MachineFloat has LINEXP INT
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix MachineFloat,vec: Vector Machin
--R reducedSystem : Matrix % -> Matrix MachineFloat
--R regularRepresentation : (%,Vector %) -> Matrix MachineFloat
--R regularRepresentation : % -> Matrix MachineFloat
--R ?rem? : (%,%) -> % if MachineFloat has EUCDOM
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if MachineFloat has FF
--R represents : (Vector MachineFloat,Vector %) -> %
```

```
--R represents : Vector MachineFloat -> %
--R retract : % -> Fraction Integer if MachineFloat has RETRACT FRAC INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if MachineFloat has RETRACT FRAC INT
--R retractIfCan : % -> Union(MachineFloat,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R sec : % -> % if MachineFloat has TRANFUN
--R sech : % -> % if MachineFloat has TRANFUN
--R sin : % -> % if MachineFloat has TRANFUN
--R sinh : % -> % if MachineFloat has TRANFUN
--R size : () -> NonNegativeInteger if MachineFloat has FINITE
--R sizeLess? : (%,%) -> Boolean if MachineFloat has EUCDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R sqrt : % -> % if MachineFloat has RADCAT and MachineFloat has TRANFUN
--R squareFree : % -> Factored % if MachineFloat has EUCDOM and MachineFloat has PFECAT or MachineFloat
--R squareFreePart : % -> % if MachineFloat has EUCDOM and MachineFloat has PFECAT or MachineFloat has F
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Mach
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if MachineFloat has
--R tan : % -> % if MachineFloat has TRANFUN
--R tanh : % -> % if MachineFloat has TRANFUN
--R traceMatrix : Vector % -> Matrix MachineFloat
--R traceMatrix : () -> Matrix MachineFloat
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

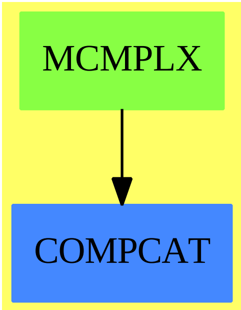
)spool
)lisp (bye)
```

— **MachineComplex.help** —

```
====================================================================
MachineComplex examples
====================================================================

See Also:
o )show MachineComplex
```

14.1.1 MachineComplex (MCMPLX)



See

⇒ "MachineInteger" (MINT) 14.3.1 on page 1521
⇒ "MachineFloat" (MFLOAT) 14.2.1 on page 1511

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
argument	asec	asech
asin	asinh	associates?
atan	atanh	basis
characteristic	characteristicPolynomial	charthRoot
coerce	complex	conditionP
conjugate	convert	coordinates
cos	cosh	cot
coth	createPrimitiveElement	csc
csch	D	definingPolynomial
derivationCoordinates	differentiate	discreteLog
discriminant	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	gcdPolynomial
gcd	factorsOfCyclicGroupSize	generator
hash	imag	imaginary
index	init	inv
latex	lcm	lift
log	lookup	map
max	min	minimalPolynomial
multiEuclidean	nextItem	norm
nthRoot	one?	order
patternMatch	pi	polarCoordinates
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
rank	rational	rational?
rationalIfCan	real	recip
reduce	reducedSystem	regularRepresentation
representationType	represents	retract
retractIfCan	sample	sec
sech	sin	sinh
size	solveLinearPolynomialEquation	sizeLess?
sqrt	squareFree	squareFreePart
squareFreePolynomial	tableForDiscreteLogarithm	subtractIfCan
tan	tanh	trace
traceMatrix	traceMatrix	unit?
unitCanonical	unitNormal	zero?
?*?	?**?	?+?
?-?	-?	?<?
?<=?	?=?	?>?
?>=?	?^?	?~=?
?/?	?.?	?quo?
?rem?		

— domain MCMPLX MachineComplex —

```
)abbrev domain MCMPLX MachineComplex
++ Date Created:  December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, FortranMachineTypeCategory, MachineInteger,
++  MachineFloat
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain which models the complex number representation
++ used by machines in the AXIOM-NAG link.

MachineComplex():Exports == Implementation where

  Exports ==> Join (FortranMachineTypeCategory,
                    ComplexCategory(MachineFloat)) with
    coerce : Complex Float -> $
      ++ coerce(u) transforms u into a MachineComplex
    coerce : Complex Integer -> $
      ++ coerce(u) transforms u into a MachineComplex
    coerce : Complex MachineFloat -> $
      ++ coerce(u) transforms u into a MachineComplex
    coerce : Complex MachineInteger -> $
      ++ coerce(u) transforms u into a MachineComplex
    coerce : $ -> Complex Float
      ++ coerce(u) transforms u into a COmplex Float

  Implementation ==> Complex MachineFloat add

    coerce(u:Complex Float):$ ==
      complex(real(u)::MachineFloat,imag(u)::MachineFloat)

    coerce(u:Complex Integer):$ ==
      complex(real(u)::MachineFloat,imag(u)::MachineFloat)

    coerce(u:Complex MachineInteger):$ ==
      complex(real(u)::MachineFloat,imag(u)::MachineFloat)

    coerce(u:Complex MachineFloat):$ ==
      complex(real(u),imag(u))

    coerce(u:$):Complex Float ==
      complex(real(u)::Float,imag(u)::Float)
```

———-

— MCMPLX.dotabb —

```
"MCMPLX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MCMPLX"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"MCMPLX" -> "COMPCAT"
```

———-

14.2 domain MFLOAT MachineFloat

— MachineFloat.input —

```
)set break resume
)sys rm -f MachineFloat.output
)spool MachineFloat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MachineFloat
--R MachineFloat  is a domain constructor
--R Abbreviation for MachineFloat is MFLOAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MFLOAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Fraction Integer) -> %
--R ?**? : (%,Integer) -> %              ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              1 : () -> %
--R 0 : () -> %                          ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %       abs : % -> %
--R associates? : (%,%) -> Boolean       base : () -> PositiveInteger
--R bits : () -> PositiveInteger         ceiling : % -> %
--R coerce : MachineInteger -> %         coerce : % -> Float
--R coerce : Float -> %                  coerce : Fraction Integer -> %
--R coerce : Integer -> %                coerce : Fraction Integer -> %
```

```
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             convert : % -> Pattern Float
--R convert : % -> DoubleFloat           convert : % -> Float
--R digits : () -> PositiveInteger       exponent : % -> Integer
--R factor : % -> Factored %             float : (Integer,Integer) -> %
--R floor : % -> %                       fractionPart : % -> %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger            inv : % -> %
--R latex : % -> String                  lcm : List % -> %
--R lcm : (%,%) -> %                     mantissa : % -> Integer
--R max : (%,%) -> %                     maximumExponent : () -> Integer
--R min : (%,%) -> %                     minimumExponent : () -> Integer
--R negative? : % -> Boolean             norm : % -> %
--R nthRoot : (%,Integer) -> %           one? : % -> Boolean
--R order : % -> Integer                 positive? : % -> Boolean
--R precision : () -> PositiveInteger    prime? : % -> Boolean
--R ?quo? : (%,%) -> %                   recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                   retract : % -> Float
--R retract : % -> Fraction Integer      retract : % -> Integer
--R round : % -> %                       sample : () -> %
--R sign : % -> Integer                  sizeLess? : (%,%) -> Boolean
--R sqrt : % -> %                        squareFree : % -> Factored %
--R squareFreePart : % -> %              truncate : % -> %
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R wholePart : % -> Integer             zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R base : PositiveInteger -> PositiveInteger
--R bits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R changeBase : (Integer,Integer,PositiveInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R decreasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R digits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R float : (Integer,Integer,PositiveInteger) -> %
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R increasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R max : () -> % if not has($,arbitraryExponent) and not has($,arbitraryPrecision)
--R maximumExponent : Integer -> Integer
--R min : () -> % if not has($,arbitraryExponent) and not has($,arbitraryPrecision)
--R minimumExponent : Integer -> Integer
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
```

```
--R precision : PositiveInteger -> PositiveInteger
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retractIfCan : % -> Union(Float,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

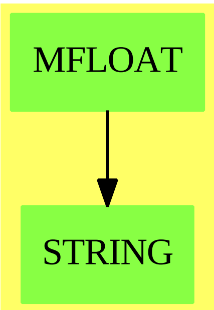
— MachineFloat.help —

```
====================================================================
MachineFloat examples
====================================================================

See Also:
o )show MachineFloat
```

———-

14.2.1 MachineFloat (MFLOAT)



See

⇒ "MachineInteger" (MINT) 14.3.1 on page 1521
⇒ "MachineComplex" (MCMPLX) 14.1.1 on page 1506

Exports:

1	0	abs	associates?
base	bits	ceiling	coerce
changeBase	characteristic	convert	decreasePrecision
digits	exponent	divide	euclideanSize
expressIdealMember	exquo	extendedEuclidean	factor
float	floor	fractionPart	gcd
gcdPolynomial	hash	increasePrecision	inv
latex	lcm	mantissa	max
maximumExponent	min	minimumExponent	multiEuclidean
negative?	norm	nthRoot	one?
order	patternMatch	positive?	precision
prime?	principalIdeal	recip	retract
retractIfCan	round	sample	sign
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	truncate	unit?	unitCanonical
unitNormal	wholePart	zero?	?*?
?**?	?+?	?-?	-?
?/?	?<?	?<=?	?=?
?>?	?>=?	?~=?	?^?
?quo?	?rem?		

— domain MFLOAT MachineFloat —

```
)abbrev domain MFLOAT MachineFloat
++ Author: Mike Dewar
++ Date Created:  December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, FortranMachineTypeCategory, MachineInteger,
++  MachineComplex
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain which models the floating point representation
++ used by machines in the AXIOM-NAG link.

MachineFloat(): Exports == Implementation where

  PI  ==> PositiveInteger
  NNI ==> NonNegativeInteger
  F   ==> Float
  I   ==> Integer
  S   ==> String
  FI  ==> Fraction Integer
```

```
SUP ==> SparseUnivariatePolynomial
SF  ==> DoubleFloat

Exports ==> Join(FloatingPointSystem,FortranMachineTypeCategory,Field,
    RetractableTo(Float),RetractableTo(Fraction(Integer)),CharacteristicZero) with
  precision : PI -> PI
    ++ precision(p) sets the number of digits in the model to p
  precision : () -> PI
    ++ precision() returns the number of digits in the model
  base   : PI -> PI
    ++ base(b) sets the base of the model to b
  base   : () -> PI
    ++ base() returns the base of the model
  maximumExponent : I -> I
    ++ maximumExponent(e) sets the maximum exponent in the model to e
  maximumExponent : () -> I
    ++ maximumExponent() returns the maximum exponent in the model
  minimumExponent : I -> I
    ++ minimumExponent(e) sets the minimum exponent in the model to e
  minimumExponent : () -> I
    ++ minimumExponent() returns the minimum exponent in the model
  coerce : $ -> F
    ++ coerce(u) transforms a MachineFloat to a standard Float
  coerce : MachineInteger -> $
    ++ coerce(u) transforms a MachineInteger into a MachineFloat
  mantissa  : $ -> I
    ++ mantissa(u) returns the mantissa of u
  exponent  : $ -> I
    ++ exponent(u) returns the exponent of u
  changeBase : (I,I,PI) -> $
    ++ changeBase(exp,man,base) is not documented

Implementation ==> add

  import F
  import FI

  Rep := Record(mantissa:I,exponent:I)

  -- Parameters of the Floating Point Representation
  P : PI := 16      -- Precision
  B : PI := 2       -- Base
  EMIN : I := -1021 -- Minimum Exponent
  EMAX : I :=  1024 -- Maximum Exponent

  -- Useful constants
  POWER : PI := 53  -- The maximum power of B which will yield P
                    -- decimal digits.
  MMAX  : PI := B**POWER
```

```
-- locals
locRound:(FI)->I
checkExponent:($)->$
normalise:($)->$
newPower:(PI,PI)->Void

retractIfCan(u:$):Union(FI,"failed") ==
  mantissa(u)*(B/1)**(exponent(u))

wholePart(u:$):Integer ==
   man:I:=mantissa u
   exp:I:=exponent u
   f:=
       positive? exp => man*B**(exp pretend PI)
       zero? exp => man
       wholePart(man/B**((-exp) pretend PI))
normalise(u:$):$ ==
  -- We want the largest possible mantissa, to ensure a canonical
  -- representation.
  exp : I  := exponent u
  man : I  := mantissa u
  BB  : I  := B pretend I
  sgn : I := sign man ; man := abs man
  zero? man => [0,0]$Rep
  if man < MMAX then
    while man < MMAX repeat
      exp := exp - 1
      man := man * BB
  if man > MMAX then
    q1:FI:= man/1
    BBF:FI:=BB/1
    while wholePart(q1) > MMAX repeat
      q1:= q1 / BBF
      exp:=exp + 1
    man := locRound(q1)
  positive?(sgn) => checkExponent [man,exp]$Rep
  checkExponent [-man,exp]$Rep

mantissa(u:$):I == elt(u,mantissa)$Rep
exponent(u:$):I == elt(u,exponent)$Rep

newPower(base:PI,prec:PI):Void ==
  power   : PI := 1
  target  : PI := 10**prec
  current : PI := base
  while (current := current*base) < target repeat power := power+1
  POWER := power
  MMAX  := B**POWER
  void()
```

```
changeBase(exp:I,man:I,base:PI):$ ==
  newExp : I  := 0
  f      : FI := man*(base pretend I)::FI**exp
  sign   : I  := sign f
  f      : FI := abs f
  newMan : I  := wholePart f
  zero? f => [0,0]$Rep
  BB     : FI := (B pretend I)::FI
  if newMan < MMAX then
    while newMan < MMAX repeat
      newExp := newExp - 1
      f := f*BB
      newMan := wholePart f
  if newMan > MMAX then
    while newMan > MMAX repeat
      newExp := newExp + 1
      f := f/BB
      newMan := wholePart f
  [sign*newMan,newExp]$Rep

checkExponent(u:$):$ ==
  exponent(u) < EMIN or exponent(u) > EMAX =>
    message :S := concat(["Exponent out of range: ",
                          convert(EMIN)@S, "..", convert(EMAX)@S])$S
    error message
  u

coerce(u:$):OutputForm ==
  coerce(u::F)

coerce(u:MachineInteger):$ ==
  checkExponent changeBase(0,retract(u)@Integer,10)

coerce(u:$):F ==
  oldDigits : PI := digits(P)$F
  r : F := float(mantissa u,exponent u,B)$Float
  digits(oldDigits)$F
  r

coerce(u:F):$ ==
  checkExponent changeBase(exponent(u)$F,mantissa(u)$F,base()$F)

coerce(u:I):$ ==
   checkExponent changeBase(0,u,10)

coerce(u:FI):$ == (numer u)::$/(denom u)::$

retract(u:$):FI ==
  value : Union(FI,"failed") := retractIfCan(u)
```

```
      value case "failed" => error "Cannot retract to a Fraction Integer"
      value::FI

    retract(u:$):F == u::F

    retractIfCan(u:$):Union(F,"failed") == u::F::Union(F,"failed")

    retractIfCan(u:$):Union(I,"failed") ==
      value:FI := mantissa(u)*(B pretend I)::FI**exponent(u)
      zero? fractionPart(value) => wholePart(value)::Union(I,"failed")
      "failed"::Union(I,"failed")

    retract(u:$):I ==
      result : Union(I,"failed") := retractIfCan u
      result = "failed" => error "Not an Integer"
      result::I

    precision(p: PI):PI ==
      old : PI := P
      newPower(B,p)
      P := p
      old

    precision():PI == P

    base(b:PI):PI ==
      old : PI := b
      newPower(b,P)
      B := b
      old

    base():PI == B

    maximumExponent(u:I):I ==
      old : I := EMAX
      EMAX := u
      old

    maximumExponent():I == EMAX

    minimumExponent(u:I):I ==
      old : I := EMIN
      EMIN := u
      old

    minimumExponent():I == EMIN

    0 == [0,0]$Rep
    1 == changeBase(0,1,10)
```

```
    zero?(u:$):Boolean == u=[0,0]$Rep



    f1:$
    f2:$


    locRound(x:FI):I ==
      abs(fractionPart(x)) >= 1/2 => wholePart(x)+sign(x)
      wholePart(x)

    recip f1 ==
      zero? f1 => "failed"
      normalise [ locRound(B**(2*POWER)/mantissa f1),-(exponent f1 + 2*POWER)]

    f1 * f2 ==
      normalise [mantissa(f1)*mantissa(f2),exponent(f1)+exponent(f2)]$Rep

    f1 **(p:FI) ==
      ((f1::F)**p)::%

--inline
    f1 / f2 ==
      zero? f2 => error "division by zero"
      zero? f1 => 0
      f1=f2 => 1
      normalise [locRound(mantissa(f1)*B**(2*POWER)/mantissa(f2)),
         exponent(f1)-(exponent f2 + 2*POWER)]

    inv(f1) == 1/f1

    f1 exquo f2 == f1/f2

    divide(f1,f2) == [ f1/f2,0]

    f1 quo f2 == f1/f2
    f1 rem f2 == 0
    u:I * f1 ==
      normalise [u*mantissa(f1),exponent(f1)]$Rep

    f1 = f2 == mantissa(f1)=mantissa(f2) and exponent(f1)=exponent(f2)

    f1 + f2 ==
      m1 : I := mantissa f1
      m2 : I := mantissa f2
      e1 : I := exponent f1
      e2 : I := exponent f2
      e1 > e2 =>
--insignificance
```

```
        e1 > e2 + POWER + 2 =>
              zero? f1 => f2
              f1
        normalise [m1*(B pretend I)**((e1-e2) pretend NNI)+m2,e2]$Rep
     e2 > e1 + POWER +2 =>
              zero? f2 => f1
              f2
     normalise [m2*(B pretend I)**((e2-e1) pretend NNI)+m1,e1]$Rep

   - f1 == [- mantissa f1,exponent f1]$Rep

   f1 - f2 == f1 + (-f2)

   f1 < f2 ==
     m1 : I := mantissa f1
     m2 : I := mantissa f2
     e1 : I := exponent f1
     e2 : I := exponent f2
     sign(m1) = sign(m2) =>
       e1 < e2 => true
       e1 = e2 and m1 < m2 => true
       false
     sign(m1) = 1 => false
     sign(m1) = 0 and sign(m2) = -1 => false
     true

   characteristic():NNI == 0
```

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— MFLOAT.dotabb —

```
"MFLOAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MFLOAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"MFLOAT" -> "STRING"
```

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14.3 domain MINT MachineInteger

— MachineInteger.input —

```
)set break resume
)sys rm -f MachineInteger.output
```

```
)spool MachineInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MachineInteger
--R MachineInteger  is a domain constructor
--R Abbreviation for MachineInteger is MINT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MINT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R D : (%,NonNegativeInteger) -> %       D : % -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        abs : % -> %
--R addmod : (%,%,%) -> %                 associates? : (%,%) -> Boolean
--R base : () -> %                        binomial : (%,%) -> %
--R bit? : (%,%) -> Boolean               coerce : Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              convert : % -> Integer
--R convert : % -> InputForm              convert : % -> Pattern Integer
--R convert : % -> Float                  convert : % -> DoubleFloat
--R copy : % -> %                         dec : % -> %
--R differentiate : % -> %                even? : % -> Boolean
--R factor : % -> Factored %              factorial : % -> %
--R gcd : (%,%) -> %                      gcd : List % -> %
--R hash : % -> %                         hash : % -> SingleInteger
--R inc : % -> %                          init : () -> %
--R invmod : (%,%) -> %                   latex : % -> String
--R lcm : (%,%) -> %                      lcm : List % -> %
--R length : % -> %                       mask : % -> %
--R max : (%,%) -> %                      maxint : () -> PositiveInteger
--R min : (%,%) -> %                      mulmod : (%,%,%) -> %
--R negative? : % -> Boolean              odd? : % -> Boolean
--R one? : % -> Boolean                   permutation : (%,%) -> %
--R positive? : % -> Boolean              positiveRemainder : (%,%) -> %
--R powmod : (%,%,%) -> %                 prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    random : () -> %
--R random : % -> %                       rational : % -> Fraction Integer
--R rational? : % -> Boolean              recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> Integer
--R sample : () -> %                      shift : (%,%) -> %
--R sign : % -> Integer                   sizeLess? : (%,%) -> Boolean
```

```
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R submod : (%,%,%) -> %                symmetricRemainder : (%,%) -> %
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coerce : Expression Integer -> Expression %
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R maxint : PositiveInteger -> PositiveInteger
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R rationalIfCan : % -> Union(Fraction Integer,"failed")
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
--R reducedSystem : Matrix % -> Matrix Integer
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— MachineInteger.help —

```
====================================================================
MachineInteger examples
====================================================================

See Also:
o )show MachineInteger
```

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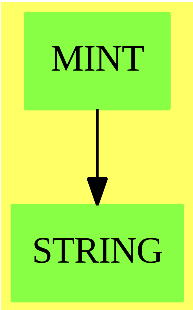
14.3.1 MachineInteger (MINT)



See

⇒ “MachineFloat” (MFLOAT) 14.2.1 on page 1511
⇒ “MachineComplex” (MCMPLX) 14.1.1 on page 1506

Exports:

0	1	abs	addmod	associates?
base	binomial	bit?	characteristic	coerce
convert	copy	D	dec	differentiate
divide	euclideanSize	even?	expressIdealMember	exquo
extendedEuclidean	factor	factorial	gcd	gcdPolynomial
hash	inc	init	invmod	latex
lcm	length	mask	max	maxint
min	mulmod	multiEuclidean	negative?	nextItem
odd?	one?	patternMatch	permutation	positive?
positiveRemainder	powmod	prime?	principalIdeal	random
rational	rationalIfCan	rational?	recip	reducedSystem
retract	retractIfCan	sample	shift	sign
sizeLess?	squareFree	squareFreePart	submod	subtractIfCan
symmetricRemainder	unit?	unitCanonical	unitNormal	zero?
?~=?	?*?	?**?	?^?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?	?quo?	?rem?	

— domain MINT MachineInteger —

```
)abbrev domain MINT MachineInteger
++ Author: Mike Dewar
++ Date Created:  December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, FortranMachineTypeCategory, MachineFloat,
++  MachineComplex
++ AMS Classifications:
++ Keywords:
```

```
++ Examples:
++ References:
++ Description:
++ A domain which models the integer representation
++ used by machines in the AXIOM-NAG link.

MachineInteger(): Exports == Implementation where

  S ==> String

  Exports ==> Join(FortranMachineTypeCategory,IntegerNumberSystem) with
    maxint : PositiveInteger -> PositiveInteger
     ++ maxint(u) sets the maximum integer in the model to u
    maxint : () -> PositiveInteger
     ++ maxint() returns the maximum integer in the model
    coerce : Expression Integer -> Expression $
     ++ coerce(x) returns x with coefficients in the domain

  Implementation  ==> Integer add

    MAXINT : PositiveInteger := 2**32

    maxint():PositiveInteger == MAXINT

    maxint(new:PositiveInteger):PositiveInteger ==
      old := MAXINT
      MAXINT := new
      old

    coerce(u:Expression Integer):Expression($) ==
      map(coerce,u)$ExpressionFunctions2(Integer,$)

    coerce(u:Integer):$ ==
      import S
      abs(u) > MAXINT =>
        message: S := concat [convert(u)@S,"  > MAXINT(",convert(MAXINT)@S,")"]
        error message
      u pretend $

    retract(u:$):Integer == u pretend Integer

    retractIfCan(u:$):Union(Integer,"failed") == u pretend Integer
```

— **MINT.dotabb** —

```
"MINT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MINT"]
```

```
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"MINT" -> "STRING"
```

———-

14.4 domain MAGMA Magma

— Magma.input —

```
)set break resume
)sys rm -f Magma.output
)spool Magma.output
)set message test on
)set message auto off
)clear all
--S 1 of 22
x:Symbol :='x
--R
--R
--R   (1)  x
--R                                                            Type: Symbol
--E 1

--S 2 of 22
y:Symbol :='y
--R
--R
--R   (2)  y
--R                                                            Type: Symbol
--E 2

--S 3 of 22
z:Symbol :='z
--R
--R
--R   (3)  z
--R                                                            Type: Symbol
--E 3

--S 4 of 22
word := OrderedFreeMonoid(Symbol)
--R
--R
--R   (4)  OrderedFreeMonoid Symbol
--R                                                            Type: Domain
--E 4
```

```
--S 5 of 22
tree := Magma(Symbol)
--R
--R
--R   (5)  Magma Symbol
--R                                                          Type: Domain
--E 5

--S 6 of 22
a:tree := x*x
--R
--R
--R   (6)  [x,x]
--R                                                    Type: Magma Symbol
--E 6

--S 7 of 22
b:tree := y*y
--R
--R
--R   (7)  [y,y]
--R                                                    Type: Magma Symbol
--E 7

--S 8 of 22
c:tree := a*b
--R
--R
--R   (8)  [[x,x],[y,y]]
--R                                                    Type: Magma Symbol
--E 8

--S 9 of 22
left c
--R
--R
--R   (9)  [x,x]
--R                                                    Type: Magma Symbol
--E 9

--S 10 of 22
right c
--R
--R
--R   (10)  [y,y]
--R                                                    Type: Magma Symbol
--E 10

--S 11 of 22
```

```
length c
--R
--R
--R   (11)  4
--R                                                      Type: PositiveInteger
--E 11

--S 12 of 22
c::word
--R
--R
--R          2 2
--R   (12)  x y
--R                                             Type: OrderedFreeMonoid Symbol
--E 12

--S 13 of 22
a < b
--R
--R
--R   (13)  true
--R                                                              Type: Boolean
--E 13

--S 14 of 22
a < c
--R
--R
--R   (14)  true
--R                                                              Type: Boolean
--E 14

--S 15 of 22
b < c
--R
--R
--R   (15)  true
--R                                                              Type: Boolean
--E 15

--S 16 of 22
first c
--R
--R
--R   (16)  x
--R                                                               Type: Symbol
--E 16

--S 17 of 22
rest c
```

```
--R
--R
--R   (17)  [x,[y,y]]
--R                                                          Type: Magma Symbol
--E 17

--S 18 of 22
rest rest c
--R
--R
--R   (18)  [y,y]
--R                                                          Type: Magma Symbol
--E 18

--S 19 of 22
ax:tree := a*x
--R
--R
--R   (19)  [[x,x],x]
--R                                                          Type: Magma Symbol
--E 19

--S 20 of 22
xa:tree := x*a
--R
--R
--R   (20)  [x,[x,x]]
--R                                                          Type: Magma Symbol
--E 20

--S 21 of 22
xa < ax
--R
--R
--R   (21)  true
--R                                                               Type: Boolean
--E 21

--S 22 of 22
lexico(xa,ax)
--R
--R
--R   (22)  false
--R                                                               Type: Boolean
--E 22
)spool
)lisp (bye)
```

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— Magma.help —

```
====================================================================
Magma examples
====================================================================

Initialisations

  x:Symbol :='x
    x
                         Type: Symbol

  y:Symbol :='y
    y
                         Type: Symbol

  z:Symbol :='z
    z
                         Type: Symbol

  word := OrderedFreeMonoid(Symbol)
    OrderedFreeMonoid Symbol
                         Type: Domain

  tree := Magma(Symbol)
    Magma Symbol
                         Type: Domain

Let's make some trees

  a:tree := x*x
    [x,x]
                         Type: Magma Symbol

  b:tree := y*y
    [y,y]
                         Type: Magma Symbol

  c:tree := a*b
    [[x,x],[y,y]]
                         Type: Magma Symbol

Query the trees

  left c
    [x,x]
                         Type: Magma Symbol

  right c
```

```
    [y,y]
                         Type: Magma Symbol

  length c
    4
                         Type: PositiveInteger
```

Coerce to the monoid

```
  c::word
     2 2
    x y
                         Type: OrderedFreeMonoid Symbol
```

Check ordering

```
  a < b
    true
                         Type: Boolean

  a < c
    true
                         Type: Boolean

  b < c
    true
                         Type: Boolean
```

Navigate the tree

```
  first c
    x
                         Type: Symbol

  rest c
    [x,[y,y]]
                         Type: Magma Symbol

  rest rest c
    [y,y]
                         Type: Magma Symbol
```

Check ordering

```
  ax:tree := a*x
    [[x,x],x]
                         Type: Magma Symbol

  xa:tree := x*a
    [x,[x,x]]
```

```
                         Type: Magma Symbol

  xa < ax
    true
                         Type: Boolean

  lexico(xa,ax)
    false
                         Type: Boolean

See Also:
o )show Magma
```

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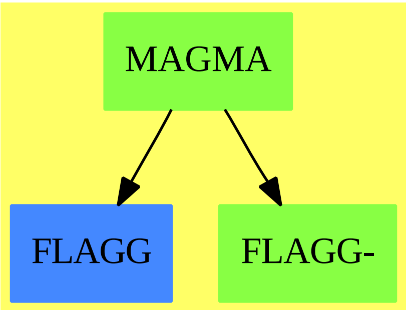
14.4.1 Magma (MAGMA)



Exports:

coerce	first	hash	latex	left
length	lexico	max	min	mirror
rest	retract	retractIfCan	retractable?	right
varList	?~=?	?*?	?<?	?<=?
?=?	?>?	?>=?		

— domain MAGMA Magma —

```
)abbrev domain MAGMA Magma
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
```

```
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type is the basic representation of
++ parenthesized words (binary trees over arbitrary symbols)
++ useful in \spadtype{LiePolynomial}.

Magma(VarSet:OrderedSet):Public == Private where
   WORD ==> OrderedFreeMonoid(VarSet)
   EX   ==> OutputForm

   Public == Join(OrderedSet,RetractableTo VarSet) with
      "*"           : ($,$) -> $
        ++ \axiom{x*y} returns the tree \axiom{[x,y]}.
      coerce        : $ -> WORD
        ++ \axiom{coerce(x)} returns the element of
        ++\axiomType{OrderedFreeMonoid}(VarSet)
        ++ corresponding to \axiom{x} by removing parentheses.
      first         : $ -> VarSet
        ++ \axiom{first(x)} returns the first entry of the tree \axiom{x}.
      left          : $ -> $
        ++ \axiom{left(x)} returns left subtree of \axiom{x} or
        ++ error if retractable?(x) is true.
      length        : $ -> PositiveInteger
        ++ \axiom{length(x)} returns the number of entries in \axiom{x}.
      lexico        : ($,$) -> Boolean
        ++ \axiom{lexico(x,y)} returns \axiom{true} iff  \axiom{x} is smaller
        ++ than \axiom{y} w.r.t. the lexicographical ordering induced by
        ++ \axiom{VarSet}.
        ++ N.B. This operation does not take into account the tree structure of
        ++ its arguments. Thus this is not a total ordering.
      mirror        : $ -> $
        ++ \axiom{mirror(x)} returns the reversed word of \axiom{x}.
        ++ That is \axiom{x} itself if retractable?(x) is true and
        ++ \axiom{mirror(z) * mirror(y)} if \axiom{x} is \axiom{y*z}.
      rest          : $ -> $
        ++ \axiom{rest(x)} return \axiom{x} without the first entry or
        ++ error if retractable?(x) is true.
      retractable?  : $ -> Boolean
        ++ \axiom{retractable?(x)} tests if \axiom{x} is a tree with
        ++ only one entry.
      right         : $ -> $
        ++ \axiom{right(x)} returns right subtree of \axiom{x} or
        ++ error if retractable?(x) is true.
      varList       : $ -> List VarSet
        ++ \axiom{varList(x)} returns the list of distinct entries of \axiom{x}.

   Private == add
    -- representation
```

```
    VWORD := Record(left:$ ,right:$)
    Rep:= Union(VarSet,VWORD)

    recursif: ($,$) -> Boolean

  -- define
    x:$ = y:$ ==
      x case VarSet =>
         y case VarSet => x::VarSet = y::VarSet
         false
      y case VWORD => x::VWORD = y::VWORD
      false

    varList x ==
      x case VarSet => [x::VarSet]
      lv: List VarSet := setUnion(varList x.left, varList x.right)
      sort_!(lv)

    left x ==
      x case VarSet => error "x has only one entry"
      x.left

    right x ==
      x case VarSet => error "x has only one entry"
      x.right
    retractable? x == (x case VarSet)

    retract x ==
       x case VarSet => x::VarSet
       error "Not retractable"

    retractIfCan x == (retractable? x => x::VarSet ; "failed")
    coerce(l:VarSet):$  == l

    mirror x ==
      x case VarSet => x
      [mirror x.right, mirror x.left]$VWORD

    coerce(x:$): WORD ==
      x case VarSet => x::VarSet::WORD
      x.left::WORD * x.right::WORD

    coerce(x:$):EX ==
       x case VarSet => x::VarSet::EX
       bracket [x.left::EX, x.right::EX]

    x * y == [x,y]$VWORD

    first x ==
       x case VarSet => x::VarSet
```

```
         first x.left

      rest x ==
         x case VarSet => error "rest$Magma: inexistant rest"
         lx:$ := x.left
         lx case VarSet => x.right
         [rest lx , x.right]$VWORD

      length x ==
         x case VarSet => 1
         length(x.left) + length(x.right)

      recursif(x,y) ==
         x case VarSet =>
            y case VarSet => x::VarSet < y::VarSet
            true
         y case VarSet => false
         x.left =  y.left =>  x.right <  y.right
         x.left < y.left

      lexico(x,y) ==      -- peut etre amelioree !!!!!!!!!!!
         x case VarSet =>
            y case VarSet => x::VarSet < y::VarSet
            x::VarSet <= first y
         y case VarSet => first x < retract y
         fx:VarSet := first x ; fy:VarSet := first y
         fx = fy => lexico(rest x , rest y)
         fx < fy

      x < y ==           -- recursif par longueur
         lx,ly: PositiveInteger
         lx:= length x ; ly:= length y
         lx = ly => recursif(x,y)
         lx < ly
```

— **MAGMA.dotabb** —

```
"MAGMA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MAGMA"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"MAGMA" -> "FLAGG"
"MAGMA" -> "FLAGG-"
```

14.5 domain MKCHSET MakeCachableSet

— MakeCachableSet.input —

```
)set break resume
)sys rm -f MakeCachableSet.output
)spool MakeCachableSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MakeCachableSet
--R MakeCachableSet S: SetCategory  is a domain constructor
--R Abbreviation for MakeCachableSet is MKCHSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MKCHSET
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              coerce : S -> %
--R coerce : % -> S                      coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R max : (%,%) -> %                     min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R position : % -> NonNegativeInteger
--R setPosition : (%,NonNegativeInteger) -> Void
--R
--E 1

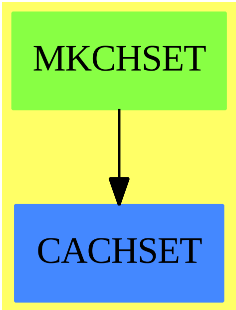
)spool
)lisp (bye)
```

— MakeCachableSet.help —

```
====================================================================
MakeCachableSet examples
====================================================================

See Also:
o )show MakeCachableSet
```

14.5.1 MakeCachableSet (MKCHSET)



See

⇒ "Kernel" (KERNEL) 12.1.1 on page 1368

Exports:

coerce	hash	latex	max	min	position
setPosition	?~=?	?<?	?<=?	?=?	?>?
?>=?					

— domain MKCHSET MakeCachableSet —

```
)abbrev domain MKCHSET MakeCachableSet
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ MakeCachableSet(S) returns a cachable set which is equal to S as a set.

MakeCachableSet(S:SetCategory): Exports == Implementation where
  Exports ==> Join(CachableSet, CoercibleTo S) with
    coerce: S -> %
      ++ coerce(s) returns s viewed as an element of %.

  Implementation ==> add
    import SortedCache(%)

    Rep := Record(setpart: S, pos: NonNegativeInteger)

    clearCache()

    position x              == x.pos
    setPosition(x, n)       == (x.pos := n; void)
    coerce(x:%):S           == x.setpart
    coerce(x:%):OutputForm == x::S::OutputForm
    coerce(s:S):%           == enterInCache([s, 0]$Rep, x+->(s = x::S))

    x < y ==
```

```
      if position(x) = 0 then enterInCache(x, x1+->(x::S = x1::S))
      if position(y) = 0 then enterInCache(y, x1+->(y::S = x1::S))
      position(x) < position(y)

    x = y ==
      if position(x) = 0 then enterInCache(x, x1+->(x::S = x1::S))
      if position(y) = 0 then enterInCache(y, x1+->(y::S = x1::S))
      position(x) = position(y)
```

— **MKCHSET.dotabb** —

```
"MKCHSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MKCHSET"]
"CACHSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=CACHSET"]
"MKCHSET" -> "CACHSET"
```

14.6 domain MMLFORM MathMLFormat

— **MathMLFormat.input** —

```
)set break resume
)sys rm -f MathMLFormat.output
)spool MathMLFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MathMLFormat
--R MathMLFormat  is a domain constructor
--R Abbreviation for MathMLFormat is MMLFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MMLFORM
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : OutputForm -> String
--R coerce : % -> OutputForm              coerceL : OutputForm -> String
--R coerceS : OutputForm -> String        display : String -> Void
--R exprex : OutputForm -> String         hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R
```

```
--E 1

)spool
)lisp (bye)
```

———-

— **MathMLFormat.help** —

```
====================================================================
MathMLFormat examples
====================================================================

See Also:
o )show MathMLFormat
```

———-

Both this code and documentation are still under development and I don't pretend they are anywhere close to perfect or even finished. However the code does work and I hope it might be useful to somebody both for it's ability to output MathML from Axiom and as an example of how to write a new output form.

14.6.1 Introduction to Mathematical Markup Language

MathML exists in two forms: presentation and content. At this time (2007-02-11) the package only has a presentation package. A content package is in the works however it is more difficult. Unfortunately Axiom does not make its semantics easily available. The **OutputForm** domain mediates between the individual Axiom domains and the user visible output but **OutputForm** does not provide full semantic information. From my currently incomplete understanding of Axiom it appears that remedying this would entail going back to the individual domains and rewriting a lot of code. However some semantics are conveyed directly by **OutputForm** and other things can be deduced from **OutputForm** or from the original user command.

14.6.2 Displaying MathML

The MathML string produced by ")set output mathml on" can be pasted directly into an appropriate xhtml page and then viewed in Firefox or some other MathML aware browser. The boiler plate code needed for a test page, testmathml.xml, is:

```
<?xml version="1.0" ?>

<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.1 plus MathML 2.0//EN"
                      "http://www.w3.org/Math/DTD/mathml2/xhtml-math11-f.dtd" [
```

```
<!ENTITY mathml "http://www.w3.org/1998/Math/MathML">
]>

<html xmlns="http://www.w3.org/1999/xhtml"
      xmlns:xlink="http://www.w3.org/1999/xlink" >


  <head>
    <title>MathML Test </title>
  </head>

  <body>

  </body>
</html>
```

Paste the MathML string into the body element and it should display nicely in Firefox.

14.6.3 Test Cases

Here's a list of test cases that currently format correctly:

1. (x+y)**2
2. integrate(x**x,x)
3. integral(x**x,x)
4. (5 + sqrt 63 + sqrt 847)**(1/3)
5. set [1,2,3]
6. multiset [x rem 5 for x in primes(2,1000)]
7. series(sin(a*x),x=0)
8. matrix [[x**i + y**j for i in 1..10] for j in 1..10]
9. y := operator 'y a. D(y(x,z),[x,x,z,x]) b. D(y x,x,2)
10. x := series 'x a. sin(1+x)
11. series(1/log(y),y=1)
12. y:UTS(FLOAT,'z,0) := exp(z)
13. a. c := continuedFraction(314159/100000) b. c := continuedFraction(314159/100000)

The **TexFormat** domain has the capability to format an object with subscripts, superscripts, presubscripts and presuperscripts however I don't know of any Axiom command that produces such an object. In fact at present I see the case of "SUPERSUB" being used for putting primes in the superscript position to denote ordinary differentiation. I also only see the "SUB" case being used to denote partial derivatives.

14.6.4)set output mathml on

Making mathml appear as output during a normal Axiom session by invoking ")set output mathml on" proved to be a bit tedious and seems to be undocumented. I document my experience here in case it proves useful to somebody else trying to get a new output format from Axiom.

In **MathMLFormat** the functions *coerce(expr : OutputForm) : String* and *display(s : String) : Void* provide the desired mathml output. Note that this package was constructed by close examination of Robert Sutor's **TexFormat** domain and much remains from that source. To have mathml displayed as output we need to get Axiom to call display(coerce(expr)) at the appropriate place. Here's what I did to get that to happen. Note that my starting point here was an attempt by Andrey Grozin to do the same. To figure things out I searched through files for "tex" to see what was done for the **TexFormat** domain, and used grep to find which files had mention of **TexFormat**.

14.6.5 File src/interp/setvars.boot.pamphlet

Create an output mathml section by analogy to the tex section. Remember to add the code chunk "outputmathmlCode" at the end.

setvars.boot is a bootstrap file which means that it has to be precompiled into lisp code and then that code has to be inserted back into setvars.boot. To do this extract the boot code by running "notangle" on it. I did this from the "tmp" directory. From inside axiom run ")lisp (boottran::boottocl "tmp/setvars.boot") which put "setvars.clisp" into "int/interp/setvars.clisp". Then replace the lisp in "setvars.boot.pamphlet" with that in the newly generated "setvars.clisp".

The relevant code chunks appearing in "setvars.boot.pamphlet" are:

```
outputmathmlCode
setOutputMathml
describeSetOutputMathml
```

and the relevant variables are:

```
setOutputMathml
$mathmlOutputStream
$mathmlOutputFile
$mathmlFormat
describeSetOutputMathml
```

14.6.6 File setvart.boot.pamphlet

Create an output mathml section in "setvart.boot.pamphlet" again patterned after the tex section. I changed the default file extension from ".stex" to ".smml".

To the "sectionoutput" table I added the line

```
mathml                    created output in MathML style      Off:CONSOLE
```

Added the code chunk "outputmathml" to the code chunk "output" in "sectionoutput".

Relevant code chunks:

```
outputmathml
```

Relevant variables:

```
setOutputMathml
$mathmlFormat
$mathmlOutputFile
```

Note when copying the tex stuff I changed occurrences of "tex" to "mathml", "Tex" to "Mathml" and "TeX" to "MathML".

14.6.7 File src/algebra/Makefile.pamphlet

The file "src/algebra/tex.spad.pamphlet" contains the domain **TexFormat** (TEX) and the package **TexFormat1** (TEX1). However the sole function of **TexFormat1** is to *coerce* objects from a domain into **OutputForm** and then apply **TexFormat** to them. It is to save programmers the trouble of doing the coercion themselves from inside spad code. It does not appear to be used for the main purpose of delivering Axiom output in TeX format. In order to keep the mathml package as simple as possible, and because I didn't see much use for this, I didn't copy the **TexFormat1** package. So no analog of the TEX1 entries in "Makefile.pamphlet" were needed. One curiosity I don't understand is why TEX1 appears in layer 4 when it seems to depend on TEX which appears in layer 14.

Initially I added "$OUT/MMLFORM.o" to layer 14 and "mathml.spad.pamphlet" to completed spad files in layer 14. When trying to compile the build failed at MMLFORM. It left "MMLFORM.erlib" in "int/algebra" instead of "MMLFORM.NRLIB" which confused me at first because mathml.spad compiled under a running axiom. By examining the file "obj/tmp/trace" I saw that a new dependency had been introduced, compared to TexFormat, with the function eltName depending on the domain FSAGG in layer 16. So the lines had to be moved from layer 14 to layer 17.

Added appropriate lines to "SPADFILES" and "DOCFILES".

14.6.8 File src/algebra/exposed.lsp.pamphlet

Add the line "(|MathMLFormat| . MMLFORM)"

14.6.9 File src/algebra/Lattice.pamphlet

I don't see that this file is used anywhere but I made the appropriate changes anyway by searching for "TEX" and mimicing everything for MMLFORM.

14.6.10 File src/doc/axiom.bib.pamphlet

Added mathml.spad subsection to "src/doc/axiom.bib.pamphlet".

14.6.11 File interp/i-output.boot.pamphlet

This is where the *coerce* and *display* functions from MathMLFormat actually get called. The following was added:

```
mathmlFormat expr ==
  mml := '(MathMLFormat)
  mmlrep := '(String)
  formatFn := getFunctionFromDomain("coerce",mml,[$OutputForm])
  displayFn := getFunctionFromDomain("display",mml,[mmlrep])
  SPADCALL(SPADCALL(expr,formatFn),displayFn)
  TERPRI $mathmlOutputStream
  FORCE_-OUTPUT $mathmlOutputStream
  NIL
```

Note that compared to the texFormat function there are a couple of differences. Since **MathMLFormat** is currently a package rather than a domain there is the "mmlrep" variable whereas in texFormat the argument of the "display" function is an instance of the domain. Also the *coerce* function here only has one argument, namely "$OutputForm".

Also for the function "output(expr,domain)" add lines for mathml, e.g. "if $mathmlFormat then mathmlFormat expr".

After these changes Axiom compiled with mathml enabled under)set output.

14.6.12 Public Declarations

The declarations

```
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String
  US     ==> UniversalSegment(Integer)
```

provide abbreviations for domains used heavily in the code. The publicly exposed functions are:

coerce: E -> S This function is the main one for converting an expression in domain OutputForm into a MathML string.

coerceS: E -> S This function is for use from the command line. It converts an OutputForm expression into a MathML string and does some formatting so that the output is not one long line. If you take the output from this function, stick it in an emacs buffer in nxml-mode and then indent according to mode, you'll get something that's nicer to look at than what comes from coerce. Note that coerceS returns the same value as coerce but invokes a display function as well so that the result will be printed twice in different formats. The need for this is that the output from coerce is automatically formatted with line breaks by Axiom's output routine that are not in the right place.

coerceL: E -> S Similar to coerceS except that the displayed result is the MathML string in one long line. These functions can be used, for instance, to get the MathML for the previous result by typing coerceL(

exprex: E -> S Converts **OutputForm** to **String** with the structure preserved with braces. This is useful in developing this package. Actually this is not quite accurate. The function *precondition* is first applied to the **OutputForm** expression before *exprex*. Raw **OutputForm** and the nature of the *precondition* function is still obscure to me at the time of this writing (2007-02-14), however I probably need to understand it to make sure I'm not missing any semantics. The spad function *precondition* is just a wrapper for the lisp function outputTran$Lisp, which I guess is compiled from boot.

display: S -> Void This one prints the string returned by coerce as one long line, adding "math" tags: $...$. Thus the output from this can be stuck directly into an appropriate html/xhtml page and will be displayed nicely by a MathML aware browser.

displayF: S -> Void This function doesn't exist yet but it would be nice to have a humanly readable formatted output as well. The basics do exist in the coerceS function however the formatting still needs some work to be really good.

— public declarations —

```
)abbrev domain MMLFORM MathMLFormat
++ Author: Arthur C. Ralfs
++ Date: January 2007
++ Basic Operations: coerce, coerceS, coerceL, exprex, display
++ Description:
++ This package is based on the TeXFormat domain by Robert S. Sutor
++ \spadtype{MathMLFormat} provides a coercion from \spadtype{OutputForm}
++ to MathML format.

MathMLFormat(): public == private where
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String
  US     ==> UniversalSegment(Integer)

  public == SetCategory with
    coerce:    E -> S
      ++ coerceS(o) changes o in the standard output format to MathML
      ++ format.
    coerceS:   E -> S
      ++ coerceS(o) changes o in the standard output format to MathML
```

```
    ++ format and displays formatted result.
  coerceL:   E -> S
    ++ coerceS(o) changes o in the standard output format to MathML
    ++ format and displays result as one long string.
  exprex:    E -> S
    ++ coverts \spadtype{OutputForm} to \spadtype{String} with the
    ++ structure preserved with braces.  Actually this is not quite
    ++ accurate.  The function \spadfun{precondition} is first
    ++ applied to the
    ++ \spadtype{OutputForm} expression before \spadfun{exprex}.
    ++ The raw \spadtype{OutputForm} and
    ++ the nature of the \spadfun{precondition} function is
    ++ still obscure to me
    ++ at the time of this writing (2007-02-14).
  display:   S -> Void
    ++ prints the string returned by coerce, adding <math ...> tags.
```

——–

14.6.13 Private Constant Declarations

— private constant declarations —

```
private == add
  import OutputForm
  import Character
  import Integer
  import List OutputForm
  import List String

  -- local variable declarations and definitions

  expr: E
  prec,opPrec: I
  str:  S
  blank         : S := " \  "

  maxPrec       : I   := 1000000
  minPrec       : I   := 0

  unaryOps      : L S := ["-","^"]$(L S)
  unaryPrecs    : L I := [700,260]$(L I)

  -- the precedence of / in the following is relatively low because
  -- the bar obviates the need for parentheses.
  binaryOps     : L S := ["+->","|","**","/","<",">","=","OVER"]$(L S)
  binaryPrecs   : L I := [0,0,900, 700,400,400,400,   700]$(L I)
```

```
naryOps       : L S := ["-","+","*",blank,",",";"," ","ROW","",
   " \cr ","&","</mtd></mtr><mtr><mtd>"]$(L S)
naryPrecs     : L I := [700,700,800,  800,110,110,  0,    0, 0,
         0,  0,   0]$(L I)
naryNGOps     : L S := ["ROW","&"]$(L S)

plexOps       : L S := ["SIGMA","SIGMA2","PI","PI2","INTSIGN","INDEFINTEGRAL"]$(L S)
plexPrecs     : L I := [    700, 800,     700, 800 , 700,      700]$(L I)

specialOps    : L S := ["MATRIX","BRACKET","BRACE","CONCATB","VCONCAT",  _
                        "AGGLST","CONCAT","OVERBAR","ROOT","SUB","TAG", _
                        "SUPERSUB","ZAG","AGGSET","SC","PAREN", _
                        "SEGMENT","QUOTE","theMap","SLASH" ]

-- the next two lists provide translations for some strings for
-- which MML provides special macros.

specialStrings : L S :=
  ["cos", "cot", "csc", "log", "sec", "sin", "tan",
    "cosh", "coth", "csch", "sech", "sinh", "tanh",
      "acos","asin","atan","erf","...","$","infinity"]
specialStringsInMML : L S :=
  ["<mo>cos</mo>","<mo>cot</mo>","<mo>csc</mo>","<mo>log</mo>","<mo>sec</mo>","<mo>sin</mo>","<mo>ta
    "<mo>cosh</mo>","<mo>coth</mo>","<mo>csch</mo>","<mo>sech</mo>","<mo>sinh</mo>","<mo>tanh</mo>",
      "<mo>arccos</mo>","<mo>arcsin</mo>","<mo>arctan</mo>","<mo>erf</mo>","<mo>&#x2026;</mo>","<mo>
```

———

14.6.14 Private Function Declarations

These are the local functions:

addBraces:S -> S

addBrackets:S -> S

atomize:E -> L E

displayElt:S -> Void function for recursively displaying mathml nicely formatted

eltLimit:(S,I,S) -> I demarcates end postion of mathml element with name:S starting at position i:I in mathml string s:S and returns end of end tag as i:I position in mathml string, i.e. find start and end of substring: <name ...>...</name>

eltName:(I,S) -> S find name of mathml element starting at position i:I in string s:S

group:S -> S

formatBinary:(S,L E, I) -> S

formatFunction:(S,L E, I) -> S

formatMatrix:L E -> S

formatNary:(S,L E, I) -> S

formatNaryNoGroup:(S,L E, I) -> S

formatNullary:S -> S

formatPlex:(S,L E, I) -> S

formatSpecial:(S,L E, I) -> S

formatUnary:(S, E, I) -> S

formatMml:(E,I) -> S

newWithNum:I -> $ this is a relic from tex.spad and is not used here so far. I'll probably remove it.

parenthesize:S -> S

precondition:E -> E this function is applied to the OutputForm expression before doing anything else.

postcondition:S -> S this function is applied after all other OutputForm -> MathML transformations. In the TexFormat domain the ungroup function first peels off the outermost set of braces however I have replaced braces with <mrow>s here and sometimes the outermost set of <mrow>s is necessary to get proper display in Firefox. For instance with getting the correct size of brackets on a matrix the whole expression needs to be enclosed in a mrow element. It also checks for +− and removes the +.

stringify:E -> S

tagEnd:(S,I,S) -> I finds closing ">" of start or end tag for mathML element for formatting MathML string for human readability. No analog in TexFormat.

ungroup:S -> S

— private function declarations —

```
-- local function signatures

addBraces:      S -> S
addBrackets:    S -> S
atomize:        E -> L E
displayElt:     S -> Void
  ++ function for recursively displaying mathml nicely formatted
eltLimit:       (S,I,S) -> I
  ++ demarcates end postion of mathml element with name:S starting at
  ++ position i:I in mathml string s:S and returns end of end tag as
  ++  i:I position in mathml string, i.e. find start and end of
  ++  substring:  <name ...>...</name>
eltName:        (I,S) -> S
  ++ find name of mathml element starting at position i:I in string s:S
group:          S -> S
formatBinary:   (S,L E, I) -> S
```

```
formatFunction: (S,L E, I) -> S
formatIntSign:  (L E, I) -> S
formatMatrix:   L E -> S
formatNary:     (S,L E, I) -> S
formatNaryNoGroup: (S,L E, I) -> S
formatNullary:  S -> S
formatPlex:     (S,L E, I) -> S
formatSpecial:  (S,L E, I) -> S
formatSub:      (E, L E, I) -> S
formatSuperSub: (E, L E, I) -> S
formatSuperSub1: (E, L E, I) -> S
formatUnary:    (S,  E, I) -> S
formatMml:      (E,I) -> S
formatZag:      L E -> S
formatZag1:     L E -> S
newWithNum:     I -> $
parenthesize:   S -> S
precondition:   E -> E
postcondition:  S -> S
stringify:      E -> S
tagEnd:         (S,I,S) -> I
  ++  finds closing ">" of start or end tag for mathML element
ungroup:        S -> S
```

———-

14.6.15 Public Function Definitions

Note that I use the function sayTeX$Lisp much as I would printf in a C program. I've noticed in grepping the code that there are other "say" functions, sayBrightly and sayMessage for instance, but I have no idea what the difference is between them at this point. sayTeX$Lisp does the job so for the time being I'll use that until I learn more.

The functions coerceS and coerceL should probably be changed to display functions, *i.e.* *displayS* and *display L*, returning Void. I really only need the one coerce function.

— public function definitions —

```
-- public function definitions

coerce(expr : E): S ==
  s : S := postcondition formatMml(precondition expr, minPrec)
  s

coerceS(expr : E): S ==
  s : S := postcondition formatMml(precondition expr, minPrec)
  sayTeX$Lisp "<math xmlns=_"http://www.w3.org/1998/Math/MathML_" mathsize=_"big_" display=_"block_"
```

```
      displayElt(s)
      sayTeX$Lisp "</math>"
      s

    coerceL(expr : E): S ==
      s : S := postcondition formatMml(precondition expr, minPrec)
      sayTeX$Lisp "<math xmlns=_"http://www.w3.org/1998/Math/MathML_" mathsize=_"big_" displa
      sayTeX$Lisp s
      sayTeX$Lisp "</math>"
      s

    display(mathml : S): Void ==
      sayTeX$Lisp "<math xmlns=_"http://www.w3.org/1998/Math/MathML_" mathsize=_"big_" displa
      sayTeX$Lisp mathml
      sayTeX$Lisp "</math>"
      void()$Void



    exprex(expr : E): S ==
      -- This breaks down an expression into atoms and returns it as
      -- a string.  It's for developmental purposes to help understand
      -- the expressions.
      a : E
      expr := precondition expr
--      sayTeX$Lisp "0: "stringify expr
      (ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
        concat ["{",stringify expr,"}"]
      le : L E := (expr pretend L E)
      op := first le
      sop : S := exprex op
      args : L E := rest le
      nargs : I := #args
--      sayTeX$Lisp concat ["1: ",stringify first le," : ",string(nargs)$S]
      s : S := concat ["{",sop]
      if nargs > 0  then
        for a in args repeat
--        sayTeX$Lisp concat ["2: ",stringify a]
          s1 : S := exprex a
          s := concat [s,s1]
      s := concat [s,"}"]
```

———

14.6.16 Private Function Definitions

Display Functions

displayElt(mathml:S):Void

eltName(pos:I,mathml:S):S

eltLimit(name:S,pos:I,mathml:S):I

tagEnd(name:S,pos:I,mathml:S):I

— display functions —

```
    displayElt(mathML:S): Void ==
      -- Takes a string of syntactically complete mathML
      -- and formats it for display.
--      sayTeX$Lisp "****displayElt1****"
--      sayTeX$Lisp mathML
      enT:I -- marks end of tag, e.g. "<name>"
      enE:I -- marks end of element, e.g. "<name> ... </name>"
      end:I -- marks end of mathML string
      u:US
      end := #mathML
      length:I := 60
--      sayTeX$Lisp "****displayElt1.1****"
      name:S := eltName(1,mathML)
--      sayTeX$Lisp name
--      sayTeX$Lisp concat("****displayElt1.2****",name)
      enE := eltLimit(name,2+#name,mathML)
--      sayTeX$Lisp "****displayElt2****"
      if enE < length then
--        sayTeX$Lisp "****displayElt3****"
        u := segment(1,enE)$US
        sayTeX$Lisp mathML.u
      else
--        sayTeX$Lisp "****displayElt4****"
        enT := tagEnd(name,1,mathML)
        u := segment(1,enT)$US
        sayTeX$Lisp mathML.u
        u := segment(enT+1,enE-#name-3)$US
        displayElt(mathML.u)
        u := segment(enE-#name-2,enE)$US
        sayTeX$Lisp mathML.u
      if end > enE then
--        sayTeX$Lisp "****displayElt5****"
        u := segment(enE+1,end)$US
        displayElt(mathML.u)

      void()$Void
```

```
eltName(pos:I,mathML:S): S ==
  -- Assuming pos is the position of "<" for a start tag of a mathML
  -- element finds and returns the element's name.
  i:I := pos+1
  --sayTeX$Lisp "eltName:mathmML string: "mathML
  while member?(mathML.i,lowerCase()$CharacterClass)$CharacterClass repeat
     i := i+1
  u:US := segment(pos+1,i-1)
  name:S := mathML.u

eltLimit(name:S,pos:I,mathML:S): I ==
  -- Finds the end of a mathML element like "<name ...> ... </name>"
  -- where pos is the position of the space after name in the start tag
  -- although it could point to the closing ">".  Returns the position
  -- of the ">" in the end tag.
  pI:I := pos
  startI:I
  endI:I
  startS:S := concat ["<",name]
  endS:S := concat ["</",name,">"]
  level:I := 1
  --sayTeX$Lisp "eltLimit: element name: "name
  while (level > 0) repeat
    startI := position(startS,mathML,pI)$String

    endI := position(endS,mathML,pI)$String

    if (startI = 0) then
      level := level-1
      --sayTeX$Lisp "****eltLimit 1******"
      pI := tagEnd(name,endI,mathML)
    else
      if (startI < endI) then
        level := level+1
        pI := tagEnd(name,startI,mathML)
      else
        level := level-1
        pI := tagEnd(name,endI,mathML)
  pI


tagEnd(name:S,pos:I,mathML:S):I ==
  -- Finds the closing ">" for either a start or end tag of a mathML
  -- element, so the return value is the position of ">" in mathML.
  pI:I := pos
  while  (mathML.pI ^= char ">") repeat
    pI := pI+1
  u:US := segment(pos,pI)$US
  --sayTeX$Lisp "tagEnd: "mathML.u
  pI
```

———-

Formatting Functions

Still need to format `\zag` in formatSpecial!

In formatPlex the case op = "INTSIGN" is now passed off to formatIntSign which is a change from the TexFormat domain. This is done here for presentation mark up to replace the ugly bound variable that Axiom delivers. For content mark up this has to be done anyway.

The formatPlex function also allows for op = "INDEFINTEGRAL". However I don't know what Axiom command gives rise to this case. The INTSIGN case already allows for both definite and indefinite integrals.

In the function formatSpecial various cases are handled including SUB and SUPERSUB. These cases are now caught in formatMml and so the code in formatSpecial doesn't get executed. The only cases I know of using these are partial derivatives for SUB and ordinary derivatives or SUPERSUB however in TexFormat the capability is there to handle multiscripts, i.e. an object with subscripts, superscripts, pre-subscripts and pre-superscripts but I am so far unaware of any Axiom command that produces such a multiscripted object.

Another question is how to represent derivatives. At present I have differential notation for partials and prime notation for ordinary derivatives, but it would be nice to allow for different derivative notations in different circumstances, maybe some options to)set output mathml on.

Ordinary derivatives are formatted in formatSuperSub and there are 2 versions, formatSuperSub and formatSuperSub1, which at this point have to be switched by swapping names.

— **formatting functions** —

```
atomize(expr : E): L E ==
  -- This breaks down an expression into a flat list of atomic expressions.
  -- expr should be preconditioned.
  le : L E := nil()
  a : E
  letmp : L E
  (ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
    le := append(le,list(expr))
  letmp := expr pretend L E
  for a in letmp repeat
    le := append(le,atomize a)
  le

ungroup(str: S): S ==
  len : I := #str
  len < 14 => str
  lrow : S :=  "<mrow>"
```

```
      rrow : S :=  "</mrow>"
      -- drop leading and trailing mrows
      u1 : US := segment(1,6)$US
      u2 : US := segment(len-6,len)$US
      if (str.u1 =$S lrow) and (str.u2 =$S rrow) then
        u : US := segment(7,len-7)$US
        str := str.u
      str

    postcondition(str: S): S ==
--      str := ungroup str
      len : I := #str
      plusminus : S := "<mo>+</mo><mo>-</mo>"
      pos : I := position(plusminus,str,1)
      if pos > 0 then
        ustart:US := segment(1,pos-1)$US
        uend:US := segment(pos+20,len)$US
        str := concat [str.ustart,"<mo>-</mo>",str.uend]
        if pos < len-18 then
          str := postcondition(str)
      str

    stringify expr == (mathObject2String$Lisp expr)@S

    group str ==
      concat ["<mrow>",str,"</mrow>"]

    addBraces str ==
      concat ["<mo>{</mo>",str,"<mo>}</mo>"]

    addBrackets str ==
      concat ["<mo>[</mo>",str,"<mo>]</mo>"]

    parenthesize str ==
      concat ["<mo>(</mo>",str,"<mo>)</mo>"]

    precondition expr ==
      outputTran$Lisp expr

    formatSpecial(op : S, args : L E, prec : I) : S ==
      arg : E
      prescript : Boolean := false
      op = "theMap" => "<mtext>theMap(...)</mtext>"
      op = "AGGLST" =>
        formatNary(",",args,prec)
      op = "AGGSET" =>
        formatNary(";",args,prec)
      op = "TAG" =>
        group concat [formatMml(first args,prec),
                      "<mo>&#x02192;</mo>",
```

```
                   formatMml(second args,prec)]
                --RightArrow
      op = "SLASH" =>
        group concat [formatMml(first args,prec),
          "<mo>/</mo>",formatMml(second args,prec)]
      op = "VCONCAT" =>
        group concat("<mtable><mtr>",
                     concat(concat([concat("<mtd>",concat(formatMml(u, minPrec),"</mtd>"))
                                    for u in args]::L S),
                            "</mtr></mtable>"))
      op = "CONCATB" =>
        formatNary(" ",args,prec)
      op = "CONCAT" =>
        formatNary("",args,minPrec)
      op = "QUOTE" =>
        group concat("<mo>'</mo>",formatMml(first args, minPrec))
      op = "BRACKET" =>
        group addBrackets ungroup formatMml(first args, minPrec)
      op = "BRACE" =>
        group addBraces ungroup formatMml(first args, minPrec)
      op = "PAREN" =>
        group parenthesize ungroup formatMml(first args, minPrec)
      op = "OVERBAR" =>
        null args => ""
        group concat ["<mover accent='true'><mrow>",_
                      formatMml(first args,minPrec),_
                      "</mrow><mo stretchy='true'>&#x000AF;</mo></mover>"]
        --OverBar
      op = "ROOT" =>
        null args => ""
        tmp : S := group formatMml(first args, minPrec)
        null rest args => concat ["<msqrt>",tmp,"</msqrt>"]
        group concat
          ["<mroot><mrow>",tmp,"</mrow>",_
            formatMml(first rest args, minPrec),"</mroot>"]
      op = "SEGMENT" =>
        tmp : S := concat [formatMml(first args, minPrec),"<mo>..</mo>"]
        group
          null rest args =>  tmp
          concat [tmp,formatMml(first rest args, minPrec)]
      -- SUB should now be diverted in formatMml although I'll leave
      -- the code here for now.
      op = "SUB" =>
        group concat ["<msub>",formatMml(first args, minPrec),
          formatSpecial("AGGLST",rest args,minPrec),"</msub>"]
      -- SUPERSUB should now be diverted in formatMml although I'll leave
      -- the code here for now.
      op = "SUPERSUB" =>
        base:S := formatMml(first args, minPrec)
        args := rest args
```

```
        if #args = 1 then
           "<msub><mrow>"base"</mrow><mrow>"_
            formatMml(first args, minPrec)"</mrow></msub>"
        else if #args = 2 then
        -- it would be nice to substitue &#x2032; for , in the case of
        -- an ordinary derivative, it looks a lot better.
          "<msubsup><mrow>"base"</mrow><mrow>"_
           formatMml(first args,minPrec)_
            "</mrow><mrow>"_
             formatMml(first rest args, minPrec)_
              "</mrow></msubsup>"
        else if #args = 3 then
          "<mmultiscripts><mrow>"base"</mrow><mrow>"_
           formatMml(first args,minPrec)"</mrow><mrow>"_
            formatMml(first rest args,minPrec)"</mrow><mprescripts/><mrow>"_
             formatMml(first rest rest args,minPrec)_
              "</mrow><none/></mmultiscripts>"
        else if #args = 4 then
         "<mmultiscripts><mrow>"base"</mrow><mrow>"_
           formatMml(first args,minPrec)"</mrow><mrow>"_
            formatMml(first rest args,minPrec)"</mrow><mprescripts/><mrow>"_
             formatMml(first rest rest args,minPrec)_
              "</mrow><mrow>"formatMml(first rest rest rest args,minPrec)_
               "</mrow></mmultiscripts>"
        else
          "<mtext>Problem with multiscript object</mtext>"
      op = "SC" =>
        -- need to handle indentation someday
        null args => ""
        tmp := formatNaryNoGroup("</mtd></mtr><mtr><mtd>", args, minPrec)
        group concat ["<mtable><mtr><mtd>",tmp,"</mtd></mtr></mtable>"]
      op = "MATRIX" => formatMatrix rest args
      op = "ZAG" =>
-- {{+}{3}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}
-- {{ZAG}{1}{25}}{{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
-- to format continued fraction traditionally need to intercept it at the
-- formatNary of the "+"
        concat [" \zag{",formatMml(first args, minPrec),"}{",
          formatMml(first rest args,minPrec),"}"]
      concat ["<mtext>not done yet for: ",op,"</mtext>"]

    formatSub(expr : E, args : L E, opPrec : I) : S ==
      -- This one produces differential notation partial derivatives.
      -- It doesn't work in all cases and may not be workable, use
      -- formatSub1 below for now.
      -- At this time this is only to handle partial derivatives.
      -- If the SUB case handles anything else I'm not aware of it.
      -- This an example of the 4th partial of y(x,z) w.r.t. x,x,z,x
      -- {{{SUB}{y}{{CONCAT}{{CONCAT}{{CONCAT}{{CONCAT}{,}{1}}
      -- {{CONCAT}{,}{1}}}{{CONCAT}{,}{2}}}{{CONCAT}{,}{1}}}}}{x}{z}}
```

```
      atomE : L E := atomize(expr)
      op : S := stringify first atomE
      op ^= "SUB" => "<mtext>Mistake in formatSub: no SUB</mtext>"
      stringify first rest rest atomE ^= "CONCAT" => _
         "<mtext>Mistake in formatSub: no CONCAT</mtext>"
      -- expecting form for atomE like
      --[{SUB}{func}{CONCAT}...{CONCAT}{,}{n}{CONCAT}{,}{n}...{CONCAT}{,}{n}],
      --counting the first CONCATs before the comma gives the number of
      --derivatives
      ndiffs : I := 0
      tmpLE : L E := rest rest atomE
      while stringify first tmpLE = "CONCAT" repeat
        ndiffs := ndiffs+1
        tmpLE := rest tmpLE
      numLS : L S := nil
      i : I := 1
      while i < ndiffs repeat
        numLS := append(numLS,list(stringify first rest tmpLE))
        tmpLE := rest rest rest tmpLE
        i := i+1
      numLS := append(numLS,list(stringify first rest tmpLE))
      -- numLS contains the numbers of the bound variables as strings
      -- for the differentiations, thus for the differentiation [x,x,z,x]
      -- for y(x,z) numLS = ["1","1","2","1"]
      posLS : L S := nil
      i := 0
--      sayTeX$Lisp "formatSub: nargs = "string(#args)
      while i < #args repeat
        posLS := append(posLS,list(string(i+1)))
        i := i+1
      -- posLS contains the positions of the bound variables in args
      -- as a list of strings, e.g. for the above example ["1","2"]
      tmpS: S := stringify atomE.2
      if ndiffs = 1 then
        s : S := "<mfrac><mo>&#x02202;</mo><mi>"tmpS"</mi><mrow>"
      else
        s : S := "<mfrac><mrow><msup><mo>&#x02202;</mo><mn>"string(ndiffs)"</mn></msup><mi>"tmpS"</mi></
      -- need to find the order of the differentiation w.r.t. the i-th
      -- variable
      i := 1
      j : I
      k : I
      tmpS: S
      while i < #posLS+1 repeat
        j := 0
        k := 1
        while k < #numLS + 1 repeat
          if numLS.k = string i then j := j + 1
          k := k+1
        if j > 0 then
```

```
      tmpS := stringify args.i
      if j = 1 then
        s := s"<mo>&#x02202;</mo><mi>"tmpS"</mi>"
      else
        s := s"<mo>&#x02202;</mo><msup><mi>"tmpS_
              "</mi><mn>"string(j)"</mn></msup>"
    i := i + 1
  s := s"</mrow></mfrac><mo>(</mo>"
  i := 1
  while i < #posLS+1 repeat
    tmpS := stringify args.i
    s := s"<mi>"tmpS"</mi>"
    if i < #posLS then s := s"<mo>,</mo>"
    i := i+1
  s := s"<mo>)</mo>"

formatSub1(expr : E, args : L E, opPrec : I) : S ==
  -- This one produces partial derivatives notated by ",n" as
  -- subscripts.
  -- At this time this is only to handle partial derivatives.
  -- If the SUB case handles anything else I'm not aware of it.
  -- This an example of the 4th partial of y(x,z) w.r.t. x,x,z,x
  -- {{{SUB}{y}{{CONCAT}{{CONCAT}{{CONCAT}{{CONCAT}{,}{1}}
  -- {{CONCAT}{,}{1}}}{{CONCAT}{,}{2}}}{{CONCAT}{,}{1}}}}{x}{z}},
  -- here expr is everything in the first set of braces and
  -- args is {{x}{z}}
  atomE : L E := atomize(expr)
  op : S := stringify first atomE
  op ^= "SUB" => "<mtext>Mistake in formatSub: no SUB</mtext>"
  stringify first rest rest atomE ^= "CONCAT" => "<mtext>Mistake in formatSub: no CONCAT
  -- expecting form for atomE like
  --[{SUB}{func}{CONCAT}...{CONCAT}{,}{n}{CONCAT}{,}{n}...{CONCAT}{,}{n}],
  --counting the first CONCATs before the comma gives the number of
  --derivatives
  ndiffs : I := 0
  tmpLE : L E := rest rest atomE
  while stringify first tmpLE = "CONCAT" repeat
    ndiffs := ndiffs+1
    tmpLE := rest tmpLE
  numLS : L S := nil
  i : I := 1
  while i < ndiffs repeat
    numLS := append(numLS,list(stringify first rest tmpLE))
    tmpLE := rest rest rest tmpLE
    i := i+1
  numLS := append(numLS,list(stringify first rest tmpLE))
  -- numLS contains the numbers of the bound variables as strings
  -- for the differentiations, thus for the differentiation [x,x,z,x]
  -- for y(x,z) numLS = ["1","1","2","1"]
  posLS : L S := nil
```

```
      i := 0
 --     sayTeX$Lisp "formatSub: nargs = "string(#args)
      while i < #args repeat
        posLS := append(posLS,list(string(i+1)))
        i := i+1
      -- posLS contains the positions of the bound variables in args
      -- as a list of strings, e.g. for the above example ["1","2"]
      funcS: S := stringify atomE.2
      s : S := "<msub><mi>"funcS"</mi><mrow>"
      i := 1
      while i < #numLS+1 repeat
        s := s"<mo>,</mo><mn>"numLS.i"</mn>"
        i := i + 1
      s := s"</mrow></msub><mo>(</mo>"
      i := 1
      while i < #posLS+1 repeat
--        tmpS := stringify args.i
        tmpS := formatMml(first args,minPrec)
        args := rest args
        s := s"<mi>"tmpS"</mi>"
        if i < #posLS then s := s"<mo>,</mo>"
        i := i+1
      s := s"<mo>)</mo>"

    formatSuperSub(expr : E, args : L E, opPrec : I) : S ==
      -- this produces prime notation ordinary derivatives.
      -- first have to divine the semantics, add cases as needed
--      WriteLine$Lisp "SuperSub1 begin"
      atomE : L E := atomize(expr)
      op : S := stringify first atomE
--      WriteLine$Lisp "op: "op
      op ^= "SUPERSUB" => _
          "<mtext>Mistake in formatSuperSub: no SUPERSUB1</mtext>"
      #args ^= 1 => "<mtext>Mistake in SuperSub1: #args <> 1</mtext>"
      var : E := first args
      -- should be looking at something like {{SUPERSUB}{var}{ }{,,...,}} for
      -- example here's the second derivative of y w.r.t. x
      -- {{{SUPERSUB}{y}{ }{,,}}{x}}, expr is the first {} and args is the
      -- {x}
      funcS : S := stringify first rest atomE
--      WriteLine$Lisp "funcS: "funcS
      bvarS : S := stringify first args
--      WriteLine$Lisp "bvarS: "bvarS
      -- count the number of commas
      commaS : S := stringify first rest rest rest atomE
      commaTest : S := ","
      i : I := 0
      while position(commaTest,commaS,1) > 0 repeat
        i := i+1
        commaTest := commaTest","
```

```
      s : S := "<msup><mi>"funcS"</mi><mrow>"
--      WriteLine$Lisp "s: "s
      j : I := 0
      while j < i repeat
        s := s"<mo>&#x02032;</mo>"
        j := j + 1
      s := s"</mrow></msup><mo>&#x02061;</mo><mo>(</mo>"_
            formatMml(first args,minPrec)"<mo>)</mo>"

    formatSuperSub1(expr : E, args : L E, opPrec : I) : S ==
      -- This one produces ordinary derivatives with differential notation,
      -- it needs a little more work yet.
      -- first have to divine the semantics, add cases as needed
--      WriteLine$Lisp "SuperSub begin"
      atomE : L E := atomize(expr)
      op : S := stringify first atomE
      op ^= "SUPERSUB" => _
         "<mtext>Mistake in formatSuperSub: no SUPERSUB</mtext>"
      #args ^= 1 => "<mtext>Mistake in SuperSub: #args <> 1</mtext>"
      var : E := first args
      -- should be looking at something like {{SUPERSUB}{var}{ }{,,...,}} for
      -- example here's the second derivative of y w.r.t. x
      -- {{{SUPERSUB}{y}{ }{,,}}{x}}, expr is the first {} and args is the
      -- {x}
      funcS : S := stringify first rest atomE
      bvarS : S := stringify first args
      -- count the number of commas
      commaS : S := stringify first rest rest rest atomE
      commaTest : S := ","
      ndiffs : I := 0
      while position(commaTest,commaS,1) > 0 repeat
        ndiffs := ndiffs+1
        commaTest := commaTest","
      s : S := "<mfrac><mrow><msup><mo>&#x02146;</mo><mn>"string(ndiffs)_
       "</mn></msup><mi>"funcS"</mi></mrow><mrow><mo>&#x02146;</mo><msup><mi>"_
        formatMml(first args,minPrec)"</mi><mn>"string(ndiffs)_
         "</mn></msup></mrow></mfrac><mo>&#x02061;</mo><mo>(</mo><mi>"_
          formatMml(first args,minPrec)"</mi><mo>)</mo>"

    formatPlex(op : S, args : L E, prec : I) : S ==
      checkarg:Boolean := false
      hold : S
      p : I := position(op,plexOps)
      p < 1 => error "unknown plex op"
      op = "INTSIGN" => formatIntSign(args,minPrec)
      opPrec := plexPrecs.p
      n : I := #args
      (n ^= 2) and (n ^= 3) => error "wrong number of arguments for plex"
      s : S :=
        op = "SIGMA"   =>
```

```
         checkarg := true
         "<mo>&#x02211;</mo>"
        -- Sum
        op = "SIGMA2"   =>
         checkarg := true
         "<mo>&#x02211;</mo>"
        -- Sum
        op = "PI"      =>
         checkarg := true
         "<mo>&#x0220F;</mo>"
        -- Product
        op = "PI2"     =>
         checkarg := true
         "<mo>&#x0220F;</mo>"
        -- Product
--        op = "INTSIGN" => "<mo>&#x0222B;</mo>"
        -- Integral, int
        op = "INDEFINTEGRAL" => "<mo>&#x0222B;</mo>"
        -- Integral, int
        "????"
      hold := formatMml(first args,minPrec)
      args := rest args
      if op ^= "INDEFINTEGRAL" then
        if hold ^= "" then
          s := concat ["<munderover>",s,group hold]
        else
          s := concat ["<munderover>",s,group " "]
        if not null rest args then
          hold := formatMml(first args,minPrec)
          if hold ^= "" then
            s := concat [s,group hold,"</munderover>"]
          else
            s := concat [s,group " ","</munderover>"]
          args := rest args
        -- if checkarg true need to test op arg for "+" at least
        -- and wrap parentheses if so
        if checkarg then
          la : L E := (first args pretend L E)
          opa : S := stringify first la
          if opa = "+" then
            s :=
             concat [s,"<mo>(</mo>",formatMml(first args,minPrec),"<mo>)</mo>"]
          else s := concat [s,formatMml(first args,minPrec)]
        else s := concat [s,formatMml(first args,minPrec)]
      else
        hold := group concat [hold,formatMml(first args,minPrec)]
        s := concat [s,hold]
--      if opPrec < prec then s := parenthesize s
-- getting ugly parentheses on fractions
      group s
```

```
formatIntSign(args : L E, opPrec : I) : S ==
  -- the original OutputForm expression looks something like this:
  -- {{INTSIGN}{NOTHING or lower limit?}
  -- {bvar or upper limit?}{{*}{integrand}{{CONCAT}{d}{axiom var}}}}
  -- the args list passed here consists of the rest of this list, i.e.
  -- starting at the NOTHING or ...
  (stringify first args) = "NOTHING" =>
    -- the bound variable is the second one in the argument list
    bvar : E := first rest args
    bvarS : S := stringify bvar
    tmpS : S
    i : I := 0
    u1 : US
    u2 : US
    -- this next one atomizes the integrand plus differential
    atomE : L E := atomize(first rest rest args)
    -- pick out the bound variable used by axiom
    varRS : S := stringify last(atomE)
    tmpLE : L E := ((first rest rest args) pretend L E)
    integrand : S := formatMml(first rest tmpLE,minPrec)
    -- replace the bound variable, i.e. axiom uses someting of the form
    -- %A for the bound variable and puts the original variable used
    -- in the input command as a superscript on the integral sign.
    -- I'm assuming that the axiom variable is 2 characters.
    while (i := position(varRS,integrand,i+1)) > 0 repeat
      u1 := segment(1,i-1)$US
      u2 := segment(i+2,#integrand)$US
      integrand := concat [integrand.u1,bvarS,integrand.u2]
    concat ["<mrow><mo>&#x0222B;</mo>" integrand _
            "<mo>&#x02146;</mo><mi>" bvarS "</mi></mrow>"]

  lowlim : S := stringify first args
  highlim : S := stringify first rest args
  bvar : E := last atomize(first rest rest args)
  bvarS : S := stringify bvar
  tmpLE : L E := ((first rest rest args) pretend L E)
  integrand : S := formatMml(first rest tmpLE,minPrec)
  concat ["<mrow><munderover><mo>&#x0222B;</mo><mi>" lowlim "</mi><mi>" highlim "</mi></


formatMatrix(args : L E) : S ==
  -- format for args is [[ROW ...],[ROW ...],[ROW ...]]
  -- generate string for formatting columns (centered)
  group addBrackets concat
    ["<mtable><mtr><mtd>",formatNaryNoGroup("</mtd></mtr><mtr><mtd>",args,minPrec),
      "</mtd></mtr></mtable>"]

formatFunction(op : S, args : L E, prec : I) : S ==
  group concat ["<mo>",op,"</mo>",parenthesize formatNary(",",args,minPrec)]
```

```
    formatNullary(op : S) ==
      op = "NOTHING" => ""
      group concat ["<mo>",op,"</mo><mo>(</mo><mo>)</mo>"]

    formatUnary(op : S, arg : E, prec : I) ==
      p : I := position(op,unaryOps)
      p < 1 => error "unknown unary op"
      opPrec := unaryPrecs.p
      s : S := concat ["<mo>",op,"</mo>",formatMml(arg,opPrec)]
      opPrec < prec => group parenthesize s
      op = "-" => s
      group s

    formatBinary(op : S, args : L E, prec : I) : S ==
      p : I := position(op,binaryOps)
      p < 1 => error "unknown binary op"
      opPrec := binaryPrecs.p
      -- if base op is product or sum need to add parentheses
      if ATOM(first args)$Lisp@Boolean then
        opa:S := stringify first args
      else
        la : L E := (first args pretend L E)
        opa : S := stringify first la
      if (opa = "SIGMA" or opa = "SIGMA2" or opa = "PI" or opa = "PI2") _
         and op = "**" then
        s1:S:=concat ["<mo>(</mo>",formatMml(first args, opPrec),"<mo>)</mo>"]
      else
       s1 : S := formatMml(first args, opPrec)
      s2 : S := formatMml(first rest args, opPrec)
      op :=
        op = "|"     =>  s := concat ["<mrow>",s1,"</mrow><mo>",op,"</mo><mrow>",s2,"</mrow>"]
        op = "**"    =>  s := concat ["<msup><mrow>",s1,"</mrow><mrow>",s2,"</mrow></msup>"]
        op = "/"     =>  s := concat ["<mfrac><mrow>",s1,"</mrow><mrow>",s2,"</mrow></mfrac>"]
        op = "OVER"  =>  s := concat ["<mfrac><mrow>",s1,"</mrow><mrow>",s2,"</mrow></mfrac>"]
        op = "+->"   =>  s := concat ["<mrow>",s1,"</mrow><mo>",op,"</mo><mrow>",s2,"</mrow>"]
        s := concat ["<mrow>",s1,"</mrow><mo>",op,"</mo><mrow>",s2,"</mrow>"]
      group
        op = "OVER" => s
--        opPrec < prec => parenthesize s
-- ugly parentheses?
        s

    formatNary(op : S, args : L E, prec : I) : S ==
      group formatNaryNoGroup(op, args, prec)

    formatNaryNoGroup(op : S, args : L E, prec : I) : S ==
      checkargs:Boolean := false
      null args => ""
      p : I := position(op,naryOps)
```

```
        p < 1 => error "unknown nary op"
        -- need to test for "ZAG" case and divert it here
        -- ex 1. continuedFraction(314159/100000)
        -- {{+}{3}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}
        -- {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
        -- this is the preconditioned output form
        -- including "op", the args list would be the rest of this
        -- i.e op = '+' and args = {{3}{{ZAG}{1}{7}}{{ZAG}{1}{15}}
        -- {{ZAG}{1}{1}}{{ZAG}{1}{25}}{{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
        -- ex 2. continuedFraction(14159/100000)
        -- this one doesn't have the leading integer
        -- {{+}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}
        -- {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
        --
        -- ex 3. continuedFraction(3,repeating [1], repeating [3,6])
        -- {{+}{3}{{ZAG}{1}{3}}{{ZAG}{1}{6}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}
        -- {{ZAG}{1}{3}}{{ZAG}{1}{6}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}
        -- {{ZAG}{1}{3}}{{ZAG}{1}{6}}{...}}
        -- In each of these examples the args list consists of the terms
        -- following the '+' op
        -- so the first arg could be a "ZAG" or something
        -- else, but the second arg looks like it has to be "ZAG", so maybe
        -- test for #args > 1 and args.2 contains "ZAG".
        -- Note that since the resulting MathML <mfrac>s are nested we need
        -- to handle the whole continued fraction at once, i.e. we can't
        -- just look for, e.g., {{ZAG}{1}{6}}
        (#args > 1) and (position("ZAG",stringify first rest args,1) > 0) =>
             tmpS : S := stringify first args
             position("ZAG",tmpS,1) > 0 => formatZag(args)
--           position("ZAG",tmpS,1) > 0 => formatZag1(args)
             concat [formatMml(first args,minPrec) "<mo>+</mo>" _
                     formatZag(rest args)]
        -- At least for the ops "*","+","-" we need to test to see if a sigma
        -- or pi is one of their arguments because we might need parentheses
        -- as indicated by the problem with
        -- summation(operator(f)(i),i=1..n)+1 versus
        -- summation(operator(f)(i)+1,i=1..n) having identical displays as
        -- of 2007-21-21
        op :=
          op = ","     => "<mo>,</mo>" --originally , \:
          op = ";"     => "<mo>;</mo>" --originally ; \: should figure these out
          op = "*"     => "<mspace width='0.3em'/>"
          -- InvisibleTimes
          op = " "     => "<mspace width='0.5em'/>"
          op = "ROW"   => "</mtd><mtd>"
          op = "+"     =>
              checkargs := true
              "<mo>+</mo>"
          op = "-"     =>
              checkargs := true
```

```
          "<mo>-</mo>"
        op
      l : L S := nil
      opPrec := naryPrecs.p
      -- if checkargs is true check each arg except last one to see if it's
      -- a sigma or pi and if so add parentheses. Other op's may have to be
      -- checked for in future
      count:I := 1
      for a in args repeat
--          WriteLine$Lisp "checking args"
        if checkargs then
          if count < #args then
            -- check here for sum or product
            if ATOM(a)$Lisp@Boolean then
              opa:S := stringify a
            else
              la : L E := (a pretend L E)
              opa : S := stringify first la
            if opa = "SIGMA" or opa = "SIGMA2" or _
               opa = "PI" or opa = "PI2" then
              l := concat(op,concat(_
                    concat ["<mo>(</mo>",formatMml(a,opPrec),_
                            "<mo>)</mo>"],l)$L(S))$L(S)
            else l := concat(op,concat(formatMml(a,opPrec),l)$L(S))$L(S)
          else l := concat(op,concat(formatMml(a,opPrec),l)$L(S))$L(S)
        else l := concat(op,concat(formatMml(a,opPrec),l)$L(S))$L(S)
        count := count + 1
      s : S := concat reverse rest l
      opPrec < prec => parenthesize s
      s

    formatZag(args : L E) : S ==
    -- args will be a list of things like this {{ZAG}{1}{7}}, the ZAG
    -- must be there, the '1' and '7' could conceivably be more complex
    -- expressions
      tmpZag : L E := first args pretend L E
      -- may want to test that tmpZag contains 'ZAG'
      #args > 1 => "<mfrac>"formatMml(first rest tmpZag,minPrec)"<mrow><mn>"formatMml(first rest rest tm
      -- EQUAL(tmpZag, "...")$Lisp => "<mo>&#x2026;</mo>"
      (first args = "..."::E)@Boolean => "<mo>&#x2026;</mo>"
      position("ZAG",stringify first args,1) > 0 =>
          "<mfrac>"formatMml(first rest tmpZag,minPrec)formatMml(first rest rest tmpZag,minPrec)"</mfrac
      "<mtext>formatZag: Unexpected kind of ZAG</mtext>"


    formatZag1(args : L E) : S ==
    -- make alternative ZAG format without diminishing fonts, maybe
    -- use a table
    -- {{ZAG}{1}{7}}
      tmpZag : L E := first args pretend L E
```

```
  #args > 1 => "<mfrac>"formatMml(first rest tmpZag,minPrec)"<mrow><mn>"formatMml(first r
  (first args = "...":: E)@Boolean => "<mo>&#x2026;</mo>"
  error "formatZag1: Unexpected kind of ZAG"


formatMml(expr : E,prec : I) ==
  i,len : Integer
  intSplitLen : Integer := 20
  ATOM(expr)$Lisp@Boolean =>
    str := stringify expr
    len := #str
    -- this bit seems to deal with integers
    INTEGERP$Lisp expr =>
      i := expr pretend Integer
      if (i < 0) or (i > 9)
        then
          group
             nstr : String := ""
             -- insert some blanks into the string, if too long
             while ((len := #str) > intSplitLen) repeat
               nstr := concat [nstr," ",
                 elt(str,segment(1,intSplitLen)$US)]
               str := elt(str,segment(intSplitLen+1)$US)
             empty? nstr => concat ["<mn>",str,"</mn>"]
             nstr :=
               empty? str => nstr
               concat [nstr," ",str]
             concat ["<mn>",elt(nstr,segment(2)$US),"</mn>"]
        else str := concat ["<mn>",str,"</mn>"]
    str = "%pi" => "<mi>&#x003C0;</mi>"
    -- pi
    str = "%e"  => "<mi>&#x02147;</mi>"
    -- ExponentialE
    str = "%i"  => "<mi>&#x02148;</mi>"
    -- ImaginaryI
    len > 0 and str.1 = char "%" => concat(concat("<mi>",str),"</mi>")
     -- should handle floats
    len > 1 and digit? str.1 => concat ["<mn>",str,"</mn>"]
    -- presumably this is a literal string
    len > 0 and str.1 = char "_"" =>
      concat(concat("<mtext>",str),"</mtext>")
    len = 1 and str.1 = char " " => " "
    (i := position(str,specialStrings)) > 0 =>
      specialStringsInMML.i
    (i := position(char " ",str)) > 0 =>
      -- We want to preserve spacing, so use a roman font.
      -- What's this for?  Leave the \rm in for now so I can see
      -- where it arises.  Removed 2007-02-14
      concat(concat("<mtext>",str),"</mtext>")
    -- if we get to here does that mean it's a variable?
```

```
    concat ["<mi>",str,"</mi>"]
  l : L E := (expr pretend L E)
  null l => blank
  op : S := stringify first l
  args : L E := rest l
  nargs : I := #args
  -- need to test here in case first l is SUPERSUB case and then
  -- pass first l and args to formatSuperSub.
  position("SUPERSUB",op,1) > 0 =>
    formatSuperSub(first l,args,minPrec)
  -- now test for SUB
  position("SUB",op,1) > 0 =>
    formatSub1(first l,args,minPrec)

  -- special cases
  member?(op, specialOps) => formatSpecial(op,args,prec)
  member?(op, plexOps)    => formatPlex(op,args,prec)

  -- nullary case
  0 = nargs => formatNullary op

  -- unary case
  (1 = nargs) and member?(op, unaryOps) =>
    formatUnary(op, first args, prec)

  -- binary case
  (2 = nargs) and member?(op, binaryOps) =>
    formatBinary(op, args, prec)

  -- nary case
  member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
  member?(op,naryOps) => formatNary(op,args, prec)

  op := formatMml(first l,minPrec)
  formatFunction(op,args,prec)
```

———-

14.6.17 Mathematical Markup Language Form

— MathMLFormat.input —

```
)set break resume
)spool MathMLFormat.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 5
)set output mathml on

--R
--E 1

--S 2 of 5
1/2
--R
--R
--R        1
--R   (1)  -
--R        2
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mfrac><mrow><mn>1</mn></mrow><mrow><mn>2</mn></mrow></mfrac></mrow>
--R</math>
--R
--R                                                        Type: Fraction Integer
--E 2

--S 3 of 5
1/(x+5)
--R
--R
--R          1
--R   (2)  -----
--R        x + 5
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mfrac><mrow><mn>1</mn></mrow><mrow><mrow><mi>x</mi><mo>+</mo><mn>5</mn></mrow></mro
--R</math>
--R
--R                                             Type: Fraction Polynomial Integer
--E 3

--S 4 of 5
(x+3)/(y-5)
--R
--R
--R        x + 3
--R   (3)  -----
--R        y - 5
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mfrac><mrow><mrow><mi>x</mi><mo>+</mo><mn>3</mn></mrow></mrow><mrow><mrow><mi>y</mi
--R</math>
--R
--R                                             Type: Fraction Polynomial Integer
--E 4

--S 5 of 5
```

```
)show MathMLFormat
--R MathMLFormat  is a domain constructor
--R Abbreviation for MathMLFormat is MMLFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MMLFORM
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : OutputForm -> String
--R coerce : % -> OutputForm              coerceL : OutputForm -> String
--R coerceS : OutputForm -> String        display : String -> Void
--R exprex : OutputForm -> String         hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R
--E 5

)spool
)lisp (bye)
```

———-

— MathMLFormat.help —

```
====================================================================
MathMLFormat examples
====================================================================

MathML is an HTML-like markup language for mathematics. It uses the
"knuckle" syntax of HTML such as "<mo>" to introduce a math operator
and "</mo>" to mark the end of the operator. Axiom can generate
MathML output and does so when it communicates to the browser front end.

This output is enabled by

  )set output mathml on

after which you'll see the MathML markup as well as the algebra.
Note that you can turn off the algebra output with

  )set output algebra off

but we don't do that here so you can compare the output.

  1/2

    1/2

  <math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big"
        display="block">
```

```
 <mrow>
  <mn>1</mn>
  <mo>/</mo>
  <mn>2</mn>
 </mrow>
</math>
```

```
1/(x+5)
```

```
  1/(x + 5)
```

```
<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big"
      display="block">
 <mrow>
  <mn>1</mn>
  <mo>/</mo>
  <mrow>
   <mo>(</mo>
   <mi>x</mi>
   <mo>+</mo>
   <mn>5</mn>
   <mo>)</mo>
  </mrow>
 </mrow>
</math>
```

```
(x+3)/(y-5)
```

```
   (x + 3)/(y - 5)
```

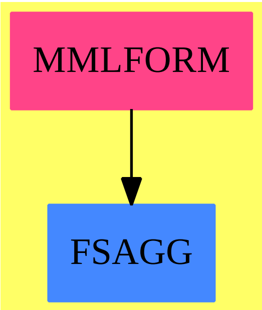
```
<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big"
      display="block">
 <mrow>
  <mrow>
   <mo>(</mo>
   <mi>x</mi>
   <mo>+</mo>
   <mn>3</mn>
   <mo>)</mo>
  </mrow>
  <mo>/</mo>
  <mrow>
   <mo>(</mo>
   <mi>y</mi>
   <mo>-</mo>
   <mn>5</mn>
   <mo>)</mo>
  </mrow>
 </mrow>
```

```
  R</math>

See Also:
o )show MathMLFormat
```

———-

14.6.18 MathMLForm (MMLFORM)



Exports:

coerce	coerceL	coerceS	display	exprex
hash	latex	?=?	?~=?	

— **domain MMLFORM MathMLFormat** —

```
\getchunk{public declarations}
\getchunk{private constant declarations}
\getchunk{private function declarations}
\getchunk{public function definitions}
\getchunk{display functions}
\getchunk{formatting functions}
```

———-

— **MMLFORM.dotabb** —

```
"MMLFORM" [color="#FF4488",href="bookvol10.4.pdf#nameddest=MMLFORM"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"MMLFORM" -> "FSAGG"
```

———-

14.7 domain MATRIX Matrix

— Matrix.input —

```
)set break resume
)sys rm -f Matrix.output
)spool Matrix.output
)set message test on
)set message auto off
)clear all
--S 1 of 38
m : Matrix(Integer) := new(3,3,0)
--R
--R
--R        +0  0  0+
--R        |       |
--R   (1)  |0  0  0|
--R        |       |
--R        +0  0  0+
--R                                                    Type: Matrix Integer
--E 1

--S 2 of 38
setelt(m,2,3,5)
--R
--R
--R   (2)  5
--R                                                    Type: PositiveInteger
--E 2

--S 3 of 38
m(1,2) := 10
--R
--R
--R   (3)  10
--R                                                    Type: PositiveInteger
--E 3

--S 4 of 38
m
--R
--R
--R        +0  10  0+
--R        |        |
--R   (4)  |0  0   5|
--R        |        |
--R        +0  0   0+
--R                                                    Type: Matrix Integer
```

```
--E 4

--S 5 of 38
matrix [ [1,2,3,4],[0,9,8,7] ]
--R
--R
--R        +1  2  3  4+
--R   (5)  |          |
--R        +0  9  8  7+
--R                                                     Type: Matrix Integer
--E 5

--S 6 of 38
dm := diagonalMatrix [1,x**2,x**3,x**4,x**5]
--R
--R
--R        +1  0   0   0   0 +
--R        |                 |
--R        |    2            |
--R        |0  x   0   0   0 |
--R        |                 |
--R        |        3        |
--R   (6)  |0  0   x   0   0 |
--R        |                 |
--R        |            4    |
--R        |0  0   0   x   0 |
--R        |                 |
--R        |                5|
--R        +0  0   0   0   x +
--R                                          Type: Matrix Polynomial Integer
--E 6

--S 7 of 38
setRow!(dm,5,vector [1,1,1,1,1])
--R
--R
--R        +1  0   0   0   0+
--R        |                |
--R        |    2           |
--R        |0  x   0   0   0|
--R        |                |
--R   (7)  |        3       |
--R        |0  0   x   0   0|
--R        |                |
--R        |            4   |
--R        |0  0   0   x   0|
--R        |                |
--R        +1  1   1   1   1+
--R                                          Type: Matrix Polynomial Integer
--E 7
```

```
--S 8 of 38
setColumn!(dm,2,vector [y,y,y,y,y])
--R
--R
--R        +1  y  0   0   0+
--R        |               |
--R        |0  y  0   0   0|
--R        |               |
--R        |       3       |
--R   (8)  |0  y  x   0   0|
--R        |               |
--R        |           4   |
--R        |0  y  0   x   0|
--R        |               |
--R        +1  y  1   1   1+
--R                                          Type: Matrix Polynomial Integer
--E 8

--S 9 of 38
cdm := copy(dm)
--R
--R
--R        +1  y  0   0   0+
--R        |               |
--R        |0  y  0   0   0|
--R        |               |
--R        |       3       |
--R   (9)  |0  y  x   0   0|
--R        |               |
--R        |           4   |
--R        |0  y  0   x   0|
--R        |               |
--R        +1  y  1   1   1+
--R                                          Type: Matrix Polynomial Integer
--E 9

--S 10 of 38
setelt(dm,4,1,1-x**7)
--R
--R
--R             7
--R   (10)  - x  + 1
--R                                                 Type: Polynomial Integer
--E 10

--S 11 of 38
[dm,cdm]
--R
--R
```

```
--R          +   1      y  0   0   0+ +1  y  0   0   0+
--R          |                     | |                |
--R          |   0      y  0   0   0| |0  y  0   0   0|
--R          |                     | |                |
--R          |             3       | |       3        |
--R   (11)  [|   0      y  x   0   0|,|0  y  x   0   0|]
--R          |                     | |                |
--R          |   7               4   | |          4   |
--R          |- x  + 1  y  0   x   0| |0  y  0   x   0|
--R          |                     | |                |
--R          +   1      y  1   1   1+ +1  y  1   1   1+
--R                                  Type: List Matrix Polynomial Integer
--E 11

--S 12 of 38
subMatrix(dm,2,3,2,4)
--R
--R
--R         +y  0   0+
--R   (12)  |        |
--R         |    3   |
--R         +y  x   0+
--R                                       Type: Matrix Polynomial Integer
--E 12

--S 13 of 38
d := diagonalMatrix [1.2,-1.3,1.4,-1.5]
--R
--R
--R         +1.2   0.0   0.0   0.0 +
--R         |                      |
--R         |0.0  - 1.3  0.0   0.0 |
--R   (13)  |                      |
--R         |0.0   0.0   1.4   0.0 |
--R         |                      |
--R         +0.0   0.0   0.0  - 1.5+
--R                                                    Type: Matrix Float
--E 13

--S 14 of 38
e := matrix [ [6.7,9.11],[-31.33,67.19] ]
--R
--R
--R         +  6.7    9.11 +
--R   (14)  |              |
--R         +- 31.33  67.19+
--R                                                    Type: Matrix Float
--E 14

--S 15 of 38
```

```
setsubMatrix!(d,1,2,e)
--R
--R
--R         +1.2    6.7    9.11    0.0 +
--R         |                          |
--R         |0.0  - 31.33  67.19   0.0 |
--R   (15)  |                          |
--R         |0.0    0.0     1.4    0.0 |
--R         |                          |
--R         +0.0    0.0     0.0   - 1.5+
--R                                                    Type: Matrix Float
--E 15

--S 16 of 38
d
--R
--R
--R         +1.2    6.7    9.11    0.0 +
--R         |                          |
--R         |0.0  - 31.33  67.19   0.0 |
--R   (16)  |                          |
--R         |0.0    0.0     1.4    0.0 |
--R         |                          |
--R         +0.0    0.0     0.0   - 1.5+
--R                                                    Type: Matrix Float
--E 16

--S 17 of 38
a := matrix [ [1/2,1/3,1/4],[1/5,1/6,1/7] ]
--R
--R
--R         +1  1  1+
--R         |-  -  -|
--R         |2  3  4|
--R   (17)  |       |
--R         |1  1  1|
--R         |-  -  -|
--R         +5  6  7+
--R                                           Type: Matrix Fraction Integer
--E 17

--S 18 of 38
b := matrix [ [3/5,3/7,3/11],[3/13,3/17,3/19] ]
--R
--R
--R         +3   3    3+
--R         |-   -   --|
--R         |5   7   11|
--R   (18)  |          |
--R         | 3   3   3|
```

```
--R         |--  --  --|
--R         +13  17  19+
--R                                                  Type: Matrix Fraction Integer
--E 18

--S 19 of 38
horizConcat(a,b)
--R
--R
--R         +1  1  1  3   3    3+
--R         |-  -  -  -   -   --|
--R         |2  3  4  5   7   11|
--R   (19)  |                   |
--R         |1  1  1   3   3   3|
--R         |-  -  -  --  --  --|
--R         +5  6  7  13  17  19+
--R                                                  Type: Matrix Fraction Integer
--E 19

--S 20 of 38
vab := vertConcat(a,b)
--R
--R
--R         +1   1   1 +
--R         |-   -   - |
--R         |2   3   4 |
--R         |          |
--R         |1   1   1 |
--R         |-   -   - |
--R         |5   6   7 |
--R   (20)  |          |
--R         |3   3    3|
--R         |-   -   --|
--R         |5   7   11|
--R         |          |
--R         | 3   3   3|
--R         |--  --  --|
--R         +13  17  19+
--R                                                  Type: Matrix Fraction Integer
--E 20

--S 21 of 38
transpose vab
--R
--R
--R         +1  1  3    3+
--R         |-  -  -   --|
--R         |2  5  5   13|
--R         |            |
--R         |1  1  3    3|
```

```
--R   (21)  |-  -  -   --|
--R         |3  6  7   17|
--R         |            |
--R         |1  1   3   3|
--R         |-  -  --  --|
--R         +4  7  11  19+
--R                                          Type: Matrix Fraction Integer
--E 21

--S 22 of 38
m := matrix [ [1,2],[3,4] ]
--R
--R
--R         +1  2+
--R   (22)  |    |
--R         +3  4+
--R                                                    Type: Matrix Integer
--E 22

--S 23 of 38
4 * m * (-5)
--R
--R
--R         +- 20  - 40+
--R   (23)  |          |
--R         +- 60  - 80+
--R                                                    Type: Matrix Integer
--E 23

--S 24 of 38
n := matrix([ [1,0,-2],[-3,5,1] ])
--R
--R
--R         + 1   0  - 2+
--R   (24)  |           |
--R         +- 3  5   1 +
--R                                                    Type: Matrix Integer
--E 24

--S 25 of 38
m * n
--R
--R
--R         +- 5  10   0 +
--R   (25)  |            |
--R         +- 9  20  - 2+
--R                                                    Type: Matrix Integer
--E 25

--S 26 of 38
```

```
vec := column(n,3)
--R
--R
--R   (26)  [- 2,1]
--R                                                   Type: Vector Integer
--E 26

--S 27 of 38
vec * m
--R
--R
--R   (27)  [1,0]
--R                                                   Type: Vector Integer
--E 27

--S 28 of 38
m * vec
--R
--R
--R   (28)  [0,- 2]
--R                                                   Type: Vector Integer
--E 28

--S 29 of 38
hilb := matrix([ [1/(i + j) for i in 1..3] for j in 1..3])
--R
--R
--R         +1  1  1+
--R         |-  -  -|
--R         |2  3  4|
--R         |       |
--R         |1  1  1|
--R   (29)  |-  -  -|
--R         |3  4  5|
--R         |       |
--R         |1  1  1|
--R         |-  -  -|
--R         +4  5  6+
--R                                          Type: Matrix Fraction Integer
--E 29

--S 30 of 38
inverse(hilb)
--R
--R
--R         + 72    - 240   180 +
--R         |                   |
--R   (30)  |- 240   900   - 720|
--R         |                   |
--R         + 180   - 720   600 +
```

```
--R                                    Type: Union(Matrix Fraction Integer,...)
--E 30

--S 31 of 38
mm := matrix([ [1,2,3,4], [5,6,7,8], [9,10,11,12], [13,14,15,16] ])
--R
--R
--R          +1   2   3   4 +
--R          |              |
--R          |5   6   7   8 |
--R   (31)   |              |
--R          |9   10  11  12|
--R          |              |
--R          +13  14  15  16+
--R                                                    Type: Matrix Integer
--E 31

--S 32 of 38
inverse(mm)
--R
--R
--R   (32)  "failed"
--R                                                 Type: Union("failed",...)
--E 32

--S 33 of 38
determinant(mm)
--R
--R
--R   (33)  0
--R                                                 Type: NonNegativeInteger
--E 33

--S 34 of 38
trace(mm)
--R
--R
--R   (34)  34
--R                                                    Type: PositiveInteger
--E 34

--S 35 of 38
rank(mm)
--R
--R
--R   (35)  2
--R                                                    Type: PositiveInteger
--E 35

--S 36 of 38
```

```
nullity(mm)
--R
--R
--R   (36)  2
--R                                                    Type: PositiveInteger
--E 36

--S 37 of 38
nullSpace(mm)
--R
--R
--R   (37)  [[1,- 2,1,0],[2,- 3,0,1]]
--R                                                 Type: List Vector Integer
--E 37

--S 38 of 38
rowEchelon(mm)
--R
--R
--R         +1  2  3  4 +
--R         |           |
--R         |0  4  8  12|
--R   (38)  |           |
--R         |0  0  0  0 |
--R         |           |
--R         +0  0  0  0 +
--R                                                     Type: Matrix Integer
--E 38
)spool
)lisp (bye)
```

— Matrix.help —

```
====================================================================
Matrix examples
====================================================================

The Matrix domain provides arithmetic operations on matrices
and standard functions from linear algebra.
This domain is similar to the TwoDimensionalArray domain, except
that the entries for Matrix must belong to a  Ring.

====================================================================
Creating Matrices
====================================================================

There are many ways to create a matrix from a collection of values or
```

from existing matrices.

If the matrix has almost all items equal to the same value, use new to create a matrix filled with that value and then reset the entries that are different.

```
m : Matrix(Integer) := new(3,3,0)
  +0  0  0+
  |       |
  |0  0  0|
  |       |
  +0  0  0+
                  Type: Matrix Integer
```

To change the entry in the second row, third column to 5, use setelt.

```
setelt(m,2,3,5)
  5
                  Type: PositiveInteger
```

An alternative syntax is to use assignment.

```
m(1,2) := 10
  10
                  Type: PositiveInteger
```

The matrix was destructively modified.

```
m
  +0  10  0+
  |        |
  |0  0   5|
  |        |
  +0  0   0+
                  Type: Matrix Integer
```

If you already have the matrix entries as a list of lists, use matrix.

```
matrix [ [1,2,3,4],[0,9,8,7] ]
  +1  2  3  4+
  |          |
  +0  9  8  7+
                  Type: Matrix Integer
```

If the matrix is diagonal, use diagonalMatrix.

```
dm := diagonalMatrix [1,x**2,x**3,x**4,x**5]
      +1  0   0   0   0 +
      |                 |
      |    2            |
```

```
        |0  x   0   0   0 |
        |                 |
        |        3        |
        |0  0   x   0   0 |
        |                 |
        |            4    |
        |0  0   0   x   0 |
        |                 |
        |                5|
        +0  0   0   0   x +
                    Type: Matrix Polynomial Integer
```

Use setRow and setColumn to change a row or column of a matrix.

```
  setRow!(dm,5,vector [1,1,1,1,1])
        +1  0   0   0   0+
        |                |
        |    2           |
        |0  x   0   0   0|
        |                |
        |        3       |
        |0  0   x   0   0|
        |                |
        |            4   |
        |0  0   0   x   0|
        |                |
        +1  1   1   1   1+
                   Type: Matrix Polynomial Integer

  setColumn!(dm,2,vector [y,y,y,y,y])
        +1  y  0   0   0+
        |               |
        |0  y  0   0   0|
        |               |
        |       3       |
        |0  y  x   0   0|
        |               |
        |           4   |
        |0  y  0   x   0|
        |               |
        +1  y  1   1   1+
                   Type: Matrix Polynomial Integer
```

Use copy to make a copy of a matrix.

```
  cdm := copy(dm)
        +1  y  0   0   0+
        |               |
        |0  y  0   0   0|
        |               |
```

```
        |       3       |
        |0  y  x   0   0|
        |               |
        |           4   |
        |0  y  0   x   0|
        |               |
        +1  y  1   1   1+
                    Type: Matrix Polynomial Integer
```

This is useful if you intend to modify a matrix destructively but want a copy of the original.

```
  setelt(dm,4,1,1-x**7)
        7
     - x  + 1
                    Type: Polynomial Integer
```

```
  [dm,cdm]
          +   1      y  0   0   0+ +1  y  0   0   0+
          |                      | |               |
          |   0      y  0   0   0| |0  y  0   0   0|
          |                      | |               |
          |             3        | |       3       |
         [|   0      y  x   0   0|,|0  y  x   0   0|]
          |                      | |               |
          |   7                 4   | |           4   |
          |- x  + 1  y  0   x   0| |0  y  0   x   0|
          |                      | |               |
          +   1      y  1   1   1+ +1  y  1   1   1+
                     Type: List Matrix Polynomial Integer
```

Use subMatrix to extract part of an existing matrix. The syntax is subMatrix(m, firstrow, lastrow, firstcol, lastcol).

```
  subMatrix(dm,2,3,2,4)
         +y  0   0+
         |        |
         |    3   |
         +y  x   0+
                     Type: Matrix Polynomial Integer
```

To change a submatrix, use setsubMatrix.

```
  d := diagonalMatrix [1.2,-1.3,1.4,-1.5]
         +1.2   0.0   0.0   0.0 +
         |                      |
         |0.0  - 1.3  0.0   0.0 |
         |                      |
         |0.0   0.0   1.4   0.0 |
         |                      |
```

```
        +0.0   0.0   0.0  - 1.5+
                    Type: Matrix Float
```

If e is too big to fit where you specify, an error message is displayed. Use subMatrix to extract part of e, if necessary.

```
  e := matrix [ [6.7,9.11],[-31.33,67.19] ]
        +  6.7    9.11 +
        |              |
        +- 31.33  67.19+
                     Type: Matrix Float
```

This changes the submatrix of d whose upper left corner is at the first row and second column and whose size is that of e.

```
  setsubMatrix!(d,1,2,e)
        +1.2    6.7    9.11    0.0 +
        |                          |
        |0.0  - 31.33  67.19   0.0 |
        |                          |
        |0.0    0.0     1.4    0.0 |
        |                          |
        +0.0    0.0     0.0   - 1.5+
                      Type: Matrix Float

  d
        +1.2    6.7    9.11    0.0 +
        |                          |
        |0.0  - 31.33  67.19   0.0 |
        |                          |
        |0.0    0.0     1.4    0.0 |
        |                          |
        +0.0    0.0     0.0   - 1.5+
                       Type: Matrix Float
```

Matrices can be joined either horizontally or vertically to make new matrices.

```
  a := matrix [ [1/2,1/3,1/4],[1/5,1/6,1/7] ]
        +1  1  1+
        |-  -  -|
        |2  3  4|
        |       |
        |1  1  1|
        |-  -  -|
        +5  6  7+
                        Type: Matrix Fraction Integer

  b := matrix [ [3/5,3/7,3/11],[3/13,3/17,3/19] ]
        +3   3    3+
```

```
       |-   -   --|
       |5   7   11|
       |          |
       | 3   3   3|
       |--  --  --|
       +13  17  19+
                          Type: Matrix Fraction Integer
```

Use horizConcat to append them side to side. The two matrices must have the same number of rows.

```
  horizConcat(a,b)
       +1  1  1  3   3    3+
       |-  -  -  -   -   --|
       |2  3  4  5   7   11|
       |                   |
       |1  1  1   3   3   3|
       |-  -  -  --  --  --|
       +5  6  7  13  17  19+
                          Type: Matrix Fraction Integer
```

Use vertConcat to stack one upon the other. The two matrices must have the same number of columns.

```
  vab := vertConcat(a,b)
       +1   1   1 +
       |-   -   - |
       |2   3   4 |
       |          |
       |1   1   1 |
       |-   -   - |
       |5   6   7 |
       |          |
       |3   3    3|
       |-   -   --|
       |5   7   11|
       |          |
       | 3   3   3|
       |--  --  --|
       +13  17  19+
                          Type: Matrix Fraction Integer
```

The operation transpose is used to create a new matrix by reflection across the main diagonal.

```
  transpose vab
       +1  1  3    3+
       |-  -  -   --|
       |2  5  5   13|
       |            |
```

```
        |1  1  3    3|
        |-  -  -   --|
        |3  6  7   17|
        |            |
        |1  1   3   3|
        |-  -  --  --|
        +4  7  11  19+
                         Type: Matrix Fraction Integer
```

```
====================================================================
Operations on Matrices
====================================================================
```

Axiom provides both left and right scalar multiplication.

```
  m := matrix [ [1,2],[3,4] ]
        +1  2+
        |    |
        +3  4+
                          Type: Matrix Integer

  4 * m * (-5)
        +- 20  - 40+
        |          |
        +- 60  - 80+
                          Type: Matrix Integer
```

You can add, subtract, and multiply matrices provided, of course, that the matrices have compatible dimensions. If not, an error message is displayed.

```
  n := matrix([ [1,0,-2],[-3,5,1] ])
        + 1   0  - 2+
        |           |
        +- 3  5   1 +
                          Type: Matrix Integer
```

This following product is defined but n * m is not.

```
  m * n
        +- 5  10   0 +
        |            |
        +- 9  20  - 2+
                          Type: Matrix Integer
```

The operations nrows and ncols return the number of rows and columns of a matrix. You can extract a row or a column of a matrix using the operations row and column. The object returned is a Vector.

Here is the third column of the matrix n.

```
  vec := column(n,3)
     [- 2,1]
                              Type: Vector Integer
```

You can multiply a matrix on the left by a "row vector" and on the right by a "column vector".

```
  vec * m
     [1,0]
                              Type: Vector Integer
```

Of course, the dimensions of the vector and the matrix must be compatible or an error message is returned.

```
  m * vec
    [0,- 2]
                              Type: Vector Integer
```

The operation inverse computes the inverse of a matrix if the matrix is invertible, and returns "failed" if not.

This Hilbert matrix is invertible.

```
  hilb := matrix([ [1/(i + j) for i in 1..3] for j in 1..3])
         +1  1  1+
         |-  -  -|
         |2  3  4|
         |       |
         |1  1  1|
         |-  -  -|
         |3  4  5|
         |       |
         |1  1  1|
         |-  -  -|
         +4  5  6+
                              Type: Matrix Fraction Integer

  inverse(hilb)
         + 72    - 240   180 +
         |                   |
         |- 240   900   - 720|
         |                   |
         + 180   - 720   600 +
                              Type: Union(Matrix Fraction Integer,...)
```

This matrix is not invertible.

```
  mm := matrix([ [1,2,3,4], [5,6,7,8], [9,10,11,12], [13,14,15,16] ])
         +1   2   3   4 +
```

```
        |              |
        |5   6   7   8 |
        |              |
        |9   10  11  12|
        |              |
        +13  14  15  16+
                           Type: Matrix Integer

  inverse(mm)
     "failed"
                           Type: Union("failed",...)
```

The operation determinant computes the determinant of a matrix provided that the entries of the matrix belong to a CommutativeRing.

The above matrix mm is not invertible and, hence, must have determinant 0.

```
  determinant(mm)
    0
                           Type: NonNegativeInteger
```

The operation trace computes the trace of a square matrix.

```
  trace(mm)
    34
                           Type: PositiveInteger
```

The operation rank computes the rank of a matrix: the maximal number of linearly independent rows or columns.

```
  rank(mm)
    2
                           Type: PositiveInteger
```

The operation nullity computes the nullity of a matrix: the dimension of its null space.

```
  nullity(mm)
    2
                           Type: PositiveInteger
```

The operation nullSpace returns a list containing a basis for the null space of a matrix. Note that the nullity is the number of elements in a basis for the null space.

```
  nullSpace(mm)
    [[1,- 2,1,0],[2,- 3,0,1]]
                           Type: List Vector Integer
```

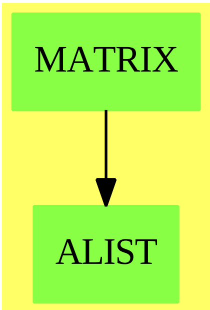
The operation rowEchelon returns the row echelon form of a matrix. It

```
is easy to see that the rank of this matrix is two and that its
nullity is also two.

  rowEchelon(mm)
         +1  2  3  4 +
         |           |
         |0  4  8  12|
         |           |
         |0  0  0  0 |
         |           |
         +0  0  0  0 +
                           Type: Matrix Integer

See Also
o )help OneDimensionalArray
o )help TwoDimensionalArray
o )help Vector
o )help Permanent
o )show Matrix
```

14.7.1 Matrix (MATRIX)



See

⇒ "IndexedMatrix" (IMATRIX) 10.12.1 on page 1204
⇒ "RectangularMatrix" (RMATRIX) 19.4.1 on page 2205
⇒ "SquareMatrix" (SQMATRIX) 20.27.1 on page 2505

Exports:

antisymmetric?	any?	coerce	column	convert
copy	count	determinant	diagonal?	diagonalMatrix
elt	empty	empty?	eq?	eval
every?	exquo	fill!	hash	horizConcat
inverse	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minordet	minRowIndex	more?
ncols	new	nrows	nullSpace	nullity
parts	qelt	qsetelt!	rank	row
rowEchelon	sample	scalarMatrix	setColumn!	setRow!
setelt	setelt	setsubMatrix!	size?	square?
squareTop	subMatrix	swapColumns!	swapRows!	symmetric?
transpose	vertConcat	zero	#?	?**?
?/?	?=?	?~=?	?*?	?+?
-?	?-?			

— domain MATRIX Matrix —

```
)abbrev domain MATRIX Matrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: IndexedMatrix, RectangularMatrix, SquareMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{Matrix} is a matrix domain where 1-based indexing is used
++ for both rows and columns.

Matrix(R): Exports == Implementation where
  R : Ring
  Row ==> Vector R
  Col ==> Vector R
  mnRow ==> 1
  mnCol ==> 1
  MATLIN ==> MatrixLinearAlgebraFunctions(R,Row,Col,$)
  MATSTOR ==> StorageEfficientMatrixOperations(R)

  Exports ==> MatrixCategory(R,Row,Col) with
    diagonalMatrix: Vector R -> $
      ++ \spad{diagonalMatrix(v)} returns a diagonal matrix where the elements
      ++ of v appear on the diagonal.

    if R has ConvertibleTo InputForm then ConvertibleTo InputForm
```

```
    if R has Field then
      inverse: $ -> Union($,"failed")
        ++ \spad{inverse(m)} returns the inverse of the matrix m.
        ++ If the matrix is not invertible, "failed" is returned.
        ++ Error: if the matrix is not square.
--     matrix: Vector Vector R -> $
--       ++ \spad{matrix(v)} converts the vector of vectors v to a matrix, where
--       ++ the vector of vectors is viewed as a vector of the rows of the
--       ++ matrix
--     diagonalMatrix: Vector $ -> $
--       ++ \spad{diagonalMatrix([m1,...,mk])} creates a block diagonal matrix
--       ++ M with block matrices m1,...,mk down the diagonal,
--       ++ with 0 block matrices elsewhere.
--     vectorOfVectors: $ -> Vector Vector R
--       ++ \spad{vectorOfVectors(m)} returns the rows of the matrix m as a
--       ++ vector of vectors

  Implementation ==>
   InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col) add
    minr ==> minRowIndex
    maxr ==> maxRowIndex
    minc ==> minColIndex
    maxc ==> maxColIndex
    mini ==> minIndex
    maxi ==> maxIndex

    minRowIndex x == mnRow
    minColIndex x == mnCol

    swapRows_!(x,i1,i2) ==
        (i1 < minRowIndex(x)) or (i1 > maxRowIndex(x)) or _
           (i2 < minRowIndex(x)) or (i2 > maxRowIndex(x)) =>
             error "swapRows!: index out of range"
        i1 = i2 => x
        minRow := minRowIndex x
        xx := x pretend PrimitiveArray(PrimitiveArray(R))
        n1 := i1 - minRow; n2 := i2 - minRow
        row1 := qelt(xx,n1)
        qsetelt_!(xx,n1,qelt(xx,n2))
        qsetelt_!(xx,n2,row1)
        xx pretend $

    positivePower:($,Integer,NonNegativeInteger) -> $
    positivePower(x,n,nn) ==
--      one? n => x
      (n = 1) => x
      -- no need to allocate space for 3 additional matrices
      n = 2 => x * x
      n = 3 => x * x * x
      n = 4 => (y := x * x; y * y)
```

```
      a := new(nn,nn,0) pretend Matrix(R)
      b := new(nn,nn,0) pretend Matrix(R)
      c := new(nn,nn,0) pretend Matrix(R)
      xx := x pretend Matrix(R)
      power_!(a,b,c,xx,n :: NonNegativeInteger)$MATSTOR pretend $

    x:$ ** n:NonNegativeInteger ==
      not((nn := nrows x) = ncols x) =>
        error "**: matrix must be square"
      zero? n => scalarMatrix(nn,1)
      positivePower(x,n,nn)

    if R has commutative("*") then

        determinant x == determinant(x)$MATLIN
        minordet    x == minordet(x)$MATLIN

    if R has EuclideanDomain then

        rowEchelon  x == rowEchelon(x)$MATLIN

    if R has IntegralDomain then

        rank        x == rank(x)$MATLIN
        nullity     x == nullity(x)$MATLIN
        nullSpace   x == nullSpace(x)$MATLIN

    if R has Field then

        inverse     x == inverse(x)$MATLIN

        x:$ ** n:Integer ==
          nn := nrows x
          not(nn = ncols x) =>
            error "**: matrix must be square"
          zero? n => scalarMatrix(nn,1)
          positive? n => positivePower(x,n,nn)
          (xInv := inverse x) case "failed" =>
            error "**: matrix must be invertible"
          positivePower(xInv :: $,-n,nn)

--     matrix(v: Vector Vector R) ==
--       (rows := # v) = 0 => new(0,0,0)
--       -- error check: this is a top level function
--       cols := # v.mini(v)
--       for k in (mini(v) + 1)..maxi(v) repeat
--         cols ^= # v.k => error "matrix: rows of different lengths"
--       ans := new(rows,cols,0)
--       for i in minr(ans)..maxr(ans) for k in mini(v)..maxi(v) repeat
--         vv := v.k
```

```
--        for j in minc(ans)..maxc(ans) for l in mini(vv)..maxi(vv) repeat
--          ans(i,j) := vv.l
--      ans

    diagonalMatrix(v: Vector R) ==
      n := #v; ans := zero(n,n)
      for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) _
          for k in mini(v)..maxi(v) repeat qsetelt_!(ans,i,j,qelt(v,k))
      ans

--     diagonalMatrix(vec: Vector $) ==
--       rows : NonNegativeInteger := 0
--       cols : NonNegativeInteger := 0
--       for r in mini(vec)..maxi(vec) repeat
--         mat := vec.r
--         rows := rows + nrows mat; cols := cols + ncols mat
--       ans := zero(rows,cols)
--       loR := minr ans; loC := minc ans
--       for r in mini(vec)..maxi(vec) repeat
--         mat := vec.r
--         hiR := loR + nrows(mat) - 1; hiC := loC + nrows(mat) - 1
--         for i in loR..hiR for k in minr(mat)..maxr(mat) repeat
--           for j in loC..hiC for l in minc(mat)..maxc(mat) repeat
--             ans(i,j) := mat(k,l)
--         loR := hiR + 1; loC := hiC + 1
--       ans

--     vectorOfVectors x ==
--       vv : Vector Vector R := new(nrows x,0)
--       cols := ncols x
--       for k in mini(vv)..maxi(vv) repeat
--         vv.k := new(cols,0)
--       for i in minr(x)..maxr(x) for k in mini(vv)..maxi(vv) repeat
--         v := vv.k
--         for j in minc(x)..maxc(x) for l in mini(v)..maxi(v) repeat
--           v.l := x(i,j)
--       vv

    if R has ConvertibleTo InputForm then
      convert(x:$):InputForm ==
         convert [convert("matrix"::Symbol)@InputForm,
                  convert listOfLists x]$List(InputForm)
```

———-

— MATRIX.dotabb —

```
"MATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MATRIX"]
```

```
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"MATRIX" -> "ALIST"
```

———-

14.8 domain MODMON ModMonic

— ModMonic.input —

```
)set break resume
)sys rm -f ModMonic.output
)spool ModMonic.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModMonic
--R ModMonic(R: Ring,Rep: UnivariatePolynomialCategory R)  is a domain constructor
--R Abbreviation for ModMonic is MODMON
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                     ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R An : % -> Vector R                   D : (%,(R -> R)) -> %
--R D : % -> %                           D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                          UnVectorise : Vector R -> %
--R Vectorise : % -> Vector R            0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coefficients : % -> List R
--R coerce : Rep -> %                    coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger     differentiate : % -> %
--R ?.? : (%,%) -> %                     ?.? : (%,R) -> R
--R eval : (%,List %,List %) -> %        eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %           eval : (%,List Equation %) -> %
--R ground : % -> R                      ground? : % -> Boolean
--R hash : % -> SingleInteger            init : () -> % if R has STEP
--R latex : % -> String                  leadingCoefficient : % -> R
--R leadingMonomial : % -> %             lift : % -> Rep
--R map : ((R -> R),%) -> %              modulus : () -> Rep
--R monomial? : % -> Boolean             monomials : % -> List %
```

```
--R one? : % -> Boolean                   pow : () -> PrimitiveArray %
--R primitiveMonomials : % -> List %      pseudoRemainder : (%,%) -> %
--R recip : % -> Union(%,"failed")        reduce : Rep -> %
--R reductum : % -> %                     retract : % -> R
--R sample : () -> %                      setPoly : Rep -> Rep
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R computePowers : () -> PrimitiveArray %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT IN
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
```

```
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R frobenius : % -> % if R has FFIELDC
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R index : PositiveInteger -> % if R has FINITE
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R lookup : % -> PositiveInteger if R has FINITE
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
```

```
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R random : () -> % if R has FINITE
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
```

```
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)
```

———-

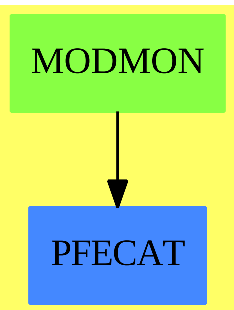
— **ModMonic.help** —

```
====================================================================
ModMonic examples
====================================================================

See Also:
o )show ModMonic
```

———-

14.8.1 ModMonic (MODMON)



Exports:

0	1	An	associates?
binomThmExpt	characteristic	charthRoot	coefficient
coefficients	coerce	composite	computePowers
conditionP	content	convert	D
degree	differentiate	discriminant	divide
divideExponents	elt	euclideanSize	eval
expressIdealMember	exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	frobenius	gcd
gcdPolynomial	ground	ground?	hash
index	init	integrate	isExpt
isPlus	isTimes	karatsubaDivide	latex
lcm	leadingCoefficient	leadingMonomial	lift
lookup	mainVariable	makeSUP	map
mapExponents	max	min	minimumDegree
modulus	monicDivide	monomial	monomial?
monomials	multiEuclidean	multiplyExponents	multivariate
nextItem	numberOfMonomials	one?	order
patternMatch	pomopo!	pow	prime?
primitiveMonomials	primitivePart	principalIdeal	pseudoDivide
pseudoQuotient	pseudoRemainder	random	recip
reduce	reducedSystem	reductum	resultant
retract	retractIfCan	sample	separate
setPoly	shiftLeft	shiftRight	size
sizeLess?	solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subResultantGcd	subtractIfCan	totalDegree
unit?	unitCanonical	unitNormal	univariate
unmakeSUP	UnVectorise	variables	vectorise
Vectorise	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?.?	?~=?	?/?
?<?	?<=?	?>?	?>=?
?quo?	?rem?		

— domain MODMON ModMonic —

```
)abbrev domain MODMON ModMonic
++ Author: Mark Botch
++ Description:
++ This package has not been documented
-- following line prevents caching ModMonic
)bo PUSH('ModMonic, $mutableDomains)

ModMonic(R,Rep): C == T
 where
  R: Ring
  Rep: UnivariatePolynomialCategory(R)
  C == UnivariatePolynomialCategory(R) with
```

```
--operations
  setPoly : Rep -> Rep
      ++ setPoly(x) is not documented
  modulus : -> Rep
      ++ modulus() is not documented
  reduce: Rep -> %
      ++ reduce(x) is not documented
  lift: % -> Rep --reduce lift = identity
      ++ lift(x) is not documented
  coerce: Rep -> %
      ++ coerce(x) is not documented
  Vectorise: % -> Vector(R)
      ++ Vectorise(x) is not documented
  UnVectorise: Vector(R) -> %
      ++ UnVectorise(v) is not documented
  An: % -> Vector(R)
      ++ An(x) is not documented
  pow : -> PrimitiveArray(%)
      ++ pow() is not documented
  computePowers : -> PrimitiveArray(%)
      ++ computePowers() is not documented
  if R has FiniteFieldCategory then
     frobenius: % -> %
      ++ frobenius(x) is not documented
  --LinearTransf: (%,Vector(R)) -> SquareMatrix<deg> R
--assertions
  if R has Finite then Finite
T == add
  --constants
    m:Rep := monomial(1,1)$Rep --| degree(m) > 0 and LeadingCoef(m) = R$1
    d := degree(m)$Rep
    d1 := (d-1):NonNegativeInteger
    twod := 2*d1+1
    frobenius?:Boolean := R has FiniteFieldCategory
    --VectorRep:= DirectProduct(d:NonNegativeInteger,R)
  --declarations
    x,y: %
    p: Rep
    d,n: Integer
    e,k1,k2: NonNegativeInteger
    c: R
    --vect: Vector(R)
    power:PrimitiveArray(%)
    frobeniusPower:PrimitiveArray(%)
    computeFrobeniusPowers : () -> PrimitiveArray(%)
  --representations
  --mutable m    --take this out??
  --define
    power := new(0,0)
    frobeniusPower := new(0,0)
```

```
setPoly (mon : Rep) ==
  mon =$Rep m => mon
  oldm := m
  leadingCoefficient mon ^= 1 => error "polynomial must be monic"
  -- following copy code needed since FFPOLY can modify mon
  copymon:Rep:= 0
  while not zero? mon repeat
     copymon := monomial(leadingCoefficient mon, degree mon)$Rep + copymon
     mon := reductum mon
  m := copymon
  d := degree(m)$Rep
  d1 := (d-1)::NonNegativeInteger
  twod := 2*d1+1
  power := computePowers()
  if frobenius? then
    degree(oldm)>1 and not((oldm exquo$Rep m) case "failed") =>
        for i in 1..d1 repeat
          frobeniusPower(i) := reduce lift frobeniusPower(i)
    frobeniusPower := computeFrobeniusPowers()
  m
modulus == m
if R has Finite then
   size == d * size$R
   random == UnVectorise([random()$R for i in 0..d1])
0 == 0$Rep
1 == 1$Rep
c * x == c *$Rep x
n * x == (n::R) *$Rep x
coerce(c:R):% == monomial(c,0)$Rep
coerce(x:%):OutputForm == coerce(x)$Rep
coefficient(x,e):R == coefficient(x,e)$Rep
reductum(x) == reductum(x)$Rep
leadingCoefficient x == (leadingCoefficient x)$Rep
degree x == (degree x)$Rep
lift(x) == x pretend Rep
reduce(p) == (monicDivide(p,m)$Rep).remainder
coerce(p) == reduce(p)
x = y == x =$Rep y
x + y == x +$Rep y
- x == -$Rep x
x * y ==
  p := x *$Rep y
  ans:=0$Rep
  while (n:=degree p)>d1 repeat
     ans:=ans + leadingCoefficient(p)*power.(n-d)
     p := reductum p
  ans+p
Vectorise(x) == [coefficient(lift(x),i) for i in 0..d1]
UnVectorise(vect) ==
  reduce(+/[monomial(vect.(i+1),i) for i in 0..d1])
```

```
computePowers ==
     mat : PrimitiveArray(%):= new(d,0)
     mat.0:= reductum(-m)$Rep
     w: % := monomial$Rep (1,1)
     for i in 1..d1 repeat
        mat.i := w *$Rep mat.(i-1)
        if degree mat.i=d then
          mat.i:= reductum mat.i + leadingCoefficient mat.i * mat.0
     mat
if frobenius? then
    computeFrobeniusPowers() ==
      mat : PrimitiveArray(%):= new(d,1)
      mat.1:= mult := monomial(1, size$R)$%
      for i in 2..d1 repeat
         mat.i := mult * mat.(i-1)
      mat

    frobenius(a:%):% ==
      aq:% := 0
      while a^=0 repeat
        aq:= aq + leadingCoefficient(a)*frobeniusPower(degree a)
        a := reductum a
      aq

pow == power
monomial(c,e)==
   if e<d then monomial(c,e)$Rep
   else
      if e<=twod then
         c * power.(e-d)
      else
         k1:=e quo twod
         k2 := (e-k1*twod)::NonNegativeInteger
         reduce((power.d1 **k1)*monomial(c,k2))
if R has Field then

   (x:% exquo y:%):Union(%, "failed") ==
      uv := extendedEuclidean(y, modulus(), x)$Rep
      uv case "failed" => "failed"
      return reduce(uv.coef1)

   recip(y:%):Union(%, "failed") ==  1 exquo y
   divide(x:%, y:%) ==
      (q := (x exquo y)) case "failed" => error "not divisible"
      [q, 0]

--     An(MM) == Vectorise(-(reduce(reductum(m))::MM))
--     LinearTransf(vect,MM) ==
--       ans:= 0::SquareMatrix<d>(R)
--       for i in 1..d do setelt(ans,i,1,vect.i)
```

```
--      for j in 2..d do
--         setelt(ans,1,j, elt(ans,d,j-1) * An(MM).1)
--         for i in 2..d do
--           setelt(ans,i,j, elt(ans,i-1,j-1) + elt(ans,d,j-1) * An(MM).i)
--      ans
```

———-

— MODMON.dotabb —

```
"MODMON" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODMON"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"MODMON" -> "PFECAT"
```

———-

14.9 domain MODFIELD ModularField

— ModularField.input —

```
)set break resume
)sys rm -f ModularField.output
)spool ModularField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModularField
--R ModularField(R: CommutativeRing,Mod: AbelianMonoid,reduction: ((R,Mod) -> R),merge: ((Mod
--R Abbreviation for ModularField is MODFIELD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODFIELD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean       coerce : % -> R
```

```
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R factor : % -> Factored %              gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                      lcm : (%,%) -> %
--R modulus : % -> Mod                    one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        reduce : (R,Mod) -> %
--R ?rem? : (%,%) -> %                     sample : () -> %
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R exQuo : (%,%) -> Union(%,"failed")
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

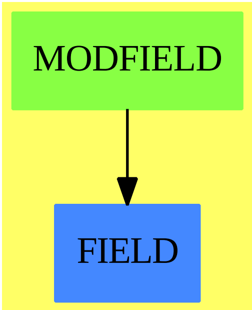
)spool
)lisp (bye)
```

— ModularField.help —

```
====================================================================
ModularField examples
====================================================================

See Also:
o )show ModularField
```

14.9.1 ModularField (MODFIELD)



See

⇒ "ModularRing" (MODRING) 14.10.1 on page 1604
⇒ "EuclideanModularRing" (EMR) 6.3.1 on page 670

Exports:

0	1	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	exQuo
extendedEuclidean	factor	gcd	gcdPolynomial	hash
inv	latex	lcm	modulus	multiEuclidean
one?	prime?	principalIdeal	recip	reduce
sample	sizeLess?	squareFree	squareFreePart	subtractIfCan
unit?	unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?	?/?
?=?	?^?	?~=?	?quo?	?rem?

— domain MODFIELD ModularField —

```
)abbrev domain MODFIELD ModularField
++ Author: Mark Botch
++ Description:
++ These domains are used for the factorization and gcds
++ of univariate polynomials over the integers in order to work modulo
++ different  primes.
++ See \spadtype{ModularRing}, \spadtype{EuclideanModularRing}

ModularField(R,Mod,reduction:(R,Mod) -> R,
               merge:(Mod,Mod) -> Union(Mod,"failed"),
                      exactQuo : (R,R,Mod) -> Union(R,"failed")) : C == T
 where
  R    :  CommutativeRing
  Mod  :  AbelianMonoid

  C == Field with
    modulus: % -> Mod
      ++ modulus(x) is not documented
```

```
    coerce: % -> R
      ++ coerce(x) is not documented
    reduce: (R,Mod) -> %
      ++ reduce(r,m) is not documented
    exQuo: (%,%) -> Union(%,"failed")
      ++ exQuo(x,y) is not documented

  T == ModularRing(R,Mod,reduction,merge,exactQuo)
```

———-

— MODFIELD.dotabb —

```
"MODFIELD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODFIELD"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"MODFIELD" -> "FIELD"
```

———-

14.10 domain MODRING ModularRing

— ModularRing.input —

```
)set break resume
)sys rm -f ModularRing.output
)spool ModularRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModularRing
--R ModularRing(R: CommutativeRing,Mod: AbelianMonoid,reduction: ((R,Mod) -> R),merge: ((Mod,Mod) -> Uni
--R Abbreviation for ModularRing is MODRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> R
```

```
--R coerce : Integer -> %                coerce : % -> OutputForm
--R hash : % -> SingleInteger            inv : % -> %
--R latex : % -> String                  modulus : % -> Mod
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reduce : (R,Mod) -> %                sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exQuo : (%,%) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

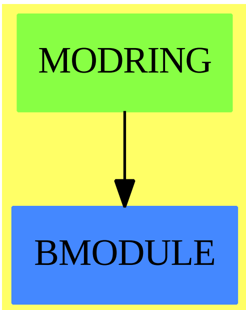
)spool
)lisp (bye)
```

— **ModularRing.help** —

```
====================================================================
ModularRing examples
====================================================================

See Also:
o )show ModularRing
```

14.10.1 ModularRing (MODRING)



See

⇒ "EuclideanModularRing" (EMR) 6.3.1 on page 670
⇒ "ModularField" (MODFIELD) 14.9.1 on page 1602

Exports:

0	1	characteristic	coerce	exQuo
hash	inv	latex	modulus	one?
recip	reduce	sample	subtractIfCan	zero?
?~=?	?*?	?**?	?^?	?+?
?-?	-?	?=?		

— domain MODRING ModularRing —

```
)abbrev domain MODRING ModularRing
++ Author: P.Gianni, B.Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ These domains are used for the factorization and gcds
++ of univariate polynomials over the integers in order to work modulo
++ different  primes.
++ See \spadtype{EuclideanModularRing} ,\spadtype{ModularField}

ModularRing(R,Mod,reduction:(R,Mod) -> R,
               merge:(Mod,Mod) -> Union(Mod,"failed"),
                      exactQuo : (R,R,Mod) -> Union(R,"failed")) : C == T
 where
  R    :  CommutativeRing
  Mod  :  AbelianMonoid

  C == Ring with
    modulus: % -> Mod
      ++ modulus(x) is not documented
    coerce: % -> R
      ++ coerce(x) is not documented
    reduce: (R,Mod) -> %
      ++ reduce(r,m) is not documented
    exQuo: (%,%) -> Union(%,"failed")
      ++ exQuo(x,y) is not documented
    recip: % -> Union(%,"failed")
      ++ recip(x) is not documented
    inv: % -> %
      ++ inv(x) is not documented

  T == add
```

```
    --representation
      Rep:= Record(val:R,modulo:Mod)
    --declarations
      x,y: %

    --define
      modulus(x)   == x.modulo
      coerce(x)    == x.val
      coerce(i:Integer):% == [i::R,0]$Rep
      i:Integer * x:% == (i::%)*x
      coerce(x):OutputForm == (x.val)::OutputForm
      reduce (a:R,m:Mod) == [reduction(a,m),m]$Rep

      characteristic():NonNegativeInteger == characteristic()$R
      0 == [0$R,0$Mod]$Rep
      1 == [1$R,0$Mod]$Rep
      zero? x == zero? x.val
--      one? x == one? x.val
      one? x == (x.val = 1)

      newmodulo(m1:Mod,m2:Mod) : Mod ==
        r:=merge(m1,m2)
        r case "failed" => error "incompatible moduli"
        r::Mod

      x=y ==
        x.val = y.val => true
        x.modulo = y.modulo => false
        (x-y).val = 0
      x+y == reduce((x.val +$R y.val),newmodulo(x.modulo,y.modulo))
      x-y == reduce((x.val -$R y.val),newmodulo(x.modulo,y.modulo))
      -x  == reduce ((-$R x.val),x.modulo)
      x*y == reduce((x.val *$R y.val),newmodulo(x.modulo,y.modulo))

      exQuo(x,y) ==
        xm:=x.modulo
        if xm ^=$Mod y.modulo then xm:=newmodulo(xm,y.modulo)
        r:=exactQuo(x.val,y.val,xm)
        r case "failed"=> "failed"
        [r::R,xm]$Rep

      --if R has EuclideanDomain then
      recip x ==
        r:=exactQuo(1$R,x.val,x.modulo)
        r case "failed" => "failed"
        [r,x.modulo]$Rep

      inv x ==
        if (u:=recip x) case "failed" then error("not invertible")
        else u::%
```

— MODRING.dotabb —

```
"MODRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODRING"]
"BMODULE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BMODULE"]
"MODRING" -> "BMODULE"
```

14.11 domain MODMONOM ModuleMonomial

— ModuleMonomial.input —

```
)set break resume
)sys rm -f ModuleMonomial.output
)spool ModuleMonomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModuleMonomial
--R ModuleMonomial(IS: OrderedSet,E: SetCategory,ff: ((Record(index: IS,exponent: E),Record(index: IS,ex
--R Abbreviation for ModuleMonomial is MODMONOM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODMONOM
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R construct : (IS,E) -> %               exponent : % -> E
--R hash : % -> SingleInteger             index : % -> IS
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      ?~=? : (%,%) -> Boolean
--R coerce : % -> Record(index: IS,exponent: E)
--R coerce : Record(index: IS,exponent: E) -> %
--R
--E 1

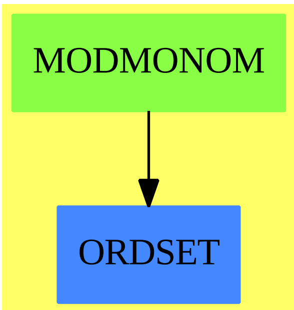
)spool
)lisp (bye)
```

— **ModuleMonomial.help** —

```
====================================================================
ModuleMonomial examples
====================================================================

See Also:
o )show ModuleMonomial
```

14.11.1 ModuleMonomial (MODMONOM)



See

⇒ "GeneralModulePolynomial" (GMODPOL) 8.2.1 on page 1025

Exports:

coerce	construct	exponent	hash	index
latex	max	min	?~=?	?<?
?<=?	?=?	?>?	?>=?	

— **domain MODMONOM ModuleMonomial** —

```
)abbrev domain MODMONOM ModuleMonomial
++ Author: Mark Botch
++ Description:
++ This package has no documentation

ModuleMonomial(IS: OrderedSet,
               E: SetCategory,
               ff:(MM, MM) -> Boolean): T == C where

   MM ==> Record(index:IS, exponent:E)
```

```
  T == OrderedSet  with
   exponent: $ -> E
     ++ exponent(x) is not documented
   index: $ -> IS
     ++ index(x) is not documented
   coerce: MM -> $
     ++ coerce(x) is not documented
   coerce: $ -> MM
     ++ coerce(x) is not documented
   construct: (IS, E) -> $
     ++ construct(i,e) is not documented
  C == MM  add
       Rep:= MM
       x:$ < y:$ == ff(x::Rep, y::Rep)
       exponent(x:$):E == x.exponent
       index(x:$): IS == x.index
       coerce(x:$):MM == x::Rep::MM
       coerce(x:MM):$ == x::Rep::$
       construct(i:IS, e:E):$ == [i, e]$MM::Rep::$
```

———-

— **MODMONOM.dotabb** —

```
"MODMONOM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODMONOM"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"MODMONOM" -> "ORDSET"
```

———-

14.12 domain MODOP ModuleOperator

— **ModuleOperator.input** —

```
)set break resume
)sys rm -f ModuleOperator.output
)spool ModuleOperator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModuleOperator
```

```
--R ModuleOperator(R: Ring,M: LeftModule R)  is a domain constructor
--R Abbreviation for ModuleOperator is MODOP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODOP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coerce : BasicOperator -> %           coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R ?.? : (%,M) -> M                      evaluate : (%,(M -> M)) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   opeval : (BasicOperator,M) -> M
--R recip : % -> Union(%,"failed")        retract : % -> BasicOperator
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,R) -> % if R has COMRING
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (BasicOperator,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R adjoint : (%,%) -> % if R has COMRING
--R adjoint : % -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R conjug : R -> R if R has COMRING
--R evaluateInverse : (%,(M -> M)) -> %
--R makeop : (R,FreeGroup BasicOperator) -> %
--R retractIfCan : % -> Union(BasicOperator,"failed")
--R retractIfCan : % -> Union(R,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

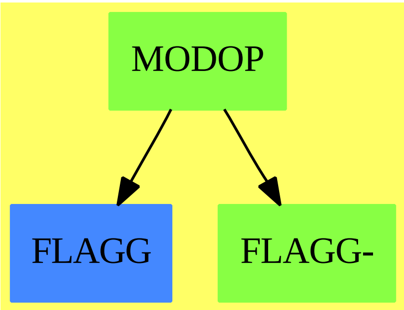
— ModuleOperator.help —

```
====================================================================
ModuleOperator examples
====================================================================
```

```
See Also:
o )show ModuleOperator
```

———-

14.12.1 ModuleOperator (MODOP)



See

⇒ "Operator" (OP) 16.10.1 on page 1766

Exports:

0	1	adjoint	characteristic	charthRoot
coerce	conjug	evaluate	evaluateInverse	hash
latex	makeop	one?	opeval	recip
retract	retractIfCan	sample	subtractIfCan	zero?
?**?	?***?	?+?	?-?	-?
?=?	?^?	?.?	?~=?	

— **domain MODOP ModuleOperator** —

```
)abbrev domain MODOP ModuleOperator
++ Author: Manuel Bronstein
++ Date Created: 15 May 1990
++ Date Last Updated: 17 June 1993
++ Description:
++ Algebra of ADDITIVE operators on a module.

ModuleOperator(R: Ring, M:LeftModule(R)): Exports == Implementation where
  O    ==> OutputForm
  OP   ==> BasicOperator
  FG   ==> FreeGroup OP
  RM   ==> Record(coef:R, monom:FG)
  TERM ==> List RM
  FAB  ==> FreeAbelianGroup TERM
```

```
OPADJ   ==> "%opAdjoint"
OPEVAL  ==> "%opEval"
INVEVAL ==> "%invEval"

Exports ==> Join(Ring, RetractableTo R, RetractableTo OP,
                 Eltable(M, M)) with
  if R has CharacteristicZero then CharacteristicZero
  if R has CharacteristicNonZero then CharacteristicNonZero
  if R has CommutativeRing then
    Algebra(R)
    adjoint: $ -> $
      ++ adjoint(op) returns the adjoint of the operator \spad{op}.
    adjoint: ($, $) -> $
      ++ adjoint(op1, op2) sets the adjoint of op1 to be op2.
      ++ op1 must be a basic operator
    conjug  : R -> R
      ++ conjug(x)should be local but conditional
  evaluate: ($, M -> M) -> $
    ++ evaluate(f, u +-> g u) attaches the map g to f.
    ++ f must be a basic operator
    ++ g MUST be additive, i.e. \spad{g(a + b) = g(a) + g(b)} for
    ++ any \spad{a}, \spad{b} in M.
    ++ This implies that \spad{g(n a) = n g(a)} for
    ++ any \spad{a} in M and integer \spad{n > 0}.
  evaluateInverse: ($, M -> M) -> $
    ++ evaluateInverse(x,f) is not documented
  "**": (OP, Integer) -> $
    ++ op**n is not documented
  "**": ($, Integer) -> $
    ++ op**n is not documented
  opeval  : (OP, M) -> M
    ++ opeval should be local but conditional
  makeop   : (R, FG) -> $
    ++ makeop should be local but conditional

Implementation ==> FAB add
  import NoneFunctions1($)
  import BasicOperatorFunctions1(M)

  Rep := FAB

  inv      : TERM -> $
  termeval : (TERM, M) -> M
  rmeval   : (RM, M) -> M
  monomeval: (FG, M) -> M
  opInvEval: (OP, M) -> M
  mkop     : (R, FG) -> $
  termprod0: (Integer, TERM, TERM) -> $
  termprod : (Integer, TERM, TERM) -> TERM
  termcopy : TERM -> TERM
```

```
    trm2O    : (Integer, TERM) -> O
    term2O   : TERM -> O
    rm2O     : (R, FG) -> O
    nocopy   : OP -> $

    1                    == makeop(1, 1)
    coerce(n:Integer):$ == n::R::$
    coerce(r:R):$       == (zero? r => 0; makeop(r, 1))
    coerce(op:OP):$     == nocopy copy op
    nocopy(op:OP):$     == makeop(1, op::FG)
    elt(x:$, r:M)       == +/[t.exp * termeval(t.gen, r) for t in terms x]
    rmeval(t, r)        == t.coef * monomeval(t.monom, r)
    termcopy t          == [[rm.coef, rm.monom] for rm in t]
    characteristic()    == characteristic()$R
    mkop(r, fg)         == [[r, fg]$RM]$TERM :: $
    evaluate(f, g)   == nocopy setProperty(retract(f)@OP,OPEVAL,g pretend None)

    if R has OrderedSet then
      makeop(r, fg) == (r >= 0 => mkop(r, fg); - mkop(-r, fg))
    else makeop(r, fg) == mkop(r, fg)

    inv(t:TERM):$ ==
      empty? t => 1
      c := first(t).coef
      m := first(t).monom
      inv(rest t) * makeop(1, inv m) * (recip(c)::R::$)

    x:$ ** i:Integer ==
      i = 0 => 1
      i > 0 => expt(x,i pretend PositiveInteger)$RepeatedSquaring($)
      (inv(retract(x)@TERM)) ** (-i)

    evaluateInverse(f, g) ==
      nocopy setProperty(retract(f)@OP, INVEVAL, g pretend None)

    coerce(x:$):O ==
      zero? x => (0$R)::O
      reduce(_+, [trm2O(t.exp, t.gen) for t in terms x])$List(O)

    trm2O(c, t) ==
--      one? c => term2O t
      (c = 1) => term2O t
      c = -1 => - term2O t
      c::O * term2O t

    term2O t ==
      reduce(_*, [rm2O(rm.coef, rm.monom) for rm in t])$List(O)

    rm2O(c, m) ==
--      one? c => m::O
```

```
      (c = 1) => m::O
--      one? m => c::O
      (m = 1) => c::O
      c::O * m::O

    x:$ * y:$ ==
      +/[ +/[termprod0(t.exp * s.exp, t.gen, s.gen) for s in terms y]
          for t in terms x]

    termprod0(n, x, y) ==
      n >= 0 => termprod(n, x, y)::$
      - (termprod(-n, x, y)::$)

    termprod(n, x, y) ==
      lc := first(xx := termcopy x)
      lc.coef := n * lc.coef
      rm := last xx
--      one?(first(y).coef) =>
      ((first(y).coef) = 1) =>
        rm.monom := rm.monom * first(y).monom
        concat_!(xx, termcopy rest y)
--      one?(rm.monom) =>
      ((rm.monom) = 1) =>
        rm.coef := rm.coef * first(y).coef
        rm.monom := first(y).monom
        concat_!(xx, termcopy rest y)
      concat_!(xx, termcopy y)

    if M has ExpressionSpace then
      opeval(op, r) ==
        (func := property(op, OPEVAL)) case "failed" => kernel(op, r)
        ((func::None) pretend (M -> M)) r

    else
      opeval(op, r) ==
        (func := property(op, OPEVAL)) case "failed" =>
          error "eval: operator has no evaluation function"
        ((func::None) pretend (M -> M)) r

    opInvEval(op, r) ==
      (func := property(op, INVEVAL)) case "failed" =>
         error "eval: operator has no inverse evaluation function"
      ((func::None) pretend (M -> M)) r

    termeval(t, r)  ==
      for rm in reverse t repeat r := rmeval(rm, r)
      r

    monomeval(m, r) ==
      for rec in reverse_! factors m repeat
```

```
        e := rec.exp
        g := rec.gen
        e > 0 =>
          for i in 1..e repeat r := opeval(g, r)
        e < 0 =>
          for i in 1..(-e) repeat r := opInvEval(g, r)
      r

    recip x ==
      (r := retractIfCan(x)@Union(R, "failed")) case "failed" => "failed"
      (r1 := recip(r::R)) case "failed" => "failed"
      r1::R::$

    retractIfCan(x:$):Union(R, "failed") ==
      (r:= retractIfCan(x)@Union(TERM,"failed")) case "failed" => "failed"
      empty?(t := r::TERM) => 0$R
      empty? rest t =>
        rm := first t
--        one?(rm.monom) => rm.coef
        (rm.monom = 1) => rm.coef
        "failed"
      "failed"

    retractIfCan(x:$):Union(OP, "failed") ==
      (r:= retractIfCan(x)@Union(TERM,"failed")) case "failed" => "failed"
      empty?(t := r::TERM) => "failed"
      empty? rest t =>
        rm := first t
--        one?(rm.coef) => retractIfCan(rm.monom)
        (rm.coef = 1) => retractIfCan(rm.monom)
        "failed"
      "failed"

    if R has CommutativeRing then
      termadj  : TERM -> $
      rmadj    : RM -> $
      monomadj : FG -> $
      opadj    : OP -> $

      r:R * x:$        == r::$ * x
      x:$ * r:R        == x * (r::$)
      adjoint x        == +/[t.exp * termadj(t.gen) for t in terms x]
      rmadj t          == conjug(t.coef) * monomadj(t.monom)
      adjoint(op, adj) == nocopy setProperty(retract(op)@OP, OPADJ, adj::None)

      termadj t ==
        ans:$ := 1
        for rm in t repeat ans := rmadj(rm) * ans
        ans
```

```
    monomadj m ==
      ans:$ := 1
      for rec in factors m repeat ans := (opadj(rec.gen) ** rec.exp) * ans
      ans

    opadj op ==
      (adj := property(op, OPADJ)) case "failed" =>
         error "adjoint: operator does not have a defined adjoint"
      (adj::None) pretend $

    if R has conjugate:R -> R then conjugate r == conjugate r else conjug r == r
```

———-

— MODOP.dotabb —

```
"MODOP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODOP"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"MODOP" -> "FLAGG"
"MODOP" -> "FLAGG-"
```

———-

14.13 domain MOEBIUS MoebiusTransform

— MoebiusTransform.input —

```
)set break resume
)sys rm -f MoebiusTransform.output
)spool MoebiusTransform.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MoebiusTransform
--R MoebiusTransform F: Field  is a domain constructor
--R Abbreviation for MoebiusTransform is MOEBIUS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MOEBIUS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,Integer) -> %
```

```
--R ?**? : (%,PositiveInteger) -> %       ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm              commutator : (%,%) -> %
--R conjugate : (%,%) -> %                eval : (%,F) -> F
--R hash : % -> SingleInteger             inv : % -> %
--R latex : % -> String                   moebius : (F,F,F,F) -> %
--R one? : % -> Boolean                   recip : % -> %
--R recip : () -> %                       recip : % -> Union(%,"failed")
--R sample : () -> %                      scale : (%,F) -> %
--R scale : F -> %                        shift : (%,F) -> %
--R shift : F -> %                        ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R eval : (%,OnePointCompletion F) -> OnePointCompletion F
--R
--E 1

)spool
)lisp (bye)
```

———-

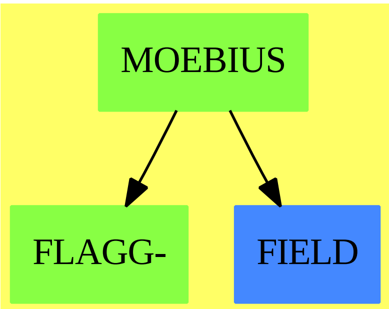
— MoebiusTransform.help —

```
====================================================================
MoebiusTransform examples
====================================================================

See Also:
o )show MoebiusTransform
```

———-

14.13.1 MoebiusTransform (MOEBIUS)



Exports:

1	coerce	commutator	conjugate	eval
hash	inv	latex	moebius	one?
recip	sample	scale	shift	?~=?
?**?	?^?	?*?	?/?	?=?
?^?				

— domain MOEBIUS MoebiusTransform —

```
)abbrev domain MOEBIUS MoebiusTransform
++ Author: Stephen "Say" Watt
++ Date Created: January 1987
++ Date Last Updated: 11 April 1990
++ Keywords:
++ Examples:
++ References:
++ Description:
++ MoebiusTransform(F) is the domain of fractional linear (Moebius)
++ transformations over F. This a domain of 2-by-2 matrices acting on P1(F).

MoebiusTransform(F): Exports == Implementation where
  F : Field
  OUT ==> OutputForm
  P1F ==> OnePointCompletion F        -- projective 1-space over F

  Exports ==> Group with

    moebius: (F,F,F,F) -> %
      ++ moebius(a,b,c,d) returns \spad{matrix [[a,b],[c,d]]}.
    shift: F -> %
      ++ shift(k) returns \spad{matrix [[1,k],[0,1]]} representing the map
      ++ \spad{x -> x + k}.
    scale: F -> %
      ++ scale(k) returns \spad{matrix [[k,0],[0,1]]} representing the map
      ++ \spad{x -> k * x}.
    recip: () -> %
      ++ recip() returns \spad{matrix [[0,1],[1,0]]} representing the map
      ++ \spad{x -> 1 / x}.
    shift: (%,F) -> %
      ++ shift(m,h) returns \spad{shift(h) * m}
      ++ (see shift from MoebiusTransform).
    scale: (%,F) -> %
      ++ scale(m,h) returns \spad{scale(h) * m}
      ++ (see shift from MoebiusTransform).
    recip: % -> %
      ++ recip(m) = recip() * m
    eval: (%,F) -> F
      ++ eval(m,x) returns \spad{(a*x + b)/(c*x + d)}
```

```
    ++ where \spad{m = moebius(a,b,c,d)}
    ++ (see moebius from MoebiusTransform).
  eval: (%,P1F) -> P1F
    ++ eval(m,x) returns \spad{(a*x + b)/(c*x + d)}
    ++ where \spad{m = moebius(a,b,c,d)}
    ++ (see moebius from MoebiusTransform).

Implementation ==> add

  Rep := Record(a: F,b: F,c: F,d: F)

  moebius(aa,bb,cc,dd) == [aa,bb,cc,dd]

  a(t:%):F == t.a
  b(t:%):F == t.b
  c(t:%):F == t.c
  d(t:%):F == t.d

  1 == moebius(1,0,0,1)
  t * s ==
    moebius(b(t)*c(s) + a(t)*a(s), b(t)*d(s) + a(t)*b(s), _
            d(t)*c(s) + c(t)*a(s), d(t)*d(s) + c(t)*b(s))
  inv t == moebius(d(t),-b(t),-c(t),a(t))

  shift f == moebius(1,f,0,1)
  scale f == moebius(f,0,0,1)
  recip() == moebius(0,1,1,0)

  shift(t,f) == moebius(a(t) + f*c(t), b(t) + f*d(t), c(t), d(t))
  scale(t,f) == moebius(f*a(t),f*b(t),c(t),d(t))
  recip t    == moebius(c(t),d(t),a(t),b(t))

  eval(t:%,f:F) == (a(t)*f + b(t))/(c(t)*f + d(t))
  eval(t:%,f:P1F) ==
    (ff := retractIfCan(f)@Union(F,"failed")) case "failed" =>
      (a(t)/c(t)) :: P1F
    zero?(den := c(t) * (fff := ff :: F) + d(t)) => infinity()
    ((a(t) * fff + b(t))/den) :: P1F

  coerce t ==
    var := "%x" :: OUT
    num := (a(t) :: OUT) * var + (b(t) :: OUT)
    den := (c(t) :: OUT) * var + (d(t) :: OUT)
    rarrow(var,num/den)

  proportional?: (List F,List F) -> Boolean
  proportional?(list1,list2) ==
    empty? list1 => empty? list2
    empty? list2 => false
    zero? (x1 := first list1) =>
```

```
      (zero? first list2) and proportional?(rest list1,rest list2)
    zero? (x2 := first list2) => false
    map((f1:F):F +-> f1/x1, list1) = map((g1:F):F +-> g1/x2, list2)

  t = s ==
    list1 : List F := [a(t),b(t),c(t),d(t)]
    list2 : List F := [a(s),b(s),c(s),d(s)]
    proportional?(list1,list2)
```

———-

— **MOEBIUS.dotabb** —

```
"MOEBIUS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MOEBIUS"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"MOEBIUS" -> "FIELD"
"MOEBIUS" -> "FLAGG-"
```

———-

14.14 domain MRING MonoidRing

— **MonoidRing.input** —

```
)set break resume
)sys rm -f MonoidRing.output
)spool MonoidRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonoidRing
--R MonoidRing(R: Ring,M: Monoid)  is a domain constructor
--R Abbreviation for MonoidRing is MRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coefficient : (%,M) -> R
--R coefficients : % -> List R           coerce : R -> %
--R coerce : M -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  map : ((R -> R),%) -> %
--R monomial : (R,M) -> %                monomial? : % -> Boolean
--R monomials : % -> List %              one? : % -> Boolean
--R recip : % -> Union(%,"failed")       retract : % -> R
--R retract : % -> M                     sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,R) -> % if R has COMRING
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : List Record(coef: R,monom: M) -> %
--R index : PositiveInteger -> % if M has FINITE and R has FINITE
--R leadingCoefficient : % -> R if M has ORDSET
--R leadingMonomial : % -> M if M has ORDSET
--R lookup : % -> PositiveInteger if M has FINITE and R has FINITE
--R numberOfMonomials : % -> NonNegativeInteger
--R random : () -> % if M has FINITE and R has FINITE
--R reductum : % -> % if M has ORDSET
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(M,"failed")
--R size : () -> NonNegativeInteger if M has FINITE and R has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List Record(coef: R,monom: M)
--R
--E 1

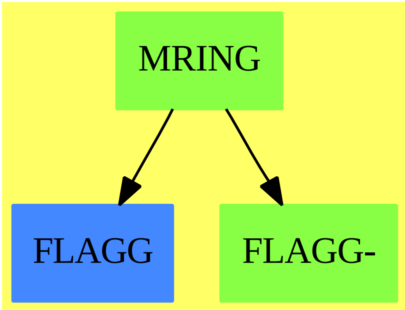
)spool
)lisp (bye)
```

— **MonoidRing.help** —

```
====================================================================
MonoidRing examples
====================================================================

See Also:
o )show MonoidRing
```

14.14.1 MonoidRing (MRING)



Exports:

0	1	characteristic	charthRoot	coefficient
coefficients	coerce	hash	index	latex
leadingCoefficient	leadingMonomial	lookup	map	monomial
monomial?	monomials	numberOfMonomials	one?	random
recip	reductum	retract	retractIfCan	sample
size	subtractIfCan	terms	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?			

— domain MRING MonoidRing —

```
)abbrev domain MRING MonoidRing
++ Authors: Stephan M. Watt; revised by Johannes Grabmeier
++ Date Created: January 1986
++ Date Last Updated: 14 December 1995, Mike Dewar
++ Basic Operations: *, +, monomials, coefficients
++ Related Constructors: Polynomial
++ Also See:
++ AMS Classifications:
++ Keywords: monoid ring, group ring, polynomials in non-commuting
++  indeterminates
++ References:
++ Description:
++ \spadtype{MonoidRing}(R,M), implements the algebra
++ of all maps from the monoid M to the commutative ring R with
++ finite support.
++ Multiplication of two maps f and g is defined
++ to map an element c of M to the (convolution) sum over f(a)g(b)
++ such that ab = c. Thus M can be identified with a canonical
++ basis and the maps can also be considered as formal linear combinations
++ of the elements in M. Scalar multiples of a basis element are called
++ monomials. A prominent example is the class of polynomials
++ where the monoid is a direct product of the natural numbers
++ with pointwise addition. When M is
```

```
++ \spadtype{FreeMonoid Symbol}, one gets polynomials
++ in infinitely many non-commuting variables. Another application
++ area is representation theory of finite groups G, where modules
++ over \spadtype{MonoidRing}(R,G) are studied.

MonoidRing(R: Ring, M: Monoid): MRcategory == MRdefinition where
    Term ==> Record(coef: R, monom: M)

    MRcategory ==> Join(Ring, RetractableTo M, RetractableTo R) with
        monomial         : (R, M) -> %
          ++ monomial(r,m) creates a scalar multiple of the basis element m.
        coefficient : (%, M) -> R
          ++ coefficient(f,m) extracts the coefficient of m in f with respect
          ++ to the canonical basis M.
        coerce:   List Term -> %
          ++ coerce(lt) converts a list of terms and
          ++ coefficients to a member of the domain.
        terms       : % -> List Term
          ++ terms(f) gives the list of non-zero coefficients combined
          ++ with their corresponding basis element as records.
          ++ This is the internal representation.
        map         : (R -> R, %) -> %
          ++ map(fn,u) maps function fn onto the coefficients
          ++ of the non-zero monomials of u.
        monomial?   : % -> Boolean
          ++ monomial?(f) tests if f is a single monomial.
        coefficients: % -> List R
          ++ coefficients(f) lists all non-zero coefficients.
        monomials: % -> List %
           ++ monomials(f) gives the list of all monomials whose
           ++ sum is f.
        numberOfMonomials: % -> NonNegativeInteger
           ++ numberOfMonomials(f) is the number of non-zero coefficients
           ++ with respect to the canonical basis.
        if R has CharacteristicZero then CharacteristicZero
        if R has CharacteristicNonZero then CharacteristicNonZero
        if R has CommutativeRing then Algebra(R)
        if (R has Finite and M has Finite) then Finite
        if M has OrderedSet then
          leadingMonomial   : % -> M
            ++ leadingMonomial(f) gives the monomial of f whose
            ++ corresponding monoid element is the greatest
            ++ among all those with non-zero coefficients.
          leadingCoefficient: % -> R
            ++ leadingCoefficient(f) gives the coefficient of f, whose
            ++ corresponding monoid element is the greatest
            ++ among all those with non-zero coefficients.
          reductum          : % -> %
            ++ reductum(f) is f minus its leading monomial.
```

```
MRdefinition ==> add
    Ex ==> OutputForm
    Cf ==> coef
    Mn ==> monom

    Rep  := List Term

    coerce(x: List Term): % == x :: %

    monomial(r:R, m:M)  ==
      r = 0 => empty()
      [[r, m]]

    if (R has Finite and M has Finite) then
      size() == size()$R ** size()$M

      index k ==
        -- use p-adic decomposition of k
        -- coefficient of p**j determines coefficient of index(i+p)$M
        i:Integer := k rem size()
        p:Integer := size()$R
        n:Integer := size()$M
        ans:% := 0
        for j in 0.. while i > 0 repeat
          h := i rem p
          -- we use index(p) = 0$R
          if h ^= 0 then
            c : R := index(h :: PositiveInteger)$R
            m : M := index((j+n) :: PositiveInteger)$M
            --ans := ans + c *$% m
            ans := ans + monomial(c, m)$%
          i := i quo p
        ans

      lookup(z : %) : PositiveInteger ==
        -- could be improved, if M has OrderedSet
        -- z = index lookup z, n = lookup index n
        -- use p-adic decomposition of k
        -- coefficient of p**j determines coefficient of index(i+p)$M
        zero?(z) => size()$% pretend PositiveInteger
        liTe : List Term := terms z  -- all non-zero coefficients
        p  : Integer := size()$R
        n  : Integer := size()$M
        res : Integer := 0
        for te in liTe repeat
          -- assume that lookup(p)$R = 0
          l:NonNegativeInteger:=lookup(te.Mn)$M
          ex : NonNegativeInteger := (n=l => 0;l)
          co : Integer := lookup(te.Cf)$R
          res := res + co * p ** ex
```

```
            res pretend PositiveInteger

          random() == index( (1+(random()$Integer rem size()$%) )_
            pretend PositiveInteger)$%

        0                   == empty()
        1                   == [[1, 1]]
        terms a             == (copy a) pretend List(Term)
        monomials a         == [[t] for t in a]
        coefficients a      == [t.Cf for t in a]
        coerce(m:M):%       == [[1, m]]
        coerce(r:R): % ==
        -- coerce of ring
          r = 0 => 0
          [[r,    1]]
        coerce(n:Integer): % ==
        -- coerce of integers
          n = 0 => 0
          [[n::R, 1]]
        - a                 == [[ -t.Cf, t.Mn] for t in a]
        if R has noZeroDivisors
           then
            (r:R) * (a:%) ==
              r = 0 => 0
              [[r*t.Cf, t.Mn] for t in a]
           else
            (r:R) * (a:%) ==
              r = 0 => 0
              [[rt, t.Mn] for t in a | (rt:=r*t.Cf) ^= 0]
        if R has noZeroDivisors
           then
            (n:Integer) * (a:%) ==
              n = 0 => 0
              [[n*t.Cf, t.Mn] for t in a]
           else
            (n:Integer) * (a:%) ==
              n = 0 => 0
              [[nt, t.Mn] for t in a | (nt:=n*t.Cf) ^= 0]
        map(f, a)           == [[ft, t.Mn] for t in a | (ft:=f(t.Cf)) ^= 0]
        numberOfMonomials a == #a

        retractIfCan(a:%):Union(M, "failed") ==
--          one?(#a) and one?(a.first.Cf) => a.first.Mn
          ((#a) = 1) and ((a.first.Cf) = 1) => a.first.Mn
          "failed"

        retractIfCan(a:%):Union(R, "failed") ==
--          one?(#a) and one?(a.first.Mn) => a.first.Cf
          ((#a) = 1) and ((a.first.Mn) = 1) => a.first.Cf
          "failed"
```

```
if R has noZeroDivisors then
  if M has Group then
    recip a ==
      lt := terms a
      #lt ^= 1 => "failed"
      (u := recip lt.first.Cf) case "failed" => "failed"
      --(u::R) * inv lt.first.Mn
      monomial((u::R), inv lt.first.Mn)$%
  else
    recip a ==
      #a ^= 1 or a.first.Mn ^= 1 => "failed"
      (u := recip a.first.Cf) case "failed" => "failed"
      u::R::%

mkTerm(r:R, m:M):Ex ==
    r=1 => m::Ex
    r=0 or m=1 => r::Ex
    r::Ex * m::Ex

coerce(a:%):Ex ==
    empty? a => (0$Integer)::Ex
    empty? rest a => mkTerm(a.first.Cf, a.first.Mn)
    reduce(_+, [mkTerm(t.Cf, t.Mn) for t in a])$List(Ex)

if M has OrderedSet then -- we mean totally ordered
    -- Terms are stored in decending order.
    leadingCoefficient a == (empty? a => 0; a.first.Cf)
    leadingMonomial a    == (empty? a => 1; a.first.Mn)
    reductum a           == (empty? a => a; rest a)

    a = b ==
        #a ^= #b => false
        for ta in a for tb in b repeat
            ta.Cf ^= tb.Cf or ta.Mn ^= tb.Mn => return false
        true

    a + b ==
        c:% := empty()
        while not empty? a and not empty? b repeat
          ta := first a; tb := first b
          ra := rest a;  rb := rest b
          c :=
            ta.Mn > tb.Mn => (a := ra; concat_!(c, ta))
            ta.Mn < tb.Mn => (b := rb; concat_!(c, tb))
            a := ra; b := rb
            not zero?(r := ta.Cf+tb.Cf) =>
                                concat_!(c, [r, ta.Mn])
            c
        concat_!(c, concat(a, b))
```

```
coefficient(a, m) ==
    for t in a repeat
        if t.Mn = m then return t.Cf
        if t.Mn < m then return 0
    0


if M has OrderedMonoid then

-- we use that multiplying an ordered list of monoid elements
-- by a single element respects the ordering

  if R has noZeroDivisors then
    a:% * b:% ==
      +/[[[ta.Cf*tb.Cf, ta.Mn*tb.Mn]$Term
        for tb in b ] for ta in reverse a]
  else
    a:% * b:% ==
      +/[[[r, ta.Mn*tb.Mn]$Term
        for tb in b | not zero?(r := ta.Cf*tb.Cf)]
          for ta in reverse a]
else -- M hasn't OrderedMonoid

-- we cannot assume that mutiplying an ordered list of
-- monoid elements by a single element respects the ordering:
-- we have to order and to collect equal terms
  ge : (Term,Term) -> Boolean
  ge(s,t) == t.Mn <= s.Mn

  sortAndAdd : List Term -> List Term
  sortAndAdd(liTe) ==  -- assume liTe not empty
    liTe := sort(ge,liTe)
    m : M :=  (first liTe).Mn
    cf : R := (first liTe).Cf
    res : List Term := []
    for te in rest liTe repeat
      if m = te.Mn then
        cf := cf + te.Cf
      else
        if not zero? cf then res := cons([cf,m]$Term, res)
        m := te.Mn
        cf := te.Cf
    if not zero? cf then res := cons([cf,m]$Term, res)
    reverse res


  if R has noZeroDivisors then
    a:% * b:% ==
      zero? a => a
```

```
            zero? b => b  -- avoid calling sortAndAdd with []
            +/[sortAndAdd [[ta.Cf*tb.Cf, ta.Mn*tb.Mn]$Term
              for tb in b ] for ta in reverse a]
        else
          a:% * b:% ==
            zero? a => a
            zero? b => b  -- avoid calling sortAndAdd with []
            +/[sortAndAdd [[r, ta.Mn*tb.Mn]$Term
              for tb in b | not zero?(r := ta.Cf*tb.Cf)]
                for ta in reverse a]


else -- M hasn't OrderedSet
    -- Terms are stored in random order.
  a = b ==
    #a ^= #b => false
    brace(a pretend List(Term)) =$Set(Term) brace(b pretend List(Term))

  coefficient(a, m) ==
    for t in a repeat
      t.Mn = m => return t.Cf
    0

  addterm(Tabl: AssociationList(M,R), r:R, m:M):R ==
      (u := search(m, Tabl)) case "failed" => Tabl.m := r
      zero?(r := r + u::R) => (remove_!(m, Tabl); 0)
      Tabl.m := r

  a + b ==
      Tabl := table()$AssociationList(M,R)
      for t in a repeat
          Tabl t.Mn := t.Cf
      for t in b repeat
          addterm(Tabl, t.Cf, t.Mn)
      [[Tabl m, m]$Term for m in keys Tabl]

  a:% * b:% ==
      Tabl := table()$AssociationList(M,R)
      for ta in a repeat
          for tb in (b pretend List(Term)) repeat
              addterm(Tabl, ta.Cf*tb.Cf, ta.Mn*tb.Mn)
      [[Tabl.m, m]$Term for m in keys Tabl]
```

———

— MRING.dotabb —

```
"MRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MRING"]
```

```
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"MRING" -> "FLAGG"
"MRING" -> "FLAGG-"
```

———-

14.15 domain MSET Multiset

— Multiset.input —

```
)set break resume
)sys rm -f Multiset.output
)spool Multiset.output
)set message test on
)set message auto off
)clear all
--S 1 of 14
s := multiset [1,2,3,4,5,4,3,2,3,4,5,6,7,4,10]
--R
--R
--R   (1)  {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
--R                                               Type: Multiset PositiveInteger
--E 1

--S 2 of 14
insert!(3,s)
--R
--R
--R   (2)  {1,2: 2,4: 3,4: 4,2: 5,6,7,10}
--R                                               Type: Multiset PositiveInteger
--E 2

--S 3 of 14
remove!(3,s,1)
--R
--R
--R   (3)  {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
--R                                               Type: Multiset PositiveInteger
--E 3

--S 4 of 14
s
--R
--R
--R   (4)  {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
```

```
--R                                                   Type: Multiset PositiveInteger
--E 4

--S 5 of 14
remove!(5,s)
--R
--R
--R   (5)  {1,2: 2,3: 3,4: 4,6,7,10}
--R                                                   Type: Multiset PositiveInteger
--E 5

--S 6 of 14
s
--R
--R
--R   (6)  {1,2: 2,3: 3,4: 4,6,7,10}
--R                                                   Type: Multiset PositiveInteger
--E 6

--S 7 of 14
count(5,s)
--R
--R
--R   (7)  0
--R                                                         Type: NonNegativeInteger
--E 7

--S 8 of 14
t := multiset [2,2,2,-9]
--R
--R
--R   (8)  {3: 2,- 9}
--R                                                           Type: Multiset Integer
--E 8

--S 9 of 14
U := union(s,t)
--R
--R
--R   (9)  {1,5: 2,3: 3,4: 4,6,7,10,- 9}
--R                                                           Type: Multiset Integer
--E 9

--S 10 of 14
I := intersect(s,t)
--R
--R
--R   (10)  {5: 2}
--R                                                           Type: Multiset Integer
--E 10
```

```
--S 11 of 14
difference(s,t)
--R
--R
--R   (11)  {1,3: 3,4: 4,6,7,10}
--R                                                 Type: Multiset Integer
--E 11

--S 12 of 14
S := symmetricDifference(s,t)
--R
--R
--R   (12)  {1,3: 3,4: 4,6,7,10,- 9}
--R                                                 Type: Multiset Integer
--E 12

--S 13 of 14
(U = union(S,I))@Boolean
--R
--R
--R   (13)  true
--R                                                       Type: Boolean
--E 13

--S 14 of 14
t1 := multiset [1,2,2,3]; [t1 < t, t1 < s, t < s, t1 <= s]
--R
--R
--R   (14)  [false,true,false,true]
--R                                                    Type: List Boolean
--E 14
)spool
)lisp (bye)
```

— **Multiset.help** —

```
====================================================================
Multiset examples
====================================================================

The domain Multiset(R) is similar to Set(R) except that multiplicities
(counts of duplications) are maintained and displayed.  Use the
operation multiset to create multisets from lists.  All the standard
operations from sets are available for multisets.  An element with
multiplicity greater than one has the multiplicity displayed first,
then a colon, and then the element.
```

Create a multiset of integers.

```
  s := multiset [1,2,3,4,5,4,3,2,3,4,5,6,7,4,10]
    {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
                          Type: Multiset PositiveInteger
```

The operation insert! adds an element to a multiset.

```
  insert!(3,s)
    {1,2: 2,4: 3,4: 4,2: 5,6,7,10}
                          Type: Multiset PositiveInteger
```

Use remove! to remove an element. If a third argument is present, it specifies how many instances to remove. Otherwise all instances of the element are removed. Display the resulting multiset.

```
  remove!(3,s,1); s

    {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
                          Type: Multiset PositiveInteger

  remove!(5,s); s
    {1,2: 2,3: 3,4: 4,6,7,10}
                          Type: Multiset PositiveInteger
```

The operation count returns the number of copies of a given value.

```
  count(5,s)
    0
                          Type: NonNegativeInteger
```

A second multiset.

```
  t := multiset [2,2,2,-9]
    {3: 2,- 9}
                          Type: Multiset Integer
```

The union of two multisets is additive.

```
  U := union(s,t)
    {1,5: 2,3: 3,4: 4,6,7,10,- 9}
                          Type: Multiset Integer
```

The intersect operation gives the elements that are in common, with additive multiplicity.

```
  I := intersect(s,t)
    {5: 2}
                          Type: Multiset Integer
```

```
The difference of s and t consists of the elements that s has but t
does not.  Elements are regarded as indistinguishable, so that if s
and t have any element in common, the difference does not contain that
element.

  difference(s,t)
    {1,3: 3,4: 4,6,7,10}
                          Type: Multiset Integer

The symmetricDifference is the union of difference(s, t) and difference(t, s).

  S := symmetricDifference(s,t)
    {1,3: 3,4: 4,6,7,10,- 9}
                          Type: Multiset Integer

Check that the union of the symmetricDifference and the intersect
equals the union of the elements.

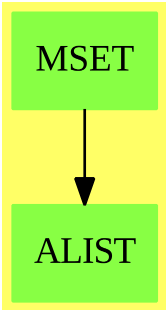
  (U = union(S,I))@Boolean
    true
                          Type: Boolean

Check some inclusion relations.

  t1 := multiset [1,2,2,3]; [t1 < t, t1 < s, t < s, t1 <= s]
    [false,true,false,true]
                          Type: List Boolean

See Also:
o )show Multiset
```

14.15.1 Multiset (MSET)



Exports:

any?	bag	brace	coerce	construct
convert	copy	count	dictionary	difference
duplicates	empty	empty?	eq?	eval
every?	extract!	find	hash	insert!
inspect	intersect	latex	less?	map
map!	members	member?	more?	multiset
parts	reduce	remove	remove!	removeDuplicates
sample	select	select!	set	size?
subset?	symmetricDifference	union	#?	?<?
?=?	?~=?			

— domain MSET Multiset —

```
)abbrev domain MSET Multiset
++ Author:Stephen M. Watt, William H. Burge, Richard D. Jenks, Frederic Lehobey
++ Date Created:NK
++ Date Last Updated: 14 June 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A multiset is a set with multiplicities.

Multiset(S: SetCategory): MultisetAggregate S with
        finiteAggregate
        shallowlyMutable
        multiset: () -> %
          ++ multiset()$D creates an empty multiset of domain D.
        multiset: S -> %
          ++ multiset(s) creates a multiset with singleton s.
```

```
    multiset: List S -> %
      ++ multiset(ls) creates a multiset with elements from \spad{ls}.
    members: % -> List S
      ++ members(ms) returns a list of the elements of \spad{ms}
      ++ without their multiplicity. See also \spadfun{parts}.
    remove: (S,%,Integer) -> %
      ++ remove(x,ms,number) removes at most \spad{number} copies of
      ++ element x if \spad{number} is positive, all of them if
      ++ \spad{number} equals zero, and all but at most \spad{-number} if
      ++ \spad{number} is negative.
    remove: ( S -> Boolean ,%,Integer) -> %
      ++ remove(p,ms,number) removes at most \spad{number} copies of
      ++ elements x such that \spad{p(x)} is \spadfun{true}
      ++ if \spad{number} is positive, all of them if
      ++ \spad{number} equals zero, and all but at most \spad{-number} if
      ++ \spad{number} is negative.
    remove_!: (S,%,Integer) -> %
      ++ remove!(x,ms,number) removes destructively at most \spad{number}
      ++ copies of element x if \spad{number} is positive, all
      ++ of them if \spad{number} equals zero, and all but at most
      ++ \spad{-number} if \spad{number} is negative.
    remove_!: ( S -> Boolean ,%,Integer) -> %
      ++ remove!(p,ms,number) removes destructively at most \spad{number}
      ++ copies of elements x such that \spad{p(x)} is
      ++ \spadfun{true} if \spad{number} is positive, all of them if
      ++ \spad{number} equals zero, and all but at most \spad{-number} if
      ++ \spad{number} is negative.

== add

    Tbl ==> Table(S, Integer)
    tbl ==> table$Tbl
    Rep := Record(count: Integer, table: Tbl)

    n: Integer
    ms, m1, m2: %
    t,  t1, t2: Tbl
    D ==> Record(entry: S, count: NonNegativeInteger)
    K ==> Record(key: S, entry: Integer)

    elt(t:Tbl, s:S):Integer ==
      a := search(s,t)$Tbl
      a case "failed" => 0
      a::Integer

    empty():% == [0,tbl()]
    multiset():% == empty()
    dictionary():% == empty() -- DictionaryOperations
    set():% == empty()
    brace():% == empty()
```

```
construct(l:List S):% ==
    t := tbl()
    n := 0
    for e in l repeat
      t.e := inc t.e
      n := inc n
    [n, t]
multiset(l:List S):% == construct l
bag(l:List S):% == construct l         -- BagAggregate
dictionary(l:List S):% == construct l -- DictionaryOperations
set(l:List S):% == construct l
brace(l:List S):% == construct l

multiset(s:S):% == construct [s]

if S has ConvertibleTo InputForm then
  convert(ms:%):InputForm ==
    convert [convert("multiset"::Symbol)@InputForm,
     convert(parts ms)@InputForm]

members(ms:%):List S == keys ms.table

coerce(ms:%):OutputForm ==
    l: List OutputForm := empty()
    t := ms.table
    colon := ": " :: OutputForm
    for e in keys t repeat
        ex := e::OutputForm
        n := t.e
        item :=
          n > 1 => hconcat [n :: OutputForm,colon, ex]
          ex
        l := cons(item,l)
    brace l

duplicates(ms:%):List D == -- MultiDictionary
  ld : List D := empty()
  t := ms.table
  for e in keys t | (n := t.e) > 1 repeat
    ld := cons([e,n::NonNegativeInteger],ld)
  ld

extract_!(ms:%):S ==         -- BagAggregate
  empty? ms => error "extract: Empty multiset"
  ms.count := dec ms.count
  t := ms.table
  e := inspect(t).key
  if (n := t.e) > 1 then t.e := dec n
   else remove_!(e,t)
```

```
  e

inspect(ms:%):S == inspect(ms.table).key  -- BagAggregate

insert_!(e:S,ms:%):% ==                  -- BagAggregate
    ms.count   := inc ms.count
    ms.table.e := inc ms.table.e
    ms

member?(e:S,ms:%):Boolean == member?(e,keys ms.table)

empty?(ms:%):Boolean == ms.count = 0

#(ms:%):NonNegativeInteger == ms.count::NonNegativeInteger

count(e:S, ms:%):NonNegativeInteger == ms.table.e::NonNegativeInteger

remove_!(e:S, ms:%, max:Integer):% ==
  zero? max => remove_!(e,ms)
  t := ms.table
  if member?(e, keys t) then
    ((n := t.e) <= max) =>
      remove_!(e,t)
      ms.count := ms.count-n
    max > 0 =>
      t.e := n-max
      ms.count := ms.count-max
    (n := n+max) > 0 =>
      t.e := -max
      ms.count := ms.count-n
  ms

remove_!(p: S -> Boolean, ms:%, max:Integer):% ==
  zero? max => remove_!(p,ms)
  t := ms.table
  for e in keys t | p(e) repeat
    ((n := t.e) <= max) =>
      remove_!(e,t)
      ms.count := ms.count-n
    max > 0 =>
      t.e := n-max
      ms.count := ms.count-max
    (n := n+max) > 0 =>
      t.e := -max
      ms.count := ms.count-n
  ms

remove(e:S, ms:%, max:Integer):% == remove_!(e, copy ms, max)

remove(p: S -> Boolean,ms:%,max:Integer):% == remove_!(p, copy ms, max)
```

```
remove_!(e:S, ms:%):% == -- DictionaryOperations
  t := ms.table
  if member?(e, keys t) then
    ms.count := ms.count-t.e
    remove_!(e, t)
  ms

remove_!(p:S ->Boolean, ms:%):% == -- DictionaryOperations
  t := ms.table
  for e in keys t | p(e) repeat
    ms.count := ms.count-t.e
    remove_!(e, t)
  ms

select_!(p: S -> Boolean, ms:%):% == -- DictionaryOperations
  remove_!((s1:S):Boolean+->not p(s1), ms)

removeDuplicates_!(ms:%):% == -- MultiDictionary
  t := ms.table
  l := keys t
  for e in l repeat t.e := 1
  ms.count := #l
  ms

insert_!(e:S,ms:%,more:NonNegativeInteger):% == -- MultiDictionary
    ms.count   := ms.count+more
    ms.table.e := ms.table.e+more
    ms

map_!(f: S->S, ms:%):% == -- HomogeneousAggregate
  t := ms.table
  t1 := tbl()
  for e in keys t repeat
    t1.f(e) := t.e
    remove_!(e, t)
  ms.table := t1
  ms

map(f: S -> S, ms:%):% == map_!(f, copy ms) -- HomogeneousAggregate

parts(m:%):List S ==
  l := empty()$List(S)
  t := m.table
  for e in keys t repeat
    for i in 1..t.e repeat
      l := cons(e,l)
  l

union(m1:%, m2:%):% ==
```

```
            t := tbl()
            t1:= m1.table
            t2:= m2.table
            for e in keys t1 repeat t.e := t1.e
            for e in keys t2 repeat t.e := t2.e + t.e
            [m1.count + m2.count, t]

        intersect(m1:%, m2:%):% ==
--          if #m1 > #m2 then intersect(m2, m1)
            t := tbl()
            t1:= m1.table
            t2:= m2.table
            n := 0
            for e in keys t1 repeat
              m := min(t1.e,t2.e)
              m > 0 =>
                m := t1.e + t2.e
                t.e := m
                n := n + m
            [n, t]

        difference(m1:%, m2:%):% ==
            t := tbl()
            t1:= m1.table
            t2:= m2.table
            n := 0
            for e in keys t1 repeat
              k1 := t1.e
              k2 := t2.e
              k1 > 0 and k2 = 0 =>
                t.e := k1
                n := n + k1
            n = 0 => empty()
            [n, t]

        symmetricDifference(m1:%, m2:%):% ==
            union(difference(m1,m2), difference(m2,m1))

        m1 = m2 ==
            m1.count ^= m2.count => false
            t1 := m1.table
            t2 := m2.table
            for e in keys t1 repeat
                t1.e ^= t2.e => return false
            for e in keys t2 repeat
                t1.e ^= t2.e => return false
            true

        m1 < m2 ==
            m1.count >= m2.count => false
```

```
            t1 := m1.table
            t2 := m2.table
            for e in keys t1 repeat
                t1.e > t2.e => return false
            m1.count < m2.count

        subset?(m1:%, m2:%):Boolean ==
            m1.count > m2.count => false
            t1 := m1.table
            t2 := m2.table
            for e in keys t1 repeat t1.e > t2.e => return false
            true
```

— MSET.dotabb —

```
"MSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MSET"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"MSET" -> "ALIST"
```

14.16 domain MPOLY MultivariatePolynomial

— MultivariatePolynomial.input —

```
)set break resume
)sys rm -f MultivariatePolynomial.output
)spool MultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 9
m : MPOLY([x,y],INT) := (x^2 - x*y^3 +3*y)^2
--R
--R
--R         4     3 3     6       2     4      2
--R   (1)  x  - 2y x  + (y  + 6y)x  - 6y x + 9y
--R                                Type: MultivariatePolynomial([x,y],Integer)
--E 1

--S 2 of 9
m :: MPOLY([y,x],INT)
```

```
--R
--R
--R         2 6       4     3 3     2     2     4
--R   (2)  x y  - 6x y  - 2x y  + 9y  + 6x y + x
--R                                    Type: MultivariatePolynomial([y,x],Integer)
--E 2

--S 3 of 9
p : MPOLY([x,y],POLY INT)
--R
--R                                                                   Type: Void
--E 3

--S 4 of 9
p :: POLY INT
--R
--R
--R   (4)  p
--R                                                    Type: Polynomial Integer
--E 4

--S 5 of 9
% :: MPOLY([a,b],POLY INT)
--R
--R
--R   (5)  p
--R                       Type: MultivariatePolynomial([a,b],Polynomial Integer)
--E 5

--S 6 of 9
q : UP(x, FRAC MPOLY([y,z],INT))
--R
--R                                                                   Type: Void
--E 6

--S 7 of 9
q := (x^2 - x*(z+1)/y +2)^2
--R
--R
--R                             2    2
--R         4   - 2z - 2  3   4y  + z  + 2z + 1  2   - 4z - 4
--R   (7)  x  + -------- x  + ----------------- x  + -------- x + 4
--R                 y                  2                 y
--R                                   y
--R Type: UnivariatePolynomial(x,Fraction MultivariatePolynomial([y,z],Integer))
--E 7

--S 8 of 9
q :: UP(z, FRAC MPOLY([x,y],INT))
--R
```

```
--R
--R   (8)
--R    2            3     2            2 4      3      2      2           2
--R   x   2   - 2y x  + 2x  - 4y x     y x  - 2y x  + (4y  + 1)x  - 4y x + 4y
--R   -- z  + ------------------- z + -------------------------------------
--R    2              2                                 2
--R   y              y                                 y
--R Type: UnivariatePolynomial(z,Fraction MultivariatePolynomial([x,y],Integer))
--E 8

--S 9 of 9
q :: MPOLY([x,z], FRAC UP(y,INT))
--R
--R
--R                                                  2
--R         4      2     2  3     1  2    2     4y  + 1  2      4     4
--R   (9)  x  + (- - z - -)x  + (-- z  + -- z + -------)x  + (- - z - -)x + 4
--R                y     y        2       2         2           y     y
--R                              y       y         y
--R Type: MultivariatePolynomial([x,z],Fraction UnivariatePolynomial(y,Integer))
--E 9
)spool
)lisp (bye)
```

———-

— MultivariatePolynomial.help —

```
====================================================================
MultivariatePolynomial examples
====================================================================

The domain constructor MultivariatePolynomial is similar to Polynomial
except that it specifies the variables to be used.  Polynomial are
available for MultivariatePolynomial.  The abbreviation for
MultivariatePolynomial is MPOLY.  The type expressions

  MultivariatePolynomial([x,y],Integer)
  MPOLY([x,y],INT)

refer to the domain of multivariate polynomials in the variables x and
y where the coefficients are restricted to be integers.  The first
variable specified is the main variable and the display of the polynomial
reflects this.

This polynomial appears with terms in descending powers of the variable x.

  m : MPOLY([x,y],INT) := (x^2 - x*y^3 +3*y)^2
     4     3 3     6       2     4      2
```

```
    x  - 2y x  + (y  + 6y)x  - 6y x + 9y
                     Type: MultivariatePolynomial([x,y],Integer)
```

It is easy to see a different variable ordering by doing a conversion.

```
  m :: MPOLY([y,x],INT)
     2 6       4     3 3     2     2     4
    x y  - 6x y  - 2x y  + 9y  + 6x y + x
                     Type: MultivariatePolynomial([y,x],Integer)
```

You can use other, unspecified variables, by using Polynomial in the coefficient type of MPOLY.

```
  p : MPOLY([x,y],POLY INT)
                     Type: Void
```

Conversions can be used to re-express such polynomials in terms of the other variables. For example, you can first push all the variables into a polynomial with integer coefficients.

```
  p :: POLY INT
    p
                     Type: Polynomial Integer
```

Now pull out the variables of interest.

```
  % :: MPOLY([a,b],POLY INT)
    p
                     Type: MultivariatePolynomial([a,b],Polynomial Integer)
```

Restriction:
  Axiom does not allow you to create types where MultivariatePolynomial
  is contained in the coefficient type of Polynomial. Therefore,
  MPOLY([x,y],POLY INT) is legal but POLY MPOLY([x,y],INT) is not.

Multivariate polynomials may be combined with univariate polynomials to create types with special structures.

```
  q : UP(x, FRAC MPOLY([y,z],INT))
                          Type: Void
```

This is a polynomial in x whose coefficients are quotients of polynomials in y and z.

```
  q := (x^2 - x*(z+1)/y +2)^2
                                 2    2
        4   - 2z - 2  3   4y  + z  + 2z + 1  2   - 4z - 4
   (7)  x  + -------- x  + ---------------- x  + -------- x + 4
                y                  2                 y
                                  y
```

```
 Type: UnivariatePolynomial(x,Fraction MultivariatePolynomial([y,z],Integer))

Use conversions for structural rearrangements. z does not appear in a
denominator and so it can be made the main variable.

  q :: UP(z, FRAC MPOLY([x,y],INT))
    2           3     2             2 4       3      2      2            2
   x   2   - 2y x  + 2x  - 4y x     y x  - 2y x  + (4y  + 1)x  - 4y x + 4y
   -- z  + ------------------- z + --------------------------------------
    2              2                                  2
   y              y                                  y
 Type: UnivariatePolynomial(z,Fraction MultivariatePolynomial([x,y],Integer))

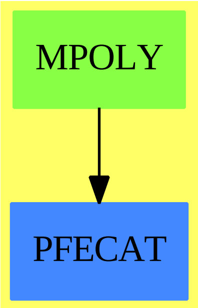
Or you can make a multivariate polynomial in x and z whose
coefficients are fractions in polynomials in y.

  q :: MPOLY([x,z], FRAC UP(y,INT))
    4      2     2  3     1  2    2     4y  + 1  2      4     4
   x  + (- - z - -)x  + (-- z  + -- z + -------)x  + (- - z - -)x + 4
           y     y        2      2         2           y     y
                         y      y         y
 Type: MultivariatePolynomial([x,z],Fraction UnivariatePolynomial(y,Integer))

A conversion like q :: MPOLY([x,y], FRAC UP(z,INT)) is not possible in
this example because y appears in the denominator of a fraction.  As
you can see, Axiom provides extraordinary flexibility in the
manipulation and display of expressions via its conversion facility.

See Also:
o )help DistributedMultivariatePolynomial
o )help UnivariatePolynomial
o )help Polynomial
o )show MultivariatePolynomial
```

14.16.1 MultivariatePolynomial (MPOLY)



See

⇒ "Polynomial" (POLY) 17.25.1 on page 2037
⇒ "SparseMultivariatePolynomial" (SMP) 20.14.1 on page 2381
⇒ "IndexedExponents" (INDE) 10.9.1 on page 1183

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	isExpt
isPlus	isTimes	latex
lcm	leadingCoefficient	leadingMonomial
mainVariable	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multivariate	numberOfMonomials
one?	patternMatch	pomopo!
prime?	primitivePart	primitiveMonomials
recip	reducedSystem	reductum
resultant	retract	retractIfCan
sample	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?<?	?<=?
?>?	?>=?	

— domain MPOLY MultivariatePolynomial —

```
)abbrev domain MPOLY MultivariatePolynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd
++ Related Constructors: SparseMultivariatePolynomial, Polynomial
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate
++ References:
++ Description:
++ This type is the basic representation of sparse recursive multivariate
++ polynomials whose variables are from a user specified list of symbols.
++ The ordering is specified by the position of the variable in the list.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.
```

```
MultivariatePolynomial(vl:List Symbol, R:Ring)
   ==  SparseMultivariatePolynomial(--SparseUnivariatePolynomial,
           R, OrderedVariableList vl)
```

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— MPOLY.dotabb —

```
"MPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MPOLY"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"MPOLY" -> "PFECAT"
```

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14.17 domain MYEXPR MyExpression

— MyExpression.input —

```
)set break resume
)sys rm -f MyExpression.output
)spool MyExpression.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MyExpression
--R MyExpression(q: Symbol,R: Join(Ring,OrderedSet,IntegralDomain))  is a domain constructor
--R Abbreviation for MyExpression is MYEXPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MYEXPR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?*? : (Integer,%) -> %
--R ?*? : (%,%) -> %                     ?**? : (%,%) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R -? : % -> %                          ?-? : (%,%) -> %
--R ?/? : (%,%) -> %                     ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean              ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean               ?>=? : (%,%) -> Boolean
--R D : (%,Symbol) -> %                  D : (%,List Symbol) -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       applyQuote : (Symbol,%,%) -> %
```

```
--R applyQuote : (Symbol,%) -> %          associates? : (%,%) -> Boolean
--R belong? : BasicOperator -> Boolean    binomial : (%,%) -> %
--R box : List % -> %                     box : % -> %
--R coerce : Integer -> %                 coerce : % -> %
--R coerce : R -> %                       coerce : Symbol -> %
--R coerce : Kernel % -> %                coerce : % -> OutputForm
--R denominator : % -> %                  differentiate : (%,Symbol) -> %
--R distribute : (%,%) -> %               distribute : % -> %
--R elt : (BasicOperator,%,%) -> %        elt : (BasicOperator,%) -> %
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
--R eval : (%,Kernel %,%) -> %            factorial : % -> %
--R factorials : % -> %                   factorials : (%,Symbol) -> %
--R freeOf? : (%,Symbol) -> Boolean       freeOf? : (%,%) -> Boolean
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             height : % -> NonNegativeInteger
--R is? : (%,Symbol) -> Boolean           kernel : (BasicOperator,%) -> %
--R kernels : % -> List Kernel %          latex : % -> String
--R map : ((% -> %),Kernel %) -> %        max : (%,%) -> %
--R min : (%,%) -> %                      numerator : % -> %
--R one? : % -> Boolean                   paren : List % -> %
--R paren : % -> %                        permutation : (%,%) -> %
--R product : (%,Symbol) -> %             recip : % -> Union(%,"failed")
--R retract : % -> Integer                retract : % -> R
--R retract : % -> Symbol                 retract : % -> Kernel %
--R sample : () -> %                      subst : (%,Equation %) -> %
--R summation : (%,Symbol) -> %           tower : % -> List Kernel %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R variables : % -> List Symbol          zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (%,R) -> % if R has COMRING
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (%,Fraction Integer) -> % if R has INTDOM
--R ?*? : (Fraction Integer,%) -> % if R has INTDOM
--R ?**? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (SparseMultivariatePolynomial(R,Kernel %),SparseMultivariatePolynomial(R,Kernel %))
--R D : (%,Symbol,NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> %
--R ?^? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R ?^? : (%,NonNegativeInteger) -> %
--R applyQuote : (Symbol,List %) -> %
--R applyQuote : (Symbol,%,%,%,%) -> %
--R applyQuote : (Symbol,%,%,%) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has INTDOM or R has RETRACT FRAC INT
--R coerce : Fraction MyUnivariatePolynomial(q,R) -> %
--R coerce : MyUnivariatePolynomial(q,R) -> %
```

```
--R coerce : Polynomial R -> % if R has RING
--R coerce : Fraction Polynomial R -> % if R has INTDOM
--R coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
--R coerce : Polynomial Fraction R -> % if R has INTDOM
--R coerce : Fraction R -> % if R has INTDOM
--R coerce : SparseMultivariatePolynomial(R,Kernel %) -> % if R has RING
--R commutator : (%,%) -> % if R has GROUP
--R conjugate : (%,%) -> % if R has GROUP
--R convert : % -> InputForm if R has KONVERT INFORM
--R convert : Factored % -> % if R has INTDOM
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
--R differentiate : (%,List Symbol) -> %
--R differentiate : (%,Symbol,NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R euclideanSize : % -> NonNegativeInteger if R has INTDOM
--R eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
--R eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
--R eval : (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> % if R has RING
--R eval : (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> % if R has RING
--R eval : (%,List BasicOperator,List %,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
--R eval : % -> % if R has KONVERT INFORM
--R eval : (%,List Symbol) -> % if R has KONVERT INFORM
--R eval : (%,Symbol) -> % if R has KONVERT INFORM
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %
--R eval : (%,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has INTDOM
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has INTDOM
--R factor : % -> Factored % if R has INTDOM
--R gcd : (%,%) -> % if R has INTDOM
--R gcd : List % -> % if R has INTDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R inv : % -> % if R has GROUP or R has INTDOM
```

```
--R is? : (%,BasicOperator) -> Boolean
--R isExpt : (%,Symbol) -> Union(Record(var: Kernel %,exponent: Integer),"failed") if R has R
--R isExpt : (%,BasicOperator) -> Union(Record(var: Kernel %,exponent: Integer),"failed") if
--R isExpt : % -> Union(Record(var: Kernel %,exponent: Integer),"failed") if R has SGROUP
--R isMult : % -> Union(Record(coef: Integer,var: Kernel %),"failed") if R has ABELSG
--R isPlus : % -> Union(List %,"failed") if R has ABELSG
--R isPower : % -> Union(Record(val: %,exponent: Integer),"failed") if R has RING
--R isTimes : % -> Union(List %,"failed") if R has SGROUP
--R kernel : (BasicOperator,List %) -> %
--R lcm : (%,%) -> % if R has INTDOM
--R lcm : List % -> % if R has INTDOM
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has INTDOM
--R numer : % -> SparseMultivariatePolynomial(R,Kernel %) if R has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R prime? : % -> Boolean if R has INTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has INTDOM
--R product : (%,SegmentBinding %) -> %
--R ?quo? : (%,%) -> % if R has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT and R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R) if R has RING
--R ?rem? : (%,%) -> % if R has INTDOM
--R retract : % -> Fraction Integer if R has INTDOM and R has RETRACT INT or R has RETRACT FR
--R retract : % -> Fraction MyUnivariatePolynomial(q,R)
--R retract : % -> MyUnivariatePolynomial(q,R)
--R retract : % -> Polynomial R if R has RING
--R retract : % -> Fraction Polynomial R if R has INTDOM
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has INTDOM and R has RETRACT IN
--R retractIfCan : % -> Union(Integer,"failed")
--R retractIfCan : % -> Union(MyUnivariatePolynomial(q,R),"failed")
--R retractIfCan : % -> Union(Polynomial R,"failed") if R has RING
--R retractIfCan : % -> Union(Fraction Polynomial R,"failed") if R has INTDOM
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R sizeLess? : (%,%) -> Boolean if R has INTDOM
--R squareFree : % -> Factored % if R has INTDOM
--R squareFreePart : % -> % if R has INTDOM
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R summation : (%,SegmentBinding %) -> %
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

```
--R univariate : (%,Kernel %) -> Fraction SparseUnivariatePolynomial % if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

———-

— MyExpression.help —

```
====================================================================
MyExpression examples
====================================================================

See Also:
o )show MyExpression
```

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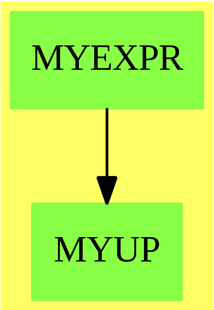
14.17.1 MyExpression (MYEXPR)



See

⇒ "MyUnivariatePolynomial" (MYUP) 14.18.1 on page 1658

Exports:

0	1	applyQuote	associates?
belong?	binomial	box	characteristic
charthRoot	coerce	commutator	conjugate
convert	D	definingPolynomial	denom
denominator	differentiate	distribute	divide
elt	euclideanSize	eval	even?
expressIdealMember	exquo	extendedEuclidean	factor
factorial	factorials	freeOf?	gcd
gcdPolynomial	ground	ground?	hash
height	inv	is?	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	numer
numerator	odd?	one?	operator
operators	paren	patternMatch	permutation
prime?	principalIdeal	product	recip
reducedSystem	retract	retractIfCan	sample
sizeLess?	squareFree	squareFreePart	subst
subtractIfCan	summation	tower	unit?
unitCanonical	unitNormal	univariate	variables
zero?	?*?	?**?	?+?
-?	?-?	?/?	?<?
?<=?	?=?	?>?	?>=?
?^?	?~=?	?quo?	?rem?

— domain MYEXPR MyExpression —

```
)abbrev domain MYEXPR MyExpression
++ Author: Mark Botch
++ Description:
++ This domain has no description

MyExpression(q: Symbol, R): Exports == Implementation where

  R: Join(Ring, OrderedSet, IntegralDomain)
  UP ==> MyUnivariatePolynomial(q, R)

  Exports == Join(FunctionSpace R, IntegralDomain,
               RetractableTo UP, RetractableTo Symbol,
               RetractableTo Integer, CombinatorialOpsCategory,
               PartialDifferentialRing Symbol) with
              _* : (%,%) -> %
              _/ : (%,%) -> %
              _*_* : (%,%) -> %
              numerator : % -> %
              denominator : % -> %
              ground? : % -> Boolean
```

```
            coerce: Fraction UP -> %
            retract: % -> Fraction UP

  Implementation == Expression R add
    Rep := Expression R

    iunivariate(p: Polynomial R): UP ==
        poly: SparseUnivariatePolynomial(Polynomial R)
             := univariate(p, q)$(Polynomial R)
        map((z1:Polynomial R):R +-> retract(z1), poly)_
          $UnivariatePolynomialCategoryFunctions2(Polynomial R,
                    SparseUnivariatePolynomial Polynomial R,
                    R, UP)

    retract(p: %): Fraction UP ==
        poly: Fraction Polynomial R := retract p
        upoly: UP := iunivariate numer poly
        vpoly: UP := iunivariate denom poly

        upoly / vpoly

    retract(p: %): UP == iunivariate retract p

    coerce(r: Fraction UP): % ==
       num: SparseUnivariatePolynomial R := makeSUP numer r
       den: SparseUnivariatePolynomial R := makeSUP denom r
       u: Polynomial R := multivariate(num, q)
       v: Polynomial R := multivariate(den, q)

       quot: Fraction Polynomial R := u/v

       quot::(Expression R)
```

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— **MYEXPR.dotabb** —

```
"MYEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MYEXPR"]
"MYUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MYUP"]
"MYEXPR" -> "MYUP"
```

———

14.18 domain MYUP MyUnivariatePolynomial

— MyUnivariatePolynomial.input —

```
)set break resume
)sys rm -f MyUnivariatePolynomial.output
)spool MyUnivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MyUnivariatePolynomial
--R MyUnivariatePolynomial(x: Symbol,R: Ring)  is a domain constructor
--R Abbreviation for MyUnivariatePolynomial is MYUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MYUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                     ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                D : % -> %
--R D : (%,NonNegativeInteger) -> %      1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> List R           coerce : Symbol -> %
--R coerce : Polynomial R -> %           coerce : Variable x -> %
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             degree : % -> NonNegativeInteger
--R differentiate : % -> %               ?.? : (%,%) -> %
--R ?.? : (%,R) -> R                     eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %                  eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %      ground : % -> R
--R ground? : % -> Boolean               hash : % -> SingleInteger
--R init : () -> % if R has STEP         latex : % -> String
--R leadingCoefficient : % -> R          leadingMonomial : % -> %
--R map : ((R -> R),%) -> %              monomial? : % -> Boolean
--R monomials : % -> List %              one? : % -> Boolean
--R primitiveMonomials : % -> List %     pseudoRemainder : (%,%) -> %
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R retract : % -> Symbol                retract : % -> R
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
```

```
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT INFORM
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has KONVERT PA
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has KONVERT PA
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
```

```
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R fmecg : (%,NonNegativeInteger,R,%) -> %
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"fai
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
```

```
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R ha
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)
```

———-

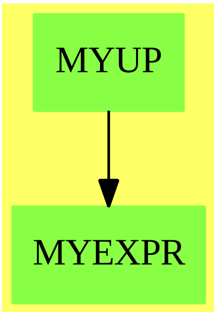
— **MyUnivariatePolynomial.help** —

```
====================================================================
MyUnivariatePolynomial examples
====================================================================

See Also:
o )show MyUnivariatePolynomial
```

———-

14.18.1 MyUnivariatePolynomial (MYUP)



See

⇒ "MyExpression" (MYEXPR) 14.17.1 on page 1651

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
composite	conditionP	content
convert	D	degree
differentiate	discriminant	divide
divideExponents	elt	euclideanSize
eval	expressIdealMember	exquo
extendedEuclidean	factor	factorPolynomial
factorSquareFreePolynomial	fmecg	gcd
gcdPolynomial	ground	ground?
hash	init	integrate
isExpt	isPlus	isTimes
karatsubaDivide	latex	lcm
leadingCoefficient	leadingMonomial	mainVariable
makeSUP	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multiEuclidean	multiplyExponents
multivariate	nextItem	numberOfMonomials
one?	order	patternMatch
pomopo!	prime?	primitiveMonomials
primitivePart	principalIdeal	pseudoDivide
pseudoQuotient	pseudoRemainder	recip
reducedSystem	reductum	resultant
retract	retractIfCan	sample
separate	shiftLeft	shiftRight
sizeLess?	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subResultantGcd
subtractIfCan	totalDegree	unit?
unitCanonical	unitNormal	univariate
unmakeSUP	variables	vectorise
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?.?
?~=?	?/?	?<?
?<=?	?>?	?>=?
?^?	?quo?	?rem?

— domain MYUP MyUnivariatePolynomial —

```
)abbrev domain MYUP MyUnivariatePolynomial
++ Author: Mark Botch
++ Description:
++ This domain has no description

MyUnivariatePolynomial(x:Symbol, R:Ring):
```

```
  UnivariatePolynomialCategory(R) with
    RetractableTo Symbol;
    coerce: Variable(x) -> %
      ++ coerce(x) converts the variable x to a univariate polynomial.
    fmecg: (%,NonNegativeInteger,R,%) -> %
        ++ fmecg(p1,e,r,p2) finds x : p1 - r * x**e * p2
    if R has univariate: (R, Symbol) -> SparseUnivariatePolynomial R
    then coerce: R -> %
    coerce: Polynomial R -> %
   == SparseUnivariatePolynomial(R) add
    Rep := SparseUnivariatePolynomial(R)
    coerce(p: %):OutputForm  == outputForm(p, outputForm x)
    coerce(x: Symbol): % == monomial(1, 1)
    coerce(v: Variable(x)):% == monomial(1, 1)
    retract(p: %): Symbol ==
        retract(p)@SingletonAsOrderedSet
        x
    if R has univariate: (R, Symbol) -> SparseUnivariatePolynomial R
    then coerce(p: R): % == univariate(p, x)$R

    coerce(p: Polynomial R): % ==
        poly: SparseUnivariatePolynomial(Polynomial R)
             := univariate(p, x)$(Polynomial R)
        map((z1:Polynomial R):R +-> retract(z1), poly)_
             $UnivariatePolynomialCategoryFunctions2(Polynomial R,
                    SparseUnivariatePolynomial Polynomial R, R, %)
```

— **MYUP.dotabb** —

```
"MYUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MYUP"]
"MYEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MYEXPR"]
"MYUP" -> "MYEXPR"
```

Chapter 15

Chapter N

15.1 domain NSDPS NeitherSparseOrDensePowerSeries

— NeitherSparseOrDensePowerSeries.input —

```
)set break resume
)sys rm -f NeitherSparseOrDensePowerSeries.output
)spool NeitherSparseOrDensePowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NeitherSparseOrDensePowerSeries
--R NeitherSparseOrDensePowerSeries K: Field  is a domain constructor
--R Abbreviation for NeitherSparseOrDensePowerSeries is NSDPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NSDPS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,K) -> %                      ?*? : (K,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        center : % -> K
--R children : % -> List %                coefOfFirstNonZeroTerm : % -> K
--R coefficient : (%,Integer) -> K        coerce : Fraction Integer -> %
```

```
--R coerce : % -> %                     coerce : Integer -> %
--R coerce : % -> OutputForm            complete : % -> %
--R concat : (%,%) -> %                 concat : List % -> %
--R copy : % -> %                       cycleEntry : % -> %
--R cycleTail : % -> %                  cyclic? : % -> Boolean
--R degree : % -> Integer               delay : (() -> %) -> %
--R delete : (%,Integer) -> %           distance : (%,%) -> Integer
--R ?.rest : (%,rest) -> %              ?.? : (%,Integer) -> K
--R empty : () -> %                     empty? : % -> Boolean
--R eq? : (%,%) -> Boolean              explicitEntries? : % -> Boolean
--R explicitlyEmpty? : % -> Boolean     explicitlyFinite? : % -> Boolean
--R extend : (%,Integer) -> %           factor : % -> Factored %
--R filterUpTo : (%,Integer) -> %       findCoef : (%,Integer) -> K
--R gcd : List % -> %                   gcd : (%,%) -> %
--R hash : % -> SingleInteger           index? : (Integer,%) -> Boolean
--R indices : % -> List Integer         insert : (%,%,Integer) -> %
--R inv : % -> %                        latex : % -> String
--R lazy? : % -> Boolean                lazyEvaluate : % -> %
--R lcm : List % -> %                   lcm : (%,%) -> %
--R leadingCoefficient : % -> K         leadingMonomial : % -> %
--R leaf? : % -> Boolean                map : ((K -> K),%) -> %
--R monomial : (K,Integer) -> %         monomial? : % -> Boolean
--R nodes : % -> List %                 one? : % -> Boolean
--R order : % -> Integer                order : % -> Integer
--R order : (%,Integer) -> Integer      pole? : % -> Boolean
--R posExpnPart : % -> %                possiblyInfinite? : % -> Boolean
--R prime? : % -> Boolean               printInfo : () -> Boolean
--R printInfo : Boolean -> Boolean      ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")      reductum : % -> %
--R ?rem? : (%,%) -> %                  removeFirstZeroes : % -> %
--R removeZeroes : % -> %               removeZeroes : (Integer,%) -> %
--R rest : % -> %                       rst : % -> %
--R sample : () -> %                    sbt : (%,%) -> %
--R series : (Integer,K,%) -> %         shift : (%,Integer) -> %
--R sizeLess? : (%,%) -> Boolean        squareFree : % -> Factored %
--R squareFreePart : % -> %             tail : % -> %
--R truncate : (%,Integer) -> %         unit? : % -> Boolean
--R unitCanonical : % -> %              variable : % -> Symbol
--R zero? : % -> Boolean                ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,K) -> % if K has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has 
--R D : (%,Symbol,NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has PDRING SYM
--R D : (%,List Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R D : (%,Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R D : % -> % if K has *: (Integer,K) -> K
--R ?^? : (%,NonNegativeInteger) -> %
```

```
--R any? : ((Record(k: Integer,c: K) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R approximate : (%,Integer) -> K if K has **: (K,Integer) -> K and K has coerce: Symbol -> K
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if K has CHARNZ
--R child? : (%,%) -> Boolean if Record(k: Integer,c: K) has SETCAT
--R coerce : % -> Stream Record(k: Integer,c: K)
--R coerce : Stream Record(k: Integer,c: K) -> %
--R coerce : K -> % if K has COMRING
--R concat : (Record(k: Integer,c: K),%) -> %
--R concat : (%,Record(k: Integer,c: K)) -> %
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R concat! : (%,Record(k: Integer,c: K)) -> % if $ has shallowlyMutable
--R construct : List Record(k: Integer,c: K) -> %
--R convert : % -> InputForm if Record(k: Integer,c: K) has KONVERT INFORM
--R count : ((Record(k: Integer,c: K) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Record(k: Integer,c: K),%) -> NonNegativeInteger if $ has finiteAggregate and Record(k: Int
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has PDRING SYM
--R differentiate : (%,List Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R differentiate : % -> % if K has *: (Integer,K) -> K
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.value : (%,value) -> Record(k: Integer,c: K)
--R ?.first : (%,first) -> Record(k: Integer,c: K)
--R ?.last : (%,last) -> Record(k: Integer,c: K)
--R ?.? : (%,UniversalSegment Integer) -> %
--R ?.? : (%,Integer) -> Record(k: Integer,c: K)
--R elt : (%,Integer,Record(k: Integer,c: K)) -> Record(k: Integer,c: K)
--R ?.? : (%,%) -> % if Integer has SGROUP
--R entries : % -> List Record(k: Integer,c: K)
--R entry? : (Record(k: Integer,c: K),%) -> Boolean if $ has finiteAggregate and Record(k: Integer,c: K)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,List Equation Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K) has EVALAB Record(k
--R eval : (%,Equation Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K) has EVALAB Record(k: Int
--R eval : (%,Record(k: Integer,c: K),Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K) has EVALA
--R eval : (%,List Record(k: Integer,c: K),List Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K)
--R eval : (%,K) -> Stream K if K has **: (K,Integer) -> K
--R every? : ((Record(k: Integer,c: K) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R fill! : (%,Record(k: Integer,c: K)) -> % if $ has shallowlyMutable
--R find : ((Record(k: Integer,c: K) -> Boolean),%) -> Union(Record(k: Integer,c: K),"failed")
--R findTerm : (%,Integer) -> Record(k: Integer,c: K)
--R first : % -> Record(k: Integer,c: K)
```

```
--R first : (%,NonNegativeInteger) -> %
--R frst : % -> Record(k: Integer,c: K)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R insert : (Record(k: Integer,c: K),%,Integer) -> %
--R last : % -> Record(k: Integer,c: K)
--R last : (%,NonNegativeInteger) -> %
--R leaves : % -> List Record(k: Integer,c: K)
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Record(k: Integer,c: K) -> Record(k: Integer,c: K)),%) -> %
--R map : (((Record(k: Integer,c: K),Record(k: Integer,c: K)) -> Record(k: Integer,c: K)),%,%
--R map! : ((Record(k: Integer,c: K) -> Record(k: Integer,c: K)),%) -> % if $ has shallowlyMu
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Record(k: Integer,c: K),%) -> Boolean if $ has finiteAggregate and Record(k: I
--R members : % -> List Record(k: Integer,c: K) if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial2series : (List %,List NonNegativeInteger,Integer) -> %
--R more? : (%,NonNegativeInteger) -> Boolean
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R multiplyExponents : (%,PositiveInteger) -> %
--R new : (NonNegativeInteger,Record(k: Integer,c: K)) -> %
--R node? : (%,%) -> Boolean if Record(k: Integer,c: K) has SETCAT
--R numberOfComputedEntries : % -> NonNegativeInteger
--R orderIfNegative : % -> Union(Integer,"failed")
--R parts : % -> List Record(k: Integer,c: K) if $ has finiteAggregate
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R qelt : (%,Integer) -> Record(k: Integer,c: K)
--R qsetelt! : (%,Integer,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shall
--R reduce : (((Record(k: Integer,c: K),Record(k: Integer,c: K)) -> Record(k: Integer,c: K))
--R reduce : (((Record(k: Integer,c: K),Record(k: Integer,c: K)) -> Record(k: Integer,c: K))
--R reduce : (((Record(k: Integer,c: K),Record(k: Integer,c: K)) -> Record(k: Integer,c: K))
--R remove : (Record(k: Integer,c: K),%) -> % if $ has finiteAggregate and Record(k: Integer
--R remove : ((Record(k: Integer,c: K) -> Boolean),%) -> %
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(k: Integer,c: K) has SETCA
--R rest : (%,NonNegativeInteger) -> %
--R second : % -> Record(k: Integer,c: K)
--R select : ((Record(k: Integer,c: K) -> Boolean),%) -> %
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallowly
--R setelt : (%,first,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallowly
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,last,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallowlyMu
--R setelt : (%,UniversalSegment Integer,Record(k: Integer,c: K)) -> Record(k: Integer,c: K)
--R setelt : (%,Integer,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallow
--R setfirst! : (%,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallowlyMuta
--R setlast! : (%,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallowlyMutab
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,Record(k: Integer,c: K)) -> Record(k: Integer,c: K) if $ has shallowlyMuta
--R size? : (%,NonNegativeInteger) -> Boolean
```

```
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R terms : % -> Stream Record(k: Integer,c: K)
--R third : % -> Record(k: Integer,c: K)
--R truncate : (%,Integer,Integer) -> %
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R value : % -> Record(k: Integer,c: K)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———-

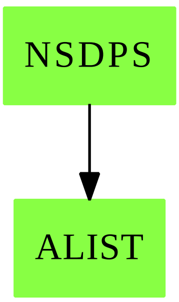
— **NeitherSparseOrDensePowerSeries.help** —

```
====================================================================
NeitherSparseOrDensePowerSeries examples
====================================================================

See Also:
o )show NeitherSparseOrDensePowerSeries
```

———-

15.1.1 NeitherSparseOrDensePowerSeries (NSDPS)



Exports:

0	1	#?
-?	?**?	?*?
?+?	?-?	?.?
?.first	?.last	?.rest
?.value	?/?	?=?
?^?	?~=?	?quo?
?rem?	D	any?
approximate	associates?	center
characteristic	charthRoot	child?
children	coefOfFirstNonZeroTerm	coefficient
coerce	complete	concat
concat!	construct	convert
copy	count	cycleEntry
cycleLength	cycleSplit!	cycleTail
cyclic?	degree	delay
delete	differentiate	distance
divide	elt	empty
empty?	entries	entry?
eq?	euclideanSize	eval
every?	explicitEntries?	explicitlyEmpty?
explicitlyFinite?	expressIdealMember	exquo
extend	extendedEuclidean	factor
fill!	filterUpTo	find
findCoef	findTerm	first
frst	gcd	gcdPolynomial
hash	index?	indices
insert	inv	last
latex	lazy?	lazyEvaluate
lcm	leadingCoefficient	leadingMonomial
leaf?	leaves	less?
map	map!	maxIndex
member?	members	minIndex
monomial	monomial2series	monomial?
more?	multiEuclidean	multiplyExponents
new	node?	nodes
numberOfComputedEntries	one?	order
orderIfNegative	parts	pole?
posExpnPart	possiblyInfinite?	prime?
principalIdeal	printInfo	qelt
qsetelt!	recip	reduce
reductum	remove	removeDuplicates
removeFirstZeroes	removeZeroes	rest
rst	sample	sbt
second	select	series
setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!
shift	size?	sizeLess?
split!	squareFree	squareFreePart
subtractIfCan	swap!	tail
terms	third	truncate
unit?	unitCanonical	unitNormal
value	variable	variables
zero?		

— domain NSDPS NeitherSparseOrDensePowerSeries —

```
)abbrev domain NSDPS NeitherSparseOrDensePowerSeries
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
NeitherSparseOrDensePowerSeries(K):Exports == Implementation where
  K:Field

  SI   ==> SingleInteger
  INT  ==> Integer
  TERM ==> Record(k:INT,c:K)
  SER  ==> Stream(TERM)
  NNI  ==> NonNegativeInteger

  Exports ==> Join(LocalPowerSeriesCategory(K),LazyStreamAggregate(TERM)) with

    findTerm: (%,Integer) -> TERM

  Implementation ==> SER add

    Rep:=SER

    var : Symbol := 't

    multC: (K,INT,%) -> %

    orderIfNegative(s:%)==
      zero?(s) => "failed"
      f:=frst(s)
      f.k >= 0 => "failed"
      zero?(f.c) => orderIfNegative(rest(s))
      f.k

    posExpnPart(s)==
      zero?(s) => 0
      o:=order s
      (o >= 0) => s
      posExpnPart(rst s)

    findTerm(s,n)==
      empty?(s) =>  [n,0]$TERM
      f:=frst(s)
      f.k > n => [n,0]$TERM
      f.k = n => f
      findTerm(rst(s),n)
```

```
findCoef(s,i)==findTerm(s,i).c

coerce(s:%):SER == s::Rep

coerce(s:SER):%==s

localVarForPrintInfo:Boolean:=false()

printInfo==localVarForPrintInfo

printInfo(flag)==localVarForPrintInfo:=flag

outTerm: TERM -> OutputForm

removeZeroes(s)== delay
  zero?(s) => 0
  f:=frst(s)
  zero?(f.c) => removeZeroes(rst(s))
  concat(f,removeZeroes(rst(s)))

inv(ra)==
  a:=removeFirstZeroes ra
  o:=-order(a)
  aa:=shift(a,o)
  aai:=recip aa
  aai case "failed" => _
    error "Big problem in inv function from CreateSeries"
  shift(aai,o)

iDiv: (%,%,K) -> %
iDiv(x,y,ry0) == delay
 empty? x => 0$%
 sx:TERM:=frst x
 c0:K:=ry0 * sx.c
 nT:TERM:=[sx.k, c0]
 tc0:%:=series(sx.k,c0,0$%)
 concat(nT,iDiv(rst x - tc0 * rst y,y,ry0))

recip x ==
  empty? x => "failed"
  rh1:TERM:=frst x
  ^zero?(rh1.k) => "failed"
  ic:K:= inv(rh1.c)
  delay
    concat([0,ic]$TERM,iDiv(- ic * rst x,x,ic))

removeFirstZeroes(s)==
  zero?(s) => 0
  f:=frst(s)
  zero?(f.c) => removeFirstZeroes(rst(s))
```

```
    s

  sbt(sa,sbb)== delay
    sb:=removeFirstZeroes(sbb)
    o:=order sb
    ^(o > 0) => _
       error "Cannot substitute by a series of order less than 1  !!!!!"
    empty?(sa) or empty?(sb) => 0
    fa:TERM:=frst(sa)
    fb:TERM:=frst(sb)
    firstElem:TERM:=[fa.k*fb.k, fa.c*(fb.c**fa.k)]
    zero?(fa.c) => sbt(rst(sa),sb)
    concat(firstElem,  rest((fa.c) * sb ** (fa.k)) + sbt(rst(sa),sb)  )

  coerce(s:%):OutputForm==
    zero?(s) => "0" :: OutputForm
    count:SI:= _$streamCount$Lisp
    lstTerm:List TERM:=empty()
    rs:%:= s
    for i in 1..count while  ^empty?(rs) repeat
      fs:=frst rs
      rs:=rst rs
      lstTerm:=concat(lstTerm,fs)
    listOfOutTerm:List OutputForm:=_
      [outTerm(t) for t in lstTerm | ^zero?(t.c) ]
    out:OutputForm:=
      if empty?(listOfOutTerm) then
        "0" :: OutputForm
      else
        reduce("+", listOfOutTerm)
    empty?(rs) => out
    out +  ("..." :: OutputForm)

  outTerm(t)==
    ee:=t.k
    cc:=t.c
    oe:OutputForm:=ee::OutputForm
    oc:OutputForm:=cc::OutputForm
    symb:OutputForm:= var :: OutputForm
    one?(cc) and one?(ee) => symb
    zero?(ee) => oc
    one?(cc) => symb ** oe
    one?(ee) => oc * symb
    oc * symb ** oe

  removeZeroes(n,s)== delay
    n < 0 => s
    zero?(s) => 0
    f:=frst(s)
    zero?(f.c) => removeZeroes(n-1, rst(s))
```

```
  concat(f,removeZeroes(n-1, rst(s)))

order(s:%)==
  zero?(s) => error _
   "From order (PlaneCurveLocalPowerSeries): cannot compute the order of 0"
  f:=frst(s)
  zero?(f.c) => order(rest(s))
  f.k

monomial2series(lpar,lexp,sh)==
  shift(reduce("*",[s**e for s in lpar for e in lexp]),sh)

coefOfFirstNonZeroTerm(s:%)==
  zero?(s) => error _
   "From order (PlaneCurveLocalPowerSeries): cannot find the coefOfFirstNonZeroTerm"
  f:=frst(s)
  zero?(f.c) => coefOfFirstNonZeroTerm(rest(s))
  f.c

degreeOfTermLower?: (TERM,INT) -> Boolean
degreeOfTermLower?(t,n)== t.k < n

filterUpTo(s,n)==filterWhile(degreeOfTermLower?(#1,n),s)

series(exp,coef,s)==cons([exp,coef]$TERM,s)

a:% ** n:NNI == -- delay
  zero?(n) => 1
  expt(a,n :: PositiveInteger)$RepeatedSquaring(%)

0 == empty()

1 == construct([[0,1]$TERM])

zero?(a)==empty?(a::Rep)

shift(s,n)== delay
  zero?(s) => 0
  fs:=frst(s)
  es:=fs.k
  concat([es+n,fs.c]$TERM,shift(rest(s),n))

a:% + b:% == delay
    zero?(a) => b
    zero?(b) => a
    fa:=frst(a)
    fb:=frst(b)
    ea:=fa.k
    eb:=fb.k
    nc:K
```

```
        ea = eb => concat([ea,fa.c+fb.c]$TERM,rest(a) + rest(b))
        ea > eb => concat([eb,fb.c]$TERM,a + rest(b))
        eb > ea => concat([ea,fa.c]$TERM,rest(a) + b)

    - a:% == --delay
      multC( (-1) :: K , 0 , a)

    a:% - b:% == --delay
      a+(-b)

    multC(coef,n,s)== delay
        zero?(coef) => 0
        zero?(s) => 0
        f:=frst(s)
        concat([f.k+n,coef*f.c]$TERM,multC(coef,n,rest(s)))

    coef:K * s:% == delay
        zero?(coef) => 0
        zero?(s) => 0
        f:=frst(s)
        concat([f.k,coef*f.c]$TERM, coef *$% rest(s))

    s:% * coef:K == coef * s

    s1:% * s2:%== delay
        zero?(s1) or zero?(s2) => 0
        f1:TERM:=frst(s1)
        f2:TERM:=frst(s2)
        e1:INT:=f1.k; e2:INT:=f2.k
        c1:K:=f1.c;   c2:K:=f2.c
        concat([e1+e2,c1*c2]$TERM,_
               multC(c1,e1,rest(s2))+multC(c2,e2,rest(s1))+rest(s1)*rest(s2))
```

———-

— NSDPS.dotabb —

```
"NSDPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSDPS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"NSDPS" -> "ALIST"
```

———-

15.2 domain NSMP NewSparseMultivariatePolynomial

Based on the **PseudoRemainderSequence** package, the domain constructor **NewSparseMultivariatePolynomial** extends the constructor **SparseMultivariatePolynomial**. It also provides some additional operations related to polynomial system solving by means of triangular sets.

— **NewSparseMultivariatePolynomial.input** —

```
)set break resume
)sys rm -f NewSparseMultivariatePolynomial.output
)spool NewSparseMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NewSparseMultivariatePolynomial
--R NewSparseMultivariatePolynomial(R: Ring,VarSet: OrderedSet)  is a domain constructor
--R Abbreviation for NewSparseMultivariatePolynomial is NSMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NSMP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                     ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R D : (%,List VarSet) -> %             D : (%,VarSet) -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coefficients : % -> List R
--R coerce : VarSet -> %                 coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R deepestInitial : % -> %              deepestTail : % -> %
--R differentiate : (%,VarSet) -> %      eval : (%,VarSet,%) -> %
--R eval : (%,VarSet,R) -> %             eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %                  eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %      ground : % -> R
--R ground? : % -> Boolean               hash : % -> SingleInteger
--R head : % -> %                        headReduce : (%,%) -> %
--R headReduced? : (%,%) -> Boolean      infRittWu? : (%,%) -> Boolean
--R init : % -> %                        initiallyReduce : (%,%) -> %
--R iteratedInitials : % -> List %       latex : % -> String
--R lazyPquo : (%,%,VarSet) -> %         lazyPquo : (%,%) -> %
--R lazyPrem : (%,%,VarSet) -> %         lazyPrem : (%,%) -> %
--R leadingCoefficient : % -> R          leadingMonomial : % -> %
--R leastMonomial : % -> %               mainCoefficients : % -> List %
--R mainMonomial : % -> %                mainMonomials : % -> List %
```

```
--R map : ((R -> R),%) -> %              mdeg : % -> NonNegativeInteger
--R monic? : % -> Boolean                monicModulo : (%,%) -> %
--R monomial? : % -> Boolean             monomials : % -> List %
--R mvar : % -> VarSet                   normalized? : (%,%) -> Boolean
--R one? : % -> Boolean                  pquo : (%,%,VarSet) -> %
--R pquo : (%,%) -> %                    prem : (%,%,VarSet) -> %
--R prem : (%,%) -> %                    primitiveMonomials : % -> List %
--R quasiMonic? : % -> Boolean           recip : % -> Union(%,"failed")
--R reduced? : (%,List %) -> Boolean     reduced? : (%,%) -> Boolean
--R reductum : (%,VarSet) -> %           reductum : % -> %
--R retract : % -> VarSet                retract : % -> R
--R sample : () -> %                     supRittWu? : (%,%) -> Boolean
--R tail : % -> %                        variables : % -> List VarSet
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List VarSet,List NonNegativeInteger) -> %
--R D : (%,VarSet,NonNegativeInteger) -> %
--R LazardQuotient : (%,%,NonNegativeInteger) -> % if R has INTDOM
--R LazardQuotient2 : (%,%,%,NonNegativeInteger) -> % if R has INTDOM
--R RittWuCompare : (%,%) -> Union(Boolean,"failed")
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List VarSet,List NonNegativeInteger) -> %
--R coefficient : (%,VarSet,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents VarSet) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseMultivariatePolynomial(R,VarSet) -> %
--R coerce : % -> SparseMultivariatePolynomial(R,VarSet)
--R coerce : % -> Polynomial R if VarSet has KONVERT SYMBOL
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,VarSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> Polynomial R if VarSet has KONVERT SYMBOL
--R convert : % -> String if R has RETRACT INT and VarSet has KONVERT SYMBOL
--R convert : Polynomial R -> % if VarSet has KONVERT SYMBOL
--R convert : Polynomial Integer -> % if R has ALGEBRA INT and VarSet has KONVERT SYMBOL and not has(R,A
--R convert : Polynomial Fraction Integer -> % if R has ALGEBRA FRAC INT and VarSet has KONVERT SYMBOL
--R convert : % -> InputForm if R has KONVERT INFORM and VarSet has KONVERT INFORM
```

```
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT and VarSet has KONVERT PATTE
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT and VarSet has KONVERT PATTE
--R degree : (%,List VarSet) -> List NonNegativeInteger
--R degree : (%,VarSet) -> NonNegativeInteger
--R degree : % -> IndexedExponents VarSet
--R differentiate : (%,List VarSet,List NonNegativeInteger) -> %
--R differentiate : (%,VarSet,NonNegativeInteger) -> %
--R differentiate : (%,List VarSet) -> %
--R discriminant : (%,VarSet) -> % if R has COMRING
--R eval : (%,List VarSet,List %) -> %
--R eval : (%,List VarSet,List R) -> %
--R exactQuotient : (%,%) -> % if R has INTDOM
--R exactQuotient : (%,R) -> % if R has INTDOM
--R exactQuotient! : (%,%) -> % if R has INTDOM
--R exactQuotient! : (%,R) -> % if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %) if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePo
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcd : (R,%) -> R if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %) if R has INTDOM
--R halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %) if R has INTDOM
--R headReduced? : (%,List %) -> Boolean
--R initiallyReduced? : (%,List %) -> Boolean
--R initiallyReduced? : (%,%) -> Boolean
--R isExpt : % -> Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lastSubResultant : (%,%) -> % if R has INTDOM
--R lazyPremWithDefault : (%,%,VarSet) -> Record(coef: %,gap: NonNegativeInteger,remainder: 
--R lazyPremWithDefault : (%,%) -> Record(coef: %,gap: NonNegativeInteger,remainder: %)
--R lazyPseudoDivide : (%,%,VarSet) -> Record(coef: %,gap: NonNegativeInteger,quotient: %,re
--R lazyPseudoDivide : (%,%) -> Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder
--R lazyResidueClass : (%,%) -> Record(polnum: %,polden: %,power: NonNegativeInteger)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R leadingCoefficient : (%,VarSet) -> %
--R mainContent : % -> % if R has GCDDOM
--R mainPrimitivePart : % -> % if R has GCDDOM
--R mainSquareFreePart : % -> % if R has GCDDOM
--R mainVariable : % -> Union(VarSet,"failed")
--R mapExponents : ((IndexedExponents VarSet -> IndexedExponents VarSet),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List VarSet) -> List NonNegativeInteger
```

```
--R minimumDegree : (%,VarSet) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents VarSet
--R monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List VarSet,List NonNegativeInteger) -> %
--R monomial : (%,VarSet,NonNegativeInteger) -> %
--R monomial : (R,IndexedExponents VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
--R nextsubResultant2 : (%,%,%,%) -> % if R has INTDOM
--R normalized? : (%,List %) -> Boolean
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has
--R pomopo! : (%,R,IndexedExponents VarSet,%) -> %
--R primPartElseUnitCanonical : % -> % if R has INTDOM
--R primPartElseUnitCanonical! : % -> % if R has INTDOM
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,VarSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R primitivePart! : % -> % if R has GCDDOM
--R pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%) -> % if R has INTDOM
--R resultant : (%,%,VarSet) -> % if R has COMRING
--R retract : % -> SparseMultivariatePolynomial(R,VarSet)
--R retract : Polynomial R -> % if VarSet has KONVERT SYMBOL and not has(R,Algebra Fraction Integer) and
--R retract : Polynomial Integer -> % if R has ALGEBRA INT and VarSet has KONVERT SYMBOL and not has(R,A
--R retract : Polynomial Fraction Integer -> % if R has ALGEBRA FRAC INT and VarSet has KONVERT SYMBOL
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseMultivariatePolynomial(R,VarSet),"failed")
--R retractIfCan : Polynomial R -> Union(%,"failed") if VarSet has KONVERT SYMBOL and not has(R,Algebra
--R retractIfCan : Polynomial Integer -> Union(%,"failed") if R has ALGEBRA INT and VarSet has KONVERT S
--R retractIfCan : Polynomial Fraction Integer -> Union(%,"failed") if R has ALGEBRA FRAC INT and VarSet
--R retractIfCan : % -> Union(VarSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R ha
--R subResultantChain : (%,%) -> List % if R has INTDOM
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List VarSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
```

```
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,VarSet) -> SparseUnivariatePolynomial %
--R
--E 1

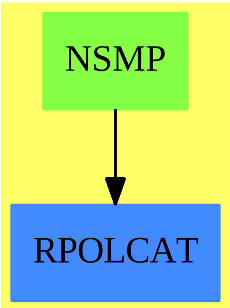
)spool
)lisp (bye)
```

— **NewSparseMultivariatePolynomial.help** —

```
====================================================================
NewSparseMultivariatePolynomial examples
====================================================================

See Also:
o )show NewSparseMultivariatePolynomial
```

15.2.1 NewSparseMultivariatePolynomial (NSMP)



See

⇒ “NewSparseUnivariatePolynomial” (NSUP) 15.3.1 on page 1691

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	D
degree	deepestInitial	deepestTail
differentiate	discriminant	eval
exactQuotient	exactQuotient!	exquo
extendedSubResultantGcd	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
halfExtendedSubResultantGcd1	ground	ground?
halfExtendedSubResultantGcd2	hash	head
headReduce	headReduced?	infRittWu?
init	initiallyReduce	initiallyReduced?
isExpt	isPlus	isTimes
iteratedInitials	lastSubResultant	latex
LazardQuotient	LazardQuotient2	lazyPremWithDefault
lazyPquo	lazyPrem	lazyPseudoDivide
lazyResidueClass	lcm	leadingCoefficient
leadingMonomial	leastMonomial	mainCoefficients
mainContent	mainMonomial	mainMonomials
mainPrimitivePart	mainSquareFreePart	mainVariable
map	mapExponents	max
mdeg	min	minimumDegree
monic?	monicDivide	monicModulo
monomial	monomial?	monomials
multivariate	mvar	nextsubResultant2
normalized?	numberOfMonomials	one?
patternMatch	pomopo!	pquo
primPartElseUnitCanonical!	primPartElseUnitCanonical	prem
prime?	primitiveMonomials	primitivePart
primitivePart!	pseudoDivide	reducedSystem
resultant	retract	retractIfCan
RittWuCompare	quasiMonic?	recip
reduced?	reductum	retract
sample	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subResultantChain
subResultantGcd	supRittWu?	subtractIfCan
tail	totalDegree	unit?
unitCanonical	unitNormal	univariate
univariate	variables	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

— domain NSMP NewSparseMultivariatePolynomial —

```
)abbrev domain NSMP NewSparseMultivariatePolynomial
++ Author: Marc Moreno Maza
++ Date Created: 22/04/94
++ Date Last Updated: 14/12/1998
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A post-facto extension for \axiomType{SMP} in order
++ to speed up operations related to pseudo-division and gcd.
++ This domain is based on the \axiomType{NSUP} constructor which is
++ itself a post-facto extension of the \axiomType{SUP} constructor.

NewSparseMultivariatePolynomial(R,VarSet) : Exports == Implementation where
  R:Ring
  VarSet:OrderedSet
  N ==> NonNegativeInteger
  Z ==> Integer
  SUP ==> NewSparseUnivariatePolynomial
  SMPR ==> SparseMultivariatePolynomial(R, VarSet)
  SUP2 ==> NewSparseUnivariatePolynomialFunctions2($,$)

  Exports == Join(RecursivePolynomialCategory(R,IndexedExponents VarSet,_
                  VarSet), CoercibleTo(SMPR),RetractableTo(SMPR))

  Implementation ==  SparseMultivariatePolynomial(R, VarSet) add

     D := NewSparseUnivariatePolynomial($)
     VPoly:=  Record(v:VarSet,ts:D)
     Rep:= Union(R,VPoly)

    --local function
     PSimp: (D,VarSet) -> %

     PSimp(up,mv) ==
       if degree(up) = 0 then leadingCoefficient(up) else [mv,up]$VPoly

     coerce (p:$):SMPR ==
       p pretend SMPR

     coerce (p:SMPR):$ ==
       p pretend $
```

```
retractIfCan (p:$) : Union(SMPR,"failed") ==
  (p pretend SMPR)::Union(SMPR,"failed")

mvar p ==
  p case R => error" Error in mvar from NSMP : #1 has no variables."
  p.v

mdeg p ==
  p case R => 0$N
  degree(p.ts)$D

init p ==
  p case R => error" Error in init from NSMP : #1 has no variables."
  leadingCoefficient(p.ts)$D

head p ==
  p case R => p
  ([p.v,leadingMonomial(p.ts)$D]$VPoly)::Rep

tail p ==
  p case R => 0$$
  red := reductum(p.ts)$D
  ground?(red)$D => (ground(red)$D)::Rep
  ([p.v,red]$VPoly)::Rep

iteratedInitials p ==
  p case R => []
  p := leadingCoefficient(p.ts)$D
  cons(p,iteratedInitials(p))

localDeepestInitial (p : $) : $ ==
  p case R => p
  localDeepestInitial leadingCoefficient(p.ts)$D

deepestInitial p ==
  p case R =>
    error"Error in deepestInitial from NSMP : #1 has no variables."
  localDeepestInitial leadingCoefficient(p.ts)$D

mainMonomial p ==
  zero? p =>
    error"Error in mainMonomial from NSMP : the argument is zero"
  p case R => 1$$
  monomial(1$$,p.v,degree(p.ts)$D)

leastMonomial p ==
  zero? p =>
    error"Error in leastMonomial from NSMP : the argument is zero"
  p case R => 1$$
  monomial(1$$,p.v,minimumDegree(p.ts)$D)
```

```
    mainCoefficients p ==
      zero? p =>
        error"Error in mainCoefficients from NSMP : the argument is zero"
      p case R => [p]
      coefficients(p.ts)$D

    leadingCoefficient(p:$,x:VarSet):$ ==
      (p case R) => p
      p.v = x => leadingCoefficient(p.ts)$D
      zero? (d := degree(p,x)) => p
      coefficient(p,x,d)

    localMonicModulo(a:$,b:$):$ ==
      -- b is assumed to have initial 1
      a case R => a
      a.v < b.v => a
      mM: $
      if a.v > b.v
        then
          m : D := map((a1:%):% +-> localMonicModulo(a1,b),a.ts)$SUP2
        else
          m : D := monicModulo(a.ts,b.ts)$D
      if ground?(m)$D
         then
           mM := (ground(m)$D)::Rep
         else
           mM := ([a.v,m]$VPoly)::Rep
      mM

    monicModulo (a,b) ==
      b case R => error"Error in monicModulo from NSMP : #2 is constant"
      ib : $ := init(b)@$
      not ground?(ib)$$ =>
        error"Error in monicModulo from NSMP : #2 is not monic"
      mM : $
--      if not one?(ib)$$
      if not ((ib) = 1)$$
        then
          r : R := ground(ib)$$
          rec : Union(R,"failed"):= recip(r)$R
          (rec case "failed") =>
            error"Error in monicModulo from NSMP : #2 is not monic"
          a case R => a
          a := (rec::R) * a
          b := (rec::R) * b
          mM := ib * localMonicModulo (a,b)
        else
          mM := localMonicModulo (a,b)
      mM
```

```
prem(a:$, b:$): $ ==
  -- with pseudoRemainder$NSUP
  b case R =>
    error "in prem$NSMP: ground? #2"
  db: N := degree(b.ts)$D
  lcb: $ := leadingCoefficient(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  delta: Z := max(test + 1$Z, 0$Z)
  (a case R) or (a.v < b.v) => lcb ** (delta::N) * a
  a.v = b.v =>
    r: D := pseudoRemainder(a.ts,b.ts)$D
    ground?(r) => return (ground(r)$D)::Rep
    ([a.v,r]$VPoly)::Rep
  while not zero?(a) and not negative?(test) repeat
    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    delta := delta - 1$Z
    test := degree(a,b.v)::Z - db
  lcb ** (delta::N) * a

pquo (a:$, b:$)  : $ ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

pseudoDivide(a:$, b:$): Record (quotient : $, remainder : $) ==
  -- from RPOLCAT
  cPS := lazyPseudoDivide(a,b)
  c := (cPS.coef) ** (cPS.gap)
  [c * cPS.quotient, c * cPS.remainder]

lazyPrem(a:$, b:$): $ ==
  -- with lazyPseudoRemainder$NSUP
  -- Uses leadingCoefficient: ($, V) -> $
  b case R =>
    error "in lazyPrem$NSMP: ground? #2"
  (a case R) or (a.v < b.v) =>  a
  a.v = b.v => PSimp(lazyPseudoRemainder(a.ts,b.ts)$D,a.v)
  db: N := degree(b.ts)$D
  lcb: $ := leadingCoefficient(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  while not zero?(a) and not negative?(test) repeat
    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    test := degree(a,b.v)::Z - db
  a

lazyPquo (a:$, b:$) : $ ==
  -- with lazyPseudoQuotient$NSUP
```

```
  b case R =>
    error " in lazyPquo$NSMP: #2 is conctant"
  (a case R) or (a.v < b.v) => 0
  a.v = b.v => PSimp(lazyPseudoQuotient(a.ts,b.ts)$D,a.v)
  db: N := degree(b.ts)$D
  lcb: $ := leadingCoefficient(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  q := 0$$
  test: Z := degree(a,b.v)::Z - db
  while not zero?(a) and not negative?(test) repeat
    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    q := lcb * q + term
    test := degree(a,b.v)::Z - db
  q

lazyPseudoDivide(a:$, b:$): Record(coef:$, gap: N,quotient:$, remainder:$) ==
  -- with lazyPseudoDivide$NSUP
  b case R =>
    error " in lazyPseudoDivide$NSMP: #2 is conctant"
  (a case R) or (a.v < b.v) => [1$$,0$N,0$$,a]
  a.v = b.v =>
    cgqr := lazyPseudoDivide(a.ts,b.ts)
    [cgqr.coef, cgqr.gap, PSimp(cgqr.quotient,a.v), PSimp(cgqr.remainder,a.v)]
  db: N := degree(b.ts)$D
  lcb: $ := leadingCoefficient(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  q := 0$$
  delta: Z := max(test + 1$Z, 0$Z)
  while not zero?(a) and not negative?(test) repeat
    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    q := lcb * q + term
    delta := delta - 1$Z
    test := degree(a,b.v)::Z - db
  [lcb, (delta::N), q, a]

lazyResidueClass(a:$, b:$): Record(polnum:$, polden:$, power:N) ==
  -- with lazyResidueClass$NSUP
  b case R =>
    error " in lazyResidueClass$NSMP: #2 is conctant"
  lcb: $ := leadingCoefficient(b.ts)$D
  (a case R) or (a.v < b.v) => [a,lcb,0]
  a.v = b.v =>
    lrc := lazyResidueClass(a.ts,b.ts)$D
    [PSimp(lrc.polnum,a.v), lrc.polden, lrc.power]
  db: N := degree(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  pow: N := 0
  while not zero?(a) and not negative?(test) repeat
```

```
        term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
        a := lcb * a - term * b
        pow := pow + 1
        test := degree(a,b.v)::Z - db
      [a, lcb, pow]

    if R has IntegralDomain
    then

      packD := PseudoRemainderSequence($,D)

      exactQuo(x:$, y:$):$ ==
        ex: Union($,"failed") := x exquo$$ y
        (ex case $) => ex::$
        error "in exactQuotient$NSMP: bad args"

      LazardQuotient(x:$, y:$, n: N):$ ==
        zero?(n) => error("LazardQuotient$NSMP : n = 0")
--        one?(n) => x
        (n = 1) => x
        a: N := 1
        while n >= (b := 2*a) repeat a := b
        c: $ := x
        n := (n - a)::N
        repeat
--          one?(a) => return c
          (a = 1) => return c
          a := a quo 2
          c := exactQuo(c*c,y)
          if n >= a then ( c := exactQuo(c*x,y) ; n := (n - a)::N )

      LazardQuotient2(p:$, a:$, b:$, n: N) ==
        zero?(n) => error " in LazardQuotient2$NSMP: bad #4"
--        one?(n) => p
        (n = 1) => p
        c: $  := LazardQuotient(a,b,(n-1)::N)
        exactQuo(c*p,b)

      next_subResultant2(p:$, q:$, z:$, s:$) ==
        PSimp(next_sousResultant2(p.ts,q.ts,z.ts,s)$packD,p.v)

      subResultantGcd(a:$, b:$): $ ==
        (a case R) or (b case R) =>
          error "subResultantGcd$NSMP: one arg is constant"
        a.v ~= b.v =>
          error "subResultantGcd$NSMP: mvar(#1) ~= mvar(#2)"
        PSimp(subResultantGcd(a.ts,b.ts),a.v)

      halfExtendedSubResultantGcd1(a:$,b:$): Record (gcd: $, coef1: $) ==
        (a case R) or (b case R) =>
```

```
        error "halfExtendedSubResultantGcd1$NSMP: one arg is constant"
      a.v ~= b.v =>
        error "halfExtendedSubResultantGcd1$NSMP: mvar(#1) ~= mvar(#2)"
      hesrg := halfExtendedSubResultantGcd1(a.ts,b.ts)$D
      [PSimp(hesrg.gcd,a.v), PSimp(hesrg.coef1,a.v)]

    halfExtendedSubResultantGcd2(a:$,b:$): Record (gcd: $, coef2: $) ==
      (a case R) or (b case R) =>
        error "halfExtendedSubResultantGcd2$NSMP: one arg is constant"
      a.v ~= b.v =>
        error "halfExtendedSubResultantGcd2$NSMP: mvar(#1) ~= mvar(#2)"
      hesrg := halfExtendedSubResultantGcd2(a.ts,b.ts)$D
      [PSimp(hesrg.gcd,a.v), PSimp(hesrg.coef2,a.v)]

    extendedSubResultantGcd(a:$,b:$): Record (gcd: $, coef1: $, coef2: $) ==
      (a case R) or (b case R) =>
        error "extendedSubResultantGcd$NSMP: one arg is constant"
      a.v ~= b.v =>
        error "extendedSubResultantGcd$NSMP: mvar(#1) ~= mvar(#2)"
      esrg := extendedSubResultantGcd(a.ts,b.ts)$D
      [PSimp(esrg.gcd,a.v),PSimp(esrg.coef1,a.v),PSimp(esrg.coef2,a.v)]

    resultant(a:$, b:$): $ ==
      (a case R) or (b case R) =>
        error "resultant$NSMP: one arg is constant"
      a.v ~= b.v =>
        error "resultant$NSMP: mvar(#1) ~= mvar(#2)"
      resultant(a.ts,b.ts)$D

    subResultantChain(a:$, b:$): List $ ==
      (a case R) or (b case R) =>
        error "subResultantChain$NSMP: one arg is constant"
      a.v ~= b.v =>
        error "subResultantChain$NSMP: mvar(#1) ~= mvar(#2)"
      [PSimp(up,a.v) for up in subResultantsChain(a.ts,b.ts)]

    lastSubResultant(a:$, b:$): $ ==
      (a case R) or (b case R) =>
        error "lastSubResultant$NSMP: one arg is constant"
      a.v ~= b.v =>
        error "lastSubResultant$NSMP: mvar(#1) ~= mvar(#2)"
      PSimp(lastSubResultant(a.ts,b.ts),a.v)

    if R has EuclideanDomain
    then

      exactQuotient (a:$,b:R) ==
--          one? b => a
        (b = 1) => a
        a case R => (a::R quo$R b)::$
```

```
          ([a.v, map((a1:%):% +-> exactQuotient(a1,b),a.ts)$SUP2]$VPoly)::Rep

        exactQuotient! (a:$,b:R) ==
--          one? b => a
          (b = 1) => a
          a case R => (a::R quo$R b)::$
          a.ts := map((a1:%):% +-> exactQuotient!(a1,b),a.ts)$SUP2
          a

      else

        exactQuotient (a:$,b:R) ==
--          one? b => a
          (b = 1) => a
          a case R => ((a::R exquo$R b)::R)::$
          ([a.v, map((a1:%):% +-> exactQuotient(a1,b),a.ts)$SUP2]$VPoly)::Rep

        exactQuotient! (a:$,b:R) ==
--          one? b => a
          (b = 1) => a
          a case R => ((a::R exquo$R b)::R)::$
          a.ts := map((a1:%):% +-> exactQuotient!(a1,b),a.ts)$SUP2
          a

    if R has GcdDomain
    then

      localGcd(r:R,p:$):R ==
        p case R => gcd(r,p::R)$R
        gcd(r,content(p))$R

      gcd(r:R,p:$):R ==
--        one? r => r
        (r = 1) => r
        zero? p => r
        localGcd(r,p)

      content p ==
        p case R => p
        up : D := p.ts
        r := 0$R
--        while (not zero? up) and (not one? r) repeat
        while (not zero? up) and (not (r = 1)) repeat
          r := localGcd(r,leadingCoefficient(up))
          up := reductum up
        r

      primitivePart! p ==
        zero? p => p
        p case R => 1$$
```

```
        cp := content(p)
        p.ts :=
          unitCanonical(map((a1:%):% +-> exactQuotient!(a1,cp),p.ts)$SUP2)$D
        p
```

———-

— NSMP.dotabb —

```
"NSMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSMP"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"NSMP" -> "RPOLCAT"
```

———-

15.3 domain NSUP NewSparseUnivariatePolynomial

Based on the **PseudoRemainderSequence** package, the domain constructor **NewSparseUnivariatePolynomial** extends the constructur **SparseUnivariatePolynomial**.

— NewSparseUnivariatePolynomial.input —

```
)set break resume
)sys rm -f NewSparseUnivariatePolynomial.output
)spool NewSparseUnivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NewSparseUnivariatePolynomial
--R NewSparseUnivariatePolynomial R: Ring  is a domain constructor
--R Abbreviation for NewSparseUnivariatePolynomial is NSUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NSUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                         ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                         ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %           ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                         ?-? : (%,%) -> %
--R -? : % -> %                              ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                    D : % -> %
--R D : (%,NonNegativeInteger) -> %          1 : () -> %
```

```
--R 0 : () -> %                         ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> List R          coerce : R -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger    differentiate : % -> %
--R ?.? : (%,%) -> %                    ?.? : (%,R) -> R
--R eval : (%,List %,List %) -> %       eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %          eval : (%,List Equation %) -> %
--R ground : % -> R                     ground? : % -> Boolean
--R hash : % -> SingleInteger           init : () -> % if R has STEP
--R latex : % -> String                 lazyPseudoQuotient : (%,%) -> %
--R leadingCoefficient : % -> R         leadingMonomial : % -> %
--R map : ((R -> R),%) -> %             monicModulo : (%,%) -> %
--R monomial? : % -> Boolean            monomials : % -> List %
--R one? : % -> Boolean                 primitiveMonomials : % -> List %
--R pseudoRemainder : (%,%) -> %        recip : % -> Union(%,"failed")
--R reductum : % -> %                   retract : % -> R
--R sample : () -> %                    zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseUnivariatePolynomial R -> %
--R coerce : % -> SparseUnivariatePolynomial R
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
```

```
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R ha
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R ha
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R extendedResultant : (%,%) -> Record(resultant: R,coef1: %,coef2: %) if R has INTDOM
--R extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %) if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePo
--R fmecg : (%,NonNegativeInteger,R,%) -> %
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R halfExtendedResultant1 : (%,%) -> Record(resultant: R,coef1: %) if R has INTDOM
--R halfExtendedResultant2 : (%,%) -> Record(resultant: R,coef2: %) if R has INTDOM
--R halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %) if R has INTDOM
```

```
--R halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %) if R has INTDOM
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lastSubResultant : (%,%) -> % if R has INTDOM
--R lazyPseudoDivide : (%,%) -> Record(coef: R,gap: NonNegativeInteger,quotient: %,remainder: %)
--R lazyPseudoRemainder : (%,%) -> %
--R lazyResidueClass : (%,%) -> Record(polnum: %,polden: R,power: NonNegativeInteger)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Singl
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SparseUnivariatePolynomial R
```

```
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseUnivariatePolynomial R,"failed")
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subResultantsChain : (%,%) -> List % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

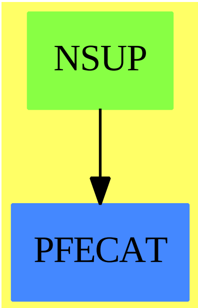
)spool
)lisp (bye)
```

— NewSparseUnivariatePolynomial.help —

```
====================================================================
NewSparseUnivariatePolynomial examples
====================================================================

See Also:
o )show NewSparseUnivariatePolynomial
```

15.3.1 NewSparseUnivariatePolynomial (NSUP)



See

⇒ "NewSparseMultivariatePolynomial" (NSMP) 15.2.1 on page 1676

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
composite	conditionP	content
convert	D	degree
differentiate	discriminant	divide
divideExponents	elt	euclideanSize
eval	expressIdealMember	exquo
extendedEuclidean	extendedResultant	extendedSubResultantGcd
factor	factorPolynomial	factorSquareFreePolynomial
fmecg	gcd	gcdPolynomial
ground	ground?	halfExtendedResultant1
halfExtendedResultant2	halfExtendedSubResultantGcd1	halfExtendedSubResultantGcd2
hash	init	integrate
isExpt	isPlus	isTimes
karatsubaDivide	lastSubResultant	latex
lazyPseudoDivide	lazyPseudoQuotient	lazyPseudoRemainder
lazyResidueClass	lcm	leadingCoefficient
leadingMonomial	mainVariable	makeSUP
map	mapExponents	max
min	minimumDegree	monicDivide
monicModulo	monomial	monomial?
monomials	multiEuclidean	multiplyExponents
multivariate	nextItem	numberOfMonomials
one?	order	patternMatch
pomopo!	prime?	primitiveMonomials
primitivePart	principalIdeal	pseudoDivide
pseudoQuotient	pseudoRemainder	recip
reducedSystem	reductum	resultant
retract	retractIfCan	sample
separate	shiftLeft	shiftRight
sizeLess?	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subResultantGcd
subResultantsChain	subtractIfCan	totalDegree
totalDegree	unit?	unitCanonical
unitNormal	univariate	unmakeSUP
variables	vectorise	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?.?	?~=?
?/?	?<?	?<=?
?>?	?>=?	?quo?
?rem?		

— domain NSUP NewSparseUnivariatePolynomial —

```
)abbrev domain NSUP NewSparseUnivariatePolynomial
++ Author: Marc Moreno Maza
++ Date Created: 23/07/98
++ Date Last Updated: 14/12/98
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A post-facto extension for \axiomType{SUP} in order
++ to speed up operations related to pseudo-division and gcd for
++ both \axiomType{SUP} and, consequently, \axiomType{NSMP}.

NewSparseUnivariatePolynomial(R): Exports == Implementation where

  R:Ring
  NNI ==> NonNegativeInteger
  SUPR ==> SparseUnivariatePolynomial R

  Exports == Join(UnivariatePolynomialCategory(R),
   CoercibleTo(SUPR),RetractableTo(SUPR)) with
     fmecg : (%,NNI,R,%) -> %
        ++ \axiom{fmecg(p1,e,r,p2)} returns \axiom{p1 - r * x**e * p2}
        ++ where \axiom{x} is \axiom{monomial(1,1)}
     monicModulo : ($, $) -> $
        ++ \axiom{monicModulo(a,b)} returns \axiom{r} such that \axiom{r} is
        ++ reduced w.r.t. \axiom{b} and \axiom{b} divides \axiom{a -r}
        ++ where \axiom{b} is monic.
     lazyResidueClass: ($,$) -> Record(polnum:$, polden:R, power:NNI)
        ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[r,c,n]} such that
        ++ \axiom{r} is reduced w.r.t. \axiom{b} and \axiom{b} divides
        ++ \axiom{c^n * a - r} where \axiom{c} is \axiom{leadingCoefficient(b)}
        ++ and \axiom{n} is as small as possible with the previous properties.
     lazyPseudoRemainder: ($,$) -> $
        ++ \axiom{lazyPseudoRemainder(a,b)} returns \axiom{r} if
        ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[r,c,n]}.
        ++ This lazy pseudo-remainder is computed by means of the
        ++ fmecg from NewSparseUnivariatePolynomial operation.
     lazyPseudoDivide: ($,$) -> Record(coef:R,gap:NNI,quotient:$,remainder:$)
        ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]} such that
        ++ \axiom{c^n * a = q*b +r} and \axiom{lazyResidueClass(a,b)} returns
        ++ \axiom{[r,c,n]} where
        ++ \axiom{n + g = max(0, degree(b) - degree(a) + 1)}.
     lazyPseudoQuotient: ($,$) -> $
        ++ \axiom{lazyPseudoQuotient(a,b)} returns \axiom{q} if
        ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}
     if R has IntegralDomain
```

```
    then
      subResultantsChain: ($, $) -> List $
        ++ \axiom{subResultantsChain(a,b)} returns the list of the non-zero
        ++ sub-resultants of \axiom{a} and \axiom{b} sorted by increasing
        ++ degree.
      lastSubResultant: ($, $) -> $
        ++ \axiom{lastSubResultant(a,b)} returns \axiom{resultant(a,b)}
        ++ if \axiom{a} and \axiom{b} has no non-trivial gcd
        ++ in \axiom{R^(-1) P}
        ++ otherwise the non-zero sub-resultant with smallest index.
      extendedSubResultantGcd: ($, $) -> Record(gcd: $, coef1: $, coef2: $)
        ++ \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca, cb]}
        ++ such that \axiom{g} is a gcd of \axiom{a} and \axiom{b} in
        ++ \axiom{R^(-1) P} and \axiom{g = ca * a + cb * b}
      halfExtendedSubResultantGcd1: ($, $) -> Record(gcd: $, coef1: $)
        ++ \axiom{halfExtendedSubResultantGcd1(a,b)} returns \axiom{[g,ca]}
        ++ such that \axiom{extendedSubResultantGcd(a,b)} returns
        ++ \axiom{[g,ca, cb]}
      halfExtendedSubResultantGcd2: ($, $) -> Record(gcd: $, coef2: $)
        ++ \axiom{halfExtendedSubResultantGcd2(a,b)} returns \axiom{[g,cb]}
        ++ such that \axiom{extendedSubResultantGcd(a,b)} returns
        ++ \axiom{[g,ca, cb]}
      extendedResultant: ($, $) -> Record(resultant: R, coef1: $, coef2: $)
        ++ \axiom{extendedResultant(a,b)} returns  \axiom{[r,ca,cb]} such that
        ++ \axiom{r} is the resultant of \axiom{a} and \axiom{b} and
        ++ \axiom{r = ca * a + cb * b}
      halfExtendedResultant1: ($, $) -> Record(resultant: R, coef1: $)
        ++ \axiom{halfExtendedResultant1(a,b)} returns \axiom{[r,ca]}
        ++ such that \axiom{extendedResultant(a,b)}  returns
        ++ \axiom{[r,ca, cb]}
      halfExtendedResultant2: ($, $) -> Record(resultant: R, coef2: $)
        ++ \axiom{halfExtendedResultant2(a,b)} returns \axiom{[r,ca]} such
        ++ that \axiom{extendedResultant(a,b)} returns \axiom{[r,ca, cb]}

  Implementation == SparseUnivariatePolynomial(R) add

    Term == Record(k:NonNegativeInteger,c:R)
    Rep ==> List Term

    rep(s:$):Rep == s pretend Rep
    per(l:Rep):$ == l pretend $

    coerce (p:$):SUPR ==
      p pretend SUPR

    coerce (p:SUPR):$ ==
      p pretend $

    retractIfCan (p:$) : Union(SUPR,"failed") ==
      (p pretend SUPR)::Union(SUPR,"failed")
```

```
    monicModulo(x,y) ==
               zero? y =>
                  error "in monicModulo$NSUP: division by 0"
               ground? y =>
                  error "in monicModulo$NSUP: ground? #2"
               yy := rep y
--               not one? (yy.first.c) =>
               not ((yy.first.c) = 1) =>
                  error "in monicModulo$NSUP: not monic #2"
               xx := rep x; empty? xx => x
               e := yy.first.k; y := per(yy.rest)
               -- while (not empty? xx) repeat
               repeat
                 if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
                 xx:= rep fmecg(per rest(xx), u, xx.first.c, y)
                 if empty? xx then break
               per xx

    lazyResidueClass(x,y) ==
               zero? y =>
                  error "in lazyResidueClass$NSUP: division by 0"
               ground? y =>
                  error "in lazyResidueClass$NSUP: ground? #2"
               yy := rep y; co := yy.first.c; xx: Rep := rep x
               empty? xx => [x, co, 0]
               pow: NNI := 0; e := yy.first.k; y := per(yy.rest);
               repeat
                 if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
                 xx:= rep fmecg(co * per rest(xx), u, xx.first.c, y)
                 pow := pow + 1
                 if empty? xx then break
               [per xx, co, pow]

    lazyPseudoRemainder(x,y) ==
               zero? y =>
                  error "in lazyPseudoRemainder$NSUP: division by 0"
               ground? y =>
                  error "in lazyPseudoRemainder$NSUP: ground? #2"
               ground? x => x
               yy := rep y; co := yy.first.c
--               one? co => monicModulo(x,y)
               (co = 1) => monicModulo(x,y)
               (co = -1) => - monicModulo(-x,-y)
               xx:= rep x; e := yy.first.k; y := per(yy.rest)
               repeat
                 if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
                 xx:= rep fmecg(co * per rest(xx), u, xx.first.c, y)
                 if empty? xx then break
               per xx
```

```
lazyPseudoDivide(x,y) ==
          zero? y =>
             error "in lazyPseudoDivide$NSUP: division by 0"
          ground? y =>
             error "in lazyPseudoDivide$NSUP: ground? #2"
          yy := rep y; e := yy.first.k;
          xx: Rep := rep x; co := yy.first.c
          (empty? xx) or (xx.first.k < e) => [co,0,0,x]
          pow: NNI := subtractIfCan(xx.first.k,e)::NNI + 1
          qq: Rep := []; y := per(yy.rest)
          repeat
            if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
            qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
            xx := rep fmecg(co * per rest(xx), u, xx.first.c, y)
            pow := subtractIfCan(pow,1)::NNI
            if empty? xx then break
          [co, pow, per reverse qq, per xx]

lazyPseudoQuotient(x,y) ==
          zero? y =>
             error "in lazyPseudoQuotient$NSUP: division by 0"
          ground? y =>
             error "in lazyPseudoQuotient$NSUP: ground? #2"
          yy := rep y; e := yy.first.k; xx: Rep := rep x
          (empty? xx) or (xx.first.k < e) => 0
          qq: Rep := []; co := yy.first.c; y := per(yy.rest)
          repeat
            if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
            qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
            xx := rep fmecg(co * per rest(xx), u, xx.first.c, y)
            if empty? xx then break
          per reverse qq

if R has IntegralDomain
then
  pack ==> PseudoRemainderSequence(R, %)

  subResultantGcd(p1,p2) == subResultantGcd(p1,p2)$pack

  subResultantsChain(p1,p2) == chainSubResultants(p1,p2)$pack

  lastSubResultant(p1,p2) == lastSubResultant(p1,p2)$pack

  resultant(p1,p2) == resultant(p1,p2)$pack

  extendedResultant(p1,p2) ==
     re: Record(coef1: $, coef2: $, resultant: R) := resultantEuclidean(p1,p2)$pack
     [re.resultant, re.coef1, re.coef2]
```

```
halfExtendedResultant1(p1:$, p2: $): Record(resultant: R, coef1: $) ==
   re: Record(coef1: $, resultant: R) := semiResultantEuclidean1(p1, p2)$pack
   [re.resultant, re.coef1]

halfExtendedResultant2(p1:$, p2: $): Record(resultant: R, coef2: $) ==
   re: Record(coef2: $, resultant: R) := semiResultantEuclidean2(p1, p2)$pack
   [re.resultant, re.coef2]

extendedSubResultantGcd(p1,p2) ==
   re: Record(coef1: $, coef2: $, gcd: $) := subResultantGcdEuclidean(p1,p2)$pack
   [re.gcd, re.coef1, re.coef2]

halfExtendedSubResultantGcd1(p1:$, p2: $): Record(gcd: $, coef1: $) ==
   re: Record(coef1: $, gcd: $) := semiSubResultantGcdEuclidean1(p1, p2)$pack
   [re.gcd, re.coef1]

halfExtendedSubResultantGcd2(p1:$, p2: $): Record(gcd: $, coef2: $) ==
   re: Record(coef2: $, gcd: $) := semiSubResultantGcdEuclidean2(p1, p2)$pack
   [re.gcd, re.coef2]

pseudoDivide(x,y) ==
         zero? y =>
            error "in pseudoDivide$NSUP: division by 0"
         ground? y =>
            error "in pseudoDivide$NSUP: ground? #2"
         yy := rep y; e := yy.first.k
         xx: Rep := rep x; co := yy.first.c
         (empty? xx) or (xx.first.k < e) => [co,0,x]
         pow: NNI := subtractIfCan(xx.first.k,e)::NNI + 1
         qq: Rep := []; y := per(yy.rest)
         repeat
           if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
           qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
           xx := rep fmecg(co * per rest(xx), u, xx.first.c, y)
           pow := subtractIfCan(pow,1)::NNI
           if empty? xx then break
         zero? pow => [co, per reverse qq, per xx]
         default: R := co ** pow
         q := default * (per reverse qq)
         x := default * (per xx)
         [co, q, x]

pseudoQuotient(x,y) ==
         zero? y =>
            error "in pseudoDivide$NSUP: division by 0"
         ground? y =>
            error "in pseudoDivide$NSUP: ground? #2"
         yy := rep y; e := yy.first.k; xx: Rep := rep x
         (empty? xx) or (xx.first.k < e) => 0
         pow: NNI := subtractIfCan(xx.first.k,e)::NNI + 1
```

```
        qq: Rep := []; co := yy.first.c; y := per(yy.rest)
        repeat
          if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
          qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
          xx := rep fmecg(co * per rest(xx), u, xx.first.c, y)
          pow := subtractIfCan(pow,1)::NNI
          if empty? xx then break
        zero? pow => per reverse qq
        (co ** pow) * (per reverse qq)
```

— NSUP.dotabb —

```
"NSUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSUP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"NSUP" -> "PFECAT"
```

15.4 domain NONE None

— None.input —

```
)set break resume
)sys rm -f None.output
)spool None.output
)set message test on
)set message auto off
)clear all
--S 1 of 3
[ ]
--R
--R
--R   (1)  []
--R                                                          Type: List None
--E 1

--S 2 of 3
[ ] :: List Float
--R
--R
--R   (2)  []
--R                                                         Type: List Float
```

```
--E 2

--S 3 of 3
[ ]$List(NonNegativeInteger)
--R
--R
--R   (3)  []
--R                                                  Type: List NonNegativeInteger
--E 3
)spool
)lisp (bye)
```

— **None.help** —

```
====================================================================
None examples
====================================================================

The None domain is not very useful for interactive work but it is
provided nevertheless for completeness of the Axiom type system.

Probably the only place you will ever see it is if you enter an
empty list with no type information.

  [ ]
    []
                         Type: List None

Such an empty list can be converted into an empty list of any other
type.

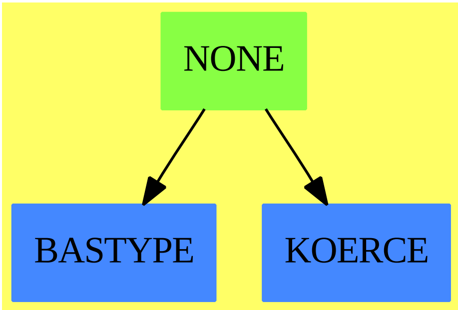
  [ ] :: List Float
    []
                         Type: List Float

If you wish to produce an empty list of a particular type directly,
such as List NonNegativeInteger, do it this way.

  [ ]$List(NonNegativeInteger)
    []
                         Type: List NonNegativeInteger

See Also:
o )show None
```

15.4.1 None (NONE)



See

⇒ “Any” (ANY) 2.9.1 on page 50

Exports:
coerce hash latex ?=? ?˜=?

— domain NONE None —

```
)abbrev domain NONE None
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Functions: coerce
++ Related Constructors: NoneFunctions1
++ Also See: Any
++ AMS Classification:
++ Keywords: none, empty
++ Description:
++ \spadtype{None} implements a type with no objects. It is mainly
++ used in technical situations where such a thing is needed (e.g.
++ the interpreter and some of the internal \spadtype{Expression} code).

None():SetCategory == add
    coerce(none:%):OutputForm == "NONE" :: OutputForm
    x:% = y:% == EQ(x,y)$Lisp
```

———-

— NONE.dotabb —

```
"NONE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NONE"]
```

```
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"NONE" -> "BASTYPE"
"NONE" -> "KOERCE"
```

———-

15.5 domain NNI NonNegativeInteger

— **NonNegativeInteger.input** —

```
)set break resume
)sys rm -f NonNegativeInteger.output
)spool NonNegativeInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NonNegativeInteger
--R NonNegativeInteger  is a domain constructor
--R Abbreviation for NonNegativeInteger is NNI
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NNI
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (%,%) -> %                      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm              gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R one? : % -> Boolean                   ?quo? : (%,%) -> %
--R random : % -> %                       recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R shift : (%,Integer) -> %              sup : (%,%) -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R exquo : (%,%) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
```

```
--R
--E 1

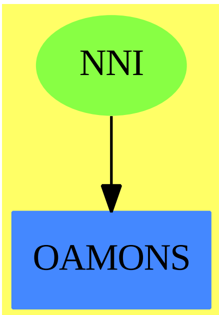
)spool
)lisp (bye)
```

— **NonNegativeInteger.help** —

```
====================================================================
NonNegativeInteger examples
====================================================================

See Also:
o )show NonNegativeInteger
```

15.5.1 NonNegativeInteger (NNI)



See

⇒ "Integer" (INT) 10.30.1 on page 1325
⇒ "PositiveInteger" (PI) 17.28.1 on page 2060
⇒ "RomanNumeral" (ROMAN) 19.12.1 on page 2286

Exports:

0	1	coerce	divide	exquo
gcd	hash	latex	max	min
one?	random	recip	sample	shift
subtractIfCan	sup	zero?	?~=?	?*?
?**?	?^?	?+?	?<?	?<=?
?=?	?>?	?>=?	?quo?	?rem?

— **domain NNI NonNegativeInteger** —

```
)abbrev domain NNI NonNegativeInteger
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations:
++ Related Constructors:
++ Keywords: integer
++ Description:
++ \spadtype{NonNegativeInteger} provides functions for non-negative integers.

NonNegativeInteger: Join(OrderedAbelianMonoidSup,Monoid) with
            _quo : (%,%) -> %
              ++ a quo b returns the quotient of \spad{a} and b, forgetting
              ++ the remainder.
            _rem : (%,%) -> %
              ++ a rem b returns the remainder of \spad{a} and b.
            gcd  : (%,%) -> %
              ++ gcd(a,b) computes the greatest common divisor of two
              ++ non negative integers \spad{a} and b.
            divide: (%,%) -> Record(quotient:%,remainder:%)
              ++ divide(a,b) returns a record containing both
              ++ remainder and quotient.
            _exquo: (%,%) -> Union(%,"failed")
              ++ exquo(a,b) returns the quotient of \spad{a} and b, or "failed"
              ++ if b is zero or \spad{a} rem b is zero.
            shift: (%, Integer) -> %
              ++ shift(a,i) shift \spad{a} by i bits.
            random   : % -> %
              ++ random(n) returns a random integer from 0 to \spad{n-1}.
            commutative("*")
              ++ commutative("*") means multiplication is commutative,
              ++ that is, \spad{x*y = y*x}.

  == SubDomain(Integer,#1 >= 0) add
      x,y:%
      sup(x,y) == MAX(x,y)$Lisp
      shift(x:%, n:Integer):% == ASH(x,n)$Lisp
      subtractIfCan(x, y) ==
        c:Integer := (x pretend Integer) - (y pretend Integer)
        c < 0 => "failed"
        c pretend %
```

— NNI.dotabb —

```
"NNI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NNI",shape=ellipse]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
```

```
"NNI" -> "OAMONS"
```

——————-

15.6 domain NOTTING NottinghamGroup

— NottinghamGroup.input —

```
)set break resume
)sys rm -f NottinghamGroup.output
)spool NottinghamGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 8
x:=monomial(1,1)$UFPS PF 1783
--R
--R
--R   (1)  x
--R                              Type: UnivariateFormalPowerSeries PrimeField 1783
--E 1

--S 2 of 8
s:=retract(sin x)$NOTTING PF 1783
--R
--R
--R                  3        5       7       9      11
--R   (2)  x + 297x  + 1679x  + 427x  + 316x  + O(x  )
--R                                          Type: NottinghamGroup PrimeField 1783
--E 2

--S 3 of 8
s^2
--R
--R
--R                  3       5        7        9      11
--R   (3)  x + 594x  + 535x  + 1166x  + 1379x  + O(x  )
--R                                          Type: NottinghamGroup PrimeField 1783
--E 3

--S 4 of 8
s^-1
--R
--R
--R                   3       5       7        9      11
```

```
--R   (4)  x + 1486x  + 847x  + 207x  + 1701x  + O(x  )
--R                                         Type: NottinghamGroup PrimeField 1783
--E 4

--S 5 of 8
s^-1*s
--R
--R
--R               11
--R   (5)  x + O(x  )
--R                                         Type: NottinghamGroup PrimeField 1783
--E 5

--S 6 of 8
s*s^-1
--R
--R
--R               11
--R   (6)  x + O(x  )
--R                                         Type: NottinghamGroup PrimeField 1783
--E 6

--S 7 of 8
sample()$NOTTING(PF(1783))
--R
--R   (7)  x
--R                                         Type: NottinghamGroup PrimeField 1783
--E 7

--S 8 of 8
)show NottinghamGroup
--R
--R NottinghamGroup F: FiniteFieldCategory  is a domain constructor
--R Abbreviation for NottinghamGroup is NOTTING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NOTTING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm             commutator : (%,%) -> %
--R conjugate : (%,%) -> %               hash : % -> SingleInteger
--R inv : % -> %                         latex : % -> String
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R sample : () -> %                     ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R retract : UnivariateFormalPowerSeries F -> %
```

```
--R
--E 8

)spool
)lisp (bye)
```

———-

— NottinghamGroup.help —

```
====================================================================
NottinghamGroup examples
====================================================================
                             n
If F is a finite field with p  elements, then we may form the group
of all formal power series {t(1+a t+a t+...)} where u(0)=0 and
 ,                                1   2
u (0)=1 and $a is an element of F.
              i
The group operation is substitution (composition).
This is called the Nottingham Group.

The Nottingham Group is the projective limit of finite p-groups.
Every finite p-group can be embedded in the Nottingham Group.

   x:=monomial(1,1)$UFPS PF 1783
     x

   s:=retract(sin x)$NOTTING PF 1783
             3        5       7       9      11
     x + 297x  + 1679x  + 427x  + 316x  + O(x  )

   s^2
             3       5        7        9      11
     x + 594x  + 535x  + 1166x  + 1379x  + O(x  )

   s^-1
              3       5       7        9      11
     x + 1486x  + 847x  + 207x  + 1701x  + O(x  )

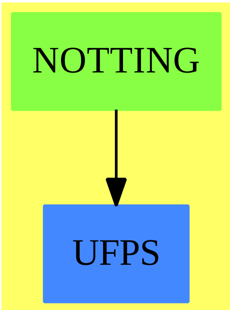
   s^-1*s
            11
     x + O(x  )

   s*s^-1
               11
        x + O(x  )
```

```
See Also:
o )show NottinghamGroup
o )show UnivariateFormalPowerSeries
```

———-

15.6.1 NottinghamGroup (NOTTING)



Exports:

1	coerce	commutator	conjugate	hash
inv	latex	one?	recip	sample
^=	retract	*	**	/
=	^			

— domain NOTTING NottinghamGroup —

```
)abbrev domain NOTTING NottinghamGroup
++ Author: Mark Botch
++ Description:
++ This is an implmenentation of the Nottingham Group

NottinghamGroup(F:FiniteFieldCategory): Group with
  retract: UnivariateFormalPowerSeries F -> %
 == add
  Rep:=UnivariateFormalPowerSeries F

  coerce f == coerce(f::Rep)$UnivariateFormalPowerSeries(F)

  retract f ==
    if zero? coefficient(f,0) and one? coefficient(f,1)
    then f::Rep
    else error"The leading term must be x"

  1 == monomial(1,1)
```

```
  f*g == f.g

  inv f == revert f
```

———-

— NOTTING.dotabb —

```
"NOTTING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NOTTING"]
"UFPS" [color="#4488FF",href="bookvol10.3.pdf#nameddest=UFPS"]
"NOTTING" -> "UFPS"
```

———-

15.7 domain NIPROB NumericalIntegrationProblem

— NumericalIntegrationProblem.input —

```
)set break resume
)sys rm -f NumericalIntegrationProblem.output
)spool NumericalIntegrationProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalIntegrationProblem
--R NumericalIntegrationProblem  is a domain constructor
--R Abbreviation for NumericalIntegrationProblem is NIPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NIPROB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R coerce : Union(nia: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ordered
--R coerce : Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion DoubleFl
--R coerce : Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R retract : % -> Union(nia: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

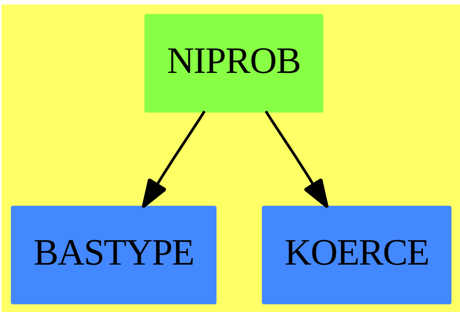
— NumericalIntegrationProblem.help —

```
====================================================================
NumericalIntegrationProblem examples
====================================================================

See Also:
o )show NumericalIntegrationProblem
```

———-

15.7.1 NumericalIntegrationProblem (NIPROB)



Exports:
coerce hash latex retract ?=? ?~=?

— domain NIPROB NumericalIntegrationProblem —

```
)abbrev domain NIPROB NumericalIntegrationProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalIntegrationProblem} is a \axiom{domain}
++ for the representation of Numerical Integration problems for use
++ by ANNA.
```

```
++
++ The representation is a Union of two record types - one for integration of
++ a function of one variable:
++
++ \axiomType{Record}(var:\axiomType{Symbol},\br
++ fn:\axiomType{Expression DoubleFloat},\br
++ range:\axiomType{Segment OrderedCompletion DoubleFloat},\br
++ abserr:\axiomType{DoubleFloat},\br
++ relerr:\axiomType{DoubleFloat},)
++
++ and one for multivariate integration:
++
++ \axiomType{Record}(fn:\axiomType{Expression DoubleFloat},\br
++ range:\axiomType{List Segment OrderedCompletion DoubleFloat},\br
++ abserr:\axiomType{DoubleFloat},\br
++ relerr:\axiomType{DoubleFloat},).
++

NumericalIntegrationProblem(): EE == II where

 EDFA   ==> Expression DoubleFloat
 SOCDFA ==> Segment OrderedCompletion DoubleFloat
 DFA    ==> DoubleFloat
 NIAA   ==> Record(var:Symbol,fn:EDFA,range:SOCDFA,abserr:DFA,relerr:DFA)
 MDNIAA ==> Record(fn:EDFA,range:List SOCDFA,abserr:DFA,relerr:DFA)

 EE ==> SetCategory with
    coerce: NIAA -> %
      ++ coerce(x) is not documented
    coerce: MDNIAA -> %
      ++ coerce(x) is not documented
    coerce: Union(nia:NIAA,mdnia:MDNIAA) -> %
      ++ coerce(x) is not documented
    coerce: % -> OutputForm
      ++ coerce(x) is not documented
    retract: % -> Union(nia:NIAA,mdnia:MDNIAA)
      ++ retract(x) is not documented

 II ==> add
      Rep := Union(nia:NIAA,mdnia:MDNIAA)

      coerce(s:NIAA) == [s]
      coerce(s:MDNIAA) == [s]
      coerce(s:Union(nia:NIAA,mdnia:MDNIAA)) == s
      coerce(x:%):OutputForm ==
        (x) case nia => (x.nia)::OutputForm
        (x.mdnia)::OutputForm
      retract(x:%):Union(nia:NIAA,mdnia:MDNIAA) ==
        (x) case nia => [x.nia]
        [x.mdnia]
```

— NIPROB.dotabb —

```
"NIPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NIPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"NIPROB" -> "BASTYPE"
"NIPROB" -> "KOERCE"
```

15.8 domain ODEPROB NumericalODEProblem

— NumericalODEProblem.input —

```
)set break resume
)sys rm -f NumericalODEProblem.output
)spool NumericalODEProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalODEProblem
--R NumericalODEProblem  is a domain constructor
--R Abbreviation for NumericalODEProblem is ODEPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODEPROB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R coerce : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat,yinit: List D
--R retract : % -> Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat,yinit:
--R
--E 1

)spool
)lisp (bye)
```

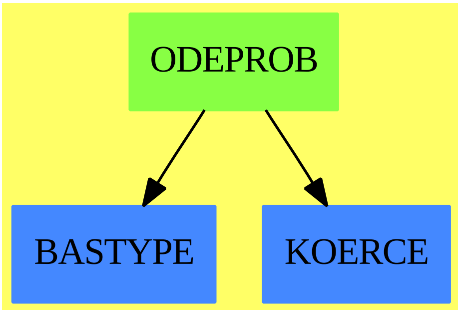
— **NumericalODEProblem.help** —

```
====================================================================
NumericalODEProblem examples
====================================================================

See Also:
o )show NumericalODEProblem
```

————

15.8.1 NumericalODEProblem (ODEPROB)



Exports:
coerce hash latex retract ?=? ?~=?

— **domain ODEPROB NumericalODEProblem** —

```
)abbrev domain ODEPROB NumericalODEProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalODEProblem} is a \axiom{domain}
++ for the representation of Numerical ODE problems for use
++ by ANNA.
++
++ The representation is of type:
++
++ \axiomType{Record}(xinit:\axiomType{DoubleFloat},\br
++ xend:\axiomType{DoubleFloat},\br
```

```
++ fn:\axiomType{Vector Expression DoubleFloat},\br
++ yinit:\axiomType{List DoubleFloat},intvals:\axiomType{List DoubleFloat},\br
++ g:\axiomType{Expression DoubleFloat},abserr:\axiomType{DoubleFloat},\br
++ relerr:\axiomType{DoubleFloat})
++

NumericalODEProblem(): EE == II where

 DFB   ==> DoubleFloat
 VEDFB ==> Vector Expression DoubleFloat
 LDFB  ==> List DoubleFloat
 EDFB  ==> Expression DoubleFloat
 ODEAB ==> Record(xinit:DFB,xend:DFB,fn:VEDFB,yinit:LDFB,intvals:LDFB,g:EDFB,abserr:DFB,relerr:DFB)

 EE ==> SetCategory with
    coerce: ODEAB -> %
      ++ coerce(x) is not documented
    coerce: % -> OutputForm
      ++ coerce(x) is not documented
    retract: % -> ODEAB
      ++ retract(x) is not documented

 II ==> add
      Rep := ODEAB

      coerce(s:ODEAB) == s
      coerce(x:%):OutputForm ==
        (retract(x))::OutputForm
      retract(x:%):ODEAB == x :: Rep
```

— **ODEPROB.dotabb** —

```
"ODEPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODEPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"ODEPROB" -> "BASTYPE"
"ODEPROB" -> "KOERCE"
```

15.9 domain OPTPROB NumericalOptimizationProblem

— **NumericalOptimizationProblem.input** —

```
)set break resume
)sys rm -f NumericalOptimizationProblem.output
)spool NumericalOptimizationProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalOptimizationProblem
--R NumericalOptimizationProblem  is a domain constructor
--R Abbreviation for NumericalOptimizationProblem is OPTPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OPTPROB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R coerce : Union(noa: Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List Ord
--R coerce : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> %
--R coerce : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedComplet
--R retract : % -> Union(noa: Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: Li
--R
--E 1

)spool
)lisp (bye)
```

———-

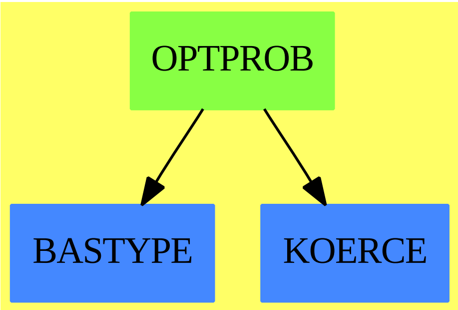
— **NumericalOptimizationProblem.help** —

```
====================================================================
NumericalOptimizationProblem examples
====================================================================

See Also:
o )show NumericalOptimizationProblem
```

———-

15.9.1 NumericalOptimizationProblem (OPTPROB)



Exports:
coerce hash latex retract ?=? ?~=?

— domain OPTPROB NumericalOptimizationProblem —

```
)abbrev domain OPTPROB NumericalOptimizationProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalOptimizationProblem} is a \axiom{domain}
++ for the representation of Numerical Optimization problems for use
++ by ANNA.
++
++ The representation is a Union of two record types - one for otimization of
++ a single function of one or more variables:
++
++ \axiomType{Record}(\br
++ fn:\axiomType{Expression DoubleFloat},\br
++ init:\axiomType{List DoubleFloat},\br
++ lb:\axiomType{List OrderedCompletion DoubleFloat},\br
++ cf:\axiomType{List Expression DoubleFloat},\br
++ ub:\axiomType{List OrderedCompletion DoubleFloat})
++
++ and one for least-squares problems i.e. optimization of a set of
++ observations of a data set:
++
++ \axiomType{Record}(lfn:\axiomType{List Expression DoubleFloat},\br
++ init:\axiomType{List DoubleFloat}).

NumericalOptimizationProblem(): EE == II where

 LDFD     ==> List DoubleFloat
```

```
LEDFD    ==> List Expression DoubleFloat
LSAD     ==> Record(lfn:LEDFD, init:LDFD)
UNOALSAD ==> Union(noa:NOAD,lsa:LSAD)
EDFD     ==> Expression DoubleFloat
LOCDFD   ==> List OrderedCompletion DoubleFloat
NOAD     ==> Record(fn:EDFD, init:LDFD, lb:LOCDFD, cf:LEDFD, ub:LOCDFD)

EE ==> SetCategory with
   coerce: NOAD -> %
     ++ coerce(x) is not documented
   coerce: LSAD -> %
     ++ coerce(x) is not documented
   coerce: UNOALSAD -> %
     ++ coerce(x) is not documented
   coerce: % -> OutputForm
     ++ coerce(x) is not documented
   retract: % -> UNOALSAD
     ++ retract(x) is not documented

II ==> add
     Rep := UNOALSAD

     coerce(s:NOAD) == [s]
     coerce(s:LSAD) == [s]
     coerce(x:UNOALSAD) == x
     coerce(x:%):OutputForm ==
       (x) case noa => (x.noa)::OutputForm
       (x.lsa)::OutputForm
     retract(x:%):UNOALSAD ==
       (x) case noa => [x.noa]
       [x.lsa]
```

— **OPTPROB.dotabb** —

```
"OPTPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OPTPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"OPTPROB" -> "BASTYPE"
"OPTPROB" -> "KOERCE"
```

15.10 domain PDEPROB NumericalPDEProblem

— NumericalPDEProblem.input —

```
)set break resume
)sys rm -f NumericalPDEProblem.output
)spool NumericalPDEProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalPDEProblem
--R NumericalPDEProblem  is a domain constructor
--R Abbreviation for NumericalPDEProblem is PDEPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PDEPROB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R coerce : Record(pde: List Expression DoubleFloat,constraints: List Record(start: DoubleFloat,finish:
--R retract : % -> Record(pde: List Expression DoubleFloat,constraints: List Record(start: DoubleFloat,f
--R
--E 1

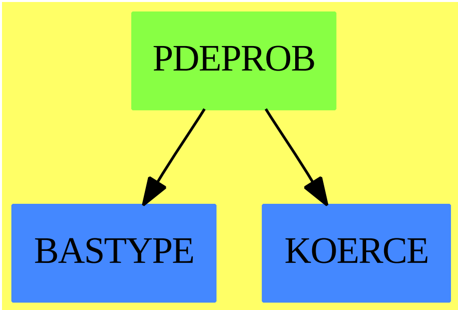
)spool
)lisp (bye)
```

— NumericalPDEProblem.help —

```
====================================================================
NumericalPDEProblem examples
====================================================================

See Also:
o )show NumericalPDEProblem
```

15.10.1 NumericalPDEProblem (PDEPROB)



Exports:
coerce hash latex retract ?=? ?~=?

— **domain PDEPROB NumericalPDEProblem** —

```
)abbrev domain PDEPROB NumericalPDEProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalPDEProblem} is a \axiom{domain}
++ for the representation of Numerical PDE problems for use
++ by ANNA.
++
++ The representation is of type:
++
++ \axiomType{Record}(pde:\axiomType{List Expression DoubleFloat}, \br
++ constraints:\axiomType{List PDEC}, \br
++ f:\axiomType{List List Expression DoubleFloat},\br
++ st:\axiomType{String},\br
++ tol:\axiomType{DoubleFloat})
++
++ where \axiomType{PDEC} is of type:
++
++ \axiomType{Record}(start:\axiomType{DoubleFloat}, \br
++ finish:\axiomType{DoubleFloat},\br
++ grid:\axiomType{NonNegativeInteger},\br
++ boundaryType:\axiomType{Integer},\br
++ dStart:\axiomType{Matrix DoubleFloat}, \br
++ dFinish:\axiomType{Matrix DoubleFloat})

NumericalPDEProblem(): EE == II where
```

```
DFC   ==> DoubleFloat
NNIC  ==> NonNegativeInteger
INTC  ==> Integer
MDFC  ==> Matrix DoubleFloat
PDECC ==> Record(start:DFC, finish:DFC, grid:NNIC, boundaryType:INTC,
               dStart:MDFC, dFinish:MDFC)
LEDFC ==> List Expression DoubleFloat
PDEBC ==> Record(pde:LEDFC, constraints:List PDECC, f:List LEDFC,
                  st:String, tol:DFC)

EE ==> SetCategory with
   coerce: PDEBC -> %
     ++ coerce(x) is not documented
   coerce: % -> OutputForm
     ++ coerce(x) is not documented
   retract: % -> PDEBC
     ++ retract(x) is not documented

II ==> add
     Rep := PDEBC

     coerce(s:PDEBC) == s
     coerce(x:%):OutputForm ==
       (retract(x))::OutputForm
     retract(x:%):PDEBC == x :: Rep
```

———-

— PDEPROB.dotabb —

```
"PDEPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PDEPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"PDEPROB" -> "BASTYPE"
"PDEPROB" -> "KOERCE"
```

———-

Chapter 16

Chapter O

16.1 domain OCT Octonion

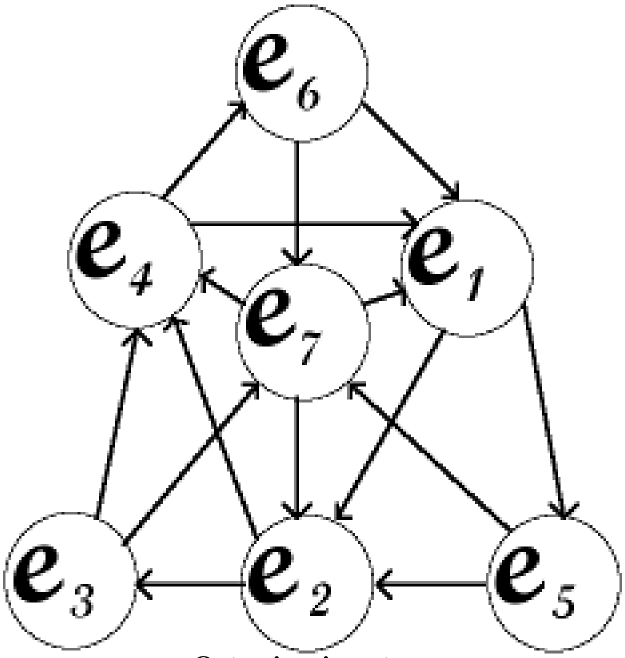
The octonions have the following multiplication table:

1	i	j	k	E	I	J	K
i	-1	k	$-j$	I	$-E$	$-K$	J
j	$-k$	-1	i	J	K	$-E$	$-I$
k	j	$-i$	-1	K	$-J$	I	$-E$
E	$-I$	$-J$	$-K$	-1	i	j	k
I	E	$-K$	J	$-i$	-1	$-k$	j
J	K	E	$-I$	$-j$	k	-1	$-i$
K	$-J$	I	E	$-k$	$-j$	i	-1

There are 3 different kinds of associativity. An algebra is

- **power-associative** if the subalgebra generated by any one element is associative. That is, given any element e then $e*(e*e) = (e*e)*e$.
- **alternative** if the subalgebra generated by any two elements is associative That is, given any two elements, a and b then $a*(a*b) = (a*a)*b$ and $a*(b*a) = (a*b)*a$ and $b*(a*a) = (b*a)*a$.
- **associative** if the subalgebra generated by any three elements is associative. That is, given any three elements a, b, and c then $a*(b*c) = (a*b)*c$.

The Octonions are power-associative and alternative but not associative, since $I*(J*K) \neq (I*J)*K$.



— **Octonion.input** —

```
)set break resume
)sys rm -f Octonion.output
)spool Octonion.output
)set message test on
)set message auto off
)clear all
--S 1 of 15
oci1 := octon(1,2,3,4,5,6,7,8)
--R
--R
--R   (1)  1 + 2i + 3j + 4k + 5E + 6I + 7J + 8K
--R                                                   Type: Octonion Integer
--E 1

--S 2 of 15
oci2 := octon(7,2,3,-4,5,6,-7,0)
--R
--R
--R   (2)  7 + 2i + 3j - 4k + 5E + 6I - 7J
--R                                                   Type: Octonion Integer
--E 2

--S 3 of 15
oci3 := octon(quatern(-7,-12,3,-10), quatern(5,6,9,0))
```

```
--R
--R
--R   (3)  - 7 - 12i + 3j - 10k + 5E + 6I + 9J
--R                                                     Type: Octonion Integer
--E 3

--S 4 of 15
(oci1 * oci2) * oci3 - oci1 * (oci2 * oci3)
--R
--R
--R   (4)  2696i - 2928j - 4072k + 16E - 1192I + 832J + 2616K
--R                                                     Type: Octonion Integer
--E 4

--S 5 of 15
[real oci1, imagi oci1, imagj oci1, imagk oci1, _
 imagE oci1, imagI oci1, imagJ oci1, imagK oci1]
--R
--R
--R   (5)  [1,2,3,4,5,6,7,8]
--R                                                 Type: List PositiveInteger
--E 5

--S 6 of 15
q : Quaternion Polynomial Integer := quatern(q1, qi, qj, qk)
--R
--R
--R   (6)  q1 + qi i + qj j + qk k
--R                                        Type: Quaternion Polynomial Integer
--E 6

--S 7 of 15
E : Octonion Polynomial Integer:= octon(0,0,0,0,1,0,0,0)
--R
--R
--R   (7)  E
--R                                          Type: Octonion Polynomial Integer
--E 7

--S 8 of 15
q * E
--R
--R
--R   (8)  q1 E + qi I + qj J + qk K
--R                                          Type: Octonion Polynomial Integer
--E 8

--S 9 of 15
E * q
--R
```

```
--R
--R   (9)  q1 E - qi I - qj J - qk K
--R                                            Type: Octonion Polynomial Integer
--E 9

--S 10 of 15
q * 1$(Octonion Polynomial Integer)
--R
--R
--R   (10)  q1 + qi i + qj j + qk k
--R                                            Type: Octonion Polynomial Integer
--E 10

--S 11 of 15
1$(Octonion Polynomial Integer) * q
--R
--R
--R   (11)  q1 + qi i + qj j + qk k
--R                                            Type: Octonion Polynomial Integer
--E 11

--S 12 of 15
o : Octonion Polynomial Integer := octon(o1, oi, oj, ok, oE, oI, oJ, oK)
--R
--R
--R   (12)  o1 + oi i + oj j + ok k + oE E + oI I + oJ J + oK K
--R                                            Type: Octonion Polynomial Integer
--E 12

--S 13 of 15
norm o
--R
--R
--R            2     2     2     2     2     2     2     2
--R   (13)  ok  + oj  + oi  + oK  + oJ  + oI  + oE  + o1
--R                                                     Type: Polynomial Integer
--E 13

--S 14 of 15
p : Octonion Polynomial Integer := octon(p1, pi, pj, pk, pE, pI, pJ, pK)
--R
--R
--R   (14)  p1 + pi i + pj j + pk k + pE E + pI I + pJ J + pK K
--R                                            Type: Octonion Polynomial Integer
--E 14

--S 15 of 15
norm(o*p)-norm(p)*norm(o)
--R
--R
```

```
--R   (15)  0
--R                                                     Type: Polynomial Integer
--E 15
)spool
)lisp (bye)
```

———-

— Octonion.help —

```
====================================================================
Octonion examples
====================================================================

The Octonions, also called the Cayley-Dixon algebra, defined over a
commutative ring are an eight-dimensional non-associative algebra.
Their construction from quaternions is similar to the construction
of quaternions from complex numbers.

As Octonion creates an eight-dimensional algebra, you have to give
eight components to construct an octonion.

  oci1 := octon(1,2,3,4,5,6,7,8)
    1 + 2i + 3j + 4k + 5E + 6I + 7J + 8K
                              Type: Octonion Integer

  oci2 := octon(7,2,3,-4,5,6,-7,0)
    7 + 2i + 3j - 4k + 5E + 6I - 7J
                              Type: Octonion Integer

Or you can use two quaternions to create an octonion.

  oci3 := octon(quatern(-7,-12,3,-10), quatern(5,6,9,0))
    - 7 - 12i + 3j - 10k + 5E + 6I + 9J
                              Type: Octonion Integer

You can easily demonstrate the non-associativity of multiplication.

  (oci1 * oci2) * oci3 - oci1 * (oci2 * oci3)
    2696i - 2928j - 4072k + 16E - 1192I + 832J + 2616K
                              Type: Octonion Integer

As with the quaternions, we have a real part, the imaginary parts
i, j, k, and four additional imaginary parts E, I, J and K. These
parts correspond to the canonical basis (1,i,j,k,E,I,J,K).

For each basis element there is a component operation to extract the
coefficient of the basis element for a given octonion.
```

```
  [real oci1, imagi oci1, imagj oci1, imagk oci1, _
   imagE oci1, imagI oci1, imagJ oci1, imagK oci1]
    [1,2,3,4,5,6,7,8]
                              Type: List PositiveInteger
```

A basis with respect to the quaternions is given by (1,E). However, you might ask, what then are the commuting rules? To answer this, we create some generic elements.

We do this in Axiom by simply changing the ground ring from Integer to Polynomial Integer.

```
  q : Quaternion Polynomial Integer := quatern(q1, qi, qj, qk)
    q1 + qi i + qj j + qk k
                             Type: Quaternion Polynomial Integer

  E : Octonion Polynomial Integer:= octon(0,0,0,0,1,0,0,0)
    E
                             Type: Octonion Polynomial Integer
```

Note that quaternions are automatically converted to octonions in the obvious way.

```
  q * E
    q1 E + qi I + qj J + qk K
                             Type: Octonion Polynomial Integer

  E * q
    q1 E - qi I - qj J - qk K
                             Type: Octonion Polynomial Integer

  q * 1$(Octonion Polynomial Integer)
    q1 + qi i + qj j + qk k
                             Type: Octonion Polynomial Integer

  1$(Octonion Polynomial Integer) * q
    q1 + qi i + qj j + qk k
                             Type: Octonion Polynomial Integer
```

Finally, we check that the norm, defined as the sum of the squares of the coefficients, is a multiplicative map.

```
  o : Octonion Polynomial Integer := octon(o1, oi, oj, ok, oE, oI, oJ, oK)
    o1 + oi i + oj j + ok k + oE E + oI I + oJ J + oK K
                             Type: Octonion Polynomial Integer

  norm o
      2     2     2     2     2     2     2     2
    ok  + oj  + oi  + oK  + oJ  + oI  + oE  + o1
                             Type: Polynomial Integer
```

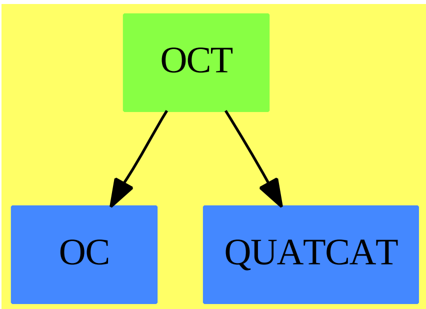
```
  p : Octonion Polynomial Integer := octon(p1, pi, pj, pk, pE, pI, pJ, pK)
    p1 + pi i + pj j + pk k + pE E + pI I + pJ J + pK K
                              Type: Octonion Polynomial Integer

Since the result is 0, the norm is multiplicative.

  norm(o*p)-norm(p)*norm(o)
    0
                              Type: Polynomial Integer

See Also:
o )help Quaternion
o )show Octonion
```

16.1.1 Octonion (OCT)



Exports:

0	1	abs	characteristic	charthRoot
coerce	conjugate	convert	eval	hash
imagE	imagI	imagJ	imagK	imagi
imagj	imagk	index	inv	latex
lookup	map	max	min	norm
octon	one?	random	rational	rational?
rationalIfCan	real	recip	retract	retractIfCan
sample	size	subtractIfCan	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?	?>?
?>=?	?.?			

— domain OCT Octonion —

```
)abbrev domain OCT Octonion
++ Author: R. Wisbauer, J. Grabmeier
++ Date Created: 05 September 1990
++ Date Last Updated: 20 September 1990
++ Basic Operations: _+,_*,octon,image,imagi,imagj,imagk,
++  imagE,imagI,imagJ,imagK
++ Related Constructors: Quaternion
++ Also See: AlgebraGivenByStructuralConstants
++ AMS Classifications:
++ Keywords: octonion, non-associative algebra, Cayley-Dixon
++ References: e.g. I.L Kantor, A.S. Solodovnikov:
++  Hypercomplex Numbers, Springer Verlag Heidelberg, 1989,
++  ISBN 0-387-96980-2
++ Description:
++ Octonion implements octonions (Cayley-Dixon algebra) over a
++ commutative ring, an eight-dimensional non-associative
++ algebra, doubling the quaternions in the same way as doubling
++ the complex numbers to get the quaternions
++ the main constructor function is octon which takes 8
++ arguments: the real part, the i imaginary part, the j
++ imaginary part, the k imaginary part, (as with quaternions)
++ and in addition the imaginary parts E, I, J, K.

--)boot $noSubsumption := true
Octonion(R:CommutativeRing): export == impl where

  QR ==> Quaternion R

  export ==> Join(OctonionCategory R, FullyRetractableTo QR)  with
    octon: (QR,QR) -> %
      ++ octon(qe,qE) constructs an octonion from two quaternions
      ++ using the relation O = Q + QE.
  impl ==> add
    Rep := Record(e: QR,E: QR)

    0 == [0,0]
    1 == [1,0]

    a,b,c,d,f,g,h,i : R
    p,q : QR
    x,y : %

    real  x == real (x.e)
    imagi x == imagI (x.e)
    imagj x == imagJ (x.e)
    imagk x == imagK (x.e)
    imagE x == real (x.E)
    imagI x == imagI (x.E)
    imagJ x == imagJ (x.E)
    imagK x == imagK (x.E)
```

```
    octon(a,b,c,d,f,g,h,i) == [quatern(a,b,c,d)$QR,quatern(f,g,h,i)$QR]
    octon(p,q) == [p,q]
    coerce(q) == [q,0$QR]
    retract(x):QR ==
      not(zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
        error "Cannot retract octonion to quaternion."
      quatern(real x, imagi x,imagj x, imagk x)$QR
    retractIfCan(x):Union(QR,"failed") ==
      not(zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
        "failed"
      quatern(real x, imagi x,imagj x, imagk x)$QR
    x * y == [x.e*y.e-(conjugate y.E)*x.E, y.E*x.e + x.E*(conjugate y.e)]
```

— **OCT.dotabb** —

```
"OCT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OCT"]
"OC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OC"]
"QUATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=QUATCAT"]
"OCT" -> "OC"
"OCT" -> "QUATCAT"
```

16.2 domain ODEIFTBL ODEIntensityFunctionsTable

— **ODEIntensityFunctionsTable.input** —

```
)set break resume
)sys rm -f ODEIntensityFunctionsTable.output
)spool ODEIntensityFunctionsTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ODEIntensityFunctionsTable
--R ODEIntensityFunctionsTable  is a domain constructor
--R Abbreviation for ODEIntensityFunctionsTable is ODEIFTBL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODEIFTBL
--R
--R------------------------------- Operations --------------------------------
```

```
--R clearTheIFTable : () -> Void          showTheIFTable : () -> %
--R iFTable : List Record(key: Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expres
--R insert! : Record(key: Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression D
--R keys : % -> List Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression Double
--R showIntensityFunctions : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expressio
--R
--E 1

)spool
)lisp (bye)
```

———-

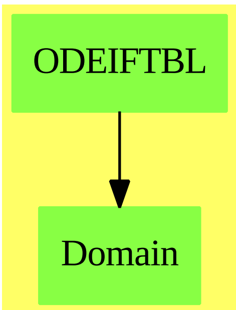
— **ODEIntensityFunctionsTable.help** —

```
====================================================================
ODEIntensityFunctionsTable examples
====================================================================

See Also:
o )show ODEIntensityFunctionsTable
```

———-

16.2.1 ODEIntensityFunctionsTable (ODEIFTBL)



Exports:
clearTheIFTable iFTable insert! keys showIntensityFunctions showTheIFTable

— **domain ODEIFTBL ODEIntensityFunctionsTable** —

```
)abbrev domain ODEIFTBL ODEIntensityFunctionsTable
++ Author: Brian Dupee
```

```
++ Date Created: May 1994
++ Date Last Updated: January 1996
++ Basic Operations: showTheIFTable, insert!
++ Description:
++ \axiom{ODEIntensityFunctionsTable()} provides a dynamic table and a set of
++ functions to store details found out about sets of ODE's.

ODEIntensityFunctionsTable(): E == I where
  LEDF  ==> List Expression DoubleFloat
  LEEDF  ==> List Equation Expression DoubleFloat
  EEDF  ==> Equation Expression DoubleFloat
  VEDF  ==> Vector Expression DoubleFloat
  MEDF  ==> Matrix Expression DoubleFloat
  MDF  ==> Matrix DoubleFloat
  EDF  ==> Expression DoubleFloat
  DF  ==> DoubleFloat
  F  ==> Float
  INT  ==> Integer
  CDF  ==> Complex DoubleFloat
  LDF  ==> List DoubleFloat
  LF  ==> List Float
  S  ==> Symbol
  LS  ==> List Symbol
  MFI  ==> Matrix Fraction Integer
  LFI  ==> List Fraction Integer
  FI  ==> Fraction Integer
  ODEA  ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,g:EDF,abserr:DF,relerr:DF)
  ON  ==> Record(additions:INT,multiplications:INT,exponentiations:INT,functionCalls:INT)
  RVE   ==> Record(val:EDF,exponent:INT)
  RSS  ==> Record(stiffnessFactor:F,stabilityFactor:F)
  ATT  ==> Record(stiffness:F,stability:F,expense:F,accuracy:F,intermediateResults:F)
  ROA  ==> Record(key:ODEA,entry:ATT)

  E ==> with
    showTheIFTable:() -> $
      ++ showTheIFTable() returns the current table of intensity functions.
    clearTheIFTable : () -> Void
      ++ clearTheIFTable() clears the current table of intensity functions.
    keys : $ -> List(ODEA)
      ++ keys(tab) returns the list of keys of f
    iFTable: List Record(key:ODEA,entry:ATT) -> $
      ++ iFTable(l) creates an intensity-functions table from the elements
      ++ of l.
    insert!:Record(key:ODEA,entry:ATT) -> $
      ++ insert!(r) inserts an entry r into theIFTable
    showIntensityFunctions:ODEA -> Union(ATT,"failed")
      ++ showIntensityFunctions(k) returns the entries in the
      ++ table of intensity functions k.

  I ==> add
```

```
Rep := Table(ODEA,ATT)
import Rep

theIFTable:$ := empty()$Rep

showTheIFTable():$ ==
  theIFTable

clearTheIFTable():Void ==
  theIFTable := empty()$Rep
  void()$Void

iFTable(l:List Record(key:ODEA,entry:ATT)):$ ==
  theIFTable := table(l)$Rep

insert!(r:Record(key:ODEA,entry:ATT)):$ ==
  insert!(r,theIFTable)$Rep

keys(t:$):List ODEA ==
  keys(t)$Rep

showIntensityFunctions(k:ODEA):Union(ATT,"failed") ==
  search(k,theIFTable)$Rep
```

— ODEIFTBL.dotabb —

```
"ODEIFTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODEIFTBL"]
"Domain" [color="#88FF44"]
"ODEIFTBL" -> "Domain"
```

16.3 domain ARRAY1 OneDimensionalArray

— OneDimensionalArray.input —

```
)set break resume
)sys rm -f OneDimensionalArray.output
)spool OneDimensionalArray.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 9
oneDimensionalArray [i**2 for i in 1..10]
--R
--R
--R   (1)  [1,4,9,16,25,36,49,64,81,100]
--R                                   Type: OneDimensionalArray PositiveInteger
--E 1

--S 2 of 9
a : ARRAY1 INT := new(10,0)
--R
--R
--R   (2)  [0,0,0,0,0,0,0,0,0,0]
--R                                           Type: OneDimensionalArray Integer
--E 2

--S 3 of 9
for i in 1..10 repeat a.i := i; a
--R
--R
--R   (3)  [1,2,3,4,5,6,7,8,9,10]
--R                                           Type: OneDimensionalArray Integer
--E 3

--S 4 of 9
map!(i +-> i ** 2,a); a
--R
--R
--R   (4)  [1,4,9,16,25,36,49,64,81,100]
--R                                           Type: OneDimensionalArray Integer
--E 4

--S 5 of 9
reverse! a
--R
--R
--R   (5)  [100,81,64,49,36,25,16,9,4,1]
--R                                           Type: OneDimensionalArray Integer
--E 5

--S 6 of 9
swap!(a,4,5); a
--R
--R
--R   (6)  [100,81,64,36,49,25,16,9,4,1]
--R                                           Type: OneDimensionalArray Integer
--E 6

--S 7 of 9
sort! a
```

```
--R
--R
--R   (7)  [1,4,9,16,25,36,49,64,81,100]
--R                                              Type: OneDimensionalArray Integer
--E 7

--S 8 of 9
b := a(6..10)
--R
--R
--R   (8)  [36,49,64,81,100]
--R                                              Type: OneDimensionalArray Integer
--E 8

--S 9 of 9
copyInto!(a,b,1)
--R
--R
--R   (9)  [36,49,64,81,100,36,49,64,81,100]
--R                                              Type: OneDimensionalArray Integer
--E 9
)spool
)lisp (bye)
```

— **OneDimensionalArray.help** —

```
====================================================================
OneDimensionalArray examples
====================================================================

The OneDimensionalArray domain is used for storing data in a
one-dimensional indexed data structure.  Such an array is a
homogeneous data structure in that all the entries of the array must
belong to the same Axiom domain.  Each array has a fixed length
specified by the user and arrays are not extensible.  The indexing of
one-dimensional arrays is one-based.  This means that the "first"
element of an array is given the index 1.

To create a one-dimensional array, apply the operation
oneDimensionalArray to a list.

  oneDimensionalArray [i**2 for i in 1..10]
    [1,4,9,16,25,36,49,64,81,100]
                     Type: OneDimensionalArray PositiveInteger

Another approach is to first create a, a one-dimensional array of 10
0's.  OneDimensionalArray has the convenient abbreviation ARRAY1.
```

```
  a : ARRAY1 INT := new(10,0)
    [0,0,0,0,0,0,0,0,0,0]
                     Type: OneDimensionalArray Integer
```

Set each i-th element to i, then display the result.

```
  for i in 1..10 repeat a.i := i; a
    [1,2,3,4,5,6,7,8,9,10]
                     Type: OneDimensionalArray Integer
```

Square each element by mapping the function i +-> i^2 onto each element.

```
  map!(i +-> i ** 2,a); a
    [1,4,9,16,25,36,49,64,81,100]
                     Type: OneDimensionalArray Integer
```

Reverse the elements in place.

```
  reverse! a
    [100,81,64,49,36,25,16,9,4,1]
                     Type: OneDimensionalArray Integer
```

Swap the 4th and 5th element.

```
  swap!(a,4,5); a
    [100,81,64,36,49,25,16,9,4,1]
                     Type: OneDimensionalArray Integer
```

Sort the elements in place.

```
  sort! a
    [1,4,9,16,25,36,49,64,81,100]
                     Type: OneDimensionalArray Integer
```

Create a new one-dimensional array b containing the last 5 elements of a.

```
  b := a(6..10)
    [36,49,64,81,100]
                     Type: OneDimensionalArray Integer
```

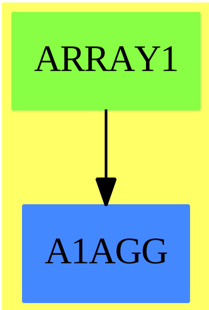
Replace the first 5 elements of a with those of b.

```
  copyInto!(a,b,1)
    [36,49,64,81,100,36,49,64,81,100]
                     Type: OneDimensionalArray Integer
```

See Also:
o)help Vector
o)help FlexibleArray

```
o )show OneDimensionalArray
```

16.3.1 OneDimensionalArray (ARRAY1)



See

⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
⇒ “IndexedFlexibleArray” (IFARRAY) 10.10.1 on page 1187
⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	new	oneDimensionalArray	parts	position
qelt	qsetelt!	reduce	remove	removeDuplicates
reverse	reverse!	sample	select	setelt
size?	sort	sort!	sorted?	swap!
#?	?<?	?<=?	?=?	?>?
?>=?	?~=?	?.?		

— domain ARRAY1 OneDimensionalArray —

```
)abbrev domain ARRAY1 OneDimensionalArray
++ Author: Mark Botch
++ Description:
```

```
++ This is the domain of 1-based one dimensional arrays

OneDimensionalArray(S:Type): Exports == Implementation where
  ARRAYMININDEX ==> 1       -- if you want to change this, be my guest
  Exports == OneDimensionalArrayAggregate S with
   oneDimensionalArray: List S -> %
    ++ oneDimensionalArray(l) creates an array from a list of elements l
    ++
    ++X oneDimensionalArray [i**2 for i in 1..10]

   oneDimensionalArray: (NonNegativeInteger, S) -> %
    ++ oneDimensionalArray(n,s) creates an array from n copies of element s
    ++
    ++X oneDimensionalArray(10,0.0)

  Implementation == IndexedOneDimensionalArray(S, ARRAYMININDEX) add
    oneDimensionalArray(u) ==
      n := #u
      n = 0 => empty()
      a := new(n, first u)
      for i in 2..n for x in rest u repeat a.i := x
      a
    oneDimensionalArray(n,s) == new(n,s)
```

— **ARRAY1.dotabb** —

```
"ARRAY1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ARRAY1"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ARRAY1" -> "A1AGG"
```

16.4 domain ONECOMP OnePointCompletion

— **OnePointCompletion.input** —

```
)set break resume
)sys rm -f OnePointCompletion.output
)spool OnePointCompletion.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show OnePointCompletion
--R OnePointCompletion R: SetCategory  is a domain constructor
--R Abbreviation for OnePointCompletion is ONECOMP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ONECOMP
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has ABELGRP          ?=? : (%,%) -> Boolean
--R 1 : () -> % if R has ORDRING          0 : () -> % if R has ABELGRP
--R coerce : R -> %                       coerce : % -> OutputForm
--R finite? : % -> Boolean                hash : % -> SingleInteger
--R infinite? : % -> Boolean              infinity : () -> %
--R latex : % -> String                   retract : % -> R
--R ?~=? : (%,%) -> Boolean
--R ?*? : (PositiveInteger,%) -> % if R has ABELGRP
--R ?*? : (NonNegativeInteger,%) -> % if R has ABELGRP
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?*? : (%,%) -> % if R has ORDRING
--R ?**? : (%,NonNegativeInteger) -> % if R has ORDRING
--R ?**? : (%,PositiveInteger) -> % if R has ORDRING
--R ?+? : (%,%) -> % if R has ABELGRP
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDRING
--R ?<=? : (%,%) -> Boolean if R has ORDRING
--R ?>? : (%,%) -> Boolean if R has ORDRING
--R ?>=? : (%,%) -> Boolean if R has ORDRING
--R ?^? : (%,NonNegativeInteger) -> % if R has ORDRING
--R ?^? : (%,PositiveInteger) -> % if R has ORDRING
--R abs : % -> % if R has ORDRING
--R characteristic : () -> NonNegativeInteger if R has ORDRING
--R coerce : Integer -> % if R has ORDRING or R has RETRACT INT
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R max : (%,%) -> % if R has ORDRING
--R min : (%,%) -> % if R has ORDRING
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has ORDRING
--R positive? : % -> Boolean if R has ORDRING
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R recip : % -> Union(%,"failed") if R has ORDRING
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R sample : () -> % if R has ABELGRP
--R sign : % -> Integer if R has ORDRING
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
--R zero? : % -> Boolean if R has ABELGRP
--R
--E 1

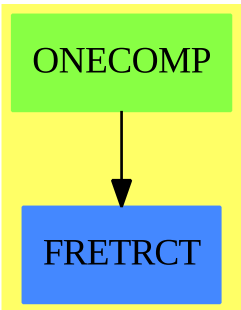
)spool
)lisp (bye)
```

— **OnePointCompletion.help** —

```
====================================================================
OnePointCompletion examples
====================================================================

See Also:
o )show OnePointCompletion
```

16.4.1 OnePointCompletion (ONECOMP)



See

⇒ "OrderedCompletion" (ORDCOMP) 16.12.1 on page 1772

Exports:

0	1	abs	characteristic	coerce
coerce	finite?	hash	infinite?	infinity
latex	max	min	negative?	one?
positive?	rational	rational?	rationalIfCan	recip
retract	retractIfCan	sample	sign	subtractIfCan
zero?	-?	?=?	?~=?	?*?
?**?	?+?	?-?	?<?	?<=?
?>?	?>=?	?^?		

— domain ONECOMP OnePointCompletion —

```
)abbrev domain ONECOMP OnePointCompletion
++ Author: Manuel Bronstein
++ Date Created: 4 Oct 1989
++ Date Last Updated: 1 Nov 1989
++ Description:
++ Completion with infinity.
++ Adjunction of a complex infinity to a set.

OnePointCompletion(R:SetCategory): Exports == Implementation where
  B ==> Boolean

  Exports ==> Join(SetCategory, FullyRetractableTo R) with
    infinity : () -> %
      ++  infinity() returns infinity.
    finite?  : %  -> B
      ++ finite?(x) tests if x is finite.
    infinite?: %  -> B
      ++ infinite?(x) tests if x is infinite.
    if R has AbelianGroup then AbelianGroup
    if R has OrderedRing then OrderedRing
    if R has IntegerNumberSystem then
      rational?: % -> Boolean
        ++ rational?(x) tests if x is a finite rational number.
      rational : % -> Fraction Integer
        ++ rational(x) returns x as a finite rational number.
        ++ Error: if x is not a rational number.
      rationalIfCan: % -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(x) returns x as a finite rational number if
        ++ it is one, "failed" otherwise.

  Implementation ==> add
    Rep := Union(R, "infinity")

    coerce(r:R):%          == r
    retract(x:%):R         == (x case R => x::R; error "Not finite")
    finite? x              == x case R
    infinite? x            == x case "infinity"
    infinity()             == "infinity"
    retractIfCan(x:%):Union(R, "failed") == (x case R => x::R; "failed")

    coerce(x:%):OutputForm ==
      x case "infinity" => "infinity"::OutputForm
      x::R::OutputForm

    x = y ==
      x case "infinity" => y case "infinity"
      y case "infinity" => false
```

```
      x::R = y::R

    if R has AbelianGroup then
      0 == 0$R

      n:Integer * x:% ==
        x case "infinity" =>
          zero? n => error "Undefined product"
          infinity()
        n * x::R

      - x ==
        x case "infinity" => error "Undefined inverse"
        - (x::R)

      x + y ==
        x case "infinity" => x
        y case "infinity" => y
        x::R + y::R

    if R has OrderedRing then
      fininf: R -> %

      1                == 1$R
      characteristic() == characteristic()$R

      fininf r ==
        zero? r => error "Undefined product"
        infinity()

      x:% * y:% ==
        x case "infinity" =>
          y case "infinity" => y
          fininf(y::R)
        y case "infinity" => fininf(x::R)
        x::R * y::R

      recip x ==
        x case "infinity" => 0
        zero?(x::R) => infinity()
        (u := recip(x::R)) case "failed" => "failed"
        u::R::%

      x < y ==
        x case "infinity" => false     -- do not change the order
        y case "infinity" => true      -- of those two tests
        x::R < y::R

    if R has IntegerNumberSystem then
      rational? x == finite? x
```

```
      rational  x == rational(retract(x)@R)

      rationalIfCan x ==
        (r:= retractIfCan(x)@Union(R,"failed")) case "failed" =>"failed"
        rational(r::R)
```

————-

— ONECOMP.dotabb —

```
"ONECOMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ONECOMP"]
"FRETRCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRETRCT"]
"ONECOMP" -> "FRETRCT"
```

————-

16.5 domain OMCONN OpenMathConnection

— OpenMathConnection.input —

```
)set break resume
)sys rm -f OpenMathConnection.output
)spool OpenMathConnection.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathConnection
--R OpenMathConnection  is a domain constructor
--R Abbreviation for OpenMathConnection is OMCONN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMCONN
--R
--R------------------------------- Operations --------------------------------
--R OMcloseConn : % -> Void               OMmakeConn : SingleInteger -> %
--R OMbindTCP : (%,SingleInteger) -> Boolean
--R OMconnInDevice : % -> OpenMathDevice
--R OMconnOutDevice : % -> OpenMathDevice
--R OMconnectTCP : (%,String,SingleInteger) -> Boolean
--R
--E 1

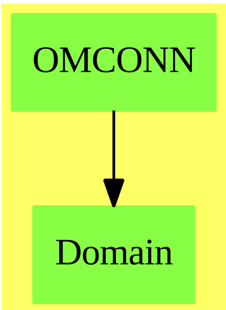
)spool
)lisp (bye)
```

— OpenMathConnection.help —

```
====================================================================
OpenMathConnection examples
====================================================================

See Also:
o )show OpenMathConnection
```

16.5.1 OpenMathConnection (OMCONN)



See

⇒ "OpenMathEncoding" (OMENC) 16.7.1 on page 1751
⇒ "OpenMathDevice" (OMDEV) 16.6.1 on page 1746

Exports:

OMbindTCP	OMcloseConn	OMconnectTCP	OMconnInDevice
OMconnOutDevice	OMmakeConn		

— domain OMCONN OpenMathConnection —

```
)abbrev domain OMCONN OpenMathConnection
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ \spadtype{OpenMathConnection} provides low-level functions
++ for handling connections to and from \spadtype{OpenMathDevice}s.

OpenMathConnection(): with
  OMmakeConn    : SingleInteger -> % ++ \spad{OMmakeConn}
  OMcloseConn   : % -> Void ++ \spad{OMcloseConn}
  OMconnInDevice: %-> OpenMathDevice ++ \spad{OMconnInDevice:}
  OMconnOutDevice: %-> OpenMathDevice ++ \spad{OMconnOutDevice:}
  OMconnectTCP  : (%, String, SingleInteger) -> Boolean ++ \spad{OMconnectTCP}
  OMbindTCP     : (%, SingleInteger) -> Boolean ++ \spad{OMbindTCP}
 == add
  OMmakeConn(timeout: SingleInteger): % == OM_-MAKECONN(timeout)$Lisp
  OMcloseConn(conn: %): Void == OM_-CLOSECONN(conn)$Lisp

  OMconnInDevice(conn: %): OpenMathDevice ==
    OM_-GETCONNINDEV(conn)$Lisp
  OMconnOutDevice(conn: %): OpenMathDevice ==
    OM_-GETCONNOUTDEV(conn)$Lisp

  OMconnectTCP(conn: %, host: String, port: SingleInteger): Boolean ==
    OM_-CONNECTTCP(conn, host, port)$Lisp
  OMbindTCP(conn: %, port: SingleInteger): Boolean ==
    OM_-BINDTCP(conn, port)$Lisp
```

— **OMCONN.dotabb** —

```
"OMCONN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMCONN"]
"Domain" [color="#88FF44"]
"OMCONN" -> "Domain"
```

16.6 domain OMDEV OpenMathDevice

— **OpenMathDevice.input** —

```
)set break resume
)sys rm -f OpenMathDevice.output
)spool OpenMathDevice.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show OpenMathDevice
--R OpenMathDevice  is a domain constructor
--R Abbreviation for OpenMathDevice is OMDEV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMDEV
--R
--R------------------------------ Operations -------------------------------
--R OMclose : % -> Void                   OMgetApp : % -> Void
--R OMgetAtp : % -> Void                  OMgetAttr : % -> Void
--R OMgetBVar : % -> Void                 OMgetBind : % -> Void
--R OMgetEndApp : % -> Void               OMgetEndAtp : % -> Void
--R OMgetEndAttr : % -> Void              OMgetEndBVar : % -> Void
--R OMgetEndBind : % -> Void              OMgetEndError : % -> Void
--R OMgetEndObject : % -> Void            OMgetError : % -> Void
--R OMgetFloat : % -> DoubleFloat         OMgetInteger : % -> Integer
--R OMgetObject : % -> Void               OMgetString : % -> String
--R OMgetType : % -> Symbol               OMgetVariable : % -> Symbol
--R OMputApp : % -> Void                  OMputAtp : % -> Void
--R OMputAttr : % -> Void                 OMputBVar : % -> Void
--R OMputBind : % -> Void                 OMputEndApp : % -> Void
--R OMputEndAtp : % -> Void               OMputEndAttr : % -> Void
--R OMputEndBVar : % -> Void              OMputEndBind : % -> Void
--R OMputEndError : % -> Void             OMputEndObject : % -> Void
--R OMputError : % -> Void                OMputObject : % -> Void
--R OMputString : (%,String) -> Void
--R OMgetSymbol : % -> Record(cd: String,name: String)
--R OMopenFile : (String,String,OpenMathEncoding) -> %
--R OMopenString : (String,OpenMathEncoding) -> %
--R OMputFloat : (%,DoubleFloat) -> Void
--R OMputInteger : (%,Integer) -> Void
--R OMputSymbol : (%,String,String) -> Void
--R OMputVariable : (%,Symbol) -> Void
--R OMsetEncoding : (%,OpenMathEncoding) -> Void
--R
--E 1

)spool
)lisp (bye)
```

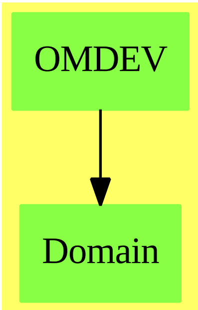
— **OpenMathDevice.help** —

```
====================================================================
OpenMathDevice examples
====================================================================
```

```
See Also:
o )show OpenMathDevice
```

———-

16.6.1 OpenMathDevice (OMDEV)



See

⇒ "OpenMathEncoding" (OMENC) 16.7.1 on page 1751
⇒ "OpenMathConnection" (OMCONN) 16.5.1 on page 1743

Exports:

OMclose	OMgetApp	OMgetAtp	OMgetAttr
OMgetBVar	OMgetBind	OMgetEndApp	OMgetEndAtp
OMgetEndAttr	OMgetEndBVar	OMgetEndBind	OMgetEndError
OMgetEndObject	OMgetError	OMgetFloat	OMgetInteger
OMgetObject	OMgetString	OMgetType	OMgetVariable
OMputApp	OMputAtp	OMputAttr	OMputBVar
OMputBind	OMputEndApp	OMputEndAtp	OMputEndAttr
OMputEndBVar	OMputEndBind	OMputEndError	OMputEndObject
OMputError	OMputObject	OMputString	OMgetSymbol
OMopenFile	OMopenString	OMputFloat	OMputInteger
OMputSymbol	OMputVariable	OMsetEncoding	

— **domain OMDEV OpenMathDevice** —

```
)abbrev domain OMDEV OpenMathDevice
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMathDevice} provides support for reading
++ and writing openMath objects to files, strings etc.  It also provides
++ access to low-level operations from within the interpreter.

OpenMathDevice(): with
  OMopenFile    : (String, String, OpenMathEncoding) -> %
  ++ OMopenFile(f,mode,enc) opens file \axiom{f} for reading or writing
  ++ OpenMath objects (depending on \axiom{mode} which can be "r", "w"
  ++ or "a" for read, write and append respectively), in the encoding
  ++ \axiom{enc}.
  OMopenString  : (String, OpenMathEncoding) -> %
  ++ OMopenString(s,mode) opens the string \axiom{s} for reading or writing
  ++ OpenMath objects in encoding \axiom{enc}.
  OMclose       : % -> Void
  ++ OMclose(dev) closes \axiom{dev}, flushing output if necessary.
  OMsetEncoding : (%, OpenMathEncoding) -> Void
  ++ OMsetEncoding(dev,enc) sets the encoding used for reading or writing
  ++ OpenMath objects to or from \axiom{dev} to \axiom{enc}.
  OMputApp      : % -> Void
  ++ OMputApp(dev) writes a begin application token to \axiom{dev}.
  OMputAtp      : % -> Void
  ++ OMputAtp(dev) writes a begin attribute pair token to \axiom{dev}.
  OMputAttr     : % -> Void
  ++ OMputAttr(dev) writes a begin attribute token to \axiom{dev}.
  OMputBind     : % -> Void
  ++ OMputBind(dev) writes a begin binder token to \axiom{dev}.
  OMputBVar     : % -> Void
  ++ OMputBVar(dev) writes a begin bound variable list token to \axiom{dev}.
  OMputError    : % -> Void
  ++ OMputError(dev) writes a begin error token to \axiom{dev}.
  OMputObject   : % -> Void
  ++ OMputObject(dev) writes a begin object token to \axiom{dev}.
  OMputEndApp   : % -> Void
  ++ OMputEndApp(dev) writes an end application token to \axiom{dev}.
  OMputEndAtp   : % -> Void
  ++ OMputEndAtp(dev) writes an end attribute pair token to \axiom{dev}.
  OMputEndAttr  : % -> Void
  ++ OMputEndAttr(dev) writes an end attribute token to \axiom{dev}.
  OMputEndBind  : % -> Void
  ++ OMputEndBind(dev) writes an end binder token to \axiom{dev}.
  OMputEndBVar  : % -> Void
  ++ OMputEndBVar(dev) writes an end bound variable list token to \axiom{dev}.
  OMputEndError : % -> Void
  ++ OMputEndError(dev) writes an end error token to \axiom{dev}.
  OMputEndObject: % -> Void
  ++ OMputEndObject(dev) writes an end object token to \axiom{dev}.
  OMputInteger  : (%, Integer) -> Void
```

```
++ OMputInteger(dev,i) writes the integer \axiom{i} to \axiom{dev}.
OMputFloat    : (%, DoubleFloat) -> Void
++ OMputFloat(dev,i) writes the float \axiom{i} to \axiom{dev}.
OMputVariable : (%, Symbol) -> Void
++ OMputVariable(dev,i) writes the variable \axiom{i} to \axiom{dev}.
OMputString   : (%, String) -> Void
++ OMputString(dev,i) writes the string \axiom{i} to \axiom{dev}.
OMputSymbol   : (%, String, String) -> Void
++ OMputSymbol(dev,cd,s) writes the symbol \axiom{s} from CD \axiom{cd}
++ to \axiom{dev}.

OMgetApp      : % -> Void
++ OMgetApp(dev) reads a begin application token from \axiom{dev}.
OMgetAtp      : % -> Void
++ OMgetAtp(dev) reads a begin attribute pair token from \axiom{dev}.
OMgetAttr     : % -> Void
++ OMgetAttr(dev) reads a begin attribute token from \axiom{dev}.
OMgetBind     : % -> Void
++ OMgetBind(dev) reads a begin binder token from \axiom{dev}.
OMgetBVar     : % -> Void
++ OMgetBVar(dev) reads a begin bound variable list token from \axiom{dev}.
OMgetError    : % -> Void
++ OMgetError(dev) reads a begin error token from \axiom{dev}.
OMgetObject   : % -> Void
++ OMgetObject(dev) reads a begin object token from \axiom{dev}.
OMgetEndApp   : % -> Void
++ OMgetEndApp(dev) reads an end application token from \axiom{dev}.
OMgetEndAtp   : % -> Void
++ OMgetEndAtp(dev) reads an end attribute pair token from \axiom{dev}.
OMgetEndAttr  : % -> Void
++ OMgetEndAttr(dev) reads an end attribute token from \axiom{dev}.
OMgetEndBind  : % -> Void
++ OMgetEndBind(dev) reads an end binder token from \axiom{dev}.
OMgetEndBVar  : % -> Void
++ OMgetEndBVar(dev) reads an end bound variable list token from \axiom{dev}.
OMgetEndError : % -> Void
++ OMgetEndError(dev) reads an end error token from \axiom{dev}.
OMgetEndObject: % -> Void
++ OMgetEndObject(dev) reads an end object token from \axiom{dev}.
OMgetInteger  : % -> Integer
++ OMgetInteger(dev) reads an integer from \axiom{dev}.
OMgetFloat    : % -> DoubleFloat
++ OMgetFloat(dev) reads a float from \axiom{dev}.
OMgetVariable : % -> Symbol
++ OMgetVariable(dev) reads a variable from \axiom{dev}.
OMgetString   : % -> String
++ OMgetString(dev) reads a string from \axiom{dev}.
OMgetSymbol   : % -> Record(cd:String, name:String)
++ OMgetSymbol(dev) reads a symbol from \axiom{dev}.
```

```
 OMgetType     : % -> Symbol
 ++ OMgetType(dev) returns the type of the next object on \axiom{dev}.
== add
 OMopenFile(fname: String, fmode: String, enc: OpenMathEncoding): % ==
   OM_-OPENFILEDEV(fname, fmode, enc)$Lisp
 OMopenString(str: String, enc: OpenMathEncoding): % ==
   OM_-OPENSTRINGDEV(str, enc)$Lisp
 OMclose(dev: %): Void ==
   OM_-CLOSEDEV(dev)$Lisp
 OMsetEncoding(dev: %, enc: OpenMathEncoding): Void ==
   OM_-SETDEVENCODING(dev, enc)$Lisp

 OMputApp(dev: %): Void == OM_-PUTAPP(dev)$Lisp
 OMputAtp(dev: %): Void == OM_-PUTATP(dev)$Lisp
 OMputAttr(dev: %): Void == OM_-PUTATTR(dev)$Lisp
 OMputBind(dev: %): Void == OM_-PUTBIND(dev)$Lisp
 OMputBVar(dev: %): Void == OM_-PUTBVAR(dev)$Lisp
 OMputError(dev: %): Void == OM_-PUTERROR(dev)$Lisp
 OMputObject(dev: %): Void == OM_-PUTOBJECT(dev)$Lisp
 OMputEndApp(dev: %): Void == OM_-PUTENDAPP(dev)$Lisp
 OMputEndAtp(dev: %): Void == OM_-PUTENDATP(dev)$Lisp
 OMputEndAttr(dev: %): Void == OM_-PUTENDATTR(dev)$Lisp
 OMputEndBind(dev: %): Void == OM_-PUTENDBIND(dev)$Lisp
 OMputEndBVar(dev: %): Void == OM_-PUTENDBVAR(dev)$Lisp
 OMputEndError(dev: %): Void == OM_-PUTENDERROR(dev)$Lisp
 OMputEndObject(dev: %): Void == OM_-PUTENDOBJECT(dev)$Lisp
 OMputInteger(dev: %, i: Integer): Void == OM_-PUTINT(dev, i)$Lisp
 OMputFloat(dev: %, f: DoubleFloat): Void == OM_-PUTFLOAT(dev, f)$Lisp
 --OMputByteArray(dev: %, b: Array Byte): Void == OM_-PUTBYTEARRAY(dev, b)$Lisp
 OMputVariable(dev: %, v: Symbol): Void == OM_-PUTVAR(dev, v)$Lisp
 OMputString(dev: %, s: String): Void == OM_-PUTSTRING(dev, s)$Lisp
 OMputSymbol(dev: %, cd: String, nm: String): Void == OM_-PUTSYMBOL(dev, cd, nm)$Lisp

 OMgetApp(dev: %): Void == OM_-GETAPP(dev)$Lisp
 OMgetAtp(dev: %): Void == OM_-GETATP(dev)$Lisp
 OMgetAttr(dev: %): Void == OM_-GETATTR(dev)$Lisp
 OMgetBind(dev: %): Void == OM_-GETBIND(dev)$Lisp
 OMgetBVar(dev: %): Void == OM_-GETBVAR(dev)$Lisp
 OMgetError(dev: %): Void == OM_-GETERROR(dev)$Lisp
 OMgetObject(dev: %): Void == OM_-GETOBJECT(dev)$Lisp
 OMgetEndApp(dev: %): Void == OM_-GETENDAPP(dev)$Lisp
 OMgetEndAtp(dev: %): Void == OM_-GETENDATP(dev)$Lisp
 OMgetEndAttr(dev: %): Void == OM_-GETENDATTR(dev)$Lisp
 OMgetEndBind(dev: %): Void == OM_-GETENDBIND(dev)$Lisp
 OMgetEndBVar(dev: %): Void == OM_-GETENDBVAR(dev)$Lisp
 OMgetEndError(dev: %): Void == OM_-GETENDERROR(dev)$Lisp
 OMgetEndObject(dev: %): Void == OM_-GETENDOBJECT(dev)$Lisp
 OMgetInteger(dev: %): Integer == OM_-GETINT(dev)$Lisp
 OMgetFloat(dev: %): DoubleFloat == OM_-GETFLOAT(dev)$Lisp
 --OMgetByteArray(dev: %): Array Byte == OM_-GETBYTEARRAY(dev)$Lisp
```

```
  OMgetVariable(dev: %): Symbol == OM_-GETVAR(dev)$Lisp
  OMgetString(dev: %): String == OM_-GETSTRING(dev)$Lisp
  OMgetSymbol(dev: %): Record(cd:String, name:String) == OM_-GETSYMBOL(dev)$Lisp

  OMgetType(dev: %): Symbol == OM_-GETTYPE(dev)$Lisp
```

———-

— OMDEV.dotabb —

```
"OMDEV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMDEV"]
"Domain" [color="#88FF44"]
"OMDEV" -> "Domain"
```

———-

16.7 domain OMENC OpenMathEncoding

— OpenMathEncoding.input —

```
)set break resume
)sys rm -f OpenMathEncoding.output
)spool OpenMathEncoding.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathEncoding
--R OpenMathEncoding  is a domain constructor
--R Abbreviation for OpenMathEncoding is OMENC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMENC
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                OMencodingBinary : () -> %
--R OMencodingSGML : () -> %              OMencodingUnknown : () -> %
--R OMencodingXML : () -> %               coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

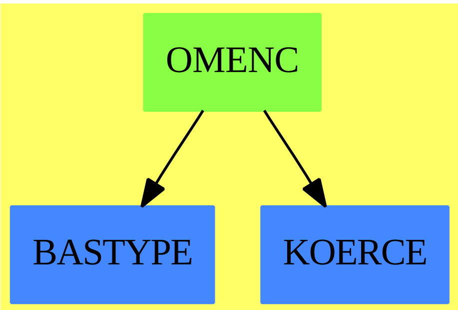
— OpenMathEncoding.help —

```
====================================================================
OpenMathEncoding examples
====================================================================

See Also:
o )show OpenMathEncoding
```

———-

16.7.1 OpenMathEncoding (OMENC)



See

⇒ “OpenMathDevice” (OMDEV) 16.6.1 on page 1746
⇒ “OpenMathConnection” (OMCONN) 16.5.1 on page 1743

Exports:

coerce	hash	latex	OMencodingBinary	OMencodingSGML
OMencodingUnknown	OMencodingXML	?=?	?~=?	

— domain OMENC OpenMathEncoding —

```
)abbrev domain OMENC OpenMathEncoding
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ \spadtype{OpenMathEncoding} is the set of valid OpenMath encodings.

OpenMathEncoding(): SetCategory with
  OMencodingUnknown : () -> %
  ++ OMencodingUnknown() is the constant for unknown encoding types. If this
  ++ is used on an input device, the encoding will be autodetected.
  ++ It is invalid to use it on an output device.
  OMencodingXML     : () -> %
  ++ OMencodingXML() is the constant for the OpenMath XML encoding.
  OMencodingSGML    : () -> %
  ++ OMencodingSGML() is the constant for the deprecated OpenMath SGML encoding.
  OMencodingBinary  : () -> %
  ++ OMencodingBinary() is the constant for the OpenMath binary encoding.
 == add
  Rep := SingleInteger

  =(u,v) == (u=v)$Rep

  import Rep

  coerce(u) ==
    u::Rep = 0$Rep => "Unknown"::OutputForm
    u::Rep = 1$Rep => "Binary"::OutputForm
    u::Rep = 2::Rep => "XML"::OutputForm
    u::Rep = 3::Rep => "SGML"::OutputForm
    error "Bogus OpenMath Encoding Type"

  OMencodingUnknown(): % == 0::Rep
  OMencodingBinary(): % == 1::Rep
  OMencodingXML(): % == 2::Rep
  OMencodingSGML(): % == 3::Rep
```

———

— OMENC.dotabb —

```
"OMENC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMENC"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"OMENC" -> "BASTYPE"
"OMENC" -> "KOERCE"
```

———

16.8 domain OMERR OpenMathError

— OpenMathError.input —

```
)set break resume
)sys rm -f OpenMathError.output
)spool OpenMathError.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathError
--R OpenMathError  is a domain constructor
--R Abbreviation for OpenMathError is OMERR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMERR
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R errorInfo : % -> List Symbol          hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R errorKind : % -> OpenMathErrorKind
--R omError : (OpenMathErrorKind,List Symbol) -> %
--R
--E 1

)spool
)lisp (bye)
```

———

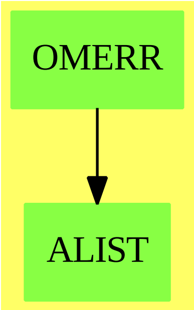
— OpenMathError.help —

```
====================================================================
OpenMathError examples
====================================================================

See Also:
o )show OpenMathError
```

———

16.8.1 OpenMathError (OMERR)



See

⇒ "OpenMathErrorKind" (OMERRK) 16.9.1 on page 1756

Exports:

coerce	errorInfo	errorKind	hash	latex
omError	?=?	? =?		

— domain OMERR OpenMathError —

```
)abbrev domain OMERR OpenMathError
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMathError} is the domain of OpenMath errors.

OpenMathError() : SetCategory with
  errorKind : % -> OpenMathErrorKind
  ++ errorKind(u) returns the type of error which u represents.
  errorInfo : % -> List Symbol
  ++ errorInfo(u) returns information about the error u.
  omError   : (OpenMathErrorKind, List Symbol) -> %
  ++ omError(k,l) creates an instance of OpenMathError.
 == add
  Rep := Record(err:OpenMathErrorKind, info:List Symbol)

  import List String

  coerce(e:%):OutputForm ==
    OMParseError? e.err => message "Error parsing OpenMath object"
```

```
    infoSize := #(e.info)
    OMUnknownCD? e.err =>
--      not one? infoSize => error "Malformed info list in OMUnknownCD"
      not (infoSize = 1) => error "Malformed info list in OMUnknownCD"
      message concat("Cannot handle CD ",string first e.info)
    OMUnknownSymbol? e.err =>
      not 2=infoSize => error "Malformed info list in OMUnknownSymbol"
      message concat ["Cannot handle Symbol ",
                      string e.info.2, " from CD ", string e.info.1]
    OMReadError? e.err =>
      message "OpenMath read error"
    error "Malformed OpenMath Error"

  omError(e:OpenMathErrorKind,i:List Symbol):% == [e,i]$Rep

  errorKind(e:%):OpenMathErrorKind == e.err
  errorInfo(e:%):List Symbol == e.info
```

———-

— **OMERR.dotabb** —

```
"OMERR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMERR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OMERR" -> "ALIST"
```

———-

16.9 domain OMERRK OpenMathErrorKind

— **OpenMathErrorKind.input** —

```
)set break resume
)sys rm -f OpenMathErrorKind.output
)spool OpenMathErrorKind.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathErrorKind
--R OpenMathErrorKind  is a domain constructor
--R Abbreviation for OpenMathErrorKind is OMERRK
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMERRK
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                  OMParseError? : % -> Boolean
--R OMReadError? : % -> Boolean             OMUnknownCD? : % -> Boolean
--R OMUnknownSymbol? : % -> Boolean         coerce : Symbol -> %
--R coerce : % -> OutputForm                hash : % -> SingleInteger
--R latex : % -> String                     ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

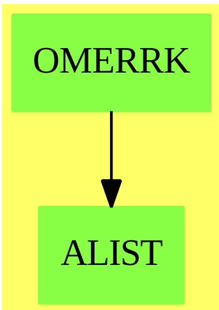
— **OpenMathErrorKind.help** —

```
====================================================================
OpenMathErrorKind examples
====================================================================

See Also:
o )show OpenMathErrorKind
```

———-

16.9.1 OpenMathErrorKind (OMERRK)



See

⇒ "OpenMathError" (OMERR) 16.8.1 on page 1754

Exports:
OMParseError? OMReadError? OMUnknownCD? OMUnknownSymbol? coerce hash

— domain OMERRK OpenMathErrorKind —

```
)abbrev domain OMERRK OpenMathErrorKind
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMathErrorKind} represents different kinds
++ of OpenMath errors: specifically parse errors, unknown CD or symbol
++ errors, and read errors.

OpenMathErrorKind() : SetCategory with
  coerce          : Symbol -> %
  ++ coerce(u) creates an OpenMath error object of an appropriate type if
  ++ \axiom{u} is one of \axiom{OMParseError}, \axiom{OMReadError},
  ++ \axiom{OMUnknownCD} or \axiom{OMUnknownSymbol}, otherwise it
  ++ raises a runtime error.
  OMParseError?    : % -> Boolean
  ++ OMParseError?(u) tests whether u is an OpenMath parsing error.
  OMUnknownCD?     : % -> Boolean
  ++ OMUnknownCD?(u) tests whether u is an OpenMath unknown CD error.
  OMUnknownSymbol? : % -> Boolean
  ++ OMUnknownSymbol?(u) tests whether u is an OpenMath unknown symbol error.
  OMReadError?     : % -> Boolean
  ++ OMReadError?(u) tests whether u is an OpenMath read error.
 == add
  Rep := Union(parseError:"OMParseError", unknownCD:"OMUnknownCD",
               unknownSymbol:"OMUnknownSymbol",readError:"OMReadError")

  OMParseError?(u) == (u case parseError)$Rep
  OMUnknownCD?(u) == (u case unknownCD)$Rep
  OMUnknownSymbol?(u) == (u case unknownSymbol)$Rep
  OMReadError?(u) == (u case readError)$Rep

  coerce(s:Symbol):% ==
    s = OMParseError    => ["OMParseError"]$Rep
    s = OMUnknownCD     => ["OMUnknownCD"]$Rep
    s = OMUnknownSymbol => ["OMUnknownSymbol"]$Rep
    s = OMReadError     => ["OMReadError"]$Rep
    error concat(string s, " is not a valid OpenMathErrorKind.")

  a = b == (a=b)$Rep
```

```
  coerce(e:%):OutputForm == coerce(e)$Rep
```

———-

— OMERRK.dotabb —

```
"OMERRK" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMERRK"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OMERRK" -> "ALIST"
```

———-

16.10 domain OP Operator

— Operator.input —

```
)set break resume
)sys rm -f Operator.output
)spool Operator.output
)set message test on
)set message auto off
)clear all
--S 1 of 21
R := SQMATRIX(2, INT)
--R
--R
--R   (1)  SquareMatrix(2,Integer)
--R                                                            Type: Domain
--E 1

--S 2 of 21
t := operator("tilde") :: OP(R)
--R
--R
--R   (2)  tilde
--R                                          Type: Operator SquareMatrix(2,Integer)
--E 2

--S 3 of 21
)set expose add constructor Operator
--R
--I   Operator is now explicitly exposed in frame frame0
--E 3
```

```
--S 4 of 21
evaluate(t, m +-> transpose m)
--R
--R
--R   (3)  tilde
--R                                      Type: Operator SquareMatrix(2,Integer)
--E 4

--S 5 of 21
s : R := matrix [ [0, 1], [1, 0] ]
--R
--R
--R        +0  1+
--R   (4)  |    |
--R        +1  0+
--R                                               Type: SquareMatrix(2,Integer)
--E 5

--S 6 of 21
rho := t * s
--R
--R
--R             +0  1+
--R   (5)  tilde|    |
--R             +1  0+
--R                                      Type: Operator SquareMatrix(2,Integer)
--E 6

--S 7 of 21
z := rho**4 - 1
--R
--R
--R                   +0  1+     +0  1+     +0  1+     +0  1+
--R   (6)  - 1 + tilde|    |tilde|    |tilde|    |tilde|    |
--R                   +1  0+     +1  0+     +1  0+     +1  0+
--R                                      Type: Operator SquareMatrix(2,Integer)
--E 7

--S 8 of 21
m:R := matrix [ [1, 2], [3, 4] ]
--R
--R
--R        +1  2+
--R   (7)  |    |
--R        +3  4+
--R                                               Type: SquareMatrix(2,Integer)
--E 8

--S 9 of 21
z m
```

```
--R
--R
--R        +0  0+
--R   (8)  |    |
--R        +0  0+
--R                                                   Type: SquareMatrix(2,Integer)
--E 9

--S 10 of 21
rho m
--R
--R
--R        +3  1+
--R   (9)  |    |
--R        +4  2+
--R                                                   Type: SquareMatrix(2,Integer)
--E 10

--S 11 of 21
rho rho m
--R
--R
--R         +4  3+
--R   (10)  |    |
--R         +2  1+
--R                                                   Type: SquareMatrix(2,Integer)
--E 11

--S 12 of 21
(rho^3) m
--R
--R
--R         +2  4+
--R   (11)  |    |
--R         +1  3+
--R                                                   Type: SquareMatrix(2,Integer)
--E 12

--S 13 of 21
b := t * s - s * t
--R
--R
--R            +0  1+             +0  1+
--R   (12)  - |    |tilde + tilde|    |
--R            +1  0+             +1  0+
--R                                          Type: Operator SquareMatrix(2,Integer)
--E 13

--S 14 of 21
b m
```

```
--R
--R
--R         +1  - 3+
--R   (13)  |      |
--R         +3  - 1+
--R                                                Type: SquareMatrix(2,Integer)
--E 14

--S 15 of 21
L n ==
  n = 0 => 1
  n = 1 => x
  (2*n-1)/n * x * L(n-1) - (n-1)/n * L(n-2)
--R
--R                                                                   Type: Void
--E 15

--S 16 of 21
dx := operator("D") :: OP(POLY FRAC INT)
--R
--R
--R   (15)  D
--R                                   Type: Operator Polynomial Fraction Integer
--E 16

--S 17 of 21
evaluate(dx, p +-> D(p, 'x))
--R
--R
--R   (16)  D
--R                                   Type: Operator Polynomial Fraction Integer
--E 17

--S 18 of 21
E n == (1 - x**2) * dx**2 - 2 * x * dx + n*(n+1)
--R
--R                                                                   Type: Void
--E 18

--S 19 of 21
L 15
--R
--R   Compiling function L with type Integer -> Polynomial Fraction
--R      Integer
--R   Compiling function L as a recurrence relation.
--R
--R   (18)
--R     9694845  15   35102025  13   50702925  11   37182145  9   14549535  7
--R     ------- x   - -------- x   + -------- x   - -------- x  + -------- x
--R      2048          2048          2048          2048         2048
```

```
--R   +
--R       2909907  5   255255  3   6435
--R     - ------- x  + ------ x  - ---- x
--R         2048        2048       2048
--R                                              Type: Polynomial Fraction Integer
--E 19

--S 20 of 21
E 15
--R
--R   Compiling function E with type PositiveInteger -> Operator
--R      Polynomial Fraction Integer
--R
--R                          2      2
--R   (19)  240 - 2x D - (x  - 1)D
--R                                     Type: Operator Polynomial Fraction Integer
--E 20

--S 21 of 21
(E 15)(L 15)
--R
--R
--R   (20)  0
--R                                              Type: Polynomial Fraction Integer
--E 21
)spool
)lisp (bye)
```

— Operator.help —

```
====================================================================
Operator examples
====================================================================

Given any ring R, the ring of the Integer-linear operators over R is
called Operator(R).  To create an operator over R, first create a
basic operator using the operation operator, and then convert it to
Operator(R) for the R you want.

We choose R to be the two by two matrices over the integers.

  R := SQMATRIX(2, INT)
    SquareMatrix(2,Integer)
                        Type: Domain

Create the operator tilde on R.
```

```
  t := operator("tilde") :: OP(R)
    tilde
                          Type: Operator SquareMatrix(2,Integer)
```

Since Operator is unexposed we must either package-call operations
from it, or expose it explicitly. For convenience we will do the latter.

Expose Operator.

```
  )set expose add constructor Operator
```

To attach an evaluation function (from R to R) to an operator over R,
use evaluate(op, f) where op is an operator over R and f is a function
R -> R. This needs to be done only once when the operator is defined.
Note that f must be Integer-linear (that is, f(ax+y) = a f(x) + f(y)
for any integer a, and any x and y in R).

We now attach the transpose map to the above operator t.

```
  evaluate(t, m +-> transpose m)
    tilde
                     Type: Operator SquareMatrix(2,Integer)
```

Operators can be manipulated formally as in any ring: + is the
pointwise addition and * is composition. Any element x of R can be
converted to an operator op(x) over R, and the evaluation function of
op(x) is left-multiplication by x.

Multiplying on the left by this matrix swaps the two rows.

```
  s : R := matrix [ [0, 1], [1, 0] ]
        +0  1+
        |    |
        +1  0+
                     Type: SquareMatrix(2,Integer)
```

Can you guess what is the action of the following operator?

```
  rho := t * s
             +0  1+
        tilde|    |
             +1  0+
                     Type: Operator SquareMatrix(2,Integer)
```

Hint: applying rho four times gives the identity, so rho^4-1 should
return 0 when applied to any two by two matrix.

```
  z := rho**4 - 1
                    +0  1+     +0  1+     +0  1+     +0  1+
        - 1 + tilde|    |tilde|    |tilde|    |tilde|    |
```

```
                  +1  0+     +1  0+     +1  0+     +1  0+
                      Type: Operator SquareMatrix(2,Integer)
```

Now check with this matrix.

```
m:R := matrix [ [1, 2], [3, 4] ]
      +1  2+
      |    |
      +3  4+
                      Type: SquareMatrix(2,Integer)
```

```
z m
      +0  0+
      |    |
      +0  0+
                      Type: SquareMatrix(2,Integer)
```

As you have probably guessed by now, rho acts on matrices by rotating the elements clockwise.

```
rho m
      +3  1+
      |    |
      +4  2+
                      Type: SquareMatrix(2,Integer)
```

```
rho rho m
       +4  3+
       |    |
       +2  1+
                      Type: SquareMatrix(2,Integer)
```

```
(rho^3) m
       +2  4+
       |    |
       +1  3+
                      Type: SquareMatrix(2,Integer)
```

Do the swapping of rows and transposition commute? We can check by computing their bracket.

```
b := t * s - s * t
         +0  1+            +0  1+
       - |    |tilde + tilde|    |
         +1  0+            +1  0+
                      Type: Operator SquareMatrix(2,Integer)
```

Now apply it to m.

```
b m
```

```
+1  - 3+
|      |
+3  - 1+
                Type: SquareMatrix(2,Integer)
```

Next we demonstrate how to define a differential operator on a polynomial ring.

This is the recursive definition of the n-th Legendre polynomial.

```
L n ==
  n = 0 => 1
  n = 1 => x
  (2*n-1)/n * x * L(n-1) - (n-1)/n * L(n-2)
                       Type: Void
```

Create the differential operator d/dx on polynomials in x over the rational numbers.

```
dx := operator("D") :: OP(POLY FRAC INT)
   D
                       Type: Operator Polynomial Fraction Integer
```

Now attach the map to it.

```
evaluate(dx, p +-> D(p, 'x))
  D
                       Type: Operator Polynomial Fraction Integer
```

This is the differential equation satisfied by the n-th Legendre polynomial.

```
E n == (1 - x**2) * dx**2 - 2 * x * dx + n*(n+1)
                       Type: Void
```

Now we verify this for n = 15. Here is the polynomial.

```
L 15
   9694845  15   35102025  13   50702925  11   37182145  9   14549535  7
   ------- x   - -------- x   + -------- x   - -------- x  + -------- x
     2048          2048           2048           2048          2048
 +
     2909907  5   255255  3   6435
   - ------- x  + ------ x  - ---- x
       2048         2048      2048
                       Type: Polynomial Fraction Integer
```

Here is the operator.

```
E 15
  240 - 2x D - (x  - 1)D
                       Type: Operator Polynomial Fraction Integer
```

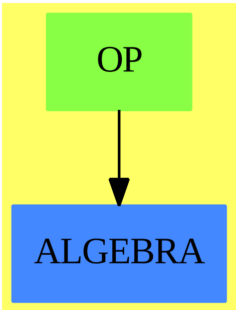
```
Here is the evaluation.

  (E 15)(L 15)
    0
                        Type: Polynomial Fraction Integer

See Also:
o )show Operator
```

———-

16.10.1 Operator (OP)



See

⇒ "ModuleOperator" (MODOP) 14.12.1 on page 1611

Exports:

0	1	adjoint	characteristic	charthRoot
coerce	conjug	evaluate	evaluateInverse	makeop
hash	latex	one?	opeval	recip
retract	retractIfCan	sample	subtractIfCan	zero?
?~=?	?*?	?**?	?^?	?.?
?+?	?-?	-?	?=?	

— domain OP Operator —

```
)abbrev domain OP Operator
++ Author: Manuel Bronstein
++ Date Created: 15 May 1990
++ Date Last Updated: 12 February 1993
++ Description:
++ Algebra of ADDITIVE operators over a ring.
```

```
Operator(R: Ring) == ModuleOperator(R,R)
```

———-

— OP.dotabb —

```
"OP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OP"]
"ALGEBRA" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ALGEBRA"]
"OP" -> "ALGEBRA"
```

———-

16.11 domain OMLO OppositeMonogenicLinearOperator

— OppositeMonogenicLinearOperator.input —

```
)set break resume
)sys rm -f OppositeMonogenicLinearOperator.output
)spool OppositeMonogenicLinearOperator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OppositeMonogenicLinearOperator
--R OppositeMonogenicLinearOperator(P: MonogenicLinearOperator R,R: Ring)  is a domain constructor
--R Abbreviation for OppositeMonogenicLinearOperator is OMLO
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMLO
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : % -> % if P has DIFRING           1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger      hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R one? : % -> Boolean                   op : P -> %
```

```
--R po : % -> P                          recip : % -> Union(%,"failed")
--R reductum : % -> %                    sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if P has DIFRING
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : R -> % if R has COMRING
--R differentiate : % -> % if P has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if P has DIFRING
--R minimumDegree : % -> NonNegativeInteger
--R monomial : (R,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

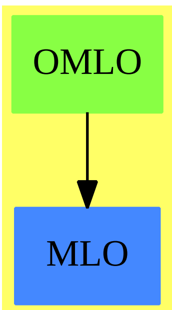
— OppositeMonogenicLinearOperator.help —

```
====================================================================
OppositeMonogenicLinearOperator examples
====================================================================

See Also:
o )show OppositeMonogenicLinearOperator
```

———-

16.11.1 OppositeMonogenicLinearOperator (OMLO)



See

⇒ "OrdinaryDifferentialRing" (ODR) 16.18.1 on page 1820
⇒ "DirectProductModule" (DPMO) 5.11.1 on page 542
⇒ "DirectProductMatrixModule" (DPMM) 5.10.1 on page 538

Exports:

0	1	characteristic	coefficient	coerce
D	degree	differentiate	hash	latex
leadingCoefficient	minimumDegree	monomial	one?	op
po	recip	reductum	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

— domain OMLO OppositeMonogenicLinearOperator —

```
)abbrev domain OMLO OppositeMonogenicLinearOperator
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains: MonogenicLinearOperator
++ Also See:
++ AMS Classifications:
++ Keywords: opposite ring
++ Examples:
++ References:
++ Description:
++ This constructor creates the \spadtype{MonogenicLinearOperator} domain
++ which is ``opposite'' in the ring sense to P.
++ That is, as sets \spad{P = $} but \spad{a * b} in \spad{$} is equal to
++ \spad{b * a} in P.

OppositeMonogenicLinearOperator(P, R): OPRcat == OPRdef where
   P: MonogenicLinearOperator(R)
   R: Ring

   OPRcat == MonogenicLinearOperator(R) with
        if P has DifferentialRing then DifferentialRing
        op: P -> $  ++ op(p) creates a value in $ equal to p in P.
        po: $ -> P  ++ po(q) creates a value in P equal to q in $.

   OPRdef  == P add
        Rep := P
        x, y: $
        a: P
        op a == a: $
        po x == x: P
        x*y == (y:P) *$P (x:P)
        coerce(x): OutputForm == prefix(op::OutputForm, [coerce(x:P)$P])
```

— **OMLO.dotabb** —

```
"OMLO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMLO"]
"MLO" [color="#4488FF",href="bookvol10.2.pdf#nameddest=MLO"]
"OMLO" -> "MLO"
```

16.12 domain ORDCOMP OrderedCompletion

— **OrderedCompletion.input** —

```
)set break resume
)sys rm -f OrderedCompletion.output
)spool OrderedCompletion.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedCompletion
--R OrderedCompletion R: SetCategory  is a domain constructor
--R Abbreviation for OrderedCompletion is ORDCOMP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ORDCOMP
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if R has ABELGRP         ?=? : (%,%) -> Boolean
--R 1 : () -> % if R has ORDRING         0 : () -> % if R has ABELGRP
--R coerce : R -> %                      coerce : % -> OutputForm
--R finite? : % -> Boolean               hash : % -> SingleInteger
--R infinite? : % -> Boolean             latex : % -> String
--R minusInfinity : () -> %              plusInfinity : () -> %
--R retract : % -> R                     whatInfinity : % -> SingleInteger
--R ?~=? : (%,%) -> Boolean
--R ?*? : (PositiveInteger,%) -> % if R has ABELGRP
--R ?*? : (NonNegativeInteger,%) -> % if R has ABELGRP
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?*? : (%,%) -> % if R has ORDRING
--R ?**? : (%,NonNegativeInteger) -> % if R has ORDRING
--R ?**? : (%,PositiveInteger) -> % if R has ORDRING
```

```
--R ?+? : (%,%) -> % if R has ABELGRP
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDRING
--R ?<=? : (%,%) -> Boolean if R has ORDRING
--R ?>? : (%,%) -> Boolean if R has ORDRING
--R ?>=? : (%,%) -> Boolean if R has ORDRING
--R ?^? : (%,NonNegativeInteger) -> % if R has ORDRING
--R ?^? : (%,PositiveInteger) -> % if R has ORDRING
--R abs : % -> % if R has ORDRING
--R characteristic : () -> NonNegativeInteger if R has ORDRING
--R coerce : Integer -> % if R has ORDRING or R has RETRACT INT
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R max : (%,%) -> % if R has ORDRING
--R min : (%,%) -> % if R has ORDRING
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has ORDRING
--R positive? : % -> Boolean if R has ORDRING
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R recip : % -> Union(%,"failed") if R has ORDRING
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R sample : () -> % if R has ABELGRP
--R sign : % -> Integer if R has ORDRING
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
--R zero? : % -> Boolean if R has ABELGRP
--R
--E 1

)spool
)lisp (bye)
```

———-

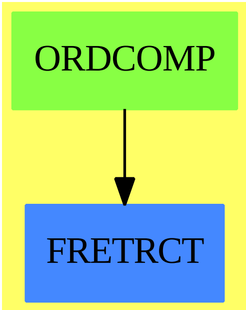
— **OrderedCompletion.help** —

```
====================================================================
OrderedCompletion examples
====================================================================

See Also:
o )show OrderedCompletion
```

———-

16.12.1 OrderedCompletion (ORDCOMP)



See

⇒ "OnePointCompletion" (ONECOMP) 16.4.1 on page 1739

Exports:

0	1	abs	characteristic	coerce
finite?	hash	infinite?	latex	max
min	minusInfinity	negative?	one?	plusInfinity
positive?	rational	rational?	rationalIfCan	recip
retract	retractIfCan	sample	sign	subtractIfCan
whatInfinity	zero?	?~=?	?*?	?**?
?+?	?-?	?<?	?<=?	?>?
?>=?	?^?	-?	?=?	

— domain ORDCOMP OrderedCompletion —

```
)abbrev domain ORDCOMP OrderedCompletion
++ Author: Manuel Bronstein
++ Date Created: 4 Oct 1989
++ Date Last Updated: 1 Nov 1989
++ Description:
++ Completion with + and - infinity.
++ Adjunction of two real infinites quantities to a set.

OrderedCompletion(R:SetCategory): Exports == Implementation where
  B ==> Boolean

  Exports ==> Join(SetCategory, FullyRetractableTo R) with
    plusInfinity : () -> %        ++ plusInfinity() returns +infinity.
    minusInfinity: () -> %        ++ minusInfinity() returns  -infinity.
    finite?      : %  -> B
      ++ finite?(x) tests if x is finite.
    infinite?    : %  -> B
      ++ infinite?(x) tests if x is +infinity or -infinity,
    whatInfinity : %  -> SingleInteger
      ++ whatInfinity(x) returns 0 if x is finite,
      ++ 1 if x is +infinity, and -1 if x is -infinity.
```

```
    if R has AbelianGroup then AbelianGroup
    if R has OrderedRing then OrderedRing
    if R has IntegerNumberSystem then
      rational?: % -> Boolean
        ++ rational?(x) tests if x is a finite rational number.
      rational : % -> Fraction Integer
        ++ rational(x) returns x as a finite rational number.
        ++ Error: if x cannot be so converted.
      rationalIfCan: % -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(x) returns x as a finite rational number if
        ++ it is one and "failed" otherwise.

  Implementation ==> add
    Rep := Union(fin:R, inf:B)  -- true = +infinity, false = -infinity

    coerce(r:R):%          == [r]
    retract(x:%):R         == (x case fin => x.fin; error "Not finite")
    finite? x              == x case fin
    infinite? x            == x case inf
    plusInfinity()         == [true]
    minusInfinity()        == [false]

    retractIfCan(x:%):Union(R, "failed") ==
      x case fin => x.fin
      "failed"

    coerce(x:%):OutputForm ==
      x case fin => (x.fin)::OutputForm
      e := "infinity"::OutputForm
      x.inf => empty() + e
      - e

    whatInfinity x ==
      x case fin => 0
      x.inf => 1
      -1

    x = y ==
      x case inf =>
        y case inf => not xor(x.inf, y.inf)
        false
      y case inf => false
      x.fin = y.fin

    if R has AbelianGroup then
      0 == [0$R]

      n:Integer * x:% ==
        x case inf =>
          n > 0 => x
```

```
        n < 0 => [not(x.inf)]
        error "Undefined product"
      [n * x.fin]

    - x ==
      x case inf => [not(x.inf)]
      [- (x.fin)]

    x + y ==
      x case inf =>
        y case fin => x
        xor(x.inf, y.inf) => error "Undefined sum"
        x
      y case inf => y
      [x.fin + y.fin]

  if R has OrderedRing then
    fininf: (B, R) -> %

    1               == [1$R]
    characteristic() == characteristic()$R

    fininf(b, r) ==
      r > 0 => [b]
      r < 0 => [not b]
      error "Undefined product"

    x:% * y:% ==
      x case inf =>
        y case inf =>
          xor(x.inf, y.inf) => minusInfinity()
          plusInfinity()
        fininf(x.inf, y.fin)
      y case inf => fininf(y.inf, x.fin)
      [x.fin * y.fin]

    recip x ==
      x case inf => 0
      (u := recip(x.fin)) case "failed" => "failed"
      [u::R]

    x < y ==
      x case inf =>
        y case inf =>
          xor(x.inf, y.inf) => y.inf
          false
        not(x.inf)
      y case inf => y.inf
      x.fin < y.fin
```

```
    if R has IntegerNumberSystem then
      rational? x == finite? x
      rational  x == rational(retract(x)@R)

      rationalIfCan x ==
        (r:= retractIfCan(x)@Union(R,"failed")) case "failed" =>"failed"
        rational(r::R)
```

———-

— ORDCOMP.dotabb —

```
"ORDCOMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ORDCOMP"]
"FRETRCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRETRCT"]
"ORDCOMP" -> "FRETRCT"
```

———-

16.13 domain ODP OrderedDirectProduct

— OrderedDirectProduct.input —

```
)set break resume
)sys rm -f OrderedDirectProduct.output
)spool OrderedDirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedDirectProduct
--R OrderedDirectProduct(dim: NonNegativeInteger,S: OrderedAbelianMonoidSup,f: ((Vector S,Vector S) -> B
--R Abbreviation for OrderedDirectProduct is ODP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODP
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if S has RING            1 : () -> % if S has MONOID
--R 0 : () -> % if S has CABMON          coerce : % -> Vector S
--R copy : % -> %                        directProduct : Vector S -> %
--R ?.? : (%,Integer) -> S               elt : (%,Integer,S) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List S                eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean      indices : % -> List Integer
```

```
--R map : ((S -> S),%) -> %              qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (PositiveInteger,%) -> % if S has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if S has CABMON
--R ?*? : (S,%) -> % if S has RING
--R ?*? : (%,S) -> % if S has RING
--R ?*? : (%,%) -> % if S has MONOID
--R ?*? : (Integer,%) -> % if S has RING
--R ?**? : (%,PositiveInteger) -> % if S has MONOID
--R ?**? : (%,NonNegativeInteger) -> % if S has MONOID
--R ?+? : (%,%) -> % if S has ABELSG
--R ?-? : (%,%) -> % if S has RING
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : % -> % if S has DIFRING and S has RING
--R ?^? : (%,PositiveInteger) -> % if S has MONOID
--R ?^? : (%,NonNegativeInteger) -> % if S has MONOID
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : S -> % if S has SETCAT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : % -> % if S has DIFRING and S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
```

```
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R index : PositiveInteger -> % if S has FINITE
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has MONOID
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(%,"failed") if S has MONOID
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R retract : % -> S if S has SETCAT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S,"failed") if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT and S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed") if S has CABMON
--R sup : (%,%) -> % if S has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R zero? : % -> Boolean if S has CABMON
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
```

```
)lisp (bye)
```

———-

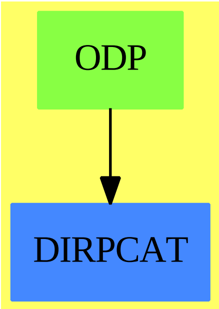
— **OrderedDirectProduct.help** —

```
====================================================================
OrderedDirectProduct examples
====================================================================

See Also:
o )show OrderedDirectProduct
```

———-

16.13.1 OrderedDirectProduct (ODP)



See

⇒ "HomogeneousDirectProduct" (HDP) 9.9.1 on page 1138
⇒ "SplitHomogeneousDirectProduct" (SHDP) 20.23.1 on page 2467

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
#?	?*?	?**?	?+?	?-?
?/?	?<?	?<=?	?=?	?>?
?>=?	?^?	?~=?	-?	?.?

— domain ODP OrderedDirectProduct —

```
)abbrev domain ODP OrderedDirectProduct
-- all direct product category domains must be compiled
-- without subsumption, set SourceLevelSubset to EQUAL
--)bo $noSubsumption := true

++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See: HomogeneousDirectProduct, SplitHomogeneousDirectProduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
++ underlying ordered component type. The ordering on the type is determined
++ by its third argument which represents the less than function on
++ vectors. This type is a suitable third argument for
++ \spadtype{GeneralDistributedMultivariatePolynomial}.

OrderedDirectProduct(dim:NonNegativeInteger,
                     S:OrderedAbelianMonoidSup,
                      f:(Vector(S),Vector(S))->Boolean):T
                             == C where
   T == DirectProductCategory(dim,S)
   C == DirectProduct(dim,S) add
        Rep:=Vector(S)
        x:% < y:% == f(x::Rep,y::Rep)
```

— ODP.dotabb —

```
"ODP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODP"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"ODP" -> "DIRPCAT"
```

16.14 domain OFMONOID OrderedFreeMonoid

— OrderedFreeMonoid.input —

```
)set break resume
)sys rm -f OrderedFreeMonoid.output
)spool OrderedFreeMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 24
m1:=(x*y*y*z)$OFMONOID(Symbol)
--R
--R
--R            2
--R   (1)  x y z
--R                                                 Type: OrderedFreeMonoid Symbol
--E 1

--S 2 of 24
m2:=(x*y)$OFMONOID(Symbol)
--R
--R
--R   (2)  x y
--R                                                 Type: OrderedFreeMonoid Symbol
--E 2

--S 3 of 24
lquo(m1,m2)
--R
--R
--R   (3)  y z
--R                                      Type: Union(OrderedFreeMonoid Symbol,...)
--E 3
```

```
--S 4 of 24
m3:=(y*y)$OFMONOID(Symbol)
--R
--R
--R        2
--R   (4)  y
--R                                               Type: OrderedFreeMonoid Symbol
--E 4

--S 5 of 24
divide(m1,m2)
--R
--R
--R   (5)  [lm= y z,rm= "failed"]
--RType: Union(Record(lm: Union(OrderedFreeMonoid Symbol,"failed"),rm: Union(OrderedFreeMonoid Symbol,"f
--E 5

--S 6 of 24
divide(m1,m3)
--R
--R
--R   (6)  [lm= "failed",rm= "failed"]
--RType: Union(Record(lm: Union(OrderedFreeMonoid Symbol,"failed"),rm: Union(OrderedFreeMonoid Symbol,"f
--E 6

--S 7 of 24
m4:=(y^3)$OFMONOID(Symbol)
--R
--R
--R        3
--R   (7)  y
--R                                               Type: OrderedFreeMonoid Symbol
--E 7

--S 8 of 24
divide(m1,m4)
--R
--R
--R   (8)  [lm= "failed",rm= "failed"]
--RType: Union(Record(lm: Union(OrderedFreeMonoid Symbol,"failed"),rm: Union(OrderedFreeMonoid Symbol,"f
--E 8

)set function compile on


-- Build the non-commutative algebra h=k[x,y] and then make computations
-- in h using some predefined rules for x and y. For example, giving
--   x*y*x=y*x*y
--   x*x=a*x+b
```

```
--   y*y=a*y+b
-- where a dn b are generic elements of k.
-- Then reduce the polynomials in x and y according to the previous rules.
-- That is, given
--   (x+y)^2   ( = x^2+x*y+y*x+y^2)
-- should reduce to
--   a*(x+y)+2*b+x*y+y*x

-- Generic elements of k (OVAR is OrderedVariableList)

--S 9 of 24
C ==> OVAR [a,b]
--R
--R                                                                   Type: Void
--E 9

-- Commutative Field: k = Q[a,b]
-- Q = Fraction Integer
-- SMP = SparseMultivariatePolynomials

--S 10 of 24
K ==> SMP(FRAC INT,C)
--R
--R                                                                   Type: Void
--E 10

-- Non-commutative variables

--S 11 of 24
V ==> OVAR [x,y]
--R
--R                                                                   Type: Void
--E 11

-- Non-commuative algebra k=k[x,y]
-- XDPOLY XDistributedPolynomial

--S 12 of 24
H ==> XDPOLY(V,K)
--R
--R                                                                   Type: Void
--E 12

-- Free Monoid

--S 13 of 24
M ==> OFMONOID V
--R
--R                                                                   Type: Void
--E 13
```

```
-- Substitution rules are applied to words from the monoid over the
-- variables and retun polynomials

--S 14 of 24
subs(w:M):H ==
  -- x*y*x = y*x*y
  n1:=lquo(w,(x::V*y::V*x::V)$M)$M
  n1 case "failed" => monom(w,1)$H
      -- x*x = a*x+b
    n2:=lquo(w,(x::V^2)$M)$M
    n2 case "failed" => monom(w,1)$H
      -- y*y = a*y+b
      n3:lquo(w,(y::V^2)$M)$M
      n3 case "failed" => monom(w,1)$H
      monom(n3,1)$H * (a::K*y::V+b::K)$M * monom(n3,1)$H
    monom(n2,1)$H * (a::K*x::V+b::K)$H * monom(n2,1)$H
  monom(n1,1)$H * (y::V*x::V*y::V)$H * monom(n1,1)$H
--R
--R   Function declaration subs : OrderedFreeMonoid OrderedVariableList [x
--R      ,y] -> XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R      [a,b])) has been added to workspace.
--R                                                               Type: Void
--E 14

-- Apply rules to a term. Keep coefficients
--S 15 of 24
newterm(x:Record(k:M,c:K)):H == x.c*subs(x,k)
--R
--R   Function declaration newterm : Record(k: OrderedFreeMonoid
--R      OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R      Fraction Integer,OrderedVariableList [a,b])) ->
--R      XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R      [a,b])) has been added to workspace.
--R                                                               Type: Void
--E 15

-- Reconstruct the polynomial term by term

--S 16 of 24
newpoly(t:H):H == reduce(+,map(newterm,listOfTerms(t)))
--R
--R   Function declaration newpoly : XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) has been added to workspace.
--R                                                               Type: Void
```

```
--E 16

-- Example calcuations

--S 17 of 24
p1:(x::V+y::V)$H^2
--R
--R
--RDaly Bug
--R   Category, domain or package constructor ^ is not available.
--E 17

--S 18 of 24
newpoly(p1)
--R
--R   Compiling function newpoly with type XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b]))
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                                 )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Frac
--R                                  Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--R   AXIOM will attempt to step through and interpret the code.
--R   Compiling function newterm with type Record(k: OrderedFreeMonoid
--R      OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R      Fraction Integer,OrderedVariableList [a,b])) ->
--R      XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R      [a,b]))
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                                 )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R
--RDaly Bug
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Frac
--R                                  Variable k
```

```
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--E 18

--S 19 of 24
p2:=(x::V+y::V)$H^3
--R
--R
--R          3    2               2      2            2     3
--R   (17)  y  + y x + y x y + y x  + x y  + x y x + x y + x
--RType: XDistributedPolynomial(OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction Integer,
--E 19

--S 20 of 24
pNew:=newpoly(p2)
--R
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                               )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fraction Intege
--R                                Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--R   AXIOM will attempt to step through and interpret the code.
--R   Compiling function newterm with type Record(k: OrderedFreeMonoid
--R      OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R      Fraction Integer,OrderedVariableList [a,b])) ->
--R      XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R      [a,b]))
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                               )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R
--RDaly Bug
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fraction Intege
--R                                Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
```

```
--E 20

-- But the rules should be applied more than once
--S 21 of 24
while pNew ~= p2 repeat
  p2 := pNew
  pNew := newpoly(p2)
--R
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                                 )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Frac
--R                                  Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--R   AXIOM will attempt to step through and interpret the code.
--R   Compiling function newterm with type Record(k: OrderedFreeMonoid
--R      OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R      Fraction Integer,OrderedVariableList [a,b])) ->
--R      XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R      [a,b]))
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                                 )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R
--RDaly Bug
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Frac
--R                                  Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--E 21

--S 22 of 24
pNew
--R
--R
--R   (18)  pNew
--R                                                          Type: Variable pNew
--E 22
```

```
--S 23 of 24
reduce(p:H):H ==
  p2 := newpoly(p)
  p3 := newpoly(p2)
  while p3 ~= p2 repeat
    p2 := p3
    p3 := newpoly(p2)
  p3
--R
--R   Function declaration reduce : XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) has been added to workspace.
--R   Compiled code for newpoly has been cleared.
--R                                                            Type: Void
--E 23

--S 24 of 24
reduce(p2)
--R
--R   Compiling function newpoly with type XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b]))
--R   Compiling function reduce with type XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R      OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R      Integer,OrderedVariableList [a,b]))
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                                 )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fraction Intege
--R                                  Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--R   AXIOM will attempt to step through and interpret the code.
--R   Compiling function newterm with type Record(k: OrderedFreeMonoid
--R      OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R      Fraction Integer,OrderedVariableList [a,b])) ->
--R      XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
```

```
--R      [a,b]))
--R   There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R                                 )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R
--RDaly Bug
--R   Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Frac
--R                                  Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--E 24

)spool
)lisp (bye)
```

— **OrderedFreeMonoid.help** —

```
====================================================================
OrderedFreeMonoid examples
====================================================================

m1:=(x*y*y*z)$OFMONOID(Symbol)

m2:=(x*y)$OFMONOID(Symbol)

lquo(m1,m2)

m3:=(y*y)$OFMONOID(Symbol)

div(m1,m2)

div(m1,m3)

m4:=(y^3)$OFMONOID(Symbol)

div(m1,m4)

Build the non-commutative algebra h=k[x,y] and then make computations
in h using some predefined rules for x and y. For example, giving

   x*y*x=y*x*y
   x*x=a*x+b
```

```
   y*y=a*y+b
```

where a dn b are generic elements of k.

Then reduce the polynomials in x and y according to the previous rules.
That is, given

```
   (x+y)^2   ( = x^2+x*y+y*x+y^2)
```

should reduce to

```
   a*(x+y)+2*b+x*y+y*x
```

We can reduce the clutter of the work by defining macros for the types
of the domains we need to create. So first we create those macros.

We create generic elements of k. First we define a macro for the domain of
ordered variables (OVAR is OrderedVariableList)

```
  C ==> OVAR [a,b]
```

Next we define a macro for the commutative field domain k = Q[a,b]
where Q is Fraction(Integer) and SMP is SparseMultivariatePolynomials

```
  K ==> SMP(FRAC INT,C)
```

Now we need some non-commutative variables so we create a macro for that.

```
  V ==> OVAR [x,y]
```

And now we need to define the non-commuative algebra k=k[x,y].
We use the domain XDistributedPolynomial (XDPOLY) as a macro.

```
  H ==> XDPOLY(V,K)
```

The non-commutative variables are in an Ordered Free Monoid

```
  M ==> OFMONOID V
```

We have three rules to apply. So we create a function that examines one
term. Substitution rules are applied to words from the monoid over the
variables and retun polynomials. If any rule matches we construct the
substitution, create a new monomial term and return it.

```
  subs(w:M):H ==
    -- x*y*x = y*x*y
    n1:=lquo(w,(x::V*y::V*x::V)$M)$M
    n1 case "failed" => monom(w,1)$H
        -- x*x = a*x+b
      n2:=lquo(w,(x::V^2)$M)$M
```

```
      n2 case "failed" => monom(w,1)$H
        -- y*y = a*y+b
        n3:lquo(w,(y::V^2)$M)$M
        n3 case "failed" => monom(w,1)$H
        monom(n3,1)$H * (a::K*y::V+b::K)$M * monom(n3,1)$H
      monom(n2,1)$H * (a::K*x::V+b::K)$H * monom(n2,1)$H
    monom(n1,1)$H * (y::V*x::V*y::V)$H * monom(n1,1)$H

We apply these rules to a term, remembering the coefficient

  newterm(x:Record(k:M,c:K)):H == x.c*subs(x,k)


And now we create a function to reconstruct the polynomial term by term.

  newpoly(t:H):H == reduce(+,map(newterm,listOfTerms(t)))


For example,

  p1:(x::V+y::V)$H^2

  newpoly(p1)

  p2:=(x::V+y::V)$H^3

  pNew:=newpoly(p2)

But the rules should be applied more than once so we create a function
to iterate the rules until nothing changes.

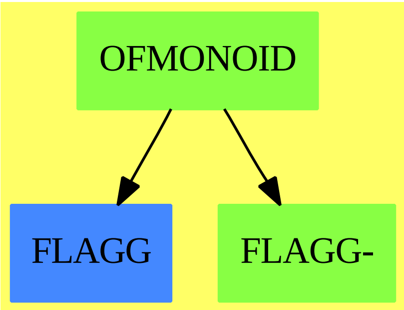
  reduce(p:H):H ==
    p2 := newpoly(p)
    p3 := newpoly(p2)
    while p3 ~= p2 repeat
      p2 := p3
      p3 := newpoly(p2)
    p3

And we can see it work

  reduce(p2)

See Also:
o )show OrderedFreeMonoid
```

16.14.1 OrderedFreeMonoid (OFMONOID)



Exports:

1	coerce	factors	first	hash
hclf	hcrf	latex	length	lexico
lquo	max	min	mirror	nthExpon
nthFactor	one?	overlap	recip	rest
retract	retractIfCan	rquo	sample	size
varList	?*?	?**?	?<?	?<=?
?=?	?>?	?>=?	?^?	?~=?
?div?				

— domain OFMONOID OrderedFreeMonoid —

```
)abbrev domain OFMONOID OrderedFreeMonoid
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The free monoid on a set \spad{S} is the monoid of finite products of
++ the form \spad{reduce(*,[si ** ni])} where the si's are in S, and the ni's
++ are non-negative integers. The multiplication is not commutative.
++ For two elements \spad{x} and \spad{y} the relation \spad{x < y}
++ holds if either \spad{length(x) < length(y)} holds or if these lengths
++ are equal and if \spad{x} is smaller than \spad{y} w.r.t. the
++ lexicographical ordering induced by \spad{S}.
++ This domain inherits implementation from \spadtype{FreeMonoid}.

OrderedFreeMonoid(S: OrderedSet): OFMcategory == OFMdefinition where
  NNI ==> NonNegativeInteger
```

```
REC ==> Record(gen:S, exp:NNI)
RESULT ==> Union(Record(lm:Union(%,"failed"),rm:Union(%,"failed")),"failed")

OFMcategory == Join(OrderedMonoid, RetractableTo S) with
  "*":    (S, %) -> %
    ++ \spad{s*x} returns the product of \spad{x} by \spad{s} on the left.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X x*m1
  "*":    (%, S) -> %
    ++ \spad{x*s} returns the product of \spad{x} by \spad{s} on the right.
    ++
    ++X m1:=(y**3)$OFMONOID(Symbol)
    ++X m1*x
  "**":   (S, NNI) -> %
    ++ \spad{s**n} returns the product of \spad{s} by itself \spad{n} times.
    ++
    ++X m1:=(y**3)$OFMONOID(Symbol)
  first: % -> S
    ++ \spad{first(x)} returns the first letter of \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X first m1
  rest:  % -> %
    ++ \spad{rest(x)} returns \spad{x} except the first letter.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X rest m1
  mirror: % -> %
    ++ \spad{mirror(x)} returns the reversed word of \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X mirror m1
  lexico: (%,%) -> Boolean
    ++ \spad{lexico(x,y)} returns \spad{true}
    ++ iff \spad{x} is smaller than \spad{y}
    ++ w.r.t. the pure lexicographical ordering induced by \spad{S}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X m2:=(x*y)$OFMONOID(Symbol)
    ++X lexico(m1,m2)
    ++X lexico(m2,m1)
  hclf:   (%, %) -> %
    ++ \spad{hclf(x, y)} returns the highest common left factor
    ++ of \spad{x} and \spad{y},
    ++ that is the largest \spad{d} such that \spad{x = d a}
    ++ and \spad{y = d b}.
    ++
    ++X m1:=(x*y*z)$OFMONOID(Symbol)
    ++X m2:=(x*y)$OFMONOID(Symbol)
```

```
    ++X hclf(m1,m2)
  hcrf:   (%, %) -> %
    ++ \spad{hcrf(x, y)} returns the highest common right
    ++ factor of \spad{x} and \spad{y},
    ++ that is the largest \spad{d} such that \spad{x = a d}
    ++ and \spad{y = b d}.
    ++
    ++X m1:=(x*y*z)$OFMONOID(Symbol)
    ++X m2:=(y*z)$OFMONOID(Symbol)
    ++X hcrf(m1,m2)
  lquo:   (%, %) -> Union(%, "failed")
    ++ \spad{lquo(x, y)} returns the exact left quotient of \spad{x}
    ++  by \spad{y} that is \spad{q} such that \spad{x = y * q},
    ++ "failed" if \spad{x} is not of the form \spad{y * q}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X m2:=(x*y)$OFMONOID(Symbol)
    ++X lquo(m1,m2)
  rquo:   (%, %) -> Union(%, "failed")
    ++ \spad{rquo(x, y)} returns the exact right quotient of \spad{x}
    ++ by \spad{y} that is \spad{q} such that \spad{x = q * y},
    ++ "failed" if \spad{x} is not of the form \spad{q * y}.
    ++
    ++X m1:=(q*y^3)$OFMONOID(Symbol)
    ++X m2:=(y^2)$OFMONOID(Symbol)
    ++X lquo(m1,m2)
  lquo:   (%, S) -> Union(%, "failed")
    ++ \spad{lquo(x, s)} returns the exact left quotient of \spad{x}
    ++ by \spad{s}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X lquo(m1,x)
  rquo:   (%, S) -> Union(%, "failed")
    ++ \spad{rquo(x, s)} returns the exact right quotient
    ++ of \spad{x} by \spad{s}.
    ++
    ++X m1:=(x*y)$OFMONOID(Symbol)
    ++X div(m1,y)
  divide: (%, %) -> RESULT
    ++ \spad{divide(x,y)} returns the left and right exact quotients of
    ++ \spad{x} by \spad{y}, that is \spad{[l,r]} such that \spad{x = l*y*r}.
    ++ "failed" is returned iff \spad{x} is not of the form \spad{l * y * r}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X m2:=(x*y)$OFMONOID(Symbol)
    ++X divide(m1,m2)
  overlap: (%, %) -> Record(lm: %, mm: %, rm: %)
    ++ \spad{overlap(x, y)} returns \spad{[l, m, r]} such that
    ++ \spad{x = l * m} and \spad{y = m * r} hold and such that
    ++ \spad{l} and \spad{r} have no overlap,
```

```
    ++ that is \spad{overlap(l, r) = [l, 1, r]}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X m2:=(x*y)$OFMONOID(Symbol)
    ++X overlap(m1,m2)
  size:   % -> NNI
    ++ \spad{size(x)} returns the number of monomials in \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X size(m1,2)
  nthExpon:  (%, Integer) -> NNI
    ++ \spad{nthExpon(x, n)} returns the exponent of the
    ++ \spad{n-th} monomial of \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X nthExpon(m1,2)
  nthFactor: (%, Integer) -> S
    ++ \spad{nthFactor(x, n)} returns the factor of the \spad{n-th}
    ++ monomial of \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X nthFactor(m1,2)
  factors: % -> List REC
    ++ \spad{factors(a1\^e1,...,an\^en)} returns
    ++ \spad{[[a1, e1],...,[an, en]]}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X factors m1
  length: % -> NNI
    ++ \spad{length(x)} returns the length of \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X length m1
  varList: % -> List S
    ++ \spad{varList(x)} returns the list of variables of \spad{x}.
    ++
    ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
    ++X varList m1

OFMdefinition == FreeMonoid(S) add
  Rep := ListMonoidOps(S, NNI, 1)

  -- definitions
  lquo(w:%, l:S) ==
    x: List REC := listOfMonoms(w)$Rep
    null x        => "failed"
    fx: REC := first x
    fx.gen ^= l  => "failed"
    fx.exp = 1   => makeMulti rest(x)
    makeMulti [[fx.gen, (fx.exp - 1)::NNI ]$REC, :rest x]
```

```
rquo(w:%, l:S) ==
  u:% := reverse w
  (r := lquo (u,l)) case "failed" => "failed"
  reverse_! (r::%)

divide(left:%,right:%) ==
  a:=lquo(left,right)
  b:=rquo(left,right)
  [a,b]

length x == reduce("+" ,[f.exp for f in listOfMonoms x], 0)

varList x ==
  le: List S := [t.gen for t in listOfMonoms x]
  sort_! removeDuplicates(le)

first w ==
  x: List REC := listOfMonoms w
  null x => error "empty word !!!"
  x.first.gen

rest w ==
  x: List REC := listOfMonoms w
  null x => error "empty word !!!"
  fx: REC := first x
  fx.exp = 1 => makeMulti rest x
  makeMulti [[fx.gen , (fx.exp - 1)::NNI ]$REC , :rest x]

lexico(a,b) ==         --  ordre lexicographique
  la := listOfMonoms a
  lb := listOfMonoms b
  while (not null la) and (not null lb) repeat
    la.first.gen > lb.first.gen => return false
    la.first.gen < lb.first.gen => return true
    if la.first.exp = lb.first.exp then
      la:=rest la
      lb:=rest lb
    else if la.first.exp > lb.first.exp then
      la:=concat([la.first.gen,
             (la.first.exp - lb.first.exp)::NNI], rest lb)
      lb:=rest lb
    else
      lb:=concat([lb.first.gen,
               (lb.first.exp-la.first.exp)::NNI], rest la)
      la:=rest la
  empty? la and not empty? lb


a < b ==               --  ordre lexicographique par longueur
```

```
      la:NNI := length a; lb:NNI := length b
      la = lb =>  lexico(a,b)
      la < lb

    mirror x == reverse(x)$Rep
```

———-

— OFMONOID.dotabb —

```
"OFMONOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OFMONOID"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"OFMONOID" -> "FLAGG"
"OFMONOID" -> "FLAGG-"
```

———-

16.15 domain OVAR OrderedVariableList

— OrderedVariableList.input —

```
)set break resume
)sys rm -f OrderedVariableList.output
)spool OrderedVariableList.output
)set message test on
)set message auto off
)clear all
--S 1 of 5
ls:List Symbol:=['x,'a,'z]
--R
--R
--R   (1)  [x,a,z]
--R                                                              Type: List Symbol
--E 1

--S 2 of 5
Z:=OVAR ls
--R
--R
--R   (2)  OrderedVariableList [x,a,z]
--R                                                                   Type: Domain
--E 2
```

```
--S 3 of 5
size()$Z
--R
--R
--R   (3)  3
--R                                                      Type: NonNegativeInteger
--E 3

--S 4 of 5
lv:=[index(i::PI)$Z for i in 1..size()$Z]
--R
--I   Compiling function G1408 with type Integer -> Boolean
--I   Compiling function G1572 with type NonNegativeInteger -> Boolean
--R
--R   (4)  [x,a,z]
--R                                        Type: List OrderedVariableList [x,a,z]
--E 4

--S 5 of 5
sorted?(>,lv)
--R
--R
--R   (5)  true
--R                                                                 Type: Boolean
--E 5
)spool
)lisp (bye)
```

— **OrderedVariableList.help** —

```
====================================================================
OrderedVariableList examples
====================================================================

The domain OrderedVariableList provides symbols which are restricted
to a particular list and have a definite ordering. Those two features
are specified by a List Symbol object that is the argument to the
domain.

This is a sample ordering of three symbols.

  ls:List Symbol:=['x,'a,'z]
    [x,a,z]
                                Type: List Symbol

Let's build the domain
```

```
  Z:=OVAR ls
    OrderedVariableList [x,a,z]
                                Type: Domain

How many variables does it have?

  size()$Z
    3
                                Type: NonNegativeInteger

They are (in the imposed order)

  lv:=[index(i::PI)$Z for i in 1..size()$Z]
    [x,a,z]
                                Type: List OrderedVariableList [x,a,z]

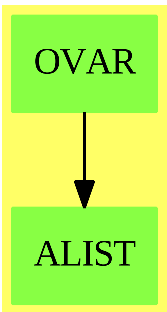
Check that the ordering is right

  sorted?(>,lv)
    true
                                Type: Boolean

See Also:
o )show OrderedVariableList
```

———–

16.15.1 OrderedVariableList (OVAR)



Exports:

coerce	convert	hash	index	latex
lookup	max	min	random	size
variable	?~=?	?<?	?<=?	?=?
?>?	?>=?			

— domain OVAR OrderedVariableList —

```
)abbrev domain OVAR OrderedVariableList
++ Author: Mark Botch
++ Description:
++ This domain implements ordered variables

OrderedVariableList(VariableList:List Symbol):
  Join(OrderedFinite, ConvertibleTo Symbol, ConvertibleTo InputForm,
       ConvertibleTo Pattern Float, ConvertibleTo Pattern Integer) with
         variable: Symbol -> Union(%,"failed")
                ++ variable(s) returns a member of the variable set or failed
    == add
       VariableList := removeDuplicates VariableList
       Rep := PositiveInteger
       s1,s2:%
       convert(s1):Symbol == VariableList.((s1::Rep)::PositiveInteger)
       coerce(s1):OutputForm == (convert(s1)@Symbol)::OutputForm
       convert(s1):InputForm == convert(convert(s1)@Symbol)
       convert(s1):Pattern(Integer) == convert(convert(s1)@Symbol)
       convert(s1):Pattern(Float) == convert(convert(s1)@Symbol)
       index i   == i::%
       lookup j  == j :: Rep
       size ()   == #VariableList
       variable(exp:Symbol) ==
            for i in 1.. for exp2 in VariableList repeat
                if exp=exp2 then return i::PositiveInteger::%
            "failed"
       s1 < s2 == s2 <$Rep s1
       s1 = s2 == s1 =$Rep s2
       latex(x:%):String      == latex(convert(x)@Symbol)
```

— OVAR.dotabb —

```
"OVAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OVAR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OVAR" -> "ALIST"
```

16.16 domain ODPOL OrderlyDifferentialPolynomial

— OrderlyDifferentialPolynomial.input —

```
)set break resume
)sys rm -f OrderlyDifferentialPolynomial.output
)spool OrderlyDifferentialPolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 36
dpol:= ODPOL(FRAC INT)
--R
--R
--R   (1)  OrderlyDifferentialPolynomial Fraction Integer
--R                                                                 Type: Domain
--E 1

--S 2 of 36
w := makeVariable('w)$dpol
--R
--R
--R   (2)  theMap(DPOLCAT-;makeVariable;AM;17!0,0)
--R Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
--E 2

--S 3 of 36
z := makeVariable('z)$dpol
--R
--R
--R   (3)  theMap(DPOLCAT-;makeVariable;AM;17!0,0)
--R Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
--E 3

--S 4 of 36
w.5
--R
--R
--R   (4)  w
--R         5
--R                            Type: OrderlyDifferentialPolynomial Fraction Integer
--E 4

--S 5 of 36
w 0
--R
--R
--R   (5)  w
--R                            Type: OrderlyDifferentialPolynomial Fraction Integer
--E 5

--S 6 of 36
```

```
[z.i for i in 1..5]
--R
--R
--R   (6)  [z ,z ,z ,z ,z ]
--R          1  2  3  4  5
--R                    Type: List OrderlyDifferentialPolynomial Fraction Integer
--E 6

--S 7 of 36
f:= w.4 - w.1 * w.1 * z.3
--R
--R
--R               2
--R   (7)  w  - w  z
--R         4    1  3
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 7

--S 8 of 36
g:=(z.1)**3 * (z.2)**2 - w.2
--R
--R
--R          3  2
--R   (8)  z  z   - w
--R         1  2     2
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 8

--S 9 of 36
D(f)
--R
--R
--R               2
--R   (9)  w  - w  z  - 2w w z
--R         5    1  4     1 2 3
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 9

--S 10 of 36
D(f,4)
--R
--R
--R   (10)
--R             2                               2
--R     w  - w  z  - 8w w z  + (- 12w w  - 12w  )z  - 2w z w
--R      8    1  7     1 2 6         1 3      2   5     1 3 5
--R   +
--R                                          2
--R     (- 8w w  - 24w w )z  - 8w z w  - 6w  z
--R          1 4      2 3  4     2 3 4     3  3
```

```
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 10

--S 11 of 36
df:=makeVariable(f)$dpol
--R
--R
--R   (11)  theMap(DPOLCAT-;makeVariable;AM;17!0,0)
--R Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
--E 11

--S 12 of 36
df.4
--R
--R
--R   (12)
--R            2                              2
--R     w  - w  z  - 8w w z  + (- 12w w  - 12w  )z  - 2w z w
--R      8    1  7     1 2 6         1 3      2   5     1 3 5
--R   +
--R                                        2
--R     (- 8w w  - 24w w )z  - 8w z w  - 6w  z
--R          1 4      2 3  4     2 3 4     3  3
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 12

--S 13 of 36
order(g)
--R
--R
--R   (13)  2
--R                                                          Type: PositiveInteger
--E 13

--S 14 of 36
order(g, 'w)
--R
--R
--R   (14)  2
--R                                                          Type: PositiveInteger
--E 14

--S 15 of 36
differentialVariables(g)
--R
--R
--R   (15)  [z,w]
--R                                                              Type: List Symbol
--E 15
```

```
--S 16 of 36
degree(g)
--R
--R
--R            2  3
--R   (16)  z  z
--R          2  1
--R                      Type: IndexedExponents OrderlyDifferentialVariable Symbol
--E 16

--S 17 of 36
degree(g, 'w)
--R
--R
--R   (17)  1
--R                                                         Type: PositiveInteger
--E 17

--S 18 of 36
weights(g)
--R
--R
--R   (18)  [7,2]
--R                                                 Type: List NonNegativeInteger
--E 18

--S 19 of 36
weights(g,'w)
--R
--R
--R   (19)  [2]
--R                                                 Type: List NonNegativeInteger
--E 19

--S 20 of 36
weight(g)
--R
--R
--R   (20)  7
--R                                                         Type: PositiveInteger
--E 20

--S 21 of 36
isobaric?(g)
--R
--R
--R   (21)  false
--R                                                                 Type: Boolean
--E 21
```

```
--S 22 of 36
eval(g,['w::Symbol],[f])
--R
--R
--R                  2                            2        3   2
--R   (22)  - w  + w  z  + 4w w z  + (2w w  + 2w  )z  + z  z
--R            6    1  5     1 2 4      1 3     2   3    1  2
--R                            Type: OrderlyDifferentialPolynomial Fraction Integer
--E 22

--S 23 of 36
eval(g,variables(w.0),[f])
--R
--R
--R            3  2
--R   (23)  z  z   - w
--R          1  2     2
--R                            Type: OrderlyDifferentialPolynomial Fraction Integer
--E 23

--S 24 of 36
monomials(g)
--R
--R
--R             3  2
--R   (24)  [z  z  ,- w ]
--R           1  2     2
--R                       Type: List OrderlyDifferentialPolynomial Fraction Integer
--E 24

--S 25 of 36
variables(g)
--R
--R
--R   (25)  [z ,w ,z ]
--R           2  2  1
--R                                   Type: List OrderlyDifferentialVariable Symbol
--E 25

--S 26 of 36
gcd(f,g)
--R
--R
--R   (26)  1
--R                            Type: OrderlyDifferentialPolynomial Fraction Integer
--E 26

--S 27 of 36
groebner([f,g])
--R
```

```
--R
--R                  2     3  2
--R   (27)  [w  - w  z ,z  z   - w ]
--R           4    1  3  1  2     2
--R                    Type: List OrderlyDifferentialPolynomial Fraction Integer
--E 27

--S 28 of 36
lg:=leader(g)
--R
--R
--R   (28)  z
--R          2
--R                                     Type: OrderlyDifferentialVariable Symbol
--E 28

--S 29 of 36
sg:=separant(g)
--R
--R
--R            3
--R   (29)  2z  z
--R           1  2
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 29

--S 30 of 36
ig:=initial(g)
--R
--R
--R           3
--R   (30)  z
--R          1
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 30

--S 31 of 36
g1 := D g
--R
--R
--R            3               2  3
--R   (31)  2z  z z  - w  + 3z  z
--R           1  2 3    3     1  2
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 31

--S 32 of 36
lg1:= leader g1
--R
--R
```

```
--R   (32)  z
--R          3
--R                                     Type: OrderlyDifferentialVariable Symbol
--E 32

--S 33 of 36
pdf:=D(f, lg1)
--R
--R
--R             2
--R   (33)  - w
--R            1
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 33

--S 34 of 36
prf:=sg * f- pdf * g1
--R
--R
--R            3         2        2  2  3
--R   (34)  2z  z w  - w  w  + 3w  z  z
--R           1  2 4    1  3     1  1  2
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 34

--S 35 of 36
lcf:=leadingCoefficient univariate(prf, lg)
--R
--R
--R            2  2
--R   (35)  3w  z
--R           1  1
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 35

--S 36 of 36
ig * prf - lcf * g * lg
--R
--R
--R            6         2  3        2  2
--R   (36)  2z  z w  - w  z  w  + 3w  z  w z
--R           1  2 4    1  1  3     1  1  2 2
--R                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 36
)spool
)lisp (bye)
```

———

— OrderlyDifferentialPolynomial.help —

```
====================================================================
OrderlyDifferentialPolynomial examples
====================================================================

Many systems of differential equations may be transformed to equivalent
systems of ordinary differential equations where the equations are expressed
polynomially in terms of the unknown functions.  In Axiom, the domain
constructors OrderlyDifferentialPolynomial (abbreviated ODPOL) and
SequentialDifferentialPolynomial (abbreviation SDPOL) implement two
domains of ordinary differential polynomials over any differential
ring.  In the simplest case, this differential ring is usually either
the ring of integers, or the field of rational numbers.  However,
Axiom can handle ordinary differential polynomials over a field of
rational functions in a single indeterminate.

The two domains ODPOL and SDPOL are almost identical, the only difference
being the choice of a different ranking, which is an ordering of the
derivatives of the indeterminates.  The first domain uses an orderly ranking,
that is, derivatives of higher order are ranked higher, and derivatives of
the same order are ranked alphabetically.  The second domain uses a
sequential ranking, where derivatives are ordered first alphabetically
by the differential indeterminates, and then by order.  A more general
domain constructor, DifferentialSparseMultivariatePolynomial (abbreviation
DSMP) allows both a user-provided list of differential indeterminates
as well as a user-defined ranking.  We shall illustrate ODPOL(FRAC INT),
which constructs a domain of ordinary differential polynomials in an
arbitrary number of differential indeterminates with rational numbers
as coefficients.

  dpol:= ODPOL(FRAC INT)
    OrderlyDifferentialPolynomial Fraction Integer
                            Type: Domain

A differential indeterminate w may be viewed as an infinite sequence of
algebraic indeterminates, which are the derivatives of w. To facilitate
referencing these, Axiom provides the operation makeVariable to convert
an element of type Symbol to a map from the natural numbers to the
differential polynomial ring.

  w := makeVariable('w)$dpol
    theMap(DPOLCAT-;makeVariable;AM;17!0,0)
 Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)

  z := makeVariable('z)$dpol
    theMap(DPOLCAT-;makeVariable;AM;17!0,0)
 Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)

The fifth derivative of w can be obtained by applying the map w to the
```

number 5. Note that the order of differentiation is given as a subscript (except when the order is 0).

```
  w.5
    w
     5
                      Type: OrderlyDifferentialPolynomial Fraction Integer

  w 0
    w
                      Type: OrderlyDifferentialPolynomial Fraction Integer
```

The first five derivatives of z can be generated by a list.

```
  [z.i for i in 1..5]
    [z ,z ,z ,z ,z ]
      1  2  3  4  5
                    Type: List OrderlyDifferentialPolynomial Fraction Integer
```

The usual arithmetic can be used to form a differential polynomial from the derivatives.

```
  f:= w.4 - w.1 * w.1 * z.3
            2
    w  - w  z
     4    1  3
                    Type: OrderlyDifferentialPolynomial Fraction Integer

  g:=(z.1)**3 * (z.2)**2 - w.2
      3  2
    z  z   - w
     1  2     2
                    Type: OrderlyDifferentialPolynomial Fraction Integer
```

The operation D computes the derivative of any differential polynomial.

```
  D(f)
            2
    w  - w  z  - 2w w z
     5    1  4     1 2 3
                    Type: OrderlyDifferentialPolynomial Fraction Integer
```

The same operation can compute higher derivatives, like the fourth derivative.

```
  D(f,4)
             2                                2
     w  - w  z  - 8w w z  + (- 12w w  - 12w  )z  - 2w z w
      8    1  7     1 2 6         1 3      2   5     1 3 5
   +
                                           2
```

```
  (- 8w w  - 24w w )z  - 8w z w  - 6w  z
       1 4      2 3  4     2 3 4     3  3
                      Type: OrderlyDifferentialPolynomial Fraction Integer
```

The operation makeVariable creates a map to facilitate referencing the derivatives of f, similar to the map w.

```
df:=makeVariable(f)$dpol
  theMap(DPOLCAT-;makeVariable;AM;17!0,0)
Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
```

The fourth derivative of f may be referenced easily.

```
df.4
          2                              2
   w  - w  z  - 8w w z  + (- 12w w  - 12w  )z  - 2w z w
    8    1  7     1 2 6         1 3      2   5     1 3 5
 +
                                       2
   (- 8w w  - 24w w )z  - 8w z w  - 6w  z
        1 4      2 3  4     2 3 4     3  3
                  Type: OrderlyDifferentialPolynomial Fraction Integer
```

The operation order returns the order of a differential polynomial, or the order in a specified differential indeterminate.

```
order(g)
  2
                  Type: PositiveInteger

order(g, 'w)
  2
                  Type: PositiveInteger
```

The operation differentialVariables returns a list of differential indeterminates occurring in a differential polynomial.

```
differentialVariables(g)
  [z,w]
                  Type: List Symbol
```

The operation degree returns the degree, or the degree in the differential indeterminate specified.

```
degree(g)
     2  3
   z  z
    2  1
                  Type: IndexedExponents OrderlyDifferentialVariable Symbol
```

```
degree(g, 'w)
  1
                Type: PositiveInteger
```

The operation weights returns a list of weights of differential monomials appearing in differential polynomial, or a list of weights in a specified differential indeterminate.

```
weights(g)
  [7,2]
                Type: List NonNegativeInteger

weights(g,'w)
  [2]
                Type: List NonNegativeInteger
```

The operation weight returns the maximum weight of all differential monomials appearing in the differential polynomial.

```
weight(g)
  7
                Type: PositiveInteger
```

A differential polynomial is isobaric if the weights of all differential monomials appearing in it are equal.

```
isobaric?(g)
  false
                Type: Boolean
```

To substitute differentially, use eval. Note that we must coerce 'w to Symbol, since in ODPOL, differential indeterminates belong to the domain Symbol. Compare this result to the next, which substitutes algebraically (no substitution is done since w.0 does not appear in g).

```
eval(g,['w::Symbol],[f])
            2                            2        3  2
   - w  + w  z  + 4w w z  + (2w w  + 2w  )z  + z  z
      6    1  5     1 2 4      1 3     2   3    1  2
                Type: OrderlyDifferentialPolynomial Fraction Integer

eval(g,variables(w.0),[f])
     3  2
   z  z   - w
    1  2     2
                Type: OrderlyDifferentialPolynomial Fraction Integer
```

Since OrderlyDifferentialPolynomial belongs to PolynomialCategory, all the operations defined in the latter category, or in packages for the latter category, are available.

```
monomials(g)
      3  2
   [z  z  ,- w ]
     1  2     2
                  Type: List OrderlyDifferentialPolynomial Fraction Integer

variables(g)
  [z ,w ,z ]
    2  2  1
                  Type: List OrderlyDifferentialVariable Symbol

gcd(f,g)
  1
                  Type: OrderlyDifferentialPolynomial Fraction Integer

groebner([f,g])
           2     3  2
   [w  - w  z ,z  z   - w ]
     4    1  3  1  2     2
                  Type: List OrderlyDifferentialPolynomial Fraction Integer
```

The next three operations are essential for elimination procedures in differential polynomial rings. The operation leader returns the leader of a differential polynomial, which is the highest ranked derivative of the differential indeterminates that occurs.

```
lg:=leader(g)
   z
    2
                  Type: OrderlyDifferentialVariable Symbol
```

The operation separant returns the separant of a differential polynomial, which is the partial derivative with respect to the leader.

```
sg:=separant(g)
     3
  2z  z
    1  2
                  Type: OrderlyDifferentialPolynomial Fraction Integer
```

The operation initial returns the initial, which is the leading coefficient when the given differential polynomial is expressed as a polynomial in the leader.

```
ig:=initial(g)
    3
  z
   1
                  Type: OrderlyDifferentialPolynomial Fraction Integer
```

Using these three operations, it is possible to reduce f modulo the differential ideal generated by g. The general scheme is to first reduce the order, then reduce the degree in the leader. First, eliminate z.3 using the derivative of g.

```
g1 := D g
     3               2  3
  2z  z z  - w  + 3z  z
    1  2 3    3     1  2
                   Type: OrderlyDifferentialPolynomial Fraction Integer
```

Find its leader.

```
lg1:= leader g1
  z
   3
                   Type: OrderlyDifferentialVariable Symbol
```

Differentiate f partially with respect to this leader.

```
pdf:=D(f, lg1)
      2
  - w
     1
                   Type: OrderlyDifferentialPolynomial Fraction Integer
```

Compute the partial remainder of f with respect to g.

```
prf:=sg * f- pdf * g1
     3         2        2  2  3
  2z  z w  - w  w  + 3w  z  z
    1  2 4    1  3     1  1  2
                    Type: OrderlyDifferentialPolynomial Fraction Integer
```

Note that high powers of lg still appear in prf. Compute the leading coefficient of prf as a polynomial in the leader of g.

```
lcf:=leadingCoefficient univariate(prf, lg)
     2  2
  3w  z
    1  1
                    Type: OrderlyDifferentialPolynomial Fraction Integer
```

Finally, continue eliminating the high powers of lg appearing in prf to obtain the (pseudo) remainder of f modulo g and its derivatives.

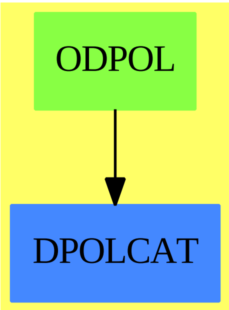
```
ig * prf - lcf * g * lg
     6         2  3        2  2
  2z  z w  - w  z  w  + 3w  z  w z
```

```
      1  2 4    1  1  3     1  1  2 2
                     Type: OrderlyDifferentialPolynomial Fraction Integer
```

```
See Also:
o )show OrderlyDifferentialPolynomial
```

———–

16.16.1 OrderlyDifferentialPolynomial (ODPOL)



See

⇒ "OrderlyDifferentialVariable" (ODVAR) 16.17.1 on page 1816
⇒ "SequentialDifferentialVariable" (SDVAR) 20.7.1 on page 2348
⇒ "DifferentialSparseMultivariatePolynomial" (DSMP) 5.8.1 on page 526
⇒ "SequentialDifferentialPolynomial" (SDPOL) 20.6.1 on page 2345

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	D
degree	differentialVariables	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	initial
isExpt	isobaric?	isPlus
isTimes	latex	lcm
leader	leadingCoefficient	leadingMonomial
mainVariable	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multivariate	numberOfMonomials
one?	order	patternMatch
pomopo!	prime?	primitiveMonomials
primitivePart	recip	reducedSystem
reductum	resultant	retract
retractIfCan	sample	separant
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	totalDegree
totalDegree	unit?	unitCanonical
unitNormal	univariate	univariate
variables	weight	weights
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?<?	?<=?
?>?	?>=?	

— domain ODPOL OrderlyDifferentialPolynomial —

```
)abbrev domain ODPOL OrderlyDifferentialPolynomial
++ Author:  William Sit
++ Date Created: 24 September, 1991
++ Date Last Updated: 7 February, 1992
++ Basic Operations:DifferentialPolynomialCategory
++ Related Constructors: DifferentialSparseMultivariatePolynomial
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++           order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
```

```
++ \spadtype{OrderlyDifferentialPolynomial} implements
++ an ordinary differential polynomial ring in arbitrary number
++ of differential indeterminates, with coefficients in a
++ ring.  The ranking on the differential indeterminate is orderly.
++ This is analogous to the domain \spadtype{Polynomial}.

OrderlyDifferentialPolynomial(R):
     Exports == Implementation where
  R: Ring
  S  ==> Symbol
  V  ==> OrderlyDifferentialVariable S
  E   ==> IndexedExponents(V)
  SMP ==> SparseMultivariatePolynomial(R, S)
  Exports ==> Join(DifferentialPolynomialCategory(R,S,V,E),
                  RetractableTo SMP)

  Implementation ==> DifferentialSparseMultivariatePolynomial(R,S,V)
```

— **ODPOL.dotabb** —

```
"ODPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODPOL"]
"DPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DPOLCAT"]
"ODPOL" -> "DPOLCAT"
```

16.17 domain ODVAR OrderlyDifferentialVariable

— **OrderlyDifferentialVariable.input** —

```
)set break resume
)sys rm -f OrderlyDifferentialVariable.output
)spool OrderlyDifferentialVariable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderlyDifferentialVariable
--R OrderlyDifferentialVariable S: OrderedSet  is a domain constructor
--R Abbreviation for OrderlyDifferentialVariable is ODVAR
--R This constructor is not exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODVAR
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : S -> %
--R coerce : % -> OutputForm             differentiate : % -> %
--R hash : % -> SingleInteger            latex : % -> String
--R max : (%,%) -> %                     min : (%,%) -> %
--R order : % -> NonNegativeInteger      retract : % -> S
--R variable : % -> S                    weight : % -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R differentiate : (%,NonNegativeInteger) -> %
--R makeVariable : (S,NonNegativeInteger) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

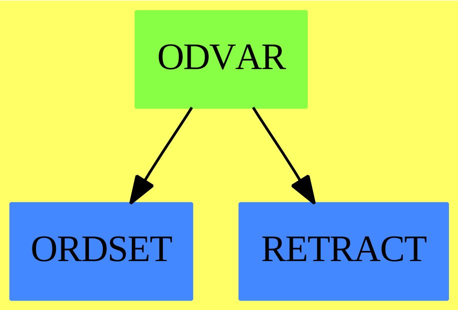
— **OrderlyDifferentialVariable.help** —

```
====================================================================
OrderlyDifferentialVariable examples
====================================================================

See Also:
o )show OrderlyDifferentialVariable
```

———-

16.17.1 OrderlyDifferentialVariable (ODVAR)



See

⇒ "SequentialDifferentialVariable" (SDVAR) 20.7.1 on page 2348
⇒ "DifferentialSparseMultivariatePolynomial" (DSMP) 5.8.1 on page 526
⇒ "OrderlyDifferentialPolynomial" (ODPOL) 16.16.1 on page 1813
⇒ "SequentialDifferentialPolynomial" (SDPOL) 20.6.1 on page 2345

Exports:

coerce	differentiate	hash	latex	makeVariable
max	min	order	retract	retractIfCan
variable	weight	?~=?	?<?	?<=?
?=?	?>?	?>=?		

— domain ODVAR OrderlyDifferentialVariable —

```
)abbrev domain ODVAR OrderlyDifferentialVariable
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:differentiate, order, variable,<
++ Related Domains: OrderedVariableList,
++                  SequentialDifferentialVariable.
++ See Also: DifferentialVariableCategory
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, orderly ranking.
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
++ \spadtype{OrderlyDifferentialVariable} adds a commonly used orderly
++ ranking to the set of derivatives of an ordered list of differential
++ indeterminates.  An orderly ranking is a ranking \spadfun{<} of the
++ derivatives with the property that for two derivatives u and v,
++ u \spadfun{<} v if the \spadfun{order} of u is less than that of v.
++ This domain belongs to \spadtype{DifferentialVariableCategory}.  It
++ defines \spadfun{weight} to be just \spadfun{order}, and it
++ defines an orderly ranking \spadfun{<} on derivatives u via the
++ lexicographic order on the pair
++ (\spadfun{order}(u), \spadfun{variable}(u)).

OrderlyDifferentialVariable(S:OrderedSet):DifferentialVariableCategory(S)
  == add
    Rep := Record(var:S, ord:NonNegativeInteger)
    makeVariable(s,n) == [s, n]
    variable v     == v.var
    order v        == v.ord
```

———

— ODVAR.dotabb —

```
"ODVAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODVAR"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"RETRACT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RETRACT"]
"ODVAR" -> "ORDSET"
"ODVAR" -> "RETRACT"
```

———-

16.18 domain ODR OrdinaryDifferentialRing

— OrdinaryDifferentialRing.input —

```
)set break resume
)sys rm -f OrdinaryDifferentialRing.output
)spool OrdinaryDifferentialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdinaryDifferentialRing
--R OrdinaryDifferentialRing(Kernels: SetCategory,R: PartialDifferentialRing Kernels,var: Ker
--R Abbreviation for OrdinaryDifferentialRing is ODR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?=? : (%,%) -> Boolean               D : % -> %
--R D : (%,NonNegativeInteger) -> %      1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R coerce : % -> R                      coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R differentiate : % -> %               hash : % -> SingleInteger
--R inv : % -> % if R has FIELD          latex : % -> String
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has FIELD
--R ?*? : (Fraction Integer,%) -> % if R has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
```

```
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if R has FIELD
--R ?^? : (%,Integer) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has FIELD
--R characteristic : () -> NonNegativeInteger
--R coerce : % -> % if R has FIELD
--R coerce : Fraction Integer -> % if R has FIELD
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has FIELD
--R gcd : (%,%) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R lcm : (%,%) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R prime? : % -> Boolean if R has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R ?quo? : (%,%) -> % if R has FIELD
--R ?rem? : (%,%) -> % if R has FIELD
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R squareFree : % -> Factored % if R has FIELD
--R squareFreePart : % -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has FIELD
--R unitCanonical : % -> % if R has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has FIELD
--R
--E 1

)spool
)lisp (bye)
```

———-

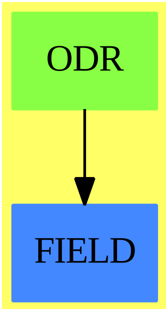
— OrdinaryDifferentialRing.help —

```
====================================================================
OrdinaryDifferentialRing examples
====================================================================

See Also:
```

o `)show OrdinaryDifferentialRing`

16.18.1 OrdinaryDifferentialRing (ODR)



See

⇒ “OppositeMonogenicLinearOperator” (OMLO) 16.11.1 on page 1768
⇒ “DirectProductModule” (DPMO) 5.11.1 on page 542
⇒ “DirectProductMatrixModule” (DPMM) 5.10.1 on page 538

Exports:

0	1	associates?	characteristic
coerce	D	differentiate	divide
euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash
inv	latex	lcm	multiEuclidean
one?	prime?	principalIdeal	recip
sample	sizeLess?	squareFree	squareFreePart
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?	?quo?	?rem?

— domain ODR OrdinaryDifferentialRing —

```
)abbrev domain ODR OrdinaryDifferentialRing
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: differential ring
```

```
++ Examples:
++ References:
++ Description:
++ This constructor produces an ordinary differential ring from
++ a partial differential ring by specifying a variable.

OrdinaryDifferentialRing(Kernels,R,var): DRcategory == DRcapsule where
    Kernels:SetCategory
    R: PartialDifferentialRing(Kernels)
    var : Kernels
    DRcategory == Join(BiModule($,$), DifferentialRing) with
        if R has Field then Field
        coerce: R -> $
            ++ coerce(r) views r as a value in the ordinary differential ring.
        coerce: $ -> R
            ++ coerce(p) views p as a valie in the partial differential ring.
    DRcapsule == R add
        n: Integer
        Rep := R
        coerce(u:R):$ == u::Rep::$
        coerce(p:$):R == p::Rep::R
        differentiate p       == differentiate(p, var)

        if R has Field then
            p / q     == ((p::R) /$R (q::R))::$
            p ** n    == ((p::R) **$R n)::$
            inv(p)    == (inv(p::R)$R)::$
```

— **ODR.dotabb** —

```
"ODR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODR"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"ODR" -> "FIELD"
```

16.19 domain OWP OrdinaryWeightedPolynomials

— **OrdinaryWeightedPolynomials.input** —

```
)set break resume
)sys rm -f OrdinaryWeightedPolynomials.output
```

```
)spool OrdinaryWeightedPolynomials.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdinaryWeightedPolynomials
--R OrdinaryWeightedPolynomials(R: Ring,vl: List Symbol,wl: List NonNegativeInteger,wtlevel:
--R Abbreviation for OrdinaryWeightedPolynomials is OWP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OWP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Polynomial R -> %
--R coerce : % -> Polynomial R            coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,R) -> % if R has COMRING
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> Union(%,"failed") if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R changeWeightLevel : NonNegativeInteger -> Void
--R characteristic : () -> NonNegativeInteger
--R coerce : R -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

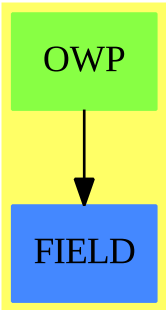
— OrdinaryWeightedPolynomials.help —

```
====================================================================
OrdinaryWeightedPolynomials examples
====================================================================

See Also:
```

```
o )show OrdinaryWeightedPolynomials
```

16.19.1 OrdinaryWeightedPolynomials (OWP)



Exports:

0	1	changeWeightLevel	characteristic	coerce
hash	latex	one?	recip	sample
subtractIfCan	zero?	?**?	?**?	?+?
?-?	-?	?=?	?^?	?~=?
?/?				

— domain OWP OrdinaryWeightedPolynomials —

```
)abbrev domain OWP OrdinaryWeightedPolynomials
++ Author: James Davenport
++ Date Created:  17 April 1992
++ Date Last Updated: 12 July 1992
++ Basic Functions: Ring, changeWeightLevel
++ Related Constructors: WeightedPolynomials
++ Also See: PolynomialRing
++ AMS classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents truncated weighted polynomials over the
++ "Polynomial" type. The variables must be
++ specified, as must the weights.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.

OrdinaryWeightedPolynomials(R:Ring,
                             vl:List Symbol, wl:List NonNegativeInteger,
                              wtlevel:NonNegativeInteger):
```

```
      Ring with
        if R has CommutativeRing then Algebra(R)
        coerce: $ -> Polynomial(R)
          ++ coerce(p) converts back into a Polynomial(R), ignoring weights
        coerce: Polynomial(R) -> $
          ++ coerce(p) coerces a Polynomial(R) into Weighted form,
          ++ applying weights and ignoring terms
        if R has Field then "/": ($,$) -> Union($,"failed")
          ++ x/y division (only works if minimum weight
          ++ of divisor is zero, and if R is a Field)
        changeWeightLevel: NonNegativeInteger -> Void
          ++ changeWeightLevel(n) This changes the weight level to the
          ++ new value given:
          ++ NB: previously calculated terms are not affected
   == WeightedPolynomials(R,Symbol,IndexedExponents(Symbol),
                         Polynomial(R),
                          vl,wl,wtlevel)
```

— **OWP.dotabb** —

```
"OWP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OWP"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"OWP" -> "FIELD"
```

16.20 domain OSI OrdSetInts

— **OrdSetInts.input** —

```
)set break resume
)sys rm -f OrdSetInts.output
)spool OrdSetInts.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdSetInts
--R OrdSetInts  is a domain constructor
--R Abbreviation for OrdSetInts is OSI
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OSI
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      value : % -> Integer
--R ?~=? : (%,%) -> Boolean
--R
--E 1

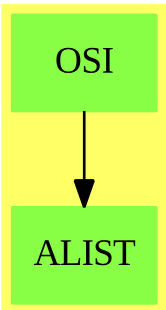
)spool
)lisp (bye)
```

— **OrdSetInts.help** —

```
====================================================================
OrdSetInts examples
====================================================================

See Also:
o )show OrdSetInts
```

16.20.1 OrdSetInts (OSI)



See

⇒ “Commutator” (COMM) 4.7.1 on page 395
⇒ “FreeNilpotentLie” (FNLA) 7.33.1 on page 993

Exports:

coerce	hash	latex	max	min
value	?~=?	?<?	?<=?	?=?
?>?	?>=?			

— domain OSI OrdSetInts —

```
)abbrev domain OSI OrdSetInts
++ Author: Larry Lambe
++ Date created: 14 August 1988
++ Date Last Updated: 11 March 1991
++ Description:
++ A domain used in order to take the free R-module on the
++ Integers I.  This is actually the forgetful functor from OrderedRings
++ to OrderedSets applied to I

OrdSetInts: Export == Implement where
   I  ==> Integer
   L  ==> List
   O  ==> OutputForm

   Export == OrderedSet with
     coerce : Integer -> %
        ++ coerce(i) returns the element corresponding to i
     value  : % -> I
        ++ value(x) returns the integer associated with x

   Implement == add
     Rep := Integer
     x,y: %

     x = y == x =$Rep y
     x < y == x <$Rep y

     coerce(i:Integer):% == i

     value(x) == x:Rep

     coerce(x):O ==
       sub(e::Symbol::O, coerce(x)$Rep)$O
```

———-

— OSI.dotabb —

```
"OSI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OSI"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OSI" -> "ALIST"
```

16.21 domain OUTFORM OutputForm

— OutputForm.input —

```
)set break resume
)sys rm -f OutputForm.output
)spool OutputForm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OutputForm
--R OutputForm  is a domain constructor
--R Abbreviation for OutputForm is OUTFORM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OUTFORM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?**? : (%,%) -> %
--R ?+? : (%,%) -> %                       -? : % -> %
--R ?-? : (%,%) -> %                       ?/? : (%,%) -> %
--R ?<? : (%,%) -> %                       ?<=? : (%,%) -> %
--R ?=? : (%,%) -> %                       ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> %                       ?>=? : (%,%) -> %
--R ?SEGMENT : % -> %                      ?..? : (%,%) -> %
--R ?^=? : (%,%) -> %                      ?and? : (%,%) -> %
--R assign : (%,%) -> %                    binomial : (%,%) -> %
--R blankSeparate : List % -> %            box : % -> %
--R brace : List % -> %                    brace : % -> %
--R bracket : List % -> %                  bracket : % -> %
--R center : % -> %                        center : (%,Integer) -> %
--R coerce : % -> OutputForm               commaSeparate : List % -> %
--R ?div? : (%,%) -> %                     dot : % -> %
--R ?.? : (%,List %) -> %                  empty : () -> %
--R exquo : (%,%) -> %                     hash : % -> SingleInteger
--R hconcat : List % -> %                  hconcat : (%,%) -> %
--R height : () -> Integer                 height : % -> Integer
--R hspace : Integer -> %                  infix : (%,%,%) -> %
--R infix : (%,List %) -> %                infix? : % -> Boolean
--R int : (%,%,%) -> %                     int : (%,%) -> %
--R int : % -> %                           label : (%,%) -> %
--R latex : % -> String                    left : % -> %
```

```
--R left : (%,Integer) -> %              matrix : List List % -> %
--R message : String -> %                messagePrint : String -> Void
--R not? : % -> %                        ?or? : (%,%) -> %
--R outputForm : DoubleFloat -> %        outputForm : String -> %
--R outputForm : Symbol -> %             outputForm : Integer -> %
--R over : (%,%) -> %                    overbar : % -> %
--R overlabel : (%,%) -> %               paren : List % -> %
--R paren : % -> %                       pile : List % -> %
--R postfix : (%,%) -> %                 prefix : (%,List %) -> %
--R presub : (%,%) -> %                  presuper : (%,%) -> %
--R prime : % -> %                       print : % -> Void
--R prod : (%,%,%) -> %                  prod : (%,%) -> %
--R prod : % -> %                        ?quo? : (%,%) -> %
--R quote : % -> %                       rarrow : (%,%) -> %
--R ?rem? : (%,%) -> %                   right : % -> %
--R right : (%,Integer) -> %             root : (%,%) -> %
--R root : % -> %                        rspace : (Integer,Integer) -> %
--R scripts : (%,List %) -> %            semicolonSeparate : List % -> %
--R slash : (%,%) -> %                   string : % -> %
--R sub : (%,%) -> %                     subHeight : % -> Integer
--R sum : (%,%,%) -> %                   sum : (%,%) -> %
--R sum : % -> %                         super : (%,%) -> %
--R superHeight : % -> Integer           supersub : (%,List %) -> %
--R vconcat : List % -> %                vconcat : (%,%) -> %
--R vspace : Integer -> %                width : () -> Integer
--R width : % -> Integer                 zag : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R differentiate : (%,NonNegativeInteger) -> %
--R dot : (%,NonNegativeInteger) -> %
--R prime : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

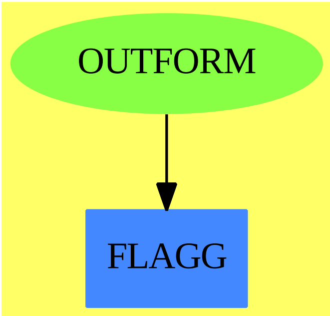
— OutputForm.help —

```
====================================================================
OutputForm examples
====================================================================

See Also:
o )show OutputForm
```

———-

16.21.1 OutputForm (OUTFORM)



Exports:

assign	binomial	blankSeparate	box	brace
bracket	center	coerce	commaSeparate	differentiate
dot	empty	exquo	hash	hconcat
height	hspace	infix	infix?	int
label	latex	left	matrix	message
messagePrint	not?	outputForm	over	overbar
overlabel	paren	pile	postfix	prefix
presub	presuper	prime	print	prod
quote	rarrow	right	root	rspace
scripts	semicolonSeparate	slash	string	sub
subHeight	sum	super	superHeight	supersub
vconcat	vspace	width	zag	?*?
?**?	?+?	-?	?-?	?/?
?<?	?<=?	?=?	?>?	?>=?
?SEGMENT	?..?	?^=?	?and?	?div?
?.?	?or?	?quo?	?rem?	?~=?

— domain OUTFORM OutputForm —

```
)abbrev domain OUTFORM OutputForm
++ Keywords: output, I/O, expression
++ SMW March/88
++ Description:
++ This domain is used to create and manipulate mathematical expressions
++ for output.  It is intended to provide an insulating layer between
++ the expression rendering software (e.g.FORTRAN, TeX, or Script) and
++ the output coercions in the various domains.

OutputForm(): SetCategory with
        --% Printing
        print  : $ -> Void
          ++ print(u) prints the form u.
        message: String -> $
```

```
  ++ message(s) creates an form with no string quotes
  ++ from string s.
messagePrint: String -> Void
  ++ messagePrint(s) prints s without string quotes. Note:
  ++ \spad{messagePrint(s)} is equivalent to \spad{print message(s)}.
--% Creation of atomic forms
outputForm: Integer -> $
  ++ outputForm(n) creates an form for integer n.
outputForm: Symbol  -> $
  ++ outputForm(s) creates an form for symbol s.
outputForm: String  -> $
  ++ outputForm(s) creates an form for string s.
outputForm: DoubleFloat  -> $
  ++ outputForm(sf) creates an form for small float sf.
empty   : () -> $
  ++ empty() creates an empty form.

--% Sizings
width: $ -> Integer
  ++ width(f) returns the width of form f (an integer).
height: $ -> Integer
  ++ height(f) returns the height of form f (an integer).
width: -> Integer
  ++ width() returns the width of the display area (an integer).
height: -> Integer
  ++ height() returns the height of the display area (an integer).
subHeight: $ -> Integer
  ++ subHeight(f) returns the height of form f below the base line.
superHeight: $ -> Integer
  ++ superHeight(f) returns the height of form f above the base line.
 --% Space manipulations
hspace: Integer -> $  ++ hspace(n) creates white space of width n.
vspace: Integer -> $  ++ vspace(n) creates white space of height n.
rspace: (Integer,Integer) -> $
  ++ rspace(n,m) creates rectangular white space, n wide by m high.
--% Area adjustments
left: ($,Integer) -> $
  ++ left(f,n) left-justifies form f within space of width n.
right: ($,Integer) -> $
  ++ right(f,n) right-justifies form f within space of width n.
center: ($,Integer) -> $
  ++ center(f,n) centers form f within space of width n.
left: $ -> $
  ++ left(f) left-justifies form f in total space.
right: $ -> $
  ++ right(f) right-justifies form f in total space.
center:   $ -> $
  ++ center(f) centers form f in total space.

--% Area manipulations
```

```
hconcat:  ($,$) -> $
  ++ hconcat(f,g) horizontally concatenate forms f and g.
vconcat:  ($,$) -> $
  ++ vconcat(f,g) vertically concatenates forms f and g.
hconcat:  List $ -> $
  ++ hconcat(u) horizontally concatenates all forms in list u.
vconcat:  List $ -> $
  ++ vconcat(u) vertically concatenates all forms in list u.

--% Application formers
prefix:  ($, List $) -> $
  ++ prefix(f,l) creates a form depicting the n-ary prefix
  ++ application of f to a tuple of arguments given by list l.
infix:   ($, List $) -> $
  ++ infix(f,l) creates a form depicting the n-ary application
  ++ of infix operation f to a tuple of arguments l.
infix:   ($, $, $) -> $
  ++ infix(op, a, b) creates a form which prints as: a op b.
postfix: ($, $)    -> $
  ++ postfix(op, a)  creates a form which prints as: a op.
infix?: $ -> Boolean
  ++ infix?(op) returns true if op is an infix operator,
  ++ and false otherwise.
elt:     ($, List $) -> $
  ++ elt(op,l) creates a form for application of op
  ++ to list of arguments l.

--% Special forms
string:  $ -> $
  ++ string(f) creates f with string quotes.
label:   ($, $) -> $
  ++ label(n,f) gives form f an equation label n.
box:     $ -> $
  ++ box(f) encloses f in a box.
matrix:  List List $ -> $
  ++ matrix(llf) makes llf (a list of lists of forms) into
  ++ a form which displays as a matrix.
zag:     ($, $) -> $
  ++ zag(f,g) creates a form for the continued fraction form for f over g.
root:    $ -> $
  ++ root(f) creates a form for the square root of form f.
root:    ($, $) -> $
  ++ root(f,n) creates a form for the nth root of form f.
over:    ($, $) -> $
  ++ over(f,g) creates a form for the vertical fraction of f over g.
slash:   ($, $) -> $
  ++ slash(f,g) creates a form for the horizontal fraction of f over g.
assign:  ($, $) -> $
  ++ assign(f,g) creates a form for the assignment \spad{f := g}.
rarrow:  ($, $) -> $
```

```
  ++ rarrow(f,g) creates a form for the mapping \spad{f -> g}.
differentiate: ($, NonNegativeInteger) -> $
  ++ differentiate(f,n) creates a form for the nth derivative of f,
  ++ e.g. \spad{f'}, \spad{f''}, \spad{f'''},
  ++ "f super \spad{iv}".
binomial: ($, $) -> $
  ++ binomial(n,m) creates a form for the binomial coefficient of n and m.

--% Scripts
sub:      ($, $) -> $
  ++ sub(f,n) creates a form for f subscripted by n.
super:    ($, $) -> $
  ++ super(f,n) creates a form for f superscripted by n.
presub:   ($, $) -> $
  ++ presub(f,n) creates a form for f presubscripted by n.
presuper:($, $) -> $
  ++ presuper(f,n) creates a form for f presuperscripted by n.
scripts: ($, List $) -> $
    ++ \spad{scripts(f, [sub, super, presuper, presub])}
    ++  creates a form for f with scripts on all 4 corners.
supersub:($, List $) -> $
    ++ supersub(a,[sub1,super1,sub2,super2,...])
    ++ creates a form with each subscript aligned
    ++ under each superscript.

--% Diacritical marks
quote:   $ -> $
  ++ quote(f) creates the form f with a prefix quote.
dot:     $ -> $
  ++ dot(f) creates the form with a one dot overhead.
dot:     ($, NonNegativeInteger) -> $
  ++ dot(f,n) creates the form f with n dots overhead.
prime:   $ -> $
  ++ prime(f) creates the form f followed by a suffix prime (single quote).
prime:   ($, NonNegativeInteger) -> $
  ++ prime(f,n) creates the form f followed by n primes.
overbar: $ -> $
  ++ overbar(f) creates the form f with an overbar.
overlabel: ($, $) -> $
  ++ overlabel(x,f) creates the form f with "x overbar" over the top.

--% Plexes
sum:     ($)        -> $
  ++ sum(expr) creates the form prefixing expr by a capital sigma.
sum:     ($, $)     -> $
  ++ sum(expr,lowerlimit) creates the form prefixing expr by
  ++ a capital sigma with a lowerlimit.
sum:     ($, $, $) -> $
  ++ sum(expr,lowerlimit,upperlimit) creates the form prefixing expr by
  ++ a capital sigma with both a lowerlimit and upperlimit.
```

```
prod:    ($)       -> $
  ++ prod(expr) creates the form prefixing expr by a capital pi.
prod:    ($, $)    -> $
  ++ prod(expr,lowerlimit) creates the form prefixing expr by
  ++ a capital pi with a lowerlimit.
prod:    ($, $, $) -> $
  ++ prod(expr,lowerlimit,upperlimit) creates the form prefixing expr by
  ++ a capital pi with both a lowerlimit and upperlimit.
int:     ($)       -> $
  ++ int(expr) creates the form prefixing expr with an integral sign.
int:     ($, $)    -> $
  ++ int(expr,lowerlimit) creates the form prefixing expr by an
  ++ integral sign with a lowerlimit.
int:     ($, $, $) -> $
  ++ int(expr,lowerlimit,upperlimit) creates the form prefixing expr by
  ++ an integral sign with both a lowerlimit and upperlimit.

--% Matchfix forms
brace:   $ -> $
  ++ brace(f) creates the form enclosing f in braces (curly brackets).
brace:   List $ -> $
  ++ brace(lf) creates the form separating the elements of lf
  ++ by commas and encloses the result in curly brackets.
bracket: $ -> $
  ++ bracket(f) creates the form enclosing f in square brackets.
bracket: List $ -> $
  ++ bracket(lf) creates the form separating the elements of lf
  ++ by commas and encloses the result in square brackets.
paren:   $ -> $
  ++ paren(f) creates the form enclosing f in parentheses.
paren:   List $ -> $
  ++ paren(lf) creates the form separating the elements of lf
  ++ by commas and encloses the result in parentheses.

--% Separators for aggregates
pile:     List $ -> $
  ++ pile(l) creates the form consisting of the elements of l which
  ++ displays as a pile, i.e. the elements begin on a new line and
  ++ are indented right to the same margin.

commaSeparate: List $ -> $
  ++ commaSeparate(l) creates the form separating the elements of l
  ++ by commas.
semicolonSeparate:  List $ -> $
  ++ semicolonSeparate(l) creates the form separating the elements of l
  ++ by semicolons.
blankSeparate: List $ -> $
  ++ blankSeparate(l) creates the form separating the elements of l
  ++ by blanks.
--% Specific applications
```

```
        "=":     ($, $) -> $
          ++ f = g creates the equivalent infix form.
        "^=":    ($, $) -> $
          ++ f ^= g creates the equivalent infix form.
        "<":     ($, $) -> $
          ++ f < g creates the equivalent infix form.
        ">":     ($, $) -> $
          ++ f > g creates the equivalent infix form.
        "<=":    ($, $) -> $
          ++ f <= g creates the equivalent infix form.
        ">=":    ($, $) -> $
          ++ f >= g creates the equivalent infix form.
        "+":     ($, $) -> $
          ++ f + g creates the equivalent infix form.
        "-":     ($, $) -> $
          ++ f - g creates the equivalent infix form.
        "-":     ($)    -> $
          ++ - f creates the equivalent prefix form.
        "*":     ($, $) -> $
          ++ f * g creates the equivalent infix form.
        "/":     ($, $) -> $
          ++ f / g creates the equivalent infix form.
        "**":    ($, $) -> $
          ++ f ** g creates the equivalent infix form.
        "div":   ($, $) -> $
          ++ f div g creates the equivalent infix form.
        "rem":   ($, $) -> $
          ++ f rem g creates the equivalent infix form.
        "quo":   ($, $) -> $
          ++ f quo g creates the equivalent infix form.
        "exquo": ($, $) -> $
          ++ exquo(f,g) creates the equivalent infix form.
        "and":   ($, $) -> $
          ++ f and g creates the equivalent infix form.
        "or":    ($, $) -> $
          ++ f or g creates the equivalent infix form.
        "not":   ($)    -> $
          ++ not f creates the equivalent prefix form.
        SEGMENT: ($,$)  -> $
          ++ SEGMENT(x,y) creates the infix form: \spad{x..y}.
        SEGMENT: ($)    -> $
          ++ SEGMENT(x) creates the prefix form: \spad{x..}.

    == add
        import NumberFormats

        -- Todo:
        --   program forms, greek letters
        --   infix, prefix, postfix, matchfix support in OUT BOOT
        --   labove rabove, corresponding overs.
```

```
--   better super script, overmark, undermark
--   bug in product, paren blankSeparate []
--   uniformize integrals, products, etc as plexes.

cons ==> CONS$Lisp
car  ==> CAR$Lisp
cdr  ==> CDR$Lisp

Rep := List $

a, b: $
l: List $
s: String
e: Symbol
n: Integer
nn:NonNegativeInteger

sform:    String  -> $
eform:    Symbol  -> $
iform:    Integer -> $

print x              == mathprint(x)$Lisp
message s            == (empty? s => empty(); s pretend $)
messagePrint s       == print message s
(a:$ = b:$):Boolean  == EQUAL(a, b)$Lisp
(a:$ = b:$):$        == [sform "=",     a, b]
coerce(a):OutputForm  == a pretend OutputForm
outputForm n          == n pretend $
outputForm e          == e pretend $
outputForm(f:DoubleFloat) == f pretend $
sform s               == s pretend $
eform e               == e pretend $
iform n               == n pretend $

outputForm s ==
  sform concat(quote()$Character, concat(s, quote()$Character))

width(a) == outformWidth(a)$Lisp
height(a) == height(a)$Lisp
subHeight(a) == subspan(a)$Lisp
superHeight(a) == superspan(a)$Lisp
height() == 20
width() == 66

center(a,w)   == hconcat(hspace((w - width(a)) quo 2),a)
left(a,w)     == hconcat(a,hspace((w - width(a))))
right(a,w)    == hconcat(hspace(w - width(a)),a)
center(a)     == center(a,width())
left(a)       == left(a,width())
right(a)      == right(a,width())
```

```
vspace(n) ==
  n = 0 => empty()
  vconcat(sform " ",vspace(n - 1))

hspace(n) ==
  n = 0 => empty()
  sform(fillerSpaces(n)$Lisp)

rspace(n, m) ==
  n = 0 or m = 0 => empty()
  vconcat(hspace n, rspace(n, m - 1))

matrix ll ==
    lv:$ := [LIST2VEC$Lisp l for l in ll]
    CONS(eform MATRIX, LIST2VEC$Lisp lv)$Lisp

pile l              == cons(eform SC, l)
commaSeparate l     == cons(eform AGGLST,  l)
semicolonSeparate l == cons(eform AGGSET,  l)
blankSeparate l     ==
   c:=eform CONCATB
   l1:$:=[]
   for u in reverse l repeat
       if EQCAR(u,c)$Lisp
          then l1:=[:cdr u,:l1]
          else l1:=[u,:l1]
   cons(c, l1)

brace a        == [eform BRACE,   a]
brace l        == brace commaSeparate l
bracket a      == [eform BRACKET, a]
bracket l      == bracket commaSeparate l
paren a        == [eform PAREN,   a]
paren l        == paren commaSeparate l

sub     (a,b)  == [eform SUB, a, b]
super   (a, b) == [eform SUPERSUB,a,sform " ",b]
presub(a,b) == [eform SUPERSUB,a,sform " ",sform " ",sform " ",b]
presuper(a, b) == [eform SUPERSUB,a,sform " ",sform " ",b]
scripts (a, l) ==
    null l => a
    null rest l => sub(a, first l)
    cons(eform SUPERSUB, cons(a, l))
supersub(a, l) ==
    if odd?(#l) then l := append(l, [empty()])
    cons(eform ALTSUPERSUB, cons(a, l))

hconcat(a,b)  == [eform CONCAT, a, b]
hconcat l     == cons(eform CONCAT, l)
```

```
vconcat(a,b)  == [eform VCONCAT, a, b]
vconcat l     == cons(eform VCONCAT, l)

a ^= b      == [sform "^=",    a, b]
a < b       == [sform "<",     a, b]
a > b       == [sform ">",     a, b]
a <= b      == [sform "<=",    a, b]
a >= b      == [sform ">=",    a, b]

a + b       == [sform "+",     a, b]
a - b       == [sform "-",     a, b]
- a         == [sform "-",     a]
a * b       == [sform "*",     a, b]
a / b       == [sform "/",     a, b]
a ** b      == [sform "**",    a, b]
a div b     == [sform "div",   a, b]
a rem b     == [sform "rem",   a, b]
a quo b     == [sform "quo",   a, b]
a exquo b   == [sform "exquo", a, b]
a and b     == [sform "and",   a, b]
a or b      == [sform "or",    a, b]
not a       == [sform "not",   a]
SEGMENT(a,b)== [eform SEGMENT, a, b]
SEGMENT(a)  == [eform SEGMENT, a]
binomial(a,b)==[eform BINOMIAL, a, b]

empty() == [eform NOTHING]

infix? a ==
    e:$ :=
        IDENTP$Lisp a => a
        STRINGP$Lisp a => INTERN$Lisp a
        return false
    if GET(e,QUOTE(INFIXOP$Lisp)$Lisp)$Lisp then true else false

elt(a, l) ==
    cons(a, l)
prefix(a,l)   ==
    not infix? a => cons(a, l)
    hconcat(a, paren commaSeparate l)
infix(a, l) ==
    null l => empty()
    null rest l => first l
    infix? a => cons(a, l)
    hconcat [first l, a, infix(a, rest l)]
infix(a,b,c)  ==
    infix? a => [a, b, c]
    hconcat [b, a, c]
postfix(a, b) ==
    hconcat(b, a)
```

```
string a   == [eform STRING,  a]
quote  a   == [eform QUOTE,   a]
overbar a  == [eform OVERBAR, a]
dot a      == super(a, sform ".")
prime a    == super(a, sform ",")
dot(a,nn)   == (s := new(nn, char "."); super(a, sform s))
prime(a,nn) == (s := new(nn, char ","); super(a, sform s))

overlabel(a,b) == [eform OVERLABEL, a, b]
box a      == [eform BOX,     a]
zag(a,b)   == [eform ZAG,     a, b]
root a     == [eform ROOT,    a]
root(a,b)  == [eform ROOT,    a, b]
over(a,b)  == [eform OVER,    a, b]
slash(a,b) == [eform SLASH,   a, b]
assign(a,b)== [eform LET,     a, b]

label(a,b) == [eform EQUATNUM, a, b]
rarrow(a,b)== [eform TAG, a, b]
differentiate(a, nn)==
    zero? nn => a
    nn < 4 => prime(a, nn)
    r := FormatRoman(nn::PositiveInteger)
    s := lowerCase(r::String)
    super(a, paren sform s)

sum(a)     == [eform SIGMA,  empty(), a]
sum(a,b)   == [eform SIGMA,  b, a]
sum(a,b,c) == [eform SIGMA2, b, c, a]
prod(a)    == [eform PI,     empty(), a]
prod(a,b)  == [eform PI,     b, a]
prod(a,b,c)== [eform PI2,    b, c, a]
int(a)     == [eform INTSIGN,empty(), empty(), a]
int(a,b)   == [eform INTSIGN,b, empty(), a]
int(a,b,c) == [eform INTSIGN,b, c, a]
```

———-

— **OUTFORM.dotabb** —

```
"OUTFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OUTFORM",
          shape=ellipse]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"OUTFORM" -> "FLAGG"
```

———-

Chapter 17

Chapter P

17.1 domain PADIC PAdicInteger

— **PAdicInteger.input** —

```
)set break resume
)sys rm -f PAdicInteger.output
)spool PAdicInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicInteger
--R PAdicInteger p: Integer  is a domain constructor
--R Abbreviation for PAdicInteger is PADIC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PADIC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R digits : % -> Stream Integer          extend : (%,Integer) -> %
--R gcd : List % -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
```

```
--R moduloP : % -> Integer                  modulus : () -> Integer
--R one? : % -> Boolean                     order : % -> NonNegativeInteger
--R ?quo? : (%,%) -> %                      quotientByP : % -> %
--R recip : % -> Union(%,"failed")          ?rem? : (%,%) -> %
--R sample : () -> %                        sizeLess? : (%,%) -> Boolean
--R sqrt : (%,Integer) -> %                 unit? : % -> Boolean
--R unitCanonical : % -> %                  zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R root : (SparseUnivariatePolynomial Integer,Integer) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———

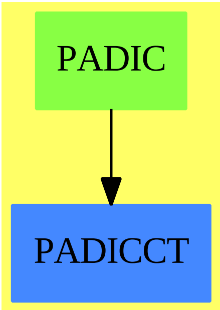
— **PAdicInteger.help** —

```
====================================================================
PAdicInteger examples
====================================================================

See Also:
o )show PAdicInteger
```

———

17.1.1 PAdicInteger (PADIC)



See

⇒ "InnerPAdicInteger" (IPADIC) 10.24.1 on page 1258
⇒ "BalancedPAdicInteger" (BPADIC) 3.2.1 on page 240
⇒ "PAdicRationalConstructor" (PADICRC) 17.3.1 on page 1850
⇒ "PAdicRational" (PADICRAT) 17.2.1 on page 1845
⇒ "BalancedPAdicRational" (BPADICRT) 3.3.1 on page 244

Exports:

0	1	approximate	associates?
characteristic	coerce	complete	digits
divide	euclideanSize	expressIdealMember	exquo
extend	extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm	lcm
moduloP	modulus	multiEuclidean	one?
order	principalIdeal	quotientByP	recip
root	sample	sizeLess?	sqrt
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?**?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?quo?	?rem?	

— domain PADIC PAdicInteger —

```
)abbrev domain PADIC PAdicInteger
++ Author: Clifton J. Williamson
++ Date Created: 20 August 1989
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: p-adic, completion
++ Examples:
++ References:
```

```
++ Description:
++ Stream-based implementation of Zp: p-adic numbers are represented as
++ sum(i = 0.., a[i] * p^i), where the a[i] lie in 0,1,...,(p - 1).

PAdicInteger(p:Integer) == InnerPAdicInteger(p,true$Boolean)
```

———-

— PADIC.dotabb —

```
"PADIC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PADIC"]
"PADICCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PADICCT"]
"PADIC" -> "PADICCT"
```

———-

17.2 domain PADICRAT PAdicRational

— PAdicRational.input —

```
)set break resume
)sys rm -f PAdicRational.output
)spool PAdicRational.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicRational
--R PAdicRational p: Integer  is a domain constructor
--R Abbreviation for PAdicRational is PADICRAT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PADICRAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,PAdicInteger p) -> %        ?*? : (PAdicInteger p,%) -> %
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
```

```
--R associates? : (%,%) -> Boolean        coerce : PAdicInteger p -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R denom : % -> PAdicInteger p           denominator : % -> %
--R factor : % -> Factored %              gcd : List % -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R numer : % -> PAdicInteger p           numerator : % -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    removeZeroes : (Integer,%) -> %
--R removeZeroes : % -> %                 retract : % -> PAdicInteger p
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %          squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (PAdicInteger p,PAdicInteger p) -> %
--R ?<? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R ?<=? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R ?>? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R ?>=? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R D : (%,(PAdicInteger p -> PAdicInteger p)) -> %
--R D : (%,(PAdicInteger p -> PAdicInteger p),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,List Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if PAdicInteger p has DIFRING
--R D : % -> % if PAdicInteger p has DIFRING
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if PAdicInteger p has OINTDOM
--R approximate : (%,Integer) -> Fraction Integer
--R ceiling : % -> PAdicInteger p if PAdicInteger p has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and PAdicInteger p has PFECAT or PAdicInteger p
--R coerce : Symbol -> % if PAdicInteger p has RETRACT SYMBOL
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and PAdicInteger p has PFECAT
--R continuedFraction : % -> ContinuedFraction Fraction Integer
--R convert : % -> DoubleFloat if PAdicInteger p has REAL
--R convert : % -> Float if PAdicInteger p has REAL
--R convert : % -> InputForm if PAdicInteger p has KONVERT INFORM
--R convert : % -> Pattern Float if PAdicInteger p has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if PAdicInteger p has KONVERT PATTERN INT
--R differentiate : (%,(PAdicInteger p -> PAdicInteger p)) -> %
--R differentiate : (%,(PAdicInteger p -> PAdicInteger p),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
```

```
--R differentiate : (%,List Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if PAdicInteger p has DIFRING
--R differentiate : % -> % if PAdicInteger p has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,PAdicInteger p) -> % if PAdicInteger p has ELTAB(PADIC p,PADIC p)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,PAdicInteger p) -> % if PAdicInteger p has IEVALAB(SYMBOL,PADIC p)
--R eval : (%,List Symbol,List PAdicInteger p) -> % if PAdicInteger p has IEVALAB(SYMBOL,PADI
--R eval : (%,List Equation PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R eval : (%,Equation PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R eval : (%,PAdicInteger p,PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R eval : (%,List PAdicInteger p,List PAdicInteger p) -> % if PAdicInteger p has EVALAB PADI
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R floor : % -> PAdicInteger p if PAdicInteger p has INS
--R fractionPart : % -> % if PAdicInteger p has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R init : () -> % if PAdicInteger p has STEP
--R map : ((PAdicInteger p -> PAdicInteger p),%) -> %
--R max : (%,%) -> % if PAdicInteger p has ORDSET
--R min : (%,%) -> % if PAdicInteger p has ORDSET
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if PAdicInteger p has OINTDOM
--R nextItem : % -> Union(%,"failed") if PAdicInteger p has STEP
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R positive? : % -> Boolean if PAdicInteger p has OINTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PAdicInteger p has INS
--R reducedSystem : Matrix % -> Matrix PAdicInteger p
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix PAdicInteger p,vec: Vector PAdi
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if PAdicInteger p has LINEXP INT
--R retract : % -> Integer if PAdicInteger p has RETRACT INT
--R retract : % -> Fraction Integer if PAdicInteger p has RETRACT INT
--R retract : % -> Symbol if PAdicInteger p has RETRACT SYMBOL
--R retractIfCan : % -> Union(Integer,"failed") if PAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if PAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Symbol,"failed") if PAdicInteger p has RETRACT SYMBOL
--R retractIfCan : % -> Union(PAdicInteger p,"failed")
--R sign : % -> Integer if PAdicInteger p has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

```
--R wholePart : % -> PAdicInteger p if PAdicInteger p has EUCDOM
--R
--E 1

)spool
)lisp (bye)
```

———-

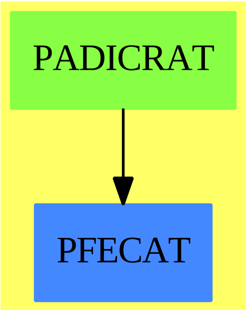
— PAdicRational.help —

```
====================================================================
PAdicRational examples
====================================================================

See Also:
o )show PAdicRational
```

———-

17.2.1 PAdicRational (PADICRAT)



See

⇒ "InnerPAdicInteger" (IPADIC) 10.24.1 on page 1258
⇒ "PAdicInteger" (PADIC) 17.1.1 on page 1841
⇒ "BalancedPAdicInteger" (BPADIC) 3.2.1 on page 240
⇒ "PAdicRationalConstructor" (PADICRC) 17.3.1 on page 1850
⇒ "BalancedPAdicRational" (BPADICRT) 3.3.1 on page 244

Exports:

0	1	abs
approximate	associates?	ceiling
characteristic	charthRoot	coerce
conditionP	continuedFraction	convert
D	denom	denominator
differentiate	divide	euclideanSize
eval	expressIdealMember	exquo
extendedEuclidean	factor	factorPolynomial
factorSquareFreePolynomial	floor	fractionPart
gcd	gcdPolynomial	hash
init	inv	latex
lcm	map	max
min	multiEuclidean	negative?
nextItem	numer	numerator
one?	patternMatch	positive?
prime?	principalIdeal	random
recip	reducedSystem	removeZeroes
retract	retractIfCan	sample
sign	sizeLess?	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	unit?	unitCanonical
unitNormal	wholePart	zero?
?*?	?**?	?+?
?-?	-?	?/?
?=?	?^?	?~=?
?<?	?<=?	?>?
?>=?	?.?	?quo?
?rem?		

— domain PADICRAT PAdicRational —

```
)abbrev domain PADICRAT PAdicRational
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Keywords: p-adic, complementation
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ Stream-based implementation of Qp: numbers are represented as
++ sum(i = k.., a[i] * p^i) where the a[i] lie in 0,1,...,(p - 1).

PAdicRational(p:Integer) == PAdicRationalConstructor(p,PAdicInteger p)
```

———-

— PADICRAT.dotabb —

```
"PADICRAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PADICRAT"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"PADICRAT" -> "PFECAT"
```

———-

17.3 domain PADICRC PAdicRationalConstructor

— PAdicRationalConstructor.input —

```
)set break resume
)sys rm -f PAdicRationalConstructor.output
)spool PAdicRationalConstructor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicRationalConstructor
--R PAdicRationalConstructor(p: Integer,PADIC: PAdicIntegerCategory p)  is a domain constructor
--R Abbreviation for PAdicRationalConstructor is PADICRC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PADICRC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,PADIC) -> %                  ?*? : (PADIC,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (PADIC,PADIC) -> %              ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : (%,(PADIC -> PADIC)) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : PADIC -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R denom : % -> PADIC                    denominator : % -> %
```

```
--R factor : % -> Factored %             gcd : List % -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R map : ((PADIC -> PADIC),%) -> %      numer : % -> PADIC
--R numerator : % -> %                   one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R removeZeroes : (Integer,%) -> %      removeZeroes : % -> %
--R retract : % -> PADIC                 sample : () -> %
--R sizeLess? : (%,%) -> Boolean         squareFree : % -> Factored %
--R squareFreePart : % -> %              unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?<? : (%,%) -> Boolean if PADIC has ORDSET
--R ?<=? : (%,%) -> Boolean if PADIC has ORDSET
--R ?>? : (%,%) -> Boolean if PADIC has ORDSET
--R ?>=? : (%,%) -> Boolean if PADIC has ORDSET
--R D : (%,(PADIC -> PADIC),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R D : (%,List Symbol) -> % if PADIC has PDRING SYMBOL
--R D : (%,Symbol) -> % if PADIC has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if PADIC has DIFRING
--R D : % -> % if PADIC has DIFRING
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if PADIC has OINTDOM
--R approximate : (%,Integer) -> Fraction Integer
--R ceiling : % -> PADIC if PADIC has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and PADIC has PFECAT or PADIC has CHA
--R coerce : Symbol -> % if PADIC has RETRACT SYMBOL
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and PADIC has PFECAT
--R continuedFraction : % -> ContinuedFraction Fraction Integer
--R convert : % -> DoubleFloat if PADIC has REAL
--R convert : % -> Float if PADIC has REAL
--R convert : % -> InputForm if PADIC has KONVERT INFORM
--R convert : % -> Pattern Float if PADIC has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if PADIC has KONVERT PATTERN INT
--R differentiate : (%,(PADIC -> PADIC)) -> %
--R differentiate : (%,(PADIC -> PADIC),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if PADIC has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if PADIC has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if PADIC has DIFRING
--R differentiate : % -> % if PADIC has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
```

```
--R ?.? : (%,PADIC) -> % if PADIC has ELTAB(PADIC,PADIC)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,PADIC) -> % if PADIC has IEVALAB(SYMBOL,PADIC)
--R eval : (%,List Symbol,List PADIC) -> % if PADIC has IEVALAB(SYMBOL,PADIC)
--R eval : (%,List Equation PADIC) -> % if PADIC has EVALAB PADIC
--R eval : (%,Equation PADIC) -> % if PADIC has EVALAB PADIC
--R eval : (%,PADIC,PADIC) -> % if PADIC has EVALAB PADIC
--R eval : (%,List PADIC,List PADIC) -> % if PADIC has EVALAB PADIC
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if PADIC ha
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R floor : % -> PADIC if PADIC has INS
--R fractionPart : % -> % if PADIC has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R init : () -> % if PADIC has STEP
--R max : (%,%) -> % if PADIC has ORDSET
--R min : (%,%) -> % if PADIC has ORDSET
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if PADIC has OINTDOM
--R nextItem : % -> Union(%,"failed") if PADIC has STEP
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if PADIC
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R positive? : % -> Boolean if PADIC has OINTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PADIC has INS
--R reducedSystem : Matrix % -> Matrix PADIC
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix PADIC,vec: Vector PADIC)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if PADIC has
--R reducedSystem : Matrix % -> Matrix Integer if PADIC has LINEXP INT
--R retract : % -> Integer if PADIC has RETRACT INT
--R retract : % -> Fraction Integer if PADIC has RETRACT INT
--R retract : % -> Symbol if PADIC has RETRACT SYMBOL
--R retractIfCan : % -> Union(Integer,"failed") if PADIC has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if PADIC has RETRACT INT
--R retractIfCan : % -> Union(Symbol,"failed") if PADIC has RETRACT SYMBOL
--R retractIfCan : % -> Union(PADIC,"failed")
--R sign : % -> Integer if PADIC has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if PADI
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R wholePart : % -> PADIC if PADIC has EUCDOM
--R
--E 1

)spool
)lisp (bye)
```

———-

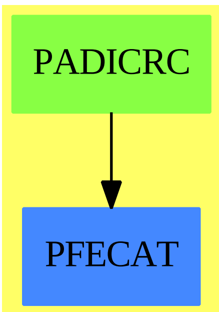
— **PAdicRationalConstructor.help** —

```
====================================================================
PAdicRationalConstructor examples
====================================================================

See Also:
o )show PAdicRationalConstructor
```

———-

17.3.1 PAdicRationalConstructor (PADICRC)



See

⇒ "InnerPAdicInteger" (IPADIC) 10.24.1 on page 1258
⇒ "PAdicInteger" (PADIC) 17.1.1 on page 1841
⇒ "BalancedPAdicInteger" (BPADIC) 3.2.1 on page 240
⇒ "PAdicRational" (PADICRAT) 17.2.1 on page 1845
⇒ "BalancedPAdicRational" (BPADICRT) 3.3.1 on page 244

Exports:

0	1	abs
approximate	associates?	ceiling
characteristic	charthRoot	coerce
conditionP	continuedFraction	convert
D	denom	denominator
differentiate	divide	euclideanSize
eval	expressIdealMember	exquo
extendedEuclidean	factor	factorPolynomial
factorSquareFreePolynomial	floor	fractionPart
gcd	gcdPolynomial	gcd
hash	init	inv
latex	lcm	map
max	min	multiEuclidean
negative?	nextItem	numer
numerator	one?	patternMatch
positive?	prime?	principalIdeal
random	recip	reducedSystem
removeZeroes	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
zero?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?.?
?quo?	?rem?	

— domain PADICRC PAdicRationalConstructor —

```
)abbrev domain PADICRC PAdicRationalConstructor
++ Author: Clifton J. Williamson
++ Date Created: 10 May 1990
++ Date Last Updated: 10 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ This is the category of stream-based representations of Qp.

PAdicRationalConstructor(p,PADIC): Exports == Implementation where
  p     :   Integer
```

```
  PADIC :   PAdicIntegerCategory p
  CF    ==> ContinuedFraction
  I     ==> Integer
  NNI   ==> NonNegativeInteger
  OUT   ==> OutputForm
  L     ==> List
  RN    ==> Fraction Integer
  ST    ==> Stream

  Exports ==> QuotientFieldCategory(PADIC) with
    approximate: (%,I) -> RN
      ++ \spad{approximate(x,n)} returns a rational number y such that
      ++ \spad{y = x (mod p^n)}.
    continuedFraction: % -> CF RN
      ++ \spad{continuedFraction(x)} converts the p-adic rational number x
      ++ to a continued fraction.
    removeZeroes: % -> %
      ++ \spad{removeZeroes(x)} removes leading zeroes from the
      ++ representation of the p-adic rational \spad{x}.
      ++ A p-adic rational is represented by (1) an exponent and
      ++ (2) a p-adic integer which may have leading zero digits.
      ++ When the p-adic integer has a leading zero digit, a 'leading zero'
      ++ is removed from the p-adic rational as follows:
      ++ the number is rewritten by increasing the exponent by 1 and
      ++ dividing the p-adic integer by p.
      ++ Note: \spad{removeZeroes(f)} removes all leading zeroes from f.
    removeZeroes: (I,%) -> %
      ++ \spad{removeZeroes(n,x)} removes up to n leading zeroes from
      ++ the p-adic rational \spad{x}.

  Implementation ==> add

    PEXPR := p :: OUT

--% representation

    Rep := Record(expon:I,pint:PADIC)

    getExpon: % -> I
    getZp   : % -> PADIC
    makeQp  : (I,PADIC) -> %

    getExpon x    == x.expon
    getZp x       == x.pint
    makeQp(r,int) == [r,int]

--% creation

    0 == makeQp(0,0)
    1 == makeQp(0,1)
```

```
    coerce(x:I)     == x :: PADIC :: %
    coerce(r:RN)    == (numer(r) :: %)/(denom(r) :: %)
    coerce(x:PADIC) == makeQp(0,x)

--% normalizations

    removeZeroes x ==
      empty? digits(xx := getZp x) => 0
      zero? moduloP xx =>
        removeZeroes makeQp(getExpon x + 1,quotientByP xx)
      x

    removeZeroes(n,x) ==
      n <= 0 => x
      empty? digits(xx := getZp x) => 0
      zero? moduloP xx =>
        removeZeroes(n - 1,makeQp(getExpon x + 1,quotientByP xx))
      x

--% arithmetic

    x = y ==
      EQ(x,y)$Lisp => true
      n := getExpon(x) - getExpon(y)
      n >= 0 =>
        (p**(n :: NNI) * getZp(x)) = getZp(y)
      (p**((- n) :: NNI) * getZp(y)) = getZp(x)

    x + y ==
      n := getExpon(x) - getExpon(y)
      n >= 0 =>
        makeQp(getExpon y,getZp(y) + p**(n :: NNI) * getZp(x))
      makeQp(getExpon x,getZp(x) + p**((-n) :: NNI) * getZp(y))

    -x == makeQp(getExpon x,-getZp(x))

    x - y ==
      n := getExpon(x) - getExpon(y)
      n >= 0 =>
        makeQp(getExpon y,p**(n :: NNI) * getZp(x) - getZp(y))
      makeQp(getExpon x,getZp(x) - p**((-n) :: NNI) * getZp(y))

    n:I * x:% == makeQp(getExpon x,n * getZp x)
    x:% * y:% == makeQp(getExpon x + getExpon y,getZp x * getZp y)

    x:% ** n:I ==
      zero? n => 1
      positive? n => expt(x,n :: PositiveInteger)$RepeatedSquaring(%)
      inv expt(x,(-n) :: PositiveInteger)$RepeatedSquaring(%)
```

```
    recip x ==
      x := removeZeroes(1000,x)
      zero? moduloP(xx := getZp x) => "failed"
      (inv := recip xx) case "failed" => "failed"
      makeQp(- getExpon x,inv :: PADIC)

    inv x ==
      (inv := recip x) case "failed" => error "inv: no inverse"
      inv :: %

    x:% / y:% == x * inv y
    x:PADIC / y:PADIC == (x :: %) / (y :: %)
    x:PADIC * y:% == makeQp(getExpon y,x * getZp y)

    approximate(x,n) ==
      k := getExpon x
      (p :: RN) ** k * approximate(getZp x,n - k)

    cfStream: % -> Stream RN
    cfStream x == delay
--    zero? x => empty()
      invx := inv x; x0 := approximate(invx,1)
      concat(x0,cfStream(invx - (x0 :: %)))

    continuedFraction x ==
      x0 := approximate(x,1)
      reducedContinuedFraction(x0,cfStream(x - (x0 :: %)))

    termOutput:(I,I) -> OUT
    termOutput(k,c) ==
      k = 0 => c :: OUT
      mon := (k = 1 => PEXPR; PEXPR ** (k :: OUT))
      c = 1 => mon
      c = -1 => -mon
      (c :: OUT) * mon

    showAll?:() -> Boolean
    -- check a global Lisp variable
    showAll?() == true

    coerce(x:%):OUT ==
      x := removeZeroes(_$streamCount$Lisp,x)
      m := getExpon x; zp := getZp x
      uu := digits zp
      l : L OUT := empty()
      empty? uu => 0 :: OUT
      n : NNI ; count : NNI := _$streamCount$Lisp
      for n in 0..count while not empty? uu repeat
        if frst(uu) ^= 0 then
```

```
        l := concat(termOutput((n :: I) + m,frst(uu)),l)
      uu := rst uu
    if showAll?() then
      for n in (count + 1).. while explicitEntries? uu and _
             not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput((n::I) + m,frst(uu)),l)
        uu := rst uu
    l :=
      explicitlyEmpty? uu => l
      eq?(uu,rst uu) and frst uu = 0 => l
      concat(prefix("O" :: OUT,[PEXPR ** ((n :: I) + m) :: OUT]),l)
    empty? l => 0 :: OUT
    reduce("+",reverse_! l)
```

— **PADICRC.dotabb** —

```
"PADICRC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PADICRC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"PADICRC" -> "PFECAT"
```

17.4 domain PALETTE Palette

— **Palette.input** —

```
)set break resume
)sys rm -f Palette.output
)spool Palette.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Palette
--R Palette  is a domain constructor
--R Abbreviation for Palette is PALETTE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PALETTE
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?=? : (%,%) -> Boolean              bright : Color -> %
--R coerce : Color -> %                 coerce : % -> OutputForm
--R dark : Color -> %                   dim : Color -> %
--R hash : % -> SingleInteger           hue : % -> Color
--R latex : % -> String                 light : Color -> %
--R pastel : Color -> %                 shade : % -> Integer
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

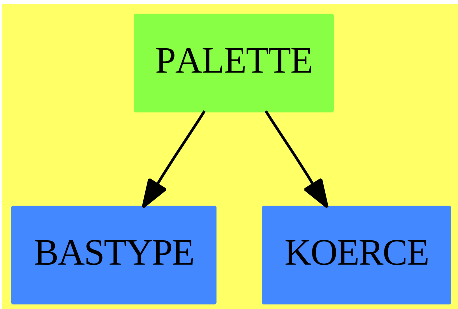
— Palette.help —

```
====================================================================
Palette examples
====================================================================

See Also:
o )show Palette
```

———-

17.4.1 Palette (PALETTE)



See

⇒ "Color" (COLOR) 4.6.1 on page 392

Exports:

bright	coerce	dark	dim	hash
hue	latex	light	pastel	shade
?~=?	?=?			

— domain PALETTE Palette —

```
)abbrev domain PALETTE Palette
++ Author: Jim Wen
++ Date Created: May 10th 1989
++ Date Last Updated: Jan 19th 1990
++ Basic Operations: dark, dim, bright, pastel, light, hue, shade, coerce
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: dim,bright,pastel,coerce
++ References:
++ Description:
++ This domain describes four groups of color shades (palettes).

Palette(): Exports == Implementation where
  I      ==> Integer
  C      ==> Color
  SHADE  ==> ["Dark","Dim","Bright","Pastel","Light"]

  Exports ==> SetCategory with
    dark   : C  -> %
      ++ dark(c) sets the shade of the indicated hue of c to it's lowest value.
    dim    : C  -> %
      ++ dim(c) sets the shade of a hue, c,  above dark, but below bright.
    bright : C  -> %
      ++ bright(c) sets the shade of a hue, c, above dim, but below pastel.
    pastel : C  -> %
      ++ pastel(c) sets the shade of a hue, c,  above bright, but below light.
    light  : C  -> %
      ++ light(c) sets the shade of a hue, c,  to it's highest value.
    hue    : %  -> C
      ++ hue(p) returns the hue field of the indicated palette p.
    shade  : %  -> I
      ++ shade(p) returns the shade index of the indicated palette p.
    coerce : C  -> %
      ++ coerce(c) sets the average shade for the palette to that of the
      ++ indicated color c.

  Implementation ==> add
    Rep := Record(shadeField:I, hueField:C)

    dark   c == [1,c]
    dim    c == [2,c]
    bright c == [3,c]
    pastel c == [4,c]
    light  c == [5,c]
    hue    p == p.hueField
    shade  p == p.shadeField
```

```
    sample() == bright(sample())
    coerce(c:Color):% == bright c
    coerce(p:%):OutputForm ==
      hconcat ["[",coerce(p.hueField),"] from the ",_
               SHADE.(p.shadeField)," palette"]
```

———-

— PALETTE.dotabb —

```
"PALETTE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PALETTE"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"PALETTE" -> "BASTYPE"
"PALETTE" -> "KOERCE"
```

———-

17.5 domain PARPCURV ParametricPlaneCurve

— ParametricPlaneCurve.input —

```
)set break resume
)sys rm -f ParametricPlaneCurve.output
)spool ParametricPlaneCurve.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ParametricPlaneCurve
--R ParametricPlaneCurve ComponentFunction: Type  is a domain constructor
--R Abbreviation for ParametricPlaneCurve is PARPCURV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PARPCURV
--R
--R------------------------------- Operations --------------------------------
--R coordinate : (%,NonNegativeInteger) -> ComponentFunction
--R curve : (ComponentFunction,ComponentFunction) -> %
--R
--E 1

)spool
)lisp (bye)
```

———–

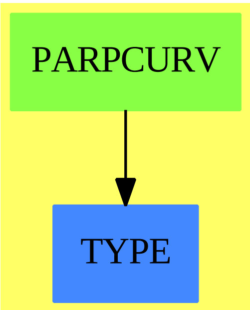
— **ParametricPlaneCurve.help** —

```
====================================================================
ParametricPlaneCurve examples
====================================================================

See Also:
o )show ParametricPlaneCurve
```

———–

17.5.1 ParametricPlaneCurve (PARPCURV)



See

⇒ "ParametricSpaceCurve" (PARSCURV) 17.6.1 on page 1861
⇒ "ParametricSurface" (PARSURF) 17.7.1 on page 1864

Exports:
coordinate curve

— **domain PARPCURV ParametricPlaneCurve** —

```
)abbrev domain PARPCURV ParametricPlaneCurve
++ Author: Clifton J. Williamson
++ Date Created: 24 May 1990
++ Date Last Updated: 24 May 1990
++ Basic Operations: curve, coordinate
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: parametric curve, graphics
++ References:
```

```
++ Description:
++ ParametricPlaneCurve is used for plotting parametric plane
++ curves in the affine plane.

ParametricPlaneCurve(ComponentFunction): Exports == Implementation where
  ComponentFunction : Type
  NNI             ==> NonNegativeInteger

  Exports ==> with
    curve: (ComponentFunction,ComponentFunction) -> %
      ++ curve(c1,c2) creates a plane curve from 2 component functions \spad{c1}
      ++ and \spad{c2}.
    coordinate: (%,NNI) -> ComponentFunction
      ++ coordinate(c,i) returns a coordinate function for c using 1-based
      ++ indexing according to i. This indicates what the function for the
      ++ coordinate component i of the plane curve is.

  Implementation ==> add

    Rep := Record(xCoord:ComponentFunction,yCoord:ComponentFunction)

    curve(x,y) == [x,y]
    coordinate(c,n) ==
      n = 1 => c.xCoord
      n = 2 => c.yCoord
      error "coordinate: index out of bounds"
```

— **PARPCURV.dotabb** —

```
"PARPCURV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PARPCURV"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"PARPCURV" -> "TYPE"
```

17.6 domain PARSCURV ParametricSpaceCurve

— **ParametricSpaceCurve.input** —

```
)set break resume
)sys rm -f ParametricSpaceCurve.output
)spool ParametricSpaceCurve.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ParametricSpaceCurve
--R ParametricSpaceCurve ComponentFunction: Type  is a domain constructor
--R Abbreviation for ParametricSpaceCurve is PARSCURV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PARSCURV
--R
--R------------------------------- Operations --------------------------------
--R coordinate : (%,NonNegativeInteger) -> ComponentFunction
--R curve : (ComponentFunction,ComponentFunction,ComponentFunction) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

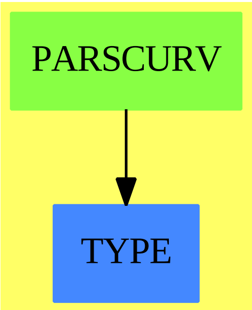
— ParametricSpaceCurve.help —

```
====================================================================
ParametricSpaceCurve examples
====================================================================

See Also:
o )show ParametricSpaceCurve
```

———-

17.6.1 ParametricSpaceCurve (PARSCURV)



See

⇒ "ParametricPlaneCurve" (PARPCURV) 17.5.1 on page 1859
⇒ "ParametricSurface" (PARSURF) 17.7.1 on page 1864

Exports:
coordinate curve

— domain PARSCURV ParametricSpaceCurve —

```
)abbrev domain PARSCURV ParametricSpaceCurve
++ Author: Clifton J. Williamson
++ Date Created: 24 May 1990
++ Date Last Updated: 24 May 1990
++ Basic Operations: curve, coordinate
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: parametric curve, graphics
++ References:
++ Description:
++ ParametricSpaceCurve is used for plotting parametric space
++ curves in affine 3-space.

ParametricSpaceCurve(ComponentFunction): Exports == Implementation where
  ComponentFunction : Type
  NNI             ==> NonNegativeInteger

  Exports ==> with
    curve: (ComponentFunction,ComponentFunction,ComponentFunction) -> %
      ++ curve(c1,c2,c3) creates a space curve from 3 component functions
      ++ \spad{c1}, \spad{c2}, and \spad{c3}.
    coordinate: (%,NNI) -> ComponentFunction
      ++ coordinate(c,i) returns a coordinate function of c using 1-based
      ++ indexing according to i. This indicates what the function for the
      ++ coordinate component, i, of the space curve is.

  Implementation ==> add

    Rep := Record(xCoord:ComponentFunction,_
                  yCoord:ComponentFunction,_
                  zCoord:ComponentFunction)

    curve(x,y,z) == [x,y,z]
    coordinate(c,n) ==
      n = 1 => c.xCoord
      n = 2 => c.yCoord
      n = 3 => c.zCoord
      error "coordinate: index out of bounds"
```

———-

— PARSCURV.dotabb —

```
"PARSCURV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PARSCURV"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"PARSCURV" -> "TYPE"
```

———-

17.7 domain PARSURF ParametricSurface

— ParametricSurface.input —

```
)set break resume
)sys rm -f ParametricSurface.output
)spool ParametricSurface.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ParametricSurface
--R ParametricSurface ComponentFunction: Type  is a domain constructor
--R Abbreviation for ParametricSurface is PARSURF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PARSURF
--R
--R------------------------------- Operations --------------------------------
--R coordinate : (%,NonNegativeInteger) -> ComponentFunction
--R surface : (ComponentFunction,ComponentFunction,ComponentFunction) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

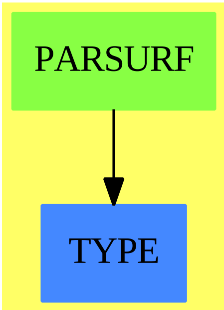
— ParametricSurface.help —

```
====================================================================
ParametricSurface examples
====================================================================
```

```
See Also:
o )show ParametricSurface
```

———-

17.7.1 ParametricSurface (PARSURF)



See

⇒ "ParametricPlaneCurve" (PARPCURV) 17.5.1 on page 1859
⇒ "ParametricSpaceCurve" (PARSCURV) 17.6.1 on page 1861

Exports:
coordinate surface

— **domain PARSURF ParametricSurface** —

```
)abbrev domain PARSURF ParametricSurface
++ Author: Clifton J. Williamson
++ Date Created: 24 May 1990
++ Date Last Updated: 24 May 1990
++ Basic Operations: surface, coordinate
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: parametric surface, graphics
++ References:
++ Description:
++ ParametricSurface is used for plotting parametric surfaces in
++ affine 3-space.

ParametricSurface(ComponentFunction): Exports == Implementation where
  ComponentFunction : Type
  NNI             ==> NonNegativeInteger
```

```
  Exports ==> with
    surface: (ComponentFunction,ComponentFunction,ComponentFunction) -> %
      ++ surface(c1,c2,c3) creates a surface from 3 parametric component
      ++ functions \spad{c1}, \spad{c2}, and \spad{c3}.
    coordinate: (%,NNI) -> ComponentFunction
      ++ coordinate(s,i) returns a coordinate function of s using 1-based
      ++ indexing according to i.  This indicates what the function for the
      ++ coordinate component, i, of the surface is.

  Implementation ==> add

    Rep := Record(xCoord:ComponentFunction,_
                  yCoord:ComponentFunction,_
                  zCoord:ComponentFunction)

    surface(x,y,z) == [x,y,z]
    coordinate(c,n) ==
      n = 1 => c.xCoord
      n = 2 => c.yCoord
      n = 3 => c.zCoord
      error "coordinate: index out of bounds"
```

———

— PARSURF.dotabb —

```
"PARSURF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PARSURF"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"PARSURF" -> "TYPE"
```

———

17.8 domain PFR PartialFraction

— PartialFraction.input —

```
)set break resume
)sys rm -f PartialFraction.output
)spool PartialFraction.output
)set message test on
)set message auto off
)clear all

--S 1 of 22
```

```
partialFraction(1,factorial 10)
--R
--R
--R        159   23   12   1
--R   (1)  --- - -- - -- + -
--R          8    4    2   7
--R         2    3    5
--R                                                  Type: PartialFraction Integer
--E 1

--S 2 of 22
f := padicFraction(%)
--R
--R
--R        1    1    1    1    1    1    2    1    2   2    2   1
--R   (2)  - + -- + -- + -- + -- + -- - -- - -- - -- - - - -- + -
--R        2    4    5    6    7    8    2    3    4   5    2   7
--R            2    2    2    2    2    3    3    3        5
--R                                                  Type: PartialFraction Integer
--E 2

--S 3 of 22
compactFraction(f)
--R
--R
--R        159   23   12   1
--R   (3)  --- - -- - -- + -
--R          8    4    2   7
--R         2    3    5
--R                                                  Type: PartialFraction Integer
--E 3

--S 4 of 22
numberOfFractionalTerms(f)
--R
--R
--R   (4)  12
--R                                                        Type: PositiveInteger
--E 4

--S 5 of 22
nthFractionalTerm(f,3)
--R
--R
--R         1
--R   (5)  --
--R         5
--R        2
--R                                                  Type: PartialFraction Integer
--E 5
```

```
--S 6 of 22
partialFraction(1,- 13 + 14 * %i)
--R
--R
--R             1         4
--R   (6)  - ------- + -------
--R          1 + 2%i   3 + 8%i
--R                                           Type: PartialFraction Complex Integer
--E 6

--S 7 of 22
% :: Fraction Complex Integer
--R
--R
--R              %i
--R   (7)  - ---------
--R          14 + 13%i
--R                                                  Type: Fraction Complex Integer
--E 7

--S 8 of 22
u : FR UP(x, FRAC INT) := reduce(*,[primeFactor(x+i,i) for i in 1..4])
--R
--R
--R                        2       3       4
--R   (8)  (x + 1)(x + 2) (x + 3) (x + 4)
--R                        Type: Factored UnivariatePolynomial(x,Fraction Integer)
--E 8

--S 9 of 22
partialFraction(1,u)
--R
--R
--R   (9)
--R     1     1      7     17  2         139   607  3   10115  2   391     44179
--R    ---    - x + --   - -- x  - 12x - ---   --- x  + ----- x  + --- x + -----
--R    648    4     16      8             8    324       432        4       324
--R   ----- + -------- + ------------------- + --------------------------------
--R   x + 1          2                  3                              4
--R           (x + 2)          (x + 3)                          (x + 4)
--R                 Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)
--E 9

--S 10 of 22
padicFraction %
--R
--R
--R   (10)
--R       1       1         1         17         3          1        607       403
```

```
--R      ---      -        --        --        -          -       ---       ---
--R      648      4        16         8        4          2       324       432
--R     ----- + ----- - -------- - ----- + -------- - -------- + ----- + --------
--R     x + 1   x + 2          2   x + 3          2          3   x + 4          2
--R                     (x + 2)            (x + 3)    (x + 3)            (x + 4)
--R   +
--R       13          1
--R       --         --
--R       36         12
--R     -------- + --------
--R            3          4
--R     (x + 4)    (x + 4)
--R              Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)
--E 10

--S 11 of 22
fraction:=Fraction(Polynomial(Integer))
--R
--R
--R   (11)  Fraction Polynomial Integer
--R                                                                  Type: Domain
--E 11

--S 12 of 22
up:=UnivariatePolynomial(y,fraction)
--R
--R
--R   (12)  UnivariatePolynomial(y,Fraction Polynomial Integer)
--R                                                                  Type: Domain
--E 12

--S 13 of 22
pfup:=PartialFraction(up)
--R
--R
--R   (13)  PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--R                                                                  Type: Domain
--E 13

--S 14 of 22
a:=x+1/(y+1)
--R
--R
--R         x y + x + 1
--R   (14)  -----------
--R            y + 1
--R                                              Type: Fraction Polynomial Integer
--E 14

--S 15 of 22
```

```
b:=partialFraction(a,y)$PartialFractionPackage(Integer)
--R
--R
--R               1
--R   (15)  x + -----
--R             y + 1
--R    Type: PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 15

--S 16 of 22
c:=b::pfup
--R
--R
--R               1
--R   (16)  x + -----
--R             y + 1
--R    Type: PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 16

--S 17 of 22
cw:=(wholePart c)::Expression(Integer)
--R
--R
--R   (17)  x
--R                                                     Type: Expression Integer
--E 17

--S 18 of 22
m:=numberOfFractionalTerms(c)
--R
--R
--R   (18)  1
--R                                                        Type: PositiveInteger
--E 18

--S 19 of 22
crList:=[nthFractionalTerm(c,i) for i in 1..m]
--R
--R
--R            1
--R   (19)  [-----]
--R          y + 1
--RType: List PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 19

--S 20 of 22
cc:=reduce(+,crList)
--R
--R
--R           1
```

```
--R   (20)  -----
--R         y + 1
--R    Type: PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 20

--S 21 of 22
ccx:=cc::(Fraction(up))::(Expression(Integer))
--R
--R
--R            1
--R   (21)  -----
--R         y + 1
--R                                                     Type: Expression Integer
--E 21

--S 22 of 22
sin(cw)*cos(ccx)+sin(ccx)*cos(cw)
--R
--R
--R                1                        1
--R   (22)  cos(-----)sin(x) + cos(x)sin(-----)
--R             y + 1                    y + 1
--R                                                     Type: Expression Integer
--E 22

)spool
)lisp (bye)
```

———-

— PartialFraction.help —

```
====================================================================
PartialFraction examples
====================================================================

A partial fraction is a decomposition of a quotient into a sum of
quotients where the denominators of the summands are powers of
primes. Most people first encounter partial fractions when they are
learning integral calculus.  For a technical discussion of partial
fractions, see, for example, Lang's Algebra. For example, the rational
number 1/6 is decomposed into 1/2-1/3.  You can compute partial
fractions of quotients of objects from domains belonging to the
category EuclideanDomain.  For example, Integer, Complex Integer, and
UnivariatePolynomial(x, Fraction Integer) all belong to EuclideanDomain.
In the examples following, we demonstrate how to decompose quotients of
each of these kinds of object into partial fractions.

It is necessary that we know how to factor the denominator when we
```

want to compute a partial fraction. Although the interpreter can often do this automatically, it may be necessary for you to include a call to factor. In these examples, it is not necessary to factor the denominators explicitly.

The main operation for computing partial fractions is called partialFraction and we use this to compute a decomposition of 1/10!. The first argument to partialFraction is the numerator of the quotient and the second argument is the factored denominator.

```
  partialFraction(1,factorial 10)
    159   23   12   1
    --- - -- - -- + -
      8    4    2   7
     2    3    5
                           Type: PartialFraction Integer
```

Since the denominators are powers of primes, it may be possible to expand the numerators further with respect to those primes. Use the operation padicFraction to do this.

```
  f := padicFraction(%)
    1    1    1    1    1    1    2    1    2   2    2   1
    - + -- + -- + -- + -- + -- - -- - -- - -- - - - -- + -
    2    4    5    6    7    8    2    3    4   5    2   7
        2    2    2    2    2    3    3    3         5
                           Type: PartialFraction Integer
```

The operation compactFraction returns an expanded fraction into the usual form. The compacted version is used internally for computational efficiency.

```
  compactFraction(f)
    159   23   12   1
    --- - -- - -- + -
      8    4    2   7
     2    3    5
                           Type: PartialFraction Integer
```

You can add, subtract, multiply and divide partial fractions. In addition, you can extract the parts of the decomposition. numberOfFractionalTerms computes the number of terms in the fractional part. This does not include the whole part of the fraction, which you get by calling wholePart. In this example, the whole part is just 0.

```
  numberOfFractionalTerms(f)
    12
                           Type: PositiveInteger
```

The operation nthFractionalTerm returns the individual terms in the decomposition. Notice that the object returned is a partial fraction itself. firstNumer and firstDenom extract the numerator and denominator of the first term of the fraction.

```
  nthFractionalTerm(f,3)
      1
     --
      5
     2
                           Type: PartialFraction Integer
```

Given two gaussian integers, you can decompose their quotient into a partial fraction.

```
  partialFraction(1,- 13 + 14 * %i)
         1         4
    - ------- + -------
      1 + 2%i   3 + 8%i
                           Type: PartialFraction Complex Integer
```

To convert back to a quotient, simply use a conversion.

```
  % :: Fraction Complex Integer
          %i
    - ---------
      14 + 13%i
                           Type: Fraction Complex Integer
```

To conclude this section, we compute the decomposition of

```
                  1
     -------------------------------
                  2       3       4
     (x + 1)(x + 2) (x + 3) (x + 4)
```

The polynomials in this object have type UnivariatePolynomial(x, Fraction Integer).

We use the primeFactor operation to create the denominator in factored form directly.

```
  u : FR UP(x, FRAC INT) := reduce(*,[primeFactor(x+i,i) for i in 1..4])
                 2       3       4
    (x + 1)(x + 2) (x + 3) (x + 4)
                  Type: Factored UnivariatePolynomial(x,Fraction Integer)
```

These are the compact and expanded partial fractions for the quotient.

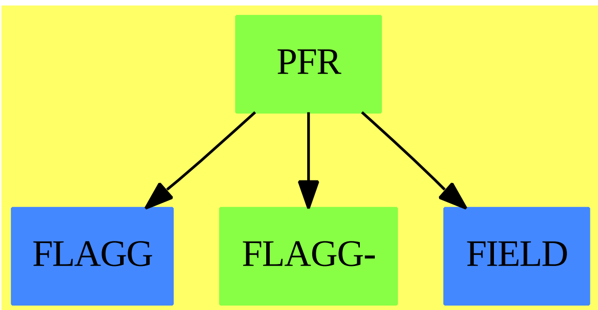
```
  partialFraction(1,u)
     1     1      7     17  2         139   607  3   10115  2   391     44179
```

```
    ---    - x + --   - -- x  - 12x - ---   --- x  + ----- x  + --- x + -----
    648    4     16      8             8   324       432        4       324
   ----- + -------- + ------------------- + ---------------------------------
   x + 1          2                3                            4
           (x + 2)          (x + 3)                      (x + 4)
               Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)

  padicFraction %
       1      1         1       17        3          1       607       403
      ---     -        --       --        -          -       ---       ---
      648     4        16        8        4          2       324       432
     ----- + ----- - -------- - ----- + -------- - -------- + ----- + --------
     x + 1   x + 2          2   x + 3          2          3   x + 4          2
                     (x + 2)            (x + 3)    (x + 3)            (x + 4)
   +
        13          1
        --         --
        36         12
     -------- + --------
            3          4
     (x + 4)    (x + 4)
               Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)
```

```
See Also:
o )help Factored
o )help Complex
o )help FullPartialFractionExpansionXmpPage
o )show PartialFraction
```

17.8.1 PartialFraction (PFR)



Exports:

0	1	associates?	characteristic
coerce	compactFraction	divide	euclideanSize
expressIdealMember	exquo	extendedEuclidean	extendedEuclidean
factor	firstDenom	firstNumer	gcd
gcdPolynomial	hash	inv	latex
lcm	multiEuclidean	nthFractionalTerm	numberOfFractionalTerms
one?	padicallyExpand	padicFraction	partialFraction
prime?	principalIdeal	recip	sample
sizeLess?	squareFree	squareFreePart	subtractIfCan
unit?	unitCanonical	unitNormal	wholePart
zero?	?*?	?**?	?+?
?-?	-?	?/?	?=?
?^?	?~=?	?quo?	?rem?

— domain PFR PartialFraction —

```
)abbrev domain PFR PartialFraction
++ Author: Robert S. Sutor
++ Date Created: 1986
++ Change History:
++   05/20/91 BMT Converted to the new library
++ Basic Operations: (Field), (Algebra),
++   coerce, compactFraction, firstDenom, firstNumer,
++   nthFractionalTerm, numberOfFractionalTerms, padicallyExpand,
++   padicFraction, partialFraction, wholePart
++ Related Constructors:
++ Also See: ContinuedFraction
++ AMS Classifications:
++ Keywords: partial fraction, factorization, euclidean domain
++ References:
++ Description:
++ The domain \spadtype{PartialFraction} implements partial fractions
++ over a euclidean domain \spad{R}.  This requirement on the
++ argument domain allows us to normalize the fractions.  Of
++ particular interest are the 2 forms for these fractions.  The
++ ``compact'' form has only one fractional term per prime in the
++ denominator, while the ``p-adic'' form expands each numerator
++ p-adically via the prime p in the denominator.  For computational
++ efficiency, the compact form is used, though the p-adic form may
++ be gotten by calling the function padicFraction}.  For a
++ general euclidean domain, it is not known how to factor the
++ denominator.  Thus the function partialFraction takes as its
++ second argument an element of \spadtype{Factored(R)}.

PartialFraction(R: EuclideanDomain): Cat == Capsule where
  FRR  ==> Factored R
  SUPR ==> SparseUnivariatePolynomial R

  Cat == Join(Field, Algebra R) with
```

```
coerce: % -> Fraction R
  ++ coerce(p) sums up the components of the partial fraction and
  ++ returns a single fraction.
  ++
  ++X a:=(13/74)::PFR(INT)
  ++X a::FRAC(INT)

coerce:  Fraction FRR -> %
  ++ coerce(f) takes a fraction with numerator and denominator in
  ++ factored form and creates a partial fraction.  It is
  ++ necessary for the parts to be factored because it is not
  ++ known in general how to factor elements of \spad{R} and
  ++ this is needed to decompose into partial fractions.
  ++
  ++X (13/74)::PFR(INT)

compactFraction: % -> %
  ++ compactFraction(p) normalizes the partial fraction \spad{p}
  ++ to the compact representation. In this form, the partial
  ++ fraction has only one fractional term per prime in the
  ++ denominator.
  ++
  ++X a:=partialFraction(1,factorial 10)
  ++X b:=padicFraction(a)
  ++X compactFraction(b)

firstDenom: % -> FRR
  ++ firstDenom(p) extracts the denominator of the first fractional
  ++ term. This returns 1 if there is no fractional part (use
  ++ wholePart from PartialFraction to get the whole part).
  ++
  ++X a:=partialFraction(1,factorial 10)
  ++X firstDenom(a)

firstNumer:  % -> R
  ++ firstNumer(p) extracts the numerator of the first fractional
  ++ term. This returns 0 if there is no fractional part (use
  ++ wholePart from PartialFraction to get the whole part).
  ++
  ++X a:=partialFraction(1,factorial 10)
  ++X firstNumer(a)

nthFractionalTerm:  (%,Integer) -> %
  ++ nthFractionalTerm(p,n) extracts the nth fractional term from
  ++ the partial fraction \spad{p}.  This returns 0 if the index
  ++ \spad{n} is out of range.
  ++
  ++X a:=partialFraction(1,factorial 10)
  ++X b:=padicFraction(a)
  ++X nthFractionalTerm(b,3)
```

```
    numberOfFractionalTerms: % -> Integer
      ++ numberOfFractionalTerms(p) computes the number of fractional
      ++ terms in \spad{p}. This returns 0 if there is no fractional
      ++ part.
      ++
      ++X a:=partialFraction(1,factorial 10)
      ++X b:=padicFraction(a)
      ++X numberOfFractionalTerms(b)

    padicallyExpand: (R,R) -> SUPR
      ++ padicallyExpand(p,x) is a utility function that expands
      ++ the second argument \spad{x} ‘‘p-adically’’ in
      ++ the first.

    padicFraction: % -> %
      ++ padicFraction(q) expands the fraction p-adically in the primes
      ++ \spad{p} in the denominator of \spad{q}. For example,
      ++ \spad{padicFraction(3/(2**2)) = 1/2 + 1/(2**2)}.
      ++ Use compactFraction from PartialFraction to
      ++ return to compact form.
      ++
      ++X a:=partialFraction(1,factorial 10)
      ++X padicFraction(a)

    partialFraction: (R, FRR) -> %
      ++ partialFraction(numer,denom) is the main function for
      ++ constructing partial fractions. The second argument is the
      ++ denominator and should be factored.
      ++
      ++X partialFraction(1,factorial 10)

    wholePart: % -> R
      ++ wholePart(p) extracts the whole part of the partial fraction
      ++ \spad{p}.
      ++
      ++X a:=(74/13)::PFR(INT)
      ++X wholePart(a)

  Capsule == add

    -- some constructor assignments and macros

    Ex     ==> OutputForm
    fTerm  ==> Record(num: R, den: FRR)          -- den should have
                                                 -- unit = 1 and only
                                                 -- 1 factor
    LfTerm ==> List Record(num: R, den: FRR)
    QR     ==> Record(quotient: R, remainder: R)
```

```
Rep    := Record(whole:R, fract: LfTerm)

-- private function signatures

copypf: % -> %
LessThan: (fTerm, fTerm) -> Boolean
multiplyFracTerms: (fTerm, fTerm) -> %
normalizeFracTerm: fTerm -> %
partialFractionNormalized: (R, FRR) -> %

-- declarations

a,b: %
n: Integer
r: R

-- private function definitions

copypf(a: %): % == [a.whole,copy a.fract]$%

LessThan(s: fTerm, t: fTerm) ==
  -- have to wait until FR has < operation
  if (GGREATERP(s.den,t.den)$Lisp : Boolean) then false
  else true

multiplyFracTerms(s : fTerm, t : fTerm) ==
  nthFactor(s.den,1) = nthFactor(t.den,1) =>
    normalizeFracTerm([s.num * t.num, s.den * t.den]$fTerm) : Rep
  i : Union(Record(coef1: R, coef2: R),"failed")
  coefs : Record(coef1: R, coef2: R)
  i := extendedEuclidean(expand t.den, expand s.den,s.num * t.num)
  i case "failed" => error "PartialFraction: not in ideal"
  coefs := (i :: Record(coef1: R, coef2: R))
  c : % := copypf 0$%
  d : %
  if coefs.coef2 ^= 0$R then
    c := normalizeFracTerm ([coefs.coef2, t.den]$fTerm)
  if coefs.coef1 ^= 0$R then
    d := normalizeFracTerm ([coefs.coef1, s.den]$fTerm)
    c.whole := c.whole + d.whole
    not (null d.fract) => c.fract := append(d.fract,c.fract)
  c

normalizeFracTerm(s : fTerm) ==
  -- makes sure num is "less than" den, whole may be non-zero
  qr : QR := divide(s.num, (expand s.den))
  qr.remainder = 0$R => [qr.quotient, nil()$LfTerm]
  -- now verify num and den are coprime
  f : R := nthFactor(s.den,1)
  nexpon : Integer := nthExponent(s.den,1)
```

```
    expon  : Integer := 0
    q : QR := divide(qr.remainder, f)
    while (q.remainder = 0$R) and (expon < nexpon) repeat
      expon := expon + 1
      qr.remainder := q.quotient
      q := divide(qr.remainder,f)
    expon = 0 => [qr.quotient,[[qr.remainder, s.den]$fTerm]$LfTerm]
    expon = nexpon => (qr.quotient + qr.remainder) :: %
    [qr.quotient,[[qr.remainder, nilFactor(f,nexpon-expon)]$fTerm]$LfTerm]

  partialFractionNormalized(nm: R, dn : FRR) ==
    -- assume unit dn = 1
    nm = 0$R   => 0$%
    dn = 1$FRR => nm :: %
    qr : QR := divide(nm, expand dn)
    c : % := [0$R,[[qr.remainder,
      nilFactor(nthFactor(dn,1), nthExponent(dn,1))]$fTerm]$LfTerm]
    d : %
    for i in 2..numberOfFactors(dn) repeat
      d :=
        [0$R,[[1$R,nilFactor(nthFactor(dn,i), nthExponent(dn,i))]$fTerm]$LfTerm]
      c := c * d
    (qr.quotient :: %) + c

  -- public function definitions

  padicFraction(a : %) ==
    b: % := compactFraction a
    null b.fract => b
    l : LfTerm := nil
    s : fTerm
    f : R
    e,d: Integer
    for s in b.fract repeat
      e := nthExponent(s.den,1)
      e = 1 => l := cons(s,l)
      f := nthFactor(s.den,1)
      d := degree(sp := padicallyExpand(f,s.num))
      while (sp ^= 0$SUPR) repeat
        l := cons([leadingCoefficient sp,nilFactor(f,e-d)]$fTerm, l)
        d := degree(sp := reductum sp)
    [b.whole, sort(LessThan,l)]$%

  compactFraction(a : %) ==
    -- only one power for each distinct denom will remain
    2 > # a.fract => a
    af : LfTerm := reverse a.fract
    bf : LfTerm := nil
    bw : R := a.whole
    b : %
```

```
    s : fTerm := [(first af).num,(first af).den]$fTerm
    f : R := nthFactor(s.den,1)
    e : Integer := nthExponent(s.den,1)
    t : fTerm
    for t in rest af repeat
      f = nthFactor(t.den,1) =>
        s.num := s.num + (t.num *
          (f **$R ((e - nthExponent(t.den,1)) : NonNegativeInteger)))
      b := normalizeFracTerm s
      bw := bw + b.whole
      if not (null b.fract) then bf := cons(first b.fract,bf)
      s := [t.num, t.den]$fTerm
      f := nthFactor(s.den,1)
      e := nthExponent(s.den,1)
    b := normalizeFracTerm s
    [bw + b.whole,append(b.fract,bf)]$%

  0 == [0$R, nil()$LfTerm]
  1 == [1$R, nil()$LfTerm]
  characteristic() == characteristic()$R

  coerce(r): % == [r, nil()$LfTerm]
  coerce(n): % == [(n :: R), nil()$LfTerm]
  coerce(a): Fraction R ==
    q : Fraction R := (a.whole :: Fraction R)
    s : fTerm
    for s in a.fract repeat
      q := q + (s.num / (expand s.den))
    q
  coerce(q: Fraction FRR): % ==
    u : R := (recip unit denom q):: R
    r1 : R := u * expand numer q
    partialFractionNormalized(r1, u * denom q)

  a exquo b ==
    b = 0$% => "failed"
    b = 1$% => a
    br : Fraction R := inv (b :: Fraction R)
    a * partialFraction(numer br,(denom br) :: FRR)
  recip a == (1$% exquo a)

  firstDenom a ==         -- denominator of 1st fractional term
    null a.fract => 1$FRR
    (first a.fract).den
  firstNumer a ==         -- numerator of 1st fractional term
    null a.fract => 0$R
    (first a.fract).num
  numberOfFractionalTerms a == # a.fract
  nthFractionalTerm(a,n) ==
    l : LfTerm := a.fract
```

```
    (n < 1) or (n > # l) => 0$%
    [0$R,[l.n]$LfTerm]$%
  wholePart a == a.whole

  partialFraction(nm: R, dn : FRR) ==
    nm = 0$R => 0$%
    -- move inv unit of den to numerator
    u : R := unit dn
    u := (recip u) :: R
    partialFractionNormalized(u * nm,u * dn)

  padicallyExpand(p : R, r : R) ==
    -- expands r as a sum of powers of p, with coefficients
    -- r = HornerEval(padicallyExpand(p,r),p)
    qr : QR := divide(r, p)
    qr.quotient = 0$R => qr.remainder :: SUPR
    (qr.remainder :: SUPR) + monomial(1$R,1$NonNegativeInteger)$SUPR *
      padicallyExpand(p,qr.quotient)

  a = b ==
    a.whole ^= b.whole => false  -- must verify this
    (null a.fract) =>
      null b.fract => a.whole = b.whole
      false
    null b.fract => false
    -- oh, no! following is temporary
    (a :: Fraction R) = (b :: Fraction R)

  - a ==
    s: fTerm
    l: LfTerm := nil
    for s in reverse a.fract repeat l := cons([- s.num,s.den]$fTerm,l)
    [- a.whole,l]

  r * a ==
    r = 0$R => 0$%
    r = 1$R => a
    b : % := (r * a.whole) :: %
    c : %
    s : fTerm
    for s in reverse a.fract repeat
      c := normalizeFracTerm [r * s.num, s.den]$fTerm
      b.whole := b.whole + c.whole
      not (null c.fract) => b.fract := append(c.fract, b.fract)
    b

  n * a == (n :: R) * a

  a + b ==
    compactFraction
```

```
        [a.whole + b.whole,
          sort(LessThan,append(a.fract,copy b.fract))]$%

    a * b ==
      null a.fract => a.whole * b
      null b.fract => b.whole * a
      af : % := [0$R, a.fract]$%   --     a - a.whole
      c: % := (a.whole * b) + (b.whole * af)
      s,t : fTerm
      for s in a.fract repeat
        for t in b.fract repeat
          c := c + multiplyFracTerms(s,t)
      c

    coerce(a): Ex ==
      null a.fract => a.whole :: Ex
      s : fTerm
      l : List Ex
      if a.whole = 0 then l := nil else l := [a.whole :: Ex]
      for s in a.fract repeat
        s.den = 1$FRR => l := cons(s.num :: Ex, l)
        l := cons(s.num :: Ex /  s.den :: Ex, l)
      # l = 1 => first l
      reduce("+", reverse l)
```

— **PFR.dotabb** —

```
"PFR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PFR"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"PFR" -> "FIELD"
"PFR" -> "FLAGG-"
"PFR" -> "FLAGG"
```

17.9 domain PRTITION Partition

— **Partition.input** —

```
)set break resume
```

```
)sys rm -f Partition.output
)spool Partition.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Partition
--R Partition  is a domain constructor
--R Abbreviation for Partition is PRTITION
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PRTITION
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              0 : () -> %
--R coerce : % -> List Integer           coerce : % -> OutputForm
--R conjugate : % -> %                   convert : % -> List Integer
--R hash : % -> SingleInteger            latex : % -> String
--R max : (%,%) -> %                     min : (%,%) -> %
--R partition : List Integer -> %        pdct : % -> Integer
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R powers : List Integer -> List List Integer
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

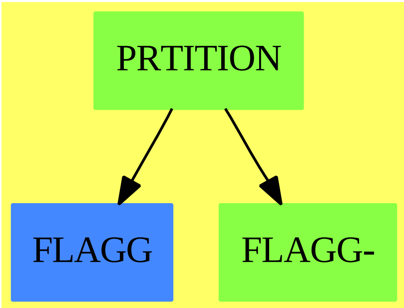
)spool
)lisp (bye)
```

— Partition.help —

```
====================================================================
Partition examples
====================================================================

See Also:
o )show Partition
```

17.9.1 Partition (PRTITION)



See

⇒ "SymmetricPolynomial" (SYMPOLY) 20.39.1 on page 2613

Exports:

0	coerce	conjugate	convert	hash
latex	max	min	partition	pdct
powers	sample	subtractIfCan	zero?	?~=?
?*?	?+?	?<?	?<=?	?=?
?>?	?>=?			

— domain PRTITION Partition —

```
)abbrev domain PRTITION Partition
++ Author: William H. Burge
++ Date Created: 29 October 1987
++ Date Last Updated: 23 Sept 1991
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Domain for partitions of positive integers
++ Partition is an OrderedCancellationAbelianMonoid which is used
++ as the basis for symmetric polynomial representation of the
++ sums of powers in SymmetricPolynomial.  Thus, \spad{(5 2 2 1)} will
++ represent \spad{s5 * s2**2 * s1}.

Partition: Exports == Implementation where
  L   ==> List
  I   ==> Integer
  OUT ==> OutputForm
  NNI ==> NonNegativeInteger
  UN  ==> Union(%,"failed")

  Exports ==> Join(OrderedCancellationAbelianMonoid,
                   ConvertibleTo List Integer) with
    partition: L I -> %
```

```
      ++ partition(li) converts a list of integers li to a partition
    powers: L I -> L L I
      ++ powers(li) returns a list of 2-element lists.  For each 2-element
      ++ list, the first element is an entry of li and the second
      ++ element is the multiplicity with which the first element
      ++ occurs in li.  There is a 2-element list for each value
      ++ occurring in l.
    pdct: % -> I
      ++ \spad{pdct(a1**n1 a2**n2 ...)} returns
      ++ \spad{n1! * a1**n1 * n2! * a2**n2 * ...}.
      ++ This function is used in the package \spadtype{CycleIndicators}.
    conjugate: % -> %
      ++ conjugate(p) returns the conjugate partition of a partition p
    coerce:% -> List Integer
      ++ coerce(p) coerces a partition into a list of integers

  Implementation ==> add

    import PartitionsAndPermutations

    Rep := List Integer
    0 == nil()

    coerce (s:%) == s pretend List Integer
    convert x == copy(x pretend L I)

    partition list == sort((i1:Integer,i2:Integer):Boolean +-> i2 < i1,list)

    x < y ==
      empty? x => not empty? y
      empty? y => false
      first x = first y => rest x < rest y
      first x < first y

    x = y ==
        EQUAL(x,y)$Lisp
--      empty? x => empty? y
--      empty? y => false
--      first x = first y => rest x = rest y
--      false

    x + y ==
      empty? x => y
      empty? y => x
      first x > first y => concat(first x,rest(x) + y)
      concat(first y,x + rest(y))
    n:NNI * x:% == (zero? n => 0; x + (subtractIfCan(n,1) :: NNI) * x)

    dp: (I,%) -> %
    dp(i,x) ==
```

```
    empty? x => 0
    first x = i => rest x
    concat(first x,dp(i,rest x))

  remv: (I,%) -> UN
  remv(i,x) == (member?(i,x) => dp(i,x); "failed")

  subtractIfCan(x, y) ==
    empty? x =>
      empty? y => 0
      "failed"
    empty? y => x
    (aa := remv(first y,x)) case "failed" => "failed"
    subtractIfCan((aa :: %), rest y)

  li1 : L I  --!! 'bite' won't compile without this
  bite: (I,L I) -> L I
  bite(i,li) ==
    empty? li => concat(0,nil())
    first li = i =>
      li1 := bite(i,rest li)
      concat(first(li1) + 1,rest li1)
    concat(0,li)

  li : L I  --!!  'powers' won't compile without this
  powers l ==
    empty? l => nil()
    li := bite(first l,rest l)
    concat([first l,first(li) + 1],powers(rest li))

  conjugate x == conjugate(x pretend Rep)$PartitionsAndPermutations

  mkterm: (I,I) -> OUT
  mkterm(i1,i2) ==
    i2 = 1 => (i1 :: OUT) ** (" " :: OUT)
    (i1 :: OUT) ** (i2 :: OUT)

  mkexp1: L L I -> L OUT
  mkexp1 lli ==
    empty? lli => nil()
    li := first lli
    empty?(rest lli) and second(li) = 1 =>
      concat(first(li) :: OUT,nil())
    concat(mkterm(first li,second li),mkexp1(rest lli))

  coerce(x:%):OUT ==
      empty? (x pretend Rep) => coerce(x pretend Rep)$Rep
      paren(reduce("*",mkexp1(powers(x pretend Rep))))

  pdct x ==
```

```
*/[factorial(second a) * (first(a) ** (second(a) pretend NNI))
        for a in powers(x pretend Rep)]
```

———-

— PRTITION.dotabb —

```
"PRTITION" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRTITION"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"PRTITION" -> "FLAGG-"
"PRTITION" -> "FLAGG"
```

———-

17.10 domain PATTERN Pattern

— Pattern.input —

```
)set break resume
)sys rm -f Pattern.output
)spool Pattern.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Pattern
--R Pattern R: SetCategory  is a domain constructor
--R Abbreviation for Pattern is PATTERN
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PATTERN
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,%) -> %
--R ?+? : (%,%) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           addBadValue : (%,Any) -> %
--R coerce : Symbol -> %                  coerce : R -> %
--R coerce : % -> OutputForm              constant? : % -> Boolean
--R convert : List % -> %                 copy : % -> %
--R depth : % -> NonNegativeInteger       generic? : % -> Boolean
--R getBadValues : % -> List Any          hasPredicate? : % -> Boolean
--R hasTopPredicate? : % -> Boolean       hash : % -> SingleInteger
```

```
--R inR? : % -> Boolean                    latex : % -> String
--R multiple? : % -> Boolean               optional? : % -> Boolean
--R predicates : % -> List Any             quoted? : % -> Boolean
--R resetBadValues : % -> %                retract : % -> Symbol
--R retract : % -> R                       symbol? : % -> Boolean
--R variables : % -> List %                ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R elt : (BasicOperator,List %) -> %
--R isExpt : % -> Union(Record(val: %,exponent: NonNegativeInteger),"failed")
--R isList : % -> Union(List %,"failed")
--R isOp : % -> Union(Record(op: BasicOperator,arg: List %),"failed")
--R isOp : (%,BasicOperator) -> Union(List %,"failed")
--R isPlus : % -> Union(List %,"failed")
--R isPower : % -> Union(Record(val: %,exponent: %),"failed")
--R isQuotient : % -> Union(Record(num: %,den: %),"failed")
--R isTimes : % -> Union(List %,"failed")
--R optpair : List % -> Union(List %,"failed")
--R patternVariable : (Symbol,Boolean,Boolean,Boolean) -> %
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(R,"failed")
--R setPredicates : (%,List Any) -> %
--R setTopPredicate : (%,List Symbol,Any) -> %
--R topPredicate : % -> Record(var: List Symbol,pred: Any)
--R withPredicates : (%,List Any) -> %
--R
--E 1

)spool
)lisp (bye)
```

———

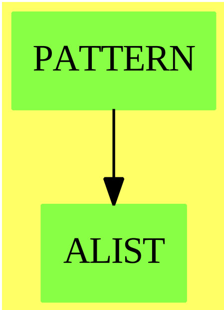
— Pattern.help —

```
====================================================================
Pattern examples
====================================================================

See Also:
o )show Pattern
```

———

17.10.1 Pattern (PATTERN)



Exports:

0	1	addBadValue	coerce	constant?
convert	copy	depth	elt	generic?
getBadValues	hasPredicate?	hasTopPredicate?	hash	inR?
isExpt	isList	isOp	isOp	isPlus
isPower	isQuotient	isTimes	latex	multiple?
optional?	optpair	patternVariable	predicates	quoted?
resetBadValues	retract	retractIfCan	setPredicates	setTopPredicate
symbol?	topPredicate	variables	withPredicates	?*?
?**?	?+?	?/?	?=?	?~=?

— domain PATTERN Pattern —

```
)abbrev domain PATTERN Pattern
++ Author: Manuel Bronstein
++ Date Created: 10 Nov 1988
++ Date Last Updated: 20 June 1991
++ Keywords: pattern, matching.
++ Description:
++ Patterns for use by the pattern matcher.
-- Not exposed.
-- Patterns are optimized for quick answers to structural questions.

Pattern(R:SetCategory): Exports == Implementation where
  B   ==> Boolean
  SI  ==> SingleInteger
  Z   ==> Integer
  SY  ==> Symbol
  O   ==> OutputForm
  BOP ==> BasicOperator
  QOT ==> Record(num:%, den:%)
  REC ==> Record(val:%, exponent:NonNegativeInteger)
  RSY ==> Record(tag:SI, val: SY, pred:List Any, bad:List Any)
  KER ==> Record(tag:SI, op:BOP, arg:List %)
  PAT ==> Union(ret:R, ker: KER, exp:REC, qot: QOT, sym:RSY)
```

```
-- the following MUST be the name of the formal exponentiation operator
  POWER ==> "%power"::Symbol

-- the 4 SYM_ constants must be disting powers of 2 (bitwise arithmetic)
  SYM_GENERIC  ==> 1::SI
  SYM_MULTIPLE ==> 2::SI
  SYM_OPTIONAL ==> 4::SI

  PAT_PLUS     ==> 1::SI
  PAT_TIMES    ==> 2::SI
  PAT_LIST     ==> 3::SI
  PAT_ZERO     ==> 4::SI
  PAT_ONE      ==> 5::SI
  PAT_EXPT     ==> 6::SI

  Exports ==> Join(SetCategory, RetractableTo R, RetractableTo SY) with
    0            : constant -> %              ++ 0
    1            : constant -> %              ++ 1
    isPlus       : %  -> Union(List %, "failed")
      ++ isPlus(p) returns \spad{[a1,...,an]} if \spad{n > 1}
      ++  and \spad{p = a1 + ... + an},
      ++ and "failed" otherwise.
    isTimes      : %  -> Union(List %, "failed")
      ++ isTimes(p) returns \spad{[a1,...,an]} if \spad{n > 1} and
      ++ \spad{p = a1 * ... * an}, and
      ++ "failed" otherwise.
    isOp         : (%, BOP) -> Union(List %, "failed")
      ++ isOp(p, op) returns \spad{[a1,...,an]} if \spad{p = op(a1,...,an)}, and
      ++ "failed" otherwise.
    isOp         : %  -> Union(Record(op:BOP, arg:List %), "failed")
      ++ isOp(p) returns \spad{[op, [a1,...,an]]} if
      ++ \spad{p = op(a1,...,an)}, and
      ++ "failed" otherwise;
    isExpt       : %  -> Union(REC, "failed")
      ++ isExpt(p) returns \spad{[q, n]} if \spad{n > 0} and \spad{p = q ** n},
      ++ and "failed" otherwise.
    isQuotient   : %  -> Union(QOT, "failed")
      ++ isQuotient(p) returns \spad{[a, b]} if \spad{p = a / b}, and
      ++ "failed" otherwise.
    isList       : %  -> Union(List %, "failed")
      ++ isList(p) returns \spad{[a1,...,an]} if \spad{p = [a1,...,an]},
      ++ "failed" otherwise;
    isPower      : %  -> Union(Record(val:%, exponent:%), "failed")
      ++ isPower(p) returns \spad{[a, b]} if \spad{p = a ** b}, and
      ++ "failed" otherwise.
    elt          : (BOP, List %) -> %
      ++ \spad{elt(op, [a1,...,an])} returns \spad{op(a1,...,an)}.
    "+"          : (%, %) -> %
      ++ \spad{a + b} returns the pattern \spad{a + b}.
```

```
"*"          : (%, %) -> %
  ++ \spad{a * b} returns the pattern \spad{a * b}.
"**"         : (%, NonNegativeInteger) -> %
  ++ \spad{a ** n} returns the pattern \spad{a ** n}.
"**"         : (%, %) -> %
  ++ \spad{a ** b} returns the pattern \spad{a ** b}.
"/"          : (%, %) -> %
  ++ \spad{a / b} returns the pattern \spad{a / b}.
depth        : % -> NonNegativeInteger
  ++ depth(p) returns the nesting level of p.
convert      : List % -> %
  ++ \spad{convert([a1,...,an])} returns the pattern \spad{[a1,...,an]}.
copy         : %  -> %
  ++ copy(p) returns a recursive copy of p.
inR?         : %  -> B
  ++ inR?(p) tests if p is an atom (i.e. an element of R).
quoted?      : %  -> B
  ++ quoted?(p) tests if p is of the form 's for a symbol s.
symbol?      : %  -> B
  ++ symbol?(p) tests if p is a symbol.
constant?    : %  -> B
  ++ constant?(p) tests if p contains no matching variables.
generic?     : %  -> B
  ++ generic?(p) tests if p is a single matching variable.
multiple?    : %  -> B
  ++ multiple?(p) tests if p is a single matching variable
  ++ allowing list matching or multiple term matching in a
  ++ sum or product.
optional?    : %  -> B
  ++ optional?(p) tests if p is a single matching variable
  ++ which can match an identity.
hasPredicate?: %  -> B
  ++ hasPredicate?(p) tests if p has predicates attached to it.
predicates   : %  -> List Any
  ++ predicates(p) returns \spad{[p1,...,pn]} such that the predicate
  ++ attached to p is p1 and ... and pn.
setPredicates: (%, List Any) -> %
  ++ \spad{setPredicates(p, [p1,...,pn])} attaches the predicate
  ++ p1 and ... and pn to p.
withPredicates:(%, List Any) -> %
  ++ \spad{withPredicates(p, [p1,...,pn])} makes a copy of p and attaches
  ++ the predicate p1 and ... and pn to the copy, which is
  ++ returned.
patternVariable: (SY, B, B, B) -> %
  ++ patternVariable(x, c?, o?, m?) creates a pattern variable x,
  ++ which is constant if \spad{c? = true}, optional if \spad{o? = true},
  ++ and multiple if \spad{m? = true}.
setTopPredicate: (%, List SY, Any) -> %
  ++ \spad{setTopPredicate(x, [a1,...,an], f)} returns x with
  ++ the top-level predicate set to \spad{f(a1,...,an)}.
```

```
    topPredicate: % -> Record(var:List SY, pred:Any)
      ++ topPredicate(x) returns \spad{[[a1,...,an], f]} where the top-level
      ++ predicate of x is \spad{f(a1,...,an)}.
      ++ Note: n is 0 if x has no top-level
      ++ predicate.
    hasTopPredicate?: % -> B
      ++ hasTopPredicate?(p) tests if p has a top-level predicate.
    resetBadValues: % -> %
      ++ resetBadValues(p) initializes the list of "bad values" for p
      ++ to \spad{[]}.
      ++ Note: p is not allowed to match any of its "bad values".
    addBadValue: (%, Any) -> %
      ++ addBadValue(p, v) adds v to the list of "bad values" for p.
      ++ Note: p is not allowed to match any of its "bad values".
    getBadValues: % -> List Any
      ++ getBadValues(p) returns the list of "bad values" for p.
      ++ Note: p is not allowed to match any of its "bad values".
    variables: % -> List %
      ++ variables(p) returns the list of matching variables
      ++ appearing in p.
    optpair: List % -> Union(List %, "failed")
      ++ optpair(l) returns l has the form \spad{[a, b]} and
      ++ a is optional, and
      ++ "failed" otherwise;

  Implementation ==> add
    Rep := Record(cons?: B, pat:PAT, lev: NonNegativeInteger,
                  topvar: List SY, toppred: Any)

    dummy:BOP := operator(new()$Symbol)
    nopred    := coerce(0$Integer)$AnyFunctions1(Integer)

    mkPat     : (B, PAT, NonNegativeInteger) -> %
    mkrsy     : (SY, B, B, B)  -> RSY
    SYM2O     : RSY -> O
    PAT2O     : PAT -> O
    patcopy   : PAT -> PAT
    bitSet?   : (SI ,  SI) -> B
    pateq?    : (PAT, PAT) -> B
    LPAT2O    : ((O, O) -> O, List %) -> O
    taggedElt : (SI, List %) -> %
    isTaggedOp: (%, SI) -> Union(List %, "failed")
    incmax    : List % -> NonNegativeInteger

    coerce(r:R):%   == mkPat(true, [r], 0)
    mkPat(c, p, l)  == [c, p, l, empty(), nopred]
    hasTopPredicate? x == not empty?(x.topvar)
    topPredicate x  == [x.topvar, x.toppred]
    setTopPredicate(x, l, f) == (x.topvar := l; x.toppred := f; x)
    constant? p     == p.cons?
```

```
depth p         == p.lev
inR? p          == p.pat case ret
symbol? p       == p.pat case sym
isPlus p        == isTaggedOp(p, PAT_PLUS)
isTimes p       == isTaggedOp(p, PAT_TIMES)
isList p        == isTaggedOp(p, PAT_LIST)
isExpt p        == (p.pat case exp => p.pat.exp; "failed")
isQuotient p    == (p.pat case qot => p.pat.qot; "failed")
hasPredicate? p == not empty? predicates p
quoted? p       == symbol? p and zero?(p.pat.sym.tag)
generic? p      == symbol? p and bitSet?(p.pat.sym.tag, SYM_GENERIC)
multiple? p     == symbol? p and bitSet?(p.pat.sym.tag,SYM_MULTIPLE)
optional? p     == symbol? p and bitSet?(p.pat.sym.tag,SYM_OPTIONAL)
bitSet?(a, b)   == And(a, b) ^= 0
coerce(p:%):O   == PAT2O(p.pat)
p1:% ** p2:%    == taggedElt(PAT_EXPT, [p1, p2])
LPAT2O(f, l)    == reduce(f, [x::O for x in l])$List(O)
retract(p:%):R  == (inR? p => p.pat.ret; error "Not retractable")
convert(l:List %):%                == taggedElt(PAT_LIST, l)
retractIfCan(p:%):Union(R,"failed") ==(inR? p => p.pat.ret;"failed")
withPredicates(p, l)               == setPredicates(copy p, l)
coerce(sy:SY):%          == patternVariable(sy, false, false, false)
copy p  == [constant? p, patcopy(p.pat), p.lev, p.topvar, p.toppred]

-- returns [a, b] if #l = 2 and optional? a, "failed" otherwise
optpair l ==
  empty? rest rest l =>
    b := first rest l
    optional?(a := first l) => l
    optional? b => reverse l
    "failed"
  "failed"

incmax l ==
  1 + reduce("max", [p.lev for p in l], 0)$List(NonNegativeInteger)

p1 = p2 ==
  (p1.cons? = p2.cons?) and (p1.lev = p2.lev) and
    (p1.topvar = p2.topvar) and
      ((EQ(p1.toppred, p2.toppred)$Lisp) pretend B) and
        pateq?(p1.pat, p2.pat)

isPower p ==
  (u := isTaggedOp(p, PAT_EXPT)) case "failed" => "failed"
  [first(u::List(%)), second(u::List(%))]

taggedElt(n, l) ==
  mkPat(every?(constant?, l), [[n, dummy, l]$KER], incmax l)

elt(o, l) ==
```

```
      is?(o, POWER) and #l = 2 => first(l) ** last(l)
      mkPat(every?(constant?, l), [[0, o, l]$KER], incmax l)

    isOp p ==
      (p.pat case ker) and zero?(p.pat.ker.tag) =>
        [p.pat.ker.op, p.pat.ker.arg]
      "failed"

    isTaggedOp(p,t) ==
      (p.pat case ker) and (p.pat.ker.tag = t) => p.pat.ker.arg
      "failed"

    if R has Monoid then
      1 == 1::R::%
    else
      1 == taggedElt(PAT_ONE,  empty())

    if R has AbelianMonoid then
      0 == 0::R::%
    else
      0 == taggedElt(PAT_ZERO, empty())

    p:% ** n:NonNegativeInteger ==
      p = 0 and n > 0 => 0
      p = 1 or zero? n => 1
--      one? n => p
      (n = 1) => p
      mkPat(constant? p, [[p, n]$REC], 1 + (p.lev))

    p1 / p2 ==
      p2 = 1 => p1
      mkPat(constant? p1 and constant? p2, [[p1, p2]$QOT],
                                  1 + max(p1.lev, p2.lev))

    p1 + p2 ==
      p1 = 0 => p2
      p2 = 0 => p1
      (u1 := isPlus p1) case List(%) =>
        (u2 := isPlus p2) case List(%) =>
          taggedElt(PAT_PLUS, concat(u1::List %, u2::List %))
        taggedElt(PAT_PLUS, concat(u1::List %, p2))
      (u2 := isPlus p2) case List(%) =>
        taggedElt(PAT_PLUS, concat(p1, u2::List %))
      taggedElt(PAT_PLUS, [p1, p2])

    p1 * p2 ==
      p1 = 0 or p2 = 0 => 0
      p1 = 1 => p2
      p2 = 1 => p1
      (u1 := isTimes p1) case List(%) =>
```

```
    (u2 := isTimes p2) case List(%) =>
      taggedElt(PAT_TIMES, concat(u1::List %, u2::List %))
    taggedElt(PAT_TIMES, concat(u1::List %, p2))
  (u2 := isTimes p2) case List(%) =>
    taggedElt(PAT_TIMES, concat(p1, u2::List %))
  taggedElt(PAT_TIMES, [p1, p2])

isOp(p, o) ==
  (p.pat case ker) and zero?(p.pat.ker.tag) and (p.pat.ker.op =o) =>
    p.pat.ker.arg
  "failed"

predicates p ==
  symbol? p => p.pat.sym.pred
  empty()

setPredicates(p, l) ==
  generic? p => (p.pat.sym.pred := l; p)
  error "Can only attach predicates to generic symbol"

resetBadValues p ==
  generic? p => (p.pat.sym.bad := empty()$List(Any); p)
  error "Can only attach bad values to generic symbol"

addBadValue(p, a) ==
  generic? p =>
    if not member?(a, p.pat.sym.bad) then
      p.pat.sym.bad := concat(a, p.pat.sym.bad)
    p
  error "Can only attach bad values to generic symbol"

getBadValues p ==
  generic? p => p.pat.sym.bad
  error "Not a generic symbol"

SYM2O p ==
  sy := (p.val)::O
  empty?(p.pred) => sy
  paren infix(" | "::O, sy,
    reduce("and",[sub("f"::O, i::O) for i in 1..#(p.pred)])$List(O))

variables p ==
  constant? p => empty()
  generic? p => [p]
  q := p.pat
  q case ret => empty()
  q case exp => variables(q.exp.val)
  q case qot => concat_!(variables(q.qot.num), variables(q.qot.den))
  q case ker => concat [variables r for r in q.ker.arg]
  empty()
```

```
PAT2O p ==
  p case ret => (p.ret)::O
  p case sym => SYM2O(p.sym)
  p case exp => (p.exp.val)::O ** (p.exp.exponent)::O
  p case qot => (p.qot.num)::O / (p.qot.den)::O
  p.ker.tag = PAT_PLUS  => LPAT2O("+", p.ker.arg)
  p.ker.tag = PAT_TIMES => LPAT2O("*", p.ker.arg)
  p.ker.tag = PAT_LIST => (p.ker.arg)::O
  p.ker.tag = PAT_ZERO => 0::Integer::O
  p.ker.tag = PAT_ONE  => 1::Integer::O
  l := [x::O for x in p.ker.arg]$List(O)
  (u:=display(p.ker.op)) case "failed" =>prefix(name(p.ker.op)::O,l)
  (u::(List O -> O)) l

patcopy p ==
  p case ret => [p.ret]
  p case sym =>
    [[p.sym.tag, p.sym.val, copy(p.sym.pred), copy(p.sym.bad)]$RSY]
  p case ker=>[[p.ker.tag,p.ker.op,[copy x for x in p.ker.arg]]$KER]
  p case qot => [[copy(p.qot.num), copy(p.qot.den)]$QOT]
  [[copy(p.exp.val), p.exp.exponent]$REC]

pateq?(p1, p2) ==
  p1 case ret => (p2 case ret) and (p1.ret = p2.ret)
  p1 case qot =>
    (p2 case qot) and (p1.qot.num = p2.qot.num)
                  and (p1.qot.den = p2.qot.den)
  p1 case sym =>
    (p2 case sym) and (p1.sym.val = p2.sym.val)
                  and {p1.sym.pred} =$Set(Any) {p2.sym.pred}
                    and {p1.sym.bad} =$Set(Any) {p2.sym.bad}
  p1 case ker =>
    (p2 case ker) and (p1.ker.tag = p2.ker.tag)
           and (p1.ker.op = p2.ker.op) and (p1.ker.arg = p2.ker.arg)
  (p2 case exp) and (p1.exp.exponent = p2.exp.exponent)
                and (p1.exp.val = p2.exp.val)

retractIfCan(p:%):Union(SY, "failed") ==
  symbol? p => p.pat.sym.val
  "failed"

mkrsy(t, c?, o?, m?) ==
  c? => [0, t, empty(), empty()]
  mlt := (m? => SYM_MULTIPLE; 0)
  opt := (o? => SYM_OPTIONAL; 0)
  [Or(Or(SYM_GENERIC, mlt), opt), t, empty(), empty()]

patternVariable(sy, c?, o?, m?) ==
  rsy := mkrsy(sy, c?, o?, m?)
```

```
    mkPat(zero?(rsy.tag), [rsy], 0)
```

——

— **PATTERN.dotabb** —

```
"PATTERN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PATTERN"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"PATTERN" -> "ALIST"
```

——

17.11 domain PATLRES PatternMatchListResult

— **PatternMatchListResult.input** —

```
)set break resume
)sys rm -f PatternMatchListResult.output
)spool PatternMatchListResult.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PatternMatchListResult
--R PatternMatchListResult(R: SetCategory,S: SetCategory,L: ListAggregate S)  is a domain co
--R Abbreviation for PatternMatchListResult is PATLRES
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PATLRES
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R failed : () -> %                      failed? : % -> Boolean
--R hash : % -> SingleInteger             latex : % -> String
--R new : () -> %                         ?~=? : (%,%) -> Boolean
--R atoms : % -> PatternMatchResult(R,S)
--R lists : % -> PatternMatchResult(R,L)
--R makeResult : (PatternMatchResult(R,S),PatternMatchResult(R,L)) -> %
--R
--E 1

)spool
)lisp (bye)
```

———–

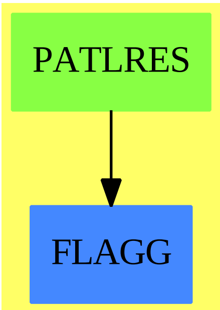
— PatternMatchListResult.help —

```
====================================================================
PatternMatchListResult examples
====================================================================

See Also:
o )show PatternMatchListResult
```

———–

17.11.1 PatternMatchListResult (PATLRES)



See

⇒ "PatternMatchResult" (PATRES) 17.12.1 on page 1900

Exports:

atoms	coerce	failed	failed?	hash
latex	lists	makeResult	new	?=?
?~=?				

— domain PATLRES PatternMatchListResult —

```
)abbrev domain PATLRES PatternMatchListResult
++ Author: Manuel Bronstein
++ Date Created: 4 Dec 1989
++ Date Last Updated: 4 Dec 1989
++ Keywords: pattern, matching, list.
++ Description:
++ A PatternMatchListResult is an object internally returned by the
++ pattern matcher when matching on lists.
++ It is either a failed match, or a pair of PatternMatchResult,
++ one for atoms (elements of the list), and one for lists.
```

```
-- not exported

PatternMatchListResult(R:SetCategory, S:SetCategory, L:ListAggregate S):
  SetCategory with
    failed?   : %  -> Boolean
      ++ failed?(r) tests if r is a failed match.
    failed    : () -> %
      ++ failed() returns a failed match.
    new       : () -> %
      ++ new() returns a new empty match result.
    makeResult: (PatternMatchResult(R,S), PatternMatchResult(R,L)) -> %
      ++ makeResult(r1,r2) makes the combined result [r1,r2].
    atoms     : %  -> PatternMatchResult(R, S)
      ++ atoms(r) returns the list of matches that match atoms
      ++ (elements of the lists).
    lists     : %  -> PatternMatchResult(R, L)
      ++ lists(r) returns the list of matches that match lists.
 == add
  Rep := Record(a:PatternMatchResult(R, S), l:PatternMatchResult(R, L))

  new()              == [new(), new()]
  atoms r            == r.a
  lists r            == r.l
  failed()           == [failed(), failed()]
  failed? r          == failed?(atoms r)
  x = y              == (atoms x = atoms y) and (lists x = lists y)

  makeResult(r1, r2) ==
    failed? r1 or failed? r2 => failed()
    [r1, r2]

  coerce(r:%):OutputForm ==
    failed? r => atoms(r)::OutputForm
    RecordPrint(r, Rep)$Lisp
```

— **PATLRES.dotabb** —

```
"PATLRES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PATLRES"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"PATLRES" -> "FLAGG"
```

17.12 domain PATRES PatternMatchResult

— PatternMatchResult.input —

```
)set break resume
)sys rm -f PatternMatchResult.output
)spool PatternMatchResult.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PatternMatchResult
--R PatternMatchResult(R: SetCategory,S: SetCategory)  is a domain constructor
--R Abbreviation for PatternMatchResult is PATRES
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PATRES
--R
--R------------------------------ Operations -------------------------------
--R ?=? : (%,%) -> Boolean                addMatch : (Pattern R,S,%) -> %
--R coerce : % -> OutputForm              failed : () -> %
--R failed? : % -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   new : () -> %
--R union : (%,%) -> %                    ?~=? : (%,%) -> Boolean
--R addMatchRestricted : (Pattern R,S,%,S) -> %
--R construct : List Record(key: Symbol,entry: S) -> %
--R destruct : % -> List Record(key: Symbol,entry: S)
--R getMatch : (Pattern R,%) -> Union(S,"failed")
--R insertMatch : (Pattern R,S,%) -> %
--R satisfy? : (%,Pattern R) -> Union(Boolean,"failed")
--R
--E 1

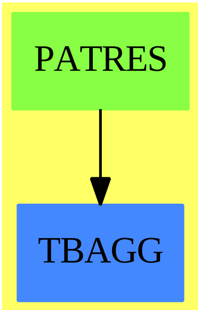
)spool
)lisp (bye)
```

— PatternMatchResult.help —

```
====================================================================
PatternMatchResult examples
====================================================================

See Also:
o )show PatternMatchResult
```

17.12.1 PatternMatchResult (PATRES)



See

⇒ "PatternMatchListResult" (PATLRES) 17.11.1 on page 1897

Exports:

addMatch	addMatchRestricted	coerce	construct	destruct
failed	failed?	getMatch	hash	insertMatch
latex	new	satisfy?	union	?=?
?~=?				

— **domain PATRES PatternMatchResult** —

```
)abbrev domain PATRES PatternMatchResult
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 5 Jul 1990
++ Keywords: pattern, matching.
++ Description:
++ A PatternMatchResult is an object internally returned by the
++ pattern matcher; It is either a failed match, or a list of
++ matches of the form (var, expr) meaning that the variable var
++ matches the expression expr.
-- not exported

PatternMatchResult(R:SetCategory, S:SetCategory): SetCategory with
  failed?           : %  -> Boolean
    ++ failed?(r) tests if r is a failed match.
  failed            : () -> %
    ++ failed() returns a failed match.
  new               : () -> %
    ++ new() returns a new empty match result.
  union             : (%, %) -> %
    ++ union(a, b) makes the set-union of two match results.
  getMatch          : (Pattern R, %) -> Union(S, "failed")
```

```
    ++ getMatch(var, r) returns the expression that var matches
    ++ in the result r, and "failed" if var is not matched in r.
  addMatch          : (Pattern R, S, %) -> %
    ++ addMatch(var, expr, r) adds the match (var, expr) in r,
    ++ provided that expr satisfies the predicates attached to var,
    ++ and that var is not matched to another expression already.
  insertMatch       : (Pattern R, S, %) -> %
    ++ insertMatch(var, expr, r) adds the match (var, expr) in r,
    ++ without checking predicates or previous matches for var.
  addMatchRestricted: (Pattern R, S, %, S) -> %
    ++ addMatchRestricted(var, expr, r, val) adds the match
    ++ (var, expr) in r,
    ++ provided that expr satisfies the predicates attached to var,
    ++ that var is not matched to another expression already,
    ++ and that either var is an optional pattern variable or that
    ++ expr is not equal to val (usually an identity).
  destruct          : % -> List Record(key:Symbol, entry:S)
    ++ destruct(r) returns the list of matches (var, expr) in r.
    ++ Error: if r is a failed match.
  construct         : List Record(key:Symbol, entry:S) -> %
    ++ construct([v1,e1],...,[vn,en]) returns the match result
    ++ containing the matches (v1,e1),...,(vn,en).
  satisfy?          : (%, Pattern R) -> Union(Boolean, "failed")
    ++ satisfy?(r, p) returns true if the matches satisfy the
    ++ top-level predicate of p, false if they don't, and "failed"
    ++ if not enough variables of p are matched in r to decide.

 == add
  LR ==> AssociationList(Symbol, S)

  import PatternFunctions1(R, S)

  Rep := Union(LR, "failed")

  new()                == empty()
  failed()             == "failed"
  failed? x            == x case "failed"
  insertMatch(p, x, l) == concat([retract p, x], l::LR)
  construct l          == construct(l)$LR
  destruct l           == entries(l::LR)$LR

-- returns "failed" if not all the variables of the pred. are matched
  satisfy?(r, p) ==
    failed? r => false
    lr := r::LR
    lv := [if (u := search(v, lr)) case "failed" then return "failed"
                    else  u::S for v in topPredicate(p).var]$List(S)
    satisfy?(lv, p)

  union(x, y) ==
```

```
    failed? x or failed? y => failed()
    removeDuplicates concat(x::LR, y::LR)

  x = y ==
    failed? x => failed? y
    failed? y => false
    x::LR =$LR y::LR

  coerce(x:%):OutputForm ==
    failed? x => "Does not match"::OutputForm
    destruct(x)::OutputForm

  addMatchRestricted(p, x, l, ident) ==
    (not optional? p) and (x = ident) => failed()
    addMatch(p, x, l)

  addMatch(p, x, l) ==
    failed?(l) or not(satisfy?(x, p)) => failed()
    al := l::LR
    sy := retract(p)@Symbol
    (r := search(sy, al)) case "failed" => insertMatch(p, x, l)
    r::S = x => l
    failed()

  getMatch(p, l) ==
    failed? l => "failed"
    search(retract(p)@Symbol, l::LR)
```

———-

— PATRES.dotabb —

```
"PATRES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PATRES"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"PATRES" -> "TBAGG"
```

———-

17.13 domain PENDTREE PendantTree

— PendantTree.input —

```
)set break resume
)sys rm -f PendantTree.output
```

```
)spool PendantTree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PendantTree
--R PendantTree S: SetCategory  is a domain constructor
--R Abbreviation for PendantTree is PENDTREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PENDTREE
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %               coerce : % -> Tree S
--R copy : % -> %                        cyclic? : % -> Boolean
--R distance : (%,%) -> Integer          ?.right : (%,right) -> %
--R ?.left : (%,left) -> %               ?.value : (%,value) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               leaf? : % -> Boolean
--R leaves : % -> List S                 left : % -> %
--R map : ((S -> S),%) -> %              nodes : % -> List %
--R ptree : (%,%) -> %                   ptree : S -> %
--R right : % -> %                       sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
```

```
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

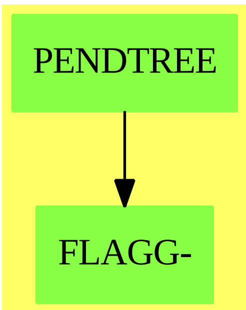
— **PendantTree.help** —

```
====================================================================
PendantTree examples
====================================================================

See Also:
o )show PendantTree
```

A PendantTree(S)is either a leaf? and is an S or has a left and a right both PendantTree(S)'s

17.13.1 PendantTree (PENDTREE)



See

⇒ "Tree" (TREE) 21.10.1 on page 2699
⇒ "BinaryTree" (BTREE) 3.11.1 on page 292
⇒ "BinarySearchTree" (BSTREE) 3.9.1 on page 285
⇒ "BinaryTournament" (BTOURN) 3.10.1 on page 289
⇒ "BalancedBinaryTree" (BBTREE) 3.1.1 on page 234

Exports:

any?	child?	children	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	left	less?	map
map!	member?	members	more?	node?
nodes	parts	ptree	right	sample
setchildren!	setelt	setleft!	setright!	setvalue!
size?	value	#?	?=?	?~=?
?.right	?.left	?.value		

— domain PENDTREE PendantTree —

```
)abbrev domain PENDTREE PendantTree
++ Author: Mark Botch
++ Description:
++ This domain has no description

PendantTree(S: SetCategory): T == C where
 T == BinaryRecursiveAggregate(S) with
   ptree : S->%
    ++ ptree(s) is a leaf? pendant tree
    ++
    ++X t1:=ptree([1,2,3])

   ptree:(%, %)->%
    ++ ptree(x,y) is not documented
    ++
    ++X t1:=ptree([1,2,3])
    ++X ptree(t1,ptree([1,2,3]))

   coerce:%->Tree S
    ++ coerce(x) is not documented
    ++
    ++X t1:=ptree([1,2,3])
    ++X t2:=ptree(t1,ptree([1,2,3]))
    ++X t2::Tree List PositiveInteger

 C == add
     Rep := Tree S
     import Tree S
     coerce (t:%):Tree S == t pretend Tree S
     ptree(n) == tree(n,[])$Rep pretend %
     ptree(l,r) == tree(value(r:Rep)$Rep,cons(l,children(r:Rep)$Rep)):%
     leaf? t == empty?(children(t)$Rep)
     t1=t2 == (t1:Rep) = (t2:Rep)
     left b ==
       leaf? b => error "ptree:no left"
       first(children(b)$Rep)
```

```
    right b ==
      leaf? b => error "ptree:no right"
      tree(value(b)$Rep,rest (children(b)$Rep))
    value b ==
      leaf? b => value(b)$Rep
      error "the pendant tree has no value"
    coerce(b:%): OutputForm ==
      leaf? b => value(b)$Rep :: OutputForm
      paren blankSeparate [left b::OutputForm,right b ::OutputForm]
```

———-

— **PENDTREE.dotabb** —

```
"PENDTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PENDTREE"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"PENDTREE" -> "FLAGG-"
```

———-

17.14 domain PERM Permutation

— **Permutation.input** —

```
)set break resume
)sys rm -f Permutation.output
)spool Permutation.output
)set message test on
)set message auto off
)clear all
--S 1 of 8
p := coercePreimagesImages([[1,2,3],[1,2,3]])
--R
--R
--R   (1)  1
--R                                         Type: Permutation PositiveInteger
--E 1

--S 2 of 8
movedPoints p    -- should return {}
--R
--R
--R   (2)  {}
--R                                                 Type: Set PositiveInteger
```

```
--E 2

--S 3 of 8
even? p          -- should return true
--R
--R
--R   (3)  true
--R                                                     Type: Boolean
--E 3

--S 4 of 8
p := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
--R
--R
--R   (4)  (1 0 3)
--R                                      Type: Permutation IntegerMod 4
--E 4

--S 5 of 8
fixedPoints p    -- should return {2}
--R
--R
--R   (5)  {2}
--R                                              Type: Set IntegerMod 4
--E 5

--S 6 of 8
q := coercePreimagesImages([[0,1,2,3],[1,0]])$PERM ZMOD 4
--R
--R
--R   (6)  (1 0)
--R                                      Type: Permutation IntegerMod 4
--E 6

--S 7 of 8
fixedPoints(p*q) -- should return {2,0}
--R
--R
--R   (7)  {2,0}
--R                                              Type: Set IntegerMod 4
--E 7

--S 8 of 8
even?(p*q)       -- should return false
--R
--R
--R   (8)  false
--R                                                     Type: Boolean
--E 8
)spool
```

```
)lisp (bye)
```

———-

— Permutation.help —

```
====================================================================
Permutation Examples
====================================================================

We represent a permutation as two lists of equal length representing preimages
and images of moved points. I.e., fixed points do not occur in either of these
lists. This enables us to compute the set of fixed points and the set of moved
points easily.

  p := coercePreimagesImages([[1,2,3],[1,2,3]])
    1
                      Type: Permutation PositiveInteger

  movedPoints p
    {}
                      Type: Set PositiveInteger

  even? p
    true
                       Type: Boolean

  p := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
    (1 0 3)
                       Type: Permutation IntegerMod 4

  fixedPoints p
    {2}
                       Type: Set IntegerMod 4

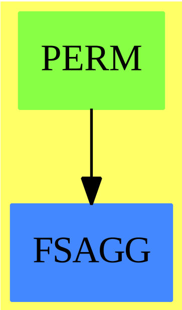
  q := coercePreimagesImages([[0,1,2,3],[1,0]])$PERM ZMOD 4
    (1 0)
                       Type: Permutation IntegerMod 4

  fixedPoints(p*q)
    {2,0}
                       Type: Set IntegerMod 4
  even?(p*q)
    false
                       Type: Boolean

See Also:
o )show Permutation
```

———-

17.14.1 Permutation (PERM)



Exports:

1	coerce	coerceImages
coerceListOfPairs	coercePreimagesImages	commutator
conjugate	cycle	cyclePartition
cycles	degree	eval
even?	fixedPoints	hash
inv	latex	listRepresentation
max	min	movedPoints
numberOfCycles	odd?	one?
orbit	order	recip
sample	sign	sort
?*?	?**?	?/?
?<?	?=?	?^?
?.?	?~=?	?<=?
?>?	?>=?	

— **domain PERM Permutation** —

```
)abbrev domain PERM Permutation
++ Authors: Johannes Grabmeier, Holger Gollan, Martin Rubey
++ Date Created: 19 May 1989
++ Date Last Updated: 2 June 2006
++ Basic Operations: _*, degree, movedPoints, cyclePartition, order,
++  numberOfCycles, sign, even?, odd?
++ Related Constructors: PermutationGroup, PermutationGroupExamples
++ Also See: RepresentationTheoryPackage1
++ AMS Classifications:
++ Keywords:
++ Reference: G. James/A. Kerber: The Representation Theory of the Symmetric
++   Group. Encycl. of Math. and its Appl., Vol. 16., Cambridge
```

```
++ Description:
++ Permutation(S) implements the group of all bijections
++ on a set S, which move only a finite number of points.
++ A permutation is considered as a map from S into S. In particular
++ multiplication is defined as composition of maps:\br
++ pi1 * pi2 = pi1 o pi2.\br
++ The internal representation of permuatations are two lists
++ of equal length representing preimages and images.

Permutation(S:SetCategory): public == private where

  B        ==> Boolean
  PI       ==> PositiveInteger
  I        ==> Integer
  L        ==> List
  NNI      ==> NonNegativeInteger
  V        ==> Vector
  PT       ==> Partition
  OUTFORM  ==> OutputForm
  RECCYPE  ==> Record(cycl: L L S, permut: %)
  RECPRIM  ==> Record(preimage: L S, image : L S)

  public ==> PermutationCategory S with

    listRepresentation:  %                ->  RECPRIM
      ++ listRepresentation(p) produces a representation rep of
      ++ the permutation p as a list of preimages and images, i.e
      ++ p maps (rep.preimage).k to (rep.image).k for all
      ++ indices k. Elements of \spad{S} not in (rep.preimage).k
      ++ are fixed points, and these are the only fixed points of the
      ++ permutation.
    coercePreimagesImages : List List S   ->  %
      ++ coercePreimagesImages(lls) coerces the representation lls
      ++ of a permutation as a list of preimages and images to a permutation.
      ++ We assume that both preimage and image do not contain repetitions.
      ++
      ++X p := coercePreimagesImages([[1,2,3],[1,2,3]])
      ++X q := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
    coerce            :  List List S      ->  %
      ++ coerce(lls) coerces a list of cycles lls to a
      ++ permutation, each cycle being a list with no
      ++ repetitions, is coerced to the permutation, which maps
      ++ ls.i to ls.i+1, indices modulo the length of the list,
      ++ then these permutations are mutiplied.
      ++ Error: if repetitions occur in one cycle.
    coerce            :  List S           ->  %
      ++ coerce(ls) coerces a cycle ls, i.e. a list with not
      ++ repetitions to a permutation, which maps ls.i to
      ++ ls.i+1, indices modulo the length of the list.
      ++ Error: if repetitions occur.
```

```
coerceListOfPairs :  List List S      ->  %
  ++ coerceListOfPairs(lls) coerces a list of pairs lls to a
  ++ permutation.
  ++ Error: if not consistent, i.e. the set of the first elements
  ++ coincides with the set of second elements.
--coerce          :  %                 ->  OUTFORM
  ++ coerce(p) generates output of the permutation p with domain
  ++ OutputForm.
degree            :  %               ->  NonNegativeInteger
  ++ degree(p) retuns the number of points moved by the
  ++ permutation p.
movedPoints       :  %               ->  Set S
  ++ movedPoints(p) returns the set of points moved by the permutation p.
  ++
  ++X p := coercePreimagesImages([[1,2,3],[1,2,3]])
  ++X movedPoints p
cyclePartition    :  %               ->  Partition
  ++ cyclePartition(p) returns the cycle structure of a permutation
  ++ p including cycles of length 1 only if S is finite.
order             :  %               ->  NonNegativeInteger
  ++ order(p) returns the order of a permutation p as a group element.
numberOfCycles    :  %               ->  NonNegativeInteger
  ++ numberOfCycles(p) returns the number of non-trivial cycles of
  ++ the permutation p.
sign              :  %               ->  Integer
  ++ sign(p) returns the signum of the permutation p, +1 or -1.
even?             :  %               ->  Boolean
  ++ even?(p) returns true if and only if p is an even permutation,
  ++ i.e. sign(p) is 1.
  ++
  ++X p := coercePreimagesImages([[1,2,3],[1,2,3]])
  ++X even? p
odd?              :  %               ->  Boolean
  ++ odd?(p) returns true if and only if p is an odd permutation
  ++ i.e. sign(p) is -1.
sort              :  L %             ->  L %
  ++ sort(lp) sorts a list of permutations lp according to
  ++ cycle structure first according to length of cycles,
  ++ second, if S has \spadtype{Finite} or S has
  ++ \spadtype{OrderedSet} according to lexicographical order of
  ++ entries in cycles of equal length.
if S has Finite then
  fixedPoints     :  %               ->  Set S
    ++ fixedPoints(p) returns the points fixed by the permutation p.
    ++X p := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
    ++X fixedPoints p
if S has IntegerNumberSystem or S has Finite then
  coerceImages    :  L S             ->  %
    ++ coerceImages(ls) coerces the list ls to a permutation
    ++ whose image is given by ls and the preimage is fixed
```

```
      ++ to be [1,...,n].
      ++ Note: {coerceImages(ls)=coercePreimagesImages([1,...,n],ls)}.
      ++ We assume that both preimage and image do not contain repetitions.

  private ==> add

    -- representation of the object:

    Rep  := V L S

    -- import of domains and packages

    import OutputForm
    import Vector List S

    -- variables

    p,q      : %
    exp      : I

    -- local functions first, signatures:

    smaller? : (S,S) -> B
    rotateCycle: L S -> L S
    coerceCycle: L L S -> %
    smallerCycle?: (L S, L S)  -> B
    shorterCycle?:(L S, L S)  -> B
    permord:(RECCYPE,RECCYPE)  -> B
    coerceToCycle:(%,B) -> L L S
    duplicates?: L S -> B

    smaller?(a:S, b:S): B ==
      S has OrderedSet => a <$S b
      S has Finite     => lookup a < lookup b
      false

    rotateCycle(cyc: L S): L S ==
      -- smallest element is put in first place
      -- doesn't change cycle if underlying set
      -- is not ordered or not finite.
      min:S := first cyc
      minpos:I := 1           -- 1 = minIndex cyc
      for i in 2..maxIndex cyc repeat
        if smaller?(cyc.i,min) then
          min  := cyc.i
          minpos := i
--      one? minpos => cyc
      (minpos = 1) => cyc
      concat(last(cyc,((#cyc-minpos+1)::NNI)),first(cyc,(minpos-1)::NNI))
```

```
coerceCycle(lls : L L S): % ==
  perm : % := 1
  for lists in reverse lls repeat
    perm := cycle lists * perm
  perm

smallerCycle?(cyca: L S, cycb: L S): B ==
  #cyca ^= #cycb =>
    #cyca < #cycb
  for i in cyca for j in cycb repeat
    i ^= j => return smaller?(i, j)
  false

shorterCycle?(cyca: L S, cycb: L S): B ==
  #cyca < #cycb

permord(pa: RECCYPE, pb : RECCYPE): B ==
  for i in pa.cycl for j in pb.cycl repeat
    i ^= j => return smallerCycle?(i, j)
  #pa.cycl < #pb.cycl

coerceToCycle(p: %, doSorting?: B): L L S ==
  preim := p.1
  im := p.2
  cycles := nil()$(L L S)
  while not null preim repeat
    -- start next cycle
    firstEltInCycle: S := first preim
    nextCycle : L S := list firstEltInCycle
    preim := rest preim
    nextEltInCycle := first im
    im      := rest im
    while nextEltInCycle ^= firstEltInCycle repeat
      nextCycle := cons(nextEltInCycle, nextCycle)
      i := position(nextEltInCycle, preim)
      preim := delete(preim,i)
      nextEltInCycle := im.i
      im := delete(im,i)
    nextCycle := reverse nextCycle
    -- check on 1-cycles, we don't list these
    if not null rest nextCycle then
      if doSorting? and (S has OrderedSet or S has Finite) then
          -- put smallest element in cycle first:
          nextCycle := rotateCycle nextCycle
      cycles := cons(nextCycle, cycles)
  not doSorting? => cycles
  -- sort cycles
  S has OrderedSet or S has Finite =>
    sort(smallerCycle?,cycles)$(L L S)
  sort(shorterCycle?,cycles)$(L L S)
```

```
duplicates? (ls : L S ): B ==
  x := copy ls
  while not null x repeat
    member? (first x ,rest x) => return true
    x := rest x
  false

-- now the exported functions

listRepresentation p ==
  s : RECPRIM := [p.1,p.2]

coercePreimagesImages preImageAndImage ==
  preImage: List S := []
  image: List S := []
  for i in preImageAndImage.1
    for pi in preImageAndImage.2 repeat
      if i ~= pi then
        preImage := cons(i, preImage)
        image := cons(pi, image)

  [preImage, image]

movedPoints p == construct p.1

degree p ==  #movedPoints p

p = q ==
  #(preimp := p.1) ^= #(preimq := q.1) => false
  for i in 1..maxIndex preimp repeat
    pos := position(preimp.i, preimq)
    pos = 0 => return false
    (p.2).i ^= (q.2).pos => return false
  true

orbit(p ,el) ==
  -- start with a 1-element list:
  out : Set S := brace list el
  el2 := eval(p, el)
  while el2 ^= el repeat
    -- be carefull: insert adds one element
    -- as side effect to out
    insert_!(el2, out)
    el2 := eval(p, el2)
  out

cyclePartition p ==
  partition([#c for c in coerceToCycle(p, false)])$Partition
```

```
order p ==
  ord: I := lcm removeDuplicates convert cyclePartition p
  ord::NNI

sign(p) ==
  even? p => 1
  - 1

even?(p) ==  even?(#(p.1) - numberOfCycles p)
  -- see the book of James and Kerber on symmetric groups
  -- for this formula.

odd?(p) ==  odd?(#(p.1) - numberOfCycles p)

pa < pb ==
  pacyc:= coerceToCycle(pa,true)
  pbcyc:= coerceToCycle(pb,true)
  for i in pacyc for j in pbcyc repeat
    i ^= j => return smallerCycle? ( i, j )
  maxIndex pacyc < maxIndex pbcyc

coerce(lls : L L S): % == coerceCycle lls

coerce(ls : L S): % == cycle ls

sort(inList : L %): L % ==
  not (S has OrderedSet or S has Finite) => inList
  ownList: L RECCYPE := nil()$(L RECCYPE)
  for sigma in inList repeat
    ownList :=
      cons([coerceToCycle(sigma,true),sigma]::RECCYPE, ownList)
  ownList := sort(permord, ownList)$(L RECCYPE)
  outList := nil()$(L %)
  for rec in ownList repeat
    outList := cons(rec.permut, outList)
  reverse outList

coerce (p: %): OUTFORM ==
  cycles: L L S := coerceToCycle(p,true)
  outfmL : L OUTFORM := nil()
  for cycle in cycles repeat
    outcycL: L OUTFORM := nil()
    for elt in cycle repeat
      outcycL := cons(elt :: OUTFORM, outcycL)
    outfmL := cons(paren blankSeparate reverse outcycL, outfmL)
  -- The identity element will be output as 1:
  null outfmL => outputForm(1@Integer)
  -- represent a single cycle in the form (a b c d)
  -- and not in the form ((a b c d)):
  null rest outfmL => first outfmL
```

```
      hconcat reverse outfmL

    cycles(vs ) == coerceCycle vs

    cycle(ls) ==
      #ls < 2 => 1
      duplicates? ls => error "cycle: the input contains duplicates"
      [ls, append(rest ls, list first ls)]

    coerceListOfPairs(loP) ==
      preim := nil()$(L S)
      im := nil()$(L S)
      for pair in loP repeat
        if first pair ^=  second pair then
          preim := cons(first pair, preim)
          im := cons(second pair, im)
      duplicates?(preim) or duplicates?(im) or brace(preim)$(Set S) _
        ^= brace(im)$(Set S) =>
        error "coerceListOfPairs: the input cannot be interpreted as a permutation"
      [preim, im]

    q * p ==
      -- use vectors for efficiency??
      preimOfp : V S := construct p.1
      imOfp : V S := construct p.2
      preimOfq := q.1
      imOfq := q.2
      preimOfqp   := nil()$(L S)
      imOfqp   := nil()$(L S)
      -- 1 = minIndex preimOfp
      for i in 1..(maxIndex preimOfp) repeat
        -- find index of image of p.i in q if it exists
        j := position(imOfp.i, preimOfq)
        if j = 0 then
          -- it does not exist
          preimOfqp := cons(preimOfp.i, preimOfqp)
          imOfqp := cons(imOfp.i, imOfqp)
        else
          -- it exists
          el := imOfq.j
          -- if the composition fixes the element, we don't
          -- have to do anything
          if el ^= preimOfp.i then
            preimOfqp := cons(preimOfp.i, preimOfqp)
            imOfqp := cons(el, imOfqp)
          -- we drop the parts of q which have to do with p
          preimOfq := delete(preimOfq, j)
          imOfq := delete(imOfq, j)
      [append(preimOfqp, preimOfq), append(imOfqp, imOfq)]
```

```
    1 == new(2,empty())$Rep

    inv p  == [p.2, p.1]

    eval(p, el) ==
      pos := position(el, p.1)
      pos = 0 => el
      (p.2).pos

    elt(p, el) == eval(p, el)

    numberOfCycles p == #coerceToCycle(p, false)


    if S has IntegerNumberSystem then

      coerceImages (image) ==
        preImage : L S := [i::S for i in 1..maxIndex image]
        coercePreimagesImages [preImage,image]

    if S has Finite then

      coerceImages (image) ==
        preImage : L S := [index(i::PI)::S for i in 1..maxIndex image]
        coercePreimagesImages [preImage,image]

      fixedPoints ( p ) == complement movedPoints p

      cyclePartition p ==
        pt := partition([#c for c in coerceToCycle(p, false)])$Partition
        pt +$PT conjugate(partition([#fixedPoints(p)])$PT)$PT
```

— PERM.dotabb —

```
"PERM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PERM"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"PERM" -> "FSAGG"
```

17.15 domain PERMGRP PermutationGroup

— PermutationGroup.input —

```
)set break resume
)sys rm -f PermutationGroup.output
)spool PermutationGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PermutationGroup
--R PermutationGroup S: SetCategory  is a domain constructor
--R Abbreviation for PermutationGroup is PERMGRP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PERMGRP
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                base : % -> List S
--R coerce : List Permutation S -> %      coerce : % -> List Permutation S
--R coerce : % -> OutputForm              degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger             latex : % -> String
--R movedPoints : % -> Set S              orbit : (%,List S) -> Set List S
--R orbit : (%,Set S) -> Set Set S        orbit : (%,S) -> Set S
--R orbits : % -> Set Set S               order : % -> NonNegativeInteger
--R random : % -> Permutation S           ?~=? : (%,%) -> Boolean
--R ?.? : (%,NonNegativeInteger) -> Permutation S
--R generators : % -> List Permutation S
--R initializeGroupForWordProblem : (%,Integer,Integer) -> Void
--R initializeGroupForWordProblem : % -> Void
--R member? : (Permutation S,%) -> Boolean
--R permutationGroup : List Permutation S -> %
--R random : (%,Integer) -> Permutation S
--R strongGenerators : % -> List Permutation S
--R wordInGenerators : (Permutation S,%) -> List NonNegativeInteger
--R wordInStrongGenerators : (Permutation S,%) -> List NonNegativeInteger
--R wordsForStrongGenerators : % -> List List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

———-

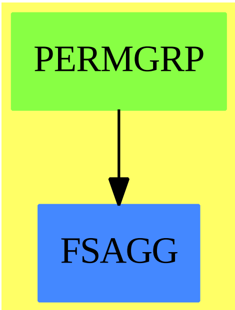
— PermutationGroup.help —

```
====================================================================
PermutationGroup examples
====================================================================
```

```
See Also:
o )show PermutationGroup
```

———-

17.15.1 PermutationGroup (PERMGRP)



Exports:

base	coerce	degree
hash	latex	movedPoints
orbit	orbits	order
random	generators	initializeGroupForWordProblem
member?	permutationGroup	random
strongGenerators	wordInGenerators	wordInStrongGenerators
wordsForStrongGenerators	?~=?	?.?
?<?	?<=?	?=?

— **domain PERMGRP PermutationGroup** —

```
)abbrev domain PERMGRP PermutationGroup
++ Authors: G. Schneider, H. Gollan, J. Grabmeier
++ Date Created: 13 February 1987
++ Date Last Updated: 24 May 1991
++ Basic Operations:
++ Related Constructors: PermutationGroupExamples, Permutation
++ Also See: RepresentationTheoryPackage1
++ AMS Classifications:
++ Keywords: permutation, permutation group, group operation, word problem
++ References:
++   C. Sims: Determining the conjugacy classes of a permutation group,
++   in Computers in Algebra and Number Theory, SIAM-AMS Proc., Vol. 4,
++    Amer. Math. Soc., Providence, R. I., 1971, pp. 191-195
++ Description:
++ PermutationGroup implements permutation groups acting
```

```
++ on a set S, i.e. all subgroups of the symmetric group of S,
++ represented as a list of permutations (generators). Note that
++ therefore the objects are not members of the \Language category
++ \spadtype{Group}.
++ Using the idea of base and strong generators by Sims,
++ basic routines and algorithms
++ are implemented so that the word problem for
++ permutation groups can be solved.
--++  Note: we plan to implement lattice operations on the subgroup
--++  lattice in a later release

PermutationGroup(S:SetCategory): public == private where

  L    ==> List
  PERM ==> Permutation
  FSET ==> Set
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  V    ==> Vector
  B    ==> Boolean
  OUT   ==> OutputForm
  SYM  ==> Symbol
  REC  ==> Record ( orb : L NNI , svc : V I )
  REC2 ==> Record(order:NNI,sgset:L V NNI,_
             gpbase:L NNI,orbs:L REC,mp:L S,wd:L L NNI)
  REC3 ==> Record(elt:V NNI,lst:L NNI)
  REC4 ==> Record(bool:B,lst:L NNI)

  public ==> SetCategory with

    coerce           : %         -> L PERM S
      ++ coerce(gp) returns the generators of the group gp.
    generators           : %         -> L PERM S
      ++ generators(gp) returns the generators of the group gp.
    elt              : (%,NNI)   -> PERM S
      ++ elt(gp,i) returns the i-th generator of the group gp.
    random           : (%,I)     -> PERM S
      ++ random(gp,i) returns a random product of maximal i generators
      ++ of the group gp.
    random           : %         -> PERM S
      ++ random(gp) returns a random product of maximal 20 generators
      ++ of the group gp.
      ++ Note: random(gp)=random(gp,20).
    order            : %         -> NNI
      ++ order(gp) returns the order of the group gp.
    degree           : %         -> NNI
      ++ degree(gp) returns the number of points moved by all permutations
      ++ of the group gp.
    base             : %         -> L S
      ++ base(gp) returns a base for the group gp.
```

```
strongGenerators : %          -> L PERM S
  ++ strongGenerators(gp) returns strong generators for
  ++ the group gp.
wordsForStrongGenerators      : %          -> L L NNI
  ++ wordsForStrongGenerators(gp) returns the words for the strong
  ++ generators of the group gp in the original generators of
  ++ gp, represented by their indices in the list, given by
  ++ generators.
coerce           : L PERM S  -> %
  ++ coerce(ls) coerces a list of permutations ls to the group
  ++ generated by this list.
permutationGroup          : L PERM S  -> %
  ++ permutationGroup(ls) coerces a list of permutations ls to
  ++ the group generated by this list.
orbit            : (%,S)      -> FSET S
  ++ orbit(gp,el) returns the orbit of the element el under the
  ++ group gp, i.e. the set of all points gained by applying
  ++ each group element to el.
orbits           : %          -> FSET FSET S
  ++ orbits(gp) returns the orbits of the group gp, i.e.
  ++ it partitions the (finite) of all moved points.
orbit            : (%,FSET S)-> FSET FSET S
  ++ orbit(gp,els) returns the orbit of the unordered
  ++ set els under the group gp.
orbit            : (%,L S)    -> FSET L S
  ++ orbit(gp,ls) returns the orbit of the ordered
  ++ list ls under the group gp.
  ++ Note: return type is L L S temporarily because FSET L S has an error.
  -- (GILT DAS NOCH?)
member?          : (PERM S, %)-> B
  ++ member?(pp,gp) answers the question, whether the
  ++ permutation pp is in the group gp or not.
wordInStrongGenerators : (PERM S, %)-> L NNI
  ++ wordInStrongGenerators(p,gp) returns the word for the
  ++ permutation p in the strong generators of the group gp,
  ++ represented by the indices of the list, given by strongGenerators.
wordInGenerators : (PERM S, %)-> L NNI
  ++ wordInGenerators(p,gp) returns the word for the permutation p
  ++ in the original generators of the group gp,
  ++ represented by the indices of the list, given by generators.
movedPoints      : %          -> FSET S
  ++ movedPoints(gp) returns the points moved by the group gp.
"<"              : (%,%)      -> B
  ++ gp1 < gp2 returns true if and only if gp1
  ++ is a proper subgroup of gp2.
"<="             : (%,%)      -> B
  ++ gp1 <= gp2 returns true if and only if gp1
  ++ is a subgroup of gp2.
  ++ Note: because of a bug in the parser you have to call this
  ++ function explicitly by gp1 <=$(PERMGRP S) gp2.
```

```
    -- (GILT DAS NOCH?)
  initializeGroupForWordProblem : %   -> Void
    ++ initializeGroupForWordProblem(gp) initializes the group gp
    ++ for the word problem.
    ++ Notes: it calls the other function of this name with parameters
    ++ 0 and 1: initializeGroupForWordProblem(gp,0,1).
    ++ Notes: (1) be careful: invoking this routine will destroy the
    ++ possibly information about your group (but will recompute it again)
    ++ (2) users need not call this function normally for the soultion of
    ++ the word problem.
  initializeGroupForWordProblem :(%,I,I) -> Void
    ++ initializeGroupForWordProblem(gp,m,n) initializes the group
    ++ gp for the word problem.
    ++ Notes: (1) with a small integer you get shorter words, but the
    ++ routine takes longer than the standard routine for longer words.
    ++ (2) be careful: invoking this routine will destroy the possibly stored
    ++ information about your group (but will recompute it again).
    ++ (3) users need not call this function normally for the soultion of
    ++ the word problem.

private ==> add

  -- representation of the object:

  Rep  := Record ( gens : L PERM S , information : REC2 )

  -- import of domains and packages

  import Permutation S
  import OutputForm
  import Symbol
  import Void

--first the local variables

  sgs               : L V NNI       := []
  baseOfGroup       : L NNI         := []
  sizeOfGroup       : NNI           := 1
  degree            : NNI           := 0
  gporb             : L REC         := []
  out               : L L V NNI     := []
  outword           : L L L NNI     := []
  wordlist          : L L NNI       := []
  basePoint         : NNI           := 0
  newBasePoint      : B             := true
  supp              : L S           := []
  ord               : NNI           := 1
  wordProblem       : B             := true

--local functions first, signatures:
```

```
    shortenWord:(L NNI, %)->L NNI
    times:(V NNI, V NNI)->V NNI
    strip:(V NNI,REC,L V NNI,L L NNI)->REC3
    orbitInternal:(%,L S )->L L S
    inv: V NNI->V NNI
    ranelt:(L V NNI,L L NNI, I)->REC3
    testIdentity:V NNI->B
    pointList: %->L S
    orbitWithSvc:(L V NNI ,NNI )->REC
    cosetRep:(NNI ,REC ,L V NNI )->REC3
    bsgs1:(L V NNI,NNI,L L NNI,I,%,I)->NNI
    computeOrbits: I->L NNI
    reduceGenerators: I->Void
    bsgs:(%, I, I)->NNI
    initialize: %->FSET PERM S
    knownGroup?: %->Void
    subgroup:(%, %)->B
    memberInternal:(PERM S, %, B)->REC4

  --local functions first, implementations:

    shortenWord ( lw : L NNI , gp : % ) : L NNI ==
      -- tries to shorten a word in the generators by removing identities
      gpgens : L PERM S := coerce gp
      orderList : L NNI := [ order gen for gen in gpgens ]
      newlw : L NNI := copy lw
      for i in 1.. maxIndex orderList repeat
        if orderList.i = 1 then
          while member?(i,newlw) repeat
          -- removing the trivial element
            pos := position(i,newlw)
            newlw := delete(newlw,pos)
      flag : B := true
      while flag repeat
        actualLength : NNI := (maxIndex newlw) pretend NNI
        pointer := actualLength
        test := newlw.pointer
        anzahl : NNI := 1
        flag := false
        while pointer > 1 repeat
          pointer := ( pointer - 1 )::NNI
          if newlw.pointer ^= test then
            -- don't get a trivial element, try next
            test := newlw.pointer
            anzahl := 1
          else
            anzahl := anzahl + 1
            if anzahl = orderList.test then
              -- we have an identity, so remove it
```

```
          for i in (pointer+anzahl)..actualLength repeat
            newlw.(i-anzahl) := newlw.i
          newlw := first(newlw, (actualLength - anzahl) :: NNI)
          flag := true
          pointer := 1
  newlw

times ( p : V NNI , q : V NNI ) : V NNI ==
  -- internal multiplication of permutations
  [ qelt(p,qelt(q,i)) for i in 1..degree ]

strip(element:V NNI,orbit:REC,group:L V NNI,words:L L NNI) : REC3 ==
  -- strip an element into the stabilizer
  actelt         := element
  schreierVector := orbit.svc
  point          := orbit.orb.1
  outlist        := nil()$(L NNI)
  entryLessZero  : B := false
  while ^entryLessZero repeat
    entry := schreierVector.(actelt.point)
    entryLessZero  := (entry < 0)
    if  ^entryLessZero then
      actelt := times(group.entry, actelt)
      if wordProblem then outlist := append ( words.(entry::NNI) , outlist )
  [ actelt , reverse outlist ]

orbitInternal ( gp : % , startList : L S ) : L L S ==
  orbitList : L L S := [ startList ]
  pos  : I := 1
  while not zero? pos  repeat
    gpset : L PERM S := gp.gens
    for gen in gpset repeat
      newList  := nil()$(L S)
      workList := orbitList.pos
      for j in #workList..1 by -1 repeat
        newList := cons ( eval ( gen , workList.j ) , newList )
      if ^member?( newList , orbitList ) then
        orbitList := cons ( newList , orbitList )
        pos  := pos + 1
    pos := pos - 1
  reverse orbitList

inv ( p : V NNI ) : V NNI ==
  -- internal inverse of a permutation
  q : V NNI := new(degree,0)$(V NNI)
  for i in 1..degree repeat q.(qelt(p,i)) := i
  q

ranelt ( group : L V NNI , word : L L NNI , maxLoops : I ) : REC3 ==
  -- generate a "random" element
```

```
    numberOfGenerators    := # group
    randomInteger : I     := 1 + (random()$Integer rem numberOfGenerators)
    randomElement : V NNI := group.randomInteger
    words                 := nil()$(L NNI)
    if wordProblem then words := word.(randomInteger::NNI)
    if maxLoops > 0 then
      numberOfLoops : I  := 1 + (random()$Integer rem maxLoops)
    else
      numberOfLoops : I := maxLoops
    while numberOfLoops > 0 repeat
      randomInteger : I := 1 + (random()$Integer rem numberOfGenerators)
      randomElement := times ( group.randomInteger , randomElement )
      if wordProblem then words := append ( word.(randomInteger::NNI) , words)
      numberOfLoops := numberOfLoops - 1
    [ randomElement , words ]

  testIdentity ( p : V NNI ) : B ==
    -- internal test for identity
    for i in 1..degree repeat qelt(p,i) ^= i => return false
    true

  pointList(group : %) : L S ==
    support : FSET S :=  brace()   -- empty set !!
    for perm in group.gens repeat
      support := union(support, movedPoints perm)
    parts support

  orbitWithSvc ( group : L V NNI , point : NNI ) : REC ==
    -- compute orbit with Schreier vector, "-2" means not in the orbit,
    -- "-1" means starting point, the PI correspond to generators
    newGroup := nil()$(L V NNI)
    for el in group repeat
      newGroup := cons ( inv el , newGroup )
    newGroup                := reverse newGroup
    orbit          : L NNI := [ point ]
    schreierVector : V I   := new ( degree , -2 )
    schreierVector.point   := -1
    position : I := 1
    while not zero? position repeat
      for i in 1..#newGroup repeat
        newPoint := orbit.position
        newPoint := newGroup.i.newPoint
        if ^ member? ( newPoint , orbit ) then
          orbit                  := cons ( newPoint , orbit )
          position               := position + 1
          schreierVector.newPoint := i
      position := position - 1
    [ reverse orbit , schreierVector ]

  cosetRep ( point : NNI , o : REC , group : L V NNI ) : REC3 ==
```

```
    ppt          := point
    xelt : V NNI := [ n for n in 1..degree ]
    word         := nil()$(L NNI)
    oorb         := o.orb
    osvc         := o.svc
    while degree > 0 repeat
      p := osvc.ppt
      p < 0 => return [ xelt , word ]
      x    := group.p
      xelt := times ( x , xelt )
      if wordProblem then word := append ( wordlist.p , word )
      ppt  := x.ppt

  bsgs1 (group:L V NNI,number1:NNI,words:L L NNI,maxLoops:I,gp:%,diff:I)_
      : NNI ==
    -- try to get a good approximation for the strong generators and base
    for i in number1..degree repeat
      ort := orbitWithSvc ( group , i )
      k   := ort.orb
      k1  := # k
      if k1 ^= 1 then leave
    gpsgs := nil()$(L V NNI)
    words2 := nil()$(L L NNI)
    gplength : NNI := #group
    for jj in 1..gplength repeat if (group.jj).i ^= i then leave
    for k in 1..gplength repeat
      el2 := group.k
      if el2.i ^= i then
        gpsgs := cons ( el2 , gpsgs )
        if wordProblem then words2 := cons ( words.k , words2 )
      else
        gpsgs := cons ( times ( group.jj , el2 ) , gpsgs )
        if wordProblem _
          then words2 := cons ( append ( words.jj , words.k ) , words2 )
    group2 := nil()$(L V NNI)
    words3 := nil()$(L L NNI)
    j : I  := 15
    while j > 0 repeat
      -- find generators for the stabilizer
      ran := ranelt ( group , words , maxLoops )
      str := strip ( ran.elt , ort , group , words )
      el2 := str.elt
      if ^ testIdentity el2 then
        if ^ member?(el2,group2) then
          group2 := cons ( el2 , group2 )
          if wordProblem then
            help : L NNI := append ( reverse str.lst , ran.lst )
            help         := shortenWord ( help , gp )
            words3       := cons ( help , words3 )
          j := j - 2
```

```
        j := j - 1
      -- this is for word length control
      if wordProblem then maxLoops    := maxLoops - diff
      if ( null group2 ) or ( maxLoops < 0 ) then
        sizeOfGroup := k1
        baseOfGroup := [ i ]
        out         := [ gpsgs ]
        outword     := [ words2 ]
        return sizeOfGroup
      k2          := bsgs1 ( group2 , i + 1 , words3 , maxLoops , gp , diff )
      sizeOfGroup := k1 * k2
      out         := append ( out , [ gpsgs ] )
      outword     := append ( outword , [ words2 ] )
      baseOfGroup := cons ( i , baseOfGroup )
      sizeOfGroup

    computeOrbits ( kkk : I ) : L NNI ==
      -- compute the orbits for the stabilizers
      sgs         := nil()
      orbitLength := nil()$(L NNI)
      gporb       := nil()
      for i in 1..#baseOfGroup repeat
        sgs         := append ( sgs , out.i )
        pt          := #baseOfGroup - i + 1
        obs         := orbitWithSvc ( sgs , baseOfGroup.pt )
        orbitLength := cons ( #obs.orb , orbitLength )
        gporb       := cons ( obs , gporb )
      gporb := reverse gporb
      reverse orbitLength

    reduceGenerators ( kkk : I ) : Void ==
      -- try to reduce number of strong generators
      orbitLength := computeOrbits ( kkk )
      sgs         := nil()
      wordlist    := nil()
      for i in 1..(kkk-1) repeat
        sgs := append ( sgs , out.i )
        if wordProblem then wordlist := append ( wordlist , outword.i )
      removedGenerator := false
      baseLength : NNI := #baseOfGroup
      for nnn in kkk..(baseLength-1) repeat
        sgs := append ( sgs , out.nnn )
        if wordProblem then wordlist := append ( wordlist , outword.nnn )
        pt  := baseLength - nnn + 1
        obs := orbitWithSvc ( sgs , baseOfGroup.pt )
        i   := 1
        while not ( i > # out.nnn ) repeat
          pos  := position ( out.nnn.i , sgs )
          sgs2 := delete(sgs, pos)
          obs2 := orbitWithSvc ( sgs2 , baseOfGroup.pt )
```

```
        if # obs2.orb = orbitLength.nnn then
          test := true
          for j in (nnn+1)..(baseLength-1) repeat
            pt2  := baseLength - j + 1
            sgs2 := append ( sgs2 , out.j )
            obs2 := orbitWithSvc ( sgs2 , baseOfGroup.pt2 )
            if # obs2.orb ^= orbitLength.j then
              test := false
              leave
          if test then
            removedGenerator := true
            sgs              := delete (sgs, pos)
            if wordProblem then wordlist    := delete(wordlist, pos)
            out.nnn          := delete (out.nnn, i)
            if wordProblem then _
              outword.nnn := delete(outword.nnn, i )
          else
            i := i + 1
        else
          i := i + 1
    if removedGenerator then orbitLength := computeOrbits ( kkk )
    void()


  bsgs ( group : % ,maxLoops : I , diff : I ) : NNI ==
    -- the MOST IMPORTANT part of the package
    supp   := pointList group
    degree := # supp
    if degree = 0 then
      sizeOfGroup := 1
      sgs         := [ [ 0 ] ]
      baseOfGroup := nil()
      gporb       := nil()
      return sizeOfGroup
    newGroup := nil()$(L V NNI)
    gp       : L PERM S := group.gens
    words := nil()$(L L NNI)
    for ggg in 1..#gp repeat
      q := new(degree,0)$(V NNI)
      for i in 1..degree repeat
        newEl := eval ( gp.ggg , supp.i )
        pos2  := position ( newEl , supp )
        q.i   := pos2 pretend NNI
      newGroup := cons ( q , newGroup )
      if wordProblem then words    := cons(list ggg, words)
    if maxLoops < 1 then
      -- try to get the (approximate) base length
      if zero? (# ((group.information).gpbase)) then
        wordProblem := false
        k           := bsgs1 ( newGroup , 1 , words , 20 , group , 0 )
```

```
          wordProblem := true
          maxLoops    := (# baseOfGroup) - 1
        else
          maxLoops    := (# ((group.information).gpbase)) - 1
      k       := bsgs1 ( newGroup , 1 , words , maxLoops , group , diff )
      kkk : I := 1
      newGroup := reverse newGroup
      noAnswer : B := true
      while noAnswer repeat
        reduceGenerators kkk
-- *** Here is former "bsgs2" *** --
        -- test whether we have a base and a strong generating set
        sgs := nil()
        wordlist := nil()
        for i in 1..(kkk-1) repeat
          sgs := append ( sgs , out.i )
          if wordProblem then wordlist := append ( wordlist , outword.i )
        noresult : B := true
        for i in kkk..#baseOfGroup while noresult repeat
          sgs    := append ( sgs , out.i )
          if wordProblem then wordlist := append ( wordlist , outword.i )
          gporbi := gporb.i
          for pt in gporbi.orb while noresult repeat
            ppp   := cosetRep ( pt , gporbi , sgs )
            y1    := inv ppp.elt
            word3 := ppp.lst
            for jjj in 1..#sgs while noresult repeat
              word         := nil()$(L NNI)
              z            := times ( sgs.jjj , y1 )
              if wordProblem then word := append ( wordlist.jjj , word )
              ppp          := cosetRep ( (sgs.jjj).pt , gporbi , sgs )
              z            := times ( ppp.elt , z )
              if wordProblem then word := append ( ppp.lst , word )
              newBasePoint := false
              for j in (i-1)..1 by -1 while noresult repeat
                s := gporb.j.svc
                p := gporb.j.orb.1
                while ( degree > 0 ) and noresult repeat
                  entry := s.(z.p)
                  if entry < 0 then
                    if entry = -1 then leave
                    basePoint := j::NNI
                    noresult := false
                  else
                    ee := sgs.entry
                    z  := times ( ee , z )
                    if wordProblem then word := append ( wordlist.entry , word )
              if noresult then
                basePoint    := 1
                newBasePoint := true
```

```
            noresult := testIdentity z
    noAnswer := not (testIdentity z)
    if noAnswer then
      -- we have missed something
      word2 := nil()$(L NNI)
      if wordProblem then
        for wd in word3 repeat
          ttt := newGroup.wd
          while not (testIdentity ttt) repeat
            word2 := cons ( wd , word2 )
            ttt   := times ( ttt , newGroup.wd )
        word := append ( word , word2 )
        word := shortenWord ( word , group )
      if newBasePoint then
        for i in 1..degree repeat
          if z.i ^= i then
            baseOfGroup := append ( baseOfGroup , [ i ] )
            leave
        out := cons (list  z, out )
        if wordProblem then outword := cons (list word , outword )
      else
        out.basePoint := cons ( z , out.basePoint )
        if wordProblem then outword.basePoint := cons(word ,outword.basePoint )
      kkk := basePoint
  sizeOfGroup  := 1
  for j in 1..#baseOfGroup repeat
    sizeOfGroup := sizeOfGroup * # gporb.j.orb
  sizeOfGroup


initialize ( group : % ) : FSET PERM S ==
  group2 := brace()$(FSET PERM S)
  gp : L PERM S := group.gens
  for gen in gp repeat
    if degree gen > 0 then insert_!(gen, group2)
  group2

knownGroup? (gp : %) : Void ==
  -- do we know the group already?
  result := gp.information
  if result.order = 0 then
    wordProblem       := false
    ord               := bsgs ( gp , 20 , 0 )
    result            := [ ord , sgs , baseOfGroup , gporb , supp , [] ]
    gp.information    := result
  else
    ord         := result.order
    sgs         := result.sgset
    baseOfGroup := result.gpbase
    gporb       := result.orbs
```

```
    supp        := result.mp
    wordlist    := result.wd
  void

subgroup ( gp1 : % , gp2 : % ) : B ==
  gpset1 := initialize gp1
  gpset2 := initialize gp2
  empty? difference (gpset1, gpset2) => true
  for el in parts gpset1 repeat
    not member? (el,  gp2) => return false
  true

memberInternal ( p : PERM S , gp : % , flag : B ) : REC4 ==
  -- internal membership testing
  supp     := pointList gp
  outlist := nil()$(L NNI)
  mP : L S := parts movedPoints p
  for x in mP repeat
    not member? (x, supp) => return [ false , nil()$(L NNI) ]
  if flag then
    member? ( p , gp.gens ) => return [ true , nil()$(L NNI) ]
    knownGroup? gp
  else
    result := gp.information
    if #(result.wd) = 0 then
      initializeGroupForWordProblem gp
    else
      ord         := result.order
      sgs         := result.sgset
      baseOfGroup := result.gpbase
      gporb       := result.orbs
      supp        := result.mp
      wordlist    := result.wd
  degree := # supp
  pp := new(degree,0)$(V NNI)
  for i in 1..degree repeat
    el   := eval ( p , supp.i )
    pos  := position ( el , supp )
    pp.i := pos::NNI
  words := nil()$(L L NNI)
  if wordProblem then
    for i in 1..#sgs repeat
      lw : L NNI := [ (#sgs - i + 1)::NNI ]
      words := cons ( lw , words )
  for i in #baseOfGroup..1 by -1 repeat
    str := strip ( pp , gporb.i , sgs , words )
    pp := str.elt
    if wordProblem then outlist := append ( outlist , str.lst )
  [ testIdentity pp , reverse outlist ]
```

```
--now the exported functions

  coerce ( gp : % ) : L PERM S == gp.gens
  generators ( gp : % ) : L PERM S == gp.gens

  strongGenerators ( group ) ==
    knownGroup? group
    degree := # supp
    strongGens := nil()$(L PERM S)
    for i in sgs repeat
      pairs := nil()$(L L S)
      for j in 1..degree repeat
        pairs := cons ( [ supp.j , supp.(i.j) ] , pairs )
      strongGens := cons ( coerceListOfPairs pairs , strongGens )
    reverse strongGens

  elt ( gp , i ) == (gp.gens).i

  movedPoints ( gp ) == brace pointList gp

  random ( group , maximalNumberOfFactors ) ==
    maximalNumberOfFactors < 1 => 1$(PERM S)
    gp : L PERM S := group.gens
    numberOfGenerators := # gp
    randomInteger : I  := 1 + (random()$Integer rem numberOfGenerators)
    randomElement      := gp.randomInteger
    numberOfLoops : I  := 1 + (random()$Integer rem maximalNumberOfFactors)
    while numberOfLoops > 0 repeat
      randomInteger : I  := 1 + (random()$Integer rem numberOfGenerators)
      randomElement := gp.randomInteger * randomElement
      numberOfLoops := numberOfLoops - 1
    randomElement

  random ( group ) == random ( group , 20 )

  order ( group ) ==
    knownGroup? group
    ord

  degree ( group ) == # pointList group

  base ( group ) ==
    knownGroup? group
    groupBase := nil()$(L S)
    for i in baseOfGroup repeat
      groupBase := cons ( supp.i , groupBase )
    reverse groupBase

  wordsForStrongGenerators ( group ) ==
    knownGroup? group
```

```
    wordlist

  coerce ( gp : L PERM S ) : % ==
    result : REC2 := [ 0 , [] , [] , [] , [] , [] ]
    group         := [ gp , result ]

  permutationGroup ( gp : L PERM S ) : % ==
    result : REC2 := [ 0 , [] , [] , [] , [] , [] ]
    group         := [ gp , result ]

  coerce(group: %) : OUT ==
    outList := nil()$(L OUT)
    gp : L PERM S := group.gens
    for i in (maxIndex gp)..1 by -1 repeat
      outList := cons(coerce gp.i, outList)
    postfix(outputForm(">":SYM),postfix(commaSeparate outList,outputForm("<":SYM)))

  orbit ( gp : % , el : S ) : FSET S ==
    elList : L S := [ el ]
    outList      := orbitInternal ( gp , elList )
    outSet       := brace()$(FSET S)
    for i in 1..#outList repeat
      insert_! ( outList.i.1 , outSet )
    outSet

  orbits ( gp ) ==
    spp    := movedPoints gp
    orbits := nil()$(L FSET S)
    while cardinality spp > 0 repeat
      el       := extract_! spp
      orbitSet := orbit ( gp , el )
      orbits   := cons ( orbitSet , orbits )
      spp      := difference ( spp , orbitSet )
    brace orbits

  member? (p,  gp) ==
    wordProblem := false
    mi := memberInternal ( p , gp , true )
    mi.bool

  wordInStrongGenerators (p, gp ) ==
    mi := memberInternal ( inv p , gp , false )
    not mi.bool => error "p is not an element of gp"
    mi.lst

  wordInGenerators (p,  gp) ==
    lll : L NNI := wordInStrongGenerators (p, gp)
    outlist := nil()$(L NNI)
    for wd in lll repeat
      outlist := append ( outlist , wordlist.wd )
```

```
    shortenWord ( outlist , gp )

  gp1 < gp2 ==
    not empty? difference ( movedPoints gp1 , movedPoints gp2 ) => false
    not subgroup ( gp1 , gp2 ) => false
    order gp1 = order gp2 => false
    true

  gp1 <= gp2 ==
    not empty?  difference ( movedPoints gp1 , movedPoints gp2 ) => false
    subgroup ( gp1 , gp2 )

  gp1 = gp2 ==
    movedPoints gp1 ^= movedPoints gp2 => false
    if #(gp1.gens) <= #(gp2.gens) then
      not subgroup ( gp1 , gp2 ) => return false
    else
      not subgroup ( gp2 , gp1 ) => return false
    order gp1 = order gp2 => true
    false

  orbit ( gp : % , startSet : FSET S ) : FSET FSET S ==
    startList : L S := parts startSet
    outList         := orbitInternal ( gp , startList )
    outSet          := brace()$(FSET FSET S)
    for i in 1..#outList repeat
      newSet : FSET S := brace outList.i
      insert_! ( newSet , outSet )
    outSet

  orbit ( gp : % , startList : L S ) : FSET L S ==
    brace orbitInternal(gp, startList)

  initializeGroupForWordProblem ( gp , maxLoops , diff ) ==
    wordProblem    := true
    ord            := bsgs ( gp , maxLoops , diff )
    gp.information := [ ord , sgs , baseOfGroup , gporb , supp , wordlist ]
    void

  initializeGroupForWordProblem ( gp ) == initializeGroupForWordProblem ( gp , 0 , 1 )
```

———

— PERMGRP.dotabb —

```
"PERMGRP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PERMGRP"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"PERMGRP" -> "FSAGG"
```

———-

17.16 domain HACKPI Pi

— Pi.input —

```
)set break resume
)sys rm -f Pi.output
)spool Pi.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Pi
--R Pi  is a domain constructor
--R Abbreviation for Pi is HACKPI
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HACKPI
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %      ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,Integer) -> %               ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean       coerce : % -> Float
--R coerce : % -> DoubleFloat            coerce : Fraction Integer -> %
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             convert : % -> InputForm
--R convert : % -> DoubleFloat           convert : % -> Float
--R factor : % -> Factored %             gcd : List % -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R one? : % -> Boolean                  pi : () -> %
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")       ?rem? : (%,%) -> %
--R retract : % -> Fraction Integer      retract : % -> Integer
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R unit? : % -> Boolean                 unitCanonical : % -> %
```

```
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R convert : % -> Fraction SparseUnivariatePolynomial Integer
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

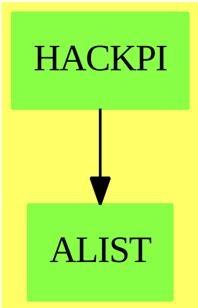
— **Pi.help** —

```
====================================================================
Pi examples
====================================================================

See Also:
o )show Pi
```

———-

17.16.1 Pi (HACKPI)



See

⇒ “Expression” (EXPR) 6.6.1 on page 691

Exports:

0	1	associates?	characteristic	coerce
convert	divide	euclideanSize	expressIdealMember	exquo
extendedEuclidean	extendedEuclidean	factor	gcd	gcdPolynomial
hash	inv	latex	lcm	multiEuclidean
one?	pi	prime?	principalIdeal	recip
retract	retractIfCan	retractIfCan	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan	unit?	unitCanonical
unitNormal	zero?	?*?	?**?	?+?
?-?	-?	?/?	?=?	?^?
?~=?	?quo?	?rem?		

— domain HACKPI Pi —

```
)abbrev domain HACKPI Pi
++ Author: Manuel Bronstein
++ Date Created: 21 Feb 1990
++ Date Last Updated: 12 Mai 1992
++ Description:
++ Symbolic fractions in %pi with integer coefficients;
++ The point for using Pi as the default domain for those fractions
++ is that Pi is coercible to the float types, and not Expression.

Pi(): Exports == Implementation where
  PZ ==> Polynomial Integer
  UP ==> SparseUnivariatePolynomial Integer
  RF ==> Fraction UP

  Exports ==> Join(Field, CharacteristicZero, RetractableTo Integer,
                   RetractableTo Fraction Integer, RealConstant,
                   CoercibleTo DoubleFloat, CoercibleTo Float,
                   ConvertibleTo RF, ConvertibleTo InputForm) with
```

```
    pi: () -> % ++ pi() returns the symbolic %pi.
  Implementation ==> RF add
    Rep := RF

    sympi := "%pi"::Symbol

    p2sf: UP -> DoubleFloat
    p2f : UP -> Float
    p2o : UP -> OutputForm
    p2i : UP -> InputForm
    p2p:  UP -> PZ

    pi()                     == (monomial(1, 1)$UP :: RF) pretend %
    convert(x:%):RF          == x pretend RF
    convert(x:%):Float       == x::Float
    convert(x:%):DoubleFloat == x::DoubleFloat
    coerce(x:%):DoubleFloat  == p2sf(numer x) / p2sf(denom x)
    coerce(x:%):Float        == p2f(numer x) / p2f(denom x)
    p2o p                    == outputForm(p, sympi::OutputForm)
    p2i p                    == convert p2p p

    p2p p ==
      ans:PZ := 0
      while p ^= 0 repeat
        ans := ans + monomial(leadingCoefficient(p)::PZ, sympi, degree p)
        p   := reductum p
      ans

    coerce(x:%):OutputForm ==
      (r := retractIfCan(x)@Union(UP, "failed")) case UP => p2o(r::UP)
      p2o(numer x) / p2o(denom x)

    convert(x:%):InputForm ==
      (r := retractIfCan(x)@Union(UP, "failed")) case UP => p2i(r::UP)
      p2i(numer x) / p2i(denom x)

    p2sf p ==
      map((x:Integer):DoubleFloat+->x::DoubleFloat, p)_
       $SparseUnivariatePolynomialFunctions2(Integer, DoubleFloat)
        (pi()$DoubleFloat)

    p2f p ==
      map((x:Integer):Float+->x::Float,p)_
       $SparseUnivariatePolynomialFunctions2(Integer, Float)
         (pi()$Float)
```

———–

— HACKPI.dotabb —

```
"HACKPI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HACKPI"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"HACKPI" -> "ALIST"
```

17.17 domain ACPLOT PlaneAlgebraicCurvePlot

— PlaneAlgebraicCurvePlot.input —

```
)set break resume
)sys rm -f PlaneAlgebraicCurvePlot.output
)spool PlaneAlgebraicCurvePlot.output
)set message test on
)set message auto off
)clear all

--S 1 of 5
sketch:=makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLOT
--R
--R   (1)                     ACPLOT
--R                         1         1      1         1
--R          y + x = 0,   - - <= x <= -,   - - <= y <= -
--R                         2         2      2         2
--R                         [0.5,- 0.5]
--R                         [- 0.5,0.5]
--R                                                  Type: PlaneAlgebraicCurvePlot
--E 1

--S 2 of 5
refined:=refine(sketch,0.1)
--R
--R   (2)                     ACPLOT
--R                         1         1      1         1
--R            y + x = 0,   - - <= x <= -,   - - <= y <= -
--R                         2         2      2         2
--R                         [0.5,- 0.5]
--R            [0.49600000000000083,- 0.49600000000000083]
--R            [0.49200000000000083,- 0.49200000000000083]
--R            [0.48800000000000082,- 0.48800000000000082]
--R            [0.48400000000000082,- 0.48400000000000082]
--R            [0.48000000000000081,- 0.48000000000000081]
--R            [0.47600000000000081,- 0.47600000000000081]
--R            [0.47200000000000081,- 0.47200000000000081]
```

```
--R         [0.4680000000000008,- 0.4680000000000008]
--R         [0.4640000000000008,- 0.4640000000000008]
--R         [0.4600000000000008,- 0.4600000000000008]
--R        [0.45600000000000079,- 0.45600000000000079]
--R        [0.45200000000000079,- 0.45200000000000079]
--R        [0.44800000000000079,- 0.44800000000000079]
--R        [0.44400000000000078,- 0.44400000000000078]
--R        [0.44000000000000078,- 0.44000000000000078]
--R        [0.43600000000000078,- 0.43600000000000078]
--R        [0.43200000000000077,- 0.43200000000000077]
--R        [0.42800000000000077,- 0.42800000000000077]
--R        [0.42400000000000077,- 0.42400000000000077]
--R        [0.42000000000000076,- 0.42000000000000076]
--R        [0.41600000000000076,- 0.41600000000000076]
--R        [0.41200000000000075,- 0.41200000000000075]
--R        [0.40800000000000075,- 0.40800000000000075]
--R        [0.40400000000000075,- 0.40400000000000075]
--R        [0.40000000000000074,- 0.40000000000000074]
--R        [0.39600000000000074,- 0.39600000000000074]
--R        [0.39200000000000074,- 0.39200000000000074]
--R        [0.38800000000000073,- 0.38800000000000073]
--R        [0.38400000000000073,- 0.38400000000000073]
--R        [0.38000000000000073,- 0.38000000000000073]
--R        [0.37600000000000072,- 0.37600000000000072]
--R        [0.37200000000000072,- 0.37200000000000072]
--R        [0.36800000000000072,- 0.36800000000000072]
--R        [0.36400000000000071,- 0.36400000000000071]
--R        [0.36000000000000071,- 0.36000000000000071]
--R         [0.3560000000000007,- 0.3560000000000007]
--R         [0.3520000000000007,- 0.3520000000000007]
--R         [0.3480000000000007,- 0.3480000000000007]
--R        [0.34400000000000069,- 0.34400000000000069]
--R        [0.34000000000000069,- 0.34000000000000069]
--R        [0.33600000000000069,- 0.33600000000000069]
--R        [0.33200000000000068,- 0.33200000000000068]
--R        [0.32800000000000068,- 0.32800000000000068]
--R        [0.32400000000000068,- 0.32400000000000068]
--R        [0.32000000000000067,- 0.32000000000000067]
--R        [0.31600000000000067,- 0.31600000000000067]
--R        [0.31200000000000067,- 0.31200000000000067]
--R        [0.30800000000000066,- 0.30800000000000066]
--R        [0.30400000000000066,- 0.30400000000000066]
--R        [0.30000000000000066,- 0.30000000000000066]
--R        [0.29600000000000065,- 0.29600000000000065]
--R        [0.29200000000000065,- 0.29200000000000065]
--R        [0.28800000000000064,- 0.28800000000000064]
--R        [0.28400000000000064,- 0.28400000000000064]
--R        [0.28000000000000064,- 0.28000000000000064]
--R        [0.27600000000000063,- 0.27600000000000063]
--R        [0.27200000000000063,- 0.27200000000000063]
```

```
--R          [0.26800000000000063,- 0.26800000000000063]
--R          [0.26400000000000062,- 0.26400000000000062]
--R          [0.26000000000000062,- 0.26000000000000062]
--R          [0.25600000000000062,- 0.25600000000000062]
--R          [0.25200000000000061,- 0.25200000000000061]
--R          [0.24800000000000061,- 0.24800000000000061]
--R          [0.24400000000000061,- 0.24400000000000061]
--R           [0.2400000000000006,- 0.2400000000000006]
--R           [0.2360000000000006,- 0.2360000000000006]
--R          [0.23200000000000059,- 0.23200000000000059]
--R          [0.22800000000000059,- 0.22800000000000059]
--R          [0.22400000000000059,- 0.22400000000000059]
--R          [0.22000000000000058,- 0.22000000000000058]
--R          [0.21600000000000058,- 0.21600000000000058]
--R          [0.21200000000000058,- 0.21200000000000058]
--R          [0.20800000000000057,- 0.20800000000000057]
--R          [0.20400000000000057,- 0.20400000000000057]
--R          [0.20000000000000057,- 0.20000000000000057]
--R          [0.19600000000000056,- 0.19600000000000056]
--R          [0.19200000000000056,- 0.19200000000000056]
--R          [0.18800000000000056,- 0.18800000000000056]
--R          [0.18400000000000055,- 0.18400000000000055]
--R          [0.18000000000000055,- 0.18000000000000055]
--R          [0.17600000000000054,- 0.17600000000000054]
--R          [0.17200000000000054,- 0.17200000000000054]
--R          [0.16800000000000054,- 0.16800000000000054]
--R          [0.16400000000000053,- 0.16400000000000053]
--R          [0.16000000000000053,- 0.16000000000000053]
--R          [0.15600000000000053,- 0.15600000000000053]
--R          [0.15200000000000052,- 0.15200000000000052]
--R          [0.14800000000000052,- 0.14800000000000052]
--R          [0.14400000000000052,- 0.14400000000000052]
--R          [0.14000000000000051,- 0.14000000000000051]
--R          [0.13600000000000051,- 0.13600000000000051]
--R          [0.13200000000000051,- 0.13200000000000051]
--R           [0.1280000000000005,- 0.1280000000000005]
--R           [0.1240000000000005,- 0.1240000000000005]
--R           [0.1200000000000005,- 0.1200000000000005]
--R          [0.11600000000000049,- 0.11600000000000049]
--R          [0.11200000000000049,- 0.11200000000000049]
--R          [0.10800000000000048,- 0.10800000000000048]
--R          [0.10400000000000048,- 0.10400000000000048]
--R          [0.10000000000000048,- 0.10000000000000048]
--R        [9.6000000000000474E-2,- 9.6000000000000474E-2]
--R         [9.200000000000047E-2,- 9.200000000000047E-2]
--R        [8.8000000000000467E-2,- 8.8000000000000467E-2]
--R        [8.4000000000000463E-2,- 8.4000000000000463E-2]
--R         [8.000000000000046E-2,- 8.000000000000046E-2]
--R        [7.6000000000000456E-2,- 7.6000000000000456E-2]
--R        [7.2000000000000453E-2,- 7.2000000000000453E-2]
```

```
--R        [6.8000000000000449E-2,- 6.8000000000000449E-2]
--R        [6.4000000000000445E-2,- 6.4000000000000445E-2]
--R        [6.0000000000000442E-2,- 6.0000000000000442E-2]
--R        [5.6000000000000438E-2,- 5.6000000000000438E-2]
--R        [5.2000000000000435E-2,- 5.2000000000000435E-2]
--R        [4.8000000000000431E-2,- 4.8000000000000431E-2]
--R        [4.4000000000000428E-2,- 4.4000000000000428E-2]
--R        [4.0000000000000424E-2,- 4.0000000000000424E-2]
--R        [3.6000000000000421E-2,- 3.6000000000000421E-2]
--R        [3.2000000000000417E-2,- 3.2000000000000417E-2]
--R        [2.8000000000000417E-2,- 2.8000000000000417E-2]
--R        [2.4000000000000417E-2,- 2.4000000000000417E-2]
--R        [2.0000000000000417E-2,- 2.0000000000000417E-2]
--R        [1.6000000000000417E-2,- 1.6000000000000417E-2]
--R        [1.2000000000000417E-2,- 1.2000000000000417E-2]
--R        [8.0000000000004165E-3,- 8.0000000000004165E-3]
--R        [4.0000000000004164E-3,- 4.0000000000004164E-3]
--R        [4.163336342344337E-16,- 4.163336342344337E-16]
--R        [- 3.9999999999995837E-3,3.9999999999995837E-3]
--R        [- 7.9999999999995838E-3,7.9999999999995838E-3]
--R        [- 1.1999999999999584E-2,1.1999999999999584E-2]
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--R                                                 Type: PlaneAlgebraicCurvePlot
--E 2

--S 3 of 5
listBranches(sketch)
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--E 3

--S 4 of 5
listBranches(refined)
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--R     ]
--R                                        Type: List List Point DoubleFloat
--E 4

--S 5 of 5
)show ACPLOT
--R PlaneAlgebraicCurvePlot  is a domain constructor
--R Abbreviation for PlaneAlgebraicCurvePlot is ACPLOT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ACPLOT
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              refine : (%,DoubleFloat) -> %
--R xRange : % -> Segment DoubleFloat     yRange : % -> Segment DoubleFloat
--R listBranches : % -> List List Point DoubleFloat
--R makeSketch : (Polynomial Integer,Symbol,Symbol,Segment Fraction Integer,Segment Fraction
--R
--E 5

)spool
)lisp (bye)
```

— **PlaneAlgebraicCurvePlot.help** —

```
====================================================================
PlaneAlgebraicCurvePlot examples
====================================================================

sketch:=makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLOT

                         ACPLOT
                      1         1      1         1
        y + x = 0,  - - <= x <= -,   - - <= y <= -
                      2         2      2         2
                        [0.5,- 0.5]
                        [- 0.5,0.5]


refined:=refine(sketch,0.1)
```

```
                    ACPLOT
                 1         1      1         1
y + x = 0,     - - <= x <= -,   - - <= y <= -
                 2         2      2         2
                  [0.5,- 0.5]
[0.49600000000000083,- 0.49600000000000083]
[0.49200000000000083,- 0.49200000000000083]
[0.48800000000000082,- 0.48800000000000082]
[0.48400000000000082,- 0.48400000000000082]
...
[- 0.48399999999999999,0.48399999999999999]
[- 0.48799999999999999,0.48799999999999999]
[- 0.49199999999999999,0.49199999999999999]
              [- 0.496,0.496]
                [- 0.5,0.5]
```

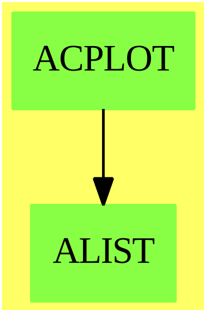
```
listBranches(sketch)
```

```
[[[0.5,- 0.5],[- 0.5,0.5]]]
```

```
listBranches(refined)
```

```
[
  [[0.5,- 0.5], [0.49600000000000083,- 0.49600000000000083],
   [0.49200000000000083,- 0.49200000000000083],
   [0.48800000000000082,- 0.48800000000000082],
   ...
   [- 0.48399999999999999,0.48399999999999999],
   [- 0.48799999999999999,0.48799999999999999],
   [- 0.49199999999999999,0.49199999999999999], [- 0.496,0.496],
```

17.17.1 PlaneAlgebraicCurvePlot (ACPLOT)



Exports:
coerce listBranches makeSketch refine xRange yRange

— domain ACPLOT PlaneAlgebraicCurvePlot —

```
)abbrev domain ACPLOT PlaneAlgebraicCurvePlot
++ Author: Clifton J. Williamson and Timothy Daly
++ Date Created: Fall 1988
++ Date Last Updated: 27 April 1990
++ Keywords: algebraic curve, non-singular, plot
++ Examples:
++ References:
++ Description:
++ Plot a NON-SINGULAR plane algebraic curve p(x,y) = 0.

PlaneAlgebraicCurvePlot(): PlottablePlaneCurveCategory _
 with

  makeSketch:(Polynomial Integer,Symbol,Symbol,Segment Fraction Integer,_
               Segment Fraction Integer) -> %
    ++ makeSketch(p,x,y,a..b,c..d) creates an ACPLOT of the
    ++ curve \spad{p = 0} in the region a <= x <= b, c <= y <= d.
    ++ More specifically, 'makeSketch' plots a non-singular algebraic curve
    ++ \spad{p = 0} in an rectangular region xMin <= x <= xMax,
    ++ yMin <= y <= yMax. The user inputs
    ++ \spad{makeSketch(p,x,y,xMin..xMax,yMin..yMax)}.
    ++ Here p is a polynomial in the variables x and y with
    ++ integer coefficients (p belongs to the domain
    ++ \spad{Polynomial Integer}). The case
    ++ where p is a polynomial in only one of the variables is
    ++ allowed.  The variables x and y are input to specify the
    ++ the coordinate axes.  The horizontal axis is the x-axis and
    ++ the vertical axis is the y-axis.  The rational numbers
    ++ xMin,...,yMax specify the boundaries of the region in
    ++ which the curve is to be plotted.
```

```
   ++
   ++X makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLOT

refine:(%,DoubleFloat) -> %
   ++ refine(p,x) is not documented
   ++
   ++X sketch:=makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLOT
   ++X refined:=refine(sketch,0.1)

== add

import PointPackage DoubleFloat
import Plot
import RealSolvePackage

BoundaryPts ==> Record(left:   List Point DoubleFloat,_
                       right:  List Point DoubleFloat,_
                       bottom: List Point DoubleFloat,_
                       top:    List Point DoubleFloat)

NewPtInfo   ==> Record(newPt: Point DoubleFloat,_
                       type:  String)

Corners     ==> Record(minXVal: DoubleFloat,_
                       maxXVal: DoubleFloat,_
                       minYVal: DoubleFloat,_
                       maxYVal: DoubleFloat)

kinte       ==> solve$RealSolvePackage()

rsolve      ==> realSolve$RealSolvePackage()

singValBetween?:(DoubleFloat,DoubleFloat,List DoubleFloat) -> Boolean

segmentInfo:(DoubleFloat -> DoubleFloat,DoubleFloat,DoubleFloat,_
             List DoubleFloat,List DoubleFloat,List DoubleFloat,_
             DoubleFloat,DoubleFloat) -> _
  Record(seg:Segment DoubleFloat,_
         left: DoubleFloat,_
         lowerVals: List DoubleFloat,_
         upperVals:List DoubleFloat)

swapCoords:Point DoubleFloat -> Point DoubleFloat

samePlottedPt?:(Point DoubleFloat,Point DoubleFloat) -> Boolean

findPtOnList:(Point DoubleFloat,List Point DoubleFloat) -> _
  Union(Point DoubleFloat,"failed")

makeCorners:(DoubleFloat,DoubleFloat,DoubleFloat,DoubleFloat) -> Corners
```

```
getXMin: Corners -> DoubleFloat

getXMax: Corners -> DoubleFloat

getYMin: Corners -> DoubleFloat

getYMax: Corners -> DoubleFloat

SFPolyToUPoly:Polynomial DoubleFloat -> _
  SparseUnivariatePolynomial DoubleFloat

RNPolyToUPoly:Polynomial Fraction Integer -> _
  SparseUnivariatePolynomial Fraction Integer

coerceCoefsToSFs:Polynomial Integer -> Polynomial DoubleFloat

coerceCoefsToRNs:Polynomial Integer -> Polynomial Fraction Integer

RNtoSF:Fraction Integer -> DoubleFloat

RNtoNF:Fraction Integer -> Float

SFtoNF:DoubleFloat -> Float

listPtsOnHorizBdry:(Polynomial Fraction Integer,Symbol,Fraction Integer,_
                    Float,Float) -> _
  List Point DoubleFloat

listPtsOnVertBdry:(Polynomial Fraction Integer,Symbol,Fraction Integer,_
                   Float,Float) -> _
  List Point DoubleFloat

listPtsInRect:(List List Float,Float,Float,Float,Float) -> _
  List Point DoubleFloat

ptsSuchThat?:(List List Float,List Float -> Boolean) -> Boolean

inRect?:(List Float,Float,Float,Float,Float) -> Boolean

onHorzSeg?:(List Float,Float,Float,Float) -> Boolean

onVertSeg?:(List Float,Float,Float,Float) -> Boolean

newX:(List List Float,List List Float,Float,Float,Float,Fraction Integer,_
      Fraction Integer) -> Fraction Integer

newY:(List List Float,List List Float,Float,Float,Float,_
      Fraction Integer,Fraction Integer) -> Fraction Integer
```

```
  makeOneVarSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                    Fraction Integer,Fraction Integer,Fraction Integer,_
                    Symbol) -> %

  makeLineSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                  Fraction Integer,Fraction Integer,Fraction Integer) -> %

  makeRatFcnSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                    Fraction Integer,Fraction Integer,Fraction Integer,_
                    Symbol) -> %

  makeGeneralSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                     Fraction Integer,Fraction Integer,Fraction Integer) -> %

  traceBranches:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
                 Polynomial DoubleFloat,Symbol,Symbol,Corners,DoubleFloat,_
                 DoubleFloat,PositiveInteger, List Point DoubleFloat,_
                 BoundaryPts) -> List List Point DoubleFloat

  dummyFirstPt:(Point DoubleFloat,Polynomial DoubleFloat,_
                Polynomial DoubleFloat,Symbol,Symbol,List Point DoubleFloat,_
                List Point DoubleFloat,List Point DoubleFloat,_
                List Point DoubleFloat) -> Point DoubleFloat

  listPtsOnSegment:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
                    Polynomial DoubleFloat,Symbol,Symbol,Point DoubleFloat,_
                    Point DoubleFloat,Corners, DoubleFloat,DoubleFloat,_
                    PositiveInteger,List Point DoubleFloat,_
                    List Point DoubleFloat) -> List List Point DoubleFloat

  listPtsOnLoop:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
                 Polynomial DoubleFloat,Symbol,Symbol,Point DoubleFloat,_
                 Corners, DoubleFloat,DoubleFloat,PositiveInteger,_
                 List Point DoubleFloat,List Point DoubleFloat) -> _
                 List List Point DoubleFloat

  computeNextPt:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
                 Polynomial DoubleFloat,Symbol,Symbol,Point DoubleFloat,_
                 Point DoubleFloat,Corners, DoubleFloat,DoubleFloat,_
                 PositiveInteger,List Point DoubleFloat,_
                 List Point DoubleFloat) -> NewPtInfo

  newtonApprox:(SparseUnivariatePolynomial DoubleFloat, DoubleFloat, _
                DoubleFloat, PositiveInteger) -> Union(DoubleFloat, "failed")

--% representation

  Rep := Record(poly    : Polynomial Integer,_
                xVar    : Symbol,_
                yVar    : Symbol,_
```

```
              minXVal : Fraction Integer,_
              maxXVal : Fraction Integer,_
              minYVal : Fraction Integer,_
              maxYVal : Fraction Integer,_
              bdryPts : BoundaryPts,_
              hTanPts : List Point DoubleFloat,_
              vTanPts : List Point DoubleFloat,_
              branches: List List Point DoubleFloat)

--% global constants

  EPSILON : Float := .000001 -- precision to which realSolve finds roots
  PLOTERR : DoubleFloat := float(1,-3,10)
    -- maximum allowable difference in each coordinate when
    -- determining if 2 plotted points are equal

--% global flags

  NADA   : String := "nothing in particular"
  BDRY   : String := "boundary point"
  CRIT   : String := "critical point"
  BOTTOM : String := "bottom"
  TOP    : String := "top"

--% hacks

  NFtoSF: Float -> DoubleFloat
  NFtoSF x == 0 + convert(x)$Float

--% points
  makePt: (DoubleFloat,DoubleFloat) -> Point DoubleFloat
  makePt(xx,yy) == point(l : List DoubleFloat := [xx,yy])

  swapCoords(pt) == makePt(yCoord pt,xCoord pt)

  samePlottedPt?(p0,p1) ==
    -- determines if p1 lies in a square with side 2 PLOTERR
    -- centered at p0
    x0 := xCoord p0; y0 := yCoord p0
    x1 := xCoord p1; y1 := yCoord p1
    (abs(x1-x0) < PLOTERR) and (abs(y1-y0) < PLOTERR)

  findPtOnList(pt,pointList) ==
    for point in pointList repeat
      samePlottedPt?(pt,point) => return point
    "failed"

--% corners

  makeCorners(xMinSF,xMaxSF,yMinSF,yMaxSF) ==
```

```
    [xMinSF,xMaxSF,yMinSF,yMaxSF]

  getXMin(corners) == corners.minXVal
  getXMax(corners) == corners.maxXVal
  getYMin(corners) == corners.minYVal
  getYMax(corners) == corners.maxYVal

--% coercions

  SFPolyToUPoly(p) ==
  -- 'p' is of type Polynomial, but has only one variable
    zero? p => 0
    monomial(leadingCoefficient p,totalDegree p) +
       SFPolyToUPoly(reductum p)

  RNPolyToUPoly(p) ==
  -- 'p' is of type Polynomial, but has only one variable
    zero? p => 0
    monomial(leadingCoefficient p,totalDegree p) +
        RNPolyToUPoly(reductum p)

  coerceCoefsToSFs(p) ==
  -- coefficients of 'p' are coerced to be DoubleFloat's
    map(coerce,p)$PolynomialFunctions2(Integer,DoubleFloat)

  coerceCoefsToRNs(p) ==
  -- coefficients of 'p' are coerced to be DoubleFloat's
    map(coerce,p)$PolynomialFunctions2(Integer,Fraction Integer)

  RNtoSF(r) == coerce(r)@DoubleFloat
  RNtoNF(r) == coerce(r)@Float
  SFtoNF(x) == convert(x)@Float

--% computation of special points

  listPtsOnHorizBdry(pRN,y,y0,xMinNF,xMaxNF) ==
  -- strict inequality here: corners on vertical boundary
    pointList : List Point DoubleFloat := nil()
    ySF := RNtoSF(y0)
    f := eval(pRN,y,y0)
    roots : List Float := kinte(f,EPSILON)
    for root in roots repeat
      if (xMinNF < root) and (root < xMaxNF) then
        pointList := cons(makePt(NFtoSF root, ySF), pointList)
    pointList

  listPtsOnVertBdry(pRN,x,x0,yMinNF,yMaxNF) ==
    pointList : List Point DoubleFloat := nil()
    xSF := RNtoSF(x0)
    f := eval(pRN,x,x0)
```

```
    roots : List Float := kinte(f,EPSILON)
    for root in roots repeat
      if (yMinNF <= root) and (root <= yMaxNF) then
        pointList := cons(makePt(xSF, NFtoSF root), pointList)
    pointList

  listPtsInRect(points,xMin,xMax,yMin,yMax) ==
    pointList : List Point DoubleFloat := nil()
    for point in points repeat
      xx := first point; yy := second point
      if (xMin<=xx) and (xx<=xMax) and (yMin<=yy) and (yy<=yMax) then
        pointList := cons(makePt(NFtoSF xx,NFtoSF yy),pointList)
    pointList

  ptsSuchThat?(points,pred) ==
    for point in points repeat
      if pred point then return true
    false

  inRect?(point,xMinNF,xMaxNF,yMinNF,yMaxNF) ==
    xx := first point; yy := second point
    xMinNF <= xx and xx <= xMaxNF and yMinNF <= yy and yy <= yMaxNF

  onHorzSeg?(point,xMinNF,xMaxNF,yNF) ==
    xx := first point; yy := second point
    yy = yNF and xMinNF <= xx and xx <= xMaxNF

  onVertSeg?(point,yMinNF,yMaxNF,xNF) ==
    xx := first point; yy := second point
    xx = xNF and yMinNF <= yy and yy <= yMaxNF

  newX(vtanPts,singPts,yMinNF,yMaxNF,xNF,xRN,horizInc) ==
    xNewNF := xNF + RNtoNF horizInc
    xRtNF := max(xNF,xNewNF); xLftNF := min(xNF,xNewNF)
--  ptsSuchThat?(singPts,inRect?(#1,xLftNF,xRtNF,yMinNF,yMaxNF)) =>
    foo : List Float -> Boolean := x +-> inRect?(x,xLftNF,xRtNF,yMinNF,yMaxNF)
    ptsSuchThat?(singPts,foo) =>
      newX(vtanPts,singPts,yMinNF,yMaxNF,xNF,xRN,_
        horizInc/2::(Fraction Integer))
--  ptsSuchThat?(vtanPts,onVertSeg?(#1,yMinNF,yMaxNF,xNewNF)) =>
    goo : List Float -> Boolean := x +-> onVertSeg?(x,yMinNF,yMaxNF,xNewNF)
    ptsSuchThat?(vtanPts,goo) =>
      newX(vtanPts,singPts,yMinNF,yMaxNF,xNF,xRN,_
        horizInc/2::(Fraction Integer))
    xRN + horizInc

  newY(htanPts,singPts,xMinNF,xMaxNF,yNF,yRN,vertInc) ==
    yNewNF := yNF + RNtoNF vertInc
    yTopNF := max(yNF,yNewNF); yBotNF := min(yNF,yNewNF)
--  ptsSuchThat?(singPts,inRect?(#1,xMinNF,xMaxNF,yBotNF,yTopNF)) =>
```

```
    foo : List Float -> Boolean := x +-> inRect?(x,xMinNF,xMaxNF,yBotNF,yTopNF)
    ptsSuchThat?(singPts,foo) =>
      newY(htanPts,singPts,xMinNF,xMaxNF,yNF,yRN,_
        vertInc/2::(Fraction Integer))
--  ptsSuchThat?(htanPts,onHorzSeg?(#1,xMinNF,xMaxNF,yNewNF)) =>
    goo : List Float -> Boolean := x +-> onHorzSeg?(x,xMinNF,xMaxNF,yNewNF)
    ptsSuchThat?(htanPts,goo) =>
      newY(htanPts,singPts,xMinNF,xMaxNF,yNF,yRN,_
        vertInc/2::(Fraction Integer))
    yRN + vertInc

--% creation of sketches

  makeSketch(p,x,y,xRange,yRange) ==
    xMin := lo xRange; xMax := hi xRange
    yMin := lo yRange; yMax := hi yRange
    -- test input for consistency
    xMax <= xMin =>
      error "makeSketch: bad range for first variable"
    yMax <= yMin =>
      error "makeSketch: bad range for second variable"
    varList := variables p
    # varList > 2 =>
      error "makeSketch: polynomial in more than 2 variables"
    # varList = 0 =>
      error "makeSketch: constant polynomial"
    -- polynomial in 1 variable
    # varList = 1 =>
      (not member?(x,varList)) and (not member?(y,varList)) =>
        error "makeSketch: bad variables"
      makeOneVarSketch(p,x,y,xMin,xMax,yMin,yMax,first varList)
    -- polynomial in 2 variables
    (not member?(x,varList)) or (not member?(y,varList)) =>
      error "makeSketch: bad variables"
    totalDegree p = 1 =>
      makeLineSketch(p,x,y,xMin,xMax,yMin,yMax)
    -- polynomial is linear in one variable
    -- y is a rational function of x
    degree(p,y) = 1 =>
      makeRatFcnSketch(p,x,y,xMin,xMax,yMin,yMax,y)
    -- x is a rational function of y
    degree(p,x) = 1 =>
      makeRatFcnSketch(p,x,y,xMin,xMax,yMin,yMax,x)
    -- the general case
    makeGeneralSketch(p,x,y,xMin,xMax,yMin,yMax)

--% special cases

  makeOneVarSketch(p,x,y,xMin,xMax,yMin,yMax,var) ==
  -- the case where 'p' is a polynomial in only one variable
```

```
-- the graph consists of horizontal or vertical lines
  if var = x then
    minVal := RNtoNF xMin
    maxVal := RNtoNF xMax
  else
    minVal := RNtoNF yMin
    maxVal := RNtoNF yMax
  lf : List Point DoubleFloat := nil()
  rt : List Point DoubleFloat := nil()
  bt : List Point DoubleFloat := nil()
  tp : List Point DoubleFloat := nil()
  htans : List Point DoubleFloat := nil()
  vtans : List Point DoubleFloat := nil()
  bran : List List Point DoubleFloat := nil()
  roots := kinte(p,EPSILON)
  sketchRoots : List DoubleFloat := nil()
  for root in roots repeat
    if (minVal <= root) and (root <= maxVal) then
        sketchRoots := cons(NFtoSF root,sketchRoots)
  null sketchRoots =>
    [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]
  if var = x then
    yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
    for rootSF in sketchRoots repeat
        tp := cons(pt1 := makePt(rootSF,yMaxSF),tp)
        bt := cons(pt2 := makePt(rootSF,yMinSF),bt)
        branch : List Point DoubleFloat := [pt1,pt2]
        bran := cons(branch,bran)
  else
    xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
    for rootSF in sketchRoots repeat
        rt := cons(pt1 := makePt(xMaxSF,rootSF),rt)
        lf := cons(pt2 := makePt(xMinSF,rootSF),lf)
        branch : List Point DoubleFloat := [pt1,pt2]
        bran := cons(branch,bran)
  [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]

makeLineSketch(p,x,y,xMin,xMax,yMin,yMax) ==
-- the case where p(x,y) = a x + b y + c with a ^= 0, b ^= 0
-- this is a line which is neither vertical nor horizontal
  xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
  yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
  -- determine the coefficients a, b, and c
  a := ground(coefficient(p,x,1)) :: DoubleFloat
  b := ground(coefficient(p,y,1)) :: DoubleFloat
  c := ground(coefficient(coefficient(p,x,0),y,0)) :: DoubleFloat
  lf : List Point DoubleFloat := nil()
  rt : List Point DoubleFloat := nil()
  bt : List Point DoubleFloat := nil()
  tp : List Point DoubleFloat := nil()
```

```
  htans : List Point DoubleFloat := nil()
  vtans : List Point DoubleFloat := nil()
  branch : List Point DoubleFloat := nil()
  bran : List List Point DoubleFloat := nil()
  -- compute x coordinate of point on line with y = yMin
  xBottom := (- b*yMinSF - c)/a
  -- compute x coordinate of point on line with y = yMax
  xTop    := (- b*yMaxSF - c)/a
  -- compute y coordinate of point on line with x = xMin
  yLeft   := (- a*xMinSF - c)/b
  -- compute y coordinate of point on line with x = xMax
  yRight  := (- a*xMaxSF - c)/b
  -- determine which of the above 4 points are in the region
  -- to be plotted and list them as a branch
  if (xMinSF < xBottom) and (xBottom < xMaxSF) then
      bt := cons(pt := makePt(xBottom,yMinSF),bt)
      branch := cons(pt,branch)
  if (xMinSF < xTop) and (xTop < xMaxSF) then
      tp := cons(pt := makePt(xTop,yMaxSF),tp)
      branch := cons(pt,branch)
  if (yMinSF <= yLeft) and (yLeft <= yMaxSF) then
      lf := cons(pt := makePt(xMinSF,yLeft),lf)
      branch := cons(pt,branch)
  if (yMinSF <= yRight) and (yRight <= yMaxSF) then
      rt := cons(pt := makePt(xMaxSF,yRight),rt)
      branch := cons(pt,branch)
  bran := cons(branch,bran)
  [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]

singValBetween?(xCurrent,xNext,xSingList) ==
  for xVal in xSingList repeat
    (xCurrent < xVal) and (xVal < xNext) => return true
  false

segmentInfo(f,lo,hi,botList,topList,singList,minSF,maxSF) ==
  repeat
    -- 'current' is the smallest element of 'topList' and 'botList'
    -- 'currentFrom' records the list from which it was taken
    if null topList then
      if null botList then
        return [segment(lo,hi),hi,nil(),nil()]
      else
        current := first botList
        botList := rest  botList
        currentFrom := BOTTOM
    else
      if null botList then
        current := first topList
        topList := rest  topList
        currentFrom := TOP
```

```
      else
        bot := first botList
        top := first topList
        if bot < top then
          current := bot
          botList := rest botList
          currentFrom := BOTTOM
        else
          current := top
          topList := rest topList
          currentFrom := TOP
    -- 'nxt' is the next smallest element of 'topList'
    --  and 'botList'
    -- 'nextFrom' records the list from which it was taken
    if null topList then
      if null botList then
        return [segment(lo,hi),hi,nil(),nil()]
      else
        nxt := first botList
        botList := rest botList
        nextFrom := BOTTOM
    else
      if null botList then
        nxt := first topList
        topList := rest topList
        nextFrom := TOP
      else
        bot := first botList
        top := first topList
        if bot < top then
          nxt := bot
          botList := rest botList
          nextFrom := BOTTOM
        else
          nxt := top
          topList := rest topList
          nextFrom := TOP
    if currentFrom = nextFrom then
      if singValBetween?(current,nxt,singList) then
        return [segment(lo,current),nxt,botList,topList]
      else
        val := f((nxt - current)/2::DoubleFloat)
        if (val <= minSF) or (val >= maxSF) then
          return [segment(lo,current),nxt,botList,topList]
    else
      if singValBetween?(current,nxt,singList) then
        return [segment(lo,current),nxt,botList,topList]

makeRatFcnSketch(p,x,y,xMin,xMax,yMin,yMax,depVar) ==
-- the case where p(x,y) is linear in x or y
```

```
-- Thus, one variable is a rational function of the other.
-- Therefore, we may use the 2-dimensional function plotting
-- package.  The only problem is determining the intervals on
-- on which the function is to be plotted.
--!! corners: e.g. upper left corner is on graph with y' > 0
  factoredP := p ::(Factored Polynomial Integer)
  numberOfFactors(factoredP) > 1 =>
      error "reducible polynomial"  --!! sketch each factor
  dpdx := differentiate(p,x)
  dpdy := differentiate(p,y)
  pRN := coerceCoefsToRNs p
  xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
  yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
  xMinNF := RNtoNF xMin; xMaxNF := RNtoNF xMax
  yMinNF := RNtoNF yMin; yMaxNF := RNtoNF yMax
  -- 'p' is of degree 1 in the variable 'depVar'.
  -- Thus, 'depVar' is a rational function of the other variable.
  num := -coefficient(p,depVar,0)
  den :=  coefficient(p,depVar,1)
  numUPolySF := SFPolyToUPoly(coerceCoefsToSFs(num))
  denUPolySF := SFPolyToUPoly(coerceCoefsToSFs(den))
  -- this is the rational function
  f : DoubleFloat -> DoubleFloat := s +-> elt(numUPolySF,s)/elt(denUPolySF,s)
  -- values of the dependent and independent variables
  if depVar = x then
    indVarMin   := yMin;   indVarMax   := yMax
    indVarMinNF := yMinNF; indVarMaxNF := yMaxNF
    indVarMinSF := yMinSF; indVarMaxSF := yMaxSF
    depVarMin   := xMin;   depVarMax   := xMax
    depVarMinSF := xMinSF; depVarMaxSF := xMaxSF
  else
    indVarMin   := xMin;   indVarMax   := xMax
    indVarMinNF := xMinNF; indVarMaxNF := xMaxNF
    indVarMinSF := xMinSF; indVarMaxSF := xMaxSF
    depVarMin   := yMin;   depVarMax   := yMax
    depVarMinSF := yMinSF; depVarMaxSF := yMaxSF
  -- Create lists of critical points.
  htanPts := rsolve([p,dpdx],[x,y],EPSILON)
  vtanPts := rsolve([p,dpdy],[x,y],EPSILON)
  htans := listPtsInRect(htanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
  vtans := listPtsInRect(vtanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
  -- Create lists which will contain boundary points.
  lf : List Point DoubleFloat := nil()
  rt : List Point DoubleFloat := nil()
  bt : List Point DoubleFloat := nil()
  tp : List Point DoubleFloat := nil()
  -- Determine values of the independent variable at the which
  -- the rational function has a pole as well as the values of
  -- the independent variable for which there is a point on the
  -- upper or lower boundary.
```

```
singList : List DoubleFloat :=
  roots : List Float := kinte(den,EPSILON)
  outList : List DoubleFloat := nil()
  for root in roots repeat
    if (indVarMinNF < root) and (root < indVarMaxNF) then
      outList := cons(NFtoSF root,outList)
  sort((x,y) +-> x < y, outList)
topList : List DoubleFloat :=
  roots : List Float := kinte(eval(pRN,depVar,depVarMax),EPSILON)
  outList : List DoubleFloat := nil()
  for root in roots repeat
    if (indVarMinNF < root) and (root < indVarMaxNF) then
      outList := cons(NFtoSF root,outList)
  sort((x,y) +-> x < y, outList)
botList : List DoubleFloat :=
  roots : List Float := kinte(eval(pRN,depVar,depVarMin),EPSILON)
  outList : List DoubleFloat := nil()
  for root in roots repeat
    if (indVarMinNF < root) and (root < indVarMaxNF) then
      outList := cons(NFtoSF root,outList)
  sort((x,y) +-> x < y, outList)
-- We wish to determine if the graph has points on the 'left'
-- and 'right' boundaries, so we compute the value of the
-- rational function at the lefthand and righthand values of
-- the dependent variable.  If the function has a singularity
-- on the left or right boundary, then 'leftVal' or 'rightVal'
-- is given a dummy valuewhich will convince the program that
-- there is no point on the left or right boundary.
denUPolyRN := RNPolyToUPoly(coerceCoefsToRNs(den))
if elt(denUPolyRN,indVarMin) = 0$(Fraction Integer) then
  leftVal  := depVarMinSF - (abs(depVarMinSF) + 1$DoubleFloat)
else
  leftVal  := f(indVarMinSF)
if elt(denUPolyRN,indVarMax) = 0$(Fraction Integer) then
  rightVal := depVarMinSF - (abs(depVarMinSF) + 1$DoubleFloat)
else
  rightVal := f(indVarMaxSF)
-- Now put boundary points on the appropriate lists.
if depVar = x then
  if (xMinSF < leftVal) and (leftVal < xMaxSF) then
    bt := cons(makePt(leftVal,yMinSF),bt)
  if (xMinSF < rightVal) and (rightVal < xMaxSF) then
    tp := cons(makePt(rightVal,yMaxSF),tp)
  for val in botList repeat
    lf := cons(makePt(xMinSF,val),lf)
  for val in topList repeat
    rt := cons(makePt(xMaxSF,val),rt)
else
  if (yMinSF < leftVal) and (leftVal < yMaxSF) then
    lf := cons(makePt(xMinSF,leftVal),lf)
```

```
    if (yMinSF < rightVal) and (rightVal < yMaxSF) then
      rt := cons(makePt(xMaxSF,rightVal),rt)
    for val in botList repeat
      bt := cons(makePt(val,yMinSF),bt)
    for val in topList repeat
      tp := cons(makePt(val,yMaxSF),tp)
  bran : List List Point DoubleFloat := nil()
  -- Determine segments on which the rational function is to
  -- be plotted.
  if (depVarMinSF < leftVal) and (leftVal < depVarMaxSF) then
    lo := indVarMinSF
  else
    if null topList then
      if null botList then
        return [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],_
                                    htans,vtans,bran]
      else
        lo := first botList
        botList := rest botList
    else
      if null botList then
        lo := first topList
        topList := rest topList
      else
        bot := first botList
        top := first topList
        if bot < top then
          lo := bot
          botList := rest botList
        else
          lo := top
          topList := rest topList
  hi := 0$DoubleFloat  -- @#$%^&* compiler
  if (depVarMinSF < rightVal) and (rightVal < depVarMaxSF) then
    hi := indVarMaxSF
  else
    if null topList then
      if null botList then
        error "makeRatFcnSketch: plot domain"
      else
        hi := last botList
        botList := remove(hi,botList)
    else
      if null botList then
        hi := last topList
        topList := remove(hi,topList)
      else
        bot := last botList
        top := last topList
        if bot > top then
```

```
          hi := bot
          botList := remove(hi,botList)
        else
          hi := top
          topList := remove(hi,topList)
    if (depVar = x) then
      (minSF := xMinSF; maxSF := xMaxSF)
    else
      (minSF := yMinSF; maxSF := yMaxSF)
    segList : List Segment DoubleFloat := nil()
    repeat
      segInfo := segmentInfo(f,lo,hi,botList,topList,singList,_
                             minSF,maxSF)
      segList := cons(segInfo.seg,segList)
      lo := segInfo.left
      botList := segInfo.lowerVals
      topList := segInfo.upperVals
      if lo = hi then break
    for segment in segList repeat
      RFPlot : Plot := plot(f,segment)
      curve := first(listBranches(RFPlot))
      if depVar = y then
        bran := cons(curve,bran)
      else
        bran := cons(map(swapCoords,curve),bran)
    [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]

--% the general case

  makeGeneralSketch(pol,x,y,xMin,xMax,yMin,yMax) ==
    --!! corners of region should not be on curve
    --!! enlarge region if necessary
    factoredPol := pol :: (Factored Polynomial Integer)
    numberOfFactors(factoredPol) > 1 =>
        error "reducible polynomial"  --!! sketch each factor
    p := nthFactor(factoredPol,1)
    dpdx := differentiate(p,x); dpdy := differentiate(p,y)
    xMinNF := RNtoNF xMin; xMaxNF := RNtoNF xMax
    yMinNF := RNtoNF yMin; yMaxNF := RNtoNF yMax
    -- compute singular points; error if singularities in region
    singPts := rsolve([p,dpdx,dpdy],[x,y],EPSILON)
--  ptsSuchThat?(singPts,inRect?(#1,xMinNF,xMaxNF,yMinNF,yMaxNF)) =>
    foo : List Float -> Boolean := s +-> inRect?(s,xMinNF,xMaxNF,yMinNF,yMaxNF)
    ptsSuchThat?(singPts,foo) =>
      error "singular pts in region of sketch"
    -- compute critical points
    htanPts := rsolve([p,dpdx],[x,y],EPSILON)
    vtanPts := rsolve([p,dpdy],[x,y],EPSILON)
    critPts := append(htanPts,vtanPts)
    -- if there are critical points on the boundary, then enlarge
```

```
    -- the region, but be sure that the new region does not contain
    -- any singular points
    hInc : Fraction Integer := (1/20) * (xMax - xMin)
    vInc : Fraction Integer := (1/20) * (yMax - yMin)
--  if ptsSuchThat?(critPts,onVertSeg?(#1,yMinNF,yMaxNF,xMinNF)) then
    foo : List Float -> Boolean := s +-> onVertSeg?(s,yMinNF,yMaxNF,xMinNF)
    if ptsSuchThat?(critPts,foo) then
      xMin := newX(critPts,singPts,yMinNF,yMaxNF,xMinNF,xMin,-hInc)
      xMinNF := RNtoNF xMin
--  if ptsSuchThat?(critPts,onVertSeg?(#1,yMinNF,yMaxNF,xMaxNF)) then
    foo : List Float -> Boolean := s +-> onVertSeg?(s,yMinNF,yMaxNF,xMaxNF)
    if ptsSuchThat?(critPts,foo) then
      xMax := newX(critPts,singPts,yMinNF,yMaxNF,xMaxNF,xMax,hInc)
      xMaxNF := RNtoNF xMax
--  if ptsSuchThat?(critPts,onHorzSeg?(#1,xMinNF,xMaxNF,yMinNF)) then
    foo : List Float -> Boolean := s +-> onHorzSeg?(s,xMinNF,xMaxNF,yMinNF)
    if ptsSuchThat?(critPts,foo) then
      yMin := newY(critPts,singPts,xMinNF,xMaxNF,yMinNF,yMin,-vInc)
      yMinNF := RNtoNF yMin
--  if ptsSuchThat?(critPts,onHorzSeg?(#1,xMinNF,xMaxNF,yMaxNF)) then
    foo : List Float -> Boolean := s +-> onHorzSeg?(s,xMinNF,xMaxNF,yMaxNF)
    if ptsSuchThat?(critPts,foo) then
      yMax := newY(critPts,singPts,xMinNF,xMaxNF,yMaxNF,yMax,vInc)
      yMaxNF := RNtoNF yMax
    htans := listPtsInRect(htanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
    vtans := listPtsInRect(vtanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
    crits := append(htans,vtans)
    -- conversions to DoubleFloats
    xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
    yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
    corners := makeCorners(xMinSF,xMaxSF,yMinSF,yMaxSF)
    pSF := coerceCoefsToSFs p
    dpdxSF := coerceCoefsToSFs dpdx
    dpdySF := coerceCoefsToSFs dpdy
    delta := min((xMaxSF - xMinSF)/25,(yMaxSF - yMinSF)/25)
    err := min(delta/100,PLOTERR/100)
    bound : PositiveInteger := 10
    -- compute points on the boundary
    pRN := coerceCoefsToRNs(p)
    lf : List Point DoubleFloat :=
      listPtsOnVertBdry(pRN,x,xMin,yMinNF,yMaxNF)
    rt : List Point DoubleFloat :=
      listPtsOnVertBdry(pRN,x,xMax,yMinNF,yMaxNF)
    bt : List Point DoubleFloat :=
      listPtsOnHorizBdry(pRN,y,yMin,xMinNF,xMaxNF)
    tp : List Point DoubleFloat :=
      listPtsOnHorizBdry(pRN,y,yMax,xMinNF,xMaxNF)
    bdPts : BoundaryPts := [lf,rt,bt,tp]
    bran := traceBranches(pSF,dpdxSF,dpdySF,x,y,corners,delta,err,_
                          bound,crits,bdPts)
```

```
  [p,x,y,xMin,xMax,yMin,yMax,bdPts,htans,vtans,bran]

refine(plot,stepFraction) ==
  p := plot.poly; x := plot.xVar; y := plot.yVar
  dpdx := differentiate(p,x); dpdy := differentiate(p,y)
  pSF := coerceCoefsToSFs p
  dpdxSF := coerceCoefsToSFs dpdx
  dpdySF := coerceCoefsToSFs dpdy
  xMin := plot.minXVal; xMax := plot.maxXVal
  yMin := plot.minYVal; yMax := plot.maxYVal
  xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
  yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
  corners := makeCorners(xMinSF,xMaxSF,yMinSF,yMaxSF)
  pSF := coerceCoefsToSFs p
  dpdxSF := coerceCoefsToSFs dpdx
  dpdySF := coerceCoefsToSFs dpdy
  delta :=
    stepFraction * min((xMaxSF - xMinSF)/25,(yMaxSF - yMinSF)/25)
  err := min(delta/100,PLOTERR/100)
  bound : PositiveInteger := 10
  crits := append(plot.hTanPts,plot.vTanPts)
  bdPts := plot.bdryPts
  bran := traceBranches(pSF,dpdxSF,dpdySF,x,y,corners,delta,err,_
                        bound,crits,bdPts)
  htans := plot.hTanPts; vtans := plot.vTanPts
  [p,x,y,xMin,xMax,yMin,yMax,bdPts,htans,vtans,bran]

traceBranches(pSF,dpdxSF,dpdySF,x,y,corners,delta,err,bound,_
                  crits,bdPts) ==
  -- for boundary points, trace curve from boundary to boundary
  -- add the branch to the list of branches
  -- update list of boundary points by deleting first and last
  -- points on this branch
  -- update list of critical points by deleting any critical
  -- points which were plotted
  lf := bdPts.left; rt := bdPts.right
  tp := bdPts.top ; bt := bdPts.bottom
  bdry := append(append(lf,rt),append(bt,tp))
  bran : List List Point DoubleFloat := nil()
  while not null bdry repeat
    pt := first bdry
    p0 := dummyFirstPt(pt,dpdxSF,dpdySF,x,y,lf,rt,bt,tp)
    segInfo := listPtsOnSegment(pSF,dpdxSF,dpdySF,x,y,p0,pt,_
                     corners,delta,err,bound,crits,bdry)
    bran  := cons(first segInfo,bran)
    crits := second segInfo
    bdry  := third segInfo
  -- trace loops beginning and ending with critical points
  -- add the branch to the list of branches
  -- update list of critical points by deleting any critical
```

```
  -- points which were plotted
  while not null crits repeat
    pt := first crits
    segInfo := listPtsOnLoop(pSF,dpdxSF,dpdySF,x,y,pt,_
                     corners,delta,err,bound,crits,bdry)
    bran  := cons(first segInfo,bran)
    crits := second segInfo
  bran

dummyFirstPt(p1,dpdxSF,dpdySF,x,y,lf,rt,bt,tp) ==
-- The function 'computeNextPt' requires 2 points, p0 and p1.
-- When computing the second point on a branch which starts
-- on the boundary, we use the boundary point as p1 and the
-- 'dummy' point returned by this function as p0.
  x1 := xCoord p1; y1 := yCoord p1
  zero := 0$DoubleFloat; one := 1$DoubleFloat
  px := ground(eval(dpdxSF,[x,y],[x1,y1]))
  py := ground(eval(dpdySF,[x,y],[x1,y1]))
  if px * py < zero then       -- positive slope at p1
    member?(p1,lf) or member?(p1,bt) =>
      makePt(x1 - one,y1 - one)
    makePt(x1 + one,y1 + one)
  else
    member?(p1,lf) or member?(p1,tp) =>
      makePt(x1 - one,y1 + one)
    makePt(x1 + one,y1 - one)


listPtsOnSegment(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                               delta,err,bound,crits,bdry) ==
-- p1 is a boundary point; p0 is a 'dummy' point
  bdry := remove(p1,bdry)
  pointList : List Point DoubleFloat := [p1]
  ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                               delta,err,bound,crits,bdry)
  p2 := ptInfo.newPt
  ptInfo.type = BDRY =>
    bdry := remove(p2,bdry)
    pointList := cons(p2,pointList)
    [pointList,crits,bdry]
  if ptInfo.type = CRIT then crits := remove(p2,crits)
  pointList := cons(p2,pointList)
  repeat
    pt0 := second pointList; pt1 := first pointList
    ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,pt0,pt1,corners,_
                                 delta,err,bound,crits,bdry)
    p2 := ptInfo.newPt
    ptInfo.type = BDRY =>
      bdry := remove(p2,bdry)
      pointList := cons(p2,pointList)
```

```
      return [pointList,crits,bdry]
    if ptInfo.type = CRIT then crits := remove(p2,crits)
    pointList := cons(p2,pointList)
  --!! delete next line (compiler bug)
  [pointList,crits,bdry]


listPtsOnLoop(pSF,dpdxSF,dpdySF,x,y,p1,corners,_
                               delta,err,bound,crits,bdry) ==
  x1 := xCoord p1; y1 := yCoord p1
  px := ground(eval(dpdxSF,[x,y],[x1,y1]))
  py := ground(eval(dpdySF,[x,y],[x1,y1]))
  p0 := makePt(x1 - 1$DoubleFloat,y1 - 1$DoubleFloat)
  pointList : List Point DoubleFloat := [p1]
  ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                               delta,err,bound,crits,bdry)
  p2 := ptInfo.newPt
  ptInfo.type = BDRY =>
    error "boundary reached while on loop"
  if ptInfo.type = CRIT then
    p1 = p2 =>
      error "first and second points on loop are identical"
    crits := remove(p2,crits)
  pointList := cons(p2,pointList)
  repeat
    pt0 := second pointList; pt1 := first pointList
    ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,pt0,pt1,corners,_
                                 delta,err,bound,crits,bdry)
    p2 := ptInfo.newPt
    ptInfo.type = BDRY =>
      error "boundary reached while on loop"
    if ptInfo.type = CRIT then
      crits := remove(p2,crits)
      p1 = p2 =>
        pointList := cons(p2,pointList)
        return [pointList,crits,bdry]
    pointList := cons(p2,pointList)
  --!! delete next line (compiler bug)
  [pointList,crits,bdry]

computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                               delta,err,bound,crits,bdry) ==
-- p0=(x0,y0) and p1=(x1,y1) are the last two points on the curve.
-- The function computes the next point on the curve.
-- The function determines if the next point is a critical point
-- or a boundary point.
-- The function returns a record of the form
-- Record(newPt:Point DoubleFloat,type:String).
-- If the new point is a boundary point, then 'type' is
-- "boundary point" and 'newPt' is a boundary point to be
```

```
-- deleted from the list of boundary points yet to be plotted.
-- Similarly, if the new point is a critical point, then 'type' is
-- "critical point" and 'newPt' is a critical point to be
-- deleted from the list of critical points yet to be plotted.
-- If the new point is neither a critical point nor a boundary
-- point, then 'type' is "nothing in particular".
  xMinSF := getXMin corners; xMaxSF := getXMax corners
  yMinSF := getYMin corners; yMaxSF := getYMax corners
  x0 := xCoord p0; y0 := yCoord p0
  x1 := xCoord p1; y1 := yCoord p1
  px := ground(eval(dpdxSF,[x,y],[x1,y1]))
  py := ground(eval(dpdySF,[x,y],[x1,y1]))
  -- let m be the slope of the tangent line at p1
  -- if |m| < 1, we will increment the x-coordinate by delta
  -- (indicated by 'incVar = x'), find an approximate
  -- y-coordinate using the tangent line, then find the actual
  -- y-coordinate using a Newton iteration
  if abs(py) > abs(px) then
    incVar0 := incVar := x
    deltaX := (if x1 > x0 then delta else -delta)
    x2Approx := x1 + deltaX
    y2Approx := y1 + (-px/py)*deltaX
  -- if |m| >= 1, we interchange the roles of the x- and y-
  -- coordinates
  else
    incVar0 := incVar := y
    deltaY := (if y1 > y0 then delta else -delta)
    x2Approx := x1 + (-py/px)*deltaY
    y2Approx := y1 + deltaY
  lookingFor := NADA
  -- See if (x2Approx,y2Approx) is out of bounds.
  -- If so, find where the line segment connecting (x1,y1) and
  -- (x2Approx,y2Approx) intersects the boundary and use this
  -- point as (x2Approx,y2Approx).
  -- If the resulting point is on the left or right boundary,
  -- we will now consider x as the 'incremented variable' and we
  -- will compute the y-coordinate using a Newton iteration.
  -- Similarly, if the point is on the top or bottom boundary,
  -- we will consider y as the 'incremented variable' and we
  -- will compute the x-coordinate using a Newton iteration.
  if x2Approx >= xMaxSF then
    incVar := x
    lookingFor := BDRY
    x2Approx := xMaxSF
    y2Approx := y1 + (-px/py)*(x2Approx - x1)
  else
    if x2Approx <= xMinSF then
      incVar := x
      lookingFor := BDRY
      x2Approx := xMinSF
```

```
    y2Approx := y1 + (-px/py)*(x2Approx - x1)
if y2Approx >= yMaxSF then
  incVar := y
  lookingFor := BDRY
  y2Approx := yMaxSF
  x2Approx := x1 + (-py/px)*(y2Approx - y1)
else
  if y2Approx <= yMinSF then
    incVar := y
    lookingFor := BDRY
    y2Approx := yMinSF
    x2Approx := x1 + (-py/px)*(y2Approx - y1)
-- set xLo = min(x1,x2Approx), xHi = max(x1,x2Approx)
-- set yLo = min(y1,y2Approx), yHi = max(y1,y2Approx)
if x1 < x2Approx then
  xLo := x1
  xHi := x2Approx
else
  xLo := x2Approx
  xHi := x1
if y1 < y2Approx then
  yLo := y1
  yHi := y2Approx
else
  yLo := y2Approx
  yHi := y1
-- check for critical points (x*,y*) with x* between
-- x1 and x2Approx or y* between y1 and y2Approx
-- store values of x2Approx and y2Approx
x2Approxx := x2Approx
y2Approxx := y2Approx
-- xPointList will contain all critical points (x*,y*)
-- with x* between x1 and x2Approx
xPointList : List Point DoubleFloat := nil()
-- yPointList will contain all critical points (x*,y*)
-- with y* between y1 and y2Approx
yPointList : List Point DoubleFloat := nil()
for pt in crits repeat
  xx := xCoord pt; yy := yCoord pt
  -- if x1 = x2Approx, then p1 is a point with horizontal
  -- tangent line
  -- in this case, we don't want critical points with
  -- x-coordinate x1
  if xx = x2Approx and not (xx = x1) then
    if min(abs(yy-yLo),abs(yy-yHi)) < delta then
      xPointList := cons(pt,xPointList)
  if ((xLo < xx) and (xx < xHi)) then
    if min(abs(yy-yLo),abs(yy-yHi)) < delta then
      xPointList := cons(pt,nil())
      x2Approx := xx
```

```
        if xx < x1 then xLo := xx else xHi := xx
    -- if y1 = y2Approx, then p1 is a point with vertical
    -- tangent line
    -- in this case, we don't want critical points with
    -- y-coordinate y1
    if yy = y2Approx and not (yy = y1) then
        yPointList := cons(pt,yPointList)
    if ((yLo < yy) and (yy < yHi)) then
      if min(abs(xx-xLo),abs(xx-xHi)) < delta then
        yPointList := cons(pt,nil())
        y2Approx := yy
        if yy < y1 then yLo := yy else yHi := yy
  -- points in both xPointList and yPointList
  if (not null xPointList) and (not null yPointList) then
    xPointList = yPointList =>
    -- this implies that the lists have only one point
      incVar := incVar0
      if incVar = x then
        y2Approx := y1 + (-px/py)*(x2Approx - x1)
      else
        x2Approx := x1 + (-py/px)*(y2Approx - y1)
      lookingFor := CRIT        -- proceed
    incVar0 = x =>
    -- first try Newton iteration with 'y' as incremented variable
      x2Temp := x1 + (-py/px)*(y2Approx - y1)
      f := SFPolyToUPoly(eval(pSF,y,y2Approx))
      x2New := newtonApprox(f,x2Temp,err,bound)
      x2New case "failed" =>
        y2Approx := y1 + (-px/py)*(x2Approx - x1)
        incVar := x
        lookingFor := CRIT      -- proceed
      y2Temp := y1 + (-px/py)*(x2Approx - x1)
      f := SFPolyToUPoly(eval(pSF,x,x2Approx))
      y2New := newtonApprox(f,y2Temp,err,bound)
      y2New case "failed" =>
        return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                      abs((x2Approx-x1)/2),err,bound,crits,bdry)
      pt1 := makePt(x2Approx,y2New :: DoubleFloat)
      pt2 := makePt(x2New :: DoubleFloat,y2Approx)
      critPt1 := findPtOnList(pt1,crits)
      critPt2 := findPtOnList(pt2,crits)
      (critPt1 case "failed") and (critPt2 case "failed") =>
        abs(x2Approx - x1) > abs(x2Temp - x1) =>
          return [pt1,NADA]
        return [pt2,NADA]
      (critPt1 case "failed") =>
        return [critPt2::(Point DoubleFloat),CRIT]
      (critPt2 case "failed") =>
        return [critPt1::(Point DoubleFloat),CRIT]
      abs(x2Approx - x1) > abs(x2Temp - x1) =>
```

```
        return [critPt2::(Point DoubleFloat),CRIT]
      return [critPt1::(Point DoubleFloat),CRIT]
    y2Temp := y1 + (-px/py)*(x2Approx - x1)
    f := SFPolyToUPoly(eval(pSF,x,x2Approx))
    y2New := newtonApprox(f,y2Temp,err,bound)
    y2New case "failed" =>
      x2Approx := x1 + (-py/px)*(y2Approx - y1)
      incVar := y
      lookingFor := CRIT      -- proceed
    x2Temp := x1 + (-py/px)*(y2Approx - y1)
    f := SFPolyToUPoly(eval(pSF,y,y2Approx))
    x2New := newtonApprox(f,x2Temp,err,bound)
    x2New case "failed" =>
      return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                   abs((y2Approx-y1)/2),err,bound,crits,bdry)
    pt1 := makePt(x2Approx,y2New :: DoubleFloat)
    pt2 := makePt(x2New :: DoubleFloat,y2Approx)
    critPt1 := findPtOnList(pt1,crits)
    critPt2 := findPtOnList(pt2,crits)
    (critPt1 case "failed") and (critPt2 case "failed") =>
      abs(y2Approx - y1) > abs(y2Temp - y1) =>
        return [pt2,NADA]
      return [pt1,NADA]
    (critPt1 case "failed") =>
      return [critPt2::(Point DoubleFloat),CRIT]
    (critPt2 case "failed") =>
      return [critPt1::(Point DoubleFloat),CRIT]
    abs(y2Approx - y1) > abs(y2Temp - y1) =>
      return [critPt1::(Point DoubleFloat),CRIT]
    return [critPt2::(Point DoubleFloat),CRIT]
  if (not null xPointList) and (null yPointList) then
    y2Approx := y1 + (-px/py)*(x2Approx - x1)
    incVar0 = x =>
      incVar := x
      lookingFor := CRIT        -- proceed
    f := SFPolyToUPoly(eval(pSF,x,x2Approx))
    y2New := newtonApprox(f,y2Approx,err,bound)
    y2New case "failed" =>
      x2Approx := x2Approxx
      y2Approx := y2Approxx     -- proceed
    pt := makePt(x2Approx,y2New::DoubleFloat)
    critPt := findPtOnList(pt,crits)
    critPt case "failed" =>
      return [pt,NADA]
    return [critPt :: (Point DoubleFloat),CRIT]
  if (null xPointList) and (not null yPointList) then
    x2Approx := x1 + (-py/px)*(y2Approx - y1)
    incVar0 = y =>
      incVar := y
      lookingFor := CRIT        -- proceed
```

```
      f := SFPolyToUPoly(eval(pSF,y,y2Approx))
      x2New := newtonApprox(f,x2Approx,err,bound)
      x2New case "failed" =>
        x2Approx := x2Approxx
        y2Approx := y2Approxx     -- proceed
      pt := makePt(x2New::DoubleFloat,y2Approx)
      critPt := findPtOnList(pt,crits)
      critPt case "failed" =>
        return [pt,NADA]
      return [critPt :: (Point DoubleFloat),CRIT]
    if incVar = x then
      x2 := x2Approx
      f := SFPolyToUPoly(eval(pSF,x,x2))
      y2New := newtonApprox(f,y2Approx,err,bound)
      y2New case "failed" =>
        return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                               abs((x2-x1)/2),err,bound,crits,bdry)
      y2 := y2New :: DoubleFloat
    else
      y2 := y2Approx
      f := SFPolyToUPoly(eval(pSF,y,y2))
      x2New := newtonApprox(f,x2Approx,err,bound)
      x2New case "failed" =>
        return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                               abs((y2-y1)/2),err,bound,crits,bdry)
      x2 := x2New :: DoubleFloat
    pt := makePt(x2,y2)
    --!! check that 'pt' is not out of bounds
    -- check if you've gotten a critical or boundary point
    lookingFor = NADA =>
      [pt,lookingFor]
    lookingFor = BDRY =>
      bdryPt := findPtOnList(pt,bdry)
      bdryPt case "failed" =>
        error "couldn't find boundary point"
      [bdryPt :: (Point DoubleFloat),BDRY]
    critPt := findPtOnList(pt,crits)
    critPt case "failed" =>
      [pt,NADA]
    [critPt :: (Point DoubleFloat),CRIT]

--% Newton iterations

  newtonApprox(f,a0,err,bound) ==
  -- Newton iteration to approximate a root of the polynomial 'f'
  -- using an initial approximation of 'a0'
  -- Newton iteration terminates when consecutive approximations
  -- are within 'err' of each other
  -- returns "failed" if this has not been achieved after 'bound'
  -- iterations
```

```
    Df := differentiate f
    oldApprox := a0
    newApprox := a0 - elt(f,a0)/elt(Df,a0)
    i : PositiveInteger := 1
    while abs(newApprox - oldApprox) > err repeat
      i = bound => return "failed"
      oldApprox := newApprox
      newApprox := oldApprox - elt(f,oldApprox)/elt(Df,oldApprox)
      i := i+1
    newApprox

--% graphics output

  listBranches(acplot) == acplot.branches

--% terminal output

  coerce(acplot:%) ==
    pp := acplot.poly :: OutputForm
    xx := acplot.xVar :: OutputForm
    yy := acplot.yVar :: OutputForm
    xLo := acplot.minXVal :: OutputForm
    xHi := acplot.maxXVal :: OutputForm
    yLo := acplot.minYVal :: OutputForm
    yHi := acplot.maxYVal :: OutputForm
    zip := message(" = 0")
    com := message(",   ")
    les := message(" <= ")
    l : List OutputForm :=
      [pp,zip,com,xLo,les,xx,les,xHi,com,yLo,les,yy,les,yHi]
    f : List OutputForm := nil()
    for branch in acplot.branches repeat
      ll : List OutputForm := [p :: OutputForm for p in branch]
      f := cons(vconcat ll,f)
    ff := vconcat(hconcat l,vconcat f)
    vconcat(message "ACPLOT",ff)
```

———-

— ACPLOT.dotabb —

```
"ACPLOT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ACPLOT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ACPLOT" -> "ALIST"
```

———-

17.18 domain PLACES Places

— Places.input —

```
)set break resume
)sys rm -f Places.output
)spool Places.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Places
--R Places K: Field  is a domain constructor
--R Abbreviation for Places is PLACES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLACES
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> Divisor %       ?+? : (%,%) -> Divisor %
--R -? : % -> Divisor %                  ?-? : (%,%) -> Divisor %
--R ?=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R create : Symbol -> %                 create : List K -> %
--R degree : % -> PositiveInteger        ?.? : (%,Integer) -> K
--R foundPlaces : () -> List %           hash : % -> SingleInteger
--R itsALeaf! : % -> Void                latex : % -> String
--R leaf? : % -> Boolean                 reduce : List % -> Divisor %
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R localParam : % -> List NeitherSparseOrDensePowerSeries K
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R setParam! : (%,List NeitherSparseOrDensePowerSeries K) -> Void
--R
--E 1

)spool
)lisp (bye)
```

———-

— Places.help —

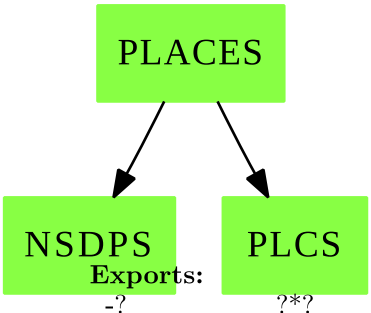
```
====================================================================
Places examples
```

```
====================================================================

See Also:
o )show Places
```

———-

17.18.1 Places (PLACES)



Exports:

-?	?*?	?+?	?-?	?.?
?=?	?~=?	coerce	create	degree
foundPlaces	hash	itsALeaf!	latex	leaf?
localParam	reduce	setDegree!	setFoundPlacesToEmpty	setParam!

— **domain PLACES Places** —

```
)abbrev domain PLACES Places
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
Places(K):Exports == Implementation where
  K:Field
  PCS    ==>  NeitherSparseOrDensePowerSeries(K)

  Exports ==> PlacesCategory(K,PCS)

  Implementation ==>   Plcs(K,PCS)
```

———-

— PLACES.dotabb —

```
"PLACES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLACES"];
"NSDPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSDPS"];
"PLCS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLCS"]
"PLACES" -> "NSDPS"
"PLACES" -> "PLCS"
```

17.19 domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField

— PlacesOverPseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f PlacesOverPseudoAlgebraicClosureOfFiniteField.output
)spool PlacesOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlacesOverPseudoAlgebraicClosureOfFiniteField
--R PlacesOverPseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory  is a domain constructor
--R Abbreviation for PlacesOverPseudoAlgebraicClosureOfFiniteField is PLACESPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLACESPS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> Divisor %       ?+? : (%,%) -> Divisor %
--R -? : % -> Divisor %                  ?-? : (%,%) -> Divisor %
--R ?=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R create : Symbol -> %                 degree : % -> PositiveInteger
--R foundPlaces : () -> List %           hash : % -> SingleInteger
--R itsALeaf! : % -> Void                latex : % -> String
--R leaf? : % -> Boolean                 reduce : List % -> Divisor %
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R create : List PseudoAlgebraicClosureOfFiniteField K -> %
--R ?.? : (%,Integer) -> PseudoAlgebraicClosureOfFiniteField K
```

```
--R localParam : % -> List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteFiel
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R setParam! : (%,List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteField K
--R
--E 1

)spool
)lisp (bye)
```

———-

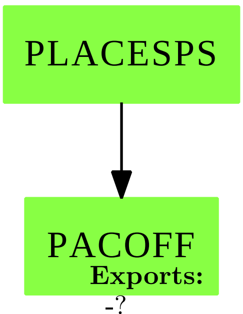
— PlacesOverPseudoAlgebraicClosureOfFiniteField.help —

```
====================================================================
PlacesOverPseudoAlgebraicClosureOfFiniteField examples
====================================================================

See Also:
o )show PlacesOverPseudoAlgebraicClosureOfFiniteField
```

———-

17.19.1 PlacesOverPseudoAlgebraicClosureOfFiniteField (PLACESPS)



Exports:

-?	?*?	?+?	?-?	?.?
?=?	?~=?	coerce	create	degree
foundPlaces	hash	itsALeaf!	latex	leaf?
localParam	reduce	setDegree!	setFoundPlacesToEmpty	setParam!

— domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField —

```
)abbrev domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
PlacesOverPseudoAlgebraicClosureOfFiniteField(K):Exports
 == Implementation where

  K:FiniteFieldCategory
  KK  ==> PseudoAlgebraicClosureOfFiniteField(K)
  PCS ==> NeitherSparseOrDensePowerSeries(KK)

  Exports ==> PlacesCategory(KK,PCS)

  Implementation ==>   Plcs(KK,PCS)
```

———

— PLACESPS.dotabb —

```
"PLACESPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLACESPS"];
"PACOFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACOFF"]
"PLACESPS" -> "PACOFF"
```

———

17.20 domain PLCS Plcs

— Plcs.input —

```
)set break resume
)sys rm -f Plcs.output
)spool Plcs.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Plcs
--R Plcs(K: Field,PCS: LocalPowerSeriesCategory K)  is a domain constructor
--R Abbreviation for Plcs is PLCS
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLCS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> Divisor %        ?+? : (%,%) -> Divisor %
--R -? : % -> Divisor %                   ?-? : (%,%) -> Divisor %
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R create : Symbol -> %                  create : List K -> %
--R degree : % -> PositiveInteger         ?.? : (%,Integer) -> K
--R foundPlaces : () -> List %            hash : % -> SingleInteger
--R itsALeaf! : % -> Void                 latex : % -> String
--R leaf? : % -> Boolean                  localParam : % -> List PCS
--R reduce : List % -> Divisor %          setParam! : (%,List PCS) -> Void
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R
--E 1

)spool
)lisp (bye)
```

———-

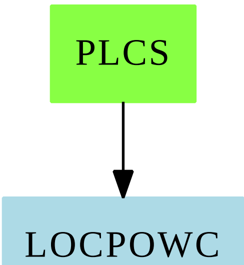
— Plcs.help —

```
====================================================================
Plcs examples
====================================================================

See Also:
o )show Plcs
```

———-

17.20.1 Plcs (PLCS)



Exports:

— domain PLCS Plcs —

```
)abbrev domain PLCS Plcs
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
Plcs(K:Field,PCS:LocalPowerSeriesCategory(K)):Exports == Implementation where

  nameOfPlace ==> Union( List(K), Symbol )

  rec ==> Record(theName:nameOfPlace,_
                locPar: List PCS,_
                deg: PositiveInteger,_
                isALeaf:Boolean,_
                inName:Symbol,_
                 actualSet:Symbol)

  Exports ==> PlacesCategory(K,PCS)

  Implementation ==>   add

    Rep:= rec

    setOfPlacesName:Symbol:=new(ActualSetOfPlacesName)$Symbol

    a:% + b:% == (a:: Divisor(%)) +$Divisor(%) (b::Divisor(%))

    a:% - b:% == (a:: Divisor(%)) -$Divisor(%) (b::Divisor(%))

    n:Integer * b:% == n *$Divisor(%) (b :: Divisor(%))
```

```
reduce(lp)==
  lpd:List Divisor(%):= [p :: Divisor(%) for p in lp]
  reduce("+", lpd, 0$Divisor(%))

d:Divisor(%) + b:% == d + (b::Divisor(%))

a:% + d:Divisor(%) == (a::Divisor(%)) + d

d:Divisor(%) - b:% == d - (b::Divisor(%))

a:% - d:Divisor(%) == (a::Divisor(%)) - d

-a:% == - ( a::Divisor(%))

outName: nameOfPlace -> OutputForm

outName(pt)==
  pt case Symbol => pt :: OutputForm
  dd:OutputForm:= ":" :: OutputForm
  llout:List(OutputForm):=[ hconcat(dd, a::OutputForm) for a in rest pt]
  lout:= cons( (first pt)::OutputForm , llout)
  out:= hconcat lout
  bracket(out)

coerce(pt:%):OutputForm ==
  nn:OutputForm:= outName(pt.theName)
  ee:OutputForm:= degree(pt) :: OutputForm
  nn ** ee

a:% = b:% ==
  ^(a.actualSet =$Symbol b.actualSet) =>
    a:String:=
     "From Places Domain: Cannot use old places with new places."
      " You have declared two different package PAFF or PAFFFF with the "
       "same arguments. This is not allowed because in that case the two "
        "packages used the same domain to represent the set of places. "
         "Two packages having the same arguments should be used in "
          "different frame"
    error a
  a.inName =$Symbol b.inName

elt(pl,n)==
  pt:= (pl :: Rep).theName
  pt case Symbol => _
    error "From Places domain : cannot return the coordinates of a leaf"
  elt(pt,n)$List(K)

leaf?(pl)==pl.isALeaf
```

```
itsALeaf_!(pl)==
  pl.isALeaf := true()
  void()

listOfFoundPlaces:List %:=[]

foundPlaces()==listOfFoundPlaces

setFoundPlacesToEmpty()==
  tmp:=copy listOfFoundPlaces
  listOfFoundPlaces:=[]
  setOfPlacesName:Symbol:=new(ActualSetOfPlacesName)$Symbol
  tmp

findInExistOnes: % -> %
findInExistOnes(pt)==
  ll:=listOfFoundPlaces
  found:Boolean:=false()
  fpl:%
  while ^found and  ^empty?(ll) repeat
    fpl:= first ll
    -- remember: the "=" test is on done on the symbolic name
    found:= pt.theName = fpl.theName
    ll:=rest ll
  ^found =>
    listOfFoundPlaces:=cons(pt,listOfFoundPlaces)
    pt
  fpl

create(pt:List(K)):%==
  newName:=new(SIMPLE)$Symbol
  newPt:%:=[pt,[],1,false(),newName,setOfPlacesName]$rec
  findInExistOnes(newPt)

create(pt:Symbol):%==
  newPt:%:=[pt,[],1,false(),pt,setOfPlacesName]$rec
  findInExistOnes(newPt)

setDegree_!(pt,d)==
  pt.deg := d
  void()

setParam_!(pt,ls)==
  pt.locPar:=ls
  void()

localParam(pt)==pt.locPar

degree(pl)==pl.deg
```

— PLCS.dotabb —

```
"PLCS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLCS"];
"LOCPOWC" [color=lightblue,href="bookvol10.2.pdf#nameddest=LOCPOWC"];
"PLCS" -> "LOCPOWC"
```

17.21 domain PLOT Plot

— Plot.input —

```
)set break resume
)sys rm -f Plot.output
)spool Plot.output
)set message test on
)set message auto off
)clear all
--S 1 of 2
fp:=(t:DFLOAT):DFLOAT +-> sin(t)
--R
--R   (1)  theMap(Closure)
--R                                           Type: (DoubleFloat -> DoubleFloat)
--E 1

--S 2 of 2
plot(fp,-1.0..1.0)$PLOT
--R
--R
--R   (2)  PLOT(x = (- 1.)..1.   y = (- 0.8414709848078965)..0.8414709848078965)
--R                                  [- 1.,- 0.8414709848078965]
--R                         [- 0.95833333333333337,- 0.81823456433427133]
--R                         [- 0.91666666666666674,- 0.79357780324894212]
--R                         [- 0.87500000000000011,- 0.76754350223602708]
--R                         [- 0.83333333333333348,- 0.74017685319603721]
--R                         [- 0.79166666666666685,- 0.7115253607990657]
--R                         [- 0.75000000000000022,- 0.68163876002333434]
--R                         [- 0.70833333333333359,- 0.65056892982223602]
--R                         [- 0.66666666666666696,- 0.61836980306973721]
--R                         [- 0.62500000000000033,- 0.58509727294046243]
--R                         [- 0.5833333333333337,- 0.55080909588697013]
```

```
--R                        [- 0.54166666666666707,- 0.51556479138264011]
--R                        [- 0.50000000000000044,- 0.47942553860420339]
--R                        [- 0.45833333333333376,- 0.44245407023325911]
--R                        [- 0.41666666666666707,- 0.40471456356112506]
--R                        [- 0.37500000000000039,- 0.3662725290860479]
--R                         [- 0.3333333333333337,- 0.3271946967961526]
--R                        [- 0.29166666666666702,- 0.28754890033552849]
--R                        [- 0.25000000000000033,- 0.24740395925452324]
--R                        [- 0.20833333333333368,- 0.20682955954864138]
--R                        [- 0.16666666666666702,- 0.16589613269341538]
--R                        [- 0.12500000000000036,- 0.12467473338522805]
--R                      [- 8.3333333333333703E-2,- 8.3236916200310623E-2]
--R                      [- 4.1666666666667039E-2,- 4.1654611386019461E-2]
--R                     [- 3.7470027081099033E-16,- 3.7470027081099033E-16]
--R                        [4.166666666666629E-2,4.1654611386018711E-2]
--R                        [8.3333333333332954E-2,8.3236916200309874E-2]
--R                          [0.12499999999999961,0.124674733385227 3]
--R                          [0.16666666666666627,0.16589613269341463]
--R                          [0.20833333333333293,0.20682955954864066]
--R                          [0.24999999999999958,0.24740395925452252]
--R                          [0.29166666666666624,0.28754890033552777]
--R                          [0.33333333333333293,0.32719469679615187]
--R                          [0.37499999999999961,0.36627252908604718]
--R                           [0.4166666666666663,0.4047145635611244]
--R                          [0.45833333333333298,0.44245407023325839]
--R                          [0.49999999999999967,0.47942553860420273]
--R                          [0.5416666666666663,0.51556479138263944]
--R                          [0.58333333333333293,0.55080909588696947]
--R                          [0.62499999999999956,0.58509727294046177]
--R                          [0.66666666666666619,0.61836980306973666]
--R                          [0.70833333333333282,0.65056892982223535]
--R                          [0.74999999999999944,0.68163876002333379]
--R                          [0.79166666666666607,0.71152536079906514]
--R                          [0.8333333333333327,0.74017685319603665]
--R                          [0.87499999999999933,0.76754350223602663]
--R                          [0.91666666666666596,0.79357780324894167]
--R                          [0.95833333333333259,0.81823456433427078]
--R                                   [1.,0.8414709848078965]
--R                                                                Type: Plot
--E 2
)spool
)lisp (bye)
```

— **Plot.help** —

```
====================================================================
Plot examples
```

```
====================================================================

The Plot (PLOT) domain supports plotting of functions defined over a
real number system. Plot is limited to 2 dimensional plots.

The function plot: (F -> F,R) -> % plots the function f(x) on the
interval a..b. So we need to define a function that maps from
DoubleFloat to DoubleFloat:

  fp:=(t:DFLOAT):DFLOAT +-> sin(t)

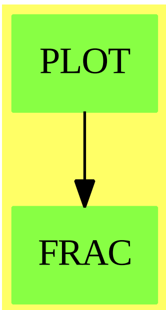
and then feed it to the plot function with a Segment DoubleFloat

  plot(fp,-1.0..1.0)$PLOT

See Also:
o )show Plot
```

———-

17.21.1 Plot (PLOT)



Exports:

adaptive?	coerce	debug	listBranches	maxPoints
minPoints	numFunEvals	parametric?	plot	plotPolar
pointPlot	refine	screenResolution	setAdaptive	setMaxPoints
setMinPoints	setScreenResolution	tRange	xRange	yRange
zoom				

— **domain PLOT Plot** —

```
)abbrev domain PLOT Plot
++ Author: Michael Monagan (revised by Clifton J. Williamson)
++ Date Created: Jan 1988
```

```
++ Date Last Updated: 30 Nov 1990 by Jonathan Steinbach
++ Basic Operations: plot, pointPlot, plotPolar, parametric?, zoom, refine,
++ tRange, minPoints, setMinPoints, maxPoints, screenResolution, adaptive?,
++ setAdaptive, numFunEvals, debug
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, function, parametric
++ References:
++ Description:
++ The Plot domain supports plotting of functions defined over a
++ real number system.  A real number system is a model for the real
++ numbers and as such may be an approximation.  For example
++ floating point numbers and infinite continued fractions.
++ The facilities at this point are limited to 2-dimensional plots
++ or either a single function or a parametric function.

Plot(): Exports == Implementation where
  B   ==> Boolean
  F   ==> DoubleFloat
  I   ==> Integer
  L   ==> List
  N   ==> NonNegativeInteger
  OUT ==> OutputForm
  P   ==> Point F
  RN  ==> Fraction Integer
  S   ==> String
  SEG ==> Segment
  R   ==> Segment F
  C   ==> Record(source: F -> P,ranges: L R,knots: L F,points: L P)

  Exports ==> PlottablePlaneCurveCategory with

--% function plots

    plot: (F -> F,R) -> %
      ++ plot(f,a..b) plots the function \spad{f(x)}
      ++ on the interval \spad{[a,b]}.
      ++
      ++X fp:=(t:DFLOAT):DFLOAT +-> sin(t)
      ++X plot(fp,-1.0..1.0)$PLOT

    plot: (F -> F,R,R) -> %
      ++ plot(f,a..b,c..d) plots the function \spad{f(x)} on the interval
      ++ \spad{[a,b]}; y-range of \spad{[c,d]} is noted in Plot object.

--% multiple function plots

    plot: (L(F -> F),R) -> %
      ++ plot([f1,...,fm],a..b) plots the functions \spad{y = f1(x)},...,
```

```
      ++ \spad{y = fm(x)} on the interval \spad{a..b}.
    plot: (L(F -> F),R,R) -> %
      ++ plot([f1,...,fm],a..b,c..d) plots the functions \spad{y = f1(x)},...,
      ++ \spad{y = fm(x)} on the interval \spad{a..b}; y-range of \spad{[c,d]} is
      ++ noted in Plot object.

--% parametric plots

    plot: (F -> F,F -> F,R) -> %
      ++ plot(f,g,a..b) plots the parametric curve \spad{x = f(t)},
      ++ \spad{y = g(t)} as t ranges over the interval \spad{[a,b]}.
    plot: (F -> F,F -> F,R,R,R) -> %
      ++ plot(f,g,a..b,c..d,e..f) plots the parametric curve \spad{x = f(t)},
      ++ \spad{y = g(t)} as t ranges over the interval \spad{[a,b]}; x-range
      ++ of \spad{[c,d]} and y-range of \spad{[e,f]} are noted in Plot object.

--% parametric plots

    pointPlot: (F -> P,R) -> %
      ++ pointPlot(t +-> (f(t),g(t)),a..b) plots the parametric curve
      ++ \spad{x = f(t)}, \spad{y = g(t)} as t ranges over the interval
      ++ \spad{[a,b]}.
    pointPlot: (F -> P,R,R,R) -> %
      ++ pointPlot(t +-> (f(t),g(t)),a..b,c..d,e..f) plots the parametric
      ++ curve \spad{x = f(t)}, \spad{y = g(t)} as t ranges over the interval
      ++ \spad{[a,b]}; x-range of \spad{[c,d]} and y-range of \spad{[e,f]}
      ++ are noted in Plot object.

--% polar plots

    plotPolar: (F -> F,R) -> %
      ++ plotPolar(f,a..b) plots the polar curve \spad{r = f(theta)} as
      ++ theta ranges over the interval \spad{[a,b]}; this is the same as
      ++ the parametric curve \spad{x = f(t)*cos(t)}, \spad{y = f(t)*sin(t)}.

    plotPolar: (F -> F) -> %
      ++ plotPolar(f) plots the polar curve \spad{r = f(theta)} as theta
      ++ ranges over the interval \spad{[0,2*%pi]}; this is the same as
      ++ the parametric curve \spad{x = f(t)*cos(t)}, \spad{y = f(t)*sin(t)}.

    plot: (%,R) -> %              -- change the range
        ++ plot(x,r) is not documented
    parametric?: % -> B
      ++ parametric? determines whether it is a parametric plot?

    zoom: (%,R) -> %
        ++ zoom(x,r) is not documented
    zoom: (%,R,R) -> %
        ++ zoom(x,r,s) is not documented
    refine: (%,R) -> %
```

```
        ++ refine(x,r) is not documented
    refine: % -> %
      ++ refine(p) performs a refinement on the plot p

    tRange: % -> R
      ++ tRange(p) returns the range of the parameter in a parametric plot p

    minPoints: () -> I
      ++ minPoints() returns the minimum number of points in a plot
    setMinPoints: I -> I
      ++ setMinPoints(i) sets the minimum number of points in a plot to i
    maxPoints: () -> I
      ++ maxPoints() returns the maximum number of points in a plot
    setMaxPoints: I -> I
      ++ setMaxPoints(i) sets the maximum number of points in a plot to i
    screenResolution: () -> I
      ++ screenResolution() returns the screen resolution
    setScreenResolution: I -> I
      ++ setScreenResolution(i) sets the screen resolution to i
    adaptive?: () -> B
      ++ adaptive?() determines whether plotting be done adaptively
    setAdaptive: B -> B
      ++ setAdaptive(true) turns adaptive plotting on
      ++ \spad{setAdaptive(false)} turns adaptive plotting off
    numFunEvals: () -> I
      ++ numFunEvals() returns the number of points computed
    debug: B -> B
      ++ debug(true) turns debug mode on
      ++ \spad{debug(false)} turns debug mode off

  Implementation ==> add
    import PointPackage(DoubleFloat)

--% local functions

    checkRange     : R -> R
      -- checks that left-hand endpoint is less than right-hand endpoint
    intersect      : (R,R) -> R
      -- intersection of two intervals
    union          : (R,R) -> R
      -- union of two intervals
    join           : (L C,I) -> R
    parametricRange: % -> R
    select         : (L P,P -> F,(F,F) -> F) -> F
    rangeRefine    : (C,R) -> C
    adaptivePlot   : (C,R,R,R,I) -> C
    basicPlot      : (F -> P,R) -> C
    basicRefine    : (C,R) -> C
    pt             : (F,F) -> P
    Fnan?           : F -> Boolean
```

```
    Pnan?           : P -> Boolean

--% representation

    Rep := Record( parametric: B, _
                   display: L R, _
                   bounds: L R, _
                   axisLabels: L S, _
                   functions: L C )

--% global constants

    ADAPTIVE: B := true
    MINPOINTS: I := 49
    MAXPOINTS: I := 1000
    NUMFUNEVALS: I := 0
    SCREENRES: I := 500
    ANGLEBOUND: F := cos inv (4::F)
    DEBUG: B := false

    Fnan?(x) == x ~= x
    Pnan?(x) == any?(Fnan?,x)

--% graphics output

    listBranches plot ==
      outList : L L P := nil()
      for curve in plot.functions repeat
        -- curve is C
        newl:L P:=nil()
        for p in curve.points repeat
          if not Pnan? p then newl:=cons(p,newl)
          else if not empty? newl then
                outList := concat(newl:=reverse! newl,outList)
                newl:=nil()
        if not empty? newl then outList := concat(newl:=reverse! newl,outList)
--      print(outList::OutputForm)
      outList

    checkRange r == (lo r > hi r => error "ranges cannot be negative"; r)
    intersect(s,t) == checkRange (max(lo s,lo t) .. min(hi s,hi t))
    union(s,t) == min(lo s,lo t) .. max(hi s,hi t)
    join(l,i) ==
      rr := first l
      u : R :=
        i = 0 => first(rr.ranges)
        i = 1 => second(rr.ranges)
        third(rr.ranges)
      for r in rest l repeat
        i = 0 => u := union(u,first(r.ranges))
```

```
      i = 1 => u := union(u,second(r.ranges))
      u := union(u,third(r.ranges))
    u
  parametricRange r == first(r.bounds)

  minPoints() == MINPOINTS
  setMinPoints n ==
    if n < 3 then error "three points minimum required"
    if MAXPOINTS < n then MAXPOINTS := n
    MINPOINTS := n
  maxPoints() == MAXPOINTS
  setMaxPoints n ==
    if n < 3 then error "three points minimum required"
    if MINPOINTS > n then MINPOINTS := n
    MAXPOINTS := n
  screenResolution() == SCREENRES
  setScreenResolution n ==
    if n < 2 then error "buy a new terminal"
    SCREENRES := n
  adaptive?() == ADAPTIVE
  setAdaptive b == ADAPTIVE := b
  parametric? p == p.parametric

  numFunEvals() == NUMFUNEVALS
  debug b == DEBUG := b

  xRange plot == second plot.bounds
  yRange plot == third plot.bounds
  tRange plot == first plot.bounds

  select(l,f,g) ==
    m := f first l
    if Fnan? m then m := 0
    for p in rest l repeat
      n := m
      m := g(m, f p)
      if Fnan? m then m := n
    m

  rangeRefine(curve,nRange) ==
    checkRange nRange; l := lo nRange; h := hi nRange
    t := curve.knots; p := curve.points; f := curve.source
    while not null t and first t < l repeat
      (t := rest t; p := rest p)
    c: L F := nil(); q: L P := nil()
    while not null t and (first t) <= h repeat
      c := concat(first t,c); q := concat(first p,q)
      t := rest t; p := rest p
    if null c then return basicPlot(f,nRange)
    if first c < h then
```

```
        c := concat(h,c)
        q := concat(f h,q)
        NUMFUNEVALS := NUMFUNEVALS + 1
      t := c := reverse_! c; p := q := reverse_! q
      s := (h-l)/(minPoints()::F-1)
      if (first t) ^= l then
        t := c := concat(l,c)
        p := q := concat(f l,p)
        NUMFUNEVALS := NUMFUNEVALS + 1
      while not null rest t repeat
        n := wholePart((second(t) - first(t))/s)
        d := (second(t) - first(t))/((n+1)::F)
        for i in 1..n repeat
          t.rest := concat(first(t) + d,rest t)
          p.rest := concat(f second t,rest p)
          NUMFUNEVALS := NUMFUNEVALS + 1
          t := rest t; p := rest p
        t := rest t
        p := rest p
      xRange := select(q,xCoord,min) .. select(q,xCoord,max)
      yRange := select(q,yCoord,min) .. select(q,yCoord,max)
      [ f, [nRange,xRange,yRange], c, q]

    adaptivePlot(curve,tRange,xRange,yRange,pixelfraction) ==
      xDiff := hi xRange - lo xRange
      yDiff := hi yRange - lo yRange
      xDiff = 0 or yDiff = 0 => curve
      l := lo tRange; h := hi tRange
      (tDiff := h-l) = 0 => curve
--      if (EQL(yDiff, _$NaNvalue$Lisp)$Lisp) then yDiff := 1::F
      t := curve.knots
      #t < 3 => curve
      p := curve.points; f := curve.source
      minLength:F := 4::F/500::F
      maxLength:F := 1::F/6::F
      tLimit := tDiff/(pixelfraction*500)::F
      while not null t and first t < l repeat (t := rest t; p := rest p)
      #t < 3 => curve
      headert := t; headerp := p

      -- jitter the input points
--      while not null rest rest t repeat
--        t0 := second(t); t1 := third(t)
--        jitter := (random()$I) :: F
--        jitter := sin (jitter)
--        val := t0 + jitter * (t1-t0)/10::F
--        t.2 := val; p.2 := f val
--        t := rest t; p := rest p
--      t := headert; p := headerp
```

```
st := t; sp := p
todot : L L F := nil()
todop : L L P := nil()
while not null rest rest st repeat
  todot := concat_!(todot, st)
  todop := concat_!(todop, sp)
  st := rest st; sp := rest sp
st := headert; sp := headerp
todo1 := todot; todo2 := todop
n : I := 0
while not null todo1 repeat
  st := first(todo1)
  t0 := first(st); t1 := second(st); t2 := third(st)
  if t2 > h then leave
  t2 - t0 < tLimit =>
      todo1 := rest todo1
      todo2 := rest todo2
      if not null todo1 then (t := first(todo1); p := first(todo2))
  sp := first(todo2)
  x0 := xCoord first(sp); y0 := yCoord first(sp)
  x1 := xCoord second(sp); y1 := yCoord second(sp)
  x2 := xCoord third(sp); y2 := yCoord third(sp)
  a1 := (x1-x0)/xDiff; b1 := (y1-y0)/yDiff
  a2 := (x2-x1)/xDiff; b2 := (y2-y1)/yDiff
  s1 := sqrt(a1**2+b1**2); s2 := sqrt(a2**2+b2**2)
  dp := a1*a2+b1*b2

  s1 < maxLength and s2 < maxLength and _
    (s1 = 0::F or s2 = 0::F or
       s1 < minLength and s2 < minLength or _
       dp/s1/s2 > ANGLEBOUND) =>
          todo1 := rest todo1
          todo2 := rest todo2
          if not null todo1 then (t := first(todo1); p := first(todo2))
  if n > MAXPOINTS then leave else n := n + 1
  st := rest t
  if not null rest rest st then
    tm := (t0+t1)/2::F
    tj := tm
    t.rest := concat(tj,rest t)
    p.rest := concat(f tj, rest p)
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    t := rest t; p := rest p
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    t := rest t; p := rest p
    todo1 := rest todo1; todo2 := rest todo2

    tm := (t1+t2)/2::F
```

```
        tj := tm
        t.rest := concat(tj, rest t)
        p.rest := concat(f tj, rest p)
        todo1 := concat_!(todo1, t)
        todo2 := concat_!(todo2, p)
        t := rest t; p := rest p
        todo1 := concat_!(todo1, t)
        todo2 := concat_!(todo2, p)
        todo1 := rest todo1
        todo2 := rest todo2
        if not null todo1 then (t := first(todo1); p := first(todo2))
      else
        tm := (t0+t1)/2::F
        tj := tm
        t.rest := concat(tj,rest t)
        p.rest := concat(f tj, rest p)
        todo1 := concat_!(todo1, t)
        todo2 := concat_!(todo2, p)
        t := rest t; p := rest p
        todo1 := concat_!(todo1, t)
        todo2 := concat_!(todo2, p)
        t := rest t; p := rest p

        tm := (t1+t2)/2::F
        tj := tm
        t.rest := concat(tj, rest t)
        p.rest := concat(f tj, rest p)
        todo1 := concat_!(todo1, t)
        todo2 := concat_!(todo2, p)
        todo1 := rest todo1
        todo2 := rest todo2
        if not null todo1 then (t := first(todo1); p := first(todo2))
    n > 0 =>
      NUMFUNEVALS := NUMFUNEVALS + n
      t := curve.knots; p := curve.points
      xRange := select(p,xCoord,min) .. select(p,xCoord,max)
      yRange := select(p,yCoord,min) .. select(p,yCoord,max)
      [ curve.source, [tRange,xRange,yRange], t, p ]
    curve

  basicPlot(f,tRange) ==
    checkRange tRange
    l := lo tRange
    h := hi tRange
    t : L F := list l
    p : L P := list f l
    s := (h-l)/(minPoints()-1)::F
    for i in 2..minPoints()-1 repeat
      l := l+s
      t := concat(l,t)
```

```
        p := concat(f l,p)
      t := reverse_! concat(h,t)
      p := reverse_! concat(f h,p)
--      print(p::OutputForm)
      xRange : R := select(p,xCoord,min) .. select(p,xCoord,max)
      yRange : R := select(p,yCoord,min) .. select(p,yCoord,max)
      [ f, [tRange,xRange,yRange], t, p ]

    zoom(p,xRange) ==
      [p.parametric, [xRange,third(p.display)], p.bounds, _
       p.axisLabels, p.functions]
    zoom(p,xRange,yRange) ==
      [p.parametric, [xRange,yRange], p.bounds, _
       p.axisLabels, p.functions]

    basicRefine(curve,nRange) ==
      tRange:R := first curve.ranges
      -- curve := copy$C curve  -- Yet another compiler bug
      curve: C := [curve.source,curve.ranges,curve.knots,curve.points]
      t := curve.knots := copy curve.knots
      p := curve.points := copy curve.points
      l := lo nRange; h := hi nRange
      f := curve.source
      while not null rest t and first t < h repeat
        second(t) < l => (t := rest t; p := rest p)
        -- insert new point between t.0 and t.1
        tm : F := (first(t) + second(t))/2::F
--         if DEBUG then output$O (tm::E)
        pm := f tm
        NUMFUNEVALS := NUMFUNEVALS + 1
        t.rest := concat(tm,rest t); t := rest rest t
        p.rest := concat(pm,rest p); p := rest rest p
      t := curve.knots; p := curve.points
      xRange := select(p,xCoord,min) .. select(p,xCoord,max)
      yRange := select(p,yCoord,min) .. select(p,yCoord,max)
      [ curve.source, [tRange,xRange,yRange], t, p ]

    refine p == refine(p,parametricRange p)
    refine(p,nRange) ==
      NUMFUNEVALS := 0
      tRange := parametricRange p
      nRange := intersect(tRange,nRange)
      curves: L C := [basicRefine(c,nRange) for c in p.functions]
      xRange := join(curves,1); yRange := join(curves,2)
      if adaptive? then
        tlimit := if parametric? p then 8 else 1
        curves := [adaptivePlot(c,nRange,xRange,yRange, _
                   tlimit) for c in curves]
        xRange := join(curves,1); yRange := join(curves,2)
--      print(NUMFUNEVALS::OUT)
```

```
      [p.parametric, p.display, [tRange,xRange,yRange], _
       p.axisLabels, curves ]

    plot(p:%,tRange:R) ==
      -- re plot p on a new range making use of the points already
      -- computed if possible
      NUMFUNEVALS := 0
      curves: L C := [rangeRefine(c,tRange) for c in p.functions]
      xRange := join(curves,1); yRange := join(curves,2)
      if adaptive? then
        tlimit := if parametric? p then 8 else 1
        curves := [adaptivePlot(c,tRange,xRange,yRange,tlimit) for c in curves]
        xRange := join(curves,1); yRange := join(curves,2)
--      print(NUMFUNEVALS::OUT)
      [ p.parametric, [xRange,yRange], [tRange,xRange,yRange],
        p.axisLabels, curves ]

    pt(xx,yy) == point(l : L F := [xx,yy])

    myTrap: (F-> F, F) -> F
    myTrap(ff:F-> F, f:F):F ==
      s := trapNumericErrors(ff(f))$Lisp :: Union(F, "failed")
      s case "failed" => _$NaNvalue$Lisp
      r:F:=s::F
      r > max()$F or r < min()$F => _$NaNvalue$Lisp
      r

    plot(f:F -> F,xRange:R) ==
      p := basicPlot((u1:F):P +-> pt(u1,myTrap(f,u1)),xRange)
      r := p.ranges
      NUMFUNEVALS := minPoints()
      if adaptive? then
        p := adaptivePlot(p,first r,second r,third r,1)
        r := p.ranges
      [ false, rest r, r, nil(), [ p ] ]

    plot(f:F -> F,xRange:R,yRange:R) ==
      p := plot(f,xRange)
      p.display := [xRange,checkRange yRange]
      p

    plot(f:F -> F,g:F -> F,tRange:R) ==
      p := basicPlot((z1:F):P +-> pt(myTrap(f,z1),myTrap(g,z1)),tRange)
      r := p.ranges
      NUMFUNEVALS := minPoints()
      if adaptive? then
        p := adaptivePlot(p,first r,second r,third r,8)
        r := p.ranges
      [ true, rest r, r, nil(), [ p ] ]
```

```
    plot(f:F -> F,g:F -> F,tRange:R,xRange:R,yRange:R) ==
      p := plot(f,g,tRange)
      p.display := [checkRange xRange,checkRange yRange]
      p

    pointPlot(f:F -> P,tRange:R) ==
      p := basicPlot(f,tRange)
      r := p.ranges
      NUMFUNEVALS := minPoints()
      if adaptive? then
        p := adaptivePlot(p,first r,second r,third r,8)
        r := p.ranges
      [ true, rest r, r, nil(), [ p ] ]

    pointPlot(f:F -> P,tRange:R,xRange:R,yRange:R) ==
      p := pointPlot(f,tRange)
      p.display := [checkRange xRange,checkRange yRange]
      p

    plot(l:L(F -> F),xRange:R) ==
      if null l then error "empty list of functions"
      t: L C :=
        [ basicPlot((z1:F):P +-> pt(z1,myTrap(f,z1)),xRange) for f in l ]
      yRange := join(t,2)
      NUMFUNEVALS := # l * minPoints()
      if adaptive? then
        t := [adaptivePlot(p,xRange,xRange,yRange,1) _
                for f in l for p in t]
        yRange := join(t,2)
--      print(NUMFUNEVALS::OUT)
      [false, [xRange,yRange], [xRange,xRange,yRange], nil(), t ]

    plot(l:L(F -> F),xRange:R,yRange:R) ==
      p := plot(l,xRange)
      p.display := [xRange,checkRange yRange]
      p

    plotPolar(f,thetaRange) ==
      plot((u1:F):F +-> f(u1) * cos(u1),
           (v1:F):F +-> f(v1) * sin(v1),thetaRange)

    plotPolar f == plotPolar(f,segment(0,2*pi()))

--% terminal output

    coerce r ==
      spaces: OUT := coerce "    "
      xSymbol := "x = " :: OUT
      ySymbol := "y = " :: OUT
      tSymbol := "t = " :: OUT
```

```
    plotSymbol := "PLOT" :: OUT
    tRange := (parametricRange r) :: OUT
    f : L OUT := nil()
    for curve in r.functions repeat
      xRange := second(curve.ranges) :: OUT
      yRange := third(curve.ranges) :: OUT
      l : L OUT := [xSymbol,xRange,spaces,ySymbol,yRange]
      if parametric? r then
        l := concat_!([tSymbol,tRange,spaces],l)
      h : OUT := hconcat l
      l := [p::OUT for p in curve.points]
      f := concat(vconcat concat(h,l),f)
    prefix("PLOT" :: OUT, reverse_! f)
```

———-

— **PLOT.dotabb** —

```
"PLOT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLOT"]
"FRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRAC"]
"PLOT" -> "FRAC"
```

———-

17.22 domain PLOT3D Plot3D

— **Plot3D.input** —

```
)set break resume
)sys rm -f Plot3D.output
)spool Plot3D.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Plot3D
--R Plot3D  is a domain constructor
--R Abbreviation for Plot3D is PLOT3D
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLOT3D
--R
--R------------------------------- Operations --------------------------------
--R adaptive3D? : () -> Boolean           coerce : % -> OutputForm
```

```
--R debug3D : Boolean -> Boolean         maxPoints3D : () -> Integer
--R minPoints3D : () -> Integer          numFunEvals3D : () -> Integer
--R refine : % -> %                      screenResolution3D : () -> Integer
--R setAdaptive3D : Boolean -> Boolean    tRange : % -> Segment DoubleFloat
--R xRange : % -> Segment DoubleFloat     yRange : % -> Segment DoubleFloat
--R zRange : % -> Segment DoubleFloat
--R listBranches : % -> List List Point DoubleFloat
--R plot : (%,Segment DoubleFloat) -> %
--R plot : ((DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(Doub
--R plot : ((DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(Doub
--R pointPlot : ((DoubleFloat -> Point DoubleFloat),Segment DoubleFloat,Segment DoubleFloat,Segment Doub
--R pointPlot : ((DoubleFloat -> Point DoubleFloat),Segment DoubleFloat) -> %
--R refine : (%,Segment DoubleFloat) -> %
--R setMaxPoints3D : Integer -> Integer
--R setMinPoints3D : Integer -> Integer
--R setScreenResolution3D : Integer -> Integer
--R tValues : % -> List List DoubleFloat
--R zoom : (%,Segment DoubleFloat,Segment DoubleFloat,Segment DoubleFloat) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

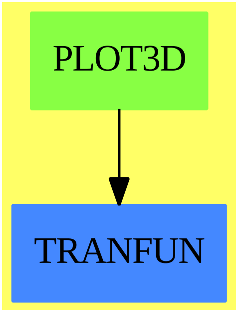
— Plot3D.help —

```
====================================================================
Plot3D examples
====================================================================

See Also:
o )show Plot3D
```

———-

17.22.1 Plot3D (PLOT3D)



Exports:

adaptive3D?	coerce	debug3D	listBranches	maxPoints3D
minPoints3D	numFunEvals3D	plot	pointPlot	refine
screenResolution3D	setAdaptive3D	setMaxPoints3D	setMinPoints3D	setScreenResolutio
tRange	tValues	xRange	yRange	zRange
zoom				

— domain PLOT3D Plot3D —

```
)abbrev domain PLOT3D Plot3D
++ Author: Clifton J. Williamson based on code by Michael Monagan
++ Date Created: Jan 1989
++ Date Last Updated: 22 November 1990 (Jon Steinbach)
++ Basic Operations: pointPlot, plot, zoom, refine, tRange, tValues,
++ minPoints3D, setMinPoints3D, maxPoints3D, setMaxPoints3D,
++ screenResolution3D, setScreenResolution3D, adaptive3D?,  setAdaptive3D,
++ numFunEvals3D, debug3D
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, parametric
++ References:
++ Description:
++ Plot3D supports parametric plots defined over a real
++ number system.  A real number system is a model for the real
++ numbers and as such may be an approximation.  For example,
++ floating point numbers and infinite continued fractions are
++ real number systems. The facilities at this point are limited
++ to 3-dimensional parametric plots.

Plot3D(): Exports == Implementation where
  B   ==> Boolean
  F   ==> DoubleFloat
  I   ==> Integer
  L   ==> List
```

```
N   ==> NonNegativeInteger
OUT ==> OutputForm
P   ==> Point F
S   ==> String
R   ==> Segment F
O   ==> OutputPackage
C   ==> Record(source: F -> P,ranges: L R, knots: L F, points: L P)

Exports ==> PlottableSpaceCurveCategory with

  pointPlot: (F -> P,R) -> %
    ++ pointPlot(f,g,h,a..b) plots {/emx = f(t), y = g(t), z = h(t)} as
    ++ t ranges over {/em[a,b]}.
  pointPlot: (F -> P,R,R,R,R) -> %
      ++ pointPlot(f,x,y,z,w) is not documented
  plot: (F -> F,F -> F,F -> F,F -> F,R) -> %
    ++ plot(f,g,h,a..b) plots {/emx = f(t), y = g(t), z = h(t)} as
    ++ t ranges over {/em[a,b]}.
  plot: (F -> F,F -> F,F -> F,F -> F,R,R,R,R,R) -> %
      ++ plot(f1,f2,f3,f4,x,y,z,w) is not documented

  plot: (%,R) -> %                        -- change the range
       ++ plot(x,r) is not documented
  zoom: (%,R,R,R) -> %
      ++ zoom(x,r,s,t) is not documented
  refine: (%,R) -> %
      ++ refine(x,r) is not documented
  refine: % -> %
      ++ refine(x) is not documented

  tRange: % -> R
    ++ tRange(p) returns the range of the parameter in a parametric plot p.
  tValues: % -> L L F
    ++ tValues(p) returns a list of lists of the values of the parameter for
    ++ which a point is computed, one list for each curve in the plot p.

  minPoints3D: () -> I
    ++ minPoints3D() returns the minimum number of points in a plot.
  setMinPoints3D: I -> I
    ++ setMinPoints3D(i) sets the minimum number of points in a plot to i.
  maxPoints3D: () -> I
    ++ maxPoints3D() returns the maximum number of points in a plot.
  setMaxPoints3D: I -> I
    ++ setMaxPoints3D(i) sets the maximum number of points in a plot to i.
  screenResolution3D: () -> I
    ++ screenResolution3D() returns the screen resolution for a 3d graph.
  setScreenResolution3D: I -> I
    ++ setScreenResolution3D(i) sets the screen resolution for a 3d graph to i.
  adaptive3D?: () -> B
    ++ adaptive3D?() determines whether plotting be done adaptively.
```

```
    setAdaptive3D: B -> B
      ++ setAdaptive3D(true) turns adaptive plotting on;
      ++ setAdaptive3D(false) turns adaptive plotting off.
    numFunEvals3D: () -> I
      ++ numFunEvals3D() returns the number of points computed.
    debug3D: B -> B
      ++ debug3D(true) turns debug mode on;
      ++ debug3D(false) turns debug mode off.

  Implementation ==> add
    import PointPackage(F)

--% local functions

    fourth         : L R -> R
    checkRange     : R -> R
      -- checks that left-hand endpoint is less than right-hand endpoint
    intersect      : (R,R) -> R
      -- intersection of two intervals
    union          : (R,R) -> R
      -- union of two intervals
    join           : (L C,I) -> R
    parametricRange: % -> R
--     setColor       : (P,F) -> F
    select         : (L P,P -> F,(F,F) -> F) -> F
--     normalizeColor : (P,F,F) -> F
    rangeRefine    : (C,R) -> C
    adaptivePlot   : (C,R,R,R,R,I,I) -> C
    basicPlot      : (F -> P,R) -> C
    basicRefine    : (C,R) -> C
    point          : (F,F,F,F) -> P

--% representation

    Rep := Record( display: L R, _
                   bounds: L R, _
                   screenres: I, _
                   axisLabels: L S, _
                   functions: L C )

--% global constants

    ADAPTIVE    : B := true
    MINPOINTS   : I := 49
    MAXPOINTS   : I := 1000
    NUMFUNEVALS : I := 0
    SCREENRES   : I := 500
    ANGLEBOUND  : F := cos inv (4::F)
    DEBUG       : B := false
```

```
    point(xx,yy,zz,col) == point(l : L F := [xx,yy,zz,col])

    fourth list == first rest rest rest list

    checkRange r == (lo r > hi r => error "ranges cannot be negative"; r)
    intersect(s,t) == checkRange (max(lo s,lo t) .. min(hi s,hi t))
    union(s:R,t:R) == min(lo s,lo t) .. max(hi s,hi t)
    join(l,i) ==
      rr := first l
      u : R :=
        i = 0 => first(rr.ranges)
        i = 1 => second(rr.ranges)
        i = 2 => third(rr.ranges)
        fourth(rr.ranges)
      for r in rest l repeat
        i = 0 => union(u,first(r.ranges))
        i = 1 => union(u,second(r.ranges))
        i = 2 => union(u,third(r.ranges))
        union(u,fourth(r.ranges))
      u
    parametricRange r == first(r.bounds)

    minPoints3D() == MINPOINTS
    setMinPoints3D n ==
      if n < 3 then error "three points minimum required"
      if MAXPOINTS < n then MAXPOINTS := n
      MINPOINTS := n
    maxPoints3D() == MAXPOINTS
    setMaxPoints3D n ==
      if n < 3 then error "three points minimum required"
      if MINPOINTS > n then MINPOINTS := n
      MAXPOINTS := n
    screenResolution3D() == SCREENRES
    setScreenResolution3D n ==
      if n < 2 then error "buy a new terminal"
      SCREENRES := n
    adaptive3D?() == ADAPTIVE
    setAdaptive3D b == ADAPTIVE := b

    numFunEvals3D() == NUMFUNEVALS
    debug3D b == DEBUG := b

--     setColor(p,c) == p.colNum := c

    xRange plot == second plot.bounds
    yRange plot == third plot.bounds
    zRange plot == fourth plot.bounds
    tRange plot == first plot.bounds

    tValues plot ==
```

```
      outList : L L F := nil()
      for curve in plot.functions repeat
        outList := concat(curve.knots,outList)
      outList

    select(l,f,g) ==
      m := f first l
      if (EQL(m, _$NaNvalue$Lisp)$Lisp) then m := 0
--      for p in rest l repeat m := g(m,fp)
      for p in rest l repeat
        fp : F := f p
        if (EQL(fp, _$NaNvalue$Lisp)$Lisp) then fp := 0
        m := g(m,fp)
      m

--     normalizeColor(p,lo,diff) ==
--       p.colNum := (p.colNum - lo)/diff

    rangeRefine(curve,nRange) ==
      checkRange nRange; l := lo nRange; h := hi nRange
      t := curve.knots; p := curve.points; f := curve.source
      while not null t and first t < l repeat
        (t := rest t; p := rest p)
      c : L F := nil(); q : L P := nil()
      while not null t and first t <= h repeat
        c := concat(first t,c); q := concat(first p,q)
        t := rest t; p := rest p
      if null c then return basicPlot(f,nRange)
      if first c < h then
        c := concat(h,c); q := concat(f h,q)
        NUMFUNEVALS := NUMFUNEVALS + 1
      t := c := reverse_! c; p := q := reverse_! q
      s := (h-l)/(MINPOINTS::F-1)
      if (first t) ^= l then
        t := c := concat(l,c); p := q := concat(f l,p)
        NUMFUNEVALS := NUMFUNEVALS + 1
      while not null rest t repeat
        n := wholePart((second(t) - first(t))/s)
        d := (second(t) - first(t))/((n+1)::F)
        for i in 1..n repeat
          t.rest := concat(first(t) + d,rest t); t1 := second t
          p.rest := concat(f t1,rest p)
          NUMFUNEVALS := NUMFUNEVALS + 1
          t := rest t; p := rest p
        t := rest t
        p := rest p
      xRange := select(q,xCoord,min) .. select(q,xCoord,max)
      yRange := select(q,yCoord,min) .. select(q,yCoord,max)
      zRange := select(q,zCoord,min) .. select(q,zCoord,max)
--       colorLo := select(q,color,min); colorHi := select(q,color,max)
```

```
--       (diff := colorHi - colorLo) = 0 =>
--         error "all points are the same color"
--       map(normalizeColor(#1,colorLo,diff),q)$ListPackage1(P)
      [f,[nRange,xRange,yRange,zRange],c,q]


    adaptivePlot(curve,tRg,xRg,yRg,zRg,pixelfraction,resolution) ==
      xDiff := hi xRg - lo xRg
      yDiff := hi yRg - lo yRg
      zDiff := hi zRg - lo zRg
--      xDiff = 0 or yDiff = 0 or zDiff = 0 => curve--!! delete this?
      if xDiff = 0::F then xDiff := 1::F
      if yDiff = 0::F then yDiff := 1::F
      if zDiff = 0::F then zDiff := 1::F
      l := lo tRg; h := hi tRg
      (tDiff := h-l) = 0 => curve
      t := curve.knots
      #t < 3 => curve
      p := curve.points; f := curve.source
      minLength:F := 4::F/resolution::F
      maxLength := 1/4::F
      tLimit := tDiff/(pixelfraction*resolution)::F
      while not null t and first t < l repeat (t := rest t; p := rest p)
      #t < 3 => curve
      headert := t; headerp := p
      st := t; sp := p
      todot : L L F := nil()
      todop : L L P := nil()
      while not null rest rest st repeat
        todot := concat_!(todot, st)
        todop := concat_!(todop, sp)
        st := rest st; sp := rest sp
      st := headert; sp := headerp
      todo1 := todot; todo2 := todop
      n : I := 0

      while not null todo1 repeat
        st := first(todo1)
        t0 := first(st); t1 := second(st); t2 := third(st)
        if t2 > h then leave
        t2 - t0 < tLimit =>
           todo1 := rest todo1
           todo2 := rest todo2;
           if not null todo1 then (t := first(todo1); p := first(todo2))
        sp := first(todo2)
        x0 := xCoord first(sp); y0 := yCoord first(sp); z0 := zCoord first(sp)
        x1 := xCoord second(sp); y1 := yCoord second(sp); z1 := zCoord second(sp)
        x2 := xCoord third(sp); y2 := yCoord third(sp); z2 := zCoord third(sp)
        a1 := (x1-x0)/xDiff; b1 := (y1-y0)/yDiff; c1 := (z1-z0)/zDiff
        a2 := (x2-x1)/xDiff; b2 := (y2-y1)/yDiff; c2 := (z2-z1)/zDiff
```

```
s1 := sqrt(a1**2+b1**2+c1**2); s2 := sqrt(a2**2+b2**2+c2**2)
dp := a1*a2+b1*b2+c1*c2
s1 < maxLength and s2 < maxLength and _
   (s1 = 0 or s2 = 0 or
      s1 < minLength and s2 < minLength or _
      dp/s1/s2 > ANGLEBOUND) =>
        todo1 := rest todo1
        todo2 := rest todo2
        if not null todo1 then (t := first(todo1); p := first(todo2))
if n = MAXPOINTS then leave else n := n + 1
--if DEBUG then
   --r : L F := [minLength,maxLength,s1,s2,dp/s1/s2,ANGLEBOUND]
   --output(r::E)$O
st := rest t
if not null rest rest st then
  tm := (t0+t1)/2::F
  tj := tm
  t.rest := concat(tj,rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := rest todo1; todo2 := rest todo2

  tm := (t1+t2)/2::F
  tj := tm
  t.rest := concat(tj, rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  todo1 := rest todo1; todo2 := rest todo2
  if not null todo1 then (t := first(todo1); p := first(todo2))
else
  tm := (t0+t1)/2::F
  tj := tm
  t.rest := concat(tj,rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
```

```
        tm := (t1+t2)/2::F
        tj := tm
        t.rest := concat(tj, rest t)
        p.rest := concat(f tj, rest p)
        todo1 := concat_!(todo1, t)
        todo2 := concat_!(todo2, p)
        todo1 := rest todo1; todo2 := rest todo2
        if not null todo1 then (t := first(todo1); p := first(todo2))
    if n > 0 then
      NUMFUNEVALS := NUMFUNEVALS + n
      t := curve.knots; p := curve.points
      xRg := select(p,xCoord,min) .. select(p,xCoord,max)
      yRg := select(p,yCoord,min) .. select(p,yCoord,max)
      zRg := select(p,zCoord,min) .. select(p,zCoord,max)
      [curve.source,[tRg,xRg,yRg,zRg],t,p]
    else curve

  basicPlot(f,tRange) ==
    checkRange tRange; l := lo tRange; h := hi tRange
    t : L F := list l; p : L P := list f l
    s := (h-l)/(MINPOINTS-1)::F
    for i in 2..MINPOINTS-1 repeat
      l := l+s; t := concat(l,t)
      p := concat(f l,p)
    t := reverse_! concat(h,t)
    p := reverse_! concat(f h,p)
    xRange : R := select(p,xCoord,min) .. select(p,xCoord,max)
    yRange : R := select(p,yCoord,min) .. select(p,yCoord,max)
    zRange : R := select(p,zCoord,min) .. select(p,zCoord,max)
    [f,[tRange,xRange,yRange,zRange],t,p]

  zoom(p,xRange,yRange,zRange) ==
   [[xRange,yRange,zRange],p.bounds,
    p.screenres,p.axisLabels,p.functions]

  basicRefine(curve,nRange) ==
    tRange:R := first curve.ranges
    -- curve := copy$C curve  -- Yet another @#$%^&* compiler bug
    curve: C := [curve.source,curve.ranges,curve.knots,curve.points]
    t := curve.knots := copy curve.knots
    p := curve.points := copy curve.points
    l := lo nRange; h := hi nRange
    f := curve.source
    while not null rest t and first(t) < h repeat
      second(t) < l => (t := rest t; p := rest p)
      -- insert new point between t.0 and t.1
      tm:F := (first(t) + second(t))/2::F
      -- if DEBUG then output$O (tm::E)
      pm := f tm
      NUMFUNEVALS := NUMFUNEVALS + 1
```

```
        t.rest := concat(tm,rest t); t := rest rest t
        p.rest := concat(pm,rest p); p := rest rest p
      t := curve.knots; p := curve.points
      xRange := select(p,xCoord,min) .. select(p,xCoord,max)
      yRange := select(p,yCoord,min) .. select(p,yCoord,max)
      zRange := select(p,zCoord,min) .. select(p,zCoord,max)
      [curve.source,[tRange,xRange,yRange,zRange],t,p]

    refine p == refine(p,parametricRange p)
    refine(p,nRange) ==
      NUMFUNEVALS := 0
      tRange := parametricRange p
      nRange := intersect(tRange,nRange)
      curves: L C := [basicRefine(c,nRange) for c in p.functions]
      xRange := join(curves,1); yRange := join(curves,2)
      zRange := join(curves,3)
      scrres := p.screenres
      if adaptive3D? then
        tlimit := 8
        curves := [adaptivePlot(c,nRange,xRange,yRange,zRange, _
                   tlimit,scrres := 2*scrres) for c in curves]
        xRange := join(curves,1); yRange := join(curves,2)
        zRange := join(curves,3)
      [p.display,[tRange,xRange,yRange,zRange], _
       scrres,p.axisLabels,curves]

    plot(p:%,tRange:R) ==
      -- re plot p on a new range making use of the points already
      -- computed if possible
      NUMFUNEVALS := 0
      curves: L C := [rangeRefine(c,tRange) for c in p.functions]
      xRange := join(curves,1); yRange := join(curves,2)
      zRange := join(curves,3)
      if adaptive3D? then
        tlimit := 8
        curves := [adaptivePlot(c,tRange,xRange,yRange,zRange,tlimit, _
                      p.screenres) for c in curves]
        xRange := join(curves,1); yRange := join(curves,2)
        zRange := join(curves,3)
--      print(NUMFUNEVALS::OUT)
      [[xRange,yRange,zRange],[tRange,xRange,yRange,zRange],
        p.screenres,p.axisLabels,curves]

    pointPlot(f:F -> P,tRange:R) ==
      p := basicPlot(f,tRange)
      r := p.ranges
      NUMFUNEVALS := MINPOINTS
      if adaptive3D? then
       p := adaptivePlot(p,first r,second r,third r,fourth r,8,SCREENRES)
--      print(NUMFUNEVALS::OUT)
```

```
--      print(p::OUT)
      [ rest r, r, SCREENRES, nil(), [ p ] ]

    pointPlot(f:F -> P,tRange:R,xRange:R,yRange:R,zRange:R) ==
      p := pointPlot(f,tRange)
      p.display:= [checkRange xRange,checkRange yRange,checkRange zRange]
      p

    myTrap: (F-> F, F) -> F
    myTrap(ff:F-> F, f:F):F ==
      s := trapNumericErrors(ff(f))$Lisp :: Union(F, "failed")
      if (s) case "failed" then
        r:F := _$NaNvalue$Lisp
      else
        r:F := s
      r

    plot(f1:F -> F,f2:F -> F,f3:F -> F,col:F -> F,tRange:R) ==
      p := basicPlot(
       (z:F):P+->point(myTrap(f1,z),myTrap(f2,z),myTrap(f3,z),col(z)),tRange)
      r := p.ranges
      NUMFUNEVALS := MINPOINTS
      if adaptive3D? then
       p := adaptivePlot(p,first r,second r,third r,fourth r,8,SCREENRES)
--      print(NUMFUNEVALS::OUT)
      [ rest r, r, SCREENRES, nil(), [ p ] ]

    plot(f1:F -> F,f2:F -> F,f3:F -> F,col:F -> F,_
             tRange:R,xRange:R,yRange:R,zRange:R) ==
      p := plot(f1,f2,f3,col,tRange)
      p.display:= [checkRange xRange,checkRange yRange,checkRange zRange]
      p

--% terminal output

    coerce r ==
      spaces := "   " :: OUT
      xSymbol := "x = " :: OUT; ySymbol := "y = " :: OUT
      zSymbol := "z = " :: OUT; tSymbol := "t = " :: OUT
      tRange := (parametricRange r) :: OUT
      f : L OUT := nil()
      for curve in r.functions repeat
        xRange := coerce curve.ranges.1
        yRange := coerce curve.ranges.2
        zRange := coerce curve.ranges.3
        l : L OUT := [xSymbol,xRange,spaces,ySymbol,yRange,_
                       spaces,zSymbol,zRange]
        l := concat_!([tSymbol,tRange,spaces],l)
        h : OUT := hconcat l
        l := [p::OUT for p in curve.points]
```

```
      f := concat(vconcat concat(h,l),f)
    prefix("PLOT" :: OUT,reverse_! f)

----% graphics output

  listBranches plot ==
    outList : L L P := nil()
    for curve in plot.functions repeat
      outList := concat(curve.points,outList)
    outList
```

———-

— PLOT3D.dotabb —

```
"PLOT3D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLOT3D"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"PLOT3D" -> "TRANFUN"
```

———-

17.23 domain PBWLB PoincareBirkhoffWittLyndonBasis

— PoincareBirkhoffWittLyndonBasis.input —

```
)set break resume
)sys rm -f PoincareBirkhoffWittLyndonBasis.output
)spool PoincareBirkhoffWittLyndonBasis.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PoincareBirkhoffWittLyndonBasis
--R PoincareBirkhoffWittLyndonBasis VarSet: OrderedSet  is a domain constructor
--R Abbreviation for PoincareBirkhoffWittLyndonBasis is PBWLB
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PBWLB
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
```

```
--R ?>=? : (%,%) -> Boolean              1 : () -> %
--R coerce : VarSet -> %                 coerce : LyndonWord VarSet -> %
--R coerce : % -> OutputForm             first : % -> LyndonWord VarSet
--R hash : % -> SingleInteger             latex : % -> String
--R length : % -> NonNegativeInteger      max : (%,%) -> %
--R min : (%,%) -> %                      rest : % -> %
--R retract : % -> LyndonWord VarSet      retractable? : % -> Boolean
--R varList : % -> List VarSet            ?~=? : (%,%) -> Boolean
--R coerce : % -> OrderedFreeMonoid VarSet
--R listOfTerms : % -> List LyndonWord VarSet
--R retractIfCan : % -> Union(LyndonWord VarSet,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

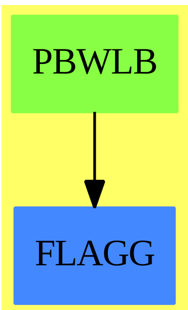
— PoincareBirkhoffWittLyndonBasis.help —

```
====================================================================
PoincareBirkhoffWittLyndonBasis examples
====================================================================

See Also:
o )show PoincareBirkhoffWittLyndonBasis
```

———-

17.23.1 PoincareBirkhoffWittLyndonBasis (PBWLB)



Exports:

1	coerce	first	hash	latex
length	listOfTerms	max	min	rest
retract	retractable?	retractIfCan	varList	?~=?
?<?	?<=?	?=?	?>?	?>=?

— domain PBWLB PoincareBirkhoffWittLyndonBasis —

```
)abbrev domain PBWLB PoincareBirkhoffWittLyndonBasis
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain provides the internal representation
++ of polynomials in non-commutative variables written
++ over the Poincare-Birkhoff-Witt basis.
++ See the \spadtype{XPBWPolynomial} domain constructor.
++ See Free Lie Algebras by C. Reutenauer
++ (Oxford science publications).

PoincareBirkhoffWittLyndonBasis(VarSet: OrderedSet): Public == Private where
   WORD    ==> OrderedFreeMonoid(VarSet)
   LWORD   ==> LyndonWord(VarSet)
   LWORDS  ==> List(LWORD)
   PI      ==> PositiveInteger
   NNI     ==> NonNegativeInteger
   EX      ==> OutputForm

   Public == Join(OrderedSet, RetractableTo LWORD) with
      1: constant -> %
         ++ \spad{1} returns the empty list.
      coerce       : $ -> WORD
         ++ \spad{coerce([l1]*[l2]*...[ln])} returns the word \spad{l1*l2*...*ln},
         ++ where \spad{[l_i]} is the backeted form of the Lyndon word \spad{l_i}.
      coerce       : VarSet -> $
         ++ \spad{coerce(v)} return \spad{v}
      first        : $ -> LWORD
         ++ \spad{first([l1]*[l2]*...[ln])} returns the Lyndon word \spad{l1}.
      length       : $ -> NNI
         ++ \spad{length([l1]*[l2]*...[ln])} returns the length of the word \spad{l1*l2*...*
      listOfTerms  : $ -> LWORDS
         ++ \spad{listOfTerms([l1]*[l2]*...[ln])} returns the list of words \spad{l1, l2, ..
      rest         : $ -> $
         ++ \spad{rest([l1]*[l2]*...[ln])} returns the list \spad{l2, .... ln}.
```

```
    retractable? : $ -> Boolean
       ++ \spad{retractable?([l1]*[l2]*...[ln])} returns true iff \spad{n}  equals \spad{1}.
    varList      : $ -> List VarSet
       ++ \spad{varList([l1]*[l2]*...[ln])} returns the list of
       ++ variables in the word \spad{l1*l2*...*ln}.

 Private == add

  -- Representation
   Rep := LWORDS

  -- Locales
   recursif: ($,$) -> Boolean

  -- Define
   1 == nil

   x = y == x =$Rep y

   varList x ==
      null x => nil
      le: List VarSet := "setUnion"/ [varList$LWORD l for l in x]

   first x == first(x)$Rep
   rest x == rest(x)$Rep

   coerce(v: VarSet):$ == [ v::LWORD ]
   coerce(l: LWORD):$ == [l]
   listOfTerms(x:$):LWORDS == x pretend LWORDS

   coerce(x:$):WORD ==
     null x => 1
     x.first :: WORD *$WORD coerce(x.rest)

   coerce(x:$):EX ==
     null x => outputForm(1$Integer)$EX
     reduce(_* ,[l :: EX for l in x])$List(EX)

   retractable? x ==
     null x => false
     null x.rest

   retract x ==
      #x ^= 1 => error "cannot convert to Lyndon word"
      x.first

   retractIfCan x ==
      retractable? x => x.first
      "failed"
```

```
    length x ==
       n: Integer := +/[ length l for l in x]
       n::NNI

    recursif(x, y) ==
      null y => false
      null x => true
      x.first = y.first => recursif(rest(x), rest(y))
      lexico(x.first, y.first)

    x < y ==
      lx: NNI := length x; ly: NNI := length y
      lx = ly => recursif(x,y)
      lx < ly
```

———-

— PBWLB.dotabb —

```
"PBWLB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PBWLB"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"PBWLB" -> "FLAGG"
```

———-

17.24 domain POINT Point

— Point.input —

```
)set break resume
)sys rm -f Point.output
)spool Point.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Point
--R Point R: Ring  is a domain constructor
--R Abbreviation for Point is POINT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for POINT
--R
--R------------------------------- Operations --------------------------------
```

```
--R -? : % -> % if R has ABELGRP         concat : List % -> %
--R concat : (%,%) -> %                  concat : (R,%) -> %
--R concat : (%,R) -> %                  construct : List R -> %
--R convert : List R -> %                copy : % -> %
--R cross : (%,%) -> %                   delete : (%,Integer) -> %
--R dimension : % -> PositiveInteger     ?.? : (%,Integer) -> R
--R elt : (%,Integer,R) -> R             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List R
--R eq? : (%,%) -> Boolean               extend : (%,List R) -> %
--R index? : (Integer,%) -> Boolean      indices : % -> List Integer
--R insert : (%,%,Integer) -> %          insert : (R,%,Integer) -> %
--R map : (((R,R) -> R),%,%) -> %        map : ((R -> R),%) -> %
--R new : (NonNegativeInteger,R) -> %    point : List R -> %
--R qelt : (%,Integer) -> R              reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (%,R) -> % if R has MONOID
--R ?*? : (R,%) -> % if R has MONOID
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?+? : (%,%) -> % if R has ABELSG
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R dot : (%,%) -> R if R has RING
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R length : % -> R if R has RADCAT and R has RING
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RADCAT and R has RING
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
```

```
--R max : (%,%) -> % if R has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R min : (%,%) -> % if R has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix R if R has RING
--R parts : % -> List R if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if $ has finiteAggregate and R has SETCAT
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if $ has finiteAggregate and R has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and R has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and R has ORDSET
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

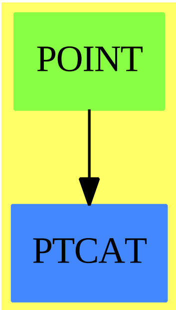
— Point.help —

```
====================================================================
Point examples
====================================================================
```

```
See Also:
o )show Point
```

———-

17.24.1 Point (POINT)



See

⇒ "SubSpaceComponentProperty" (COMPPROP) 20.34.1 on page 2583
⇒ "SubSpace" (SUBSPACE) 20.33.1 on page 2573

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	count	cross
delete	dimension	dot	elt	empty
empty?	entries	entry?	eq?	eval
every?	extend	fill!	find	first
hash	index?	indices	insert	insert
latex	length	less?	magnitude	map
map!	max	maxIndex	member?	members
merge	merge	min	minIndex	more?
new	outerProduct	parts	point	position
qelt	qsetelt!	reduce	remove	removeDuplicates
reverse	reverse!	sample	select	setelt
size?	sort	sort!	sorted?	swap!
zero	#?	?*?	?+?	?-?
?<?	?<=?	?=?	?>?	?>=?
?.?	?~=?	-?	?.?	

— **domain POINT Point** —

```
)abbrev domain POINT Point
++ Author: Mark Botch
++ Description:
```

```
++ This domain implements points in coordinate space

Point(R:Ring) : Exports == Implementation where
  -- Domains for points, subspaces and properties of components in
  -- a subspace

  Exports ==> PointCategory(R)

  Implementation ==> Vector (R) add
    PI   ==> PositiveInteger

    point(l:List R):% ==
      pt := new(#l,R)
      for x in l for i in minIndex(pt).. repeat
        pt.i := x
      pt
    dimension p == (# p)::PI  -- Vector returns NonNegativeInteger...?
    convert(l:List R):% == point(l)
    cross(p0, p1) ==
      #p0 ^=3 or #p1^=3 => error "Arguments to cross must be three dimensional"
      point [p0.2 * p1.3 - p1.2 * p0.3, _
             p1.1 * p0.3 - p0.1 * p1.3, _
             p0.1 * p1.2 - p1.1 * p0.2]
    extend(p,l) == concat(p,point l)
```

———-

— POINT.dotabb —

```
"POINT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=POINT"]
"PTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PTCAT"]
"POINT" -> "PTCAT"
```

———-

17.25 domain POLY Polynomial

— Polynomial.input —

```
)set break resume
)sys rm -f Polynomial.output
)spool Polynomial.output
)set message test on
)set message auto off
```

```
--S 1 of 46
x + 1
--R
--R
--R   (1)  x + 1
--R                                                     Type: Polynomial Integer
--E 1

--S 2 of 46
z - 2.3
--R
--R
--R   (2)  z - 2.3
--R                                                       Type: Polynomial Float
--E 2

--S 3 of 46
y**2 - z + 3/4
--R
--R
--R              2   3
--R   (3)  - z + y  + -
--R                   4
--R                                            Type: Polynomial Fraction Integer
--E 3

--S 4 of 46
y **2 + x*y + y
--R
--R
--R         2
--R   (4)  y  + (x + 1)y
--R                                                     Type: Polynomial Integer
--E 4

--S 5 of 46
% :: DMP([y,x],INT)
--R
--R
--R         2
--R   (5)  y  + y x + y
--R                         Type: DistributedMultivariatePolynomial([y,x],Integer)
--E 5

--S 6 of 46
p := (y-1)**2 * x * z
--R
--R
--R            2
--R   (6)  (x y  - 2x y + x)z
```

```
--R                                                    Type: Polynomial Integer
--E 6

--S 7 of 46
q := (y-1) * x * (z+5)
--R
--R
--R   (7)  (x y - x)z + 5x y - 5x
--R                                                    Type: Polynomial Integer
--E 7

--S 8 of 46
factor(q)
--R
--R
--R   (8)  x(y - 1)(z + 5)
--R                                           Type: Factored Polynomial Integer
--E 8

--S 9 of 46
p - q**2
--R
--R
--R   (9)
--R          2 2     2     2  2          2      2       2              2
--R     (- x y  + 2x y - x )z  + ((- 10x  + x)y  + (20x  - 2x)y - 10x  + x)z
--R   +
--R           2 2      2       2
--R     - 25x y  + 50x y - 25x
--R                                                    Type: Polynomial Integer
--E 9

--S 10 of 46
gcd(p,q)
--R
--R
--R   (10)  x y - x
--R                                                    Type: Polynomial Integer
--E 10

--S 11 of 46
factor %
--R
--R
--R   (11)  x(y - 1)
--R                                           Type: Factored Polynomial Integer
--E 11

--S 12 of 46
lcm(p,q)
```

```
--R
--R
--R              2              2         2
--R   (12)  (x y  - 2x y + x)z  + (5x y  - 10x y + 5x)z
--R                                                     Type: Polynomial Integer
--E 12

--S 13 of 46
content p
--R
--R
--R   (13)  1
--R                                                        Type: PositiveInteger
--E 13

--S 14 of 46
resultant(p,q,z)
--R
--R
--R            2 3      2 2      2      2
--R   (14)  5x y  - 15x y  + 15x y - 5x
--R                                                     Type: Polynomial Integer
--E 14

--S 15 of 46
resultant(p,q,x)
--R
--R
--R   (15)  0
--R                                                     Type: Polynomial Integer
--E 15

--S 16 of 46
mainVariable p
--R
--R
--R   (16)  z
--R                                                      Type: Union(Symbol,...)
--E 16

--S 17 of 46
mainVariable(1 :: POLY INT)
--R
--R
--R   (17)  "failed"
--R                                                    Type: Union("failed",...)
--E 17

--S 18 of 46
ground? p
```

```
--R
--R
--R   (18)  false
--R                                                               Type: Boolean
--E 18

--S 19 of 46
ground?(1 :: POLY INT)
--R
--R
--R   (19)  true
--R                                                               Type: Boolean
--E 19

--S 20 of 46
variables p
--R
--R
--R   (20)  [z,y,x]
--R                                                           Type: List Symbol
--E 20

--S 21 of 46
degree(p,x)
--R
--R
--R   (21)  1
--R                                                       Type: PositiveInteger
--E 21

--S 22 of 46
degree(p,y)
--R
--R
--R   (22)  2
--R                                                       Type: PositiveInteger
--E 22

--S 23 of 46
degree(p,z)
--R
--R
--R   (23)  1
--R                                                       Type: PositiveInteger
--E 23

--S 24 of 46
degree(p,[x,y,z])
--R
--R
```

```
--R   (24)  [1,2,1]
--R                                                 Type: List NonNegativeInteger
--E 24

--S 25 of 46
minimumDegree(p,z)
--R
--R
--R   (25)  1
--R                                                         Type: PositiveInteger
--E 25

--S 26 of 46
totalDegree p
--R
--R
--R   (26)  4
--R                                                         Type: PositiveInteger
--E 26

--S 27 of 46
leadingMonomial p
--R
--R
--R            2
--R   (27)  x y z
--R                                                      Type: Polynomial Integer
--E 27

--S 28 of 46
reductum p
--R
--R
--R   (28)  (- 2x y + x)z
--R                                                      Type: Polynomial Integer
--E 28

--S 29 of 46
p - leadingMonomial p - reductum p
--R
--R
--R   (29)  0
--R                                                      Type: Polynomial Integer
--E 29

--S 30 of 46
leadingCoefficient p
--R
--R
--R   (30)  1
```

```
--R                                                          Type: PositiveInteger
--E 30

--S 31 of 46
p
--R
--R
--R                2
--R   (31)  (x y  - 2x y + x)z
--R                                                       Type: Polynomial Integer
--E 31

--S 32 of 46
eval(p,x,w)
--R
--R
--R                2
--R   (32)  (w y  - 2w y + w)z
--R                                                       Type: Polynomial Integer
--E 32

--S 33 of 46
eval(p,x,1)
--R
--R
--R              2
--R   (33)  (y  - 2y + 1)z
--R                                                       Type: Polynomial Integer
--E 33

--S 34 of 46
eval(p,x,y^2 - 1)
--R
--R
--R              4     3
--R   (34)  (y  - 2y  + 2y - 1)z
--R                                                       Type: Polynomial Integer
--E 34

--S 35 of 46
D(p,x)
--R
--R
--R              2
--R   (35)  (y  - 2y + 1)z
--R                                                       Type: Polynomial Integer
--E 35

--S 36 of 46
D(p,y)
```

```
--R
--R
--R   (36)  (2x y - 2x)z
--R                                                     Type: Polynomial Integer
--E 36

--S 37 of 46
D(p,z)
--R
--R
--R            2
--R   (37)  x y  - 2x y + x
--R                                                     Type: Polynomial Integer
--E 37

--S 38 of 46
integrate(p,y)
--R
--R
--R          1    3      2
--R   (38)  (- x y  - x y  + x y)z
--R          3
--R                                            Type: Polynomial Fraction Integer
--E 38

--S 39 of 46
qr := monicDivide(p,x+1,x)
--R
--R
--R                       2                            2
--R   (39)  [quotient= (y  - 2y + 1)z,remainder= (- y  + 2y - 1)z]
--R     Type: Record(quotient: Polynomial Integer,remainder: Polynomial Integer)
--E 39

--S 40 of 46
qr.remainder
--R
--R
--R             2
--R   (40)  (- y  + 2y - 1)z
--R                                                     Type: Polynomial Integer
--E 40

--S 41 of 46
p - ((x+1) * qr.quotient + qr.remainder)
--R
--R
--R   (41)  0
--R                                                     Type: Polynomial Integer
--E 41
```

```
--S 42 of 46
p/q
--R
--R
--R         (y - 1)z
--R   (42)  --------
--R           z + 5
--R                                                 Type: Fraction Polynomial Integer
--E 42

--S 43 of 46
(2/3) * x**2 - y + 4/5
--R
--R
--R               2  2   4
--R   (43)  - y + - x  + -
--R               3      5
--R                                                 Type: Polynomial Fraction Integer
--E 43

--S 44 of 46
% :: FRAC POLY INT
--R
--R
--R                     2
--R         - 15y + 10x  + 12
--R   (44)  -----------------
--R                 15
--R                                                 Type: Fraction Polynomial Integer
--E 44

--S 45 of 46
% :: POLY FRAC INT
--R
--R
--R               2  2   4
--R   (45)  - y + - x  + -
--R               3      5
--R                                                 Type: Polynomial Fraction Integer
--E 45

--S 46 of 46
map(numeric,%)
--R
--R
--R                                                 2
--R   (46)  - 1.0 y + 0.6666666666 6666666667 x  + 0.8
--R                                                            Type: Polynomial Float
--E 46
```

```
)spool
)lisp (bye)
```

— Polynomial.help —

```
====================================================================
Polynomial examples
====================================================================

The domain constructor Polynomial (abbreviation: POLY) provides
polynomials with an arbitrary number of unspecified variables.

It is used to create the default polynomial domains in Axiom.  Here
the coefficients are integers.

  x + 1
    x + 1
                          Type: Polynomial Integer

Here the coefficients have type Float.

  z - 2.3
    z - 2.3
                           Type: Polynomial Float

And here we have a polynomial in two variables with coefficients which
have type Fraction Integer.

  y**2 - z + 3/4
           2   3
    - z + y  + -
               4
                           Type: Polynomial Fraction Integer

The representation of objects of domains created by Polynomial is that
of recursive univariate polynomials. The term univariate means "one
variable". The term multivariate means "possibly more than one variable".

This recursive structure is sometimes obvious from the display of a polynomial.

  y **2 + x*y + y
     2
    y  + (x + 1)y
                           Type: Polynomial Integer

In this example, you see that the polynomial is stored as a polynomial
in y with coefficients that are polynomials in x with integer coefficients.
```

In fact, you really don't need to worry about the representation unless you are working on an advanced application where it is critical. The polynomial types created from DistributedMultivariatePolynomial and NewDistributedMultivariatePolynomial are stored and displayed in a non-recursive manner.

You see a "flat" display of the above polynomial by converting to one of those types.

```
  % :: DMP([y,x],INT)
     2
    y  + y x + y
                   Type: DistributedMultivariatePolynomial([y,x],Integer)
```

We will demonstrate many of the polynomial facilities by using two polynomials with integer coefficients.

By default, the interpreter expands polynomial expressions, even if they are written in a factored format.

```
  p := (y-1)**2 * x * z
        2
    (x y  - 2x y + x)z
                   Type: Polynomial Integer
```

See Factored to see how to create objects in factored form directly.

```
  q := (y-1) * x * (z+5)
    (x y - x)z + 5x y - 5x
                   Type: Polynomial Integer
```

The fully factored form can be recovered by using factor.

```
  factor(q)
    x(y - 1)(z + 5)
                   Type: Factored Polynomial Integer
```

This is the same name used for the operation to factor integers. Such reuse of names is called overloading and makes it much easier to think of solving problems in general ways. Axiom facilities for factoring polynomials created with Polynomial are currently restricted to the integer and rational number coefficient cases.

The standard arithmetic operations are available for polynomials.

```
  p - q**2
         2 2     2     2  2          2      2       2             2
     (- x y  + 2x y - x )z  + ((- 10x  + x)y  + (20x  - 2x)y - 10x  + x)z
   +
          2 2      2       2
```

```
     - 25x y  + 50x y - 25x
                   Type: Polynomial Integer
```

The operation gcd is used to compute the greatest common divisor of two polynomials.

```
  gcd(p,q)
    x y - x
                   Type: Polynomial Integer
```

In the case of p and q, the gcd is obvious from their definitions. We factor the gcd to show this relationship better.

```
  factor %
    x(y - 1)
                   Type: Factored Polynomial Integer
```

The least common multiple is computed by using lcm.

```
  lcm(p,q)
        2             2        2
    (x y  - 2x y + x)z  + (5x y  - 10x y + 5x)z
                   Type: Polynomial Integer
```

Use content to compute the greatest common divisor of the coefficients of the polynomial.

```
  content p
    1
                   Type: PositiveInteger
```

Many of the operations on polynomials require you to specify a variable. For example, resultant requires you to give the variable in which the polynomials should be expressed.

This computes the resultant of the values of p and q, considering them as polynomials in the variable z. They do not share a root when thought of as polynomials in z.

```
  resultant(p,q,z)
       2 3      2 2      2      2
     5x y  - 15x y  + 15x y - 5x
                   Type: Polynomial Integer
```

This value is 0 because as polynomials in x the polynomials have a common root.

```
  resultant(p,q,x)
    0
                   Type: Polynomial Integer
```

```
The data type used for the variables created by Polynomial is Symbol.
As mentioned above, the representation used by Polynomial is recursive
and so there is a main variable for nonconstant polynomials.

The operation mainVariable returns this variable.  The return type is
actually a union of Symbol and "failed".

  mainVariable p
    z
                  Type: Union(Symbol,...)

The latter branch of the union is be used if the polynomial has no
variables, that is, is a constant.

  mainVariable(1 :: POLY INT)
    "failed"
                  Type: Union("failed",...)

You can also use the predicate ground? to test whether a polynomial is
in fact a member of its ground ring.

  ground? p
    false
                  Type: Boolean

  ground?(1 :: POLY INT)
    true
                  Type: Boolean

The complete list of variables actually used in a particular polynomial
is returned by variables.  For constant polynomials, this list is empty.

  variables p
    [z,y,x]
                  Type: List Symbol

The degree operation returns the degree of a polynomial in a specific variable.

  degree(p,x)
    1
                  Type: PositiveInteger

  degree(p,y)
    2
                  Type: PositiveInteger

  degree(p,z)
    1
                  Type: PositiveInteger
```

If you give a list of variables for the second argument, a list of the degrees in those variables is returned.

```
  degree(p,[x,y,z])
    [1,2,1]
                   Type: List NonNegativeInteger
```

The minimum degree of a variable in a polynomial is computed using minimumDegree.

```
  minimumDegree(p,z)
    1
                   Type: PositiveInteger
```

The total degree of a polynomial is returned by totalDegree.

```
  totalDegree p
    4
                   Type: PositiveInteger
```

It is often convenient to think of a polynomial as a leading monomial plus the remaining terms.

```
  leadingMonomial p
        2
     x y z
                   Type: Polynomial Integer
```

The reductum operation returns a polynomial consisting of the sum of the monomials after the first.

```
  reductum p
    (- 2x y + x)z
                   Type: Polynomial Integer
```

These have the obvious relationship that the original polynomial is equal to the leading monomial plus the reductum.

```
  p - leadingMonomial p - reductum p
    0
                   Type: Polynomial Integer
```

The value returned by leadingMonomial includes the coefficient of that term. This is extracted by using leadingCoefficient on the original polynomial.

```
  leadingCoefficient p
    1
                   Type: PositiveInteger
```

The operation eval is used to substitute a value for a variable in a polynomial.

```
  p
        2
    (x y  - 2x y + x)z
                   Type: Polynomial Integer
```

This value may be another variable, a constant or a polynomial.

```
  eval(p,x,w)
        2
    (w y  - 2w y + w)z
                   Type: Polynomial Integer
```

```
  eval(p,x,1)
      2
    (y  - 2y + 1)z
                   Type: Polynomial Integer
```

Actually, all the things being substituted are just polynomials, some more trivial than others.

```
  eval(p,x,y^2 - 1)
      4     3
    (y  - 2y  + 2y - 1)z
                   Type: Polynomial Integer
```

Derivatives are computed using the D operation.

```
  D(p,x)
      2
    (y  - 2y + 1)z
                   Type: Polynomial Integer
```

The first argument is the polynomial and the second is the variable.

```
  D(p,y)
    (2x y - 2x)z
                   Type: Polynomial Integer
```

Even if the polynomial has only one variable, you must specify it.

```
  D(p,z)
       2
    x y  - 2x y + x
                   Type: Polynomial Integer
```

Integration of polynomials is similar and the integrate operation is used.

Integration requires that the coefficients support division. Axiom converts polynomials over the integers to polynomials over the rational numbers before integrating them.

```
  integrate(p,y)
     1    3      2
    (- x y  - x y  + x y)z
     3
                  Type: Polynomial Fraction Integer
```

It is not possible, in general, to divide two polynomials. In our example using polynomials over the integers, the operation monicDivide divides a polynomial by a monic polynomial (that is, a polynomial with leading coefficient equal to 1). The result is a record of the quotient and remainder of the division.

You must specify the variable in which to express the polynomial.

```
  qr := monicDivide(p,x+1,x)
                  2                           2
    [quotient= (y  - 2y + 1)z,remainder= (- y  + 2y - 1)z]
     Type: Record(quotient: Polynomial Integer,remainder: Polynomial Integer)
```

The selectors of the components of the record are quotient and remainder. Issue this to extract the remainder.

```
  qr.remainder
        2
    (- y  + 2y - 1)z
                       Type: Polynomial Integer
```

Now that we can extract the components, we can demonstrate the relationship among them and the arguments to our original expression qr := monicDivide(p,x+1,x).

```
  p - ((x+1) * qr.quotient + qr.remainder)
    0
                       Type: Polynomial Integer
```

If the / operator is used with polynomials, a fraction object is created. In this example, the result is an object of type Fraction Polynomial Integer.

```
  p/q
    (y - 1)z
    --------
      z + 5
                       Type: Fraction Polynomial Integer
```

If you use rational numbers as polynomial coefficients, the resulting object is of type Polynomial Fraction Integer.

```
  (2/3) * x**2 - y + 4/5
          2  2   4
    - y + - x  + -
          3      5
                       Type: Polynomial Fraction Integer
```

This can be converted to a fraction of polynomials and back again, if required.

```
  % :: FRAC POLY INT
               2
    - 15y + 10x  + 12
    -----------------
           15
                       Type: Fraction Polynomial Integer

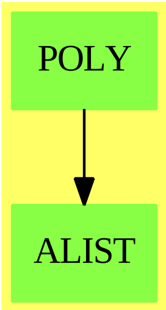
  % :: POLY FRAC INT
          2  2   4
    - y + - x  + -
          3      5
                       Type: Polynomial Fraction Integer
```

To convert the coefficients to floating point, map the numeric operation on the coefficients of the polynomial.

```
  map(numeric,%)
    - 1.0 y + 0.6666666666 6666666667 x  + 0.8
                       Type: Polynomial Float
```

See Also:
o)help Factored
o)help UnivariatePolynomial
o)help MultivariatePolynomial
o)help DistributedMultivariatePolynomial
o)help NewDistributedMultivariatePolynomial
o)show Polynomial

17.25.1 Polynomial (POLY)



See

⇒ “MultivariatePolynomial” (MPOLY) 14.16.1 on page 1645
⇒ “SparseMultivariatePolynomial” (SMP) 20.14.1 on page 2381
⇒ “IndexedExponents” (INDE) 10.9.1 on page 1183

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	integrate
isExpt	isPlus	isTimes
latex	lcm	leadingCoefficient
leadingMonomial	mainVariable	map
mapExponents	max	min
minimumDegree	monicDivide	monomial
monomial?	monomials	multivariate
one?	numberOfMonomials	patternMatch
pomopo!	prime?	primitiveMonomials
primitivePart	recip	reducedSystem
reductum	resultant	retract
retractIfCan	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	totalDegree	totalDegree
unit?	unitCanonical	unitNormal
univariate	variables	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

— domain POLY Polynomial —

```
)abbrev domain POLY Polynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd
++ Related Constructors: SparseMultivariatePolynomial, MultivariatePolynomial
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate
++ References:
++ Description:
++ This type is the basic representation of sparse recursive multivariate
++ polynomials whose variables are arbitrary symbols. The ordering
++ is alphabetic determined by the Symbol type.
++ The coefficient ring may be non commutative,
```

```
++ but the variables are assumed to commute.

Polynomial(R:Ring):
  PolynomialCategory(R, IndexedExponents Symbol, Symbol) with
   if R has Algebra Fraction Integer then
     integrate: (%, Symbol) -> %
       ++ integrate(p,x) computes the integral of \spad{p*dx}, i.e.
       ++ integrates the polynomial p with respect to the variable x.
 == SparseMultivariatePolynomial(R, Symbol) add

    import UserDefinedPartialOrdering(Symbol)

    coerce(p:%):OutputForm ==
      (r:= retractIfCan(p)@Union(R,"failed")) case R => r::R::OutputForm
      a :=
        userOrdered?() => largest variables p
        mainVariable(p)::Symbol
      outputForm(univariate(p, a), a::OutputForm)

    if R has Algebra Fraction Integer then
      integrate(p, x) == (integrate univariate(p, x)) (x::%)
```

— **POLY.dotabb** —

```
"POLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=POLY"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"POLY" -> "ALIST"
```

17.26 domain IDEAL PolynomialIdeals

— **PolynomialIdeals.input** —

```
)set break resume
)sys rm -f PolynomialIdeals.output
)spool PolynomialIdeals.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show PolynomialIdeals
--R PolynomialIdeals(F: Field,Expon: OrderedAbelianMonoidSup,VarSet: OrderedSet,DPoly: Polyno
--R Abbreviation for PolynomialIdeals is IDEAL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDEAL
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean               coerce : List DPoly -> %
--R coerce : % -> OutputForm             dimension : % -> Integer
--R element? : (DPoly,%) -> Boolean      generators : % -> List DPoly
--R groebner : % -> %                    groebner? : % -> Boolean
--R groebnerIdeal : List DPoly -> %      hash : % -> SingleInteger
--R ideal : List DPoly -> %              in? : (%,%) -> Boolean
--R inRadical? : (DPoly,%) -> Boolean    intersect : List % -> %
--R intersect : (%,%) -> %               latex : % -> String
--R leadingIdeal : % -> %                one? : % -> Boolean
--R quotient : (%,DPoly) -> %            quotient : (%,%) -> %
--R saturate : (%,DPoly) -> %            zero? : % -> Boolean
--R zeroDim? : % -> Boolean              ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R backOldPos : Record(mval: Matrix F,invmval: Matrix F,genIdeal: %) -> %
--R dimension : (%,List VarSet) -> Integer
--R generalPosition : (%,List VarSet) -> Record(mval: Matrix F,invmval: Matrix F,genIdeal: %)
--R relationsIdeal : List DPoly -> SuchThat(List Polynomial F,List Equation Polynomial F) if
--R saturate : (%,DPoly,List VarSet) -> %
--R zeroDim? : (%,List VarSet) -> Boolean
--R
--E 1

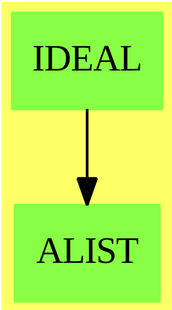
)spool
)lisp (bye)
```

— **PolynomialIdeals.help** —

```
====================================================================
PolynomialIdeals examples
====================================================================

See Also:
o )show PolynomialIdeals
```

17.26.1 PolynomialIdeals (IDEAL)



Exports:

backOldPos	coerce	dimension	element?	generalPosition
generators	groebner	groebner?	groebnerIdeal	hash
ideal	in?	inRadical?	intersect	latex
leadingIdeal	one?	quotient	relationsIdeal	saturate
zero?	zeroDim?	?~=?	?**?	?*?
?+?	?=?			

— domain IDEAL PolynomialIdeals —

```
)abbrev domain IDEAL PolynomialIdeals
++ Author: P. Gianni
++ Date Created: summer 1986
++ Date Last Updated: September 1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References: GTZ
++ Description:
++ This domain represents polynomial ideals with coefficients in any
++ field and supports the basic ideal operations, including intersection
++ sum and quotient.
++ An ideal is represented by a list of polynomials (the generators of
++ the ideal) and a boolean that is true if the generators are a Groebner
++ basis.
++ The algorithms used are based on Groebner basis computations. The
++ ordering is determined by the datatype of the input polynomials.
++ Users may use refinements of total degree orderings.

PolynomialIdeals(F,Expon,VarSet,DPoly) : C == T
 where
   F         :  Field
   Expon     :  OrderedAbelianMonoidSup
```

```
VarSet    :  OrderedSet
DPoly     :  PolynomialCategory(F,Expon,VarSet)

SUP      ==> SparseUnivariatePolynomial(DPoly)
NNI      ==> NonNegativeInteger
Z        ==> Integer
P        ==> Polynomial F
MF       ==> Matrix(F)
ST       ==> SuchThat(List P, List Equation P)

GenMPos  ==> Record(mval:MF,invmval:MF,genIdeal:Ideal)
Ideal    ==> %

C == SetCategory with

  "*"            :  (Ideal,Ideal)       -> Ideal
    ++ I*J computes the product of the ideal I and J.
  "**"           :   (Ideal,NNI)        -> Ideal
    ++ I**n computes the nth power of the ideal I.
  "+"            :  (Ideal,Ideal)       -> Ideal
    ++ I+J computes the ideal generated by the union of I and J.
  one?           :     Ideal            -> Boolean
    ++ one?(I) tests whether the ideal I is the unit ideal,
    ++ i.e. contains 1.
  zero?          :     Ideal            -> Boolean
    ++ zero?(I) tests whether the ideal I is the zero ideal
  element?       :  (DPoly,Ideal)       -> Boolean
    ++ element?(f,I) tests whether the polynomial f belongs to
    ++ the ideal I.
  in?            :  (Ideal,Ideal)       -> Boolean
    ++ in?(I,J) tests if the ideal I is contained in the ideal J.
  inRadical?     :  (DPoly,Ideal)       -> Boolean
    ++ inRadical?(f,I) tests if some power of the polynomial f
    ++ belongs to the ideal I.
  zeroDim?       : (Ideal,List VarSet)  -> Boolean
    ++ zeroDim?(I,lvar) tests if the ideal I is zero dimensional, i.e.
    ++ all its associated primes are maximal,
    ++ in the ring \spad{F[lvar]}
  zeroDim?       :     Ideal            -> Boolean
    ++ zeroDim?(I) tests if the ideal I is zero dimensional, i.e.
    ++ all its associated primes are maximal,
    ++ in the ring \spad{F[lvar]},
    ++ where lvar are the variables appearing  in I
  intersect      :  (Ideal,Ideal)       -> Ideal
    ++ intersect(I,J) computes the intersection of the ideals I and J.
  intersect      :   List(Ideal)        -> Ideal
    ++ intersect(LI) computes the intersection of the list of ideals LI.
  quotient       :  (Ideal,Ideal)       -> Ideal
    ++ quotient(I,J) computes the quotient of the ideals I and J,
    ++ \spad{(I:J)}.
```

```
quotient        :  (Ideal,DPoly)       -> Ideal
  ++ quotient(I,f) computes the quotient of the ideal I by the principal
  ++ ideal generated by the polynomial f, \spad{(I:(f))}.
groebner        :     Ideal            -> Ideal
  ++ groebner(I) returns a set of generators of I that are a
  ++ Groebner basis for I.
generalPosition :  (Ideal,List VarSet)      -> GenMPos
  ++ generalPosition(I,listvar) perform a random linear
  ++ transformation on the variables in listvar and returns
  ++ the transformed ideal along with the change of basis matrix.
backOldPos      :     GenMPos          -> Ideal
  ++ backOldPos(genPos) takes the result
  ++ produced by generalPosition from PolynomialIdeals
  ++ and performs the inverse transformation, returning the original ideal
  ++ \spad{backOldPos(generalPosition(I,listvar))} = I.
dimension       : (Ideal,List VarSet)   -> Z
  ++ dimension(I,lvar) gives the dimension of the ideal I,
  ++ in the ring \spad{F[lvar]}
dimension       :      Ideal           -> Z
  ++ dimension(I) gives the dimension of the ideal I.
  ++ in the ring \spad{F[lvar]}, where lvar are the variables
  ++ appearing  in I
leadingIdeal    :     Ideal            -> Ideal
  ++ leadingIdeal(I) is the ideal generated by the
  ++ leading terms of the elements of the ideal I.
ideal           :   List DPoly         ->  Ideal
  ++ ideal(polyList) constructs the ideal generated by the list
  ++ of polynomials polyList.
groebnerIdeal       :   List DPoly          ->  Ideal
  ++ groebnerIdeal(polyList) constructs the ideal generated by the list
  ++ of polynomials polyList which are assumed to be a Groebner
  ++ basis.
  ++ Note: this operation avoids a Groebner basis computation.
groebner?           :     Ideal             ->  Boolean
  ++ groebner?(I) tests if the generators of the ideal I are a
  ++ Groebner basis.
generators      :     Ideal            ->  List DPoly
  ++ generators(I) returns a list of generators for the ideal I.
coerce          :   List DPoly         ->  Ideal
  ++ coerce(polyList) converts the list of polynomials polyList
  ++ to an ideal.

saturate        :  (Ideal,DPoly)       -> Ideal
  ++ saturate(I,f) is the saturation of the ideal I
  ++ with respect to the multiplicative
  ++ set generated by the polynomial f.
saturate        :(Ideal,DPoly,List VarSet)  -> Ideal
  ++ saturate(I,f,lvar) is the saturation with respect to the prime
  ++ principal ideal which is generated by f in the polynomial ring
  ++ \spad{F[lvar]}.
```

```
  if VarSet has ConvertibleTo Symbol then
    relationsIdeal  :    List DPoly         -> ST
      ++ relationsIdeal(polyList) returns the ideal of relations among the
      ++ polynomials in polyList.

T  == add

---  Representation ---
  Rep := Record(idl:List DPoly,isGr:Boolean)


             ----  Local Functions  ----

  contractGrob  :    newIdeal          ->  Ideal
  npoly         :     DPoly            ->  newPoly
  oldpoly       :     newPoly          ->  Union(DPoly,"failed")
  leadterm      :   (DPoly,VarSet)     ->  DPoly
  choosel       :   (DPoly,DPoly)      ->  DPoly
  isMonic?      :   (DPoly,VarSet)     ->  Boolean
  randomat      :     List Z           ->  Record(mM:MF,imM:MF)
  monomDim      : (Ideal,List VarSet)  ->  NNI
  variables     :       Ideal          ->  List VarSet
  subset        :     List VarSet      ->  List List VarSet
  makeleast     : (List VarSet,List VarSet)  ->  List VarSet

  newExpon:  OrderedAbelianMonoidSup
  newExpon:= Product(NNI,Expon)
  newPoly := PolynomialRing(F,newExpon)

  import GaloisGroupFactorizer(SparseUnivariatePolynomial Z)
  import GroebnerPackage(F,Expon,VarSet,DPoly)
  import GroebnerPackage(F,newExpon,VarSet,newPoly)

  newIdeal ==> List(newPoly)

  npoly(f:DPoly) : newPoly ==
    f=0$DPoly => 0$newPoly
    monomial(leadingCoefficient f,makeprod(0,degree f))$newPoly +
          npoly(reductum f)

  oldpoly(q:newPoly) : Union(DPoly,"failed") ==
    q=0$newPoly => 0$DPoly
    dq:newExpon:=degree q
    n:NNI:=selectfirst (dq)
    n^=0 => "failed"
    ((g:=oldpoly reductum q) case "failed") => "failed"
    monomial(leadingCoefficient q,selectsecond dq)$DPoly + (g::DPoly)

  leadterm(f:DPoly,lvar:List VarSet) : DPoly ==
    empty?(lf:=variables f)  or lf=lvar => f
```

```
    leadterm(leadingCoefficient univariate(f,lf.first),lvar)

  choosel(f:DPoly,g:DPoly) : DPoly ==
    g=0 => f
    (f1:=f exquo g) case "failed" => f
    choosel(f1::DPoly,g)

  contractGrob(I1:newIdeal) : Ideal ==
    J1:List(newPoly):=groebner(I1)
    while (oldpoly J1.first) case "failed" repeat J1:=J1.rest
    [[(oldpoly f)::DPoly for f in J1],true]

  makeleast(fullVars: List VarSet,leastVars:List VarSet) : List VarSet ==
    n:= # leastVars
    #fullVars < n  => error "wrong vars"
    n=0 => fullVars
    append([vv for vv in fullVars| ^member?(vv,leastVars)],leastVars)

  isMonic?(f:DPoly,x:VarSet) : Boolean ==
    ground? leadingCoefficient univariate(f,x)

  subset(lv : List VarSet) : List List VarSet ==
    #lv =1 => [lv,empty()]
    v:=lv.1
    ll:=subset(rest lv)
    l1:=[concat(v,set) for set in ll]
    concat(l1,ll)

  monomDim(listm:Ideal,lv:List VarSet) : NNI ==
    monvar: List List VarSet := []
    for f in generators listm repeat
      mvset := variables f
      #mvset > 1 => monvar:=concat(mvset,monvar)
      lv:=delete(lv,position(mvset.1,lv))
    empty? lv => 0
    lsubset : List List VarSet := sort((a,b)+->#a > #b ,subset(lv))
    for subs in lsubset repeat
      ldif:List VarSet:= lv
      for mvset in monvar while ldif ^=[] repeat
        ldif:=setDifference(mvset,subs)
      if ^(empty? ldif) then  return #subs
    0

          --    Exported  Functions   ----

            ----  is  I =  J  ?  ----
  (I:Ideal = J:Ideal) == in?(I,J) and in?(J,I)

          ----  check if f is in I  ----
  element?(f:DPoly,I:Ideal) : Boolean ==
```

```
  Id:=(groebner I).idl
  empty? Id => f = 0
  normalForm(f,Id) = 0

        ---- check if I is contained in J  ----
in?(I:Ideal,J:Ideal):Boolean ==
  J:= groebner J
  empty?(I.idl) => true
  "and"/[element?(f,J) for f in I.idl ]


       ----  groebner base for an Ideal  ----
groebner(I:Ideal) : Ideal  ==
  I.isGr =>
    "or"/[^zero? f for f in I.idl] => I
    [empty(),true]
  [groebner I.idl ,true]

       ----  Intersection of two ideals  ----
intersect(I:Ideal,J:Ideal) : Ideal ==
  empty?(Id:=I.idl) => I
  empty?(Jd:=J.idl) => J
  tp:newPoly := monomial(1,makeprod(1,0$Expon))$newPoly
  tp1:newPoly:= tp-1
  contractGrob(concat([tp*npoly f for f in Id],
              [tp1*npoly f for f in Jd]))


       ----   intersection for a list of ideals  ----

intersect(lid:List(Ideal)) : Ideal == "intersect"/[l for l in lid]

          ----  quotient by an element  ----
quotient(I:Ideal,f:DPoly) : Ideal ==
  --[[(g exquo f)::DPoly for g in (intersect(I,[f]::%)).idl ],true]
   import GroebnerInternalPackage(F,Expon,VarSet,DPoly)
   [minGbasis [(g exquo f)::DPoly
            for g in (intersect(I,[f]::%)).idl ],true]

           ----  quotient of two ideals  ----
quotient(I:Ideal,J:Ideal) : Ideal ==
  Jdl := J.idl
  empty?(Jdl) => ideal [1]
  [("intersect"/[quotient(I,f) for f in Jdl ]).idl ,true]


           ----    sum of two ideals  ----
(I:Ideal + J:Ideal) : Ideal == [groebner(concat(I.idl ,J.idl )),true]

           ----   product of two ideals  ----
```

```
(I:Ideal * J:Ideal):Ideal ==
  [groebner([:[f*g for f in I.idl ] for g in J.idl ]),true]

         ----  power of an ideal  ----
(I:Ideal ** n:NNI) : Ideal ==
  n=0 => [[1$DPoly],true]
  (I * (I**(n-1):NNI))

  ----  saturation with respect to the multiplicative set f**n ----
saturate(I:Ideal,f:DPoly) : Ideal ==
  f=0 => error "f is zero"
  tp:newPoly := (monomial(1,makeprod(1,0$Expon))$newPoly * npoly f)-1
  contractGrob(concat(tp,[npoly g for g in I.idl ]))

----  saturation with respect to a prime principal ideal in lvar ---
saturate(I:Ideal,f:DPoly,lvar:List(VarSet)) : Ideal ==
  Id := I.idl
  fullVars := "setUnion"/[variables g for g in Id]
  newVars:=makeleast(fullVars,lvar)
  subVars := [monomial(1,vv,1) for vv in newVars]
  J:List DPoly:=groebner([eval(g,fullVars,subVars) for g in Id])
  ltJ:=[leadterm(g,lvar) for g in J]
  s:DPoly:=_*/[choosel(ltg,f) for ltg in ltJ]
  fullPol:=[monomial(1,vv,1) for vv in fullVars]
  [[eval(g,newVars,fullPol) for g in (saturate(J::%,s)).idl],true]

      ----  is the ideal zero dimensional?  ----
      ----      in the ring F[lvar]?        ----
zeroDim?(I:Ideal,lvar:List VarSet) : Boolean ==
  J:=(groebner I).idl
  empty? J => false
  J = [1] => false
  n:NNI := # lvar
  #J < n => false
  for f in J while ^empty?(lvar) repeat
    x:=(mainVariable f)::VarSet
    if isMonic?(f,x) then lvar:=delete(lvar,position(x,lvar))
  empty?(lvar)

      ----  is the ideal zero dimensional?  ----
zeroDim?(I:Ideal):Boolean == zeroDim?(I,"setUnion"/[variables g for g in I.idl])

          ----  test if f is in the radical of I  ----
inRadical?(f:DPoly,I:Ideal) : Boolean ==
  f=0$DPoly => true
  tp:newPoly :=(monomial(1,makeprod(1,0$Expon))$newPoly * npoly f)-1
  Id:=I.idl
  normalForm(1$newPoly,groebner concat(tp,[npoly g for g in Id])) = 0

         ----   dimension of an ideal  ----
```

```
        ----    in the ring F[lvar]   ----
dimension(I:Ideal,lvar:List VarSet) : Z ==
  I:=groebner I
  empty?(I.idl) => # lvar
  element?(1,I) => -1
  truelist:="setUnion"/[variables f for f in I.idl]
  "or"/[^member?(vv,lvar) for vv in truelist] => error "wrong variables"
  truelist:=setDifference(lvar,setDifference(lvar,truelist))
  ed:Z:=#lvar - #truelist
  leadid:=leadingIdeal(I)
  n1:Z:=monomDim(leadid,truelist)::Z
  ed+n1

dimension(I:Ideal) : Z == dimension(I,"setUnion"/[variables g for g in I.idl])

-- leading term ideal --
leadingIdeal(I : Ideal) : Ideal ==
  Idl:= (groebner I).idl
  [[(f-reductum f) for f in Idl],true]

         ---- ideal of relations among the fi  ----
if VarSet has ConvertibleTo Symbol then

  monompol(df:List NNI,lcf:F,lv:List VarSet) : P ==
    g:P:=lcf::P
    for dd in df for v in lv repeat
      g:= monomial(g,convert v,dd)
    g

  relationsIdeal(listf : List DPoly): ST ==
    empty? listf  => [empty(),empty()]$ST
    nf:=#listf
    lvint := "setUnion"/[variables g for g in listf]
    vl: List Symbol := [convert vv for vv in lvint]
    nvar:List Symbol:=[new() for i in 1..nf]
    VarSet1:=OrderedVariableList(concat(vl,nvar))
    lv1:=[variable(vv)$VarSet1::VarSet1 for vv in nvar]
    DirP:=DirectProduct(nf,NNI)
    nExponent:=Product(Expon,DirP)
    nPoly := PolynomialRing(F,nExponent)
    gp:=GroebnerPackage(F,nExponent,VarSet1,nPoly)
    lf:List nPoly :=[]
    lp:List P:=[]
    for f in listf for i in 1..  repeat
      vec2:Vector(NNI):=new(nf,0$NNI)
      vec2.i:=1
      g:nPoly:=0$nPoly
      pol:=0$P
      while f^=0 repeat
        df:=degree(f-reductum f,lvint)
```

```
        lcf:=leadingCoefficient f
        pol:=pol+monompol(df,lcf,lvint)
        g:=g+monomial(lcf,makeprod(degree f,0))$nPoly
        f:=reductum f
      lp:=concat(pol,lp)
      lf:=concat(monomial(1,makeprod(0,directProduct vec2))-g,lf)
    npol:List P :=[v::P for v in nvar]
    leq : List Equation P :=
          [p = pol for p in npol for pol in reverse lp ]
    lf:=(groebner lf)$gp
    while lf^=[] repeat
      q:=lf.first
      dq:nExponent:=degree q
      n:=selectfirst (dq)
      if n=0 then leave "done"
      lf:=lf.rest
    solsn:List P:=[]
    for q in lf repeat
      g:Polynomial F :=0
      while q^=0 repeat
        dq:=degree q
        lcq:=leadingCoefficient q
        q:=reductum q
        vdq:=(selectsecond dq):Vector NNI
        g:=g+ lcq*
           _*/[p**vdq.j for p in npol for j in 1..]
      solsn:=concat(g,solsn)
    [solsn,leq]$ST

coerce(Id:List DPoly) : Ideal == [Id,false]

coerce(I:Ideal) : OutputForm ==
  Idl := I.idl
  empty? Idl => [0$DPoly] :: OutputForm
  Idl :: OutputForm

ideal(Id:List DPoly) :Ideal  ==  [[f for f in Id|f^=0],false]

groebnerIdeal(Id:List DPoly) : Ideal == [Id,true]

generators(I:Ideal) : List DPoly  == I.idl

groebner?(I:Ideal) : Boolean  ==  I.isGr

one?(I:Ideal) : Boolean == element?(1, I)

zero?(I:Ideal) : Boolean == empty? (groebner I).idl
```

———-

— IDEAL.dotabb —

```
"IDEAL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDEAL"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"IDEAL" -> "ALIST"
```

————-

17.27 domain PR PolynomialRing

— PolynomialRing.input —

```
)set break resume
)sys rm -f PolynomialRing.output
)spool PolynomialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialRing
--R PolynomialRing(R: Ring,E: OrderedAbelianMonoid)  is a domain constructor
--R Abbreviation for PolynomialRing is PR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                    ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                    ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %      ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                    ?-? : (%,%) -> %
--R -? : % -> %                         ?=? : (%,%) -> Boolean
--R 1 : () -> %                         0 : () -> %
--R ?^? : (%,PositiveInteger) -> %      coefficient : (%,E) -> R
--R coefficients : % -> List R          coerce : R -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R degree : % -> E                     ground : % -> R
--R ground? : % -> Boolean              hash : % -> SingleInteger
--R latex : % -> String                 leadingCoefficient : % -> R
--R leadingMonomial : % -> %            map : ((R -> R),%) -> %
--R mapExponents : ((E -> E),%) -> %    minimumDegree : % -> E
--R monomial : (R,E) -> %               monomial? : % -> Boolean
--R one? : % -> Boolean                 pomopo! : (%,R,E,%) -> %
--R recip : % -> Union(%,"failed")      reductum : % -> %
--R retract : % -> R                    sample : () -> %
```

```
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : % -> % if R has INTDOM
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R fmecg : (%,E,R,%) -> % if E has CABMON and R has INTDOM
--R numberOfMonomials : % -> NonNegativeInteger
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

———-

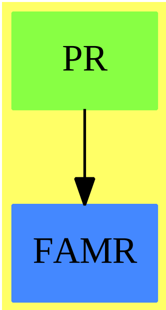
— PolynomialRing.help —

```
====================================================================
PolynomialRing examples
====================================================================

See Also:
o )show PolynomialRing
```

———-

17.27.1 PolynomialRing (PR)



See

⇒ “FreeModule” (FM) 7.30.1 on page 980
⇒ “SparseUnivariatePolynomial” (SUP) 20.18.1 on page 2425
⇒ “UnivariatePolynomial” (UP) 22.4.1 on page 2784

Exports:

0	1	associates?	binomThmExpt	characteristic
charthRoot	coerce	coefficient	coefficients	content
degree	exquo	exquo	fmecg	ground
ground?	hash	latex	leadingCoefficient	leadingMonomial
map	mapExponents	minimumDegree	monomial	monomial?
numberOfMonomials	one?	pomopo!	primitivePart	recip
reductum	retract	retractIfCan	sample	subtractIfCan
unit?	unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?/?		

— domain PR PolynomialRing —

```
)abbrev domain PR PolynomialRing
++ Author: Dave Barton, James Davenport, Barry Trager
++ Date Created:
++ Date Last Updated: 14.08.2000. Improved exponentiation [MMM/TTT]
++ Basic Functions: Ring, degree, coefficient, monomial, reductum
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents generalized polynomials with coefficients
++ (from a not necessarily commutative ring), and terms
++ indexed by their exponents (from an arbitrary ordered abelian monoid).
++ This type is used, for example,
++ by the \spadtype{DistributedMultivariatePolynomial} domain where
```

```
++ the exponent domain is a direct product of non negative integers.

PolynomialRing(R:Ring,E:OrderedAbelianMonoid): T == C
 where
  T == FiniteAbelianMonoidRing(R,E) with
    --assertions
       if R has IntegralDomain and E has CancellationAbelianMonoid then
            fmecg: (%,E,R,%) -> %
            ++ fmecg(p1,e,r,p2) finds x : p1 - r * x**e * p2
       if R has canonicalUnitNormal then canonicalUnitNormal
          ++ canonicalUnitNormal guarantees that the function
          ++ unitCanonical returns the same representative for all
          ++ associates of any particular element.

  C == FreeModule(R,E) add
    --representations
       Term:=  Record(k:E,c:R)
       Rep:=  List Term


    --declarations
       x,y,p,p1,p2: %
       n: Integer
       nn: NonNegativeInteger
       np: PositiveInteger
       e: E
       r: R
    --local operations
       1  == [[0$E,1$R]]
       characteristic  == characteristic$R
       numberOfMonomials x  == (# x)$Rep
       degree p == if null p then 0 else p.first.k
       minimumDegree p == if null p then 0 else (last p).k
       leadingCoefficient p == if null p then 0$R else p.first.c
       leadingMonomial p == if null p then 0 else [p.first]
       reductum p == if null p then p else p.rest
       retractIfCan(p:%):Union(R,"failed") ==
         null p => 0$R
         not null p.rest => "failed"
         zero?(p.first.k) => p.first.c
         "failed"
       coefficient(p,e)  ==
          for tm in p repeat
            tm.k=e => return tm.c
            tm.k < e => return 0$R
          0$R
       recip(p) ==
           null p => "failed"
           p.first.k > 0$E => "failed"
           (u:=recip(p.first.c)) case "failed" => "failed"
```

```
    (u::R)::%

coerce(r) == if zero? r then 0$% else [[0$E,r]]
coerce(n) == (n::R)::%

ground?(p): Boolean == empty? p or (empty? rest p and zero? degree p)

qsetrest!: (Rep, Rep) -> Rep
qsetrest!(l: Rep, e: Rep): Rep == RPLACD(l, e)$Lisp

times!: (R,    %) -> %
times:  (R, E, %) -> %

entireRing? := R has EntireRing

times!(r: R, x: %): % ==
  res, endcell, newend, xx: Rep
  if entireRing? then
         for tx in x repeat tx.c := r*tx.c
  else
         xx := x
         res := empty()
         while not empty? xx repeat
                 tx := first xx
                 tx.c := r * tx.c
                 if zero? tx.c then
                         xx := rest xx
                 else
                         newend := xx
                         xx := rest xx
                         if empty? res then
                                 res := newend
                                 endcell := res
                         else
                                 qsetrest!(endcell, newend)
                                 endcell := newend
         res;

 --- term * polynomial
termTimes: (R, E, Term) -> Term
termTimes(r: R, e: E, tx:Term): Term == [e+tx.k, r*tx.c]
times(tco: R, tex: E, rx: %): % ==
 if entireRing? then
         map(x1+->termTimes(tco, tex, x1), rx::Rep)
 else
         [[tex + tx.k, r] for tx in rx::Rep | not zero? (r := tco * tx.c)]



-- local addm!
```

```
addm!: (Rep, R, E, Rep) -> Rep
 -- p1 + coef*x^E * p2
 -- ‘spare’ (commented out) is for storage efficiency (not so good for
 -- performance though.
addm!(p1:Rep, coef:R, exp: E, p2:Rep): Rep ==
         --local res, newend, last: Rep
         res, newcell, endcell: Rep
         spare: List Rep
         res     := empty()
         endcell := empty()
         --spare   := empty()
         while not empty? p1 and not empty? p2 repeat
                 tx := first p1
                 ty := first p2
                 exy := exp + ty.k
                 newcell := empty();
                 if tx.k = exy then
                         newcoef := tx.c + coef * ty.c
                         if not zero? newcoef then
                                 tx.c    := newcoef
                                 newcell := p1
                         --else
                         --        spare   := cons(p1, spare)
                         p1 := rest p1
                         p2 := rest p2
                 else if tx.k > exy then
                         newcell := p1
                         p1      := rest p1
                 else
                         newcoef := coef * ty.c
                         if not entireRing? and zero? newcoef then
                                 newcell := empty()
                         --else if empty? spare then
                         --        ttt := [exy, newcoef]
                         --        newcell := cons(ttt, empty())
                         --else
                         --        newcell := first spare
                         --        spare   := rest spare
                         --        ttt := first newcell
                         --        ttt.k := exy
                         --        ttt.c := newcoef
                         else
                                 ttt := [exy, newcoef]
                                 newcell := cons(ttt, empty())
                         p2 := rest p2
                 if not empty? newcell then
                         if empty? res then
                                 res := newcell
                                 endcell := res
                         else
```

```
                                  qsetrest!(endcell, newcell)
                                  endcell := newcell
              if not empty? p1 then  -- then end is const * p1
                      newcell := p1
              else  -- then end is (coef, exp) * p2
                      newcell := times(coef, exp, p2)
              empty? res => newcell
              qsetrest!(endcell, newcell)
              res
       pomopo! (p1, r, e, p2) ==  addm!(p1, r, e, p2)
       p1 * p2 ==
              xx := p1::Rep
              empty? xx => p1
              yy := p2::Rep
              empty? yy => p2
              zero? first(xx).k => first(xx).c * p2
              zero? first(yy).k => p1 * first(yy).c
              --if #xx > #yy then
              --        (xx, yy) := (yy, xx)
              --        (p1, p2) := (p2, p1)
              xx := reverse xx
              res : Rep := empty()
              for tx in xx repeat res:=addm!(res,tx.c,tx.k,yy)
              res

--     if R has EntireRing then
--         p1 * p2  ==
--            null p1 => 0
--            null p2 => 0
--            zero?(p1.first.k) => p1.first.c * p2
--            one? p2 => p1
--            +/[[[t1.k+t2.k,t1.c*t2.c]$Term for t2 in p2]
--                   for t1 in reverse(p1)]
--                   -- This 'reverse' is an efficiency improvement:
--                   -- reduces both time and space [Abbott/Bradford/Davenport]
--        else
--         p1 * p2  ==
--            null p1 => 0
--            null p2 => 0
--            zero?(p1.first.k) => p1.first.c * p2
--            one? p2 => p1
--            +/[[[t1.k+t2.k,r]$Term for t2 in p2 | (r:=t1.c*t2.c) ^= 0]
--                 for t1 in reverse(p1)]
--                  -- This 'reverse' is an efficiency improvement:
--                  -- reduces both time and space [Abbott/Bradford/Davenport]
       if R has CommutativeRing  then
         p ** np == p ** (np pretend NonNegativeInteger)
         p ^ np  == p ** (np pretend NonNegativeInteger)
         p ^ nn  == p ** nn
```

```
       p ** nn  ==
          null p => 0
          zero? nn => 1
--          one? nn => p
          (nn = 1) => p
          empty? p.rest =>
            zero?(cc:=p.first.c ** nn) => 0
            [[nn * p.first.k, cc]]
          binomThmExpt([p.first], p.rest, nn)

     if R has Field then
       unitNormal(p) ==
          null p or (lcf:R:=p.first.c) = 1 => [1,p,1]
          a := inv lcf
          [lcf::%, [[p.first.k,1],:(a * p.rest)], a::%]
       unitCanonical(p) ==
          null p or (lcf:R:=p.first.c) = 1 => p
          a := inv lcf
          [[p.first.k,1],:(a * p.rest)]
     else if R has IntegralDomain then
       unitNormal(p) ==
          null p or p.first.c = 1 => [1,p,1]
          (u,cf,a):=unitNormal(p.first.c)
          [u::%, [[p.first.k,cf],:(a * p.rest)], a::%]
       unitCanonical(p) ==
          null p or p.first.c = 1 => p
          (u,cf,a):=unitNormal(p.first.c)
          [[p.first.k,cf],:(a * p.rest)]
     if R has IntegralDomain then
       associates?(p1,p2) ==
          null p1 => null p2
          null p2 => false
          p1.first.k = p2.first.k and
            associates?(p1.first.c,p2.first.c) and
             ((p2.first.c exquo p1.first.c)::R * p1.rest = p2.rest)
       p exquo r  ==
         [(if (a:= tm.c exquo r) case "failed"
             then return "failed" else [tm.k,a])
                for tm in p] :: Union(%,"failed")
       if E has CancellationAbelianMonoid then
         fmecg(p1:%,e:E,r:R,p2:%):% ==       -- p1 - r * X**e * p2
            rout:%:= []
            r:= - r
            for tm in p2 repeat
               e2:= e + tm.k
               c2:= r * tm.c
               c2 = 0 => "next term"
               while not null p1 and p1.first.k > e2 repeat
                 (rout:=[p1.first,:rout]; p1:=p1.rest)  --use PUSH and POP?
```

```
              null p1 or p1.first.k < e2 => rout:=[[e2,c2],:rout]
              if (u:=p1.first.c + c2) ^= 0 then rout:=[[e2, u],:rout]
              p1:=p1.rest
            NRECONC(rout,p1)$Lisp
          if R has approximate then
            p1 exquo p2  ==
              null p2 => error "Division by 0"
              p2 = 1 => p1
              p1=p2 => 1
            --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
              rout:= []@List(Term)
              while not null p1 repeat
                 (a:= p1.first.c exquo p2.first.c)
                 a case "failed" => return "failed"
                 ee:= subtractIfCan(p1.first.k, p2.first.k)
                 ee case "failed" => return "failed"
                 p1:= fmecg(p1.rest, ee, a, p2.rest)
                 rout:= [[ee,a], :rout]
              null p1 => reverse(rout)::%    -- nreverse?
              "failed"
          else -- R not approximate
            p1 exquo p2  ==
              null p2 => error "Division by 0"
              p2 = 1 => p1
            --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
              rout:= []@List(Term)
              while not null p1 repeat
                 (a:= p1.first.c exquo p2.first.c)
                 a case "failed" => return "failed"
                 ee:= subtractIfCan(p1.first.k, p2.first.k)
                 ee case "failed" => return "failed"
                 p1:= fmecg(p1.rest, ee, a, p2.rest)
                 rout:= [[ee,a], :rout]
              null p1 => reverse(rout)::%    -- nreverse?
              "failed"
      if R has Field then
         x/r == inv(r)*x
```

— **PR.dotabb** —

```
"PR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PR"]
"FAMR" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAMR"]
"PR" -> "FAMR"
```

17.28 domain PI PositiveInteger

— PositiveInteger.input —

```
)set break resume
)sys rm -f PositiveInteger.output
)spool PositiveInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PositiveInteger
--R PositiveInteger  is a domain constructor
--R Abbreviation for PositiveInteger is PI
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PI
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (%,%) -> %                      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> OutputForm
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R ?~=? : (%,%) -> Boolean
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

———–

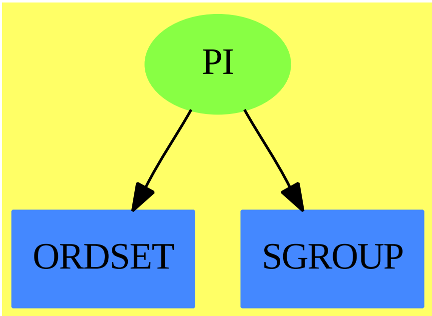
— PositiveInteger.help —

```
====================================================================
PositiveInteger examples
====================================================================

See Also:
o )show PositiveInteger
```

———-

17.28.1 PositiveInteger (PI)



See

⇒ "Integer" (INT) 10.30.1 on page 1325
⇒ "NonNegativeInteger" (NNI) 15.5.1 on page 1702
⇒ "RomanNumeral" (ROMAN) 19.12.1 on page 2286

Exports:

1	coerce	gcd	hash	latex
max	min	one?	recip	sample
?^?	?~=?	?**?	?*?	?+?
?<?	?<=?	?=?	?>?	?>=?

— **domain PI PositiveInteger** —

```
)abbrev domain PI PositiveInteger
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations:
++ Related Constructors:
++ Keywords: positive integer
++ Description:
++ \spadtype{PositiveInteger} provides functions for
++ positive integers.

PositiveInteger: Join(AbelianSemiGroup,OrderedSet,Monoid) with
    gcd: (%,%) -> %
      ++ gcd(a,b) computes the greatest common divisor of two
      ++ positive integers \spad{a} and b.
    commutative("*")
      ++ commutative("*") means multiplication is commutative : x*y = y*x
```

```
 == SubDomain(NonNegativeInteger,#1 > 0) add
     x:%
     y:%
```

———-

— PI.dotabb —

```
"PI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PI",shape=ellipse]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"SGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SGROUP"]
"PI" -> "ORDSET"
"PI" -> "SGROUP"
```

———-

17.29 domain PF PrimeField

— PrimeField.input —

```
)set break resume
)sys rm -f PrimeField.output
)spool PrimeField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrimeField
--R PrimeField p: PositiveInteger  is a domain constructor
--R Abbreviation for PrimeField is PF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R D : % -> %                            D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,PositiveInteger) -> %
```

```
--R algebraic? : % -> Boolean            associates? : (%,%) -> Boolean
--R basis : () -> Vector %               charthRoot : % -> %
--R coerce : Fraction Integer -> %       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R convert : % -> Integer               coordinates : % -> Vector %
--R createPrimitiveElement : () -> %     degree : % -> PositiveInteger
--R differentiate : % -> %               dimension : () -> CardinalNumber
--R factor : % -> Factored %             gcd : List % -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R inGroundField? : % -> Boolean        index : PositiveInteger -> %
--R init : () -> %                       inv : % -> %
--R latex : % -> String                  lcm : List % -> %
--R lcm : (%,%) -> %                     lookup : % -> PositiveInteger
--R norm : % -> %                        one? : % -> Boolean
--R order : % -> PositiveInteger         prime? : % -> Boolean
--R primeFrobenius : % -> %              primitive? : % -> Boolean
--R primitiveElement : () -> %           ?quo? : (%,%) -> %
--R random : () -> %                     recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                   represents : Vector % -> %
--R retract : % -> %                     sample : () -> %
--R size : () -> NonNegativeInteger      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R trace : % -> %                       transcendent? : % -> Boolean
--R unit? : % -> Boolean                 unitCanonical : % -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if $ has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if $ has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix % -> Union(Vector %,"failed")
--R coordinates : Vector % -> Matrix %
--R createNormalElement : () -> % if $ has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R extensionDegree : () -> PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer)
```

```
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if $ has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial %) -> % if $ has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial % if $ has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial %,"failed") if $ has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial % if $ has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial %
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if $ has FINITE
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R norm : (%,PositiveInteger) -> % if $ has FINITE
--R normal? : % -> Boolean if $ has FINITE
--R normalElement : () -> % if $ has FINITE
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R retractIfCan : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R trace : (%,PositiveInteger) -> % if $ has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

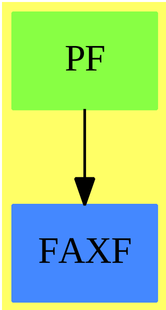
)spool
)lisp (bye)
```

— PrimeField.help —

```
====================================================================
PrimeField examples
====================================================================

See Also:
o )show PrimeField
```

17.29.1 PrimeField (PF)



See

⇒ "InnerPrimeField" (IPF) 10.25.1 on page 1267

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	conditionP	coordinates
coerce	convert	coordinates
createPrimitiveElement	createNormalElement	D
definingPolynomial	degree	differentiate
dimension	discreteLog	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	inGroundField?	index
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	?quo?
?rem?		

— domain PF PrimeField —

```
)abbrev domain PF PrimeField
++ Authors: N.N.,
++ Date Created: November 1990, 26.03.1991
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: prime characteristic, prime field, finite field
++ References:
++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ Description:
++ PrimeField(p) implements the field with p elements if p is a prime number.
```

```
++ Error: if p is not prime.
++ Note: this domain does not check that argument is a prime.

PrimeField(p:PositiveInteger): Exp == Impl where
  Exp ==> Join(FiniteFieldCategory,FiniteAlgebraicExtensionField($),_
    ConvertibleTo(Integer))
  Impl ==>  InnerPrimeField(p) add
    if not prime?(p)$IntegerPrimesPackage(Integer) then
      error "Argument to prime field must be a prime"
```

———-

— PF.dotabb —

```
"PF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PF"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"PF" -> "FAXF"
```

———-

17.30 domain PRIMARR PrimitiveArray

— PrimitiveArray.input —

```
)set break resume
)sys rm -f PrimitiveArray.output
)spool PrimitiveArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrimitiveArray
--R PrimitiveArray S: Type  is a domain constructor
--R Abbreviation for PrimitiveArray is PRIMARR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PRIMARR
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                 concat : (%,%) -> %
--R concat : (S,%) -> %                  concat : (%,S) -> %
--R construct : List S -> %              copy : % -> %
--R delete : (%,Integer) -> %            ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S             empty : () -> %
```

```
--R empty? : % -> Boolean                 entries : % -> List S
--R eq? : (%,%) -> Boolean                index? : (Integer,%) -> Boolean
--R indices : % -> List Integer           insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %           map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %               new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S               reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
```

```
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

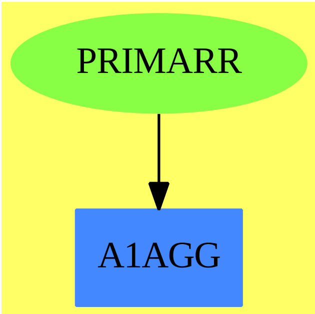
— **PrimitiveArray.help** —

```
====================================================================
PrimitiveArray examples
====================================================================

See Also:
o )show PrimitiveArray
```

———-

17.30.1 PrimitiveArray (PRIMARR)



See

⇒ "Tuple" (TUPLE) 21.12.1 on page 2711
⇒ "IndexedFlexibleArray" (IFARRAY) 10.10.1 on page 1187
⇒ "FlexibleArray" (FARRAY) 7.14.1 on page 853
⇒ "IndexedOneDimensionalArray" (IARRAY1) 10.13.1 on page 1208
⇒ "OneDimensionalArray" (ARRAY1) 16.3.1 on page 1736

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	entry?
elt	empty	empty?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	insert
latex	less?	map	map!	max
maxIndex	member?	members	merge	min
minIndex	more?	new	parts	position
qelt	qsetelt!	reduce	remove	removeDuplicates
reverse	reverse!	sample	select	setelt
size?	sort	sort!	sorted?	swap!
#?	?.?	?~=?	?<?	?<=?
?=?	?>?	?>=?		

— domain PRIMARR PrimitiveArray —

```
)abbrev domain PRIMARR PrimitiveArray
++ Author: Mark Botch
++ Description:
++ This provides a fast array type with no bound checking on elt's.
++ Minimum index is 0 in this type, cannot be changed

PrimitiveArray(S:Type): OneDimensionalArrayAggregate S == add
   Qmax ==> QVMAXINDEX$Lisp
   Qsize ==> QVSIZE$Lisp
--   Qelt ==> QVELT$Lisp
--   Qsetelt ==> QSETVELT$Lisp
```

```
Qelt ==> ELT$Lisp
Qsetelt ==> SETELT$Lisp
Qnew ==> MAKE_-ARRAY$Lisp

#x                          == Qsize x
minIndex x                  == 0
empty()                     == Qnew(0$Lisp)
new(n, x)                   == fill_!(Qnew n, x)
qelt(x, i)                  == Qelt(x, i)
elt(x:%, i:Integer)         == Qelt(x, i)
qsetelt_!(x, i, s)          == Qsetelt(x, i, s)
setelt(x:%, i:Integer, s:S) == Qsetelt(x, i, s)
fill_!(x, s)       == (for i in 0..Qmax x repeat Qsetelt(x, i, s); x)
```

— PRIMARR.dotabb —

```
"PRIMARR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRIMARR",
          shape=ellipse]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"PRIMARR" -> "A1AGG"
```

17.31 domain PRODUCT Product

— Product.input —

```
)set break resume
)sys rm -f Product.output
)spool Product.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
f:=(x:INT):INT +-> 3*x
--R
--R
--R   (1)  theMap(Closure)
--R                                              Type: (Integer -> Integer)
--E 1
```

```
--S 2 of 6
f(3)
--R
--R
--R   (2)  9
--R                                                    Type: PositiveInteger
--E 2

--S 3 of 6
g:=(x:INT):INT +-> x^3
--R
--R
--R   (3)  theMap(Closure)
--R                                                 Type: (Integer -> Integer)
--E 3

--S 4 of 6
g(3)
--R
--R
--R   (4)  27
--R                                                    Type: PositiveInteger
--E 4

--S 5 of 6
h(x:INT):Product(INT,INT) == makeprod(f x, g x)
--R
--R   Function declaration h : Integer -> Product(Integer,Integer) has
--R      been added to workspace.
--R                                                               Type: Void
--E 5

--S 6 of 6
h(3)
--R
--R   Compiling function h with type Integer -> Product(Integer,Integer)
--R
--R   (6)  (9,27)
--R                                              Type: Product(Integer,Integer)
--E 6

)spool
)lisp (bye)
```

— **Product.help** —

```
====================================================================
```

```
Product examples
====================================================================

The direct product of two functions over the same domain is
another function over that domain whose co-domain is the product
of their co-domains.

So we can define two functions, f and g, that go from INT -> INT

   f:=(x:INT):INT +-> 3*x
   g:=(x:INT):INT +-> x^3

so
    f(3) is 9
    g(3) is 27

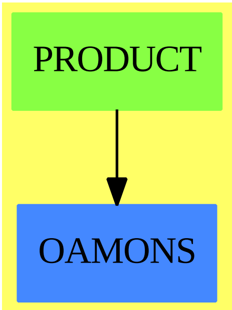
and we can construct the direct product of those functions h=f,g

   h(x:INT):Product(INT,INT) == makeprod(f x, g x)

so
   h(3) is (9,27)

See Also:
o )show Product
```

17.31.1 Product (PRODUCT)



Exports:

0	1	coerce	commutator	conjugate
hash	index	inv	latex	lookup
makeprod	max	min	one?	random
recip	sample	selectfirst	selectsecond	size
subtractIfCan	sup	zero?	?=?	?~=?
?*?	?**?	?+?	-?	?-?
?/?	?<?	?<=?	?>?	?>=?
?^?				

— domain PRODUCT Product —

```
)abbrev domain PRODUCT Product
++ Author: Mark Botch
++ Description:
++ This domain implements cartesian product

Product (A:SetCategory,B:SetCategory) : C == T
 where
  C == SetCategory  with
       if A has Finite and B has Finite then Finite
       if A has Monoid and B has Monoid then Monoid
       if A has AbelianMonoid and B has AbelianMonoid then AbelianMonoid
       if A has CancellationAbelianMonoid and
          B has CancellationAbelianMonoid then CancellationAbelianMonoid
       if A has Group  and B has Group  then  Group
       if A has AbelianGroup and B has AbelianGroup then  AbelianGroup
       if A has OrderedAbelianMonoidSup and B has OrderedAbelianMonoidSup
                                             then OrderedAbelianMonoidSup
       if A has OrderedSet and B has OrderedSet then  OrderedSet

       makeprod     : (A,B) -> %
        ++ makeprod(a,b) computes the product of two functions
        ++
        ++X f:=(x:INT):INT +-> 3*x
        ++X g:=(x:INT):INT +-> x^3
        ++X h(x:INT):Product(INT,INT) == makeprod(f x, g x)
        ++X h(3)
       selectfirst  :   %   -> A
        ++ selectfirst(x) is not documented
       selectsecond :   %   -> B
        ++ selectsecond(x) is not documented

  T == add

    --representations
       Rep := Record(acomp:A,bcomp:B)

    --declarations
       x,y: %
```

```
   i: NonNegativeInteger
   p: NonNegativeInteger
   a: A
   b: B
   d: Integer

--define
   coerce(x):OutputForm == paren [(x.acomp)::OutputForm,
                                  (x.bcomp)::OutputForm]
   x=y ==
       x.acomp = y.acomp => x.bcomp = y.bcomp
       false
   makeprod(a:A,b:B) :%   == [a,b]

   selectfirst(x:%) : A   == x.acomp

   selectsecond (x:%) : B == x.bcomp

   if A has Monoid and B has Monoid then
      1 == [1$A,1$B]
      x * y == [x.acomp * y.acomp,x.bcomp * y.bcomp]
      x ** p == [x.acomp ** p ,x.bcomp ** p]

   if A has Finite and B has Finite then
      size == size$A () * size$B ()

   if A has Group and B has Group then
      inv(x) == [inv(x.acomp),inv(x.bcomp)]

   if A has AbelianMonoid and B has AbelianMonoid then
      0 == [0$A,0$B]

      x + y == [x.acomp + y.acomp,x.bcomp + y.bcomp]

      c:NonNegativeInteger * x == [c * x.acomp,c*x.bcomp]

   if A has CancellationAbelianMonoid and
      B has CancellationAbelianMonoid then
        subtractIfCan(x, y) : Union(%,"failed") ==
          (na:= subtractIfCan(x.acomp, y.acomp)) case "failed" => "failed"
          (nb:= subtractIfCan(x.bcomp, y.bcomp)) case "failed" => "failed"
          [na::A,nb::B]

   if A has AbelianGroup and B has AbelianGroup then
      - x == [- x.acomp,-x.bcomp]
      (x - y):% == [x.acomp - y.acomp,x.bcomp - y.bcomp]
      d * x == [d * x.acomp,d * x.bcomp]

   if A has OrderedAbelianMonoidSup and B has OrderedAbelianMonoidSup then
      sup(x,y) == [sup(x.acomp,y.acomp),sup(x.bcomp,y.bcomp)]
```

```
    if A has OrderedSet and B has OrderedSet then
       x < y ==
            xa:= x.acomp ; ya:= y.acomp
            xa < ya => true
            xb:= x.bcomp ; yb:= y.bcomp
            xa = ya => (xb < yb)
            false

--     coerce(x:%):Symbol ==
--      PrintableForm()
--      formList([x.acomp::Expression,x.bcomp::Expression])$PrintableForm
```

— PRODUCT.dotabb —

```
"PRODUCT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRODUCT"]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
"PRODUCT" -> "OAMONS"
```

17.32 domain PROJPL ProjectivePlane

— ProjectivePlane.input —

```
)set break resume
)sys rm -f ProjectivePlane.output
)spool ProjectivePlane.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ProjectivePlane
--R ProjectivePlane K: Field  is a domain constructor
--R Abbreviation for ProjectivePlane is PROJPL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PROJPL
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : List K -> %
--R coerce : % -> List K                  coerce : % -> OutputForm
```

```
--R conjugate : % -> %                  definingField : % -> K
--R degree : % -> PositiveInteger        ?.? : (%,Integer) -> K
--R hash : % -> SingleInteger            homogenize : % -> %
--R homogenize : (%,Integer) -> %        lastNonNul : % -> Integer
--R lastNonNull : % -> Integer           latex : % -> String
--R list : % -> List K                   orbit : % -> List %
--R pointValue : % -> List K             projectivePoint : List K -> %
--R rational? : % -> Boolean             setelt : (%,Integer,K) -> K
--R ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

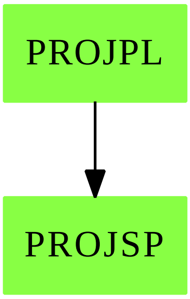
)spool
)lisp (bye)
```

— **ProjectivePlane.help** —

```
====================================================================
ProjectivePlane examples
====================================================================

See Also:
o )show ProjectivePlane
```

17.32.1 ProjectivePlane (PROJPL)



Exports:

?.?	?=?	?~=?	coerce	conjugate
definingField	degree	hash	homogenize	lastNonNul
lastNonNull	latex	list	orbit	pointValue
projectivePoint	rational?	removeConjugate	setelt	

— domain PROJPL ProjectivePlane —

```
)abbrev domain PROJPL ProjectivePlane
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
ProjectivePlane(K):Exports == Implementation where
  K:Field

  Exports ==> ProjectiveSpaceCategory(K)

  Implementation ==> ProjectiveSpace(3,K)
```

———-

— PROJPL.dotabb —

```
"PROJPL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJPL"];
"PROJSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJSP"];
"PROJPL" -> "PROJSP"
```

———-

17.33 domain PROJPLPS ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField

— ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)spool ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField
--R ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory  is a domai
--R Abbreviation for ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField is PROJPLPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PROJPLPS
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R conjugate : % -> %                    degree : % -> PositiveInteger
--R hash : % -> SingleInteger             homogenize : % -> %
--R homogenize : (%,Integer) -> %         lastNonNul : % -> Integer
--R lastNonNull : % -> Integer            latex : % -> String
--R orbit : % -> List %                   rational? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R coerce : List PseudoAlgebraicClosureOfFiniteField K -> %
--R coerce : % -> List PseudoAlgebraicClosureOfFiniteField K
--R conjugate : (%,NonNegativeInteger) -> %
--R definingField : % -> PseudoAlgebraicClosureOfFiniteField K
--R ?.? : (%,Integer) -> PseudoAlgebraicClosureOfFiniteField K
--R list : % -> List PseudoAlgebraicClosureOfFiniteField K
--R orbit : (%,NonNegativeInteger) -> List %
--R pointValue : % -> List PseudoAlgebraicClosureOfFiniteField K
--R projectivePoint : List PseudoAlgebraicClosureOfFiniteField K -> %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R setelt : (%,Integer,PseudoAlgebraicClosureOfFiniteField K) -> PseudoAlgebraicClosureOfFi
--R
--E 1

)spool
)lisp (bye)
```

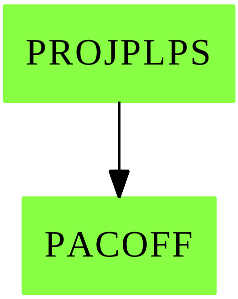
— **ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.help** —

```
====================================================================
ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField examples
====================================================================

See Also:
o )show ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField
```

———-

17.33.1 ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField (PROJPLPS)



Exports:

?.?	?=?	?~=?	coerce	conjugate
definingField	degree	hash	homogenize	lastNonNul
lastNonNull	latex	list	orbit	orbit
pointValue	projectivePoint	rational?	removeConjugate	setelt

— domain PROJPLPS ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField —

```
)abbrev domain PROJPLPS ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField(K):Exp == Impl where
  K:FiniteFieldCategory

  KK   ==> PseudoAlgebraicClosureOfFiniteField(K)

  Exp  ==> ProjectiveSpaceCategory(KK)

  Impl ==> ProjectivePlane(KK)
```

———-

— PROJPLPS.dotabb —

```
"PROJPLPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJPLPS"];
"PACOFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACOFF"];
"PROJPLPS" -> "PACOFF"
```

——–

17.34 domain PROJSP ProjectiveSpace

— ProjectiveSpace.input —

```
)set break resume
)sys rm -f ProjectiveSpace.output
)spool ProjectiveSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ProjectiveSpace
--R ProjectiveSpace(dim: NonNegativeInteger,K: Field)  is a domain constructor
--R Abbreviation for ProjectiveSpace is PROJSP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PROJSP
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : List K -> %
--R coerce : % -> List K                  coerce : % -> OutputForm
--R conjugate : % -> %                    definingField : % -> K
--R degree : % -> PositiveInteger         ?.? : (%,Integer) -> K
--R hash : % -> SingleInteger             homogenize : % -> %
--R homogenize : (%,Integer) -> %         lastNonNul : % -> Integer
--R lastNonNull : % -> Integer            latex : % -> String
--R list : % -> List K                    orbit : % -> List %
--R pointValue : % -> List K              projectivePoint : List K -> %
--R rational? : % -> Boolean              setelt : (%,Integer,K) -> K
--R ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)
```

———-

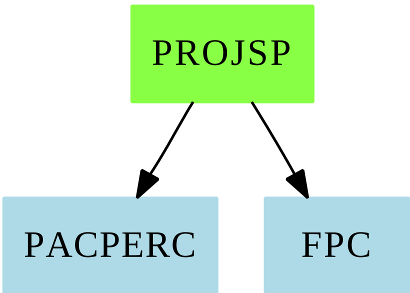
— **ProjectiveSpace.help** —

```
====================================================================
ProjectiveSpace examples
====================================================================

See Also:
o )show ProjectiveSpace
```

———-

17.34.1 ProjectiveSpace (PROJSP)



Exports:

?.?	?=?	?~=?	coerce	conjugate
definingField	degree	hash	homogenize	lastNonNul
lastNonNull	latex	list	orbit	orbit
pointValue	projectivePoint	rational?	removeConjugate	setelt

— **domain PROJSP ProjectiveSpace** —

```
)abbrev domain PROJSP ProjectiveSpace
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
ProjectiveSpace(dim,K):Exports == Implementation where

  NNI  ==> NonNegativeInteger
  LIST ==> List
```

```
dim:NNI
K:Field

Exports ==> ProjectiveSpaceCategory(K)

Implementation ==> List(K) add

  Rep:= List(K)

  coerce(pt:%):OutputForm ==
    dd:OutputForm:= ":" :: OutputForm
    llout:List(OutputForm):=[ hconcat(dd, a::OutputForm) for a in rest pt]
    lout:= cons( (first pt)::OutputForm , llout)
    out:= hconcat lout
    oo:=paren(out)
    ee:OutputForm:= degree(pt) :: OutputForm
    oo**ee

  definingField(pt)==
    K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
      maxTower(pt pretend Rep)
    1$K

  degree(pt)==
    K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
      extDegree definingField pt
    1

  coerce(pt:%):List(K) == pt pretend Rep

  projectivePoint(pt:LIST(K))==
      pt :: %

  list(ptt)==
    ptt pretend Rep

  pointValue(ptt)==
    ptt pretend Rep

  conjugate(p,e)==
    lp:Rep:=p
    pc:List(K):=[c**e for c in lp]
    projectivePoint(pc)

  homogenize(ptt,nV)==
    if K has Field then
      pt:=list(ptt)$%
      zero?(pt.nV) => error "Impossible to homogenize this point"
      divPt:=pt.nV
```

```
      ([(a/divPt) for a in pt])
    else
      ptt

  rational?(p,n)== p=conjugate(p,n)

  rational?(p)==rational?(p,characteristic()$K)

  removeConjugate(l)==removeConjugate(l,characteristic()$K)

  removeConjugate(l:LIST(%),n:NNI):LIST(%)==
    if K has FiniteFieldCategory then
      allconj:LIST(%):=empty()
      conjrem:LIST(%):=empty()
      for p in l repeat
        if ^member?(p,allconj) then
          conjrem:=cons(p,conjrem)
          allconj:=concat(allconj,orbit(p,n))
      conjrem
    else
      error "The field is not finite"

  conjugate(p)==conjugate(p,characteristic()$K)

  orbit(p)==orbit(p,characteristic()$K)

  orbit(p,e)==
    if K has FiniteFieldCategory then
      l:LIST(%):=[p]
      np:%:=conjugate(p,e)
      flag:=^(np=p)::Boolean
      while flag repeat
        l:=concat(np,l)
        np:=conjugate(np,e)
        flag:=not (np=p)::Boolean
      l
    else
      error "Cannot compute the conjugate"

  aa:% = bb:% ==
    ah:=homogenize(aa)
    bh:=homogenize(bb)
    ah =$Rep bh

  coerce(pt:LIST(K))==
      ^(dim=#pt) => error "Le point n'a pas la bonne dimension"
      reduce("and",[zero?(a) for a in pt]) => _
        error "Ce n'est pas un point projectif"
      ptt:%:= pt
      homogenize ptt
```

```
homogenize(ptt)==
  homogenize(ptt,lastNonNull(ptt))

nonZero?: K -> Boolean
nonZero?(a)==
  not(zero?(a))

lastNonNull(ptt)==
  pt:=ptt pretend Rep
  (dim pretend Integer)+1-_
    (position("nonZero?",(reverse(pt)$LIST(K)))$LIST(K))

lastNonNul(pt)==lastNonNull(pt)
```

— **PROJSP.dotabb** —

```
"PROJSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJSP"];
"PACPERC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"FPC" [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"PROJSP" -> "PACPERC"
"PROJSP" -> "FPC"
```

17.35 domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber

— **PseudoAlgebraicClosureOfAlgExtOfRationalNumber.input** —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfAlgExtOfRationalNumber.output
)spool PseudoAlgebraicClosureOfAlgExtOfRationalNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfAlgExtOfRationalNumber
--E 1
```

```
)spool
)lisp (bye)
```

———-

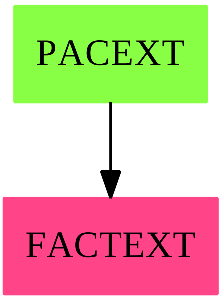
— **PseudoAlgebraicClosureOfAlgExtOfRationalNumber.help** —

```
====================================================================
PseudoAlgebraicClosureOfAlgExtOfRationalNumber examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfAlgExtOfRationalNumber
```

———-

17.35.1 PseudoAlgebraicClosureOfAlgExtOfRationalNumber (PACEXT)



Exports:

— **domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber** —

```
)abbrev domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber
-- PseudoAlgebraicClosureOfAlgExtOfRationalNumber
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain implement dynamic extension over the
++ PseudoAlgebraicClosureOfRationalNumber.
```

```
++ A tower extension T  of the ground field K is any sequence of field
++ extension (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1
++ and defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1) and
++ (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfAlgExtOfRationalNumber(downLevel:K):Exp == Impl where
  K       ==> PseudoAlgebraicClosureOfRationalNumber
  INT     ==> Integer
  NNI     ==> NonNegativeInteger
  SUP     ==> SparseUnivariatePolynomial
  BOOLEAN ==> Boolean
  PI      ==> PositiveInteger
  FACTRN  ==> FactorisationOverPseudoAlgebraicClosureOfAlgExtOfRationalNumber

  recRep ==> Record(recEl:SUP(%),_
                    recTower:SUP(%),_
                    recDeg:PI,_
                    recPrevTower:%,_
                    recName:Symbol)

  Exp == PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory with

    fullOutput: % -> OutputForm

    retractToGrn: % -> K

  Impl == add
    Rep := Union(recRep,K)

    -- signature of local function
    replaceRecEl: (%,SUP(%)) -> %

    down: % -> %

    retractPol( pol:SUP(%) ):SUP(K)==
      zero? pol => 0$SUP(K)
      lc := leadingCoefficient pol
      d := degree pol
      rlc := retractToGrn( lc )
      monomial( rlc , d )$SUP(K) + retractPol( reductum pol )

    retractToGrn(aa)==
      aa case K => aa
      a:=(aa pretend recRep)
```

```
  el:=  a.recEl
  t:=   a.recTower
  d:=   a.recDeg * extDegree downLevel
  pt:=  a.recPrevTower
  n:= a.recName
  newElement(retractPol el, retractPol t, d, retractToGrn pt, n)$K

newElement(pol,subF,inName)  ==
    -- pol is an irreducible polynomial over the field extension
    -- given by subF.
    -- The output of this function is a root of pol.
  dp:=degree pol
  listCoef: List % := coefficients pol
  a1:% := inv first listCoef
  b1:% := last listCoef
  rr:% := b1*a1
  one?(dp) =>
    one?(#listCoef) => 0
    - rr
  ground?(pol) => error "Cannot create a new element with a constant"
  d:PI := (dp pretend PI) * extDegree(subF)
  [monomial(1$%,1)$SUP(%),pol,d,subF,inName] :: Rep

coerce(a:Integer):%== (a :: K)

down(a:%) ==
  a case K => a
  aa:=(a pretend recRep)
  elel := aa.recEl
  ^ground?(elel)$SUP(%) => a
  gel:%:=ground(elel)
  down(gel)

n:INT * a:% ==
  one?(n) => a
  zero?(a) or zero?(n) => 0
  (n < 0) => - ((-n)*a)
  mm:PositiveInteger:=(n pretend PositiveInteger)
  double(mm,a)$RepeatedDoubling(%)

replaceRecEl(a,el)==
  a case K => a
  aa:=copy a
  aa.recEl := el
  aa

localTower :% := downLevel

lift(a) ==
  a case K => monomial(a,0)
```

```
  (a pretend recRep).recEl

lift(a,b)==
  extDegree a > extDegree b => _
    error "Cannot lift something at lower level !!!!!"
  extDegree a < extDegree b => monomial(a,0)$SUP(%)
  lift a

reduce(a)==
  localTower case K =>
    coefficient(a,0)
  ar:= a rem (localTower pretend recRep).recTower
  replaceRecEl(localTower,ar)

maxTower(la)==
    --return an element from the list la which is in the largest
    --extension of the ground field
    --PRECONDITION: all elements in same tower, else no meaning?
  m:="max"/[extDegree(a)$% for a in la]
  first [b for b in la | extDegree(b)=m]

ground?(a)== a case K

vectorise(a,lev)==
  da:=extDegree a
  dlev:=extDegree lev
  dlev < da => _
    error "Cannot vectorise at a lower level than the element to vectorise"
  lev case K => [a]
  pa:SUP(%)
  na:%
  ^(da = dlev) =>
    pa:=  monomial(a,0)$SUP(%)
    na:=  replaceRecEl(lev,pa)
    vectorise(na,lev)$%

  prevLev:=previousTower(lev)
  a case K => error "At this point a is not suppose to be in K"
  aEl:=(a pretend recRep).recEl
  daEl:=degree definingPolynomial(a)$%
  lv:=[vectorise(c,prevLev)$% for c in entries(vectorise(aEl,daEl)$SUP(%))]
  concat lv

retractIfCan(a:%):Union(K,"failed")==
  a case K => a
  "failed"

retractIfCan(a:%):Union(Integer,"failed")==
  a case K => retractIfCan(a)$K
  "failed"
```

```
setTower!(a) ==
  if a case K then
    localTower := downLevel
  else
    localTower:=a
  void()

definingPolynomial == definingPolynomial(localTower)

a:% + b:% ==
  (a case K) and (b case K) => a +$K b
  extDegree(a) > extDegree(b) => b + a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:=   b.recEl +$SUP(%) a.recEl
    res2:=   replaceRecEl(b,res1)
  else
    res1:=   b.recEl +$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

a:% * b:% ==
  (a case K) and (b case K) => a *$K b
  extDegree(a) > extDegree(b) => b * a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:=   b.recEl *$SUP(%) a.recEl rem b.recTower
    res2:=   replaceRecEl(b,res1)
  else
    res1:=   b.recEl *$SUP(%) monomial(a,0)$SUP(%)
    res2:=  replaceRecEl(b,res1)
  down(res2)

distinguishedRootsOf(polyZero,ee) ==
    setTower!(ee)
    zero?(polyZero) => error "to lazy to give you all the roots of 0 !!!"
    factorf: Factored SUP % :=  factor(polyZero,localTower)$FACTRN(%)
    listFact:List SUP %  := [pol.fctr for pol in factorList(factorf)]
    listOfZeros:List(%):=empty()
    for p in listFact repeat
      root:=newElement(p, new(E::Symbol)$Symbol)
      listOfZeros:List(%):=concat([ root ], listOfZeros)
    listOfZeros

1 == 1$K

0 == 0$K
```

```
newElement(poll:SUP(%),inName:Symbol)==
  newElement(poll,localTower,inName)$%

--Field operations
inv(a)==
  a case K => inv(a)$K
  aRecEl:= (a pretend recRep).recEl
  aDefPoly:= (a pretend recRep).recTower
  aInv := extendedEuclidean( aRecEl , aDefPoly, 1 )
  aInv  case "failed" => error "PACOFF : division by zero"
  -- On doit retourner un Record reprsentant l'inverse de a.
  -- Ce Record est exactement le mme que celui de a sauf
  -- qu'il faut remplacer le polynme du selecteur recEl
  -- par le polynme reprsentant l'inverse de a :
  -- C'est ce que fait la fonction replaceRecEl.
  replaceRecEl( a , aInv.coef1 )

a:% / b:% == a * inv(b)

a:K * b:%==
  (a :: %) * b

b:% * a:K == a*b

a:% - b:% ==
  a + (-b)

a:% * b:Fraction(Integer) ==
  bn:=numer b
  bd:=denom b
  ebn:%:= bn * 1$%
  ebd:%:= bd * 1$%
  a * ebn * inv(ebd)

-a:% ==
   a case K => -$K a
   [-$SUP(%) (a pretend recRep).recEl,_
    (a pretend recRep).recTower,_
    (a pretend recRep).recDeg,_
    (a pretend recRep).recPrevTower,_
    (a pretend recRep).recName ]

bb:% = aa:% ==
  b:=down bb
  a:=down aa
  ^( extDegree(b) =$NNI extDegree(a) ) => false
  (b case K)  =>  ( (retract a)@K  =$K (retract b)@K )
  rda := a :: recRep
  rdb := b :: recRep
```

```
      not (rda.recTower =$SUP(%) rdb.recTower) => false
      rdb.recEl =$SUP(%) rda.recEl

    zero?(a:%) ==
      da:=down a  -- just to be sure !!!
      ^(da case K) => false
      zero?(da)$K

    one?(a:%) ==
      da:= down a  -- just to be sure !!!
      ^(da case K) => false
      one?(da)$K

    coerce(a:K):% == a

    coerce(a:%):OutputForm ==
      a case K => ((retract a)@K) ::OutputForm
      outputForm((a pretend recRep).recEl,_
                 ((a pretend recRep).recName)::OutputForm) $SUP(%)

    fullOutput(a:%):OutputForm==
      a case K => ((retract a)@K) ::OutputForm
      (a pretend recRep)::OutputForm

    definingPolynomial(a:%): SUP % ==
      a case K => monomial(1,1)$SUP(%)
      (a pretend recRep).recTower

    extDegree(a:%): PI ==
      a case K => 1
      (a pretend recRep).recDeg

    previousTower(a:%):% ==
      a case K => error "No previous extension for ground field element"
      (a pretend recRep).recPrevTower

    name(a:%):Symbol ==
      a case K => error "No name for ground field element"
      (a pretend recRep).recName

    characteristic == characteristic()$K
```

— PACEXT.dotabb —

```
"PACEXT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACEXT"];
"FACTEXT" [color="#FF4488",href="bookvol10.4.pdf#nameddest=FACTEXT"]
```

```
"PACEXT" -> "FACTEXT"
```

———-

17.36 domain PACOFF PseudoAlgebraicClosureOfFiniteField

— PseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfFiniteField.output
)spool PseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfFiniteField
--R PseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory  is a domain constructor
--R Abbreviation for PseudoAlgebraicClosureOfFiniteField is PACOFF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PACOFF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,K) -> %                      ?*? : (K,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,K) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : % -> %
--R D : (%,NonNegativeInteger) -> %       1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        charthRoot : % -> %
--R coerce : K -> %                       coerce : Fraction Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              conjugate : % -> %
--R createPrimitiveElement : () -> %      differentiate : % -> %
--R dimension : () -> CardinalNumber      extDegree : % -> PositiveInteger
--R factor : % -> Factored %              fullOutput : % -> OutputForm
--R gcd : List % -> %                     gcd : (%,%) -> %
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R inGroundField? : % -> Boolean         index : PositiveInteger -> %
```

```
--R init : () -> %                      inv : % -> %
--R latex : % -> String                 lcm : List % -> %
--R lcm : (%,%) -> %                    lookup : % -> PositiveInteger
--R maxTower : List % -> %              one? : % -> Boolean
--R order : % -> PositiveInteger        previousTower : % -> %
--R prime? : % -> Boolean               primeFrobenius : % -> %
--R primitive? : % -> Boolean           primitiveElement : () -> %
--R ?quo? : (%,%) -> %                  random : () -> %
--R recip : % -> Union(%,"failed")      ?rem? : (%,%) -> %
--R retract : % -> K                    sample : () -> %
--R setTower! : % -> Void               size : () -> NonNegativeInteger
--R sizeLess? : (%,%) -> Boolean        squareFree : % -> Factored %
--R squareFreePart : % -> %             transcendent? : % -> Boolean
--R unit? : % -> Boolean                unitCanonical : % -> %
--R vectorise : (%,%) -> Vector %       zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if K has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if K has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix % -> Union(Vector %,"failed")
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R lift : % -> SparseUnivariatePolynomial %
--R lift : (%,%) -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R nextItem : % -> Union(%,"failed")
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
```

```
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R retractIfCan : % -> Union(K,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

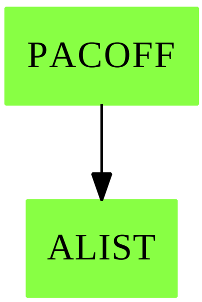
— **PseudoAlgebraicClosureOfFiniteField.help** —

```
====================================================================
PseudoAlgebraicClosureOfFiniteField examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfFiniteField
```

———-

17.36.1 PseudoAlgebraicClosureOfFiniteField (PACOFF)



Exports:

0	1	-?
?**?	?*?	?+?
?-?	?/?	?=?
?^?	?~=?	?quo?
?rem?	D	algebraic?
associates?	characteristic	charthRoot
coerce	conditionP	conjugate
createPrimitiveElement	definingPolynomial	degree
differentiate	dimension	discreteLog
distinguishedRootsOf	divide	euclideanSize
expressIdealMember	exquo	extDegree
extendedEuclidean	extensionDegree	factor
factorsOfCyclicGroupSize	Frobenius	fullOutput
gcd	gcdPolynomial	ground?
hash	inGroundField?	index
init	inv	latex
lcm	lift	lookup
maxTower	multiEuclidean	newElement
nextItem	one?	order
previousTower	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	reduce
representationType	retract	retractIfCan
sample	setTower!	size
sizeLess?	squareFree	squareFreePart
subtractIfCan	tableForDiscreteLogarithm	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	vectorise	zero?

— domain PACOFF PseudoAlgebraicClosureOfFiniteField —

```
)abbrev domain PACOFF PseudoAlgebraicClosureOfFiniteField
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain implement  dynamic extension using the simple notion of
++ tower extensions. A tower extension T  of the ground  field K is any
++ sequence of field extension (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1 and
++ defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2) are said to be related
++ if T_1 <= T_2 (or T_1 >= T_2), that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfFiniteField(K):Exports == Implementation where
```

```
K:FiniteFieldCategory

INT      ==> Integer
NNI      ==> NonNegativeInteger
SUP      ==> SparseUnivariatePolynomial
BOOLEAN  ==> Boolean
PI       ==> PositiveInteger
FFFACTSE ==> FiniteFieldFactorizationWithSizeParseBySideEffect

recRep ==> Record(recEl:SUP(%),_
                  recTower:SUP(%),_
                  recDeg:PI,_
                  recPrevTower:%,_
                  recName:Symbol)

Exports == Join(PseudoAlgebraicClosureOfFiniteFieldCategory,_
                ExtensionField(K)) with

  fullOutput: % -> OutputForm

Implementation == add
  Rep := Union(recRep,K)

  -- signature of local function
  replaceRecEl: (%,SUP(%)) -> %
  down: % -> %
  localRandom: % -> %
  repPolynomial : % -> SUP(%)

  replaceRecEl(a,el)==
    a case K => a
    aa:=copy a
    aa.recEl := el
    aa

  -- local variable
  localTower :% := 1$K

  localSize :NNI := size()$K
  -- implemetation of exported function

  degree(a)==
    da:PositiveInteger:= extDegree a
    coerce(da@PositiveInteger)$OnePointCompletion(PositiveInteger)

  repPolynomial(a)==
    a case K => error "Is in ground field"
    (a pretend recRep).recEl

  inv(a)==
```

```
    a case K => inv(a)$K
    aRecEl:= repPolynomial a
    aDefPoly:= definingPolynomial a
    aInv := extendedEuclidean( aRecEl , aDefPoly, 1 )
    aInv  case "failed" => error "PACOFF : division by zero"
    down replaceRecEl( a , aInv.coef1 )

  a:% ** n:PositiveInteger ==
    zero?(a) => 0
    expt( a , n )$RepeatedSquaring(%)

  a:% ** n:NonNegativeInteger ==
    zero?(a) and zero?(n) => error " --- 0^0 not defined "
    zero?(n) => 1$%
    a ** ( n pretend PositiveInteger )

  a:% ** n:Integer ==
    n < 0 => inv( a ** ( (-n)  pretend PositiveInteger) )
    a ** ( n pretend NonNegativeInteger )

  unitNormal(a)==
    zero? a => [1,0,1]
    [a,1,inv a]

  ground?(a)== a case K

  vectorise(a,lev)==
    da:=extDegree a
    dlev:=extDegree lev
    dlev < da => _
     error "Cannot vectorise at a lower level than the element to vectorise"
    lev case K => [a]
    pa:SUP(%)
    na:%
    ^(da = dlev) =>
      pa:=  monomial(a,0)$SUP(%)
      na:=  replaceRecEl(lev,pa)
      vectorise(na,lev)$%
    prevLev:=previousTower(lev)
    a case K => _
     error "At this point a is not suppose to be in K, big error"
    aEl:=(a pretend recRep).recEl
    daEl:=degree(definingPolynomial a)$SUP(%)
    lv:=[vectorise(c,prevLev)$% for c in entries(vectorise(aEl,daEl)$SUP(%))]
    concat lv

  size == localSize

  setTower!(a) ==
    localTower:=a
```

```
    localSize:=(size()$K)**extDegree(a)
    void()

  localRandom(a) ==
      --return a random element at the extension of a
    a case K => random()$K
    subF:=previousTower(a)
    d:=degree(a.recTower)-1
    pol:=reduce("+",[monomial(localRandom(subF),i)$SUP(%) for i in 0..d])
    down replaceRecEl(a,pol)

  a:% + b:% ==
    (a case K) and (b case K) => a +$K b
    extDegree(a) > extDegree(b) => b + a
    res1:SUP(%)
    res2:%
    if extDegree(a) = extDegree(b) then
      res1:=   b.recEl +$SUP(%) a.recEl
      res2:=   replaceRecEl(b,res1)
    else
      res1:=   b.recEl +$SUP(%) monomial(a,0)$SUP(%)
      res2:= replaceRecEl(b,res1)
    down(res2)

  a:% * b:% ==
    (a case K) and (b case K) => a *$K b
    extDegree(a) > extDegree(b) => b * a
    res1:SUP(%)
    res2:%
    if extDegree(a) = extDegree(b) then
      res1:=   b.recEl *$SUP(%) a.recEl rem b.recTower
      res2:=   replaceRecEl(b,res1)
    else
      res1:=   b.recEl *$SUP(%) monomial(a,0)$SUP(%)
      res2:=  replaceRecEl(b,res1)
    down(res2)

  distinguishedRootsOf(polyZero,ee) ==
      setTower!(ee)
      zero?(polyZero) => error "to lazy to give you all the roots of 0 !!!"
      factorf: Factored SUP % :=  factor(polyZero)$FFFACTSE(%,SUP(%))
      listFact:List SUP %  := [pol.fctr for pol in factorList(factorf)]
      listOfZeros:List(%):=empty()
      for p in listFact repeat
        root:=newElement(p, new(D::Symbol)$Symbol)
        listOfZeros:List(%):=concat([ root ], listOfZeros)
      listOfZeros

  random==
    localRandom(localTower)
```

```
extDegOfGrdField:PI :=
  i: PI := 1
  while characteristic()$K ** i < size()$K repeat
    i:= i + 1
  i

charthRoot(a : %): % ==
  --return a**(1/chararcteristic )
  a case K => charthRoot(retract a)$K
  b:NNI := extDegree(a) * extDegOfGrdField
  j := subtractIfCan(b,1)
  if (j case "failed") then b:= 0
  else b:= j
  c:= (characteristic()$K) ** b
  a**c

conjugate(a)==
  a ** size()$K

1 == 1$K

0 == 0$K

newElement(pol:SUP(%),subF:%,inName:Symbol): % ==
    -- pol is an irreducible polynomial over the field extension
    -- given by subF.
    -- The output of this function is a root of pol.
  dp:=degree pol
  one?(dp) =>
    listCoef:=coefficients(pol)
    one?(#listCoef) => 0
    - last(listCoef) / first(listCoef)
  ground?(pol) => error "Cannot create a new element with a constant"
  d:PI := (dp pretend PI) * extDegree(subF)
  [monomial(1$%,1),pol,d,subF,inName] :: Rep

newElement(poll:SUP(%),inName:Symbol)==
  newElement(poll,localTower,inName)

maxTower(la)==
    --return an element from the list la which is in the largest
    --extension of the ground field
    --PRECONDITION: all elements in same tower, else no meaning?
  m:=reduce("max",[extDegree(a) for a in la])
  first [b for b in la | extDegree(b)=m]

--Field operations

a:% / b:% == a * inv(b)
```

```
a:K * b:%==
  (a :: %) * b

b:% * a:K == a*b

a:% - b:% ==
  a + (-b)

a:% * b:Fraction(Integer) ==
  bn:=numer b
  bd:=denom b
  ebn:%:= bn * 1$%
  ebd:%:= bd * 1$%
  a * ebn * inv(ebd)

-a:% ==
   a case K => -$K a
   [-$SUP(%) (a pretend recRep).recEl,_
    (a pretend recRep).recTower,_
    (a pretend recRep).recDeg,_
    (a pretend recRep).recPrevTower,_
    (a pretend recRep).recName ]

n:INT * a:% ==
  one?(n) => a
  zero?(a) or zero?(n) => 0
  (n < 0) => - ((-n)*a)
  mm:PositiveInteger:=(n pretend PositiveInteger)
  double(mm,a)$RepeatedDoubling(%)

bb:% = aa:% ==
  b:=down bb
  a:=down aa
  ^( extDegree(b) =$NNI extDegree(a) ) => false
  (b case K)  =>  ( (retract a)  =$K (retract b) )
  rda := a :: recRep
  rdb := b :: recRep
  not (rda.recTower =$SUP(%) rdb.recTower) => false
  rdb.recEl =$SUP(%) rda.recEl

zero?(a:%) ==
  da:=down a  -- just to be sure !!!
  ^(da case K) => false
  zero?(da)$K

one?(a:%) ==
  da:= down a  -- just to be sure !!!
  ^(da case K) => false
  one?(da)$K
```

```
--Coerce Functions

coerce(a:K) == a

retractIfCan(a)==
  a case K => a
  "failed"

coerce(a:%):OutputForm ==
  a case K => (retract a)::OutputForm
  outputForm((a pretend recRep).recEl,_
            ((a pretend recRep).recName)::OutputForm) $SUP(%)

fullOutput(a:%):OutputForm==
  a case K => (retract a)::OutputForm
  (a pretend recRep)::OutputForm

definingPolynomial(a:%): SUP % ==
  a case K => 1
  (a pretend recRep).recTower

extDegree(a:%): PI ==
  a case K => 1
  (a pretend recRep).recDeg

previousTower(a:%):% ==
  a case K => error "No previous extension for ground field element"
  (a pretend recRep).recPrevTower

name(a:%):Symbol ==
  a case K => error "No name for ground field element"
  (a pretend recRep).recName

-- function related to the ground field

lookup(a:%)==
  aa:=down a
  ^(aa case K) => _
    error "From NonGlobalDynamicExtensionOfFiniteField fnc Lookup: Cannot take i-dex"
  lookup(retract aa)$K

index(i)==(index(i)$K)

fromPrimeField? == characteristic()$K = size()$K

representationType == representationType()$K

characteristic == characteristic()$K
```

```
    -- implementation of local functions

    down(a:%) ==
      a case K => a
      aa:=(a pretend recRep)
      elel := aa.recEl
      ^ground?(elel) => a
      gel:%:=ground(elel)
      down(gel)
```

— PACOFF.dotabb —

```
"PACOFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACOFF"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"PACOFF" -> "ALIST"
```

17.37 domain PACRAT PseudoAlgebraicClosureOfRationalNumber

— PseudoAlgebraicClosureOfRationalNumber.input —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfRationalNumber.output
)spool PseudoAlgebraicClosureOfRationalNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfRationalNumber
--R PseudoAlgebraicClosureOfRationalNumber  is a domain constructor
--R Abbreviation for PseudoAlgebraicClosureOfRationalNumber is PACRAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PACRAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction Integer) -> %       ?*? : (Fraction Integer,%) -> %
--R ?*? : (Fraction Integer,%) -> %       ?*? : (%,Fraction Integer) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
```

```
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,Integer) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,Fraction Integer) -> %       ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        coerce : Fraction Integer -> %
--R coerce : Fraction Integer -> %        coerce : Integer -> %
--R coerce : Fraction Integer -> %        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R conjugate : % -> %                    dimension : () -> CardinalNumber
--R extDegree : % -> PositiveInteger      factor : % -> Factored %
--R fullOutput : % -> OutputForm          gcd : List % -> %
--R gcd : (%,%) -> %                      ground? : % -> Boolean
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                     lcm : (%,%) -> %
--R maxTower : List % -> %                one? : % -> Boolean
--R previousTower : % -> %                prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> Fraction Integer
--R retract : % -> Fraction Integer       retract : % -> Integer
--R sample : () -> %                      setTower! : % -> Void
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored %
--R squareFreePart : % -> %               transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R vectorise : (%,%) -> Vector %         zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if Fraction Integer has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Fraction Integer has CHARNZ or Fraction Integer has FINITE
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction Integer has CHARNZ or Fraction
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R lift : (%,%) -> SparseUnivariatePolynomial %
```

```
--R lift : % -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %,PositiveInteger,%
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R order : % -> OnePointCompletion PositiveInteger if Fraction Integer has CHARNZ or Fracti
--R primeFrobenius : % -> % if Fraction Integer has CHARNZ or Fraction Integer has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction Integer has CHARNZ or Fraction I
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

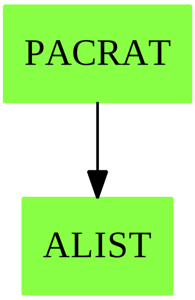
)spool
)lisp (bye)
```

— PseudoAlgebraicClosureOfRationalNumber.help —

```
====================================================================
PseudoAlgebraicClosureOfRationalNumber examples
====================================================================

See Also:
o )show PseudoAlgebraicClosureOfRationalNumber
```

17.37.1 PseudoAlgebraicClosureOfRationalNumber (PACRAT)



Exports:

0	0	1	-?	?**?
?*?	?+?	?-?	?/?	
?=?	?^?	?~=?	?quo?	
?rem?	algebraic?	associates?	characteristic	
charthRoot	coerce	conjugate	definingPolynomial	
degree	dimension	discreteLog	distinguishedRootsOf	
divide	euclideanSize	expressIdealMember	exquo	
extDegree	extendedEuclidean	extensionDegree	factor	
Frobenius	fullOutput	gcd	gcdPolynomial	
ground?	hash	inGroundField?	inv	
latex	lcm	lift	maxTower	
multiEuclidean	newElement	one?	order	
previousTower	prime?	primeFrobenius	principalIdeal	
recip	reduce	retract	retractIfCan	
sample	setTower!	sizeLess?	squareFree	
squareFreePart	subtractIfCan	transcendenceDegree	transcendent?	
unit?	unitCanonical	unitNormal	vectorise	
zero?				

— domain PACRAT PseudoAlgebraicClosureOfRationalNumber —

```
)abbrev domain PACRAT PseudoAlgebraicClosureOfRationalNumber
++ Authors: Gaetan Hache
++ Date Created: feb 1997
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain implements dynamic extension using the simple notion of
++ tower extensions. A tower extension T of the ground field K is any
++ sequence of field extension (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1 and
++ defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1) and
++ (T_2: K_02, K_12,...,K_i2,...,K_n2) are said to be related if T_1 <= T_2
```

```
++ (or T_1 >= T_2), that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfRationalNumber:Exports == Implementation where

  INT     ==> Integer
  K       ==> Fraction Integer
  NNI     ==> NonNegativeInteger
  SUP     ==> SparseUnivariatePolynomial
  BOOLEAN ==> Boolean
  PI      ==> PositiveInteger
  FACTRN  ==> FactorisationOverPseudoAlgebraicClosureOfRationalNumber

  recRep  ==> Record(recEl:SUP(%),_
                     recTower:SUP(%),_
                     recDeg:PI,_
                     recPrevTower:%,_
                     recName:Symbol)

  Exports == PseudoAlgebraicClosureOfRationalNumberCategory with

    fullOutput: % -> OutputForm

    newElement: (SUP(%), SUP(%), PI,  %, Symbol) -> %

  Implementation == add
    Rep := Union(recRep,K)

    -- signature of local function
    replaceRecEl: (%,SUP(%)) -> %
    down: % -> %

    down(a:%) ==
      a case K => a
      aa:=(a pretend recRep)
      elel := aa.recEl
      ^ground?(elel)$SUP(%) => a
      gel:%:=ground(elel)
      down(gel)

    coerce(a:Integer):%== (a :: K)

    n:INT * a:% ==
      one?(n) => a
      zero?(a) or zero?(n) => 0
      (n < 0) => - ((-n)*a)
      mm:PositiveInteger:=(n pretend PositiveInteger)
      double(mm,a)$RepeatedDoubling(%)
```

```
replaceRecEl(a,el)==
  a case K => a
  aa:=copy a
  aa.recEl := el
  aa

-- local variable
localTower :% := 1$K

-- implemetation of exported function

lift(a) ==
  a case K => monomial(a,0)
  (a pretend recRep).recEl

lift(a,b)==
  extDegree a > extDegree b => _
      error "Cannot lift something at lower level !!!!!"
  extDegree a < extDegree b => monomial(a,0)$SUP(%)
  lift a

reduce(a)==
  localTower case K =>
    coefficient(a,0)
  ar:= a rem (localTower pretend recRep).recTower
  replaceRecEl(localTower,ar)

maxTower(la)==
    --return an element from the list la which is in the largest
    --extension of the ground field
    --PRECONDITION: all elements in same tower, else no meaning?
  m:="max"/[extDegree(a)$% for a in la]
  first [b for b in la | extDegree(b)=m]

ground?(a)== a case K

vectorise(a,lev)==
  da:=extDegree a
  dlev:=extDegree lev
  dlev < da => _
    error "Cannot vectorise at a lower level than the element to vectorise"
  lev case K => [a]
  pa:SUP(%)
  na:%
  ^(da = dlev) =>
    pa:=  monomial(a,0)$SUP(%)
    na:=  replaceRecEl(lev,pa)
    vectorise(na,lev)$%
  prevLev:=previousTower(lev)
  a case K => error "At this point a is not suppose to be in K"
```

```
    aEl:=(a pretend recRep).recEl
    daEl:=degree definingPolynomial(a)$%
    lv:=[vectorise(c,prevLev)$% for c in entries(vectorise(aEl,daEl)$SUP(%))]
    concat lv

  setTower!(a) ==
    localTower:=a
    void()

  definingPolynomial == definingPolynomial(localTower)

  a:% + b:% ==
    (a case K) and (b case K) => a +$K b
    extDegree(a) > extDegree(b) => b + a
    res1:SUP(%)
    res2:%
    if extDegree(a) = extDegree(b) then
      res1:=   b.recEl +$SUP(%) a.recEl
      res2:=   replaceRecEl(b,res1)
    else
      res1:=   b.recEl +$SUP(%) monomial(a,0)$SUP(%)
      res2:= replaceRecEl(b,res1)
    down(res2)

  a:% * b:% ==
    (a case K) and (b case K) => a *$K b
    extDegree(a) > extDegree(b) => b * a
    res1:SUP(%)
    res2:%
    if extDegree(a) = extDegree(b) then
      res1:=   b.recEl *$SUP(%) a.recEl rem b.recTower
      res2:=   replaceRecEl(b,res1)
    else
      res1:=   b.recEl *$SUP(%) monomial(a,0)$SUP(%)
      res2:=  replaceRecEl(b,res1)
    down(res2)

  distinguishedRootsOf(polyZero,ee) ==
      setTower!(ee)
      zero?(polyZero) => error "to lazy to give you all the roots of 0 !!!"
      factorf: Factored SUP % :=  factor(polyZero,ee)$FACTRN(%)
      listFact:List SUP %  := [pol.fctr for pol in factorList(factorf)]
      listOfZeros:List(%):=empty()
      for p in listFact repeat
        root:=newElement(p, new(D::Symbol)$Symbol)
        listOfZeros:List(%):=concat([ root ], listOfZeros)
      listOfZeros

  1 == 1$K
```

```
0 == 0$K

newElement(pol:SUP(%),subF:%,inName:Symbol): % ==
    -- pol is an irreducible polynomial over the field extension
    -- given by subF.
    -- The output of this function is a root of pol.
  dp:=degree pol
  one?(dp) =>
    listCoef:=coefficients(pol)
    one?(#listCoef) => 0
    - last(listCoef) / first(listCoef)
  ground?(pol) => error "Cannot create a new element with a constant"
  d:PI := (dp pretend PI) * extDegree(subF)
  [monomial(1$%,1),pol,d,subF,inName] :: Rep

newElement(poll:SUP(%),inName:Symbol)==
  newElement(poll,localTower,inName)

newElement(elPol:SUP(%),pol:SUP(%),d:PI,subF:%,inName:Symbol): % ==
  [elPol, pol,d,subF,inName] :: Rep

--Field operations
inv(a)==
  a case K => inv(a)$K
  aRecEl:= (a pretend recRep).recEl
  aDefPoly:= (a pretend recRep).recTower
  aInv := extendedEuclidean( aRecEl , aDefPoly, 1 )
  aInv  case "failed" => error "PACOFF : division by zero"
  -- On doit retourner un Record reprsentant l'inverse de a.
  -- Ce Record est exactement le mme que celui de a sauf
  -- qu'il faut remplacer le polynme du selecteur recEl
  -- par le polynme reprsentant l'inverse de a :
  -- C'est ce que fait la fonction replaceRecEl.
  replaceRecEl( a , aInv.coef1 )

a:% / b:% == a * inv(b)

a:K * b:%==
  (a :: %) * b

b:% * a:K == a*b

a:% - b:% ==
  a + (-b)

a:% * b:Fraction(Integer) ==
  bn:=numer b
  bd:=denom b
  ebn:%:= bn * 1$%
  ebd:%:= bd * 1$%
```

```
  a * ebn * inv(ebd)

-a:% ==
   a case K => -$K a
   [-$SUP(%) (a pretend recRep).recEl,_
    (a pretend recRep).recTower,_
    (a pretend recRep).recDeg,_
    (a pretend recRep).recPrevTower,_
    (a pretend recRep).recName ]

bb:% = aa:% ==
  b:=down bb
  a:=down aa
  ^( extDegree(b) =$NNI extDegree(a) ) => false
  (b case K)  =>  ( (retract a)@K  =$K (retract b)@K )
  rda := a :: recRep
  rdb := b :: recRep
  not (rda.recTower =$SUP(%) rdb.recTower) => false
  rdb.recEl =$SUP(%) rda.recEl

zero?(a:%) ==
  da:=down a  -- just to be sure !!!
  ^(da case K) => false
  zero?(da)$K

one?(a:%) ==
  da:= down a  -- just to be sure !!!
  ^(da case K) => false
  one?(da)$K

--Coerce Functions

coerce(a:K):% == a

retractIfCan(a:%):Union(Integer,"failed")==
  a case K => retractIfCan(a)$K
  "failed"

retractIfCan(a:%):Union(K,"failed")==
  a case K => a
  "failed"

coerce(a:%):OutputForm ==
  a case K => ((retract a)@K) ::OutputForm
  outputForm((a pretend recRep).recEl,_
            ((a pretend recRep).recName)::OutputForm) $SUP(%)

fullOutput(a:%):OutputForm==
  a case K => ((retract a)@K) ::OutputForm
  (a pretend recRep)::OutputForm
```

```
defining Polynomial(a:%): SUP % ==
  a case K => monomial(1,1)$SUP(%)
  (a pretend recRep).recTower

extDegree(a:%): PI ==
  a case K => 1
  (a pretend recRep).recDeg

previousTower(a:%):% ==
  a case K => error "No previous extension for ground field element"
  (a pretend recRep).recPrevTower

name(a:%):Symbol ==
  a case K => error "No name for ground field element"
  (a pretend recRep).recName

-- function related to the ground field

characteristic == characteristic()$K
```

— **PACRAT.dotabb** —

```
"PACRAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRODUCT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"PACRAT" -> "ALIST"
```

Chapter 18

Chapter Q

18.1 domain QFORM QuadraticForm

— **QuadraticForm.input** —

```
)set break resume
)sys rm -f QuadraticForm.output
)spool QuadraticForm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuadraticForm
--R QuadraticForm(n: PositiveInteger,K: Field)  is a domain constructor
--R Abbreviation for QuadraticForm is QFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QFORM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 0 : () -> %                          coerce : % -> OutputForm
--R ?.? : (%,DirectProduct(n,K)) -> K    hash : % -> SingleInteger
--R latex : % -> String                  matrix : % -> SquareMatrix(n,K)
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R quadraticForm : SquareMatrix(n,K) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
```

```
--E 1

)spool
)lisp (bye)
```

———–

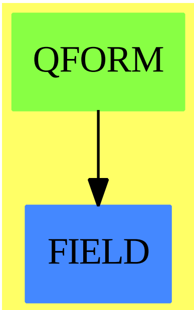
— QuadraticForm.help —

```
====================================================================
QuadraticForm examples
====================================================================

See Also:
o )show QuadraticForm
```

———–

18.1.1 QuadraticForm (QFORM)



See

⇒ "CliffordAlgebra" (CLIF) 4.5.12 on page 386

Exports:

0	coerce	hash	latex	matrix
quadraticForm	sample	subtractIfCan	zero?	?~=?
?*?	?.?	?+?	?-?	-?
?=?				

— domain QFORM QuadraticForm —

```
)abbrev domain QFORM QuadraticForm
++ Author: Stephen M. Watt
++ Date Created: August 1988
++ Date Last Updated: May 17, 1991
```

```
++ Basic Operations: quadraticForm, elt
++ Related Domains: Matrix, SquareMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: quadratic form
++ Examples:
++ References:
++
++ Description:
++ This domain provides modest support for quadratic forms.

QuadraticForm(n, K): T == Impl where
    n: PositiveInteger
    K: Field
    SM ==> SquareMatrix
    V  ==> DirectProduct

    T ==> AbelianGroup with
        quadraticForm: SM(n, K) -> %
            ++ quadraticForm(m) creates a quadratic form from a symmetric,
            ++ square matrix m.
        matrix: % -> SM(n, K)
            ++ matrix(qf) creates a square matrix from the quadratic form qf.
        elt: (%, V(n, K)) -> K
            ++ elt(qf,v) evaluates the quadratic form qf on the vector v,
            ++ producing a scalar.

    Impl ==> SM(n,K) add
        Rep := SM(n,K)

        quadraticForm m ==
            not symmetric? m =>
                error "quadraticForm requires a symmetric matrix"
            m::%
        matrix q == q pretend SM(n,K)
        elt(q,v) == dot(v, (matrix q * v))
```

———-

— QFORM.dotabb —

```
"QFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QFORM"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"QFORM" -> "FIELD"
```

———-

18.2 domain QALGSET QuasiAlgebraicSet

— QuasiAlgebraicSet.input —

```
)set break resume
)sys rm -f QuasiAlgebraicSet.output
)spool QuasiAlgebraicSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuasiAlgebraicSet
--R QuasiAlgebraicSet(R: GcdDomain,Var: OrderedSet,Expon: OrderedAbelianMonoidSup,Dpoly: Poly
--R Abbreviation for QuasiAlgebraicSet is QALGSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QALGSET
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R definingInequation : % -> Dpoly       empty : () -> %
--R empty? : % -> Boolean                 hash : % -> SingleInteger
--R idealSimplify : % -> %                latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R definingEquations : % -> List Dpoly
--R quasiAlgebraicSet : (List Dpoly,Dpoly) -> %
--R setStatus : (%,Union(Boolean,"failed")) -> %
--R simplify : % -> % if R has CHARZ and R has EUCDOM
--R status : % -> Union(Boolean,"failed")
--R
--E 1

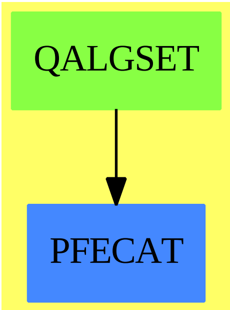
)spool
)lisp (bye)
```

— QuasiAlgebraicSet.help —

```
====================================================================
QuasiAlgebraicSet examples
====================================================================

See Also:
o )show QuasiAlgebraicSet
```

18.2.1 QuasiAlgebraicSet (QALGSET)



Exports:

coerce	definingEquations	definingInequation	empty
empty?	hash	idealSimplify	latex
quasiAlgebraicSet	setStatus	simplify	status
?=?	?~=?		

— **domain QALGSET QuasiAlgebraicSet** —

```
)abbrev domain QALGSET QuasiAlgebraicSet
++ Author:  William Sit
++ Date Created: March 13, 1992
++ Date Last Updated: June 12, 1992
++ Basic Operations:
++ Related Constructors:GroebnerPackage
++ See Also: QuasiAlgebraicSet2
++ AMS Classifications:
++ Keywords: Zariski closed sets, quasi-algebraic sets
++ References:William Sit, "An Algorithm for Parametric Linear Systems"
++            J. Sym. Comp., April, 1992
++ Description:
++ \spadtype{QuasiAlgebraicSet} constructs a domain representing
++ quasi-algebraic sets, which is the intersection of a Zariski
++ closed set, defined as the common zeros of a given list of
++ polynomials (the defining polynomials for equations), and a principal
++ Zariski open set, defined as the complement of the common
++ zeros of a polynomial f (the defining polynomial for the inequation).
++ This domain provides simplification of a user-given representation
++ using groebner basis computations.
++ There are two simplification routines: the first function
++ \spadfun{idealSimplify}  uses groebner
++ basis of ideals alone, while the second, \spadfun{simplify} uses both
++ groebner basis and factorization.  The resulting defining equations L
++ always form a groebner basis, and the resulting defining
++ inequation f is always reduced.  The function \spadfun{simplify} may
++ be applied several times if desired.   A third simplification
```

```
++ routine \spadfun{radicalSimplify} is provided in
++ \spadtype{QuasiAlgebraicSet2}  for comparison study only,
++ as it is inefficient compared to the other two, as well as is
++ restricted to only certain coefficient domains.  For detail analysis
++ and a comparison of the three methods, please consult the reference
++ cited.
++
++ A polynomial function q defined on the quasi-algebraic set
++ is equivalent to its reduced form with respect to L.  While
++ this may be obtained using the usual normal form
++ algorithm, there is no canonical form for q.
++
++ The ordering in groebner basis computation is determined by
++ the data type of the input polynomials.  If it is possible
++ we suggest to use refinements of total degree orderings.

QuasiAlgebraicSet(R, Var,Expon,Dpoly) : C == T
 where
   R          :  GcdDomain
   Expon      :  OrderedAbelianMonoidSup
   Var        :  OrderedSet
   Dpoly      :  PolynomialCategory(R,Expon,Var)
   NNI      ==> NonNegativeInteger
   newExpon ==> Product(NNI,Expon)
   newPoly  ==> PolynomialRing(R,newExpon)
   Ex       ==> OutputForm
   mrf      ==> MultivariateFactorize(Var,Expon,R,Dpoly)
   Status   ==> Union(Boolean,"failed") -- empty or not, or don't know

   C == Join(SetCategory, CoercibleTo OutputForm) with
  --- should be Object instead of SetCategory, bug in LIST Object ---
  --- equality is not implemented ---
     empty: () -> $
       ++ empty() returns the empty quasi-algebraic set
     quasiAlgebraicSet:   (List Dpoly, Dpoly) -> $
       ++ quasiAlgebraicSet(pl,q) returns the quasi-algebraic set
       ++ with defining equations p = 0 for p belonging to the list pl, and
       ++ defining inequation q ^= 0.
     status: $ -> Status
       ++ status(s) returns true if the quasi-algebraic set is empty,
       ++ false if it is not, and "failed" if not yet known
     setStatus: ($, Status) -> $
       ++ setStatus(s,t) returns the same representation for s, but
       ++ asserts the following: if t is true, then s is empty,
       ++ if t is false, then s is non-empty, and if t = "failed",
       ++ then no assertion is made (that is, "don't know").
       ++ Note: for internal use only, with care.
     empty?          :   $   -> Boolean
       ++ empty?(s) returns
       ++ true if the quasialgebraic set s has no points,
```

```
      ++ and false otherwise.
    definingEquations: $ -> List Dpoly
      ++ definingEquations(s) returns a list of defining polynomials
      ++ for equations, that is, for the Zariski closed part of s.
    definingInequation: $ -> Dpoly
      ++ definingInequation(s) returns a single defining polynomial for the
      ++ inequation, that is, the Zariski open part of s.
    idealSimplify:$ -> $
      ++ idealSimplify(s) returns a different and presumably simpler
      ++ representation of s with the defining polynomials for the
      ++ equations
      ++ forming a groebner basis, and the defining polynomial for the
      ++ inequation reduced with respect to the basis, using Buchberger's
      ++ algorithm.
    if (R has EuclideanDomain) and (R has CharacteristicZero) then
      simplify:$ -> $
        ++ simplify(s) returns a different and presumably simpler
        ++ representation of s with the defining polynomials for the
        ++ equations
        ++ forming a groebner basis, and the defining polynomial for the
        ++ inequation reduced with respect to the basis, using a heuristic
        ++ algorithm based on factoring.
  T  == add
    Rep := Record(status:Status,zero:List Dpoly, nzero:Dpoly)
    x:$

    import GroebnerPackage(R,Expon,Var,Dpoly)
    import GroebnerPackage(R,newExpon,Var,newPoly)
    import GroebnerInternalPackage(R,Expon,Var,Dpoly)

                  ----  Local Functions  ----

    minset   : List List Dpoly -> List List Dpoly
    overset? : (List Dpoly, List List Dpoly) -> Boolean
    npoly    : Dpoly            ->  newPoly
    oldpoly  : newPoly          ->  Union(Dpoly,"failed")


    if (R has EuclideanDomain) and (R has CharacteristicZero) then
      factorset (y:Dpoly):List Dpoly ==
        ground? y => []
        [j.factor for j in factors factor$mrf  y]

      simplify x ==
        if x.status case "failed" then
          x:=quasiAlgebraicSet(zro:=groebner x.zero, redPol(x.nzero,zro))
        (pnzero:=x.nzero)=0 => empty()
        nzro:=factorset pnzero
        mset:=minset [factorset p for p in x.zero]
        mset:=[setDifference(s,nzro) for s in mset]
```

```
    zro:=groebner [*/s for s in mset]
    member? (1$Dpoly, zro) => empty()
    [x.status, zro, primitivePart redPol(*/nzro, zro)]

npoly(f:Dpoly) : newPoly ==
  zero? f => 0
  monomial(leadingCoefficient f,makeprod(0,degree f))$newPoly +
        npoly(reductum f)

oldpoly(q:newPoly) : Union(Dpoly,"failed") ==
  q=0$newPoly => 0$Dpoly
  dq:newExpon:=degree q
  n:NNI:=selectfirst (dq)
  n^=0 => "failed"
  ((g:=oldpoly reductum q) case "failed") => "failed"
  monomial(leadingCoefficient q,selectsecond dq)$Dpoly + (g::Dpoly)

coerce x ==
  x.status = true => "Empty"::Ex
  bracket [[hconcat(f::Ex, " = 0"::Ex) for f in x.zero ]::Ex,
            hconcat( x.nzero::Ex, " != 0"::Ex)]

empty? x ==
  if x.status case "failed" then x:=idealSimplify x
  x.status :: Boolean

empty() == [true::Status, [1$Dpoly], 0$Dpoly]
status x == x.status
setStatus(x,t) == [t,x.zero,x.nzero]
definingEquations x == x.zero
definingInequation x == x.nzero
quasiAlgebraicSet(z0,n0) == ["failed", z0, n0]

idealSimplify x ==
  x.status case Boolean => x
  z0:= x.zero
  n0:= x.nzero
  empty? z0 => [false, z0, n0]
  member? (1$Dpoly, z0) => empty()
  tp:newPoly:=(monomial(1,makeprod(1,0$Expon))$newPoly * npoly n0)-1
  ngb:=groebner concat(tp, [npoly g for g in z0])
  member? (1$newPoly, ngb) => empty()
  gb:List Dpoly:=nil
  while not empty? ngb repeat
    if ((f:=oldpoly ngb.first) case Dpoly) then gb:=concat(f, gb)
    ngb:=ngb.rest
  [false::Status, gb, primitivePart redPol(n0, gb)]


minset lset ==
```

```
    empty? lset => lset
    [s for s  in lset | ^(overset?(s,lset))]

  overset?(p,qlist) ==
    empty? qlist => false
    or/[(brace$(Set Dpoly) q) <$(Set Dpoly) (brace$(Set Dpoly) p) for q in qlist]
```

———-

— QALGSET.dotabb —

```
"QALGSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QALGSET"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"QALGSET" -> "PFECAT"
```

———-

18.3 domain QUAT Quaternion

— Quaternion.input —

```
)set break resume
)sys rm -f Quaternion.output
)spool Quaternion.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
q := quatern(2/11,-8,3/4,1)
--R
--R
--R         2        3
--R   (1)  -- - 8i + - j + k
--R        11        4
--R                                            Type: Quaternion Fraction Integer
--E 1

--S 2 of 13
[real q, imagI q, imagJ q, imagK q]
--R
--R
--R          2     3
--R   (2)  [--,- 8,-,1]
--R         11     4
```

```
--R                                         Type: List Fraction Integer
--E 2

--S 3 of 13
inv q
--R
--R
--R         352     15488      484       1936
--R   (3)  ------ + ------ i - ----- j - ------ k
--R        126993   126993     42331     126993
--R                                   Type: Quaternion Fraction Integer
--E 3

--S 4 of 13
q^6
--R
--R
--R         2029490709319345   48251690851     144755072553     48251690851
--R   (4)  - ---------------- - ----------- i + ------------ j + ----------- k
--R             7256313856        1288408         41229056         10307264
--R                                   Type: Quaternion Fraction Integer
--E 4

--S 5 of 13
r := quatern(-2,3,23/9,-89); q + r
--R
--R
--R         20         119
--R   (5)  - -- - 5i + --- j - 88k
--R         11          36
--R                                   Type: Quaternion Fraction Integer
--E 5

--S 6 of 13
q * r - r * q
--R
--R
--R         2495              817
--R   (6)  - ---- i - 1418j - --- k
--R          18                18
--R                                   Type: Quaternion Fraction Integer
--E 6

--S 7 of 13
i:=quatern(0,1,0,0)
--R
--R
--R   (7)  i
--R                                            Type: Quaternion Integer
--E 7
```

```
--S 8 of 13
j:=quatern(0,0,1,0)
--R
--R
--R   (8)  j
--R                                                       Type: Quaternion Integer
--E 8

--S 9 of 13
k:=quatern(0,0,0,1)
--R
--R
--R   (9)  k
--R                                                       Type: Quaternion Integer
--E 9

--S 10 of 13
[i*i, j*j, k*k, i*j, j*k, k*i, q*i]
--R
--R
--R                                  2         3
--R   (10)  [- 1,- 1,- 1,k,i,j,8 + -- i + j - - k]
--R                                 11         4
--R                                         Type: List Quaternion Fraction Integer
--E 10

--S 11 of 13
norm q
--R
--R
--R         126993
--R   (11)  ------
--R          1936
--R                                                         Type: Fraction Integer
--E 11

--S 12 of 13
conjugate q
--R
--R
--R          2        3
--R   (12)  -- + 8i - - j - k
--R         11        4
--R                                              Type: Quaternion Fraction Integer
--E 12

--S 13 of 13
q * %
--R
```

```
--R
--R          126993
--R   (13)  ------
--R           1936
--R                                        Type: Quaternion Fraction Integer
--E 13
)spool
)lisp (bye)
```

— Quaternion.help —

```
====================================================================
Quaternion examples
====================================================================

The domain constructor Quaternion implements quaternions over
commutative rings.

The basic operation for creating quaternions is quatern.  This is a
quaternion over the rational numbers.

  q := quatern(2/11,-8,3/4,1)
     2        3
    -- - 8i + - j + k
    11        4
                        Type: Quaternion Fraction Integer

The four arguments are the real part, the i imaginary part, the j
imaginary part, and the k imaginary part, respectively.

  [real q, imagI q, imagJ q, imagK q]
      2     3
    [--,- 8,-,1]
     11     4
                        Type: List Fraction Integer

Because q is over the rationals (and nonzero), you can invert it.

  inv q
      352     15488      484       1936
    ------ + ------ i - ----- j - ------ k
    126993   126993     42331     126993
                        Type: Quaternion Fraction Integer

The usual arithmetic (ring) operations are available

  q^6
```

```
     2029490709319345   48251690851     144755072553     48251690851
   - ---------------- - ----------- i + ------------ j + ----------- k
        7256313856        1288408         41229056         10307264
                       Type: Quaternion Fraction Integer
```

```
  r := quatern(-2,3,23/9,-89); q + r
     20       119
   - -- - 5i + --- j - 88k
     11        36
                       Type: Quaternion Fraction Integer
```

In general, multiplication is not commutative.

```
  q * r - r * q
     2495            817
   - ---- i - 1418j - --- k
      18              18
                        Type: Quaternion Fraction Integer
```

There are no predefined constants for the imaginary i, j, and k parts, but you can easily define them.

```
  i:=quatern(0,1,0,0)
    i
                        Type: Quaternion Integer
```

```
  j:=quatern(0,0,1,0)
    j
                        Type: Quaternion Integer
```

```
  k:=quatern(0,0,0,1)
    k
                        Type: Quaternion Integer
```

These satisfy the normal identities.

```
  [i*i, j*j, k*k, i*j, j*k, k*i, q*i]
                              2        3
     [- 1,- 1,- 1,k,i,j,8 + -- i + j - - k]
                             11        4
                         Type: List Quaternion Fraction Integer
```

The norm is the quaternion times its conjugate.

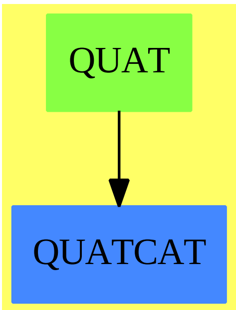
```
  norm q
    126993
    ------
     1936
                         Type: Fraction Integer
```

```
  conjugate q
      2        3
     -- + 8i - - j - k
     11        4
                            Type: Quaternion Fraction Integer

  q * %
     126993
     ------
      1936
                            Type: Quaternion Fraction Integer

See Also:
o )help Octonion
o )help Complex
o )help CliffordAlgebra
o )show Quaternion
```

18.3.1 Quaternion (QUAT)



Exports:

0	1	abs	characteristic	charthRoot
coerce	conjugate	convert	D	differentiate
eval	hash	imagI	imagJ	imagK
inv	latex	map	max	min
norm	one?	quatern	rational	rational?
rationalIfCan	real	recip	reducedSystem	retract
retractIfCan	sample	subtractIfCan	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?	?>?
?>=?	?.?			

— domain QUAT Quaternion —

```
)abbrev domain QUAT Quaternion
++ Author: Robert S. Sutor
++ Date Created: 23 May 1990
++ Change History:
++   10 September 1990
++ Basic Operations: (Algebra)
++   abs, conjugate, imagI, imagJ, imagK, norm, quatern, rational,
++   rational?, real
++ Related Constructors: QuaternionCategoryFunctions2
++ Also See: QuaternionCategory, DivisionRing
++ AMS Classifications: 11R52
++ Keywords: quaternions, division ring, algebra
++ Description:
++ \spadtype{Quaternion} implements quaternions over a
++ commutative ring. The main constructor function is \spadfun{quatern}
++ which takes 4 arguments: the real part, the i imaginary part, the j
++ imaginary part and the k imaginary part.

Quaternion(R:CommutativeRing): QuaternionCategory(R) == add
  Rep := Record(r:R,i:R,j:R,k:R)

  0 == [0,0,0,0]
  1 == [1,0,0,0]

  a,b,c,d : R
  x,y : $

  real  x == x.r
  imagI x == x.i
  imagJ x == x.j
  imagK x == x.k

  quatern(a,b,c,d) == [a,b,c,d]

  x * y == [x.r*y.r-x.i*y.i-x.j*y.j-x.k*y.k,
             x.r*y.i+x.i*y.r+x.j*y.k-x.k*y.j,
               x.r*y.j+x.j*y.r+x.k*y.i-x.i*y.k,
                 x.r*y.k+x.k*y.r+x.i*y.j-x.j*y.i]
```

———

— QUAT.dotabb —

```
"QUAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QUAT"]
"QUATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=QUATCAT"]
```

```
"QUAT" -> "QUATCAT"
```

———-

18.4 domain QEQUAT QueryEquation

— QueryEquation.input —

```
)set break resume
)sys rm -f QueryEquation.output
)spool QueryEquation.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QueryEquation
--R QueryEquation  is a domain constructor
--R Abbreviation for QueryEquation is QEQUAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QEQUAT
--R
--R------------------------------ Operations --------------------------------
--R coerce : % -> OutputForm              equation : (Symbol,String) -> %
--R value : % -> String                   variable : % -> Symbol
--R
--E 1

)spool
)lisp (bye)
```

———-

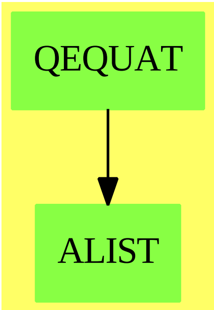
— QueryEquation.help —

```
====================================================================
QueryEquation examples
====================================================================

See Also:
o )show QueryEquation
```

———-

18.4.1 QueryEquation (QEQUAT)



See

⇒ "DataList" (DLIST) 5.2.1 on page 445
⇒ "IndexCard" (ICARD) 10.2.1 on page 1159
⇒ "Database" (DBASE) 5.1.1 on page 440

Exports:
coerce equation value variable

— domain QEQUAT QueryEquation —

```
)abbrev domain QEQUAT QueryEquation
++ Author: Mark Botch
++ Description:
++ This domain implements simple database queries

QueryEquation(): Exports == Implementation where
  Exports == CoercibleTo(OutputForm) with
    equation: (Symbol,String) -> %
      ++ equation(s,"a") creates a new equation.
    variable: % -> Symbol
      ++ variable(q) returns the variable (i.e. left hand side) of \axiom{q}.
    value: % -> String
      ++ value(q) returns the value (i.e. right hand side) of \axiom{q}.
  Implementation == add
    Rep := Record(var:Symbol, val:String)
    coerce(u) == coerce(u.var)$Symbol = coerce(u.val)$String
    equation(x,s) == [x,s]
    variable q == q.var
    value q == q.val
```

———

— QEQUAT.dotabb —

```
"QEQUAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QEQUAT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"QEQUAT" -> "ALIST"
```

———-

18.5 domain QUEUE Queue

— Queue.input —

```
)set break resume
)sys rm -f Queue.output
)spool Queue.output
)set message test on
)set message auto off
)clear all

--S 1 of 46
a:Queue INT:= queue [1,2,3,4,5]
--R
--R
--R   (1)  [1,2,3,4,5]
--R                                                    Type: Queue Integer
--E 1

--S 2 of 46
dequeue! a
--R
--R
--R   (2)  1
--R                                                  Type: PositiveInteger
--E 2

--S 3 of 46
a
--R
--R
--R   (3)  [2,3,4,5]
--R                                                    Type: Queue Integer
--E 3

--S 4 of 46
extract! a
--R
--R
--R   (4)  2
```

```
--R                                                   Type: PositiveInteger
--E 4

--S 5 of 46
a
--R
--R
--R   (5)  [3,4,5]
--R                                                     Type: Queue Integer
--E 5

--S 6 of 46
enqueue!(9,a)
--R
--R
--R   (6)  9
--R                                                   Type: PositiveInteger
--E 6

--S 7 of 46
a
--R
--R
--R   (7)  [3,4,5,9]
--R                                                     Type: Queue Integer
--E 7

--S 8 of 46
insert!(8,a)
--R
--R
--R   (8)  [3,4,5,9,8]
--R                                                     Type: Queue Integer
--E 8

--S 9 of 46
a
--R
--R
--R   (9)  [3,4,5,9,8]
--R                                                     Type: Queue Integer
--E 9

--S 10 of 46
inspect a
--R
--R
--R   (10)  3
--R                                                   Type: PositiveInteger
--E 10
```

```
--S 11 of 46
empty? a
--R
--R
--R   (11)  false
--R                                                        Type: Boolean
--E 11

--S 12 of 46
front a
--R
--R
--R   (12)  3
--R                                                Type: PositiveInteger
--E 12

--S 13 of 46
back a
--R
--R
--R   (13)  8
--R                                                Type: PositiveInteger
--E 13

--S 14 of 46
rotate! a
--R
--R
--R   (14)  [4,5,9,8,3]
--R                                                  Type: Queue Integer
--E 14

--S 15 of 46
#a
--R
--R
--R   (15)  5
--R                                                Type: PositiveInteger
--E 15

--S 16 of 46
length a
--R
--R
--R   (16)  5
--R                                                Type: PositiveInteger
--E 16

--S 17 of 46
```

```
less?(a,9)
--R
--R
--R   (17)  true
--R                                                            Type: Boolean
--E 17

--S 18 of 46
more?(a,9)
--R
--R
--R   (18)  false
--R                                                            Type: Boolean
--E 18

--S 19 of 46
size?(a,#a)
--R
--R
--R   (19)  true
--R                                                            Type: Boolean
--E 19

--S 20 of 46
size?(a,9)
--R
--R
--R   (20)  false
--R                                                            Type: Boolean
--E 20

--S 21 of 46
parts a
--R
--R
--R   (21)  [4,5,9,8,3]
--R                                                       Type: List Integer
--E 21

--S 22 of 46
bag([1,2,3,4,5])$Queue(INT)
--R
--R
--R   (22)  [1,2,3,4,5]
--R                                                      Type: Queue Integer
--E 22

--S 23 of 46
b:=empty()$(Queue INT)
--R
```

```
--R
--R   (23)  []
--R                                                      Type: Queue Integer
--E 23

--S 24 of 46
empty? b
--R
--R
--R   (24)  true
--R                                                            Type: Boolean
--E 24

--S 25 of 46
sample()$Queue(INT)
--R
--R
--R   (25)  []
--R                                                      Type: Queue Integer
--E 25

--S 26 of 46
c:=copy a
--R
--R
--R   (26)  [4,5,9,8,3]
--R                                                      Type: Queue Integer
--E 26

--S 27 of 46
eq?(a,c)
--R
--R
--R   (27)  false
--R                                                            Type: Boolean
--E 27

--S 28 of 46
eq?(a,a)
--R
--R
--R   (28)  true
--R                                                            Type: Boolean
--E 28

--S 29 of 46
(a=c)@Boolean
--R
--R
--R   (29)  true
```

```
--R                                                    Type: Boolean
--E 29

--S 30 of 46
(a=a)@Boolean
--R
--R
--R   (30)  true
--R                                                    Type: Boolean
--E 30

--S 31 of 46
a~=c
--R
--R
--R   (31)  false
--R                                                    Type: Boolean
--E 31

--S 32 of 46
any?(x+->(x=4),a)
--R
--R
--R   (32)  true
--R                                                    Type: Boolean
--E 32

--S 33 of 46
any?(x+->(x=11),a)
--R
--R
--R   (33)  false
--R                                                    Type: Boolean
--E 33

--S 34 of 46
every?(x+->(x=11),a)
--R
--R
--R   (34)  false
--R                                                    Type: Boolean
--E 34

--S 35 of 46
count(4,a)
--R
--R
--R   (35)  1
--R                                            Type: PositiveInteger
--E 35
```

```
--S 36 of 46
count(x+->(x>2),a)
--R
--R
--R   (36)  5
--R                                                     Type: PositiveInteger
--E 36

--S 37 of 46
map(x+->x+10,a)
--R
--R
--R   (37)  [14,15,19,18,13]
--R                                                      Type: Queue Integer
--E 37

--S 38 of 46
a
--R
--R
--R   (38)  [4,5,9,8,3]
--R                                                      Type: Queue Integer
--E 38

--S 39 of 46
map!(x+->x+10,a)
--R
--R
--R   (39)  [14,15,19,18,13]
--R                                                      Type: Queue Integer
--E 39

--S 40 of 46
a
--R
--R
--R   (40)  [14,15,19,18,13]
--R                                                      Type: Queue Integer
--E 40

--S 41 of 46
members a
--R
--R
--R   (41)  [14,15,19,18,13]
--R                                                       Type: List Integer
--E 41

--S 42 of 46
```

```
member?(14,a)
--R
--R
--R   (42)  true
--R                                                                Type: Boolean
--E 42

--S 43 of 46
coerce a
--R
--R
--R   (43)  [14,15,19,18,13]
--R                                                             Type: OutputForm
--E 43

--S 44 of 46
hash a
--R
--R
--I   (44)  4999531
--R                                                          Type: SingleInteger
--E 44

--S 45 of 46
latex a
--R
--R
--R   (45)  "\mbox{\bf Unimplemented}"
--R                                                                 Type: String
--E 45

--S 46 of 46
)show Queue
--R
--R Queue S: SetCategory  is a domain constructor
--R Abbreviation for Queue is QUEUE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QUEUE
--R
--R------------------------------- Operations --------------------------------
--R back : % -> S                         bag : List S -> %
--R copy : % -> %                         dequeue! : % -> S
--R empty : () -> %                       empty? : % -> Boolean
--R enqueue! : (S,%) -> S                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     front : % -> S
--R insert! : (S,%) -> %                  inspect : % -> S
--R length : % -> NonNegativeInteger      map : ((S -> S),%) -> %
--R queue : List S -> %                   rotate! : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
```

```
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 46


)spool
)lisp (bye)
```

———-

— Queue.help —

```
====================================================================
Queue examples
====================================================================

A Queue object is represented as a list ordered by first-in, first-out.
It operates like a line of people, where the "next" person is the one
at the front of the line.

Here we create an queue of integers from a list. Notice that the
order in the list is the order in the queue.

   a:Queue INT:= queue [1,2,3,4,5]
      [1,2,3,4,5]

We can remove the top of the queue using dequeue!:

   dequeue! a
```

```
      1
```

Notice that the use of dequeue! is destructive (destructive operations in Axiom usually end with ! to indicate that the underylying data structure is changed).

```
   a
      [2,3,4,5]
```

The extract! operation is another name for the pop! operation and has the same effect. This operation treats the queue as a BagAggregate:

```
   extract! a
      2
```

and you can see that it also has destructively modified the queue:

```
   a
      [3,4,5]
```

Next we use enqueue! to add a new element to the end of the queue:

```
   push!(9,a)
      9
```

Again, the push! operation is destructive so the queue is changed:

```
   a
      [3,4,5,9]
```

Another name for enqueue! is insert!, which treats the queue as a BagAggregate:

```
   insert!(8,a)
      [3,4,5,9,8]
```

and it modifies the queue:

```
   a
      [3,4,5,9,8]
```

The inspect function returns the top of the queue without modification, viewed as a BagAggregate:

```
   inspect a
      8
```

The empty? operation returns true only if there are no element on the queue, otherwise it returns false:

```
   empty? a
      false
```

The front operation returns the front of the queue without modification:

```
   front a
      3
```

The back operation returns the back of the queue without modification:

```
   back a
      8
```

The rotate! operation moves the item at the front of the queue to the back of the queue:

```
   rotate! a
      [4,5,9,8,3]
```

The # (length) operation:

```
   #a
       5
```

The length operation does the same thing:

```
   length a
       5
```

The less? predicate will compare the queue length to an integer:

```
   less?(a,9)
        true
```

The more? predicate will compare the queue length to an integer:

```
   more?(a,9)
        false
```

The size? operation will compare the queue length to an integer:

```
   size?(a,#a)
        true
```

and since the last computation must alwasy be true we try:

```
   size?(a,9)
        false
```

The parts function will return the queue as a list of its elements:

```
   parts a
        [8,9,3,4,5]
```

If we have a BagAggregate of elements we can use it to construct a queue:

```
   bag([1,2,3,4,5])$Queue(INT)
        [1,2,3,4,5]
```

The empty function will construct an empty queue of a given type:

```
   b:=empty()$(Queue INT)
        []
```

and the empty? predicate allows us to find out if a queue is empty:

```
   empty? b
        true
```

The sample function returns a sample, empty queue:

```
   sample()$Queue(INT)
        []
```

We can copy a queue and it does not share storage so subsequent modifications of the original queue will not affect the copy:

```
   c:=copy a
        [4,5,9,8,3]
```

The eq? function is only true if the lists are the same reference, so even though c is a copy of a, they are not the same:

```
   eq?(a,c)
        false
```

However, a clearly shares a reference with itself:

```
   eq?(a,a)
        true
```

But we can compare a and c for equality:

```
   (a=c)@Boolean
        true
```

and clearly a is equal to itself:

```
   (a=a)@Boolean
        true
```

and since a and c are equal, they are clearly NOT not-equal:

```
   a~=c
        false
```

We can use the any? function to see if a predicate is true for any element:

```
   any?(x+->(x=4),a)
        true
```

or false for every element:

```
   any?(x+->(x=11),a)
        false
```

We can use the every? function to check every element satisfies a predicate:

```
   every?(x+->(x=11),a)
        false
```

We can count the elements that are equal to an argument of this type:

```
   count(4,a)
        1
```

or we can count against a boolean function:

```
   count(x+->(x>2),a)
        5
```

You can also map a function over every element, returning a new queue:

```
   map(x+->x+10,a)
        [14,15,19,18,13]
```

Notice that the orignal queue is unchanged:

```
   a
        [4,5,9,8,3]
```

You can use map! to map a function over every element and change
the original queue since map! is destructive:

```
   map!(x+->x+10,a)
       [14,15,19,18,13]
o
```

Notice that the orignal queue has been changed:

```
   a
```

```
       [14,15,19,18,13]

The member function can also get the element of the queue as a list:

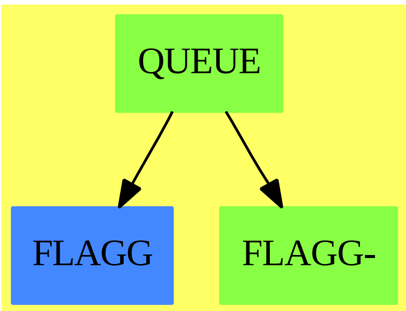
   members a
       [18,19,13,14,15]

and using member? we can test if the queue holds a given element:

   member?(14,a)
       true


See Also:
o )show Stack
o )show ArrayStack
o )show Queue
o )show Dequeue
o )show Heap
o )show BagAggregate
```

18.5.1 Queue (QUEUE)



See

⇒ "Stack" (STACK) 20.28.1 on page 2521
⇒ "ArrayStack" (ASTACK) 2.10.1 on page 65
⇒ "Dequeue" (DEQUEUE) 5.5.1 on page 497
⇒ "Heap" (HEAP) 9.2.1 on page 1100

Exports:

any?	back	bag	coerce	copy
count	dequeue!	empty	empty?	enqueue!
eq?	eval	every?	extract!	front
hash	insert!	inspect	latex	length
less?	map	map!	member?	members
more?	parts	queue	rotate!	sample
size?	#?	?=?	?~=?	

— domain QUEUE Queue —

```
)abbrev domain QUEUE Queue
++ Author: Michael Monagan and Stephen Watt
++ Date Created:June 86 and July 87
++ Date Last Updated:Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Linked List implementation of a Queue
--% Dequeue and Heap data types

Queue(S:SetCategory): QueueAggregate S with
    queue: List S -> %
      ++ queue([x,y,...,z]) creates a queue with first (top)
      ++ element x, second element y,...,and last (bottom) element z.
      ++
      ++E e:Queue INT:= queue [1,2,3,4,5]

 -- Inherited Signatures repeated for examples documentation

    dequeue_! : % -> S
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X dequeue! a
      ++X a
    extract_! : % -> S
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X extract! a
      ++X a
    enqueue_! : (S,%) -> S
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X enqueue! (9,a)
      ++X a
    insert_! : (S,%) -> %
```

```
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X insert! (8,a)
    ++X a
  inspect : % -> S
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X inspect a
  front : % -> S
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X front a
  back : % -> S
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X back a
  rotate_! : % -> %
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X rotate! a
  length : % -> NonNegativeInteger
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X length a
  less? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X less?(a,9)
  more? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X more?(a,9)
  size? : (%,NonNegativeInteger) -> Boolean
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X size?(a,5)
  bag : List S -> %
    ++
    ++X bag([1,2,3,4,5])$Queue(INT)
  empty? : % -> Boolean
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X empty? a
  empty : () -> %
    ++
    ++X b:=empty()$(Queue INT)
  sample : () -> %
    ++
    ++X sample()$Queue(INT)
  copy : % -> %
```

```
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X copy a
  eq? : (%,%) -> Boolean
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X b:=copy a
    ++X eq?(a,b)
  map :  ((S -> S),%) -> %
    ++
    ++X a:Queue INT:= queue [1,2,3,4,5]
    ++X map(x+->x+10,a)
    ++X a
  if $ has shallowlyMutable then
    map! :  ((S -> S),%) -> %
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X map!(x+->x+10,a)
      ++X a
  if S has SetCategory then
    latex : % -> String
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X latex a
    hash : % -> SingleInteger
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X hash a
    coerce : % -> OutputForm
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X coerce a
    "=": (%,%) -> Boolean
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X b:Queue INT:= queue [1,2,3,4,5]
      ++X (a=b)@Boolean
    "~=" : (%,%) -> Boolean
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X b:=copy a
      ++X (a~=b)
  if % has finiteAggregate then
    every? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X every?(x+->(x=4),a)
    any? : ((S -> Boolean),%) -> Boolean
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
```

```
      ++X any?(x+->(x=4),a)
    count :  ((S -> Boolean),%) -> NonNegativeInteger
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X count(x+->(x>2),a)
    _# : % -> NonNegativeInteger
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X #a
    parts : % -> List S
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X parts a
    members : % -> List S
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X members a
  if % has finiteAggregate and S has SetCategory then
    member? : (S,%) -> Boolean
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X member?(3,a)
    count : (S,%) -> NonNegativeInteger
      ++
      ++X a:Queue INT:= queue [1,2,3,4,5]
      ++X count(4,a)

== Stack S add
  Rep := Reference List S
  lastTail==> LAST$Lisp
  enqueue_!(e,q) ==
      if null deref q then setref(q, list e)
      else lastTail.(deref q).rest := list e
      e
  insert_!(e,q) == (enqueue_!(e,q);q)
  dequeue_! q ==
      empty? q => error "empty queue"
      e := first deref q
      setref(q,rest deref q)
      e
  extract_! q == dequeue_! q
  rotate_! q == if empty? q then q else (enqueue_!(dequeue_! q,q); q)
  length q == # deref q
  front q == if empty? q then error "empty queue" else first deref q
  inspect q == front q
  back q == if empty? q then error "empty queue" else last deref q
  queue q == ref copy q
```

———-

— **QUEUE.dotabb** —

```
"QUEUE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QUEUE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"QUEUE" -> "FLAGG"
"QUEUE" -> "FLAGG-"
```

———-

Chapter 19

Chapter R

19.1 domain RADFF RadicalFunctionField

— **RadicalFunctionField.input** —

```
)set break resume
)sys rm -f RadicalFunctionField.output
)spool RadicalFunctionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RadicalFunctionField
--R RadicalFunctionField(F: UniqueFactorizationDomain,UP: UnivariatePolynomialCategory F,UPUP: Univariat
--R Abbreviation for RadicalFunctionField is RADFF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RADFF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction UP,%) -> %           ?*? : (%,Fraction UP) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       basis : () -> Vector %
--R branchPoint? : UP -> Boolean         branchPoint? : F -> Boolean
--R coerce : Fraction UP -> %            coerce : Integer -> %
--R coerce : % -> OutputForm             convert : UPUP -> %
--R convert : % -> UPUP                  convert : Vector Fraction UP -> %
--R convert : % -> Vector Fraction UP    definingPolynomial : () -> UPUP
```

```
--R discriminant : () -> Fraction UP      elt : (%,F,F) -> F
--R generator : () -> %                   genus : () -> NonNegativeInteger
--R hash : % -> SingleInteger             integral? : (%,UP) -> Boolean
--R integral? : (%,F) -> Boolean          integral? : % -> Boolean
--R integralBasis : () -> Vector %        latex : % -> String
--R lift : % -> UPUP                      norm : % -> Fraction UP
--R one? : % -> Boolean                   primitivePart : % -> %
--R ramified? : UP -> Boolean             ramified? : F -> Boolean
--R rank : () -> PositiveInteger          rationalPoint? : (F,F) -> Boolean
--R recip : % -> Union(%,"failed")        reduce : UPUP -> %
--R represents : (Vector UP,UP) -> %      retract : % -> Fraction UP
--R sample : () -> %                      singular? : UP -> Boolean
--R singular? : F -> Boolean              trace : % -> Fraction UP
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Fraction UP has FIELD
--R ?*? : (Fraction Integer,%) -> % if Fraction UP has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if Fraction UP has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Fraction UP has FIELD
--R D : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIELD
--R D : (%,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FIELD or F
--R D : (%,Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRIN
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP
--R D : (%,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R D : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if Fraction UP has FIELD
--R ?^? : (%,Integer) -> % if Fraction UP has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R absolutelyIrreducible? : () -> Boolean
--R algSplitSimple : (%,(UP -> UP)) -> Record(num: %,den: UP,derivden: UP,gd: UP)
--R associates? : (%,%) -> Boolean if Fraction UP has FIELD
--R branchPointAtInfinity? : () -> Boolean
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UPUP
--R charthRoot : % -> Union(%,"failed") if Fraction UP has CHARNZ
--R charthRoot : % -> % if Fraction UP has FFIELDC
--R coerce : % -> % if Fraction UP has FIELD
--R coerce : Fraction Integer -> % if Fraction UP has FIELD or Fraction UP has RETRACT FRAC I
--R complementaryBasis : Vector % -> Vector %
--R conditionP : Matrix % -> Union(Vector %,"failed") if Fraction UP has FFIELDC
--R coordinates : Vector % -> Matrix Fraction UP
--R coordinates : % -> Vector Fraction UP
--R coordinates : (Vector %,Vector %) -> Matrix Fraction UP
--R coordinates : (%,Vector %) -> Vector Fraction UP
--R createPrimitiveElement : () -> % if Fraction UP has FFIELDC
--R derivationCoordinates : (Vector %,(Fraction UP -> Fraction UP)) -> Matrix Fraction UP if
--R differentiate : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction U
--R differentiate : (%,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP ha
```

```
--R differentiate : (%,Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRI
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction U
--R differentiate : (%,(UP -> UP)) -> %
--R differentiate : (%,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R differentiate : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if Fraction UP has FIELD
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction UP has FFIELDC
--R discreteLog : % -> NonNegativeInteger if Fraction UP has FFIELDC
--R discriminant : Vector % -> Fraction UP
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Fraction UP has FIELD
--R elliptic : () -> Union(UP,"failed")
--R euclideanSize : % -> NonNegativeInteger if Fraction UP has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Fraction UP has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Fraction UP has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Fraction UP has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Fraction UP has FIELD
--R factor : % -> Factored % if Fraction UP has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if Fraction UP has F
--R gcd : (%,%) -> % if Fraction UP has FIELD
--R gcd : List % -> % if Fraction UP has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R hyperelliptic : () -> Union(UP,"failed")
--R index : PositiveInteger -> % if Fraction UP has FINITE
--R init : () -> % if Fraction UP has FFIELDC
--R integralAtInfinity? : % -> Boolean
--R integralBasisAtInfinity : () -> Vector %
--R integralCoordinates : % -> Record(num: Vector UP,den: UP)
--R integralDerivationMatrix : (UP -> UP) -> Record(num: Matrix UP,den: UP)
--R integralMatrix : () -> Matrix Fraction UP
--R integralMatrixAtInfinity : () -> Matrix Fraction UP
--R integralRepresents : (Vector UP,UP) -> %
--R inv : % -> % if Fraction UP has FIELD
--R inverseIntegralMatrix : () -> Matrix Fraction UP
--R inverseIntegralMatrixAtInfinity : () -> Matrix Fraction UP
--R lcm : (%,%) -> % if Fraction UP has FIELD
--R lcm : List % -> % if Fraction UP has FIELD
--R lookup : % -> PositiveInteger if Fraction UP has FINITE
--R minimalPolynomial : % -> UPUP if Fraction UP has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Fraction UP has FIELD
--R nextItem : % -> Union(%,"failed") if Fraction UP has FFIELDC
--R nonSingularModel : Symbol -> List Polynomial F if F has FIELD
--R normalizeAtInfinity : Vector % -> Vector %
--R numberOfComponents : () -> NonNegativeInteger
--R order : % -> OnePointCompletion PositiveInteger if Fraction UP has FFIELDC
--R order : % -> PositiveInteger if Fraction UP has FFIELDC
--R prime? : % -> Boolean if Fraction UP has FIELD
--R primeFrobenius : % -> % if Fraction UP has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction UP has FFIELDC
--R primitive? : % -> Boolean if Fraction UP has FFIELDC
```

```
--R primitiveElement : () -> % if Fraction UP has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Fraction UP has FIELD
--R ?quo? : (%,%) -> % if Fraction UP has FIELD
--R ramifiedAtInfinity? : () -> Boolean
--R random : () -> % if Fraction UP has FINITE
--R rationalPoints : () -> List List F if F has FINITE
--R reduce : Fraction UPUP -> Union(%,"failed") if Fraction UP has FIELD
--R reduceBasisAtInfinity : Vector % -> Vector %
--R reducedSystem : Matrix % -> Matrix Fraction UP
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction UP,vec: Vector Fractio
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if Fraction UP has LINEXP INT
--R regularRepresentation : % -> Matrix Fraction UP
--R regularRepresentation : (%,Vector %) -> Matrix Fraction UP
--R ?rem? : (%,%) -> % if Fraction UP has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if Fraction UP has FF
--R represents : Vector Fraction UP -> %
--R represents : (Vector Fraction UP,Vector %) -> %
--R retract : % -> Fraction Integer if Fraction UP has RETRACT FRAC INT
--R retract : % -> Integer if Fraction UP has RETRACT INT
--R retractIfCan : % -> Union(Fraction UP,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if Fraction UP has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if Fraction UP has RETRACT INT
--R singularAtInfinity? : () -> Boolean
--R size : () -> NonNegativeInteger if Fraction UP has FINITE
--R sizeLess? : (%,%) -> Boolean if Fraction UP has FIELD
--R squareFree : % -> Factored % if Fraction UP has FIELD
--R squareFreePart : % -> % if Fraction UP has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Fract
--R traceMatrix : () -> Matrix Fraction UP
--R traceMatrix : Vector % -> Matrix Fraction UP
--R unit? : % -> Boolean if Fraction UP has FIELD
--R unitCanonical : % -> % if Fraction UP has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Fraction UP has FIELD
--R yCoordinates : % -> Record(num: Vector UP,den: UP)
--R
--E 1

)spool
)lisp (bye)
```

———

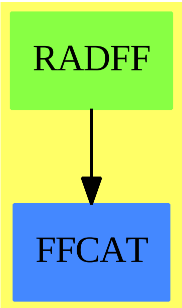
— RadicalFunctionField.help —

```
====================================================================
RadicalFunctionField examples
====================================================================
```

```
See Also:
o )show RadicalFunctionField
```

——————

19.1.1 RadicalFunctionField (RADFF)



See

⇒ "AlgebraicFunctionField" (ALGFF) 2.5.1 on page 27

Exports:

0	1	absolutelyIrreducible?
algSplitSimple	associates?	basis
branchPoint?	branchPointAtInfinity?	characteristic
characteristicPolynomial	charthRoot	coerce
complementaryBasis	conditionP	convert
coordinates	createPrimitiveElement	D
derivationCoordinates	definingPolynomial	differentiate
discreteLog	discriminant	divide
elliptic	elt	euclideanSize
expressIdealMember	exquo	extendedEuclidean
factor	factorsOfCyclicGroupSize	gcd
gcdPolynomial	generator	genus
hash	hyperelliptic	index
init	integral?	integralAtInfinity?
integralBasis	integralBasisAtInfinity	integralCoordinates
integralDerivationMatrix	integralMatrix	integralMatrixAtInfinity
integralRepresents	inv	inverseIntegralMatrix
inverseIntegralMatrixAtInfinity	latex	lcm
lift	lookup	minimalPolynomial
multiEuclidean	nextItem	nonSingularModel
norm	normalizeAtInfinity	numberOfComponents
one?	order	prime?
primeFrobenius	primitive?	primitiveElement
primitivePart	principalIdeal	ramified?
ramifiedAtInfinity?	random	rank
rationalPoint?	rationalPoints	recip
reduce	reduce	reduceBasisAtInfinity
reducedSystem	regularRepresentation	representationType
represents	retract	retractIfCan
sample	singular?	singularAtInfinity?
size	sizeLess?	squareFree
squareFreePart	subtractIfCan	tableForDiscreteLogarithm
trace	traceMatrix	unit?
unitCanonical	unitNormal	yCoordinates
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?quo?	?rem?

— domain RADFF RadicalFunctionField —

```
)abbrev domain RADFF RadicalFunctionField
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 27 July 1993
++ Keywords: algebraic, curve, radical, function, field.
++ Examples: )r RADFF INPUT
```

```
++ Description:
++ Function field defined by y**n = f(x);

RadicalFunctionField(F, UP, UPUP, radicnd, n): Exports == Impl where
  F       : UniqueFactorizationDomain
  UP      : UnivariatePolynomialCategory F
  UPUP    : UnivariatePolynomialCategory Fraction UP
  radicnd : Fraction UP
  n       : NonNegativeInteger

  N   ==> NonNegativeInteger
  Z   ==> Integer
  RF  ==> Fraction UP
  QF  ==> Fraction UPUP
  UP2 ==> SparseUnivariatePolynomial UP
  REC ==> Record(factor:UP, exponent:Z)
  MOD ==> monomial(1, n)$UPUP - radicnd::UPUP
  INIT ==> if (deref brandNew?) then startUp false

  Exports ==> FunctionFieldCategory(F, UP, UPUP)

  Impl ==> SimpleAlgebraicExtension(RF, UPUP, MOD) add
    import ChangeOfVariable(F, UP, UPUP)
    import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
    import UnivariatePolynomialCategoryFunctions2(RF, UPUP, UP, UP2)

    diag        : Vector RF -> Vector $
    startUp     : Boolean -> Void
    fullVector  : (Factored UP, N) -> PrimitiveArray UP
    iBasis      : (UP, N) -> Vector UP
    inftyBasis  : (RF, N) -> Vector RF
    basisvec    : () -> Vector RF
    char0StartUp: () -> Void
    charPStartUp: () -> Void
    getInfBasis : () -> Void
    radcand     : () -> UP
    charPintbas : (UPUP, RF, Vector RF, Vector RF) -> Void

    brandNew?:Reference(Boolean) := ref true
    discPoly:Reference(RF) := ref(0$RF)
    newrad:Reference(UP) := ref(0$UP)
    n1 := (n - 1)::N
    modulus := MOD
    ibasis:Vector(RF)     := new(n, 0)
    invibasis:Vector(RF)  := new(n, 0)
    infbasis:Vector(RF)   := new(n, 0)
    invinfbasis:Vector(RF):= new(n, 0)
    mini := minIndex ibasis

    discriminant()                   == (INIT; discPoly())
```

```
    radcand()                        == (INIT; newrad())
    integralBasis()                  == (INIT; diag ibasis)
    integralBasisAtInfinity()        == (INIT; diag infbasis)
    basisvec()                       == (INIT; ibasis)
    integralMatrix()                 == diagonalMatrix basisvec()
    integralMatrixAtInfinity()       == (INIT; diagonalMatrix infbasis)
    inverseIntegralMatrix()          == (INIT; diagonalMatrix invibasis)
    inverseIntegralMatrixAtInfinity()==(INIT;diagonalMatrix invinfbasis)
    definingPolynomial()             == modulus
    ramified?(point:F)               == zero?(radcand() point)
    branchPointAtInfinity?()  == (degree(radcand()) exquo n) case "failed"
    elliptic()     == (n = 2 and degree(radcand()) = 3 => radcand(); "failed")
    hyperelliptic() == (n=2 and odd? degree(radcand()) => radcand(); "failed")
    diag v == [reduce monomial(qelt(v,i+mini), i) for i in 0..n1]

    integralRepresents(v, d) ==
      ib := basisvec()
      represents
        [qelt(ib, i) * (qelt(v, i) /$RF d) for i in mini .. maxIndex ib]

    integralCoordinates f ==
      v  := coordinates f
      ib := basisvec()
      splitDenominator
        [qelt(v,i) / qelt(ib,i) for i in mini .. maxIndex ib]$Vector(RF)

    integralDerivationMatrix d ==
      dlogp := differentiate(radicnd, d) / (n * radicnd)
      v := basisvec()
      cd := splitDenominator(
              [(i - mini) * dlogp + differentiate(qelt(v, i), d) / qelt(v, i)
                                    for i in mini..maxIndex v]$Vector(RF))
      [diagonalMatrix(cd.num), cd.den]

-- return (d0,...,d(n-1)) s.t. (1/d0, y/d1,...,y**(n-1)/d(n-1))
-- is an integral basis for the curve y**d = p
-- requires that p has no factor of multiplicity >= d
    iBasis(p, d) ==
      pl := fullVector(squareFree p, d)
      d1 := (d - 1)::N
      [*/[pl.j ** ((i * j) quo d) for j in 0..d1] for i in 0..d1]

-- returns a vector [a0,a1,...,a_{m-1}] of length m such that
-- p = a0^0 a1^1 ... a_{m-1}^{m-1}
    fullVector(p, m) ==
      ans:PrimitiveArray(UP) := new(m, 0)
      ans.0 := unit p
      l := factors p
      for i in 1..maxIndex ans repeat
        ans.i :=
```

```
          (u := find(s+->s.exponent = i, l)) case "failed" => 1
          (u::REC).factor
      ans

-- return (f0,...,f(n-1)) s.t. (f0, y f1,..., y**(n-1) f(n-1))
-- is a local integral basis at infinity for the curve y**d = p
    inftyBasis(p, m) ==
      rt := rootPoly(p(x := inv(monomial(1, 1)$UP :: RF)), m)
      m ^= rt.exponent =>
        error "Curve not irreducible after change of variable 0 -> infinity"
      a    := (rt.coef) x
      b:RF := 1
      v    := iBasis(rt.radicand, m)
      w:Vector(RF) := new(m, 0)
      for i in mini..maxIndex v repeat
        qsetelt_!(w, i, b / ((qelt(v, i)::RF) x))
        b := b * a
      w

    charPintbas(p, c, v, w) ==
      degree(p) ^= n => error "charPintbas: should not happen"
      q:UP2 := map(s+->retract(s)@UP, p)
      ib := integralBasis()$FunctionFieldIntegralBasis(UP, UP2,
                                       SimpleAlgebraicExtension(UP, UP2, q))
      not diagonal?(ib.basis)=>
         error "charPintbas: integral basis not diagonal"
      a:RF := 1
      for i in minRowIndex(ib.basis) .. maxRowIndex(ib.basis)
        for j in minColIndex(ib.basis) .. maxColIndex(ib.basis)
          for k in mini .. maxIndex v repeat
            qsetelt_!(v, k, (qelt(ib.basis, i, j) / ib.basisDen) * a)
            qsetelt_!(w, k, qelt(ib.basisInv, i, j) * inv a)
            a := a * c
      void

    charPStartUp() ==
      r      := mkIntegral modulus
      charPintbas(r.poly, r.coef, ibasis, invibasis)
      x      := inv(monomial(1, 1)$UP :: RF)
      invmod := monomial(1, n)$UPUP - (radicnd x)::UPUP
      r      := mkIntegral invmod
      charPintbas(r.poly, (r.coef) x, infbasis, invinfbasis)

    startUp b ==
      brandNew?() := b
      if zero?(p := characteristic()$F) or p > n then char0StartUp()
                                            else charPStartUp()
      dsc:RF := ((-1)$Z ** ((n *$N n1) quo 2::N) * (n::Z)**n)$Z *
                 radicnd ** n1 *
                    */[qelt(ibasis, i) ** 2 for i in mini..maxIndex ibasis]
```

```
      discPoly() := primitivePart(numer dsc) / denom(dsc)
      void

    char0StartUp() ==
      rp          := rootPoly(radicnd, n)
      rp.exponent ^= n =>
         error "RadicalFunctionField: curve is not irreducible"
      newrad()    := rp.radicand
      ib          := iBasis(newrad(), n)
      infb        := inftyBasis(radicnd, n)
      invden:RF   := 1
      for i in mini..maxIndex ib repeat
        qsetelt_!(invibasis, i, a := qelt(ib, i) * invden)
        qsetelt_!(ibasis, i, inv a)
        invden := invden / rp.coef        -- always equals 1/rp.coef**(i-mini)
        qsetelt_!(infbasis, i, a := qelt(infb, i))
        qsetelt_!(invinfbasis, i, inv a)
      void

    ramified?(p:UP) ==
      (r := retractIfCan(p)@Union(F, "failed")) case F =>
        singular?(r::F)
      (radcand() exquo p) case UP

    singular?(p:UP) ==
      (r := retractIfCan(p)@Union(F, "failed")) case F =>
        singular?(r::F)
      (radcand() exquo(p**2)) case UP

    branchPoint?(p:UP) ==
      (r := retractIfCan(p)@Union(F, "failed")) case F =>
        branchPoint?(r::F)
      ((q := (radcand() exquo p)) case UP) and
        ((q::UP exquo p) case "failed")

    singular?(point:F) ==
      zero?(radcand()  point) and
        zero?(((radcand() exquo (monomial(1,1)$UP-point::UP))::UP) point)

    branchPoint?(point:F) ==
      zero?(radcand()  point) and not
        zero?(((radcand() exquo (monomial(1,1)$UP-point::UP))::UP) point)
```

———-

— RADFF.dotabb —

```
"RADFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RADFF"]
```

```
"FFCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FFCAT"]
"RADFF" -> "FFCAT"
```

19.2 domain RADIX RadixExpansion

— RadixExpansion.input —

```
)set break resume
)sys rm -f RadixExpansion.output
)spool RadixExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 17
111::RadixExpansion(5)
--R
--R
--R   (1)  421
--R                                                     Type: RadixExpansion 5
--E 1

--S 2 of 17
(5/24)::RadixExpansion(2)
--R
--R
--R               --
--R   (2)  0.00110
--R                                                     Type: RadixExpansion 2
--E 2

--S 3 of 17
(5/24)::RadixExpansion(3)
--R
--R
--R              --
--R   (3)  0.012
--R                                                     Type: RadixExpansion 3
--E 3

--S 4 of 17
(5/24)::RadixExpansion(8)
--R
--R
--R              --
```

```
--R   (4)  0.152
--R                                                         Type: RadixExpansion 8
--E 4

--S 5 of 17
(5/24)::RadixExpansion(10)
--R
--R
--R                _
--R   (5)  0.2083
--R                                                        Type: RadixExpansion 10
--E 5

--S 6 of 17
(5/24)::RadixExpansion(12)
--R
--R
--R   (6)  0.26
--R                                                        Type: RadixExpansion 12
--E 6

--S 7 of 17
(5/24)::RadixExpansion(16)
--R
--R
--R              _
--R   (7)  0.35
--R                                                        Type: RadixExpansion 16
--E 7

--S 8 of 17
(5/24)::RadixExpansion(36)
--R
--R
--R   (8)  0.7I
--R                                                        Type: RadixExpansion 36
--E 8

--S 9 of 17
(5/24)::RadixExpansion(38)
--R
--R
--R                      -----
--R   (9)  0 . 7 34 31 25 12
--R                                                        Type: RadixExpansion 38
--E 9

--S 10 of 17
a := (76543/210)::RadixExpansion(8)
--R
```

```
--R
--R                ----
--R   (10)  554.37307
--R                                                     Type: RadixExpansion 8
--E 10

--S 11 of 17
w := wholeRagits a
--R
--R
--R   (11)  [5,5,4]
--R                                                         Type: List Integer
--E 11

--S 12 of 17
f0 := prefixRagits a
--R
--R
--R   (12)  [3]
--R                                                         Type: List Integer
--E 12

--S 13 of 17
f1 := cycleRagits a
--R
--R
--R   (13)  [7,3,0,7]
--R                                                         Type: List Integer
--E 13

--S 14 of 17
u:RadixExpansion(8):=wholeRadix(w)+fractRadix(f0,f1)
--R
--R
--R                ----
--R   (14)  554.37307
--R                                                     Type: RadixExpansion 8
--E 14

--S 15 of 17
v: RadixExpansion(12) := fractRadix([1,2,3,11], [0])
--R
--R
--R                  _
--R   (15)  0.123B0
--R                                                    Type: RadixExpansion 12
--E 15

--S 16 of 17
fractRagits(u)
```

```
--R
--R
--R                -------
--R   (16)  [3,7,3,0,7,7]
--R                                                       Type: Stream Integer
--E 16

--S 17 of 17
a :: Fraction(Integer)
--R
--R
--R         76543
--R   (17)  -----
--R          210
--R                                                     Type: Fraction Integer
--E 17
)spool
)lisp (bye)
```

— RadixExpansion.help —

```
====================================================================
RadixExpansion examples
====================================================================

It possible to expand numbers in general bases.

Here we expand 111 in base 5. This means
  10^2+10^1+10^0 = 4 * 5^2+2 * 5^1 + 5^0

  111::RadixExpansion(5)
    421
                              Type: RadixExpansion 5

You can expand fractions to form repeating expansions.

  (5/24)::RadixExpansion(2)
           --
    0.00110
                              Type: RadixExpansion 2

  (5/24)::RadixExpansion(3)
         --
    0.012
                              Type: RadixExpansion 3

  (5/24)::RadixExpansion(8)
```

```
     --
   0.152
                              Type: RadixExpansion 8

(5/24)::RadixExpansion(10)

        -
   0.2083
                              Type: RadixExpansion 10
```

For bases from 11 to 36 the letters A through Z are used.

```
(5/24)::RadixExpansion(12)
   0.26
                              Type: RadixExpansion 12

(5/24)::RadixExpansion(16)

      -
   0.35
                              Type: RadixExpansion 16

(5/24)::RadixExpansion(36)
   0.7I
                              Type: RadixExpansion 36
```

For bases greater than 36, the ragits are separated by blanks.

```
(5/24)::RadixExpansion(38)

               -----
   0 . 7 34 31 25 12
                              Type: RadixExpansion 38
```

The RadixExpansion type provides operations to obtain the individual ragits. Here is a rational number in base 8.

```
a := (76543/210)::RadixExpansion(8)

         ----
    554.37307
                              Type: RadixExpansion 8
```

The operation wholeRagits returns a list of the ragits for the integral part of the number.

```
w := wholeRagits a
   [5,5,4]
                              Type: List Integer
```

The operations prefixRagits and cycleRagits return lists of the initial and repeating ragits in the fractional part of the number.

```
f0 := prefixRagits a
```

```
    [3]
                                Type: List Integer

  f1 := cycleRagits a
    [7,3,0,7]
                                Type: List Integer
```

You can construct any radix expansion by giving the whole, prefix and cycle parts. The declaration is necessary to let Axiom know the base of the ragits.

```
  u:RadixExpansion(8):=wholeRadix(w)+fractRadix(f0,f1)
         ----
    554.37307
                                Type: RadixExpansion 8
```

If there is no repeating part, then the list [0] should be used.

```
  v: RadixExpansion(12) := fractRadix([1,2,3,11], [0])
           -
    0.123B0
                                Type: RadixExpansion 12
```

If you are not interested in the repeating nature of the expansion, an infinite stream of ragits can be obtained using fractRagits.

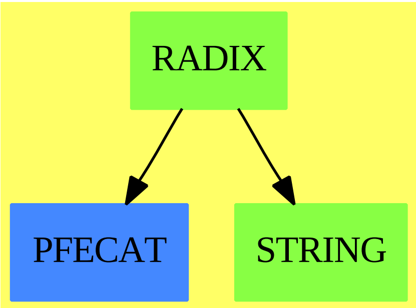
```
  fractRagits(u)
         -------
    [3,7,3,0,7,7]
                                Type: Stream Integer
```

Of course, it's possible to recover the fraction representation:

```
  a :: Fraction(Integer)
    76543
    -----
     210
                                Type: Fraction Integer
```

See Also:
o)help DecimalExpansion
o)help BinaryExpansion
o)help HexadecimalExpansion
o)show RadixExpansion

19.2.1 RadixExpansion (RADIX)



See

⇒ "BinaryExpansion" (BINARY) 3.7.1 on page 274
⇒ "DecimalExpansion" (DECIMAL) 5.3.1 on page 451
⇒ "HexadecimalExpansion" (HEXADEC) 9.3.1 on page 1108

Exports:

0	1	abs
associates?	ceiling	characteristic
charthRoot	coerce	conditionP
convert	cycleRagits	D
denom	denominator	differentiate
divide	euclideanSize	eval
expressIdealMember	exquo	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractRadix	fractRagits
fractionPart	gcd	gcdPolynomial
hash	init	inv
latex	lcm	map
max	min	multiEuclidean
negative?	nextItem	numer
numerator	one?	patternMatch
positive?	prefixRagits	prime?
principalIdeal	random	recip
reducedSystem	retract	retractIfCan
sample	sign	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	unit?
unitCanonical	unitNormal	wholePart
wholeRadix	wholeRagits	zero?
?.?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?~=?	?<?	?<=?
?>?	?>=?	?quo?
?rem?		

— domain RADIX RadixExpansion —

```
)abbrev domain RADIX RadixExpansion
++ Author: Stephen M. Watt
++ Date Created: October 1986
++ Date Last Updated: May 15, 1991
++ Basic Operations: wholeRadix, fractRadix, wholeRagits, fractRagits
++ Related Domains: BinaryExpansion, DecimalExpansion, HexadecimalExpansion,
++    RadixUtilities
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, repeating decimal
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ decimal expansions or more generally as repeating expansions in any base.
```

```
RadixExpansion(bb): Exports == Implementation where
  bb   :  Integer
  I   ==> Integer
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  RN  ==> Fraction Integer
  ST  ==> Stream Integer
  QuoRem ==> Record(quotient: Integer, remainder: Integer)

  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
      ++ coerce(rx) converts a radix expansion to a rational number.
    fractionPart: % -> Fraction Integer
      ++ fractionPart(rx) returns the fractional part of a radix expansion.
    wholeRagits: % -> List Integer
      ++ wholeRagits(rx) returns the ragits of the integer part
      ++ of a radix expansion.
    fractRagits: % -> Stream Integer
      ++ fractRagits(rx) returns the ragits of the fractional part
      ++ of a radix expansion.
    prefixRagits: % -> List Integer
      ++ prefixRagits(rx) returns the non-cyclic part of the ragits
      ++ of the fractional part of a radix expansion.
      ++ For example, if \spad{x = 3/28 = 0.10 714285 714285 ...},
      ++ then \spad{prefixRagits(x)=[1,0]}.
    cycleRagits: % -> List Integer
      ++ cycleRagits(rx) returns the cyclic part of the ragits of the
      ++ fractional part of a radix expansion.
      ++ For example, if \spad{x = 3/28 = 0.10 714285 714285 ...},
      ++ then \spad{cycleRagits(x) = [7,1,4,2,8,5]}.
    wholeRadix: List Integer -> %
      ++ wholeRadix(l) creates an integral radix expansion from a list
      ++ of ragits.
      ++ For example, \spad{wholeRadix([1,3,4])} will return \spad{134}.
    fractRadix: (List Integer, List Integer) -> %
      ++ fractRadix(pre,cyc) creates a fractional radix expansion
      ++ from a list of prefix ragits and a list of cyclic ragits.
      ++ e.g., \spad{fractRadix([1],[6])} will return \spad{0.16666666...}.

  Implementation ==> add
    -- The efficiency of arithmetic operations is poor.
    -- Could use a lazy eval where either rational rep
    -- or list of ragit rep (the current) or both are kept
    -- as demanded.

    bb < 2 => error "Radix base must be at least 2"
    Rep := Record(sgn: Integer,      int: List Integer,
                  pfx: List Integer, cyc: List Integer)

    q:     RN
```

```
qr:    QuoRem
a,b:   %
n:     I

radixInt:    (I, I)    -> List I
radixFrac:   (I, I, I) -> Record(pfx: List I, cyc: List I)
checkRagits: List I    -> Boolean

-- Arithmetic operations
characteristic() == 0
differentiate a == 0

0     == [1, nil(),  nil(), nil()]
1     == [1, [1], nil(), nil()]
- a   == (a = 0 => 0; [-a.sgn, a.int, a.pfx, a.cyc])
a + b == (a::RN + b::RN)::%
a - b == (a::RN - b::RN)@RN::%
n * a == (n     * a::RN)::%
a * b == (a::RN * b::RN)::%
a / b == (a::RN / b::RN)::%
(i:I) / (j:I) == (i/j)@RN :: %
a < b == a::RN < b::RN
a = b == a.sgn = b.sgn and a.int = b.int and
         a.pfx = b.pfx and a.cyc = b.cyc
numer a == numer(a::RN)
denom a == denom(a::RN)

-- Algebraic coercions
coerce(a):RN == (wholePart a) :: RN + fractionPart a
coerce(n):%  == n :: RN :: %
coerce(q):%  ==
  s := 1; if q < 0 then (s := -1; q := -q)
  qr      := divide(numer q,denom q)
  whole   := radixInt (qr.quotient,bb)
  fractn  := radixFrac(qr.remainder,denom q,bb)
  cycle   := (fractn.cyc = [0] => nil(); fractn.cyc)
  [s,whole,fractn.pfx,cycle]

retractIfCan(a):Union(RN,"failed") == a::RN
retractIfCan(a):Union(I,"failed") ==
  empty?(a.pfx) and empty?(a.cyc) => wholePart a
  "failed"

-- Exported constructor/destructors
ceiling a == ceiling(a::RN)
floor a == floor(a::RN)

wholePart a ==
  n0 := 0
  for r in a.int repeat n0 := bb*n0 + r
```

```
    a.sgn*n0
  fractionPart a ==
    n0 := 0
    for r in a.pfx repeat n0 := bb*n0 + r
    null a.cyc =>
        a.sgn*n0/bb**((#a.pfx)::NNI)
    n1 := n0
    for r in a.cyc repeat n1 := bb*n1 + r
    n := n1 - n0
    d := (bb**((#a.cyc)::NNI) - 1) * bb**((#a.pfx)::NNI)
    a.sgn*n/d

  wholeRagits  a == a.int
  fractRagits  a == concat(construct(a.pfx)@ST,repeating a.cyc)
  prefixRagits a == a.pfx
  cycleRagits  a == a.cyc

  wholeRadix li ==
    checkRagits li
    [1, li, nil(), nil()]
  fractRadix(lpfx, lcyc) ==
    checkRagits lpfx; checkRagits lcyc
    [1, nil(), lpfx, lcyc]

  -- Output

  ALPHAS : String := "ABCDEFGHIJKLMNOPQRSTUVWXYZ"

  intToExpr(i:I): OUT ==
    -- computes a digit for bases between 11 and 36
    i < 10 => i :: OUT
    elt(ALPHAS,(i-10) + minIndex(ALPHAS)) :: OUT

  exprgroup(le: List OUT): OUT ==
    empty? le      => error "exprgroup needs non-null list"
    empty? rest le => first le
    abs bb <= 36 => hconcat le
    blankSeparate le

  intgroup(li: List I): OUT ==
    empty? li      => error "intgroup needs non-null list"
    empty? rest li => intToExpr first(li)
    abs bb <= 10 => hconcat [i :: OUT for i in li]
    abs bb <= 36 => hconcat [intToExpr(i) for i in li]
    blankSeparate [i :: OUT for i in li]

  overBar(li: List I): OUT == overbar intgroup li

  coerce(a): OUT ==
    le : List OUT := nil()
```

```
  if not null a.cyc then le := concat(overBar  a.cyc,le)
  if not null a.pfx then le := concat(intgroup a.pfx,le)
  if not null le    then le := concat("." :: OUT,le)
  if not null a.int then le := concat(intgroup a.int,le)
  else le := concat(0 :: OUT,le)
  rex := exprgroup le
  if a.sgn < 0 then -rex else rex

-- Construction utilities
checkRagits li ==
  for i in li repeat if i < 0 or i >= bb then
    error "Each ragit (digit) must be between 0 and base-1"
  true

radixInt(n,bas) ==
  rits: List I := nil()
  while abs n ^= 0 repeat
    qr   := divide(n,bas)
    n    := qr.quotient
    rits := concat(qr.remainder,rits)
  rits

radixFrac(num,den,bas) ==
  -- Rits is the sequence of quotient/remainder pairs
  -- in calculating the radix expansion of the rational number.
  -- We wish to find p and c such that
  --    rits.i are distinct    for 0<=i<=p+c-1
  --    rits.i = rits.(i+p)    for i>p
  -- I.e. p is the length of the non-periodic prefix and c is
  -- the length of the cycle.

  -- Compute p and c using Floyd's algorithm.
  -- 1. Find smallest n s.t. rits.n = rits.(2*n)
  qr    := divide(bas * num, den)
  i : I := 0
  qr1i  := qr2i := qr
  rits: List QuoRem := [qr]
  until qr1i = qr2i repeat
    qr1i := divide(bas * qr1i.remainder,den)
    qrt  := divide(bas * qr2i.remainder,den)
    qr2i := divide(bas * qrt.remainder,den)
    rits := concat(qr2i, concat(qrt, rits))
    i    := i + 1
  rits := reverse_! rits
  n    := i
  -- 2. Find p = first i such that rits.i = rits.(i+n)
  ritsi := rits
  ritsn := rits; for i in 1..n repeat ritsn := rest ritsn
  i := 0
  while first(ritsi) ^= first(ritsn) repeat
```

```
        ritsi := rest ritsi
        ritsn := rest ritsn
        i     := i + 1
      p := i
      -- 3. Find c = first i such that rits.p = rits.(p+i)
      ritsn := rits; for i in 1..n repeat ritsn := rest ritsn
      rn    := first ritsn
      cfound:= false
      c : I := 0
      for i in 1..p while not cfound repeat
        ritsn := rest ritsn
        if rn = first(ritsn) then
          c := i
          cfound := true
      if not cfound then c := n
      -- 4. Now produce the lists of ragits.
      ritspfx: List I := nil()
      ritscyc: List I := nil()
      for i in 1..p repeat
        ritspfx := concat(first(rits).quotient, ritspfx)
        rits    := rest rits
      for i in 1..c repeat
        ritscyc := concat(first(rits).quotient, ritscyc)
        rits    := rest rits
      [reverse_! ritspfx, reverse_! ritscyc]
```

— **RADIX.dotabb** —

```
"RADIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RADIX"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"RADIX" -> "PFECAT"
"RADIX" -> "STRING"
```

19.3 domain RECLOS RealClosure

The domain constructore **RealClosure** by Renaud Rioboo (University of Paris 6, France) provides the real closure of an ordered field. The implementation is based on interval arithmetic. Moreover, the design of this constructor and its related packages allows an easy use of other codings for real algebraic numbers. ordered field

The RealClosure domain is the end-user code, it provides usual arithmetics with real algebraic numbers, along with the functionalities of a real closed field. It also provides functions to approximate a real algebraic number by an element of the base field. This approximation may either be absolute (approximate) or relative (realtivApprox).

CAVEATS

Since real algebraic expressions are stored as depending on "real roots" which are managed like variables, there is an ordering on these. This ordering is dynamical in the sense that any new algebraic takes precedence over older ones. In particular every cretaion function raises a new "real root". This has the effect that when you type something like sqrt(2) + sqrt(2) you have two new variables which happen to be equal. To avoid this name the expression such as in s2 := sqrt(2) ; s2 + s2

Also note that computing times depend strongly on the ordering you implicitly provide. Please provide algebraics in the order which most natural to you.

LIMITATIONS

The file reclos.input show some basic use of the package. This packages uses algorithms which are published in [1] and [2] which are based on field arithmetics, inparticular for polynomial gcd related algorithms. This can be quite slow for high degree polynomials and subresultants methods usually work best. Betas versions of the package try to use these techniques in a better way and work significantly faster. These are mostly based on unpublished algorithms and cannot be distributed. Please contact the author if you have a particular problem to solve or want to use these versions.

Be aware that approximations behave as post-processing and that all computations are done excatly. They can thus be quite time consuming when depending on several "real roots".

— RealClosure.input —

```
)set break resume
)sys rm -f RealClosure.output
)spool RealClosure.output
)set message test on
)set message auto off
)clear all
--S 1 of 67
Ran := RECLOS(FRAC INT)
--R
--R
--R   (1)  RealClosure Fraction Integer
--R                                                                  Type: Domain
--E 1

--S 2 of 67
fourSquares(a:Ran,b:Ran,c:Ran,d:Ran):Ran==sqrt(a)+sqrt(b)-sqrt(c)-sqrt(d)
--R
--R   Function declaration fourSquares : (RealClosure Fraction Integer,
--R      RealClosure Fraction Integer,RealClosure Fraction Integer,
```

```
--R      RealClosure Fraction Integer) -> RealClosure Fraction Integer has
--R      been added to workspace.
--R                                                               Type: Void
--E 2

--S 3 of 67
squareDiff1 := fourSquares(73,548,60,586)
--R
--R   Compiling function fourSquares with type (RealClosure Fraction
--R      Integer,RealClosure Fraction Integer,RealClosure Fraction Integer
--R      ,RealClosure Fraction Integer) -> RealClosure Fraction Integer
--R
--R           +---+    +--+    +---+    +--+
--R   (3)  - \|586  - \|60  + \|548  + \|73
--R                                         Type: RealClosure Fraction Integer
--E 3

--S 4 of 67
recip(squareDiff1)
--R
--R
--R   (4)
--R             +---+          +--+  +--+         +--+ +---+            +---+
--R     ((54602\|548  + 149602\|73 )\|60  + 49502\|73 \|548  + 9900895)\|586
--R   +
--R             +--+ +---+             +--+            +---+            +--+
--R     (154702\|73 \|548  + 30941947)\|60  + 10238421\|548  + 28051871\|73
--R                                  Type: Union(RealClosure Fraction Integer,...)
--E 4

--S 5 of 67
sign(squareDiff1)
--R
--R
--R   (5)  1
--R                                                        Type: PositiveInteger
--E 5

--S 6 of 67
squareDiff2 := fourSquares(165,778,86,990)
--R
--R
--R           +---+    +--+    +---+    +---+
--R   (6)  - \|990  - \|86  + \|778  + \|165
--R                                         Type: RealClosure Fraction Integer
--E 6

--S 7 of 67
recip(squareDiff2)
--R
```

```
--R
--R   (7)
--R                +---+           +---+  +--+          +---+ +---+
--R       ((556778\|778  + 1209010\|165 )\|86  + 401966\|165 \|778  + 144019431)
--R    *
--R        +---+
--R       \|990
--R   +
--R              +---+ +---+              +--+            +---+             +---+
--R     (1363822\|165 \|778  + 488640503)\|86  + 162460913\|778  + 352774119\|165
--R                               Type: Union(RealClosure Fraction Integer,...)
--E 7

--S 8 of 67
sign(squareDiff2)
--R
--R
--R   (8)  1
--R                                                        Type: PositiveInteger
--E 8

--S 9 of 67
squareDiff3 := fourSquares(217,708,226,692)
--R
--R
--R            +---+    +---+    +---+    +---+
--R   (9)  - \|692  - \|226  + \|708  + \|217
--R                                          Type: RealClosure Fraction Integer
--E 9

--S 10 of 67
recip(squareDiff3)
--R
--R
--R   (10)
--R               +---+         +---+  +---+         +---+ +---+             +---+
--R     ((- 34102\|708  - 61598\|217 )\|226  - 34802\|217 \|708  - 13641141)\|692
--R   +
--R              +---+ +---+             +---+            +---+            +---+
--R     (- 60898\|217 \|708  - 23869841)\|226  - 13486123\|708  - 24359809\|217
--R                               Type: Union(RealClosure Fraction Integer,...)
--E 10

--S 11 of 67
sign(squareDiff3)
--R
--R
--R   (11)  - 1
--R                                                                Type: Integer
--E 11
```

```
--S 12 of 67
squareDiff4 := fourSquares(155,836,162,820)
--R
--R
--R           +---+    +---+    +---+    +---+
--R   (12)  - \|820  - \|162  + \|836  + \|155
--R                                         Type: RealClosure Fraction Integer
--E 12

--S 13 of 67
recip(squareDiff4)
--R
--R
--R   (13)
--R              +---+         +---+  +---+         +---+ +---+             +---+
--R     ((- 37078\|836  - 86110\|155 )\|162  - 37906\|155 \|836  - 13645107)\|820
--R   +
--R             +---+ +---+             +---+            +---+            +---+
--R     (- 85282\|155 \|836  - 30699151)\|162  - 13513901\|836  - 31384703\|155
--R                              Type: Union(RealClosure Fraction Integer,...)
--E 13

--S 14 of 67
sign(squareDiff4)
--R
--R
--R   (14)  - 1
--R                                                            Type: Integer
--E 14

--S 15 of 67
squareDiff5 := fourSquares(591,772,552,818)
--R
--R
--R           +---+    +---+    +---+    +---+
--R   (15)  - \|818  - \|552  + \|772  + \|591
--R                                         Type: RealClosure Fraction Integer
--E 15

--S 16 of 67
recip(squareDiff5)
--R
--R
--R   (16)
--R            +---+         +---+  +---+         +---+ +---+             +---+
--R     ((70922\|772  + 81058\|591 )\|552  + 68542\|591 \|772  + 46297673)\|818
--R   +
--R           +---+ +---+             +---+            +---+            +---+
--R     (83438\|591 \|772  + 56359389)\|552  + 47657051\|772  + 54468081\|591
```

```
--R                                Type: Union(RealClosure Fraction Integer,...)
--E 16

--S 17 of 67
sign(squareDiff5)
--R
--R
--R   (17)  1
--R                                                        Type: PositiveInteger
--E 17

--S 18 of 67
squareDiff6 := fourSquares(434,1053,412,1088)
--R
--R
--R            +----+    +---+    +----+    +---+
--R   (18)  - \|1088  - \|412  + \|1053  + \|434
--R                                           Type: RealClosure Fraction Integer
--E 18

--S 19 of 67
recip(squareDiff6)
--R
--R
--R   (19)
--R                +----+          +---+  +---+          +---+ +----+
--R       ((115442\|1053  + 179818\|434 )\|412  + 112478\|434 \|1053  + 76037291)
--R    *
--R        +----+
--R       \|1088
--R   +
--R              +---+ +----+              +---+            +----+             +---+
--R   (182782\|434 \|1053  + 123564147)\|412  + 77290639\|1053  + 120391609\|434
--R                                Type: Union(RealClosure Fraction Integer,...)
--E 19

--S 20 of 67
sign(squareDiff6)
--R
--R
--R   (20)  1
--R                                                        Type: PositiveInteger
--E 20

--S 21 of 67
squareDiff7 := fourSquares(514,1049,446,1152)
--R
--R
--R            +----+    +---+    +----+    +---+
--R   (21)  - \|1152  - \|446  + \|1049  + \|514
```

```
--R                                         Type: RealClosure Fraction Integer
--E 21

--S 22 of 67
recip(squareDiff7)
--R
--R
--R   (22)
--R               +-----+          +----+  +----+          +----+ +-----+
--R       ((349522\|1049  + 499322\|514 )\|446  + 325582\|514 \|1049  + 239072537)
--R    *
--R        +-----+
--R       \|1152
--R   +
--R          +----+ +-----+             +----+            +-----+            +----+
--R   (523262\|514 \|1049  + 384227549)\|446  + 250534873\|1049  + 357910443\|514
--R                              Type: Union(RealClosure Fraction Integer,...)
--E 22

--S 23 of 67
sign(squareDiff7)
--R
--R
--R   (23)  1
--R                                                      Type: PositiveInteger
--E 23

--S 24 of 67
squareDiff8 := fourSquares(190,1751,208,1698)
--R
--R
--R           +-----+    +----+    +-----+    +----+
--R   (24)  - \|1698  - \|208  + \|1751  + \|190
--R                                         Type: RealClosure Fraction Integer
--E 24

--S 25 of 67
recip(squareDiff8)
--R
--R
--R   (25)
--R                    +-----+          +----+  +----+          +----+ +-----+
--R          (- 214702\|1751  - 651782\|190 )\|208  - 224642\|190 \|1751
--R        +
--R          - 129571901
--R    *
--R        +-----+
--R       \|1698
--R   +
--R              +----+ +-----+             +----+            +-----+
```

```
--R     (- 641842\|190 \|1751  - 370209881)\|208  - 127595865\|1751
--R   +
--R                   +---+
--R     - 387349387\|190
--R                                   Type: Union(RealClosure Fraction Integer,...)
--E 25

--S 26 of 67
sign(squareDiff8)
--R
--R
--R   (26)  - 1
--R                                                                 Type: Integer
--E 26

--S 27 of 67
relativeApprox(squareDiff8,10**(-3))::Float
--R
--R
--R   (27)  - 0.2340527771 5937700123 E -10
--R                                                                   Type: Float
--E 27

--S 28 of 67
l := allRootsOf((x**2-2)**2-2)$Ran
--R
--R
--R   (28)  [%A33,%A34,%A35,%A36]
--R                                         Type: List RealClosure Fraction Integer
--E 28

--S 29 of 67
removeDuplicates map(mainDefiningPolynomial,l)
--R
--R
--R           4     2
--R   (29)  [?  - 4?  + 2]
--RType: List Union(SparseUnivariatePolynomial RealClosure Fraction Integer,"failed")
--E 29

--S 30 of 67
map(mainCharacterization,l)
--R
--R
--R   (30)  [[- 2,- 1[,[- 1,0[,[0,1[,[1,2[]
--RType: List Union(RightOpenIntervalRootCharacterization(RealClosure Fraction Integer,Sparse
--E 30

--S 31 of 67
[reduce(+,l),reduce(*,l)-2]
```

```
--R
--R
--R   (31)  [0,0]
--R                                     Type: List RealClosure Fraction Integer
--E 31

--S 32 of 67
(s2, s5, s10) := (sqrt(2)$Ran, sqrt(5)$Ran, sqrt(10)$Ran)
--R
--R
--R          +--+
--R   (32)  \|10
--R                                          Type: RealClosure Fraction Integer
--E 32

--S 33 of 67
eq1:=sqrt(s10+3)*sqrt(s5+2) - sqrt(s10-3)*sqrt(s5-2) = sqrt(10*s2+10)
--R
--R
--R            +---------+ +---------+    +---------+ +---------+   +-----------+
--R            | +--+      | +-+          | +--+      | +-+         |   +-+
--R   (33)  - \|\|10  - 3 \|\|5  - 2  + \|\|10  + 3 \|\|5  + 2 = \|10\|2  + 10
--R                                 Type: Equation RealClosure Fraction Integer
--E 33

--S 34 of 67
eq1::Boolean
--R
--R
--R   (34)  true
--R                                                                Type: Boolean
--E 34

--S 35 of 67
eq2:=sqrt(s5+2)*sqrt(s2+1) - sqrt(s5-2)*sqrt(s2-1) = sqrt(2*s10+2)
--R
--R
--R            +--------+ +--------+    +--------+ +--------+   +----------+
--R            | +-+      | +-+         | +-+      | +-+        |  +--+
--R   (35)  - \|\|5  - 2 \|\|2  - 1  + \|\|5  + 2 \|\|2  + 1 = \|2\|10  + 2
--R                                 Type: Equation RealClosure Fraction Integer
--E 35

--S 36 of 67
eq2::Boolean
--R
--R
--R   (36)  true
--R                                                                Type: Boolean
--E 36
```

```
--S 37 of 67
s3 := sqrt(3)$Ran
--R
--R
--R          +-+
--R   (37)  \|3
--R                                          Type: RealClosure Fraction Integer
--E 37

--S 38 of 67
s7:= sqrt(7)$Ran
--R
--R
--R          +-+
--R   (38)  \|7
--R                                          Type: RealClosure Fraction Integer
--E 38

--S 39 of 67
e1 := sqrt(2*s7-3*s3,3)
--R
--R
--R          +-------------+
--R         3|  +-+     +-+
--R   (39)  \|2\|7  - 3\|3
--R                                          Type: RealClosure Fraction Integer
--E 39

--S 40 of 67
e2 := sqrt(2*s7+3*s3,3)
--R
--R
--R          +-------------+
--R         3|  +-+     +-+
--R   (40)  \|2\|7  + 3\|3
--R                                          Type: RealClosure Fraction Integer
--E 40

--S 41 of 67
e2-e1-s3
--R
--R
--R   (41)  0
--R                                          Type: RealClosure Fraction Integer
--E 41

--S 42 of 67
pol : UP(x,Ran) := x**4+(7/3)*x**2+30*x-(100/3)
--R
```

```
--R
--R          4   7  2         100
--R   (42)  x  + - x  + 30x - ---
--R              3             3
--R                    Type: UnivariatePolynomial(x,RealClosure Fraction Integer)
--E 42

--S 43 of 67
r1 := sqrt(7633)$Ran
--R
--R
--R          +----+
--R   (43)  \|7633
--R                                        Type: RealClosure Fraction Integer
--E 43

--S 44 of 67
alpha := sqrt(5*r1-436,3)/3
--R
--R
--R            +--------------+
--R         1 3|  +----+
--R   (44)  - \|5\|7633  - 436
--R         3
--R                                        Type: RealClosure Fraction Integer
--E 44

--S 45 of 67
beta := -sqrt(5*r1+436,3)/3
--R
--R
--R              +--------------+
--R           1 3|  +----+
--R   (45)  - - \|5\|7633  + 436
--R           3
--R                                        Type: RealClosure Fraction Integer
--E 45

--S 46 of 67
pol.(alpha+beta-1/3)
--R
--R
--R   (46)  0
--R                                        Type: RealClosure Fraction Integer
--E 46

--S 47 of 67
qol : UP(x,Ran) := x**5+10*x**3+20*x+22
--R
--R
```

```
--R          5      3
--R   (47)  x  + 10x  + 20x + 22
--R                  Type: UnivariatePolynomial(x,RealClosure Fraction Integer)
--E 47

--S 48 of 67
r2 := sqrt(153)$Ran
--R
--R
--R          +---+
--R   (48)  \|153
--R                                        Type: RealClosure Fraction Integer
--E 48

--S 49 of 67
alpha2 := sqrt(r2-11,5)
--R
--R
--R          +-----------+
--R         5| +---+
--R   (49)  \|\|153  - 11
--R                                        Type: RealClosure Fraction Integer
--E 49

--S 50 of 67
beta2 := -sqrt(r2+11,5)
--R
--R
--R            +-----------+
--R           5| +---+
--R   (50)  - \|\|153  + 11
--R                                        Type: RealClosure Fraction Integer
--E 50

--S 51 of 67
qol(alpha2+beta2)
--R
--R
--R   (51)  0
--R                                        Type: RealClosure Fraction Integer
--E 51

--S 52 of 67
dst1:=sqrt(9+4*s2)=1+2*s2
--R
--R
--R          +---------+
--R          |  +-+         +-+
--R   (52)  \|4\|2  + 9 = 2\|2  + 1
--R                               Type: Equation RealClosure Fraction Integer
```

```
--E 52

--S 53 of 67
dst1::Boolean
--R
--R
--R   (53)  true
--R                                                                Type: Boolean
--E 53

--S 54 of 67
s6:Ran:=sqrt 6
--R
--R
--R          +-+
--R   (54)  \|6
--R                                         Type: RealClosure Fraction Integer
--E 54

--S 55 of 67
dst2:=sqrt(5+2*s6)+sqrt(5-2*s6) = 2*s3
--R
--R
--R          +-----------+    +---------+
--R          |    +-+         |  +-+         +-+
--R   (55)  \|- 2\|6  + 5  + \|2\|6  + 5 = 2\|3
--R                                Type: Equation RealClosure Fraction Integer
--E 55

--S 56 of 67
dst2::Boolean
--R
--R
--R   (56)  true
--R                                                                Type: Boolean
--E 56

--S 57 of 67
s29:Ran:=sqrt 29
--R
--R
--R          +--+
--R   (57)  \|29
--R                                         Type: RealClosure Fraction Integer
--E 57

--S 58 of 67
dst4:=sqrt(16-2*s29+2*sqrt(55-10*s29)) = sqrt(22+2*s5)-sqrt(11+2*s29)+s5
--R
--R
```

```
--R   (58)
--R    +--------------------------------+
--R    |  +---------------+                   +------------+    +-----------+
--R    |  |     +--+           +--+           |  +--+           |  +-+           +-+
--R   \|2\|- 10\|29  + 55  - 2\|29  + 16 = - \|2\|29  + 11  + \|2\|5  + 22  + \|5
--R                                Type: Equation RealClosure Fraction Integer
--E 58

--S 59 of 67
dst4::Boolean
--R
--R
--R   (59)  true
--R                                                              Type: Boolean
--E 59

--S 60 of 67
dst6:=sqrt((112+70*s2)+(46+34*s2)*s5) = (5+4*s2)+(3+s2)*s5
--R
--R
--R         +--------------------------------+
--R         |    +-+       +-+      +-+           +-+      +-+     +-+
--R   (60)  \|(34\|2  + 46)\|5  + 70\|2  + 112 = (\|2  + 3)\|5  + 4\|2  + 5
--R                                Type: Equation RealClosure Fraction Integer
--E 60

--S 61 of 67
dst6::Boolean
--R
--R
--R   (61)  true
--R                                                              Type: Boolean
--E 61

--S 62 of 67
f3:Ran:=sqrt(3,5)
--R
--R
--R        5+-+
--R   (62)  \|3
--R                                         Type: RealClosure Fraction Integer
--E 62

--S 63 of 67
f25:Ran:=sqrt(1/25,5)
--R
--R
--R         +--+
--R         | 1
--R   (63)  5|--
```

```
--R         \|25
--R                                          Type: RealClosure Fraction Integer
--E 63

--S 64 of 67
f32:Ran:=sqrt(32/5,5)
--R
--R
--R          +--+
--R          |32
--R   (64)  5|--
--R         \| 5
--R                                          Type: RealClosure Fraction Integer
--E 64

--S 65 of 67
f27:Ran:=sqrt(27/5,5)
--R
--R
--R          +--+
--R          |27
--R   (65)  5|--
--R         \| 5
--R                                          Type: RealClosure Fraction Integer
--E 65

--S 66 of 67
dst5:=sqrt((f32-f27,3)) = f25*(1+f3-f3**2)
--R
--R
--R          +----------------+
--R          |   +--+    +--+                        +--+
--R          |   |27     |32       5+-+2   5+-+       | 1
--R   (66)  3|- 5|--  + 5|--  = (- \|3   + \|3  + 1) 5|--
--R         \|  \| 5    \| 5                         \|25
--R                                 Type: Equation RealClosure Fraction Integer
--E 66

--S 67 of 67
dst5::Boolean
--R
--R
--R   (67)  true
--R                                                               Type: Boolean
--E 67
)spool
)lisp (bye)
```

———-

— RealClosure.help —

```
====================================================================
RealClosure examples
====================================================================

The Real Closure 1.0 package provided by Renaud Rioboo consists of
different packages, categories and domains :

The package RealPolynomialUtilitiesPackage which needs a Field F and a
UnivariatePolynomialCategory domain with coefficients in F. It computes
some simple functions such as Sturm and Sylvester sequences sturmSequence,
sylvesterSequence.

The category RealRootCharacterizationCategory provides abstract
functions to work with "real roots" of univariate polynomials. These
resemble variables with some functionality needed to compute important
operations.

The category RealClosedField provides common operations available over
real closed fiels. These include finding all the roots of a univariate
polynomial, taking square (and higher) roots, ...

The domain RightOpenIntervalRootCharacterization is the main code that
provides the functionality of RealRootCharacterizationCategory for the
case of archimedean fields. Abstract roots are encoded with a left
closed right open interval containing the root together with a
defining polynomial for the root.

The RealClosure domain is the end-user code. It provides usual arithmetic
with real algebraic numbers, along with the functionality of a real closed
field. It also provides functions to approximate a real algebraic number
by an element of the base field. This approximation may either be absolute,
approximate or relative (relativeApprox).

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CAVEATS
====================================================================

Since real algebraic expressions are stored as depending on "real
roots" which are managed like variables, there is an ordering on
these. This ordering is dynamical in the sense that any new algebraic
takes precedence over older ones. In particular every creation
function raises a new "real root". This has the effect that when you
type something like sqrt(2) + sqrt(2) you have two new variables
which happen to be equal. To avoid this name the expression such as in
s2 := sqrt(2) ; s2 + s2

Also note that computing times depend strongly on the ordering you
```

```
implicitly provide. Please provide algebraics in the order which seems
most natural to you.

====================================================================
LIMITATIONS
====================================================================

This packages uses algorithms which are published in [1] and [2] which
are based on field arithmetics, in particular for polynomial gcd
related algorithms. This can be quite slow for high degree polynomials
and subresultants methods usually work best. Beta versions of the
package try to use these techniques in a better way and work
significantly faster. These are mostly based on unpublished algorithms
and cannot be distributed. Please contact the author if you have a
particular problem to solve or want to use these versions.

Be aware that approximations behave as post-processing and that all
computations are done exactly. They can thus be quite time consuming when
depending on several ``real roots''.

====================================================================
REFERENCES
====================================================================


[1]  R. Rioboo : Real Algebraic Closure of an ordered Field : Implementation
     in Axiom.
     In proceedings of the ISSAC'92 Conference, Berkeley 1992 pp. 206-215.

[2]  Z. Ligatsikas, R. Rioboo, M. F. Roy : Generic computation of the real
     closure of an ordered field.
     In Mathematics and Computers in Simulation Volume 42, Issue 4-6,
     November 1996.

====================================================================
EXAMPLES
====================================================================

We shall work with the real closure of the ordered field of
rational numbers.

  Ran := RECLOS(FRAC INT)
    RealClosure Fraction Integer
                                 Type: Domain

Some simple signs for square roots, these correspond to an extension
of degree 16 of the rational numbers. Examples provided by J. Abbot.

  fourSquares(a:Ran,b:Ran,c:Ran,d:Ran):Ran==sqrt(a)+sqrt(b)-sqrt(c)-sqrt(d)
                                 Type: Void
```

```
These produce values very close to zero.

  squareDiff1 := fourSquares(73,548,60,586)
       +---+    +--+    +---+    +--+
   - \|586  - \|60  + \|548  + \|73
                              Type: RealClosure Fraction Integer

  recip(squareDiff1)
            +---+          +--+  +--+         +--+ +---+            +---+
     ((54602\|548  + 149602\|73 )\|60  + 49502\|73 \|548  + 9900895)\|586
   +
            +--+ +---+             +--+            +---+            +--+
     (154702\|73 \|548  + 30941947)\|60  + 10238421\|548  + 28051871\|73
                              Type: Union(RealClosure Fraction Integer,...)

  sign(squareDiff1)
    1
                              Type: PositiveInteger

  squareDiff2 := fourSquares(165,778,86,990)
       +---+    +--+    +---+    +---+
   - \|990  - \|86  + \|778  + \|165
                              Type: RealClosure Fraction Integer

  recip(squareDiff2)
               +---+           +---+  +--+          +---+ +---+
       ((556778\|778  + 1209010\|165 )\|86  + 401966\|165 \|778  + 144019431)
    *
        +---+
       \|990
   +
             +---+ +---+              +--+             +---+             +---+
     (1363822\|165 \|778  + 488640503)\|86  + 162460913\|778  + 352774119\|165
                              Type: Union(RealClosure Fraction Integer,...)

  sign(squareDiff2)
    1
                              Type: PositiveInteger

  squareDiff3 := fourSquares(217,708,226,692)
       +---+    +---+    +---+    +---+
   - \|692  - \|226  + \|708  + \|217
                              Type: RealClosure Fraction Integer

  recip(squareDiff3)
              +---+         +---+  +---+         +---+ +---+             +---+
     ((- 34102\|708  - 61598\|217 )\|226  - 34802\|217 \|708  - 13641141)\|692
   +
             +---+ +---+             +---+           +---+            +---+
```

```
   (- 60898\|217 \|708  - 23869841)\|226  - 13486123\|708  - 24359809\|217
                            Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff3)
  - 1
                            Type: Integer

squareDiff4 := fourSquares(155,836,162,820)
     +---+    +---+    +---+    +---+
  - \|820  - \|162  + \|836  + \|155
                            Type: RealClosure Fraction Integer

recip(squareDiff4)
             +---+         +---+  +---+         +---+ +---+             +---+
   ((- 37078\|836  - 86110\|155 )\|162  - 37906\|155 \|836  - 13645107)\|820
 +
            +---+ +---+            +---+            +---+            +---+
   (- 85282\|155 \|836  - 30699151)\|162  - 13513901\|836  - 31384703\|155
                            Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff4)
  - 1
                            Type: Integer

squareDiff5 := fourSquares(591,772,552,818)
     +---+    +---+    +---+    +---+
  - \|818  - \|552  + \|772  + \|591
                            Type: RealClosure Fraction Integer

recip(squareDiff5)
           +---+         +---+  +---+         +---+ +---+             +---+
   ((70922\|772  + 81058\|591 )\|552  + 68542\|591 \|772  + 46297673)\|818
 +
          +---+ +---+            +---+            +---+            +---+
   (83438\|591 \|772  + 56359389)\|552  + 47657051\|772  + 54468081\|591
                            Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff5)
  1
                            Type: PositiveInteger

squareDiff6 := fourSquares(434,1053,412,1088)
     +----+    +---+    +----+    +---+
  - \|1088  - \|412  + \|1053  + \|434
                            Type: RealClosure Fraction Integer

recip(squareDiff6)
              +----+          +---+  +---+          +---+ +----+
     ((115442\|1053  + 179818\|434 )\|412  + 112478\|434 \|1053  + 76037291)
  *
```

```
      +----+
     \|1088
 +
        +---+ +----+              +---+            +----+             +---+
 (182782\|434 \|1053  + 123564147)\|412  + 77290639\|1053  + 120391609\|434
                           Type: Union(RealClosure Fraction Integer,...)
```

```
sign(squareDiff6)
  1
                           Type: PositiveInteger
```

```
squareDiff7 := fourSquares(514,1049,446,1152)
     +----+    +---+    +----+    +---+
  - \|1152  - \|446  + \|1049  + \|514
                           Type: RealClosure Fraction Integer
```

```
recip(squareDiff7)
             +----+          +---+  +---+          +---+ +----+
     ((349522\|1049  + 499322\|514 )\|446  + 325582\|514 \|1049  + 239072537)
  *
      +----+
     \|1152
 +
        +---+ +----+              +---+             +----+             +---+
 (523262\|514 \|1049  + 384227549)\|446  + 250534873\|1049  + 357910443\|514
                           Type: Union(RealClosure Fraction Integer,...)
```

```
sign(squareDiff7)
  1
                           Type: PositiveInteger
```

```
squareDiff8 := fourSquares(190,1751,208,1698)
     +----+    +---+    +----+    +---+
  - \|1698  - \|208  + \|1751  + \|190
                           Type: RealClosure Fraction Integer
```

```
recip(squareDiff8)
                  +----+          +---+  +---+          +---+ +----+
         (- 214702\|1751  - 651782\|190 )\|208  - 224642\|190 \|1751
       +
         - 129571901
  *
      +----+
     \|1698
 +
            +---+ +----+              +---+             +----+
   (- 641842\|190 \|1751  - 370209881)\|208  - 127595865\|1751
 +
              +---+
   - 387349387\|190
```

```
                           Type: Union(RealClosure Fraction Integer,...)

  sign(squareDiff8)
    - 1
                           Type: Integer
```

This should give three digits of precision

```
  relativeApprox(squareDiff8,10**(-3))::Float
    - 0.2340527771 5937700123 E -10
                           Type: Float
```

The sum of these 4 roots is 0

```
  l := allRootsOf((x**2-2)**2-2)$Ran
    [%A33,%A34,%A35,%A36]
                           Type: List RealClosure Fraction Integer
```

Check that they are all roots of the same polynomial

```
  removeDuplicates map(mainDefiningPolynomial,l)
      4     2
    [?  - 4?  + 2]
       Type: List Union(
                  SparseUnivariatePolynomial RealClosure Fraction Integer,
                  "failed")
```

We can see at a glance that they are separate roots

```
  map(mainCharacterization,l)
    [[- 2,- 1[,[- 1,0[,[0,1[,[1,2[]
       Type: List Union(
             RightOpenIntervalRootCharacterization(
              RealClosure Fraction Integer,
              SparseUnivariatePolynomial RealClosure Fraction Integer),
             "failed")
```

Check the sum and product

```
  [reduce(+,l),reduce(*,l)-2]
    [0,0]
                   Type: List RealClosure Fraction Integer
```

A more complicated test that involve an extension of degree 256.
This is a way of checking nested radical identities.

```
  (s2, s5, s10) := (sqrt(2)$Ran, sqrt(5)$Ran, sqrt(10)$Ran)
     +--+
    \|10
                   Type: RealClosure Fraction Integer
```

```
eq1:=sqrt(s10+3)*sqrt(s5+2) - sqrt(s10-3)*sqrt(s5-2) = sqrt(10*s2+10)
     +---------+ +---------+    +---------+ +---------+   +-----------+
     | +--+      | +-+          | +--+      | +-+         |    +-+
  - \|\|10  - 3 \|\|5  - 2  + \|\|10  + 3 \|\|5  + 2 = \|10\|2  + 10
                  Type: Equation RealClosure Fraction Integer

eq1::Boolean
  true
                  Type: Boolean

eq2:=sqrt(s5+2)*sqrt(s2+1) - sqrt(s5-2)*sqrt(s2-1) = sqrt(2*s10+2)
     +--------+ +---------+    +--------+ +---------+   +----------+
     | +-+      | +-+          | +-+      | +-+         |  +--+
  - \|\|5  - 2 \|\|2  - 1  + \|\|5  + 2 \|\|2  + 1 = \|2\|10  + 2
                  Type: Equation RealClosure Fraction Integer

eq2::Boolean
  true
                  Type: Boolean
```

Some more examples from J. M. Arnaudies

```
s3 := sqrt(3)$Ran
   +-+
  \|3
                  Type: RealClosure Fraction Integer

s7:= sqrt(7)$Ran
   +-+
  \|7
                  Type: RealClosure Fraction Integer

e1 := sqrt(2*s7-3*s3,3)
   +-------------+
  3|  +-+     +-+
  \|2\|7  - 3\|3
                  Type: RealClosure Fraction Integer

e2 := sqrt(2*s7+3*s3,3)
   +-------------+
  3|  +-+     +-+
  \|2\|7  + 3\|3
                  Type: RealClosure Fraction Integer
```

This should be null

```
e2-e1-s3
  0
                  Type: RealClosure Fraction Integer
```

```
A quartic polynomial

  pol : UP(x,Ran) := x**4+(7/3)*x**2+30*x-(100/3)
     4   7  2         100
    x  + - x  + 30x - ---
         3             3
                  Type: UnivariatePolynomial(x,RealClosure Fraction Integer)

Add some cubic roots

  r1 := sqrt(7633)$Ran
     +----+
    \|7633
                        Type: RealClosure Fraction Integer

  alpha := sqrt(5*r1-436,3)/3
       +--------------+
    1 3|  +----+
    - \|5\|7633  - 436
    3
                        Type: RealClosure Fraction Integer

  beta := -sqrt(5*r1+436,3)/3
         +--------------+
      1 3|  +----+
    - - \|5\|7633  + 436
      3
                        Type: RealClosure Fraction Integer

this should be null

  pol.(alpha+beta-1/3)
    0
                        Type: RealClosure Fraction Integer

A quintic polynomial

  qol : UP(x,Ran) := x**5+10*x**3+20*x+22
     5      3
    x  + 10x  + 20x + 22
              Type: UnivariatePolynomial(x,RealClosure Fraction Integer)

Add some cubic roots

  r2 := sqrt(153)$Ran
     +---+
    \|153
                          Type: RealClosure Fraction Integer
```

```
alpha2 := sqrt(r2-11,5)
   +-----------+
  5| +---+
  \|\|153  - 11
                         Type: RealClosure Fraction Integer

beta2 := -sqrt(r2+11,5)
     +-----------+
    5| +---+
  - \|\|153  + 11
                         Type: RealClosure Fraction Integer
```

this should be null

```
qol(alpha2+beta2)
  0
                         Type: RealClosure Fraction Integer
```

Finally, some examples from the book Computer Algebra by Davenport, Siret and Tournier (page 77). The last one is due to Ramanujan.

```
dst1:=sqrt(9+4*s2)=1+2*s2
   +---------+
   |  +-+         +-+
  \|4\|2  + 9 = 2\|2  + 1
                         Type: Equation RealClosure Fraction Integer

dst1::Boolean
  true
                         Type: Boolean

s6:Ran:=sqrt 6
   +-+
  \|6
                         Type: RealClosure Fraction Integer

dst2:=sqrt(5+2*s6)+sqrt(5-2*s6) = 2*s3
   +-----------+    +---------+
   |    +-+         |  +-+         +-+
  \|- 2\|6  + 5  + \|2\|6  + 5 = 2\|3
                         Type: Equation RealClosure Fraction Integer

dst2::Boolean
  true
                         Type: Boolean

s29:Ran:=sqrt 29
   +--+
  \|29
                         Type: RealClosure Fraction Integer
```

```
dst4:=sqrt(16-2*s29+2*sqrt(55-10*s29)) = sqrt(22+2*s5)-sqrt(11+2*s29)+s5
  +--------------------------------+
  |  +--------------+                    +-----------+    +----------+
  |  |     +--+           +--+           |  +--+          |  +-+           +-+
 \|2\|- 10\|29  + 55  - 2\|29  + 16 = - \|2\|29  + 11  + \|2\|5  + 22  + \|5
                        Type: Equation RealClosure Fraction Integer

dst4::Boolean
  true
                        Type: Boolean

dst6:=sqrt((112+70*s2)+(46+34*s2)*s5) = (5+4*s2)+(3+s2)*s5
   +--------------------------------+
   |    +-+       +-+      +-+           +-+      +-+     +-+
  \|(34\|2  + 46)\|5  + 70\|2  + 112 = (\|2  + 3)\|5  + 4\|2  + 5
                        Type: Equation RealClosure Fraction Integer

dst6::Boolean
  true
                        Type: Boolean

f3:Ran:=sqrt(3,5)
  5+-+
  \|3
                        Type: RealClosure Fraction Integer

f25:Ran:=sqrt(1/25,5)
   +--+
   | 1
  5|--
  \|25
                        Type: RealClosure Fraction Integer

f32:Ran:=sqrt(32/5,5)
   +--+
   |32
  5|--
  \| 5
                        Type: RealClosure Fraction Integer

f27:Ran:=sqrt(27/5,5)
   +--+
   |27
  5|--
  \| 5
                        Type: RealClosure Fraction Integer

dst5:=sqrt((f32-f27,3)) = f25*(1+f3-f3**2)
   +---------------+
```

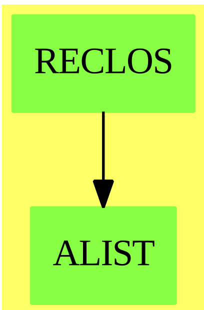
```
     |   +--+    +--+                         +--+
     |   |27     |32       5+-+2   5+-+       | 1
    3|- 5|--  + 5|--  = (- \|3   + \|3  + 1) 5|--
    \|  \| 5   \| 5                          \|25
                          Type: Equation RealClosure Fraction Integer

  dst5::Boolean
    true
                          Type: Boolean

See Also:
o )help RightOpenIntervalRootCharacterization
o )help RealClosedField
o )help RealRootCharacterizationCategory
o )help UnivariatePolynomialCategory
o )help Field
o )help RealPolynomialUtilitiesPackage
o )show RealClosure
```

———-

19.3.1 RealClosure (RECLOS)



See

⇒ "RightOpenIntervalRootCharacterization" (ROIRC) 19.11.1 on page 2270

Exports:

0	1	abs	algebraicOf
allRootsOf	approximate	associates?	characteristic
coerce	divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor	gcd
gcdPolynomial	hash	inv	latex
lcm	mainCharacterization	mainDefiningPolynomial	mainForm
mainValue	max	multiEuclidean	min
negative?	nthRoot	one?	positive?
prime?	principalIdeal	recip	relativeApprox
rename	rename!	retract	retractIfCan
rootOf	sample	sign	sizeLess?
sqrt	squareFree	squareFreePart	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?*?	?**?	?+?	?-?
-?	?/?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

— domain RECLOS RealClosure —

```
)abbrev domain RECLOS RealClosure
++ Author: Renaud Rioboo
++ Date Created: summer 1988
++ Date Last Updated: January 2004
++ Basic Functions: provides computations in an ordered real closure
++ Related Constructors: RightOpenIntervalRootCharacterization
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ This domain implements the real closure of an ordered field.
++ Note:
++ The code here is generic i.e. it does not depend of the way the operations
++ are done. The two macros PME and SEG should be passed as functorial
++ arguments to the domain. It does not help much to write a category
++ since non trivial methods cannot be placed there either.
++

RealClosure(TheField): PUB == PRIV where

   TheField   : Join(OrderedRing, Field, RealConstant)

--   ThePols    : UnivariatePolynomialCategory($)
--   PME        ==> ThePols
--   TheCharDom : RealRootCharacterizationCategory($, ThePols )
--   SEG        ==> TheCharDom
-- this does not work yet
```

```
   E          ==> OutputForm
   Z          ==> Integer
   SE         ==> Symbol
   B          ==> Boolean
   SUP        ==> SparseUnivariatePolynomial($)
   N          ==> PositiveInteger
   RN         ==> Fraction Z
   LF         ==> ListFunctions2($,N)
   PME        ==> SparseUnivariatePolynomial($)
   SEG        ==> RightOpenIntervalRootCharacterization($,PME)

   PUB == Join(RealClosedField,
                FullyRetractableTo TheField,
                Algebra TheField) with

       algebraicOf :   (SEG,E) -> $
             ++ \axiom{algebraicOf(char)} is the external number

       mainCharacterization :   $ -> Union(SEG,"failed")
             ++ \axiom{mainCharacterization(x)} is the main algebraic
             ++ quantity of \axiom{x} (\axiom{SEG})

       relativeApprox :     ($,$) -> RN
             ++ \axiom{relativeApprox(n,p)} gives a relative
             ++ approximation of \axiom{n}
             ++ that has precision \axiom{p}

   PRIV == add

-- local functions

       lessAlgebraic  : $ -> $
       newElementIfneeded : (SEG,E) -> $

-- Representation

       Rec := Record(seg: SEG, val:PME, outForm:E, order:N)
       Rep := Union(TheField,Rec)

-- global (mutable) variables

       orderOfCreation : N := 1$N
          -- it is internally used to sort the algebraic levels

       instanceName : Symbol := new()$Symbol
          -- this used to print the results, thus different instanciations
          -- use different names

-- now the code
```

```
relativeApprox(nbe,prec) ==
   nbe case TheField => retract(nbe)
   appr := relativeApprox(nbe.val, nbe.seg, prec)
   -- now appr has the good exact precision but is $
   relativeApprox(appr,prec)


approximate(nbe,prec) ==
   abs(nbe) < prec => 0
   nbe case TheField => retract(nbe)
   appr := approximate(nbe.val, nbe.seg, prec)
   -- now appr has the good exact precision but is $
   approximate(appr,prec)

newElementIfneeded(s,o) ==
  p := definingPolynomial(s)
  degree(p) = 1 =>
      - coefficient(p,0) / leadingCoefficient(p)
  res := [s, monomial(1,1), o, orderOfCreation ]$Rec
  orderOfCreation := orderOfCreation + 1
  res :: $

algebraicOf(s,o) ==
  pol := definingPolynomial(s)
  degree(pol) = 1 =>
    -coefficient(pol,0) / leadingCoefficient(pol)
  res := [s, monomial(1,1), o, orderOfCreation ]$Rec
  orderOfCreation := orderOfCreation + 1
  res :: $

rename!(x,o) ==
  x.outForm := o
  x

rename(x,o) ==
  [x.seg, x.val, o, x.order]$Rec

rootOf(pol,n) ==
 degree(pol) = 0 => "failed"
 degree(pol) = 1 =>
   if n=1
   then
     -coefficient(pol,0) / leadingCoefficient(pol)
   else
     "failed"
 r := rootOf(pol,n)$SEG
 r case "failed" => "failed"
 o := hconcat(instanceName :: E , orderOfCreation :: E)$E
 algebraicOf(r,o)
```

```
allRootsOf(pol:SUP):List($) ==
 degree(pol)=0 => []
 degree(pol)=1 => [-coefficient(pol,0) / leadingCoefficient(pol)]
 liste := allRootsOf(pol)$SEG
 res : List $ := []
 for term in liste repeat
    o := hconcat(instanceName :: E , orderOfCreation :: E)$E
    res := cons(algebraicOf(term,o), res)
 reverse! res

coerce(x:$):$ ==
   x case TheField => x
   [x.seg,x.val rem$PME definingPolynomial(x.seg),x.outForm,x.order]$Rec

positive?(x) ==
   x case TheField => positive?(x)$TheField
   positive?(x.val,x.seg)$SEG

negative?(x) ==
   x case TheField => negative?(x)$TheField
   negative?(x.val,x.seg)$SEG

abs(x) == sign(x)*x

sign(x) ==
   x case TheField => sign(x)$TheField
   sign(x.val,x.seg)$SEG

x < y == positive?(y-x)

x = y == zero?(x-y)

mainCharacterization(x) ==
   x case TheField => "failed"
   x.seg

mainDefiningPolynomial(x) ==
   x case TheField => "failed"
   definingPolynomial x.seg

mainForm(x) ==
   x case TheField => "failed"
   x.outForm

mainValue(x) ==
   x case TheField => "failed"
   x.val

coerce(x:$):E ==
   x case TheField => x::TheField :: E
```

```
      xx:$ := coerce(x)
      outputForm(univariate(xx.val),x.outForm)$SUP


   inv(x) ==
      (res:= recip x) case "failed" => error "Division by 0"
      res :: $

   recip(x) ==
     x case TheField =>
       if ((r := recip(x)$TheField) case TheField)
       then r::$
       else "failed"
     if ((r := recip(x.val,x.seg)$SEG) case "failed")
     then "failed"
     else lessAlgebraic([x.seg,r::PME,x.outForm,x.order]$Rec)

   (n:Z * x:$):$ ==
      x case TheField => n *$TheField x
      zero?(n) => 0
      one?(n) => x
      [x.seg,map(z+->n*z, x.val),x.outForm,x.order]$Rec

   (rn:TheField * x:$):$ ==
      x case TheField => rn *$TheField x
      zero?(rn) => 0
      one?(rn) => x
      [x.seg,map(z+->rn*z, x.val),x.outForm,x.order]$Rec

   (x:$ * y:$):$ ==
      (x case TheField) and (y case TheField) => x *$TheField y
      (x case TheField) => x::TheField * y
          -- x is no longer TheField
      (y case TheField) => y::TheField * x
          -- now both are algebraic
      y.order > x.order =>
        [y.seg,map(z+->x*z , y.val),y.outForm,y.order]$Rec
      x.order > y.order =>
        [x.seg,map(z+->z*y , x.val),x.outForm,x.order]$Rec
          -- now x.exp = y.exp
          -- we will multiply the polynomials and then reduce
          -- however wee need to call lessAlgebraic
      lessAlgebraic([x.seg,
                     (x.val * y.val) rem definingPolynomial(x.seg),
                     x.outForm,
                     x.order]$Rec)

   nonNull(rep:Rec):$ ==
     degree(rep.val)=0 => leadingCoefficient(rep.val)
     numberOfMonomials(rep.val) = 1 => rep
```

```
          zero?(rep.val,rep.seg)$SEG => 0
          rep

--        zero?(x) ==
--           x case TheField => zero?(x)$TheField
--           zero?(x.val,x.seg)$SEG

        zero?(x) ==
           x case TheField => zero?(x)$TheField
           false

        x + y ==
           (x case TheField) and (y case TheField) => x +$TheField y
           (x case TheField) =>
              if zero?(x)
              then
                y
              else
                nonNull([y.seg,x::PME+(y.val),y.outForm,y.order]$Rec)
              -- x is no longer TheField
           (y case TheField) =>
              if zero?(y)
              then
                x
              else
                nonNull([x.seg,(x.val)+y::PME,x.outForm,x.order]$Rec)
              -- now both are algebraic
           y.order > x.order =>
                nonNull([y.seg,x::PME+y.val,y.outForm,y.order]$Rec)
           x.order > y.order =>
                nonNull([x.seg,(x.val)+y::PME,x.outForm,x.order]$Rec)
               -- now x.exp = y.exp
               -- we simply add polynomials (since degree cannot increase)
               -- however wee need to call lessAlgebraic
           nonNull([x.seg,x.val + y.val,x.outForm,x.order])


        -x ==
           x case TheField => -$TheField (x::TheField)
           [x.seg,-$PME x.val,x.outForm,x.order]$Rec


        retractIfCan(x:$):Union(TheField,"failed") ==
           x case TheField => x
           o := x.order
           res := lessAlgebraic x
           res case TheField => res
           o = res.order => "failed"
           retractIfCan res
```

```
    retract(x:$):TheField ==
       x case TheField => x
       o := x.order
       res := lessAlgebraic x
       res case TheField => res
       o = res.order => error "Can't retract"
       retract res


    lessAlgebraic(x) ==
       x case TheField => x
       degree(x.val) = 0 => leadingCoefficient(x.val)
       def := definingPolynomial(x.seg)
       degree(def) = 1 =>
         x.val.(- coefficient(def,0) / leadingCoefficient(def))
       x

    0 == (0$TheField) :: $

    1 == (1$TheField) :: $

    coerce(rn:TheField):$ == rn :: $
```

— RECLOS.dotabb —

```
"RECLOS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RECLOS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RECLOS" -> "ALIST"
```

19.4 domain RMATRIX RectangularMatrix

— RectangularMatrix.input —

```
)set break resume
)sys rm -f RectangularMatrix.output
)spool RectangularMatrix.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show RectangularMatrix
--R RectangularMatrix(m: NonNegativeInteger,n: NonNegativeInteger,R: Ring)  is a domain const
--R Abbreviation for RectangularMatrix is RMATRIX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RMATRIX
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antisymmetric? : % -> Boolean
--R coerce : % -> Matrix R                coerce : % -> OutputForm
--R copy : % -> %                         diagonal? : % -> Boolean
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   listOfLists : % -> List List R
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R matrix : List List R -> %             maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       qelt : (%,Integer,Integer) -> R
--R rectangularMatrix : Matrix R -> %     sample : () -> %
--R square? : % -> Boolean                symmetric? : % -> Boolean
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (NonNegativeInteger,%) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R column : (%,Integer) -> DirectProduct(m,R)
--R convert : % -> InputForm if R has KONVERT INFORM
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R dimension : () -> CardinalNumber if R has FIELD
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R nullSpace : % -> List DirectProduct(m,R) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List R if $ has finiteAggregate
```

```
--R rank : % -> NonNegativeInteger if R has INTDOM
--R row : (%,Integer) -> DirectProduct(n,R)
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———–

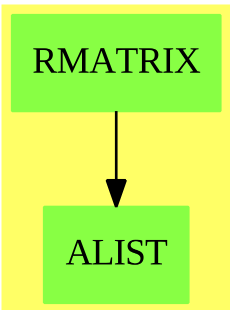
— **RectangularMatrix.help** —

```
====================================================================
RectangularMatrix examples
====================================================================

See Also:
o )show RectangularMatrix
```

———–

19.4.1 RectangularMatrix (RMATRIX)



See

⇒ "IndexedMatrix" (IMATRIX) 10.12.1 on page 1204
⇒ "Matrix" (MATRIX) 14.7.1 on page 1586
⇒ "SquareMatrix" (SQMATRIX) 20.27.1 on page 2505

Exports:

0	antisymmetric?	any?	coerce	column
convert	copy	count	diagonal?	dimension
elt	empty	empty?	eq?	eval
every?	exquo	hash	latex	less?
listOfLists	map	map!	matrix	maxColIndex
maxRowIndex	member?	members	minColIndex	minRowIndex
more?	ncols	nrows	nullSpace	nullity
parts	qelt	rank	rectangularMatrix	row
rowEchelon	sample	size?	square?	subtractIfCan
symmetric?	zero?	#?	?~=?	?*?
?/?	?+?	?-?	-?	?=?

— domain RMATRIX RectangularMatrix —

```
)abbrev domain RMATRIX RectangularMatrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: IndexedMatrix, Matrix, SquareMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{RectangularMatrix} is a matrix domain where the number of rows
++ and the number of columns are parameters of the domain.

RectangularMatrix(m,n,R): Exports == Implementation where
  m,n : NonNegativeInteger
  R   : Ring
  Row ==> DirectProduct(n,R)
  Col ==> DirectProduct(m,R)
  Exports ==> Join(RectangularMatrixCategory(m,n,R,Row,Col),_
                   CoercibleTo Matrix R) with

    if R has Field then VectorSpace R

    if R has ConvertibleTo InputForm then ConvertibleTo InputForm

    rectangularMatrix: Matrix R -> $
      ++ \spad{rectangularMatrix(m)} converts a matrix of type \spadtype{Matrix}
      ++ to a matrix of type \spad{RectangularMatrix}.
    coerce: $ -> Matrix R
      ++ \spad{coerce(m)} converts a matrix of type \spadtype{RectangularMatrix}
      ++ to a matrix of type \spad{Matrix}.

  Implementation ==> Matrix R add
```

```
minr ==> minRowIndex
maxr ==> maxRowIndex
minc ==> minColIndex
maxc ==> maxColIndex
mini ==> minIndex
maxi ==> maxIndex

ZERO := new(m,n,0)$Matrix(R) pretend $
0    == ZERO

coerce(x:$):OutputForm == coerce(x pretend Matrix R)$Matrix(R)

matrix(l: List List R) ==
  -- error check: this is a top level function
  #l ^= m => error "matrix: wrong number of rows"
  for ll in l repeat
    #ll ^= n => error "matrix: wrong number of columns"
  ans : Matrix R := new(m,n,0)
  for i in minr(ans)..maxr(ans) for ll in l repeat
    for j in minc(ans)..maxc(ans) for r in ll repeat
      qsetelt_!(ans,i,j,r)
  ans pretend $

row(x,i)    == directProduct row(x pretend Matrix(R),i)
column(x,j) == directProduct column(x pretend Matrix(R),j)

coerce(x:$):Matrix(R) == copy(x pretend Matrix(R))

rectangularMatrix x ==
  (nrows(x) ^= m) or (ncols(x) ^= n) =>
    error "rectangularMatrix: matrix of bad dimensions"
  copy(x) pretend $

if R has EuclideanDomain then

  rowEchelon x == rowEchelon(x pretend Matrix(R)) pretend $

if R has IntegralDomain then

  rank x    == rank(x pretend Matrix(R))
  nullity x == nullity(x pretend Matrix(R))
  nullSpace x ==
    [directProduct c for c in nullSpace(x pretend Matrix(R))]

if R has Field then

  dimension() == (m * n) :: CardinalNumber

if R has ConvertibleTo InputForm then
  convert(x:$):InputForm ==
```

```
        convert [convert("rectangularMatrix"::Symbol)@InputForm,
                 convert(x::Matrix(R))]$List(InputForm)
```

———-

— RMATRIX.dotabb —

```
"RMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RMATRIX"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RMATRIX" -> "ALIST"
```

———-

19.5 domain REF Reference

— Reference.input —

```
)set break resume
)sys rm -f Reference.output
)spool Reference.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Reference
--R Reference S: Type  is a domain constructor
--R Abbreviation for Reference is REF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for REF
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                deref : % -> S
--R elt : % -> S                          ref : S -> %
--R setelt : (%,S) -> S                   setref : (%,S) -> S
--R coerce : % -> OutputForm if S has SETCAT
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

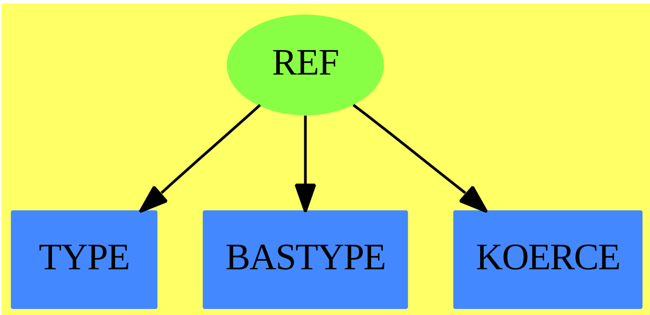
— Reference.help —

```
====================================================================
Reference examples
====================================================================

See Also:
o )show Reference
```

———-

19.5.1 Reference (REF)



See

⇒ "Boolean" (BOOLEAN) 3.15.1 on page 304
⇒ "IndexedBits" (IBITS) 10.3.1 on page 1165
⇒ "Bits" (BITS) 3.12.1 on page 297

Exports:

coerce	deref	elt	hash	latex
ref	setelt	setref	?=?	?~=?

— domain REF Reference —

```
)abbrev domain REF Reference
++ Author: Stephen M. Watt
++ Date Created:
++ Change History:
++ Basic Operations: deref, elt, ref, setelt, setref, =
++ Related Constructors:
++ Keywords:  reference
++ Description:
++ \spadtype{Reference} is for making a changeable instance
```

```
++ of something.

Reference(S:Type): Type with
        ref   : S -> %
          ++  ref(n) creates a pointer (reference) to the object n.
        elt   : % -> S
          ++ elt(n) returns the object n.
        setelt: (%, S) -> S
          ++ setelt(n,m) changes the value of the object n to m.
        -- alternates for when bugs don't allow the above
        deref : % -> S
          ++ deref(n) is equivalent to \spad{elt(n)}.
        setref: (%, S) -> S
          ++ setref(n,m) same as \spad{setelt(n,m)}.
        _=   : (%, %) -> Boolean
          ++ a=b tests if \spad{a} and b are equal.
        if S has SetCategory then SetCategory

    == add
        Rep := Record(value: S)

        p = q        == EQ(p, q)$Lisp
        ref v        == [v]
        elt p        == p.value
        setelt(p, v) == p.value := v
        deref p      == p.value
        setref(p, v) == p.value := v

        if S has SetCategory then
          coerce p ==
            prefix(message("ref"@String), [p.value::OutputForm])
```

— **REF.dotabb** —

```
"REF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=REF",shape=ellipse]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"REF" -> "TYPE"
"REF" -> "BASTYPE"
"REF" -> "KOERCE"
```

19.6 domain RGCHAIN RegularChain

— RegularChain.input —

```
)set break resume
)sys rm -f RegularChain.output
)spool RegularChain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RegularChain
--R RegularChain(R: GcdDomain,ls: List Symbol)  is a domain constructor
--R Abbreviation for RegularChain is RGCHAIN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RGCHAIN
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R collectQuasiMonic : % -> %            copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             headReduced? : % -> Boolean
--R infRittWu? : (%,%) -> Boolean         initiallyReduced? : % -> Boolean
--R latex : % -> String                   normalized? : % -> Boolean
--R purelyAlgebraic? : % -> Boolean       rest : % -> Union(%,"failed")
--R sample : () -> %                      stronglyReduced? : % -> Boolean
--R trivialIdeal? : % -> Boolean          ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraic? : (OrderedVariableList ls,%) -> Boolean
--R algebraicCoefficients? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R algebraicVariables : % -> List OrderedVariableList ls
--R any? : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> Boolean if $ ha
--R augment : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R augment : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R augment : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R augment : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R autoReduced? : (%,((NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List NewSparseMultivar
--R basicSet : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),(NewSparseMultivariatePol
--R basicSet : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),((NewSparseMultivariatePo
--R coHeight : % -> NonNegativeInteger if OrderedVariableList ls has FINITE
--R coerce : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R collect : (%,OrderedVariableList ls) -> %
--R collectUnder : (%,OrderedVariableList ls) -> %
--R collectUpper : (%,OrderedVariableList ls) -> %
--R construct : List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> %
--R convert : % -> InputForm if NewSparseMultivariatePolynomial(R,OrderedVariableList ls) has KONVERT IN
```

```
--R count : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> Nor
--R count : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NonNegativeInteg
--R eval : (%,List Equation NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> % i
--R eval : (%,Equation NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> % if New
--R eval : (%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariate
--R eval : (%,List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List NewSparseMu
--R every? : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> Bo
--R extend : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List
--R extend : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R extend : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R extend : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R extend : (%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> %
--R extendIfCan : (%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> Union(%,"f
--R find : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> Unio
--R first : % -> Union(NewSparseMultivariatePolynomial(R,OrderedVariableList ls),"failed")
--R headReduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMu
--R headReduced? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R headRemainder : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Record(n
--R initiallyReduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSpa
--R initiallyReduced? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Bool
--R initials : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R internalAugment : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> %
--R internalAugment : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> %
--R intersect : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R intersect : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> Li
--R intersect : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R intersect : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R invertible? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R invertible? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List Recor
--R invertibleElseSplit? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> U
--R invertibleSet : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R last : % -> Union(NewSparseMultivariatePolynomial(R,OrderedVariableList ls),"failed")
--R lastSubResultant : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMu
--R lastSubResultantElseSplit : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),Ne
--R less? : (%,NonNegativeInteger) -> Boolean
--R mainVariable? : (OrderedVariableList ls,%) -> Boolean
--R mainVariables : % -> List OrderedVariableList ls
--R map : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> NewSparseMultivariat
--R map! : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> NewSparseMultivaria
--R member? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean if $ h
--R members : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) if $ has fi
--R more? : (%,NonNegativeInteger) -> Boolean
--R mvar : % -> OrderedVariableList ls
--R normalized? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R parts : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) if $ has fini
--R purelyAlgebraic? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boole
--R purelyAlgebraicLeadingMonomial? : (NewSparseMultivariatePolynomial(R,OrderedVariableList
--R purelyTranscendental? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) ->
--R quasiComponent : % -> Record(close: List NewSparseMultivariatePolynomial(R,OrderedVariabl
--R reduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%,((NewSparseMultivar
```

```
--R reduce : (((NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomia
--R reduce : (((NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomia
--R reduce : (((NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomia
--R reduceByQuasiMonic : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMulti
--R reduced? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%,((NewSparseMultivariatePolyn
--R remainder : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Record(rnum: R,polnum:
--R remove : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> % if $ has fi
--R remove : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and NewSparseMultivariatePolynomial(R,OrderedVari
--R removeZero : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMultivariateP
--R retract : List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> %
--R retractIfCan : List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -
--R rewriteIdealWithRemainder : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Li
--R rewriteSetWithReduction : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%,((NewSpa
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%,OrderedVariableList ls) -> Union(NewSparseMultivariatePolynomial(R,OrderedVariableList l
--R select : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> % if $ has fi
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,OrderedVariableList ls) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List Record(val: N
--R stronglyReduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMultivari
--R stronglyReduced? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R variables : % -> List OrderedVariableList ls
--R zeroSetSplit : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),Boolean,Boolean) -> L
--R zeroSetSplit : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),Boolean) -> List %
--R zeroSetSplit : List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> List %
--R zeroSetSplitIntoTriangularSystems : List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -
--R
--E 1

)spool
)lisp (bye)
```

———-

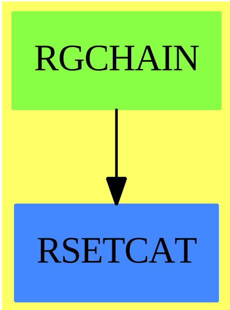
— **RegularChain.help** —

```
====================================================================
RegularChain examples
====================================================================

See Also:
o )show RegularChain
```

———

19.6.1 RegularChain (RGCHAIN)



Exports:

algebraic?	algebraicCoefficients?
algebraicVariables	any?
augment	autoReduced?
basicSet	coHeight
coerce	collect
collectQuasiMonic	collectUnder
collectUpper	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	internalAugment
intersect	invertible?
invertibleElseSplit?	invertibleSet
last	lastSubResultant
lastSubResultantElseSplit	latex
less?	mainVariable?
mainVariables	map
map!	member?
members	more?
mvar	normalized?
parts	purelyAlgebraic?
purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce
reduceByQuasiMonic	reduced?
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
squareFreePart	stronglyReduce
stronglyReduced?	triangular?
trivialIdeal?	variables
zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?~=?
?=?	

— domain RGCHAIN RegularChain —

```
)abbrev domain RGCHAIN RegularChain
++ Author: Marc Moreno Maza
++ Date Created: 01/1999
++ Date Last Updated: 23/01/1999
++ Description:
++ A domain for regular chains (i.e. regular triangular sets) over
++ a Gcd-Domain and with a fix list of variables.
++ This is just a front-end for the \spadtype{RegularTriangularSet}
++ domain constructor.

RegularChain(R,ls): Exports == Implementation where
  R : GcdDomain
  ls: List Symbol
  V ==> OrderedVariableList ls
  E ==> IndexedExponents V
  P ==> NewSparseMultivariatePolynomial(R,V)
  TS ==> RegularTriangularSet(R,E,V,P)

  Exports ==  RegularTriangularSetCategory(R,E,V,P) with
     zeroSetSplit: (List P, Boolean, Boolean) -> List $
       ++ \spad{zeroSetSplit(lp,clos?,info?)} returns a list \spad{lts} of
       ++ regular chains such that the union of the closures of their regular
       ++ zero sets equals the affine variety associated with \spad{lp}.
       ++ Moreover, if \spad{clos?} is \spad{false} then the union of the
       ++ regular zero set of the \spad{ts} (for \spad{ts} in \spad{lts})
       ++ equals this variety.
       ++ If \spad{info?} is \spad{true} then some information is
       ++ displayed during the computations. See
       ++ zeroSetSplit from RegularTriangularSet.

  Implementation == RegularTriangularSet(R,E,V,P)
```

———-

— RGCHAIN.dotabb —

```
"RGCHAIN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RGCHAIN"]
"RSETCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RSETCAT"]
"RGCHAIN" -> "RSETCAT"
```

———-

19.7 domain REGSET RegularTriangularSet

Several domain constructors implement regular triangular sets (or regular chains). Among them **RegularTriangularSet** and **SquareFreeRegularTriangularSet**. They also implement an algorithm by Marc Moreno Maza for computing triangular decompositions of polynomial systems. This method is refined in the package **LazardSetSolvingPackage** in order to produce decompositions by means of Lazard triangular sets.

— **RegularTriangularSet.input** —

```
)set break resume
)sys rm -f RegularTriangularSet.output
)spool RegularTriangularSet.output
)set message test on
)set message auto off
)clear all
--S 1 of 34
R := Integer
--R
--R
--R   (1)  Integer
--R                                                          Type: Domain
--E 1

--S 2 of 34
ls : List Symbol := [x,y,z,t]
--R
--R
--R   (2)  [x,y,z,t]
--R                                                     Type: List Symbol
--E 2

--S 3 of 34
V := OVAR(ls)
--R
--R
--R   (3)  OrderedVariableList [x,y,z,t]
--R                                                          Type: Domain
--E 3

--S 4 of 34
E := IndexedExponents V
--R
--R
--R   (4)  IndexedExponents OrderedVariableList [x,y,z,t]
--R                                                          Type: Domain
--E 4

--S 5 of 34
```

```
P := NSMP(R, V)
--R
--R
--R   (5)  NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--R                                                                  Type: Domain
--E 5

--S 6 of 34
x: P := 'x
--R
--R
--R   (6)  x
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 6

--S 7 of 34
y: P := 'y
--R
--R
--R   (7)  y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 7

--S 8 of 34
z: P := 'z
--R
--R
--R   (8)  z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 8

--S 9 of 34
t: P := 't
--R
--R
--R   (9)  t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 9

--S 10 of 34
T := REGSET(R,E,V,P)
--R
--R
--R   (10)
--R  RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],O
--R  rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV
--R  ariableList [x,y,z,t]))
--R                                                                  Type: Domain
--E 10
```

```
--S 11 of 34
p1 := x ** 31 - x ** 6 - x - y
--R
--R
--R          31    6
--R   (11)  x   - x  - x - y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 11

--S 12 of 34
p2 := x ** 8  - z
--R
--R
--R          8
--R   (12)  x  - z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 12

--S 13 of 34
p3 := x ** 10 - t
--R
--R
--R          10
--R   (13)  x   - t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 13

--S 14 of 34
lp := [p1, p2, p3]
--R
--R
--R           31    6          8      10
--R   (14)  [x   - x  - x - y,x  - z,x   - t]
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 14

--S 15 of 34
zeroSetSplit(lp)$T
--R
--R
--R            5    4      2     3     8     5    3    2   4                2
--R   (15)  [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z }]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariabl
--E 15

--S 16 of 34
lts := zeroSetSplit(lp,false)$T
--R
--R
--R   (16)
```

```
--R      5    4      2     3     8     5    3    2   4                 2
--R   [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z },
--R      3      5          2     3         2
--R    {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}, {t,z,y,x}]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],Ord
--E 16

--S 17 of 34
[coHeight(ts) for ts in lts]
--R
--R
--R   (17)  [1,0,0]
--R                                                   Type: List NonNegativeInteger
--E 17

--S 18 of 34
f1 := y**2*z+2*x*y*t-2*x-z
--R
--R
--R                            2
--R   (18)  (2t y - 2)x + z y  - z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 18

--S 19 of 34
f2:=-x**3*z+ 4*x*y**2*z+4*x**2*y*t+2*y**3*t+4*x**2-10*y**2+4*x*z-10*y*t+2
--R
--R
--R               3             2        2              3      2
--R   (19)  - z x  + (4t y + 4)x  + (4z y  + 4z)x + 2t y  - 10y  - 10t y + 2
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 19

--S 20 of 34
f3 :=  2*y*z*t+x*t**2-x-2*z
--R
--R
--R            2
--R   (20)  (t  - 1)x + 2t z y - 2z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 20

--S 21 of 34
f4:=-x*z**3+4*y*z**2*t+4*x*z*t**2+2*y*t**3+4*x*z+4*z**2-10*y*t- 10*t**2+2
--R
--R
--R              3      2                2     3            2      2
--R   (21)  (- z  + (4t  + 4)z)x + (4t z  + 2t  - 10t)y + 4z  - 10t  + 2
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 21
```

```
--S 22 of 34
lf := [f1, f2, f3, f4]
--R
--R
--R   (22)
--R                        2
--R   [(2t y - 2)x + z y  - z,
--R         3              2        2               3      2
--R    - z x  + (4t y + 4)x  + (4z y  + 4z)x + 2t y  - 10y  - 10t y + 2,
--R      2
--R    (t  - 1)x + 2t z y - 2z,
--R        3      2                2     3              2      2
--R    (- z  + (4t  + 4)z)x + (4t z  + 2t  - 10t)y + 4z  - 10t  + 2]
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 22

--S 23 of 34
zeroSetSplit(lf)$T
--R
--R
--R   (23)
--R      2      8      6       2                  3           2
--R   [{t  - 1,z  - 16z  + 256z  - 256,t y - 1,(z  - 8z)x - 8z  + 16},
--R       2      2     2
--R    {3t  + 1,z  - 7t  - 1,y + t,x + z},
--R      8      6      2         3           2
--R    {t  - 10t  + 10t  - 1,z,(t  - 5t)y - 5t  + 1,x},
--R      2      2
--R    {t  + 3,z  - 4,y + t,x - z}]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariabl
--E 23

--S 24 of 34
lts2 := zeroSetSplit(lf,false)$T
--R
--R
--R   (24)
--R      8      6      2         3           2
--R   [{t  - 10t  + 10t  - 1,z,(t  - 5t)y - 5t  + 1,x},
--R      2      8      6       2                  3           2
--R    {t  - 1,z  - 16z  + 256z  - 256,t y - 1,(z  - 8z)x - 8z  + 16},
--R       2      2     2                           2      2
--R    {3t  + 1,z  - 7t  - 1,y + t,x + z}, {t  + 3,z  - 4,y + t,x - z}]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariabl
--E 24

--S 25 of 34
[coHeight(ts) for ts in lts2]
--R
```

```
--R
--R   (25)  [0,0,0,0]
--R                                                 Type: List NonNegativeInteger
--E 25

--S 26 of 34
degrees := [degree(ts) for ts in lts2]
--R
--R
--R   (26)  [8,16,4,4]
--R                                                 Type: List NonNegativeInteger
--E 26

--S 27 of 34
reduce(+,degrees)
--R
--R
--R   (27)  32
--R                                                         Type: PositiveInteger
--E 27

--S 28 of 34
u : R := 2
--R
--R
--R   (28)  2
--R                                                                 Type: Integer
--E 28

--S 29 of 34
q1 := 2*(u-1)**2+ 2*(x-z*x+z**2)+ y**2*(x-1)**2- 2*u*x+ 2*y*t*(1-x)*(x-z)+_
      2*u*z*t*(t-y)+ u**2*t**2*(1-2*z)+ 2*u*t**2*(z-x)+ 2*u*t*y*(z-1)+_
      2*u*z*x*(y+1)+ (u**2-2*u)*z**2*t**2+ 2*u**2*z**2+ 4*u*(1-u)*z+_
      t**2*(z-x)**2
--R
--R
--R   (29)
--R       2            2  2        2                            2           2
--R     (y  - 2t y + t )x  + (- 2y  + ((2t + 4)z + 2t)y + (- 2t  + 2)z - 4t  - 2)x
--R   +
--R      2                       2       2          2
--R     y  + (- 2t z - 4t)y + (t  + 10)z  - 8z + 4t  + 2
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 29

--S 30 of 34
q2 := t*(2*z+1)*(x-z)+ y*(z+2)*(1-x)+ u*(u-2)*t+ u*(1-2*u)*z*t+_
      u*y*(x+u-z*x-1)+ u*(u+1)*z**2*t
--R
--R
```

```
--R                                               2
--R   (30)  (- 3z y + 2t z + t)x + (z + 4)y + 4t z  - 7t z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 30

--S 31 of 34
q3 := -u**2*(z-1)**2+ 2*z*(z-x)-2*(x-1)
--R
--R
--R                           2
--R   (31)  (- 2z - 2)x - 2z  + 8z - 2
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 31

--S 32 of 34
q4 := u**2+4*(z-x**2)+3*y**2*(x-1)**2- 3*t**2*(z-x)**2+_
      3*u**2*t**2*(z-1)**2+u**2*z*(z-2)+6*u*t*y*(z+x+z*x-1)
--R
--R
--R   (32)
--R        2     2      2        2                      2        2
--R     (3y  - 3t  - 4)x  + (- 6y  + (12t z + 12t)y + 6t z)x + 3y  + (12t z - 12t)y
--R   +
--R        2      2         2            2
--R     (9t  + 4)z  + (- 24t  - 4)z + 12t  + 4
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 32

--S 33 of 34
lq := [q1, q2, q3, q4]
--R
--R
--R   (33)
--R   [
--R         2           2  2
--R       (y  - 2t y + t )x
--R     +
--R           2                            2          2         2
--R       (- 2y  + ((2t + 4)z + 2t)y + (- 2t  + 2)z - 4t  - 2)x + y
--R     +
--R                         2       2          2
--R       (- 2t z - 4t)y + (t  + 10)z  - 8z + 4t  + 2
--R     ,
--R                                             2                         2
--R    (- 3z y + 2t z + t)x + (z + 4)y + 4t z  - 7t z, (- 2z - 2)x - 2z  + 8z - 2,
--R
--R          2     2      2        2                      2        2
--R       (3y  - 3t  - 4)x  + (- 6y  + (12t z + 12t)y + 6t z)x + 3y
--R     +
--R                          2     2         2            2
```

```
--R       (12t z - 12t)y + (9t  + 4)z  + (- 24t  - 4)z + 12t  + 4
--R     ]
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 33

--S 34 of 34
zeroSetSplit(lq,true,true)$T
--R
--R[1 <4,0> -> |4|; {0}]W[2 <5,0>,<3,1> -> |8|; {0}][2 <4,1>,<3,1> -> |7|; {0}][1 <3,1> -> |
--R   *** QCMPACK Statistics ***
--R      Table     size:  36
--R      Entries reused:  255
--R
--R   *** REGSETGCD: Gcd Statistics ***
--R      Table     size:  125
--R      Entries reused:  0
--R
--R   *** REGSETGCD: Inv Set Statistics ***
--R      Table     size:  30
--R      Entries reused:  0
--R
--R   (34)
--R   [
--R     {
--R                            24                    23                      22
--R         960725655771966t   + 386820897948702t   + 8906817198608181t
--R       +
--R                             21                     20                      19
--R         2704966893949428t   + 37304033340228264t   + 7924782817170207t
--R       +
--R                              18                     17                       16
--R         93126799040354990t   + 13101273653130910t   + 156146250424711858t
--R       +
--R                              15                      14                      13
--R         16626490957259119t   + 190699288479805763t   + 24339173367625275t
--R       +
--R                               12                      11                       10
--R         180532313014960135t   + 35288089030975378t   + 135054975747656285t
--R       +
--R                              9                     8                      7
--R         34733736952488540t  + 75947600354493972t  + 19772555692457088t
--R       +
--R                              6                     5                    4
--R         28871558573755428t  + 5576152439081664t  + 6321711820352976t
--R       +
--R                          3                   2
--R       438314209312320t  + 581105748367008t  - 60254467992576t + 1449115951104
--R       ,
--R
--R                                                                              23
```

```
--R                 26604210869491302385515265737052082361668474181372891857784t
--R               +
--R                                                                            22
--R                 443104378424686086067294899528296664238693556855017735265295t
--R               +
--R                                                                            21
--R                 279078393286701234679141342358988327155321305829547090310242t
--R               +
--R                                                                             20
--R                 3390276361413232465107617176615543054620626391823613392185226t
--R               +
--R                                                                            19
--R                 941478179503540575554198645220352803719793196473813837434129t
--R               +
--R                                                                              18
--R                 11547855194679475242211696749673949352585747674184320988144390t
--R               +
--R                                                                             17
--R                 1343609566765597789881701656699413216467215660333356417241432t
--R               +
--R                                                                              16
--R                 23233813868147873503933551617175640859899102987800663566699334t
--R               +
--R                                                                            15
--R                 869574020537672336950845440508790740850931336484983573386433t
--R               +
--R                                                                              14
--R                 31561554305876934875419461486969926554241750065103460820476969t
--R               +
--R                                                                             13
--R                 1271400990287717487442065952547731879554823889855386072264931t
--R               +
--R                                                                              12
--R                 31945089913863736044802526964079540198337049550503295825160523t
--R               +
--R                                                                             11
--R                 3738735704288144509871371560232845884439102270778010470931960t
--R               +
--R                                                                              10
--R                 25293997512391412026144601435771131587561905532992045692885927t
--R               +
--R                                                                             9
--R                 5210239009846067123469262799870052773410471135950175008046524t
--R               +
--R                                                                              8
--R                 15083887986930297166259870568608270427403187606238713491129188t
--R               +
--R                                                                             7
--R                 3522087234692930126383686270775779553481769125670839075109000t
--R               +
```

```
--R                                                                           6
--R             60799452003956810130865337925688864911012442474400349692885 88t
--R           +
--R                                                                           5
--R             109063485243390088819991375624798602319698772346993493360368 0t
--R           +
--R                                                                           4
--R             140581943087190710229443253753833540210283899401966748745835 2t
--R           +
--R                                                                         3
--R             8807152795032045007253667126550774887834782888493360520243 2t
--R           +
--R                                                                          2
--R             1358824894336409332297811771559777680160657654823786571294 40t
--R           +
--R             - 13957283442882262230559894607400314082516690749975646520320t
--R           +
--R             334637692973189299277258325709308472592117112855749713920
--R        *
--R           z
--R       +
--R                                                                    23
--R         85671754840439528797567259645068339321496371010905211649 36t
--R       +
--R                                                                      22
--R         1497923928642017918457083740327289424987975192516672509457 21t
--R       +
--R                                                                     21
--R         772583717836458221574108615821597641381230030741903740215 50t
--R       +
--R                                                                       20
--R         1108862254126854214498918940708612211184560556764334742191 654t
--R       +
--R                                                                      19
--R         2132504944606788652197744801068260537838157896215017326723 27t
--R       +
--R                                                                       18
--R         3668929075160666195729177894178343514501987898410131431699 882t
--R       +
--R                                                                      17
--R         1713889064710018728794901243687482363147654590395678200488 72t
--R       +
--R                                                                       16
--R         7192430746914602166660233477331022483144921771645523139658 986t
--R       +
--R                                                                        15
--R         - 128798674689690072812879965633090291959663143108437362453385t
--R       +
--R                                                                       14
--R         9553010858341425909306423132921134040856028790803526430270 671t
```

```
--R       +
--R                                                                  13
--R         - 13296096245675492874538687646300437824658458709144441096603t
--R       +
--R                                                                  12
--R         9475806805814145326383085518325333106881690568644274964864413t
--R       +
--R                                                                 11
--R         803234687925133458861659855664084927606298794799856265539336t
--R       +
--R                                                                  10
--R         7338202759292865165994622349207516400662174302614595173333825t
--R       +
--R                                                                  9
--R         1308004628480367351164369613111971668880538855640917200187108t
--R       +
--R                                                                  8
--R         4268059455741255498880229598973705747098216067697754352634748t
--R       +
--R                                                                 7
--R         892893526858514095791318775904093300103045601514470613580600t
--R       +
--R                                                                  6
--R         1679152575460683956631925852181341501981598137465328797013652t
--R       +
--R                                                                 5
--R         269757415767922980378967154143357835544113158280591408043936t
--R       +
--R                                                                 4
--R         380951527864657529033580829801282724081345372680202920198224t
--R       +
--R                                                                3
--R         19785545294228495032998826937601341132725035339452913286656t
--R       +
--R                                                                2
--R         36477412057384782942366635303396637763303928174935079178528t
--R       +
--R         - 3722212879279038648713080422224976273210890229485838670848t
--R       +
--R         89079724853114348361230634484013862024728599906874105856
--R       ,
--R         3      2                  3       2
--R      (3z  - 11z  + 8z + 4)y + 2t z  + 4t z  - 5t z - t,
--R                    2
--R      (z + 1)x + z  - 4z + 1}
--R     ]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariabl
--E 34
)spool
)lisp (bye)
```

— RegularTriangularSet.help —

```
====================================================================
RegularTriangularSet examples
====================================================================

The RegularTriangularSet domain constructor implements regular
triangular sets.  These particular triangular sets were introduced by
M. Kalkbrener (1991) in his PhD Thesis under the name regular chains.
Regular chains and their related concepts are presented in the paper
"On the Theories of Triangular sets" By P. Aubry, D. Lazard and
M. Moreno Maza (to appear in the Journal of Symbolic Computation).
The RegularTriangularSet constructor also provides a new method
(by the third author) for solving polynomial system by means of
regular chains.  This method has two ways of solving.  One has the
same specifications as Kalkbrener's algorithm (1991) and the other is
closer to Lazard's method (Discr. App. Math, 1991).  Moreover, this
new method removes redundant component from the decompositions when
this is not too expensive.  This is always the case with
square-free regular chains.  So if you want to obtain decompositions
without redundant components just use the SquareFreeRegularTriangularSet
domain constructor or the LazardSetSolvingPackage package constructor.
See also the LexTriangularPackage and ZeroDimensionalSolvePackage for the
case of algebraic systems with a finite number of (complex) solutions.

One of the main features of regular triangular sets is that they
naturally define towers of simple extensions of a field.  This allows
to perform with multivariate polynomials the same kind of operations
as one can do in an EuclideanDomain.

The RegularTriangularSet constructor takes four arguments.  The first
one, R, is the coefficient ring of the polynomials; it must belong to
the category GcdDomain.  The second one, E, is the exponent monoid of
the polynomials; it must belong to the category OrderedAbelianMonoidSup.
The third one, V, is the ordered set of variables; it must belong to the
category OrderedSet.  The last one is the polynomial ring; it must belong
to the category RecursivePolynomialCategory(R,E,V).  The abbreviation for
RegularTriangularSet is REGSET.  See also the constructor RegularChain
which only takes two arguments, the coefficient ring and the ordered
set of variables; in that case, polynomials are necessarily built with
the NewSparseMultivariatePolynomial domain constructor.

We shall explain now how to use the constructor REGSET and how to read
the decomposition of a polynomial system by means of regular sets.

Let us give some examples.  We start with an easy one
(Donati-Traverso) in order to understand the two ways of solving
```

polynomial systems provided by the REGSET constructor.

Define the coefficient ring.

```
  R := Integer
    Integer
                               Type: Domain
```

Define the list of variables,

```
  ls : List Symbol := [x,y,z,t]
    [x,y,z,t]
                               Type: List Symbol
```

and make it an ordered set;

```
  V := OVAR(ls)
    OrderedVariableList [x,y,z,t]
                               Type: Domain
```

then define the exponent monoid.

```
  E := IndexedExponents V
    IndexedExponents OrderedVariableList [x,y,z,t]
                               Type: Domain
```

Define the polynomial ring.

```
  P := NSMP(R, V)
    NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
                               Type: Domain
```

Let the variables be polynomial.

```
  x: P := 'x
    x
             Type: NewSparseMultivariatePolynomial(Integer,
                                 OrderedVariableList [x,y,z,t])
  y: P := 'y
    y
             Type: NewSparseMultivariatePolynomial(Integer,
                                 OrderedVariableList [x,y,z,t])

  z: P := 'z
    z
             Type: NewSparseMultivariatePolynomial(Integer,
                                 OrderedVariableList [x,y,z,t])

  t: P := 't
   t
```

```
            Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])
```

Now call the RegularTriangularSet domain constructor.

```
  T := REGSET(R,E,V,P)
  RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],O
  rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV
  ariableList [x,y,z,t]))
                              Type: Domain
```

Define a polynomial system.

```
  p1 := x ** 31 - x ** 6 - x - y
      31    6
     x   - x  - x - y
            Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])

  p2 := x ** 8  - z
     8
    x  - z
            Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])

  p3 := x ** 10 - t
     10
    x   - t
            Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])

  lp := [p1, p2, p3]
      31    6          8      10
    [x   - x  - x - y,x  - z,x   - t]
            Type: List NewSparseMultivariatePolynomial(Integer,
                                      OrderedVariableList [x,y,z,t])
```

First of all, let us solve this system in the sense of Kalkbrener.

```
  zeroSetSplit(lp)$T
       5    4      2     3     8     5    3    2   4                2
    [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z }]
       Type: List RegularTriangularSet(Integer,
                   IndexedExponents OrderedVariableList [x,y,z,t],
                   OrderedVariableList [x,y,z,t],
                   NewSparseMultivariatePolynomial(Integer,
                       OrderedVariableList [x,y,z,t]))
```

And now in the sense of Lazard (or Wu and other authors).

```
  lts := zeroSetSplit(lp,false)$T
     5    4      2     3     8     5    3    2   4               2
  [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z },
     3      5          2     3         2
   {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}, {t,z,y,x}]
       Type: List RegularTriangularSet(Integer,
                   IndexedExponents OrderedVariableList [x,y,z,t],
                   OrderedVariableList [x,y,z,t],
                   NewSparseMultivariatePolynomial(Integer,
                       OrderedVariableList [x,y,z,t]))
```

We can see that the first decomposition is a subset of the second.
So how can both be correct ?

Recall first that polynomials from a domain of the category
RecursivePolynomialCategory are regarded as univariate polynomials in
their main variable. For instance the second polynomial in the first
set of each decomposition has main variable y and its initial
(i.e. its leading coefficient w.r.t. its main variable) is t z.

Now let us explain how to read the second decomposition. Note that
the non-constant initials of the first set are t^4-t and t z. Then
the solutions described by this first set are the common zeros of
its polynomials that do not cancel the polynomials t^4-t and ty z.
Now the solutions of the input system lp satisfying these equations
are described by the second and the third sets of the decomposition.
Thus, in some sense, they can be considered as degenerated solutions.
The solutions given by the first set are called the generic points of
the system; they give the general form of the solutions. The first
decomposition only provides these generic points. This latter
decomposition is useful when they are many degenerated solutions
(which is sometimes hard to compute) and when one is only interested
in general informations, like the dimension of the input system.

We can get the dimensions of each component of a decomposition as follows.

```
  [coHeight(ts) for ts in lts]
    [1,0,0]
                             Type: List NonNegativeInteger
```

Thus the first set has dimension one. Indeed t can take any value,
except 0 or any third root of 1, whereas z is completely determined
from t, y is given by z and t, and finally x is given by the other
three variables. In the second and the third sets of the second
decomposition the four variables are completely determined and thus
these sets have dimension zero.

We give now the precise specifications of each decomposition. This
assume some mathematical knowledge. However, for the non-expert user,
the above explanations will be sufficient to understand the other

features of the RSEGSET constructor.

The input system lp is decomposed in the sense of Kalkbrener as finitely many regular sets T1,...,Ts such that the radical ideal generated by lp is the intersection of the radicals of the saturated ideals of T1,...,Ts. In other words, the affine variety associated with lp is the union of the closures (w.r.t. Zarisky topology) of the regular-zeros sets of T1,...,Ts.

N. B. The prime ideals associated with the radical of the saturated ideal of a regular triangular set have all the same dimension; moreover these prime ideals can be given by characteristic sets with the same main variables. Thus a decomposition in the sense of Kalkbrener is unmixed dimensional. Then it can be viewed as a lazy decomposition into prime ideals (some of these prime ideals being merged into unmixed dimensional ideals).

Now we explain the other way of solving by means of regular triangular sets. The input system lp is decomposed in the sense of Lazard as finitely many regular triangular sets T1,...,Ts such that the affine variety associated with lp is the union of the regular-zeros sets of T1,...,Ts. Thus a decomposition in the sense of Lazard is also a decomposition in the sense of Kalkbrener; the converse is false as we have seen before.

When the input system has a finite number of solutions, both ways of solving provide similar decompositions as we shall see with this second example (Caprasse).

Define a polynomial system.

```
  f1 := y**2*z+2*x*y*t-2*x-z
                     2
    (2t y - 2)x + z y  - z
           Type: NewSparseMultivariatePolynomial(Integer,
                    OrderedVariableList [x,y,z,t])

  f2:=-x**3*z+ 4*x*y**2*z+4*x**2*y*t+2*y**3*t+4*x**2-10*y**2+4*x*z-10*y*t+2
         3              2        2              3      2
    - z x  + (4t y + 4)x  + (4z y  + 4z)x + 2t y  - 10y  - 10t y + 2
           Type: NewSparseMultivariatePolynomial(Integer,
                    OrderedVariableList [x,y,z,t])

  f3 :=  2*y*z*t+x*t**2-x-2*z
      2
    (t  - 1)x + 2t z y - 2z
           Type: NewSparseMultivariatePolynomial(Integer,
                    OrderedVariableList [x,y,z,t])

  f4:=-x*z**3+4*y*z**2*t+4*x*z*t**2+2*y*t**3+4*x*z+4*z**2-10*y*t- 10*t**2+2
```

```
      3      2                2     3             2      2
   (- z  + (4t  + 4)z)x + (4t z  + 2t  - 10t)y + 4z  - 10t  + 2
          Type: NewSparseMultivariatePolynomial(Integer,
                   OrderedVariableList [x,y,z,t])

 lf := [f1, f2, f3, f4]
                  2
  [(2t y - 2)x + z y  - z,
       3            2        2            3      2
   - z x  + (4t y + 4)x  + (4z y  + 4z)x + 2t y  - 10y  - 10t y + 2,
     2
   (t  - 1)x + 2t z y - 2z,
      3      2                2     3             2      2
   (- z  + (4t  + 4)z)x + (4t z  + 2t  - 10t)y + 4z  - 10t  + 2]
          Type: List NewSparseMultivariatePolynomial(Integer,
                          OrderedVariableList [x,y,z,t])
```

First of all, let us solve this system in the sense of Kalkbrener.

```
 zeroSetSplit(lf)$T
     2      8      6      2                3           2
  [{t  - 1,z  - 16z  + 256z  - 256,t y - 1,(z  - 8z)x - 8z  + 16},
      2      2     2
   {3t  + 1,z  - 7t  - 1,y + t,x + z},
     8      6      2        3          2
   {t  - 10t  + 10t  - 1,z,(t  - 5t)y - 5t  + 1,x},
     2      2
   {t  + 3,z  - 4,y + t,x - z}]
     Type: List RegularTriangularSet(Integer,
                IndexedExponents OrderedVariableList [x,y,z,t],
                OrderedVariableList [x,y,z,t],
                NewSparseMultivariatePolynomial(Integer,
                    OrderedVariableList [x,y,z,t]))
```

And now in the sense of Lazard (or Wu and other authors).

```
 lts2 := zeroSetSplit(lf,false)$T
     8      6      2        3          2
  [{t  - 10t  + 10t  - 1,z,(t  - 5t)y - 5t  + 1,x},
     2      8      6      2                3           2
   {t  - 1,z  - 16z  + 256z  - 256,t y - 1,(z  - 8z)x - 8z  + 16},
      2      2     2                        2      2
   {3t  + 1,z  - 7t  - 1,y + t,x + z}, {t  + 3,z  - 4,y + t,x - z}]
      Type: List RegularTriangularSet(Integer,
                 IndexedExponents OrderedVariableList [x,y,z,t],
                 OrderedVariableList [x,y,z,t],
                 NewSparseMultivariatePolynomial(Integer,
                     OrderedVariableList [x,y,z,t]))
```

Up to the ordering of the components, both decompositions are identical.

Let us check that each component has a finite number of solutions.

```
  [coHeight(ts) for ts in lts2]
    [0,0,0,0]
                        Type: List NonNegativeInteger
```

Let us count the degrees of each component,

```
  degrees := [degree(ts) for ts in lts2]
    [8,16,4,4]
                        Type: List NonNegativeInteger
```

and compute their sum.

```
  reduce(+,degrees)
    32
                        Type: PositiveInteger
```

We study now the options of the zeroSetSplit operation. As we have seen yet, there is an optional second argument which is a boolean value. If this value is true (this is the default) then the decomposition is computed in the sense of Kalkbrener, otherwise it is computed in the sense of Lazard.

There is a second boolean optional argument that can be used (in that case the first optional argument must be present). This second option allows you to get some information during the computations.

Therefore, we need to understand a little what is going on during the computations. An important feature of the algorithm is that the intermediate computations are managed in some sense like the processes of a Unix system. Indeed, each intermediate computation may generate other intermediate computations and the management of all these computations is a crucial task for the efficiency. Thus any intermediate computation may be suspended, killed or resumed, depending on algebraic considerations that determine priorities for these processes. The goal is of course to go as fast as possible towards the final decomposition which means to avoid as much as possible unnecessary computations.

To follow the computations, one needs to set to true the second argument. Then a lot of numbers and letters are displayed. Between a [and a] one has the state of the processes at a given time. Just after [one can see the number of processes. Then each process is represented by two numbers between < and >. A process consists of a list of polynomial ps and a triangular set ts; its goal is to compute the common zeros of ps that belong to the regular-zeros set of ts. After the processes, the number between pipes gives the total number of polynomials in all the sets ps. Finally, the number between braces gives the number of components of a decomposition that are already computed. This number may decrease.

Let us take a third example (Czapor-Geddes-Wang) to see how this information is displayed.

Define a polynomial system.

```
  u : R := 2
    2
                          Type: Integer

  q1 := 2*(u-1)**2+ 2*(x-z*x+z**2)+ y**2*(x-1)**2- 2*u*x+ 2*y*t*(1-x)*(x-z)+_
        2*u*z*t*(t-y)+ u**2*t**2*(1-2*z)+ 2*u*t**2*(z-x)+ 2*u*t*y*(z-1)+_
        2*u*z*x*(y+1)+ (u**2-2*u)*z**2*t**2+ 2*u**2*z**2+ 4*u*(1-u)*z+_
        t**2*(z-x)**2}
       2           2  2        2                             2           2
     (y  - 2t y + t )x  + (- 2y  + ((2t + 4)z + 2t)y + (- 2t  + 2)z - 4t  - 2)x
   +
      2                      2       2           2
     y  + (- 2t z - 4t)y + (t  + 10)z  - 8z + 4t  + 2
       Type: NewSparseMultivariatePolynomial(Integer,
               OrderedVariableList [x,y,z,t])

  q2 := t*(2*z+1)*(x-z)+ y*(z+2)*(1-x)+ u*(u-2)*t+ u*(1-2*u)*z*t+_
        u*y*(x+u-z*x-1)+ u*(u+1)*z**2*t}
    (- 3z y + 2t z + t)x + (z + 4)y + 4t z  - 7t z
       Type: NewSparseMultivariatePolynomial(Integer,
               OrderedVariableList [x,y,z,t])

  q3 := -u**2*(z-1)**2+ 2*z*(z-x)-2*(x-1)
    (- 2z - 2)x - 2z  + 8z - 2
       Type: NewSparseMultivariatePolynomial(Integer,
               OrderedVariableList [x,y,z,t])

  q4 := u**2+4*(z-x**2)+3*y**2*(x-1)**2- 3*t**2*(z-x)**2+_
        3*u**2*t**2*(z-1)**2+u**2*z*(z-2)+6*u*t*y*(z+x+z*x-1)}
       2     2     2        2                     2        2
    (3y  - 3t  - 4)x  + (- 6y  + (12t z + 12t)y + 6t z)x + 3y  + (12t z - 12t)y
   +
       2      2         2            2
    (9t  + 4)z  + (- 24t  - 4)z + 12t  + 4
        Type: NewSparseMultivariatePolynomial(Integer,
                OrderedVariableList [x,y,z,t])

  lq := [q1, q2, q3, q4]
   [
         2           2  2
       (y  - 2t y + t )x
     +
           2                            2           2          2
```

```
      (- 2y  + ((2t + 4)z + 2t)y + (- 2t  + 2)z - 4t  - 2)x + y
    +
                          2       2          2
      (- 2t z - 4t)y + (t  + 10)z  - 8z + 4t  + 2
    ,
                                             2                          2
   (- 3z y + 2t z + t)x + (z + 4)y + 4t z  - 7t z, (- 2z - 2)x - 2z  + 8z - 2,

        2     2      2        2                      2        2
      (3y  - 3t  - 4)x  + (- 6y  + (12t z + 12t)y + 6t z)x + 3y
    +
                           2      2         2            2
      (12t z - 12t)y + (9t  + 4)z  + (- 24t  - 4)z + 12t  + 4
    ]
       Type: List NewSparseMultivariatePolynomial(Integer,
                    OrderedVariableList [x,y,z,t])
```

Let us try the information option. N.B. The timing should be between
1 and 10 minutes, depending on your machine.

```
  zeroSetSplit(lq,true,true)$T
     [1 <4,0> -> |4|; {0}]W[2 <5,0>,<3,1> -> |8|; {0}]
     [2 <4,1>,<3,1> -> |7|; {0}]
     [1 <3,1> -> |3|; {0}]G
     [2 <4,1>,<4,1> -> |8|; {0}]W
     [3 <5,1>,<4,1>,<3,2> -> |12|; {0}]GI
     [3 <4,2>,<4,1>,<3,2> -> |11|; {0}]GWw
     [3 <4,1>,<3,2>,<5,2> -> |12|; {0}]
     [3 <3,2>,<3,2>,<5,2> -> |11|; {0}]GIwWWWw
     [4 <3,2>,<4,2>,<5,2>,<2,3> -> |14|; {0}]
     [4 <2,2>,<4,2>,<5,2>,<2,3> -> |13|; {0}]Gwww
     [5 <3,2>,<3,2>,<4,2>,<5,2>,<2,3> -> |17|; {0}]Gwwwwww
     [8 <3,2>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |30|; {0}]Gwwwwww
     [8 <4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |31|; {0}]
     [8 <3,3>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |30|; {0}]
     [8 <2,3>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |29|; {0}]
     [8 <1,3>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |28|; {0}]
     [7 <4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |27|; {0}]
     [6 <4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |23|; {0}]
     [5 <4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |19|; {0}]GIGIWwww
     [6 <5,2>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |23|; {0}]
     [6 <4,3>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |22|; {0}]GIGI
     [6 <3,4>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |21|; {0}]
     [6 <2,4>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |20|; {0}]GGG
     [5 <4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |18|; {0}]GIGIWwwwW
     [6 <5,2>,<4,2>,<5,2>,<3,3>,<3,3>,<2,3> -> |22|; {0}]
     [6 <4,3>,<4,2>,<5,2>,<3,3>,<3,3>,<2,3> -> |21|; {0}]
     GIwwWwWWWWWWWwWWWWwwwww
     [8 <4,2>,<5,2>,<3,3>,<3,3>,<4,3>,<2,3>,<3,4>,<3,4> -> |27|; {0}]
     [8 <3,3>,<5,2>,<3,3>,<3,3>,<4,3>,<2,3>,<3,4>,<3,4> -> |26|; {0}]
```

```
[8 <2,3>,<5,2>,<3,3>,<3,3>,<4,3>,<2,3>,<3,4>,<3,4> -> |25|; {0}]
Gwwwwwwwwwwwwwwwwwwww
[9 <5,2>,<3,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,4>,<3,4> -> |29|; {0}]GI
[9 <4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,4>,<3,4> -> |28|; {0}]
[9 <3,3>,<3,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,4>,<3,4> -> |27|; {0}]
[9 <2,3>,<3,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,4>,<3,4> -> |26|; {0}]
GGwwwwwwwwwwwwwWWwwwwwww
[11 <3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
   -> |33|; {0}]
[11 <2,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |32|; {0}]
[11 <1,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |31|; {0}]GGGwwwwwwwwwwwww
[12 <2,3>,<2,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |34|; {0}]GGwwwwwwwwwwwww
[13 <3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |38|; {0}]Gwwwwwwwwwwwww
[13 <2,3>,<3,3>,<4,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |39|; {0}]GGGwwwwwwwwwwwww
[15 <3,3>,<4,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,
  <3,3>,<3,3>,<3,4>,<3,4> -> |46|; {0}]
[14 <4,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,
  <3,3>,<3,4>,<3,4> -> |43|; {0}]GIGGGGIGGI
[14 <3,4>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,
  <3,3>,<3,4>,<3,4> -> |42|; {0}]GGG
[14 <2,4>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,
  <3,3>,<3,4>,<3,4> -> |41|; {0}]
[14 <1,4>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,
  <3,3>,<3,4>,<3,4> -> |40|; {0}]GGG
[13 <3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |39|; {0}]Gwwwwwwwwwwwww
[15 <3,3>,<3,3>,<4,3>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,
  <3,3>,<3,3>,<3,4>,<3,4> -> |48|; {0}]Gwwwwwwwwwwwww
[15 <4,3>,<4,3>,<3,3>,<4,3>,<4,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,3>,
  <3,3>,<3,3>,<3,4>,<3,4> -> |49|; {0}]GIGI
[15 <3,4>,<4,3>,<3,3>,<4,3>,<4,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,3>,
  <3,3>,<3,3>,<3,4>,<3,4> -> |48|; {0}]G
[14 <4,3>,<3,3>,<4,3>,<4,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,3>,<3,3>,
  <3,3>,<3,4>,<3,4> -> |45|; {0}]
[13 <3,3>,<4,3>,<4,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |41|; {0}]Gwwwwwwwwwwwww
[13 <4,3>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |42|; {0}]GIGGGGIGGI
[13 <3,4>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |41|; {0}]GGGGGGGG
[13 <2,4>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |40|; {0}]
[13 <1,4>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4>  -> |39|; {0}]
[13 <0,4>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,
```

```
  <3,4>,<3,4> -> |38|; {0}]
[12 <4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |38|; {1}]
[11 <4,3>,<3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |34|; {1}]
[10 <3,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |30|; {1}]
[10 <2,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |29|; {1}]GGGwwwwwwwwwwwww
[11 <3,3>,<3,3>,<4,3>,<3,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |33|; {1}]GGGwwwwwwwwwwwww
[12 <4,3>,<3,3>,<4,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |38|; {1}]Gwwwwwwwwwwwww
[12 <3,3>,<4,3>,<5,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |39|; {1}]GGwwwwwwwwwwwww
[13 <5,3>,<4,3>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |44|; {1}]GIGGGGIGGIW
[13 <4,4>,<4,3>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |43|; {1}]GGW
[13 <3,4>,<4,3>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |42|; {1}]GGG
[12 <4,3>,<4,3>,<4,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |39|; {1}]Gwwwwwwwwwwwww
[12 <4,3>,<4,3>,<5,3>,<3,3>,<4,3>,<3,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |40|; {1}]Gwwwwwwwwwwwww
[13 <5,3>,<5,3>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |46|; {1}]GIGIW
[13 <4,4>,<5,3>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |45|; {1}]
[13 <3,4>,<5,3>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |44|; {1}]
[13 <2,4>,<5,3>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,
  <3,4>,<3,4> -> |43|; {1}]GG
[12 <5,3>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |41|; {1}]GIGGGGIGGIW
[12 <4,4>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |40|; {1}]GGGGGGW
[12 <3,4>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |39|; {1}]
[12 <2,4>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |38|; {1}]
[12 <1,4>,<4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,
  <3,4> -> |37|; {1}]GGG
[11 <4,3>,<5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |36|; {1}]
[10 <5,3>,<3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4>
  -> |32|; {1}]
[9 <3,3>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |27|; {1}]W
[9 <2,4>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |26|; {1}]
[9 <1,4>,<3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |25|; {1}]
```

```
  [8 <3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |24|; {1}]W
  [8 <2,4>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |23|; {1}]
  [8 <1,4>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |22|; {1}]
  [7 <4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |21|; {1}]w
  [7 <3,4>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |20|; {1}]
  [7 <2,4>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |19|; {1}]
  [7 <1,4>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |18|; {1}]
  [6 <2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |17|; {1}]GGwwwwww
  [7 <3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |21|; {1}]GIW
  [7 <2,4>,<3,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |20|; {1}]GG
  [6 <3,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |18|; {1}]Gwwwwww
  [7 <4,3>,<4,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |23|; {1}]GIW
  [7 <3,4>,<4,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |22|; {1}]
  [6 <4,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |19|; {1}]GIW
  [6 <3,4>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |18|; {1}]GGW
  [6 <2,4>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |17|; {1}]
  [6 <1,4>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |16|; {1}]GGG
  [5 <3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |15|; {1}]GIW
  [5 <2,4>,<3,3>,<3,3>,<3,4>,<3,4> -> |14|; {1}]GG
  [4 <3,3>,<3,3>,<3,4>,<3,4> -> |12|; {1}]
  [3 <3,3>,<3,4>,<3,4> -> |9|; {1}]W
  [3 <2,4>,<3,4>,<3,4> -> |8|; {1}]
  [3 <1,4>,<3,4>,<3,4> -> |7|; {1}]G
  [2 <3,4>,<3,4> -> |6|; {1}]G
  [1 <3,4> -> |3|; {1}]
  [1 <2,4> -> |2|; {1}]
  [1 <1,4> -> |1|; {1}]
*** QCMPACK Statistics ***
   Table     size:  36
   Entries reused:  255

*** REGSETGCD: Gcd Statistics ***
   Table     size:  125
   Entries reused:  0

*** REGSETGCD: Inv Set Statistics ***
   Table     size:  30
   Entries reused:  0

[
  {
                     24                    23                     22
      960725655771966t   + 386820897948702t   + 8906817198608181t
    +
                      21                     20                      19
      2704966893949428t   + 37304033340228264t   + 7924782817170207t
    +
                       18                     17                       16
      93126799040354990t   + 13101273653130910t   + 156146250424711858t
    +
```

```
                      15                       14                     13
  16626490957259119t    + 190699288479805763t    + 24339173367625275t
+
                       12                      11                     10
  18053231301496013 5t    + 3528808903097537 8t    + 13505497574765628 5t
+
                      9                     8                    7
  347337369524885 40t  + 759476003544939 72t  + 197725556924570 88t
+
                      6                    5                    4
  288715585737554 28t  + 557615243908166 4t  + 632171182035297 6t
+
                  3                   2
438314209312320t  + 581105748367008t  - 60254467992576t + 1449115951104
,

                                                                   23
      2660421086949130238551526573705208236166847418137289185778 4t
    +
                                                                    22
      443104378424686086067294899528296664238693556855017735265295t
    +
                                                                    21
      279078393286701234679141342358988327155321305829547090310242t
    +
                                                                     20
      339027636141323246510761717661554305462062639182361339218522 6t
    +
                                                                    19
      941478179503540575554198645220352803719793196473813837434129t
    +
                                                                      18
      115478551946794752422116967496739493525857476741843209881443 90t
    +
                                                                     17
      134360956676559778988170165669941321646721566033335641724143 2t
    +
                                                                      16
      232338138681478735039335516171756408598991029878006635666993 34t
    +
                                                                    15
      869574020537672336950845440508790740850931336484983573386433t
    +
                                                                      14
      315615543058769348754194614869699265542417500651034608204769 69t
    +
                                                                     13
      127140099028771748744206595254773187955482388985538607226493 1t
    +
                                                                      12
```

```
      31945089913863736044802526964079540198337049550503295825160523t
    +
                                                                  11
      3738735704288144509871371560232845884439102270778010470931960t
    +
                                                                   10
      25293997512391412026144601435771131587561905532992045692885927t
    +
                                                                  9
      5210239009846067123469262799870052773410471135950175008046524t
    +
                                                                   8
      15083887986930297166259870568608270427403187606238713491129188t
    +
                                                                  7
      3522087234692930126383686270775779553481769125670839075109000t
    +
                                                                  6
      6079945200395681013086533792568886491101244247440034969288588t
    +
                                                                  5
      1090634852433900888199913756247986023196987723469934933603680t
    +
                                                                  4
      1405819430871907102294432537538335402102838994019667487458352t
    +
                                                                3
      88071527950320450072536671265507748878347828884933605202432t
    +
                                                                 2
      135882489433640933229781177155977768016065765482378657129440t
    +
      - 13957283442882262230559894607400314082516690749975646520320t
    +
      334637692973189299277258325709308472592117112855749713920
 *
    z
+
                                                            23
  8567175484043952879756725964506833932149637101090521164936t
+
                                                              22
  149792392864201791845708374032728942498797519251667250945721t
+
                                                             21
  77258371783645822157410861582159764138123003074190374021550t
+
                                                               20
  1108862254126854214498918940708612211184560556764334742191654t
+
```

```
                                                                     19
   21325049446067886521977448010682605378381578962150173267232 7t
```

```
                                                                     19
   213250494460678865219774480106826053783815789621501732672327t
 +
                                                                      18
   366892907516066619572917789417834351450198789841013143169988 2t
 +
                                                                     17
   171388906471001872879490124368748236314765459039567820048872t
 +
                                                                      16
   719243074691460216666023347733102248314492177164552313965898 6t
 +
                                                                        15
   - 128798674689690072812879965633090291959663143108437362453385t
 +
                                                                      14
   955301085834142590930642313292113404085602879080352643027067 1t
 +
                                                                      13
   - 13296096245675492874538687646300437824658458709144441096603t
 +
                                                                      12
   947580680581414532638308551832533310688169056864427496486441 3t
 +
                                                                     11
   803234687925133458861659855664084927606298794799856265539336t
 +
                                                                      10
   733820275929286516599462234920751640066217430261459517333382 5t
 +
                                                                      9
   130800462848036735116436961311197166888053885564091720018710 8t
 +
                                                                      8
   426805945574125549888022959897370574709821606769775435263474 8t
 +
                                                                     7
   892893526858514095791318775904093300103045601514470613580600t
 +
                                                                      6
   167915257546068395663192585218134150198159813746532879701365 2t
 +
                                                                     5
   269757415767922980378967154143357835544113158280591408043936t
 +
                                                                     4
   380951527864657529033580829801282724081345372680202920198224t
 +
                                                                    3
   19785545294228495032998826937601341132725035339452913286656t
```

```
   +
                                                                         2
     36477412057384782942366635303396637763303928174935079178528t
   +
     - 372221287927903864871308042224976273210890229485838670848t
   +
     890797248531143483612306344840138620247285999068741058 56
   ,
     3     2                  3      2
  (3z  - 11z  + 8z + 4)y + 2t z  + 4t z  - 5t z - t,
             2
  (z + 1)x + z  - 4z + 1}
 ]
   Type: List RegularTriangularSet(Integer,
               IndexedExponents OrderedVariableList [x,y,z,t],
               OrderedVariableList [x,y,z,t],
               NewSparseMultivariatePolynomial(Integer,
                 OrderedVariableList [x,y,z,t]))
```

Between a sequence of processes, thus between a] and a [you can see capital letters W, G, I and lower case letters i, w. Each time a capital letter appears a non-trivial computation has be performed and its result is put in a hash-table. Each time a lower case letter appears a needed result has been found in an hash-table. The use of these hash-tables generally speed up the computations. However, on very large systems, it may happen that these hash-tables become too big to be handle by your AXIOM configuration. Then in these exceptional cases, you may prefer getting a result (even if it takes a long time) than getting nothing. Hence you need to know how to prevent the RSEGSET constructor from using these hash-tables. In that case you will be using the zeroSetSplit with five arguments. The first one is the input system lp as above. The second one is a boolean value hash? which is true iff you want to use hash-tables. The third one is boolean value clos? which is true iff you want to solve your system in the sense of Kalkbrener, the other way remaining that of Lazard. The fourth argument is boolean value info? which is true iff you want to display information during the computations. The last one is boolean value prep? which is true iff you want to use some heuristics that are performed on the input system before starting the real algorithm. The value of this flag is true when you are using zeroSetSplit with less than five arguments. Note that there is no available signature for zeroSetSplit with four arguments.

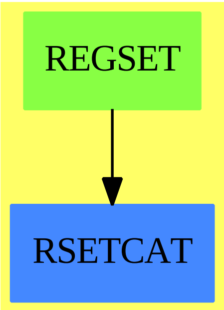
We finish this section by some remarks about both ways of solving, in the sense of Kalkbrener or in the sense of Lazard. For problems with a finite number of solutions, there are theoretically equivalent and the resulting decompositions are identical, up to the ordering of the components. However, when solving in the sense of Lazard, the algorithm behaves differently. In that case, it becomes more incremental than in the sense of Kalkbrener. That means the

```
polynomials of the input system are considered one after another
whereas in the sense of Kalkbrener the input system is treated more
globally.

This makes an important difference in positive dimension.  Indeed when
solving in the sense of Kalkbrener, the Primeidealkettensatz of Krull
is used.  That means any regular triangular containing more
polynomials than the input system can be deleted.  This is not
possible when solving in the sense of Lazard.  This explains why
Kalkbrener's decompositions usually contain less components than those
of Lazard.  However, it may happen with some examples that the
incremental process (that cannot be used when solving in the sense of
Kalkbrener) provide a more efficient way of solving than the global
one even if the Primeidealkettensatz is used.  Thus just try both,
with the various options, before concluding that you cannot solve your
favorite system with zeroSetSplit.  There exist more options at the
development level that are not currently available in this public
version.

See Also:
o )help GcdDomain
o )help OrderedAbelianMonoidSup
o )help OrderedSet
o )help RecursivePolynomialCategory
o )help RegularChain
o )help NewSparseMultivariatePolynomial
o )help ZeroDimensionalSolvePackage
o )help LexTriangularPackage
o )help LazardSetSolvingPackage
o )help SquareFreeRegularTriangularSet
o )show RegularTriangularSet
```

19.7.1 RegularTriangularSet (REGSET)



Exports:

algebraic?
algebraicCoefficients?
algebraicVariables
any?
augment
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
convert
copy
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headRemainder
headReduce
headReduced?
infRittWu?
initiallyReduce
initiallyReduced?
initials
internalAugment
internalZeroSetSplit
intersect
invertible?
invertibleElseSplit?
invertibleSet
last
lastSubResultant
lastSubResultantElseSplit
latex
less?
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
preprocess
purelyAlgebraic?
purelyAlgebraicLeadingMonomial?
purelyTranscendental?
quasiComponent
reduce
reduced?
reduceByQuasiMonic
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
squareFreePart
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?~=?
?=?

— domain REGSET RegularTriangularSet —

```
)abbrev domain REGSET RegularTriangularSet
++ Author: Marc Moreno Maza
++ Date Created: 08/25/1998
++ Date Last Updated: 16/12/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++  [1] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
++ Description:
++ This domain provides an implementation of regular chains.
++ Moreover, the operation zeroSetSplit is an implementation of a new
++ algorithm for solving polynomial systems by means of regular chains.

RegularTriangularSet(R,E,V,P) : Exports == Implementation where

  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)
  N ==> NonNegativeInteger
  Z ==> Integer
  B ==> Boolean
  LP ==> List P
  PtoP ==> P -> P
  PS ==> GeneralPolynomialSet(R,E,V,P)
  PWT ==> Record(val : P, tower : $)
  BWT ==> Record(val : Boolean, tower : $)
  LpWT ==> Record(val : (List P), tower : $)
  Split ==> List $
  iprintpack ==> InternalPrintPackage()
  polsetpack ==> PolynomialSetUtilitiesPackage(R,E,V,P)
  quasicomppack ==> QuasiComponentPackage(R,E,V,P,$)
  regsetgcdpack ==> RegularTriangularSetGcdPackage(R,E,V,P,$)
  regsetdecomppack ==> RegularSetDecompositionPackage(R,E,V,P,$)

  Exports ==  RegularTriangularSetCategory(R,E,V,P) with

     internalAugment: (P,$,B,B,B,B,B) -> List $
       ++ \axiom{internalAugment(p,ts,b1,b2,b3,b4,b5)}
       ++ is an internal subroutine, exported only for developement.
     zeroSetSplit: (LP, B, B) -> Split
       ++ \axiom{zeroSetSplit(lp,clos?,info?)} has the same specifications as
       ++ zeroSetSplit from RegularTriangularSetCategory.
```

```
      ++ Moreover, if \axiom{clos?} then solves in the sense of the Zariski closure
      ++ else solves in the sense of the regular zeros. If \axiom{info?} then
      ++ do print messages during the computations.
    zeroSetSplit: (LP, B, B, B, B) -> Split
      ++ \axiom{zeroSetSplit(lp,b1,b2.b3,b4)}
      ++ is an internal subroutine, exported only for developement.
    internalZeroSetSplit: (LP, B, B, B) -> Split
      ++ \axiom{internalZeroSetSplit(lp,b1,b2,b3)}
      ++ is an internal subroutine, exported only for developement.
    pre_process: (LP, B, B) -> Record(val: LP, towers: Split)
      ++ \axiom{pre_process(lp,b1,b2)}
      ++ is an internal subroutine, exported only for developement.

 Implementation == add

    Rep ==> LP

    rep(s:$):Rep == s pretend Rep
    per(l:Rep):$ == l pretend $

    copy ts ==
      per(copy(rep(ts))$LP)
    empty() ==
      per([])
    empty?(ts:$) ==
      empty?(rep(ts))
    parts ts ==
      rep(ts)
    members ts ==
      rep(ts)
    map (f : PtoP, ts : $) : $ ==
      construct(map(f,rep(ts))$LP)$$
    map! (f : PtoP, ts : $) : $  ==
      construct(map!(f,rep(ts))$LP)$$
    member? (p,ts) ==
      member?(p,rep(ts))$LP
    unitIdealIfCan() ==
      "failed"::Union($,"failed")
    roughUnitIdeal? ts ==
      false
    coerce(ts:$) : OutputForm ==
      lp : List(P) := reverse(rep(ts))
      brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm
    mvar ts ==
      empty? ts => error "mvar$REGSET: #1 is empty"
      mvar(first(rep(ts)))$P
    first ts ==
      empty? ts => "failed"::Union(P,"failed")
      first(rep(ts))::Union(P,"failed")
    last ts ==
```

```
      empty? ts => "failed"::Union(P,"failed")
      last(rep(ts))::Union(P,"failed")
    rest ts ==
      empty? ts => "failed"::Union($,"failed")
      per(rest(rep(ts)))::Union($,"failed")
    coerce(ts:$) : (List P) ==
      rep(ts)

    collectUpper (ts,v) ==
      empty? ts => ts
      lp := rep(ts)
      newlp : Rep := []
      while (not empty? lp) and (mvar(first(lp)) > v) repeat
        newlp := cons(first(lp),newlp)
        lp := rest lp
      per(reverse(newlp))

    collectUnder (ts,v) ==
      empty? ts => ts
      lp := rep(ts)
      while (not empty? lp) and (mvar(first(lp)) >= v) repeat
        lp := rest lp
      per(lp)

    construct(lp:List(P)) ==
      ts : $ := per([])
      empty? lp => ts
      lp := sort(infRittWu?,lp)
      while not empty? lp repeat
        eif := extendIfCan(ts,first(lp))
        not (eif case $) =>
          error"in construct : List P -> $  from REGSET : bad #1"
        ts := eif::$
        lp := rest lp
      ts

    extendIfCan(ts:$,p:P) ==
      ground? p => "failed"::Union($,"failed")
      empty? ts =>
        p := primitivePart p
        (per([p]))::Union($,"failed")
      not (mvar(ts) < mvar(p)) => "failed"::Union($,"failed")
      invertible?(init(p),ts)@Boolean =>
        (per(cons(p,rep(ts))))::Union($,"failed")
      "failed"::Union($,"failed")

    removeZero(p:P, ts:$): P ==
      (ground? p) or (empty? ts) => p
      v := mvar(p)
      ts_v_- := collectUnder(ts,v)
```

```
    if algebraic?(v,ts)
      then
        q := lazyPrem(p,select(ts,v)::P)
        zero? q => return q
        zero? removeZero(q,ts_v_-) => return 0
    empty? ts_v_- => p
    q: P := 0
    while positive? degree(p,v) repeat
       q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
       p := tail(p)
    q + removeZero(p,ts_v_-)

  internalAugment(p:P,ts:$): $ ==
    -- ASSUME that adding p to ts DOES NOT require any split
    ground? p => error "in internalAugment$REGSET: ground? #1"
    first(internalAugment(p,ts,false,false,false,false,false))

  internalAugment(lp:List(P),ts:$): $ ==
    -- ASSUME that adding p to ts DOES NOT require any split
    empty? lp => ts
    internalAugment(rest lp, internalAugment(first lp, ts))

  internalAugment(p:P,ts:$,rem?:B,red?:B,prim?:B,sqfr?:B,extend?:B): Split ==
    -- ASSUME p is not a constant
    -- ASSUME mvar(p) is not algebraic w.r.t. ts
    -- ASSUME init(p) invertible modulo ts
    -- if rem? then REDUCE p by remainder
    -- if prim? then REPLACE p by its main primitive part
    -- if sqfr? then FACTORIZE SQUARE FREE p over R
    -- if extend? DO NOT ASSUME every pol in ts_v_+ is invertible modulo ts
    v := mvar(p)
    ts_v_- := collectUnder(ts,v)
    ts_v_+ := collectUpper(ts,v)
    if rem? then p := remainder(p,ts_v_-).polnum
    -- if rem? then p := reduceByQuasiMonic(p,ts_v_-)
    if red? then p := removeZero(p,ts_v_-)
    if prim? then p := mainPrimitivePart p
    if sqfr?
      then
        lsfp := squareFreeFactors(p)$polsetpack
        lts: Split := [per(cons(f,rep(ts_v_-))) for f in lsfp]
      else
        lts: Split := [per(cons(p,rep(ts_v_-)))]
    extend? => extend(members(ts_v_+),lts)
    [per(concat(rep(ts_v_+),rep(us))) for us in lts]

  augment(p:P,ts:$): List $ ==
    ground? p => error "in augment$REGSET: ground? #1"
    algebraic?(mvar(p),ts) => error "in augment$REGSET: bad #1"
    -- ASSUME init(p) invertible modulo ts
```

```
    -- DOES NOT ASSUME anything else.
    -- THUS reduction, mainPrimitivePart and squareFree are NEEDED
    internalAugment(p,ts,true,true,true,true,true)

  extend(p:P,ts:$): List $ ==
    ground? p => error "in extend$REGSET: ground? #1"
    v := mvar(p)
    not (mvar(ts) < mvar(p)) => error "in extend$REGSET: bad #1"
    lts: List($) := []
    split: List($) := invertibleSet(init(p),ts)
    for us in split repeat
      lts := concat(augment(p,us),lts)
    lts

  invertible?(p:P,ts:$): Boolean ==
    toseInvertible?(p,ts)$regsetgcdpack

  invertible?(p:P,ts:$): List BWT ==
    toseInvertible?(p,ts)$regsetgcdpack

  invertibleSet(p:P,ts:$): Split ==
    toseInvertibleSet(p,ts)$regsetgcdpack

  lastSubResultant(p1:P,p2:P,ts:$): List PWT ==
    toseLastSubResultant(p1,p2,ts)$regsetgcdpack

  squareFreePart(p:P, ts: $): List PWT ==
    toseSquareFreePart(p,ts)$regsetgcdpack

  intersect(p:P, ts: $): List($) == decompose([p], [ts], false, false)$regsetdecomppack

  intersect(lp: LP, lts: List($)): List($) == decompose(lp, lts, false, false)$regsetdecomppack
     -- SOLVE in the regular zero sense
     -- and DO NOT PRINT info

  decompose(p:P, ts: $): List($) == decompose([p], [ts], true, false)$regsetdecomppack

  decompose(lp: LP, lts: List($)): List($) == decompose(lp, lts, true, false)$regsetdecomppack
     -- SOLVE in the closure sense
     -- and DO NOT PRINT info

  zeroSetSplit(lp:List(P)) == zeroSetSplit(lp,true,false)
     -- by default SOLVE in the closure sense
     -- and DO NOT PRINT info

  zeroSetSplit(lp:List(P), clos?: B) == zeroSetSplit(lp,clos?, false)
     -- DO NOT PRINT info

  zeroSetSplit(lp:List(P), clos?: B, info?: B) ==
    -- if clos? then SOLVE in the closure sense
```

```
  -- if info? then PRINT info
  -- by default USE hash-tables
  -- and PREPROCESS the input system
  zeroSetSplit(lp,true,clos?,info?,true)

zeroSetSplit(lp:List(P),hash?:B,clos?:B,info?:B,prep?:B) ==
  -- if hash? then USE hash-tables
  -- if info? then PRINT information
  -- if clos? then SOLVE in the closure sense
  -- if prep? then PREPROCESS the input system
  if hash?
    then
      s1, s2, s3, dom1, dom2, dom3: String
      e: String := empty()$String
      if info? then (s1,s2,s3) := ("w","g","i") else (s1,s2,s3) := (e,e,e)
      if info?
        then
          (dom1, dom2, dom3) := ("QCMPACK", "REGSETGCD: Gcd", "REGSETGCD: Inv Set")
        else
          (dom1, dom2, dom3) := (e,e,e)
      startTable!(s1,"W",dom1)$quasicomppack
      startTableGcd!(s2,"G",dom2)$regsetgcdpack
      startTableInvSet!(s3,"I",dom3)$regsetgcdpack
  lts := internalZeroSetSplit(lp,clos?,info?,prep?)
  if hash?
    then
      stopTable!()$quasicomppack
      stopTableGcd!()$regsetgcdpack
      stopTableInvSet!()$regsetgcdpack
  lts

internalZeroSetSplit(lp:LP,clos?:B,info?:B,prep?:B) ==
  -- if info? then PRINT information
  -- if clos? then SOLVE in the closure sense
  -- if prep? then PREPROCESS the input system
  if prep?
    then
      pp := pre_process(lp,clos?,info?)
      lp := pp.val
      lts := pp.towers
    else
      ts: $ := [[]]
      lts := [ts]
  lp := remove(zero?, lp)
  any?(ground?, lp) => []
  empty? lp => lts
  empty? lts => lts
  lp := sort(infRittWu?,lp)
  clos? => decompose(lp,lts, clos?, info?)$regsetdecomppack
  -- IN DIM > 0 with clos? the following is false ...
```

```
        for p in lp repeat
          lts := decompose([p],lts, clos?, info?)$regsetdecomppack
        lts

      largeSystem?(lp:LP): Boolean ==
        -- Gonnet and Gerdt and not Wu-Wang.2
        #lp > 16 => true
        #lp < 13 => false
        lts: List($) := []
        (#lp :: Z - numberOfVariables(lp,lts)$regsetdecomppack :: Z) > 3

      smallSystem?(lp:LP): Boolean ==
        -- neural, Vermeer, Liu, and not f-633 and not Hairer-2
        #lp < 5

      mediumSystem?(lp:LP): Boolean ==
        -- f-633 and not Hairer-2
        lts: List($) := []
        (numberOfVariables(lp,lts)$regsetdecomppack :: Z - #lp :: Z) < 2

--      lin?(p:P):Boolean == ground?(init(p)) and one?(mdeg(p))
      lin?(p:P):Boolean == ground?(init(p)) and (mdeg(p) = 1)

      pre_process(lp:LP,clos?:B,info?:B): Record(val: LP, towers: Split) ==
        -- if info? then PRINT information
        -- if clos? then SOLVE in the closure sense
        ts: $ := [[]];
        lts: Split := [ts]
        empty? lp => [lp,lts]
        lp1: List P := []
        lp2: List P := []
        for p in lp repeat
           ground? (tail p) => lp1 := cons(p, lp1)
           lp2 := cons(p, lp2)
        lts: Split := decompose(lp1,[ts],clos?,info?)$regsetdecomppack
        probablyZeroDim?(lp)$polsetpack =>
           largeSystem?(lp) => return [lp2,lts]
           if #lp > 7
             then
               -- Butcher (8,8) + Wu-Wang.2 (13,16)
               lp2 := crushedSet(lp2)$polsetpack
               lp2 := remove(zero?,lp2)
               any?(ground?,lp2) => return [lp2, lts]
               lp3 := [p for p in lp2 | lin?(p)]
               lp4 := [p for p in lp2 | not lin?(p)]
               if clos?
                 then
                   lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
                 else
                   lp4 := sort(infRittWu?,lp4)
```

```
              for p in lp4 repeat
                lts := decompose([p],lts, clos?, info?)$regsetdecomppack
            lp2 := lp3
          else
            lp2 := crushedSet(lp2)$polsetpack
            lp2 := remove(zero?,lp2)
            any?(ground?,lp2) => return [lp2, lts]
        if clos?
          then
            lts := decompose(lp2,lts, clos?, info?)$regsetdecomppack
          else
            lp2 := sort(infRittWu?,lp2)
            for p in lp2 repeat
              lts := decompose([p],lts, clos?, info?)$regsetdecomppack
        lp2 := []
        return [lp2,lts]
      smallSystem?(lp) => [lp2,lts]
      mediumSystem?(lp) => [crushedSet(lp2)$polsetpack,lts]
      lp3 := [p for p in lp2 | lin?(p)]
      lp4 := [p for p in lp2 | not lin?(p)]
      if clos?
        then
          lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
        else
          lp4 := sort(infRittWu?,lp4)
          for p in lp4 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
      if clos?
        then
          lts := decompose(lp3,lts, clos?, info?)$regsetdecomppack
        else
          lp3 := sort(infRittWu?,lp3)
          for p in lp3 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
      lp2 := []
      return [lp2,lts]
```

————

— **REGSET.dotabb** —

```
"REGSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=REGSET"]
"RSETCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RSETCAT"]
"REGSET" -> "RSETCAT"
```

————

19.8 domain RESRING ResidueRing

— ResidueRing.input —

```
)set break resume
)sys rm -f ResidueRing.output
)spool ResidueRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ResidueRing
--R ResidueRing(F: Field,Expon: OrderedAbelianMonoidSup,VarSet: OrderedSet,FPol: PolynomialCategory(F,Ex
--R Abbreviation for ResidueRing is RESRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RESRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,F) -> %                      ?*? : (F,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : FPol -> %
--R coerce : F -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   lift : % -> FPol
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R reduce : FPol -> %                    sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

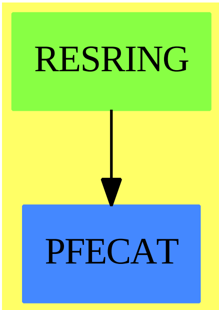
— ResidueRing.help —

```
====================================================================
```

```
ResidueRing examples
====================================================================

See Also:
o )show ResidueRing
```

19.8.1 ResidueRing (RESRING)



Exports:

0	1	characteristic	coerce	hash
latex	lift	one?	recip	reduce
sample	subtractIfCan	zero?	?~=?	?*?
?**?	?^?	?+?	?-?	-?
?=?				

— domain RESRING ResidueRing —

```
)abbrev domain RESRING ResidueRing
++ Author: P.Gianni
++ Date Created: December 1992
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ ResidueRing is the quotient of a polynomial ring by  an ideal.
++ The ideal is given as a list of generators. The elements of the domain
++ are equivalence classes expressed in terms of reduced elements

ResidueRing(F,Expon,VarSet,FPol,LFPol) : Dom  == Body
```

```
 where
   F       :  Field
   Expon   :  OrderedAbelianMonoidSup
   VarSet  :  OrderedSet
   FPol    :  PolynomialCategory(F, Expon, VarSet)
   LFPol   :  List FPol

   Dom   == Join(CommutativeRing, Algebra F) with
     reduce   :   FPol -> $
     ++ reduce(f) produces the equivalence class of f in the residue ring
     coerce   :   FPol -> $
     ++ coerce(f) produces the equivalence class of f in the residue ring
     lift     :     $  -> FPol
     ++ lift(x) return the canonical representative of the equivalence class x
   Body  ==  add
    --representation
      Rep:= FPol
      import GroebnerPackage(F,Expon,VarSet,FPol)
      relations:= groebner(LFPol)
      relations = [1] => error "the residue ring is the zero ring"
    --declarations
      x,y: $
    --definitions
      0 == 0$Rep
      1 == 1$Rep
      reduce(f : FPol) : $ == normalForm(f,relations)
      coerce(f : FPol) : $ == normalForm(f,relations)
      lift x  == x :: Rep :: FPol
      x + y == x +$Rep y
      -x == -$Rep x
      x*y == normalForm(lift(x *$Rep y),relations)
      (n : Integer) * x == n *$Rep x
      (a : F) * x == a *$Rep x
      x = y == x =$Rep y
      characteristic()      == characteristic()$F
      coerce(x) : OutputForm == coerce(x)$Rep
```

———-

— **RESRING.dotabb** —

```
"RESRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RESRING"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"RESRING" -> "PFECAT"
```

———-

19.9 domain RESULT Result

— Result.input —

```
)set break resume
)sys rm -f Result.output
)spool Result.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Result
--R Result  is a domain constructor
--R Abbreviation for Result is RESULT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RESULT
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         dictionary : () -> %
--R elt : (%,Symbol,Any) -> Any           ?.? : (%,Symbol) -> Any
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Any               eq? : (%,%) -> Boolean
--R index? : (Symbol,%) -> Boolean        indices : % -> List Symbol
--R key? : (Symbol,%) -> Boolean          keys : % -> List Symbol
--R map : ((Any -> Any),%) -> %           qelt : (%,Symbol) -> Any
--R sample : () -> %                      setelt : (%,Symbol,Any) -> Any
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R any? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregat
--R bag : List Record(key: Symbol,entry: Any) -> %
--R coerce : % -> OutputForm if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R construct : List Record(key: Symbol,entry: Any) -> %
--R convert : % -> InputForm if Record(key: Symbol,entry: Any) has KONVERT INFORM
--R count : ((Any -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Any,%) -> NonNegativeInteger if $ has finiteAggregate and Any has SETCAT
--R count : (Record(key: Symbol,entry: Any),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> NonNegativeInteger if $ has fi
--R dictionary : List Record(key: Symbol,entry: Any) -> %
--R entry? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R eval : (%,List Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,Any,Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,List Any,List Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,List Record(key: Symbol,entry: Any),List Record(key: Symbol,entry: Any)) -> % i
--R eval : (%,Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> % if Record(k
```

```
--R eval : (%,Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has EVALAB
--R eval : (%,List Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has E
--R every? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Symbol,entry: Any)
--R fill! : (%,Any) -> % if $ has shallowlyMutable
--R find : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Union(Record(key: Symbol,entry: Any),"fail
--R first : % -> Any if Symbol has ORDSET
--R hash : % -> SingleInteger if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R insert! : (Record(key: Symbol,entry: Any),%) -> %
--R inspect : % -> Record(key: Symbol,entry: Any)
--R latex : % -> String if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Any,Any) -> Any),%,%) -> %
--R map : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> %
--R map! : ((Any -> Any),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> % if $ has shallowl
--R maxIndex : % -> Symbol if Symbol has ORDSET
--R member? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R member? : (Record(key: Symbol,entry: Any),%) -> Boolean if $ has finiteAggregate and Record(key: Sym
--R members : % -> List Any if $ has finiteAggregate
--R members : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R minIndex : % -> Symbol if Symbol has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Any if $ has finiteAggregate
--R parts : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R qsetelt! : (%,Symbol,Any) -> Any if $ has shallowlyMutable
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key: Symbol,ent
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key: Symbol,ent
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key: Symbol,ent
--R remove : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate and Record(key: Symbol,ent
--R remove! : (Symbol,%) -> Union(Any,"failed")
--R remove! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Symbol,entry: Any) has SETCAT
--R search : (Symbol,%) -> Union(Any,"failed")
--R select : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R showArrayValues : Boolean -> Boolean
--R showScalarValues : Boolean -> Boolean
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Symbol,Symbol) -> Void if $ has shallowlyMutable
--R table : List Record(key: Symbol,entry: Any) -> %
--R ?~=? : (%,%) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R
--E 1

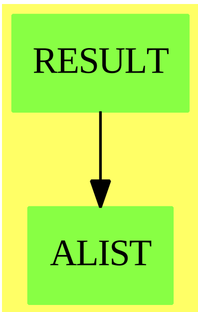
)spool
)lisp (bye)
```

— Result.help —

```
====================================================================
Result examples
====================================================================

See Also:
o )show Result
```

19.9.1 Result (RESULT)



See

⇒ “FortranCode” (FC) 7.16.1 on page 898
⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923
⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
⇒ “Switch” (SWITCH) 20.36.1 on page 2588
⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934
⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	entry?	elt
empty	empty?	entries	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	showArrayValues	showScalarValues
size?	swap!	table	#?	?=?
?~=?	?.?			

— domain RESULT Result —

```
)abbrev domain RESULT Result
++ Author: Didier Pinchon and Mike Dewar
++ Date Created:  8 April 1994
++ Date Last Updated: 28 June 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain used to return the results from a call to the NAG
++ Library.  It prints as a list of names and types, though the user may
++ choose to display values automatically if he or she wishes.

Result():Exports==Implementation where

  O  ==> OutputForm

  Exports ==> TableAggregate(Symbol,Any) with
    showScalarValues : Boolean -> Boolean
      ++ showScalarValues(true) forces the values of scalar components to be
      ++  displayed rather than just their types.
    showArrayValues : Boolean -> Boolean
      ++ showArrayValues(true) forces the values of array components to be
      ++  displayed rather than just their types.
    finiteAggregate

  Implementation ==> Table(Symbol,Any) add

    -- Constant
    colon := ": "::Symbol::O
```

```
elide := "..."::Symbol::O

-- Flags
showScalarValuesFlag : Boolean := false
showArrayValuesFlag  : Boolean := false

cleanUpDomainForm(d:SExpression):O ==
  not list? d => d::O
  #d=1 => (car d)::O
  -- If the car is an atom then we have a domain constructor, if not
  -- then we have some kind of value.  Since we often can't print these
  -- ****ers we just elide them.
  not atom? car d => elide
  prefix((car d)::O,[cleanUpDomainForm(u) for u in destruct cdr(d)]$List(O))

display(v:Any,d:SExpression):O ==
  not list? d => error "Domain form is non-list"
  #d=1 =>
   showScalarValuesFlag => objectOf v
   cleanUpDomainForm d
  car(d) = convert("Complex"::Symbol)@SExpression =>
   showScalarValuesFlag => objectOf v
   cleanUpDomainForm d
  showArrayValuesFlag => objectOf v
  cleanUpDomainForm d

makeEntry(k:Symbol,v:Any):O ==
  hconcat [k::O,colon,display(v,dom v)]

coerce(r:%):O ==
  bracket [makeEntry(key,r.key) for key in reverse! keys(r)]

showArrayValues(b:Boolean):Boolean  == showArrayValuesFlag := b
showScalarValues(b:Boolean):Boolean == showScalarValuesFlag := b
```

— **RESULT.dotabb** —

```
"RESULT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RESULT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RESULT" -> "ALIST"
```

19.10 domain RULE RewriteRule

— RewriteRule.input —

```
)set break resume
)sys rm -f RewriteRule.output
)spool RewriteRule.output
)set message test on
)set message auto off
)clear all

--S 1 of 4
)show RewriteRule
--R RewriteRule(Base: SetCategory,R: Join(Ring,PatternMatchable Base,OrderedSet,ConvertibleTo Pattern Ba
--R Abbreviation for RewriteRule is RULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RULE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : Equation F -> %
--R coerce : % -> OutputForm              elt : (%,F,PositiveInteger) -> F
--R ?.? : (%,F) -> F                      hash : % -> SingleInteger
--R latex : % -> String                   lhs : % -> F
--R pattern : % -> Pattern Base           retract : % -> Equation F
--R rhs : % -> F                          rule : (F,F,List Symbol) -> %
--R rule : (F,F) -> %                     ?~=? : (%,%) -> Boolean
--R quotedOperators : % -> List Symbol
--R retractIfCan : % -> Union(Equation F,"failed")
--R suchThat : (%,List Symbol,(List F -> Boolean)) -> %
--R
--E 1

--S 2 of 4
logrule := rule log(x) + log(y) == log(x*y)
--R
--R   (1)  log(y) + log(x) + %B == log(x y) + %B
--R                        Type: RewriteRule(Integer,Integer,Expression Integer)
--E 2

--S 3 of 4
f := log(sin(x)) + log(x)
--R
--R   (2)  log(sin(x)) + log(x)
--R                                                         Type: Expression Integer
--E 3

--S 4 of 4
logrule f
```

```
--R
--R   (3)  log(x sin(x))
--R                                                      Type: Expression Integer
--E 4

)spool
)lisp (bye)
```

— RewriteRule.help —

```
====================================================================
RewriteRule examples
====================================================================
For example:

logrule := rule log(x) + log(y) == log(x*y)
   log(y) + log(x) + %C == log(x y) + %C

f := log(sin(x)) + log(x)
   log(sin(x)) + log(x)

logrule f
   log(x sin(x))

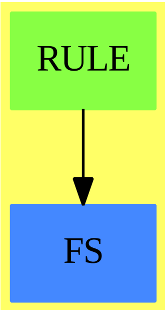

Note that you cannot write the simple form of a rule as:

rule1 := rule a*x+b == b*x+a

as this causes an infinite loop. The pattern is properly matched
but the result also properly matches and the pattern is applied
again.


See Also:
o )show ApplyRules
o )show RewriteRule
```

19.10.1 RewriteRule (RULE)



See

⇒ "Ruleset" (RULESET) 19.15.1 on page 2303

Exports:

coerce	elt	hash	latex	lhs
pattern	quotedOperators	retract	retractIfCan	rhs
rule	suchThat	?.?	?~=?	?=?

— **domain RULE RewriteRule** —

```
)abbrev domain RULE RewriteRule
++ Author: Manuel Bronstein
++ Date Created: 24 Oct 1988
++ Date Last Updated: 26 October 1993
++ Keywords: pattern, matching, rule.
++ Description:
++ Rules for the pattern matcher

RewriteRule(Base, R, F): Exports == Implementation where
  Base   : SetCategory
  R      : Join(Ring, PatternMatchable Base, OrderedSet,
                                 ConvertibleTo Pattern Base)
  F      : Join(FunctionSpace R, PatternMatchable Base,
                                 ConvertibleTo Pattern Base)

  P    ==> Pattern Base

  Exports ==>
   Join(SetCategory, Eltable(F, F), RetractableTo Equation F) with
    rule    : (F, F) -> $
      ++ rule(f, g) creates the rewrite rule: \spad{f == eval(g, g is f)},
      ++ with left-hand side f and right-hand side g.
      ++
      ++X logrule := rule log(x) + log(y) == log(x*y)
      ++X f := log(sin(x)) + log(x)
      ++X logrule f
```

```
  rule    : (F, F, List Symbol) -> $
    ++ rule(f, g, [f1,...,fn]) creates the rewrite rule
    ++ \spad{f == eval(eval(g, g is f), [f1,...,fn])},
    ++ that is a rule with left-hand side f and right-hand side g;
    ++ The symbols f1,...,fn are the operators that are considered
    ++ quoted, that is they are not evaluated during any rewrite,
    ++ but just applied formally to their arguments.
  suchThat: ($, List Symbol, List F -> Boolean) -> $
    ++ suchThat(r, [a1,...,an], f) returns the rewrite rule r with
    ++ the predicate \spad{f(a1,...,an)} attached to it.
  pattern : $ -> P
    ++ pattern(r) returns the pattern corresponding to
    ++ the left hand side of the rule r.
  lhs     : $ -> F
    ++ lhs(r) returns the left hand side of the rule r.
  rhs     : $ -> F
    ++ rhs(r) returns the right hand side of the rule r.
  elt     : ($, F, PositiveInteger) -> F
    ++ elt(r,f,n) or r(f, n) applies the rule r to f at most n times.
  quotedOperators: $ -> List Symbol
    ++ quotedOperators(r) returns the list of operators
    ++ on the right hand side of r that are considered
    ++ quoted, that is they are not evaluated during any rewrite,
    ++ but just applied formally to their arguments.

Implementation ==> add
  import ApplyRules(Base, R, F)
  import PatternFunctions1(Base, F)
  import FunctionSpaceAssertions(R, F)

  Rep := Record(pat: P, lft: F, rgt: F, qot: List Symbol)

  mkRule      : (P, F, F, List Symbol) -> $
  transformLhs: P -> Record(plus: F, times: F)
  bad?        : Union(List P, "failed") -> Boolean
  appear?     : (P, List P) -> Boolean
  opt         : F -> P
  F2Symbol    : F -> F

  pattern x               == x.pat
  lhs x                   == x.lft
  rhs x                   == x.rgt
  quotedOperators x       == x.qot
  mkRule(pt, p, s, l)     == [pt, p, s, l]
  coerce(eq:Equation F):$  == rule(lhs eq, rhs eq, empty())
  rule(l, r)              == rule(l, r, empty())
  elt(r:$, s:F) == applyRules([r pretend RewriteRule(Base, R, F)], s)

  suchThat(x, l, f) ==
    mkRule(suchThat(pattern x,l,f),  lhs x, rhs x, quotedOperators x)
```

```
    x = y ==
     (lhs x = lhs y) and (rhs x = rhs y) and
        (quotedOperators x = quotedOperators y)

    elt(r:$, s:F, n:PositiveInteger) ==
      applyRules([r pretend RewriteRule(Base, R, F)], s, n)

-- remove the extra properties from the constant symbols in f
    F2Symbol f ==
      l := select_!(z+->symbolIfCan z case Symbol, tower f)$List(Kernel F)
      eval(f, l, [symbolIfCan(k)::Symbol::F for k in l])

    retractIfCan r ==
      constant? pattern r =>
        (u:= retractIfCan(lhs r)@Union(Kernel F,"failed")) case "failed"
          => "failed"
        F2Symbol(u::Kernel(F)::F) = rhs r
      "failed"

    rule(p, s, l) ==
      lh := transformLhs(pt := convert(p)@P)
      mkRule(opt(lh.times) * (opt(lh.plus) + pt),
             lh.times * (lh.plus + p), lh.times * (lh.plus + s), l)

    opt f ==
      retractIfCan(f)@Union(R, "failed") case R => convert f
      convert optional f

-- appear?(x, [p1,...,pn]) is true if x appears as a variable in
-- a composite pattern pi.
    appear?(x, l) ==
      for p in l | p ^= x repeat
        member?(x, variables p) => return true
      false

-- a sum/product p1 @ ... @ pn is "bad" if it will not match
-- a sum/product p1 @ ... @ pn @ p(n+1)
-- in which case one should transform p1 @ ... @ pn to
-- p1 @ ... @ ?p(n+1) which does not change its meaning.
-- examples of "bad" combinations
--   sin(x) @ sin(y)     sin(x) @ x
-- examples of "good" combinations
--   sin(x) @ y
    bad? u ==
      u case List(P) =>
        for x in u::List(P) repeat
          generic? x and not appear?(x, u::List(P)) => return false
        true
      false
```

```
transformLhs p ==
  bad? isPlus p  => [new()$Symbol :: F, 1]
  bad? isTimes p => [0, new()$Symbol :: F]
  [0, 1]

coerce(x:$):OutputForm ==
  infix(" == "::Symbol::OutputForm,
        lhs(x)::OutputForm, rhs(x)::OutputForm)
```

———-

— **RULE.dotabb** —

```
"RULE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RULE"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"RULE" -> "FS"
```

———-

19.11 domain ROIRC RightOpenIntervalRootCharacterization

The domain RightOpenIntervalRootCharacterization is the main code that provides the functionalities of RealRootCharacterizationCategory for the case of archimedean fileds. Abstract roots are encoded with a left closed right open interval containing the root together with a defining polynomial for the root.

CAVEATS

Since real algebraic expressions are stored as depending on "real roots" which are managed like variables, there is an ordering on these. This ordering is dynamical in the sense that any new algebraic takes precedence over older ones. In particular every cretaion function raises a new "real root". This has the effect that when you type something like sqrt(2) + sqrt(2) you have two new variables which happen to be equal. To avoid this name the expression such as in s2 := sqrt(2) ; s2 + s2

Also note that computing times depend strongly on the ordering you implicitly provide. Please provide algebraics in the order which most natural to you.

LIMITATIONS

The file reclos.input show some basic use of the package. This packages uses algorithms which are published in [1] and [2] which are based on field arithmetics, inparticular for polynomial gcd related algorithms. This can be quite slow for high degree polynomials and subresultants methods usually work best. Betas versions of the package try to use these techniques in a better way and work significantly faster. These are mostly based on unpublished algorithms and cannot be distributed. Please contact the author if you have a particular problem to solve or want to use these versions.

Be aware that approximations behave as post-processing and that all computations are done exactly. They can thus be quite time consuming when depending on several "real roots".

— **RightOpenIntervalRootCharacterization.input** —

```
)set break resume
)sys rm -f RightOpenIntervalRootCharacterization.output
)spool RightOpenIntervalRootCharacterization.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RightOpenIntervalRootCharacterization
--R RightOpenIntervalRootCharacterization(TheField: Join(OrderedRing,Field),ThePolDom: UnivariatePolynom
--R Abbreviation for RightOpenIntervalRootCharacterization is ROIRC
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for ROIRC
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                allRootsOf : ThePolDom -> List %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   left : % -> TheField
--R middle : % -> TheField                refine : % -> %
--R right : % -> TheField                 sign : (ThePolDom,%) -> Integer
--R size : % -> TheField                  zero? : (ThePolDom,%) -> Boolean
--R ?~=? : (%,%) -> Boolean
--R approximate : (ThePolDom,%,TheField) -> TheField
--R definingPolynomial : % -> ThePolDom
--R mightHaveRoots : (ThePolDom,%) -> Boolean
--R negative? : (ThePolDom,%) -> Boolean
--R positive? : (ThePolDom,%) -> Boolean
--R recip : (ThePolDom,%) -> Union(ThePolDom,"failed")
--R relativeApprox : (ThePolDom,%,TheField) -> TheField
--R rootOf : (ThePolDom,PositiveInteger) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———

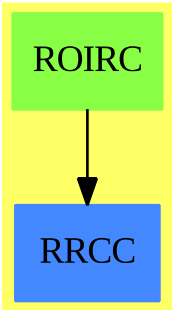
— **RightOpenIntervalRootCharacterization.help** —

```
====================================================================
RightOpenIntervalRootCharacterization examples
====================================================================

See Also:
o )show RightOpenIntervalRootCharacterization
```

———-

19.11.1 RightOpenIntervalRootCharacterization (ROIRC)



See

⇒ "RealClosure" (RECLOS) 19.3.1 on page 2196

Exports:

allRootsOf	approximate	coerce	definingPolynomial	hash
latex	left	middle	mightHaveRoots	negative?
positive?	recip	refine	relativeApprox	right
rootOf	sign	size	zero?	?=?
?~=?				

— domain ROIRC RightOpenIntervalRootCharacterization —

```
)abbrev domain ROIRC RightOpenIntervalRootCharacterization
++ Author: Renaud Rioboo
++ Date Created: summer 1992
++ Date Last Updated: January 2004
++ Basic Functions: provides computations with real roots of olynomials
++ Related Constructors: RealRootCharacterizationCategory, RealClosure
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ \axiomType{RightOpenIntervalRootCharacterization} provides work with
```

```
++ interval root coding.

RightOpenIntervalRootCharacterization(TheField,ThePolDom) : PUB == PRIV where

  TheField : Join(OrderedRing,Field)
  ThePolDom : UnivariatePolynomialCategory(TheField)


  Z           ==>  Integer
  P           ==>  ThePolDom
  N           ==>  NonNegativeInteger
  B           ==>  Boolean
  UTIL        ==>  RealPolynomialUtilitiesPackage(TheField,ThePolDom)
  RRCC        ==>  RealRootCharacterizationCategory
  O ==> OutputForm
  TwoPoints ==> Record(low:TheField , high:TheField)

  PUB == RealRootCharacterizationCategory(TheField, ThePolDom) with

      left    :             $           -> TheField
           ++ \axiom{left(rootChar)} is the left bound of the isolating
           ++ interval
      right   :             $           -> TheField
           ++ \axiom{right(rootChar)} is the right bound of the isolating
           ++ interval
      size    :             $           -> TheField
           ++ The size of the isolating interval
      middle  :             $           -> TheField
           ++ \axiom{middle(rootChar)} is the middle of the isolating
           ++ interval
      refine  :             $           ->    $
           ++ \axiom{refine(rootChar)} shrinks isolating interval around
           ++ \axiom{rootChar}
      mightHaveRoots :     (P,$)        ->    B
           ++ \axiom{mightHaveRoots(p,r)} is false if \axiom{p.r} is not 0
      relativeApprox :     (P,$,TheField) -> TheField
           ++ \axiom{relativeApprox(exp,c,p) = a} is relatively close to exp
           ++ as a polynomial in c ip to precision p

  PRIV == add



-- local functions


   makeChar:            (TheField,TheField,ThePolDom) ->     $
   refine! :                             $            ->     $
   sturmIsolate : (List(P), TheField, TheField,N,N)    -> List TwoPoints
   isolate :                           List(P)        -> List TwoPoints
```

```
   rootBound :                           P           ->   TheField
--   varStar :                              P           ->     N
   linearRecip :                      ( P , $)        -> Union(P, "failed")
   linearZero? :                   (TheField,$)       ->     B
   linearSign :                        (P,$)          ->     Z
   sturmNthRoot : (List(P), TheField, TheField,N,N,N)  -> Union(TwoPoints,"failed")
   addOne :                            P              ->      P
   minus :                             P              ->      P
   translate :                   (P,TheField)         ->      P
   dilate :                      (P,TheField)         ->      P
   invert :                            P              ->      P
   evalOne :                           P              ->   TheField
   hasVarsl:                    List(TheField)        ->      B
   hasVars:                            P              ->      B

-- Representation

   Rep:= Record(low:TheField,high:TheField,defPol:ThePolDom)

-- and now the code !


   size(rootCode) ==
     rootCode.high - rootCode.low

   relativeApprox(pval,rootCode,prec) ==
     -- beurk !
     dPol := rootCode.defPol
     degree(dPol) = 1 =>
       c := -coefficient(dPol,0)/leadingCoefficient(dPol)
       pval.c
     pval := pval rem dPol
     degree(pval) = 0 => leadingCoefficient(pval)
     zero?(pval,rootCode)  => 0
     while mightHaveRoots(pval,rootCode) repeat
          rootCode := refine(rootCode)
     dpval := differentiate(pval)
     degree(dpval) = 0 =>
       l := left(rootCode)
       r := right(rootCode)
       a := pval.l
       b := pval.r
       while ( abs(2*(a-b)/(a+b)) > prec ) repeat
         rootCode := refine(rootCode)
         l := left(rootCode)
         r := right(rootCode)
         a := pval.l
         b := pval.r
       (a+b)/(2::TheField)
     zero?(dpval,rootCode) =>
```

```
      relativeApprox(pval,
                     [left(rootCode),
                       right(rootCode),
                         gcd(dpval,rootCode.defPol)]$Rep,
                     prec)
    while mightHaveRoots(dpval,rootCode) repeat
         rootCode := refine(rootCode)
    l := left(rootCode)
    r := right(rootCode)
    a := pval.l
    b := pval.r
    while ( abs(2*(a-b)/(a+b)) > prec ) repeat
      rootCode := refine(rootCode)
      l := left(rootCode)
      r := right(rootCode)
      a := pval.l
      b := pval.r
    (a+b)/(2::TheField)

  approximate(pval,rootCode,prec) ==
    -- glurp
    dPol := rootCode.defPol
    degree(dPol) = 1 =>
      c := -coefficient(dPol,0)/leadingCoefficient(dPol)
      pval.c
    pval := pval rem dPol
    degree(pval) = 0 => leadingCoefficient(pval)
    dpval := differentiate(pval)
    degree(dpval) = 0 =>
      l := left(rootCode)
      r := right(rootCode)
      while ( abs((a := pval.l) - (b := pval.r)) > prec ) repeat
        rootCode := refine(rootCode)
        l := left(rootCode)
        r := right(rootCode)
      (a+b)/(2::TheField)
    zero?(dpval,rootCode) =>
       approximate(pval,
                  [left(rootCode),
                   right(rootCode),
                     gcd(dpval,rootCode.defPol)]$Rep,
                  prec)
    while mightHaveRoots(dpval,rootCode) repeat
         rootCode := refine(rootCode)
    l := left(rootCode)
    r := right(rootCode)
    while ( abs((a := pval.l) - (b := pval.r)) > prec ) repeat
      rootCode := refine(rootCode)
      l := left(rootCode)
      r := right(rootCode)
```

```
    (a+b)/(2::TheField)

  addOne(p) == p.(monomial(1,1)+(1::P))

  minus(p) == p.(monomial(-1,1))

  translate(p,a) == p.(monomial(1,1)+(a::P))

  dilate(p,a) == p.(monomial(a,1))

  evalOne(p) == "+" / coefficients(p)

  invert(p) ==
       d := degree(p)
       mapExponents(z +-> (d-z)::N, p)

  rootBound(p) ==
    res : TheField := 1
    raw :TheField := 1+boundOfCauchy(p)$UTIL
    while (res < raw) repeat
      res := 2*(res)
    res

  sturmNthRoot(lp,l,r,vl,vr,n) ==
   nv := (vl - vr)::N
   nv < n => "failed"
   ((nv = 1) and (n = 1)) => [l,r]
   int := (l+r)/(2::TheField)
   lt:List(TheField):=[]
   for t in lp repeat
       lt := cons(t.int , lt)
   vi := sturmVariationsOf(reverse! lt)$UTIL
   o :Z := n - vl + vi
   if o > 0
   then
      sturmNthRoot(lp,int,r,vi,vr,o::N)
   else
      sturmNthRoot(lp,l,int,vl,vi,n)

  sturmIsolate(lp,l,r,vl,vr) ==
   r <= l => error "ROIRC: sturmIsolate: bad bounds"
   n := (vl - vr)::N
   zero?(n) => []
   one?(n) => [[l,r]]
   int := (l+r)/(2::TheField)
   vi := sturmVariationsOf( [t.int for t in lp ] )$UTIL
   append(sturmIsolate(lp,l,int,vl,vi),sturmIsolate(lp,int,r,vi,vr))

  isolate(lp) ==
```

```
    b := rootBound(first(lp))
    l1,l2 : List(TheField)
    (l1,l2) := ([] , [])
    for t in reverse(lp) repeat
      if odd?(degree(t))
      then
       (l1,l2):= (cons(-leadingCoefficient(t),l1),
                  cons(leadingCoefficient(t),l2))
      else
       (l1,l2):= (cons(leadingCoefficient(t),l1),
                  cons(leadingCoefficient(t),l2))
    sturmIsolate(lp,
                 -b,
                 b,
                 sturmVariationsOf(l1)$UTIL,
                 sturmVariationsOf(l2)$UTIL)

  rootOf(pol,n) ==
   ls := sturmSequence(pol)$UTIL
   pol := unitCanonical(first(ls)) -- this one is SqFR
   degree(pol) = 0 => "failed"
   numberOfMonomials(pol) = 1 => ([0,1,monomial(1,1)]$Rep)::$
   b := rootBound(pol)
   l1,l2 : List(TheField)
   (l1,l2) := ([] , [])
   for t in reverse(ls) repeat
     if odd?(degree(t))
     then
      (l1,l2):= (cons(leadingCoefficient(t),l1),
                 cons(-leadingCoefficient(t),l2))
     else
      (l1,l2):= (cons(leadingCoefficient(t),l1),
                 cons(leadingCoefficient(t),l2))
   res := sturmNthRoot(ls,
                       -b,
                       b,
                       sturmVariationsOf(l2)$UTIL,
                       sturmVariationsOf(l1)$UTIL,
                       n)
   res case "failed" => "failed"
   makeChar(res.low,res.high,pol)

  allRootsOf(pol) ==
   ls := sturmSequence(unitCanonical pol)$UTIL
   pol := unitCanonical(first(ls)) -- this one is SqFR
   degree(pol) = 0 => []
   numberOfMonomials(pol) = 1 => [[0,1,monomial(1,1)]$Rep]
   [ makeChar(term.low,term.high,pol) for term in isolate(ls) ]
```

```
   hasVarsl(l:List(TheField)) ==
    null(l) => false
    f := sign(first(l))
    for term in rest(l) repeat
      if f*term < 0 then return(true)
    false

   hasVars(p:P) ==
    zero?(p) => error "ROIRC: hasVars: null polynonial"
    zero?(coefficient(p,0)) => true
    hasVarsl(coefficients(p))


   mightHaveRoots(p,rootChar) ==
      a := rootChar.low
      q := translate(p,a)
      not(hasVars(q)) => false
--      varStar(q) = 0 => false
      a := (rootChar.high) - a
      q := dilate(q,a)
      sign(coefficient(q,0))*sign(evalOne(q)) <= 0 => true
      q := minus(addOne(q))
      not(hasVars(q)) => false
--      varStar(q) = 0 => false
      q := invert(q)
      hasVars(addOne(q))
--      ^(varStar(addOne(q)) = 0)

   coerce(rootChar:$):O ==
     commaSeparate([ hconcat("[" :: O , (rootChar.low)::O),
                     hconcat((rootChar.high)::O,"[" ::O ) ])

   c1 = c2 ==
     mM := max(c1.low,c2.low)
     Mm := min(c1.high,c2.high)
     mM >= Mm => false
     rr : ThePolDom := gcd(c1.defPol,c2.defPol)
     degree(rr) = 0 => false
     sign(rr.mM) * sign(rr.Mm) <= 0

   makeChar(left,right,pol) ==
-- The following lines of code, which check for a possible error,
-- cause major performance problems and were removed by Renaud Rioboo,
-- the original author. They were originally inserted for debugging.
--    right <= left => error "ROIRC: makeChar: Bad interval"
--    (pol.left * pol.right) > 0 => error "ROIRC: makeChar: Bad pol"
    res :$ := [left,right,leadingMonomial(pol)+reductum(pol)]$Rep -- safe copy
    while zero?(pol.(res.high)) repeat refine!(res)
    while (res.high * res.low < 0 ) repeat refine!(res)
    zero?(pol.(res.low)) => [res.low,res.high,monomial(1,1)-(res.low)::P]
```

```
     res

   definingPolynomial(rootChar) == rootChar.defPol

   linearRecip(toTest,rootChar) ==
      c := - inv(leadingCoefficient(toTest)) * coefficient(toTest,0)
      r := recip(rootChar.defPol.c)
      if (r case "failed")
      then
        if (c - rootChar.low) * (c - rootChar.high) <= 0
        then
          "failed"
        else
          newPol := (rootChar.defPol exquo toTest)::P
          ((1$ThePolDom - inv(newPol.c)*newPol) exquo toTest)::P
      else
         ((1$ThePolDom - (r::TheField)*rootChar.defPol) exquo toTest)::P

   recip(toTest,rootChar) ==
     degree(toTest) = 0 or degree(rootChar.defPol) <= degree(toTest) =>
       error "IRC: recip: Not reduced"
     degree(rootChar.defPol) = 1 =>
       error "IRC: recip: Linear Defining Polynomial"
     degree(toTest) = 1 =>
       linearRecip(toTest, rootChar)
     d := extendedEuclidean((rootChar.defPol),toTest)
     (degree(d.generator) = 0 ) =>
         d.coef2
     d.generator := unitCanonical(d.generator)
     (d.generator.(rootChar.low) *
      d.generator.(rootChar.high)<= 0) => "failed"
     newPol := (rootChar.defPol exquo (d.generator))::P
     degree(newPol) = 1 =>
       c := - inv(leadingCoefficient(newPol)) * coefficient(newPol,0)
       inv(toTest.c)::P
     degree(toTest) = 1 =>
       c := - coefficient(toTest,0)/ leadingCoefficient(toTest)
       ((1$ThePolDom - inv(newPol.(c))*newPol) exquo toTest)::P
     d := extendedEuclidean(newPol,toTest)
     d.coef2

   linearSign(toTest,rootChar) ==
      c := - inv(leadingCoefficient(toTest)) * coefficient(toTest,0)
      ev := sign(rootChar.defPol.c)
      if zero?(ev)
      then
        if (c - rootChar.low) * (c - rootChar.high) <= 0
        then
          0
        else
```

```
          sign(toTest.(rootChar.high))
      else
        if (ev*sign(rootChar.defPol.(rootChar.high)) <= 0 )
        then
          sign(toTest.(rootChar.high))
        else
          sign(toTest.(rootChar.low))

   sign(toTest,rootChar) ==
     degree(toTest) = 0 or degree(rootChar.defPol) <= degree(toTest) =>
       error "IRC: sign: Not reduced"
     degree(rootChar.defPol) = 1 =>
       error "IRC: sign: Linear Defining Polynomial"
     degree(toTest) = 1 =>
      linearSign(toTest, rootChar)
     s := sign(leadingCoefficient(toTest))
     toTest := monomial(1,degree(toTest))+
               inv(leadingCoefficient(toTest))*reductum(toTest)
     delta := gcd(toTest,rootChar.defPol)
     newChar := [rootChar.low,rootChar.high,rootChar.defPol]$Rep
     if degree(delta) > 0
     then
       if sign(delta.(rootChar.low) * delta.(rootChar.high)) <= 0
       then
        return(0)
       else
        newChar.defPol := (newChar.defPol exquo delta) :: P
        toTest := toTest rem (newChar.defPol)
     degree(toTest) = 0 => s * sign(leadingCoefficient(toTest))
     degree(toTest) = 1 => s * linearSign(toTest, newChar)
     while mightHaveRoots(toTest,newChar) repeat
       newChar := refine(newChar)
     s*sign(toTest.(newChar.low))

   linearZero?(c,rootChar) ==
      zero?((rootChar.defPol).c) and
       (c - rootChar.low) * (c - rootChar.high) <= 0

   zero?(toTest,rootChar) ==
     degree(toTest) = 0 or degree(rootChar.defPol) <= degree(toTest) =>
       error "IRC: zero?: Not reduced"
     degree(rootChar.defPol) = 1 =>
       error "IRC: zero?: Linear Defining Polynomial"
     degree(toTest) = 1 =>
      linearZero?(- inv(leadingCoefficient(toTest)) * coefficient(toTest,0),
                  rootChar)
     toTest := monomial(1,degree(toTest))+
               inv(leadingCoefficient(toTest))*reductum(toTest)
     delta := gcd(toTest,rootChar.defPol)
     degree(delta) = 0 => false
```

```
     sign(delta.(rootChar.low) * delta.(rootChar.high)) <= 0


   refine!(rootChar) ==
     -- this is not a safe function, it can work with badly created object
     -- we do not assume (rootChar.defPol).(rootChar.high) <> 0
        int := middle(rootChar)
        s1 := sign((rootChar.defPol).(rootChar.low))
        zero?(s1) =>
          rootChar.high := int
          rootChar.defPol := monomial(1,1) - (rootChar.low)::P
          rootChar
        s2 := sign((rootChar.defPol).int)
        zero?(s2) =>
          rootChar.low := int
          rootChar.defPol := monomial(1,1) - int::P
          rootChar
        if (s1*s2 < 0)
        then
          rootChar.high := int
        else
          rootChar.low := int
        rootChar

   refine(rootChar) ==
     -- we assume (rootChar.defPol).(rootChar.high) <> 0
        int := middle(rootChar)
        s:= (rootChar.defPol).int * (rootChar.defPol).(rootChar.high)
        zero?(s) => [int,rootChar.high,monomial(1,1)-int::P]
        if s < 0
        then
          [int,rootChar.high,rootChar.defPol]
        else
          [rootChar.low,int,rootChar.defPol]

   left(rootChar) == rootChar.low

   right(rootChar) == rootChar.high

   middle(rootChar) == (rootChar.low + rootChar.high)/(2::TheField)

--   varStar(p) == -- if 0 no roots in [0,:infty[
--     res : N := 0
--     lsg := sign(coefficient(p,0))
--     l := [ sign(i) for i in reverse!(coefficients(p))]
--     for sg in l repeat
--      if (sg ^= lsg) then res := res + 1
--      lsg := sg
--     res
```

— ROIRC.dotabb —

```
"ROIRC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ROIRC"]
"RRCC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RRCC"]
"ROIRC" -> "RRCC"
```

19.12 domain ROMAN RomanNumeral

— RomanNumeral.input —

```
)set break resume
)sys rm -f RomanNumeral.output
)spool RomanNumeral.output
)set message test on
)set message auto off
)clear all
--S 1 of 15
f := operator 'f
--R
--R
--R   (1)  f
--R                                                         Type: BasicOperator
--E 1

--S 2 of 15
D(f x,x,7)
--R
--R
--R         (vii)
--R   (2)  f     (x)
--R
--R                                                    Type: Expression Integer
--E 2

--S 3 of 15
a := roman(1978 - 1965)
--R
--R
--R   (3)  XIII
--R                                                          Type: RomanNumeral
--E 3
```

```
--S 4 of 15
x : UTS(ROMAN,'x,0) := x
--R
--R
--R   (4)  x
--R                                  Type: UnivariateTaylorSeries(RomanNumeral,x,0)
--E 4

--S 5 of 15
recip(1 - x - x**2)
--R
--R
--R   (5)
--R                  2        3      4         5         6        7          8
--R     I + x + II x  + III x  + V x  + VIII x  + XIII x  + XXI x  + XXXIV x
--R   +
--R         9           10      11
--R     LV x  + LXXXIX x   + O(x  )
--R                       Type: Union(UnivariateTaylorSeries(RomanNumeral,x,0),...)
--E 5

--S 6 of 15
m : MATRIX FRAC ROMAN
--R
--R                                                                    Type: Void
--E 6

--S 7 of 15
m := matrix [ [1/(i + j) for i in 1..3] for j in 1..3]
--R
--R
--R        + I    I    I+
--R        |--   ---  --|
--R        |II   III  IV|
--R        |            |
--R        | I    I   I |
--R   (7)  |---  --   - |
--R        |III  IV   V |
--R        |            |
--R        | I    I    I|
--R        |--    -   --|
--R        +IV    V   VI+
--R                                        Type: Matrix Fraction RomanNumeral
--E 7

--S 8 of 15
inverse m
--R
--R
```

```
--R        +LXXII   - CCXL    CLXXX +
--R        |                        |
--R   (8)  |- CCXL    CM     - DCCXX|
--R        |                        |
--R        +CLXXX   - DCCXX    DC   +
--R                                    Type: Union(Matrix Fraction RomanNumeral,...)
--E 8

--S 9 of 15
y := factorial 10
--R
--R
--R   (9)  3628800
--R                                                            Type: PositiveInteger
--E 9

--S 10 of 15
roman y
--R
--R
--R   (10)
--R  ((((I))))((((I))))((((I)))) (((I)))(((I)))(((I)))(((I)))(((I)))(((I))) ((I))(
--R  (I)) MMMMMMMMDCCC
--R                                                               Type: RomanNumeral
--E 10

--S 11 of 15
a := roman(78)
--R
--R
--R   (11)  LXXVIII
--R                                                               Type: RomanNumeral
--E 11

--S 12 of 15
b := roman(87)
--R
--R
--R   (12)  LXXXVII
--R                                                               Type: RomanNumeral
--E 12

--S 13 of 15
a + b
--R
--R
--R   (13)  CLXV
--R                                                               Type: RomanNumeral
--E 13
```

```
--S 14 of 15
a * b
--R
--R
--R   (14)  MMMMMMDCCLXXXVI
--R                                                      Type: RomanNumeral
--E 14

--S 15 of 15
b rem a
--R
--R
--R   (15)  IX
--R                                                      Type: RomanNumeral
--E 15
)spool
)lisp (bye)
```

— **RomanNumeral.help** —

```
====================================================================
RomanNumeral Examples
====================================================================

The Roman numeral package was added to Axiom in MCMLXXXVI for use in
denoting higher order derivatives.

For example, let f be a symbolic operator.

  f := operator 'f
    f
                                  Type: BasicOperator

This is the seventh derivative of f with respect to x.
  D(f x,x,7)
     (vii)
    f     (x)
                                  Type: Expression Integer

You can have integers printed as Roman numerals by declaring variables to
be of type RomanNumeral (abbreviation ROMAN).

  a := roman(1978 - 1965)
    XIII
                                  Type: RomanNumeral

This package now has a small but devoted group of followers that claim
```

this domain has shown its efficacy in many other contexts. They claim that Roman numerals are every bit as useful as ordinary integers.

In a sense, they are correct, because Roman numerals form a ring and you can therefore construct polynomials with Roman numeral coefficients, matrices over Roman numerals, etc..

```
x : UTS(ROMAN,'x,0) := x
  x
                         Type: UnivariateTaylorSeries(RomanNumeral,x,0)
```

Was Fibonacci Italian or ROMAN?

```
recip(1 - x - x**2)
             2        3      4         5         6        7          8
   I + x + II x  + III x  + V x  + VIII x  + XIII x  + XXI x  + XXXIV x
 +
       9          10      11
   LV x  + LXXXIX x   + O(x  )
               Type: Union(UnivariateTaylorSeries(RomanNumeral,x,0),...)
```

You can also construct fractions with Roman numeral numerators and denominators, as this matrix Hilberticus illustrates.

```
m : MATRIX FRAC ROMAN
                         Type: Void

m := matrix [ [1/(i + j) for i in 1..3] for j in 1..3]
      + I    I    I+
      |--   ---  --|
      |II   III  IV|
      |            |
      | I    I   I |
      |---  --   - |
      |III  IV   V |
      |            |
      | I    I    I|
      |--    -   --|
      +IV    V   VI+
                         Type: Matrix Fraction RomanNumeral
```

Note that the inverse of the matrix has integral ROMAN entries.

```
inverse m
      +LXXII   - CCXL    CLXXX +
      |                        |
      |- CCXL    CM     - DCCXX|
      |                        |
      +CLXXX   - DCCXX    DC   +
                         Type: Union(Matrix Fraction RomanNumeral,...)
```

```
Unfortunately, the spoil-sports say that the fun stops when the
numbers get big---mostly because the Romans didn't establish
conventions about representing very large numbers.

  y := factorial 10
    3628800
                          Type: PositiveInteger

You work it out!

  roman y
  ((((I))))((((I))))((((I)))) (((I)))(((I)))(((I)))(((I)))(((I)))(((I))) ((I))(
  (I)) MMMMMMMMDCCC
                          Type: RomanNumeral

Issue the system command )show RomanNumeral to display the full
list of operations defined by RomanNumeral.


  a := roman(78)
    LXXVIII
                      Type: RomanNumeral

  b := roman(87)
    LXXXVII
                      Type: RomanNumeral

  a + b
    CLXV
                      Type: RomanNumeral

  a * b
    MMMMMMDCCLXXXVI
                      Type: RomanNumeral

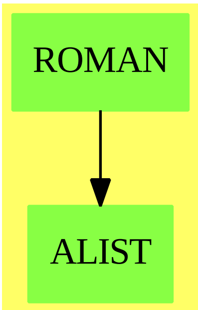
  b rem a
    IX
                      Type: RomanNumeral

See Also:
o )help Integer
o )help Complex
o )help Factored
o )help Records
o )help Fraction
o )help RadixExpansion
o )help HexadecimalExpansion
o )help BinaryExpansion
o )help DecimalExpansion
```

```
o )help IntegerNumberTheoryFunctions
o )show RomanNumeral
```

——————-

19.12.1 RomanNumeral (ROMAN)



See

⇒ "Integer" (INT) 10.30.1 on page 1325
⇒ "NonNegativeInteger" (NNI) 15.5.1 on page 1702
⇒ "PositiveInteger" (PI) 17.28.1 on page 2060

Exports:

0	1	abs	addmod
associates?	base	binomial	bit?
characteristic	coerce	convert	copy
D	dec	differentiate	divide
euclideanSize	even?	expressIdealMember	exquo
extendedEuclidean	extendedEuclidean	factor	factorial
gcd	gcdPolynomial	hash	inc
init	invmod	latex	lcm
length	mask	max	min
mulmod	multiEuclidean	negative?	nextItem
odd?	one?	patternMatch	permutation
positive?	positiveRemainder	powmod	prime?
principalIdeal	random	rational	rational?
rationalIfCan	recip	reducedSystem	retract
retractIfCan	roman	sample	shift
sign	sizeLess?	squareFree	squareFreePart
submod	subtractIfCan	symmetricRemainder	unit?
unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?
?<?	?<=?	?=?	?>?
?>=?	?^?	?~=?	?quo?
?rem?			

— domain ROMAN RomanNumeral —

```
)abbrev domain ROMAN RomanNumeral
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Related Constructors:
++ Keywords: roman numerals
++ Description:
++ \spadtype{RomanNumeral} provides functions for converting
++ integers to roman numerals.

RomanNumeral(): IntegerNumberSystem with
    canonical
      ++ mathematical equality is data structure equality.
    canonicalsClosed
      ++ two positives multiply to give positive.
    noetherian
      ++ ascending chain condition on ideals.
    convert: Symbol  -> %
      ++ convert(n) creates a roman numeral for symbol n.
    roman  : Symbol  -> %
      ++ roman(n) creates a roman numeral for symbol n.
    roman  : Integer -> %
      ++ roman(n) creates a roman numeral for n.
```

```
  == Integer add
        import NumberFormats()

        roman(n:Integer) == n::%
        roman(sy:Symbol) == convert sy
        convert(sy:Symbol):%    == ScanRoman(string sy)::%

        coerce(r:%):OutputForm ==
            n := convert(r)@Integer
            -- okay, we stretch it
            zero? n => n::OutputForm
            negative? n => - ((-r)::OutputForm)
            FormatRoman(n::PositiveInteger)::Symbol::OutputForm
```

———-

— ROMAN.dotabb —

```
"ROMAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ROMAN"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ROMAN" -> "ALIST"
```

———-

19.13 domain ROUTINE RoutinesTable

— RoutinesTable.input —

```
)set break resume
)sys rm -f RoutinesTable.output
)spool RoutinesTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RoutinesTable
--R RoutinesTable  is a domain constructor
--R Abbreviation for RoutinesTable is ROUTINE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ROUTINE
--R
--R------------------------------- Operations --------------------------------
```

```
--R concat : (%,%) -> %                    copy : % -> %
--R deleteRoutine! : (%,Symbol) -> %      dictionary : () -> %
--R elt : (%,Symbol,Any) -> Any           ?.? : (%,Symbol) -> Any
--R empty : () -> %                        empty? : % -> Boolean
--R entries : % -> List Any               eq? : (%,%) -> Boolean
--R getMeasure : (%,Symbol) -> Float      index? : (Symbol,%) -> Boolean
--R indices : % -> List Symbol            key? : (Symbol,%) -> Boolean
--R keys : % -> List Symbol               map : ((Any -> Any),%) -> %
--R qelt : (%,Symbol) -> Any              routines : () -> %
--R sample : () -> %                       selectFiniteRoutines : % -> %
--R selectNonFiniteRoutines : % -> %      selectODEIVPRoutines : % -> %
--R selectPDERoutines : % -> %            setelt : (%,Symbol,Any) -> Any
--R showTheRoutinesTable : () -> %        table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R any? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Symbol,entry: Any) -> %
--R changeMeasure : (%,Symbol,Float) -> %
--R changeThreshhold : (%,Symbol,Float) -> %
--R coerce : % -> OutputForm if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R construct : List Record(key: Symbol,entry: Any) -> %
--R convert : % -> InputForm if Record(key: Symbol,entry: Any) has KONVERT INFORM
--R count : ((Any -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Any,%) -> NonNegativeInteger if $ has finiteAggregate and Any has SETCAT
--R count : (Record(key: Symbol,entry: Any),%) -> NonNegativeInteger if $ has finiteAggregate and Record
--R count : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggrega
--R dictionary : List Record(key: Symbol,entry: Any) -> %
--R entry? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R eval : (%,List Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,Any,Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,List Any,List Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,List Record(key: Symbol,entry: Any),List Record(key: Symbol,entry: Any)) -> % if Record(ke
--R eval : (%,Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,
--R eval : (%,Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has EVALAB
--R eval : (%,List Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has E
--R every? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Symbol,entry: Any)
--R fill! : (%,Any) -> % if $ has shallowlyMutable
--R find : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Union(Record(key: Symbol,entry: Any),"fail
--R first : % -> Any if Symbol has ORDSET
--R getExplanations : (%,String) -> List String
--R hash : % -> SingleInteger if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R insert! : (Record(key: Symbol,entry: Any),%) -> %
--R inspect : % -> Record(key: Symbol,entry: Any)
--R latex : % -> String if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Any,Any) -> Any),%,%) -> %
```

```
--R map : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> %
--R map! : ((Any -> Any),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> % if $ h
--R maxIndex : % -> Symbol if Symbol has ORDSET
--R member? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R member? : (Record(key: Symbol,entry: Any),%) -> Boolean if $ has finiteAggregate and Reco
--R members : % -> List Any if $ has finiteAggregate
--R members : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R minIndex : % -> Symbol if Symbol has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Any if $ has finiteAggregate
--R parts : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R qsetelt! : (%,Symbol,Any) -> Any if $ has shallowlyMutable
--R recoverAfterFail : (%,String,Integer) -> Union(String,"failed")
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key
--R remove : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate and Record(key
--R remove! : (Symbol,%) -> Union(Any,"failed")
--R remove! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Symbol,entry: Any) has
--R search : (Symbol,%) -> Union(Any,"failed")
--R select : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R selectIntegrationRoutines : % -> %
--R selectMultiDimensionalRoutines : % -> %
--R selectOptimizationRoutines : % -> %
--R selectSumOfSquaresRoutines : % -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Symbol,Symbol) -> Void if $ has shallowlyMutable
--R table : List Record(key: Symbol,entry: Any) -> %
--R ?~=? : (%,%) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **RoutinesTable.help** —

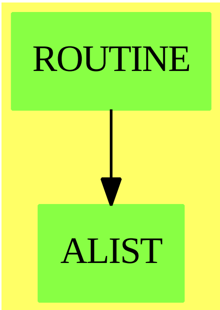
```
====================================================================
RoutinesTable examples
====================================================================

See Also:
```

```
o )show RoutinesTable
```

———-

19.13.1 RoutinesTable (ROUTINE)



Exports:

any?	bag
changeMeasure	changeThreshhold
coerce	concat
construct	convert
copy	count
deleteRoutine!	dictionary
elt	empty
empty?	entries
entry?	eq?
eval	every?
extract!	fill!
find	first
getExplanations	getMeasure
hash	index?
indices	insert!
inspect	key?
keys	latex
less?	map
map!	maxIndex
member?	members
minIndex	more?
parts	qelt
qsetelt!	recoverAfterFail
reduce	remove
remove!	removeDuplicates
routines	sample
search	select
select!	selectFiniteRoutines
selectIntegrationRoutines	selectNonFiniteRoutines
selectMultiDimensionalRoutines	selectODEIVPRoutines
selectOptimizationRoutines	selectPDERoutines
selectSumOfSquaresRoutines	setelt
showTheRoutinesTable	size?
swap!	table
#?	?=?
?~=?	?.?

— domain ROUTINE RoutinesTable —

```
)abbrev domain ROUTINE RoutinesTable
++ Author: Brian Dupee
++ Date Created: August 1994
++ Date Last Updated: December 1997
++ Basic Operations: routines, getMeasure
++ Related Constructors: TableAggregate(Symbol,Any)
++ Description:
++ \axiomType{RoutinesTable} implements a database and associated tuning
```

```
++ mechanisms for a set of known NAG routines

RoutinesTable(): E == I where
  F  ==> Float
  ST  ==> String
  LST  ==> List String
  Rec  ==> Record(key:Symbol,entry:Any)
  RList  ==> List(Record(key:Symbol,entry:Any))
  IFL  ==> List(Record(ifail:Integer,instruction:ST))
  Entry  ==> Record(chapter:ST, type:ST, domainName: ST,
                   defaultMin:F, measure:F, failList:IFL, explList:LST)

  E ==> TableAggregate(Symbol,Any) with

    concat:(%,%) -> %
      ++ concat(x,y) merges two tables x and y
    routines:() -> %
      ++ routines() initialises a database of known NAG routines
    selectIntegrationRoutines:% -> %
      ++ selectIntegrationRoutines(R) chooses only those routines from
      ++ the database which are for integration
    selectOptimizationRoutines:% -> %
      ++ selectOptimizationRoutines(R) chooses only those routines from
      ++ the database which are for integration
    selectPDERoutines:% -> %
      ++ selectPDERoutines(R) chooses only those routines from the
      ++ database which are for the solution of PDE's
    selectODEIVPRoutines:% -> %
      ++ selectODEIVPRoutines(R) chooses only those routines from the
      ++ database which are for the solution of ODE's
    selectFiniteRoutines:% -> %
      ++ selectFiniteRoutines(R) chooses only those routines from the
      ++ database which are designed for use with finite expressions
    selectSumOfSquaresRoutines:% -> %
      ++ selectSumOfSquaresRoutines(R) chooses only those routines from the
      ++ database which are designed for use with sums of squares
    selectNonFiniteRoutines:% -> %
      ++ selectNonFiniteRoutines(R) chooses only those routines from the
      ++ database which are designed for use with non-finite expressions.
    selectMultiDimensionalRoutines:% -> %
      ++ selectMultiDimensionalRoutines(R) chooses only those routines from
      ++ the database which are designed for use with multi-dimensional
      ++ expressions
    changeThreshhold:(%,Symbol,F) -> %
      ++ changeThreshhold(R,s,newValue) changes the value below which,
      ++ given a NAG routine generating a higher measure, the routines will
      ++ make no attempt to generate a measure.
    changeMeasure:(%,Symbol,F) -> %
      ++ changeMeasure(R,s,newValue) changes the maximum value for a
      ++ measure of the given NAG routine.
```

```
  getMeasure:(%,Symbol) -> F
    ++ getMeasure(R,s) gets the current value of the maximum measure for
    ++ the given NAG routine.
  getExplanations:(%,ST) -> LST
    ++ getExplanations(R,s) gets the explanations of the output parameters for
    ++ the given NAG routine.
  deleteRoutine!:(%,Symbol) -> %
    ++ deleteRoutine!(R,s) destructively deletes the given routine from
    ++ the current database of NAG routines
  showTheRoutinesTable:() -> %
    ++ showTheRoutinesTable() returns the current table of NAG routines.
  recoverAfterFail:(%,ST,Integer) ->  Union(ST,"failed")
    ++ recoverAfterFail(routs,routineName,ifailValue) acts on the
    ++ instructions given by the ifail list
  finiteAggregate

I ==> Result add

  Rep := Result
  import Rep

  theRoutinesTable:% := routines()

  showTheRoutinesTable():% == theRoutinesTable

  integrationRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
    (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
      elt(a,chapter) = "Integration"
    false

  selectIntegrationRoutines(R:%):% == select(integrationRoutine?,R)

  optimizationRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
    (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
      elt(a,chapter) = "Optimization"
    false

  selectOptimizationRoutines(R:%):% == select(optimizationRoutine?,R)

  PDERoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
    (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
      elt(a,chapter) = "PDE"
    false

  selectPDERoutines(R:%):% == select(PDERoutine?,R)

  ODERoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
    (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
      elt(a,chapter) = "ODE"
    false
```

```
selectODEIVPRoutines(R:%):% == select(ODERoutine?,R)

sumOfSquaresRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "SS"
  false

selectSumOfSquaresRoutines(R:%):% == select(sumOfSquaresRoutine?,R)

finiteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "One-dimensional finite"
  false

selectFiniteRoutines(R:%):% == select(finiteRoutine?,R)

infiniteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "One-dimensional infinite"
  false

semiInfiniteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "One-dimensional semi-infinite"
  false

nonFiniteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (semiInfiniteRoutine?(r) or infiniteRoutine?(r))

selectNonFiniteRoutines(R:%):% == select(nonFiniteRoutine?,R)

multiDimensionalRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "Multi-dimensional"
  false

selectMultiDimensionalRoutines(R:%):% == select(multiDimensionalRoutine?,R)

concat(a:%,b:%):% ==
  membersOfa := (members(a)@List(Record(key:Symbol,entry:Any)))
  membersOfb := (members(b)@List(Record(key:Symbol,entry:Any)))
  allMembers:=
    concat(membersOfa,membersOfb)$List(Record(key:Symbol,entry:Any))
  construct(allMembers)

changeThreshhold(R:%,s:Symbol,newValue:F):% ==
  (a := search(s,R)) case Any =>
    e := retract(a)$AnyFunctions1(Entry)
    e.defaultMin := newValue
```

```
      a := coerce(e)$AnyFunctions1(Entry)
      insert!([s,a],R)
    error("changeThreshhold","Cannot find routine of that name")$ErrorFunctions

  changeMeasure(R:%,s:Symbol,newValue:F):% ==
    (a := search(s,R)) case Any =>
      e := retract(a)$AnyFunctions1(Entry)
      e.measure := newValue
      a := coerce(e)$AnyFunctions1(Entry)
      insert!([s,a],R)
    error("changeMeasure","Cannot find routine of that name")$ErrorFunctions

  getMeasure(R:%,s:Symbol):F ==
    (a := search(s,R)) case Any =>
      e := retract(a)$AnyFunctions1(Entry)
      e.measure
    error("getMeasure","Cannot find routine of that name")$ErrorFunctions

  deleteRoutine!(R:%,s:Symbol):% ==
    (a := search(s,R)) case Any =>
      e:Record(key:Symbol,entry:Any) := [s,a]
      remove!(e,R)
    error("deleteRoutine!","Cannot find routine of that name")$ErrorFunctions

  routines():% ==
    f := "One-dimensional finite"
    s := "One-dimensional semi-infinite"
    i := "One-dimensional infinite"
    m := "Multi-dimensional"
    int := "Integration"
    ode := "ODE"
    pde := "PDE"
    opt := "Optimization"
    d01ajfExplList:LST := ["result:  Calculated value of the integral",
                           "iw:  iw(1) contains the actual number of sub-intervals usec
                            "w:  contains the end-points of the sub-intervals used alor
                             "abserr:  the estimate of the absolute error of the result
                              "ifail:  the error warning parameter",
                               "method:  details of the method used and measures of al]
                                "attributes:  a list of the attributes pertaining to th
    d01asfExplList:LST := ["result:  Calculated value of the integral",
                           "iw:  iw(1) contains the actual number of sub-intervals usec
                            "lst:  contains the actual number of sub-intervals used",
                             "erlst:  contains the error estimates over the sub-interva
                              "rslst:  contains the integral contributions of the sub-i
                               "ierlst:  contains the error flags corresponding to the
                                "abserr:  the estimate of the absolute error of the res
                                 "ifail:  the error warning parameter",
                                  "method:  details of the method used and measures of
                                   "attributes:  a list of the attributes pertaining to
```

```
d01fcfExplList:LST := ["result:  Calculated value of the integral",
                            "acc:  the estimate of the relative error of the result",
                             "minpts:  the number of integrand evaluations",
                              "ifail:  the error warning parameter",
                               "method:  details of the method used and measures of all methods",
                                "attributes:  a list of the attributes pertaining to the integrand
d01transExplList:LST := ["result:  Calculated value of the integral",
                              "abserr:  the estimate of the absolute error of the result",
                               "method:  details of the method and transformation used and measures
                                "d01***AnnaTypeAnswer:  the individual results from the routines",
                                 "attributes:  a list of the attributes pertaining to the integrand
d02bhfExplList:LST := ["x:  the value of x at the end of the calculation",
                             "y:  the computed values of Y\[1\]..Y\[n\] at x",
                              "tol:   the (possible) estimate of the error; this is not guarunteed"
                               "ifail:  the error warning parameter",
                                "method:  details of the method used and measures of all methods",
                                 "intensityFunctions:  a list of the attributes and values pertaini
d02bbfExplList:LST := concat(["result:  the computed values of the solution at the required points
d03eefExplList:LST := ["See the NAG On-line Documentation for D03EEF/D03EDF",
                             "u:  the computed solution u[i][j] is returned in u(i+(j-1)*ngx),for i
e04fdfExplList:LST := ["x:  the position of the minimum",
                             "objf:  the value of the objective function at x",
                              "ifail:  the error warning parameter",
                               "method:  details of the method used and measures of all methods",
                                 "attributes:  a list of the attributes pertaining to the function
e04dgfExplList:LST := concat(e04fdfExplList,
                                   ["objgrd:  the values of the derivatives at x",
                                     "iter:  the number of iterations performed"])$LST
e04jafExplList:LST := concat(e04fdfExplList,
                                   ["bu:  the values of the upper bounds used",
                                     "bl:  the values of the lower bounds used"])$LST
e04ucfExplList:LST := concat(e04dgfExplList,
                                   ["istate:  the status of every constraint at x",
                                     "clamda:  the QP multipliers for the last QP sub-problem",
                                      "For other output parameters see the NAG On-line Documentatio
e04mbfExplList:LST := concat(e04fdfExplList,
                                   ["istate:  the status of every constraint at x",
                                     "clamda:  the Lagrange multipliers for each constraint"])$LST
d01ajfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
                    [5,"delete"], [6,"delete"]]
d01akfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"]]
d01alfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
                    [5,"delete"], [6,"delete"], [7,"delete"]]
d01amfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
                    [5,"delete"], [6,"delete"]]
d01anfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
                    [5,"delete"], [6,"delete"], [7,"delete"]]
d01apfIfail:IFL :=
  [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"]]
d01aqfIfail:IFL :=
```

```
  [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"]]
d01asfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
      [5,"delete"], [6,"delete"], [7,"delete"], [8,"delete"], [9,"delete"]]
d01fcfIfail:IFL := [[1,"delete"], [2,"incrFunEvals"], [3,"delete"]]
d01gbfIfail:IFL := [[1,"delete"], [2,"incrFunEvals"]]
d02bbfIfail:IFL :=
  [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
   [4,"delete"], [5,"delete"], [6,"delete"], [7,"delete"]]
d02bhfIfail:IFL :=
  [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
   [4,"no action"], [5,"delete"], [6,"delete"], [7,"delete"]]
d02cjfIfail:IFL :=
  [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
   [4,"delete"], [5,"delete"], [6,"no action"], [7,"delete"]]
d02ejfIfail:IFL :=
  [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
   [4,"delete"], [5,"delete"], [6,"no action"], [7,"delete"], [8,"delete"],
    [9,"delete"]]
e04dgfIfail:IFL := [[3,"delete"], [4,"no action"], [6,"delete"],
                    [7,"delete"], [8,"delete"], [9,"delete"]]
e04fdfIfail:IFL :=
  [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"no action"], [6,"no action"], [7,"delete"], [8,"delete"]]
e04gcfIfail:IFL := [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
  [5,"no action"], [6,"no action"], [7,"delete"], [8,"delete"], [9,"delete"]]
e04jafIfail:IFL := [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
  [5,"no action"], [6,"no action"], [7,"delete"], [8,"delete"], [9,"delete"]]
e04mbfIfail:IFL :=
  [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"]]
e04nafIfail:IFL :=
  [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"],
    [6,"delete"], [7,"delete"], [8,"delete"], [9,"delete"]]
e04ucfIfail:IFL := [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
  [5,"delete"], [6,"delete"], [7,"delete"], [8,"delete"], [9,"delete"]]
d01ajfEntry:Entry := [int, f, "d01ajfAnnaType",0.4,0.4,d01ajfIfail,d01ajfExplList]
d01akfEntry:Entry := [int, f, "d01akfAnnaType",0.6,1.0,d01akfIfail,d01ajfExplList]
d01alfEntry:Entry := [int, f, "d01alfAnnaType",0.6,0.6,d01alfIfail,d01ajfExplList]
d01amfEntry:Entry := [int, i, "d01amfAnnaType",0.5,0.5,d01amfIfail,d01ajfExplList]
d01anfEntry:Entry := [int, f, "d01anfAnnaType",0.6,0.9,d01anfIfail,d01ajfExplList]
d01apfEntry:Entry := [int, f, "d01apfAnnaType",0.7,0.7,d01apfIfail,d01ajfExplList]
d01aqfEntry:Entry := [int, f, "d01aqfAnnaType",0.6,0.7,d01aqfIfail,d01ajfExplList]
d01asfEntry:Entry := [int, s, "d01asfAnnaType",0.6,0.9,d01asfIfail,d01asfExplList]
d01transEntry:Entry:=[int, i, "d01TransformFunctionType",0.6,0.9,[],d01transExplList]
d01gbfEntry:Entry := [int, m, "d01gbfAnnaType",0.6,0.6,d01gbfIfail,d01fcfExplList]
d01fcfEntry:Entry := [int, m, "d01fcfAnnaType",0.5,0.5,d01fcfIfail,d01fcfExplList]
d02bbfEntry:Entry := [ode, "IVP", "d02bbfAnnaType",0.7,0.5,d02bbfIfail,d02bbfExplList]
d02bhfEntry:Entry := [ode, "IVP", "d02bhfAnnaType",0.7,0.49,d02bhfIfail,d02bhfExplList]
d02cjfEntry:Entry := [ode, "IVP", "d02cjfAnnaType",0.7,0.5,d02cjfIfail,d02bbfExplList]
d02ejfEntry:Entry := [ode, "IVP", "d02ejfAnnaType",0.7,0.5,d02ejfIfail,d02bbfExplList]
d03eefEntry:Entry := [pde, "2", "d03eefAnnaType",0.6,0.5,[],d03eefExplList]
```

```
  --d03fafEntry:Entry := [pde, "3", "d03fafAnnaType",0.6,0.5,[],[]]
  e04dgfEntry:Entry := [opt, "CGA", "e04dgfAnnaType",0.4,0.4,e04dgfIfail,e04dgfExplList]
  e04fdfEntry:Entry := [opt, "SS", "e04fdfAnnaType",0.7,0.7,e04fdfIfail,e04fdfExplList]
  e04gcfEntry:Entry := [opt, "SS", "e04gcfAnnaType",0.8,0.8,e04gcfIfail,e04fdfExplList]
  e04jafEntry:Entry := [opt, "QNA", "e04jafAnnaType",0.5,0.5,e04jafIfail,e04jafExplList]
  e04mbfEntry:Entry := [opt, "LP", "e04mbfAnnaType",0.7,0.7,e04mbfIfail,e04mbfExplList]
  e04nafEntry:Entry := [opt, "QP", "e04nafAnnaType",0.7,0.7,e04nafIfail,e04mbfExplList]
  e04ucfEntry:Entry := [opt, "SQP", "e04ucfAnnaType",0.6,0.6,e04ucfIfail,e04ucfExplList]
  rl:RList :=
    [["d01apf" :: Symbol, coerce(d01apfEntry)$AnyFunctions1(Entry)],_
     ["d01aqf" :: Symbol, coerce(d01aqfEntry)$AnyFunctions1(Entry)],_
     ["d01alf" :: Symbol, coerce(d01alfEntry)$AnyFunctions1(Entry)],_
     ["d01anf" :: Symbol, coerce(d01anfEntry)$AnyFunctions1(Entry)],_
     ["d01akf" :: Symbol, coerce(d01akfEntry)$AnyFunctions1(Entry)],_
     ["d01ajf" :: Symbol, coerce(d01ajfEntry)$AnyFunctions1(Entry)],_
     ["d01asf" :: Symbol, coerce(d01asfEntry)$AnyFunctions1(Entry)],_
     ["d01amf" :: Symbol, coerce(d01amfEntry)$AnyFunctions1(Entry)],_
     ["d01transform" :: Symbol, coerce(d01transEntry)$AnyFunctions1(Entry)],_
     ["d01gbf" :: Symbol, coerce(d01gbfEntry)$AnyFunctions1(Entry)],_
     ["d01fcf" :: Symbol, coerce(d01fcfEntry)$AnyFunctions1(Entry)],_
     ["d02bbf" :: Symbol, coerce(d02bbfEntry)$AnyFunctions1(Entry)],_
     ["d02bhf" :: Symbol, coerce(d02bhfEntry)$AnyFunctions1(Entry)],_
     ["d02cjf" :: Symbol, coerce(d02cjfEntry)$AnyFunctions1(Entry)],_
     ["d02ejf" :: Symbol, coerce(d02ejfEntry)$AnyFunctions1(Entry)],_
     ["d03eef" :: Symbol, coerce(d03eefEntry)$AnyFunctions1(Entry)],_
     --["d03faf" :: Symbol, coerce(d03fafEntry)$AnyFunctions1(Entry)],
     ["e04dgf" :: Symbol, coerce(e04dgfEntry)$AnyFunctions1(Entry)],_
     ["e04fdf" :: Symbol, coerce(e04fdfEntry)$AnyFunctions1(Entry)],_
     ["e04gcf" :: Symbol, coerce(e04gcfEntry)$AnyFunctions1(Entry)],_
     ["e04jaf" :: Symbol, coerce(e04jafEntry)$AnyFunctions1(Entry)],_
     ["e04mbf" :: Symbol, coerce(e04mbfEntry)$AnyFunctions1(Entry)],_
     ["e04naf" :: Symbol, coerce(e04nafEntry)$AnyFunctions1(Entry)],_
     ["e04ucf" :: Symbol, coerce(e04ucfEntry)$AnyFunctions1(Entry)]]
  construct(rl)

getIFL(s:Symbol,l:%):Union(IFL,"failed") ==
  o := search(s,l)$%
  o case "failed" => "failed"
  e := retractIfCan(o)$AnyFunctions1(Entry)
  e case "failed" => "failed"
  e.failList

getInstruction(l:IFL,ifailValue:Integer):Union(ST,"failed") ==
  output := empty()$ST
  for i in 1..#l repeat
    if ((l.i).ifail=ifailValue)@Boolean then
      output := (l.i).instruction
  empty?(output)$ST => "failed"
  output
```

```
  recoverAfterFail(routs:%,routineName:ST,
                   ifailValue:Integer):Union(ST,"failed") ==
    name := routineName :: Symbol
    failedList := getIFL(name,routs)
    failedList case "failed" => "failed"
    empty? failedList => "failed"
    instr := getInstruction(failedList,ifailValue)
    instr case "failed" => concat(routineName," failed")$ST
    (instr = "delete")@Boolean =>
      deleteRoutine!(routs,name)
      concat(routineName," failed - trying alternatives")$ST
    instr

  getExplanations(R:%,routineName:ST):LST ==
    name := routineName :: Symbol
    (a := search(name,R)) case Any =>
      e := retract(a)$AnyFunctions1(Entry)
      e.explList
    empty()$LST
```

———-

— ROUTINE.dotabb —

```
"ROUTINE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ROUTINE"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ROUTINE" -> "ALIST"
```

———-

19.14 domain RULECOLD RuleCalled

— RuleCalled.input —

```
)set break resume
)sys rm -f RuleCalled.output
)spool RuleCalled.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RuleCalled
--R RuleCalled f: Symbol  is a domain constructor
```

```
--R Abbreviation for RuleCalled is RULECOLD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RULECOLD
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R name : % -> Symbol                    ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

————

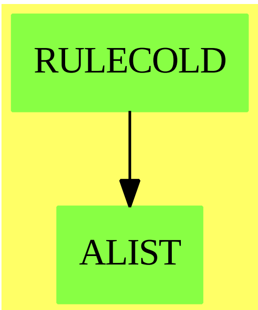
— RuleCalled.help —

```
====================================================================
RuleCalled examples
====================================================================

See Also:
o )show RuleCalled
```

————

19.14.1 RuleCalled (RULECOLD)



Exports:
coerce hash latex name ?=? ?~=?

— domain RULECOLD RuleCalled —

```
)abbrev domain RULECOLD RuleCalled
++ Author: Mark Botch
++ Description:
++ This domain implements named rules

RuleCalled(f:Symbol): SetCategory with
        name: % -> Symbol
                ++ name(x) returns the symbol
 == add
  name r                 == f
  coerce(r:%):OutputForm == f::OutputForm
  x = y                  == true
  latex(x:%):String      == latex f
```

— RULECOLD.dotabb —

```
"RULECOLD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RULECOLD"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RULECOLD" -> "ALIST"
```

19.15 domain RULESET Ruleset

— Ruleset.input —

```
)set break resume
)sys rm -f Ruleset.output
)spool Ruleset.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Ruleset
--R Ruleset(Base: SetCategory,R: Join(Ring,PatternMatchable Base,OrderedSet,ConvertibleTo Pat
--R Abbreviation for Ruleset is RULESET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RULESET
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R elt : (%,F,PositiveInteger) -> F      ?.? : (%,F) -> F
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R rules : % -> List RewriteRule(Base,R,F)
--R ruleset : List RewriteRule(Base,R,F) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

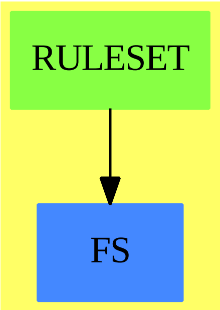
— **Ruleset.help** —

```
====================================================================
Ruleset examples
====================================================================

See Also:
o )show Ruleset
```

———-

19.15.1 Ruleset (RULESET)



See

⇒ "RewriteRule" (RULE) 19.10.1 on page 2265

Exports:

coerce	elt	hash	latex	rules
ruleset	?=?	?~=?	?.?	

— **domain RULESET Ruleset** —

```
)abbrev domain RULESET Ruleset
++ Author: Manuel Bronstein
++ Date Created: 20 Mar 1990
++ Date Last Updated: 29 Jun 1990
++ Keywords: pattern, matching, rule.
++ Description:
++ Sets of rules for the pattern matcher.
++ A ruleset is a set of pattern matching rules grouped together.

Ruleset(Base, R, F): Exports == Implementation where
  Base   : SetCategory
  R      : Join(Ring, PatternMatchable Base, OrderedSet,
                                 ConvertibleTo Pattern Base)
  F      : Join(FunctionSpace R, PatternMatchable Base,
                                 ConvertibleTo Pattern Base)

  RR ==> RewriteRule(Base, R, F)

  Exports ==> Join(SetCategory, Eltable(F, F)) with
    ruleset: List RR -> $
      ++ ruleset([r1,...,rn]) creates the rule set \spad{{r1,...,rn}}.
    rules  : $ -> List RR
      ++ rules(r) returns the rules contained in r.
    elt    : ($, F, PositiveInteger) -> F
      ++ elt(r,f,n) or r(f, n) applies all the rules of r to f at most n times.

  Implementation ==> add
    import ApplyRules(Base, R, F)

    Rep := Set RR

    ruleset l                         == {l}$Rep
    coerce(x:$):OutputForm            == coerce(x)$Rep
    x = y                             == x =$Rep y
    elt(x:$, f:F)                     == applyRules(rules x, f)
    elt(r:$, s:F, n:PositiveInteger) == applyRules(rules r, s, n)
    rules x                           == parts(x)$Rep
```

— **RULESET.dotabb** —

```
"RULESET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RULESET"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"RULESET" -> "FS"
```

Chapter 20

Chapter S

20.1 domain FORMULA ScriptFormulaFormat

— **ScriptFormulaFormat.input** —

```
)set break resume
)sys rm -f ScriptFormulaFormat.output
)spool ScriptFormulaFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ScriptFormulaFormat
--R ScriptFormulaFormat  is a domain constructor
--R Abbreviation for ScriptFormulaFormat is FORMULA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FORMULA
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : OutputForm -> %
--R coerce : % -> OutputForm              display : % -> Void
--R display : (%,Integer) -> Void         epilogue : % -> List String
--R formula : % -> List String            hash : % -> SingleInteger
--R latex : % -> String                   new : () -> %
--R prologue : % -> List String           ?~=? : (%,%) -> Boolean
--R convert : (OutputForm,Integer) -> %
--R setEpilogue! : (%,List String) -> List String
--R setFormula! : (%,List String) -> List String
--R setPrologue! : (%,List String) -> List String
--R
--E 1
```

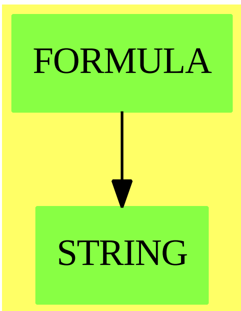
```
)spool
)lisp (bye)
```

— ScriptFormulaFormat.help —

```
====================================================================
ScriptFormulaFormat examples
====================================================================

See Also:
o )show ScriptFormulaFormat
```

20.1.1 ScriptFormulaFormat (FORMULA)



Exports:

coerce	display	epilogue	formula	hash
latex	new	prologue	convert	setEpilogue!
setFormula!	setPrologue!	?=?	?~=?	

— domain FORMULA ScriptFormulaFormat —

```
)abbrev domain FORMULA ScriptFormulaFormat
++ Author: Robert S. Sutor
++ Date Created: 1987 through 1990
++ Change History:
++ Basic Operations: coerce, convert, display, epilogue,
++   formula, new, prologue, setEpilogue!, setFormula!, setPrologue!
++ Related Constructors: ScriptFormulaFormat1
++ Also See: TexFormat
```

```
++ AMS Classifications:
++ Keywords: output, format, SCRIPT, BookMaster, formula
++ References:
++   SCRIPT Mathematical Formula Formatter User's Guide, SH20-6453,
++   IBM Corporation, Publishing Systems Information Development,
++   Dept. G68, P.O. Box 1900, Boulder, Colorado, USA 80301-9191.
++ Description:
++ \spadtype{ScriptFormulaFormat} provides a coercion from
++ \spadtype{OutputForm} to IBM SCRIPT/VS Mathematical Formula Format.
++ The basic SCRIPT formula format object consists of three parts:  a
++ prologue, a formula part and an epilogue.  The functions
++ \spadfun{prologue}, \spadfun{formula} and \spadfun{epilogue}
++ extract these parts, respectively.  The central parts of the expression
++ go into the formula part.  The other parts can be set
++ (\spadfun{setPrologue!}, \spadfun{setEpilogue!}) so that contain the
++ appropriate tags for printing.  For example, the prologue and
++ epilogue might simply contain ":df."  and ":edf."  so that the
++ formula section will be printed in display math mode.

ScriptFormulaFormat(): public == private where
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String

  public == SetCategory with
    coerce:   E -> %
      ++ coerce(o) changes o in the standard output format to
      ++ SCRIPT formula format.
    convert:  (E,I) -> %
      ++ convert(o,step) changes o in standard output format to
      ++ SCRIPT formula format and also adds the given step number.
      ++ This is useful if you want to create equations with given numbers
      ++ or have the equation numbers correspond to the interpreter step
      ++ numbers.
    display:  (%, I) -> Void
      ++ display(t,width) outputs the formatted code t so that each
      ++ line has length less than or equal to \spadvar{width}.
    display:  % -> Void
      ++ display(t) outputs the formatted code t so that each
      ++ line has length less than or equal to the value set by
      ++ the system command \spadsyscom{set output length}.
    epilogue: % -> L S
      ++ epilogue(t) extracts the epilogue section of a formatted object t.
    formula:      % -> L S
      ++ formula(t) extracts the formula section of a formatted object t.
    new:      () -> %
      ++ new() create a new, empty object. Use \spadfun{setPrologue!},
      ++ \spadfun{setFormula!} and \spadfun{setEpilogue!} to set the
      ++ various components of this object.
```

```
    prologue: % -> L S
      ++ prologue(t) extracts the prologue section of a formatted object t.
    setEpilogue!: (%, L S) -> L S
      ++ setEpilogue!(t,strings) sets the epilogue section of a
      ++ formatted object t to strings.
    setFormula!:  (%, L S) -> L S
      ++ setFormula!(t,strings) sets the formula section of a
      ++ formatted object t to strings.
    setPrologue!: (%, L S) -> L S
      ++ setPrologue!(t,strings) sets the prologue section of a
      ++ formatted object t to strings.

  private == add
    import OutputForm
    import Character
    import Integer
    import List OutputForm
    import List String

    Rep := Record(prolog : L S, formula : L S, epilog : L S)

    -- local variables declarations and definitions

    expr: E
    prec,opPrec: I
    str:  S
    blank         : S := " @@ "

    maxPrec       : I   := 1000000
    minPrec       : I   := 0

    splitChars    : S   := " <>[](){}+*=,-%"

    unaryOps      : L S := ["-","^"]$(L S)
    unaryPrecs    : L I := [700,260]$(L I)

    -- the precedence of / in the following is relatively low because
    -- the bar obviates the need for parentheses.
    binaryOps     : L S := ["+->","|","**","/","<",">","=","OVER"]$(L S)
    binaryPrecs   : L I := [0,0,900, 700,400,400,400,   700]$(L I)

    naryOps       : L S := ["-","+","*",blank,",",";"," ","ROW","",
                           " habove "," here "," labove "]$(L S)
    naryPrecs     : L I := [700,700,800,  800,110,110,  0,    0, 0,
                           0,  0,  0]$(L I)
--  naryNGOps     : L S := ["ROW"," here "]$(L S)
    naryNGOps     : L S := nil$(L S)

    plexOps       : L S := ["SIGMA","PI","INTSIGN","INDEFINTEGRAL"]$(L S)
    plexPrecs     : L I := [    700, 800,      700,            700]$(L I)
```

```
specialOps    : L S := ["MATRIX","BRACKET","BRACE","CONCATB",     _
                        "AGGLST","CONCAT","OVERBAR","ROOT","SUB", _
                        "SUPERSUB","ZAG","AGGSET","SC","PAREN"]

-- the next two lists provide translations for some strings for
-- which the formula formatter provides special variables.

specialStrings : L S :=
  ["5","..."]
specialStringsInFormula : L S :=
  [" alpha "," ellipsis "]

-- local function signatures

addBraces:      S -> S
addBrackets:    S -> S
group:          S -> S
formatBinary:   (S,L E, I) -> S
formatFunction: (S,L E, I) -> S
formatMatrix:   L E -> S
formatNary:     (S,L E, I) -> S
formatNaryNoGroup: (S,L E, I) -> S
formatNullary:  S -> S
formatPlex:     (S,L E, I) -> S
formatSpecial:  (S,L E, I) -> S
formatUnary:    (S,  E, I) -> S
formatFormula:      (E,I) -> S
parenthesize:   S -> S
precondition:   E -> E
postcondition:  S -> S
splitLong:      (S,I) -> L S
splitLong1:     (S,I) -> L S
stringify:      E -> S

-- public function definitions

new() : % == [[".eq set blank @",":df."]$(L S),
  [""]$(L S), [":edf."]$(L S)]$Rep

coerce(expr : E): % ==
  f : % := new()$%
  f.formula := [postcondition
    formatFormula(precondition expr, minPrec)]$(L S)
  f

convert(expr : E, stepNum : I): % ==
  f : % := new()$%
  f.formula := concat(["<leqno lparen ",string(stepNum)$S,
    " rparen>"], [postcondition
```

```
      formatFormula(precondition expr, minPrec)]$(L S))
  f

display(f : %, len : I) ==
  s,t : S
  for s in f.prolog repeat sayFORMULA(s)$Lisp
  for s in f.formula repeat
    for t in splitLong(s, len) repeat sayFORMULA(t)$Lisp
  for s in f.epilog repeat sayFORMULA(s)$Lisp
  void()$Void

display(f : %) ==
  display(f, _$LINELENGTH$Lisp pretend I)

prologue(f : %) == f.prolog
formula(f : %)  == f.formula
epilogue(f : %) == f.epilog

setPrologue!(f : %, l : L S) == f.prolog := l
setFormula!(f : %, l : L S)  == f.formula := l
setEpilogue!(f : %, l : L S) == f.epilog := l

coerce(f : %): E ==
  s,t : S
  l : L S := nil
  for s in f.prolog repeat l := concat(s,l)
  for s in f.formula repeat
    for t in splitLong(s, (_$LINELENGTH$Lisp pretend Integer) - 4) repeat
      l := concat(t,l)
  for s in f.epilog repeat l := concat(s,l)
  (reverse l) :: E

-- local function definitions

postcondition(str: S): S ==
  len : I := #str
  len < 4 => str
  plus : Character := char "+"
  minus: Character := char "-"
  for i in 1..(len-1) repeat
    if (str.i =$Character plus) and (str.(i+1) =$Character minus)
      then setelt(str,i,char " ")$S
  str

stringify expr == object2String(expr)$Lisp pretend S

splitLong(str : S, len : I): L S ==
  -- this blocks into lines
  if len < 20 then len := _$LINELENGTH$Lisp
  splitLong1(str, len)
```

```
splitLong1(str : S, len : I) ==
  l : List S := nil
  s : S := ""
  ls : I := 0
  ss : S
  lss : I
  for ss in split(str,char " ") repeat
    lss := #ss
    if ls + lss > len then
      l := concat(s,l)$List(S)
      s := ""
      ls := 0
    lss > len => l := concat(ss,l)$List(S)
    ls := ls + lss + 1
    s := concat(s,concat(ss," ")$S)$S
  if ls > 0 then l := concat(s,l)$List(S)
  reverse l

group str ==
  concat ["<",str,">"]

addBraces str ==
  concat ["left lbrace ",str," right rbrace"]

addBrackets str ==
  concat ["left lb ",str," right rb"]

parenthesize str ==
  concat ["left lparen ",str," right rparen"]

precondition expr ==
  outputTran(expr)$Lisp

formatSpecial(op : S, args : L E, prec : I) : S ==
  op = "AGGLST" =>
    formatNary(",",args,prec)
  op = "AGGSET" =>
    formatNary(";",args,prec)
  op = "CONCATB" =>
    formatNary(" ",args,prec)
  op = "CONCAT" =>
    formatNary("",args,prec)
  op = "BRACKET" =>
    group addBrackets formatFormula(first args, minPrec)
  op = "BRACE" =>
    group addBraces formatFormula(first args, minPrec)
  op = "PAREN" =>
    group parenthesize formatFormula(first args, minPrec)
  op = "OVERBAR" =>
```

```
        null args => ""
        group concat [formatFormula(first args, minPrec)," bar"]
      op = "ROOT" =>
        null args => ""
        tmp : S := formatFormula(first args, minPrec)
        null rest args => group concat ["sqrt ",tmp]
        group concat ["midsup adjust(u 1.5 r 9) ",
          formatFormula(first rest args, minPrec)," sqrt ",tmp]
      op = "SC" =>
        formatNary(" labove ",args,prec)
      op = "SUB" =>
        group concat [formatFormula(first args, minPrec)," sub ",
          formatSpecial("AGGLST",rest args,minPrec)]
      op = "SUPERSUB" =>
        -- variable name
        form : List S := [formatFormula(first args, minPrec)]
        -- subscripts
        args := rest args
        null args => concat form
        tmp : S := formatFormula(first args, minPrec)
        if tmp ^= "" then form := append(form,[" sub ",tmp])$(List S)
        -- superscripts
        args := rest args
        null args => group concat form
        tmp : S := formatFormula(first args, minPrec)
        if tmp ^= "" then form := append(form,[" sup ",tmp])$(List S)
        -- presuperscripts
        args := rest args
        null args => group concat form
        tmp : S := formatFormula(first args, minPrec)
        if tmp ^= "" then form := append(form,[" presup ",tmp])$(List S)
        -- presubscripts
        args := rest args
        null args => group concat form
        tmp : S := formatFormula(first args, minPrec)
        if tmp ^= "" then form := append(form,[" presub ",tmp])$(List S)
        group concat form
      op = "MATRIX" => formatMatrix rest args
--    op = "ZAG" =>
--      concat ["\zag{",formatFormula(first args, minPrec),"}{",
--        formatFormula(first rest args,minPrec),"}"]
      concat ["not done yet for ",op]

    formatPlex(op : S, args : L E, prec : I) : S ==
      hold : S
      p : I := position(op,plexOps)
      p < 1 => error "unknown Script Formula Formatter unary op"
      opPrec := plexPrecs.p
      n : I := #args
      (n ^= 2) and (n ^= 3) => error "wrong number of arguments for plex"
```

```
    s : S :=
      op = "SIGMA"   => "sum"
      op = "PI"      => "product"
      op = "INTSIGN" => "integral"
      op = "INDEFINTEGRAL" => "integral"
      "????"
    hold := formatFormula(first args,minPrec)
    args := rest args
    if op ^= "INDEFINTEGRAL" then
      if hold ^= "" then
        s := concat [s," from",group concat ["\displaystyle ",hold]]
      if not null rest args then
        hold := formatFormula(first args,minPrec)
        if hold ^= "" then
          s := concat [s," to",group concat ["\displaystyle ",hold]]
        args := rest args
      s := concat [s," ",formatFormula(first args,minPrec)]
    else
      hold := group concat [hold," ",formatFormula(first args,minPrec)]
      s := concat [s," ",hold]
    if opPrec < prec then s := parenthesize s
    group s

  formatMatrix(args : L E) : S ==
    -- format for args is [[ROW ...],[ROW ...],[ROW ...]]
    group addBrackets formatNary(" habove ",args,minPrec)

  formatFunction(op : S, args : L E, prec : I) : S ==
    group concat [op, " ", parenthesize formatNary(",",args,minPrec)]

  formatNullary(op : S) ==
    op = "NOTHING" => ""
    group concat [op,"()"]

  formatUnary(op : S, arg : E, prec : I) ==
    p : I := position(op,unaryOps)
    p < 1 => error "unknown Script Formula Formatter unary op"
    opPrec := unaryPrecs.p
    s : S := concat [op,formatFormula(arg,opPrec)]
    opPrec < prec => group parenthesize s
    op = "-" => s
    group s

  formatBinary(op : S, args : L E, prec : I) : S ==
    p : I := position(op,binaryOps)
    p < 1 => error "unknown Script Formula Formatter binary op"
    op :=
      op = "**"    => " sup "
      op = "/"     => " over "
      op = "OVER"  => " over "
```

```
      op
    opPrec := binaryPrecs.p
    s : S := formatFormula(first args, opPrec)
    s := concat [s,op,formatFormula(first rest args, opPrec)]
    group
      op = " over " => s
      opPrec < prec => parenthesize s
      s

  formatNary(op : S, args : L E, prec : I) : S ==
    group formatNaryNoGroup(op, args, prec)

  formatNaryNoGroup(op : S, args : L E, prec : I) : S ==
    null args => ""
    p : I := position(op,naryOps)
    p < 1 => error "unknown Script Formula Formatter nary op"
    op :=
      op = ","     => ", @@ "
      op = ";"     => "; @@ "
      op = "*"     => blank
      op = " "     => blank
      op = "ROW"   => " here "
      op
    l : L S := nil
    opPrec := naryPrecs.p
    for a in args repeat
      l := concat(op,concat(formatFormula(a,opPrec),l)$L(S))$L(S)
    s : S := concat reverse rest l
    opPrec < prec => parenthesize s
    s

  formatFormula(expr,prec) ==
    i : Integer
    ATOM(expr)$Lisp pretend Boolean =>
      str := stringify expr
      INTEGERP(expr)$Lisp =>
        i := expr : Integer
        if (i < 0) or (i > 9) then group str else str
      (i := position(str,specialStrings)) > 0 =>
        specialStringsInFormula.i
      str
    l : L E := (expr pretend L E)
    null l => blank
    op : S := stringify first l
    args : L E := rest l
    nargs : I := #args

    -- special cases
    member?(op, specialOps) => formatSpecial(op,args,prec)
    member?(op, plexOps)    => formatPlex(op,args,prec)
```

```
      -- nullary case
      0 = nargs => formatNullary op

      -- unary case
      (1 = nargs) and member?(op, unaryOps) =>
        formatUnary(op, first args, prec)

      -- binary case
      (2 = nargs) and member?(op, binaryOps) =>
        formatBinary(op, args, prec)

      -- nary case
      member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
      member?(op,naryOps) => formatNary(op,args, prec)
      op := formatFormula(first l,minPrec)
      formatFunction(op,args,prec)
```

———-

— FORMULA.dotabb —

```
"FORMULA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FORMULA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FORMULA" -> "STRING"
```

———-

20.2 domain SEG Segment

— Segment.input —

```
)set break resume
)sys rm -f Segment.output
)spool Segment.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
s := 3..10
--R
--R
--R   (1)  3..10
--R                                                 Type: Segment PositiveInteger
```

```
--E 1

--S 2 of 10
lo s
--R
--R
--R   (2)  3
--R                                                   Type: PositiveInteger
--E 2

--S 3 of 10
hi s
--R
--R
--R   (3)  10
--R                                                   Type: PositiveInteger
--E 3

--S 4 of 10
t := 10..3 by -2
--R
--R
--R   (4)  10..3 by - 2
--R                                           Type: Segment PositiveInteger
--E 4

--S 5 of 10
incr s
--R
--R
--R   (5)  1
--R                                                   Type: PositiveInteger
--E 5

--S 6 of 10
incr t
--R
--R
--R   (6)  - 2
--R                                                           Type: Integer
--E 6

--S 7 of 10
l := [1..3, 5, 9, 15..11 by -1]
--R
--R
--R   (7)  [1..3,5..5,9..9,15..11 by - 1]
--R                                      Type: List Segment PositiveInteger
--E 7
```

```
--S 8 of 10
expand s
--R
--R
--R   (8)  [3,4,5,6,7,8,9,10]
--R                                                       Type: List Integer
--E 8

--S 9 of 10
expand t
--R
--R
--R   (9)  [10,8,6,4]
--R                                                       Type: List Integer
--E 9

--S 10 of 10
expand l
--R
--R
--R   (10)  [1,2,3,5,9,15,14,13,12,11]
--R                                                       Type: List Integer
--E 10
)spool
)lisp (bye)
```

— Segment.help —

```
====================================================================
Segment examples
====================================================================

The Segment domain provides a generalized interval type.

Segments are created using the .. construct by indicating the
(included) end points.

  s := 3..10
    3..10
                             Type: Segment PositiveInteger

The first end point is called the lo and the second is called hi.

  lo s
    3
                             Type: PositiveInteger
```

These names are used even though the end points might belong to an unordered set.

```
  hi s
    10
                              Type: PositiveInteger
```

In addition to the end points, each segment has an integer "increment". An increment can be specified using the "by" construct.

```
  t := 10..3 by -2
    10..3 by - 2
                              Type: Segment PositiveInteger
```

This part can be obtained using the incr function.

```
  incr s
    1
                              Type: PositiveInteger
```

Unless otherwise specified, the increment is 1.

```
  incr t
    - 2
                              Type: Integer
```

A single value can be converted to a segment with equal end points. This happens if segments and single values are mixed in a list.

```
  l := [1..3, 5, 9, 15..11 by -1]
    [1..3,5..5,9..9,15..11 by - 1]
                               Type: List Segment PositiveInteger
```

If the underlying type is an ordered ring, it is possible to perform additional operations. The expand operation creates a list of points in a segment.

```
  expand s
    [3,4,5,6,7,8,9,10]
                               Type: List Integer
```

If k > 0, then expand(l..h by k) creates the list [l, l+k, ..., lN] where lN <= h < lN+k. If k < 0, then lN >= h > lN+k.

```
  expand t
    [10,8,6,4]
                               Type: List Integer
```

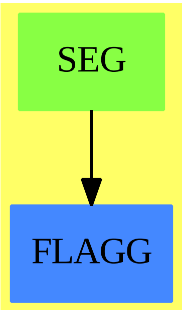
It is also possible to expand a list of segments. This is equivalent to appending lists obtained by expanding each segment individually.

```
  expand l
    [1,2,3,5,9,15,14,13,12,11]
                                    Type: List Integer

See Also:
o )help UniversalSegment
o )help SegmentBinding
o )show Segment
```

———-

20.2.1 Segment (SEG)



See

⇒ "SegmentBinding" (SEGBIND) 20.3.1 on page 2324
⇒ "UniversalSegment" (UNISEG) 22.11.1 on page 2853

Exports:

BY	coerce	convert	expand	hash
hi	high	incr	latex	lo
low	map	segment	?=?	?~=?
?..?				

— **domain SEG Segment** —

```
)abbrev domain SEG Segment
++ Author:  Stephen M. Watt
++ Date Created:  December 1986
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords: range, segment
++ Examples:
++ References:
++ Description:
++ This type is used to specify a range of values from type \spad{S}.

Segment(S:Type): SegmentCategory(S) with
    if S has SetCategory then SetCategory
    if S has OrderedRing then SegmentExpansionCategory(S, List S)
  == add

    Rep := Record(low: S, high: S, incr: Integer)

    a..b == [a,b,1]
    lo s == s.low
    low s == s.low
    hi s == s.high
    high s == s.high
    incr s == s.incr
    segment(a,b) == [a,b,1]
    BY(s, r) == [lo s, hi s, r]

    if S has SetCategory then
      (s1:%) = (s2:%) ==
        s1.low = s2.low and s1.high=s2.high and s1.incr = s2.incr

      coerce(s:%):OutputForm ==
        seg := SEGMENT(s.low::OutputForm, s.high::OutputForm)
        s.incr = 1 => seg
        infix(" by "::OutputForm, seg, s.incr::OutputForm)

    convert a == [a,a,1]

    if S has OrderedRing then
      expand(ls: List %):List S ==
        lr := nil()$List(S)
        for s in ls repeat
          l := lo s
          h := hi s
          inc := (incr s)::S
          zero? inc => error "Cannot expand a segment with an increment of zero"
          if inc > 0 then
            while l <= h repeat
              lr := concat(l, lr)
              l := l + inc
          else
            while l >= h repeat
              lr := concat(l, lr)
              l := l + inc
        reverse_! lr
```

```
      expand(s : %) == expand([s]$List(%))$%
      map(f : S->S, s : %): List S ==
        lr := nil()$List(S)
        l := lo s
        h := hi s
        inc := (incr s)::S
        if inc > 0 then
          while l <= h repeat
            lr := concat(f l, lr)
            l := l + inc
        else
          while l >= h repeat
            lr := concat(f l, lr)
            l := l + inc
        reverse_! lr
```

— **SEG.dotabb** —

```
"SEG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEG"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"SEG" -> "FLAGG"
```

20.3 domain SEGBIND SegmentBinding

— **SegmentBinding.input** —

```
)set break resume
)sys rm -f SegmentBinding.output
)spool SegmentBinding.output
)set message test on
)set message auto off
)clear all
--S 1 of 5
x = a..b
--R
--R
--R   (1)  x= a..b
--R                                                   Type: SegmentBinding Symbol
--E 1
```

```
--S 2 of 5
sum(i**2, i = 0..n)
--R
--R
--R          3     2
--R        2n  + 3n  + n
--R   (2)  -------------
--R              6
--R                                             Type: Fraction Polynomial Integer
--E 2

--S 3 of 5
sb := y = 1/2..3/2
--R
--R
--R            1    3
--R   (3)  y= (-)..(-)
--R            2    2
--R                                           Type: SegmentBinding Fraction Integer
--E 3

--S 4 of 5
variable(sb)
--R
--R
--R   (4)  y
--R                                                                   Type: Symbol
--E 4

--S 5 of 5
segment(sb)
--R
--R
--R         1    3
--R   (5)  (-)..(-)
--R         2    2
--R                                                 Type: Segment Fraction Integer
--E 5
)spool
)lisp (bye)
```

— **SegmentBinding.help** —

```
====================================================================
SegmentBinding examples
====================================================================
```

```
The SegmentBinding type is used to indicate a range for a named symbol.

First give the symbol, then an = and finally a segment of values.

  x = a..b
    x= a..b
                            Type: SegmentBinding Symbol

This is used to provide a convenient syntax for arguments to certain
operations.

  sum(i**2, i = 0..n)
       3     2
    2n  + 3n  + n
    -------------
          6
                            Type: Fraction Polynomial Integer

  draw(x**2, x = -2..2)
    TwoDimensionalViewport: "x*x"
                            Type: TwoDimensionalViewport


The left-hand side must be of type Symbol but the right-hand side can
be a segment over any type.

  sb := y = 1/2..3/2
        1    3
    y= (-)..(-)
        2    2
                            Type: SegmentBinding Fraction Integer

The left- and right-hand sides can be obtained using the variable and
segment operations.

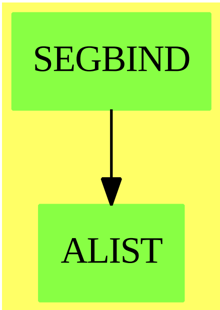
  variable(sb)
    y
                            Type: Symbol

  segment(sb)
     1    3
    (-)..(-)
     2    2
                            Type: Segment Fraction Integer

See Also:
o )help Segment
o )help UniversalSegment
o )show SegmentBinding
```

———-

20.3.1 SegmentBinding (SEGBIND)



See

⇒ "Segment" (SEG) 20.2.1 on page 2319
⇒ "UniversalSegment" (UNISEG) 22.11.1 on page 2853

Exports:

coerce	equation	hash	latex	segment
variable	?=?	?~=?		

— domain SEGBIND SegmentBinding —

```
)abbrev domain SEGBIND SegmentBinding
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: Equation, Segment, Symbol
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This domain is used to provide the function argument syntax \spad{v=a..b}.
++ This is used, for example, by the top-level \spadfun{draw} functions.

SegmentBinding(S:Type): Type with
  equation: (Symbol, Segment S) -> %
      ++ equation(v,a..b) creates a segment binding value with variable
      ++ \spad{v} and segment \spad{a..b}.  Note that the interpreter parses
```

```
      ++ \spad{v=a..b} to this form.
  variable: % -> Symbol
      ++ variable(segb) returns the variable from the left hand side of
      ++ the \spadtype{SegmentBinding}.  For example, if \spad{segb} is
      ++ \spad{v=a..b}, then \spad{variable(segb)} returns \spad{v}.
  segment : % -> Segment S
      ++ segment(segb) returns the segment from the right hand side of
      ++ the \spadtype{SegmentBinding}.  For example, if \spad{segb} is
      ++ \spad{v=a..b}, then \spad{segment(segb)} returns \spad{a..b}.

  if S has SetCategory then SetCategory
 == add
  Rep := Record(var:Symbol, seg:Segment S)
  equation(x,s) == [x, s]
  variable b    == b.var
  segment b     == b.seg

  if S has SetCategory then

     b1 = b2       == variable b1 = variable b2 and segment b1 = segment b2

     coerce(b:%):OutputForm ==
       variable(b)::OutputForm = segment(b)::OutputForm
```

— **SEGBIND.dotabb** —

```
"SEGBIND" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEGBIND"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SEGBIND" -> "ALIST"
```

20.4 domain SET Set

— **Set.input** —

```
)set break resume
)sys rm -f Set.output
)spool Set.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 20
s := set [x**2-1, y**2-1, z**2-1]
--R
--R
--R          2      2      2
--R   (1)  {x  - 1,y  - 1,z  - 1}
--R                                                 Type: Set Polynomial Integer
--E 1

--S 2 of 20
t := set [x**i - i+1 for i in 2..10 | prime? i]
--R
--R
--R          2      3      5      7
--R   (2)  {x  - 1,x  - 2,x  - 4,x  - 6}
--R                                                 Type: Set Polynomial Integer
--E 2

--S 3 of 20
i := intersect(s,t)
--R
--R
--R          2
--R   (3)  {x  - 1}
--R                                                 Type: Set Polynomial Integer
--E 3

--S 4 of 20
u := union(s,t)
--R
--R
--R          2      3      5      7      2      2
--R   (4)  {x  - 1,x  - 2,x  - 4,x  - 6,y  - 1,z  - 1}
--R                                                 Type: Set Polynomial Integer
--E 4

--S 5 of 20
difference(s,t)
--R
--R
--R          2      2
--R   (5)  {y  - 1,z  - 1}
--R                                                 Type: Set Polynomial Integer
--E 5

--S 6 of 20
symmetricDifference(s,t)
--R
--R
--R          3      5      7      2      2
```

```
--R   (6)  {x  - 2,x  - 4,x  - 6,y  - 1,z  - 1}
--R                                                 Type: Set Polynomial Integer
--E 6

--S 7 of 20
member?(y, s)
--R
--R
--R   (7)  false
--R                                                                Type: Boolean
--E 7

--S 8 of 20
member?((y+1)*(y-1), s)
--R
--R
--R   (8)  true
--R                                                                Type: Boolean
--E 8

--S 9 of 20
subset?(i, s)
--R
--R
--R   (9)  true
--R                                                                Type: Boolean
--E 9

--S 10 of 20
subset?(u, s)
--R
--R
--R   (10)  false
--R                                                                Type: Boolean
--E 10

--S 11 of 20
gs := set [g for i in 1..11 | primitive?(g := i::PF 11)]
--R
--R
--R   (11)  {2,6,7,8}
--R                                                      Type: Set PrimeField 11
--E 11

--S 12 of 20
complement gs
--R
--R
--R   (12)  {1,3,4,5,9,10,0}
--R                                                      Type: Set PrimeField 11
```

```
--E 12

--S 13 of 20
a := set [i**2 for i in 1..5]
--R
--R
--R   (13)  {1,4,9,16,25}
--R                                                  Type: Set PositiveInteger
--E 13

--S 14 of 20
insert!(32, a)
--R
--R
--R   (14)  {1,4,9,16,25,32}
--R                                                  Type: Set PositiveInteger
--E 14

--S 15 of 20
remove!(25, a)
--R
--R
--R   (15)  {1,4,9,16,32}
--R                                                  Type: Set PositiveInteger
--E 15

--S 16 of 20
a
--R
--R
--R   (16)  {1,4,9,16,32}
--R                                                  Type: Set PositiveInteger
--E 16

--S 17 of 20
b := b0 := set [i**2 for i in 1..5]
--R
--R
--R   (17)  {1,4,9,16,25}
--R                                                  Type: Set PositiveInteger
--E 17

--S 18 of 20
b := union(b, {32})
--R
--R
--R   (18)  {1,4,9,16,25,32}
--R                                                  Type: Set PositiveInteger
--E 18
```

```
--S 19 of 20
b := difference(b, {25})
--R
--R
--R   (19)  {1,4,9,16,32}
--R                                                   Type: Set PositiveInteger
--E 19

--S 20 of 20
b0
--R
--R
--R   (20)  {1,4,9,16,25}
--R                                                   Type: Set PositiveInteger
--E 20
)spool
)lisp (bye)
```

— **Set.help** —

```
====================================================================
Set examples
====================================================================

The Set domain allows one to represent explicit finite sets of values.
These are similar to lists, but duplicate elements are not allowed.

Sets can be created by giving a fixed set of values ...

  s := set [x**2-1, y**2-1, z**2-1]
      2      2      2
    {x  - 1,y  - 1,z  - 1}
                           Type: Set Polynomial Integer

or by using a collect form, just as for lists.  In either case, the
set is formed from a finite collection of values.

  t := set [x**i - i+1 for i in 2..10 | prime? i]
      2      3      5      7
    {x  - 1,x  - 2,x  - 4,x  - 6}
                           Type: Set Polynomial Integer

The basic operations on sets are intersect, union, difference, and
symmetricDifference.

  i := intersect(s,t)
      2
```

```
   {x  - 1}
                           Type: Set Polynomial Integer

 u := union(s,t)
     2      3      5      7      2      2
   {x  - 1,x  - 2,x  - 4,x  - 6,y  - 1,z  - 1}
                           Type: Set Polynomial Integer
```

The set difference(s,t) contains those members of s which are not in t.

```
 difference(s,t)
     2      2
   {y  - 1,z  - 1}
                           Type: Set Polynomial Integer
```

The set symmetricDifference(s,t) contains those elements which are in s or t but not in both.

```
 symmetricDifference(s,t)
     3      5      7      2      2
   {x  - 2,x  - 4,x  - 6,y  - 1,z  - 1}
                           Type: Set Polynomial Integer
```

Set membership is tested using the member? operation.

```
 member?(y, s)
   false
                           Type: Boolean

 member?((y+1)*(y-1), s)
   true
                           Type: Boolean
```

The subset? function determines whether one set is a subset of another.

```
 subset?(i, s)
   true
                           Type: Boolean

 subset?(u, s)
   false
                           Type: Boolean
```

When the base type is finite, the absolute complement of a set is defined. This finds the set of all multiplicative generators of PrimeField 11---the integers mod 11.

```
 gs := set [g for i in 1..11 | primitive?(g := i::PF 11)]
   {2,6,7,8}
                           Type: Set PrimeField 11
```

The following values are not generators.

```
  complement gs
    {1,3,4,5,9,10,0}
                           Type: Set PrimeField 11
```

Often the members of a set are computed individually; in addition, values can be inserted or removed from a set over the course of a computation.

There are two ways to do this:

```
  a := set [i**2 for i in 1..5]
    {1,4,9,16,25}
                           Type: Set PositiveInteger
```

One is to view a set as a data structure and to apply updating operations.

```
  insert!(32, a)
    {1,4,9,16,25,32}
                           Type: Set PositiveInteger

  remove!(25, a)
    {1,4,9,16,32}
                           Type: Set PositiveInteger

  a
    {1,4,9,16,32}
                           Type: Set PositiveInteger
```

The other way is to view a set as a mathematical entity and to create new sets from old.

```
  b := b0 := set [i**2 for i in 1..5]
    {1,4,9,16,25}
                           Type: Set PositiveInteger

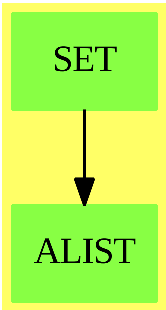
  b := union(b, {32})
    {1,4,9,16,25,32}
                           Type: Set PositiveInteger

  b := difference(b, {25})
    {1,4,9,16,32}
                           Type: Set PositiveInteger

  b0
    {1,4,9,16,25}
                           Type: Set PositiveInteger
```

```
See Also:
o )help List
o )show Set
```

20.4.1 Set (SET)



Exports:

any?	bag	brace	cardinality	coerce
complement	construct	convert	copy	count
dictionary	difference	empty	empty?	eq?
eval	every?	extract!	find	hash
index	insert!	inspect	intersect	latex
less?	lookup	map	map!	max
member?	members	min	more?	parts
random	reduce	remove	remove!	removeDuplicates
sample	select	select!	set	size
size?	subset?	symmetricDifference	union	universe
#?	?~=?	?<?	?=?	

— **domain SET Set** —

```
)abbrev domain SET Set
++ Author: Michael Monagan; revised by Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: May 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ A set over a domain D models the usual mathematical notion of a finite set
++ of elements from D.
++ Sets are unordered collections of distinct elements
++ (that is, order and duplication does not matter).
++ The notation \spad{set [a,b,c]} can be used to create
++ a set and the usual operations such as union and intersection are available
++ to form new sets.
++ In our implementation, \Language{} maintains the entries in
++ sorted order.  Specifically, the parts function returns the entries
++ as a list in ascending order and
++ the extract operation returns the maximum entry.
++ Given two sets s and t where \spad{#s = m} and \spad{#t = n},
++ the complexity of\br
++ \tab{5}\spad{s = t} is \spad{O(min(n,m))}\br
++ \tab{5}\spad{s < t} is \spad{O(max(n,m))}\br
++ \tab{5}\spad{union(s,t)}, \spad{intersect(s,t)}, \spad{minus(s,t)},\br
++ \tab{10 \spad{symmetricDifference(s,t)} is \spad{O(max(n,m))}\br
++ \tab{5}\spad{member(x,t)} is \spad{O(n log n)}\br
++ \tab{5}\spad{insert(x,t)} and \spad{remove(x,t)} is \spad{O(n)}

Set(S:SetCategory): FiniteSetAggregate S == add
   Rep := FlexibleArray(S)
   # s       == _#$Rep s
   brace()   == empty()
   set()     == empty()
   empty()   == empty()$Rep
   copy s    == copy(s)$Rep
   parts s   == parts(s)$Rep
   inspect s == (empty? s => error "Empty set"; s(maxIndex s))

   extract_! s ==
     x := inspect s
     delete_!(s, maxIndex s)
     x

   find(f, s) == find(f, s)$Rep

   map(f, s) == map_!(f,copy s)

   map_!(f,s) ==
     map_!(f,s)$Rep
     removeDuplicates_! s

   reduce(f, s) == reduce(f, s)$Rep

   reduce(f, s, x) == reduce(f, s, x)$Rep

   reduce(f, s, x, y) == reduce(f, s, x, y)$Rep
```

```
if S has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
     convert [convert("set"::Symbol)@InputForm,
                       convert(parts x)@InputForm]

if S has OrderedSet then
  s = t == s =$Rep t
  max s == inspect s
  min s == (empty? s => error "Empty set"; s(minIndex s))

  construct l ==
    zero?(n := #l) => empty()
    a := new(n, first l)
    for i in minIndex(a).. for x in l repeat a.i := x
    removeDuplicates_! sort_! a

  insert_!(x, s) ==
    n := inc maxIndex s
    k := minIndex s
    while k < n and x > s.k repeat k := inc k
    k < n and s.k = x => s
    insert_!(x, s, k)

  member?(x, s) == -- binary search
    empty? s => false
    t := maxIndex s
    b := minIndex s
    while b < t repeat
      m := (b+t) quo 2
      if x > s.m then b := m+1 else t := m
    x = s.t

  remove_!(x:S, s:%) ==
    n := inc maxIndex s
    k := minIndex s
    while k < n and x > s.k repeat k := inc k
    k < n and x = s.k => delete_!(s, k)
    s

  -- the set operations are implemented as variations of merging
  intersect(s, t) ==
    m := maxIndex s
    n := maxIndex t
    i := minIndex s
    j := minIndex t
    r := empty()
    while i <= m and j <= n repeat
      s.i = t.j => (concat_!(r, s.i); i := i+1; j := j+1)
      if s.i < t.j then i := i+1 else j := j+1
    r
```

```
difference(s:%, t:%) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i = t.j => (i := i+1; j := j+1)
    s.i < t.j => (concat_!(r, s.i); i := i+1)
    j := j+1
  while i <= m repeat (concat_!(r, s.i); i := i+1)
  r

symmetricDifference(s, t) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i < t.j => (concat_!(r, s.i); i := i+1)
    s.i > t.j => (concat_!(r, t.j); j := j+1)
    i := i+1; j := j+1
  while i <= m repeat (concat_!(r, s.i); i := i+1)
  while j <= n repeat (concat_!(r, t.j); j := j+1)
  r

subset?(s, t) ==
  m := maxIndex s
  n := maxIndex t
  m > n => false
  i := minIndex s
  j := minIndex t
  while i <= m and j <= n repeat
    s.i = t.j => (i := i+1; j := j+1)
    s.i > t.j => j := j+1
    return false
  i > m

union(s:%, t:%) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i = t.j => (concat_!(r, s.i); i := i+1; j := j+1)
    s.i < t.j => (concat_!(r, s.i); i := i+1)
    (concat_!(r, t.j); j := j+1)
```

```
       while i <= m repeat (concat_!(r, s.i); i := i+1)
       while j <= n repeat (concat_!(r, t.j); j := j+1)
       r

    else
      insert_!(x, s) ==
        for k in minIndex s .. maxIndex s repeat
          s.k = x => return s
        insert_!(x, s, inc maxIndex s)

      remove_!(x:S, s:%) ==
        n := inc maxIndex s
        k := minIndex s
        while k < n repeat
          x = s.k => return delete_!(s, k)
          k := inc k
        s
```

— **SET.dotabb** —

```
"SET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SET"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SET" -> "ALIST"
```

20.5 domain SETMN SetOfMIntegersInOneToN

— **SetOfMIntegersInOneToN.input** —

```
)set break resume
)sys rm -f SetOfMIntegersInOneToN.output
)spool SetOfMIntegersInOneToN.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SetOfMIntegersInOneToN
--R SetOfMIntegersInOneToN(m: PositiveInteger,n: PositiveInteger)  is a domain constructor
--R Abbreviation for SetOfMIntegersInOneToN is SETMN
--R This constructor is not exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SETMN
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R enumerate : () -> Vector %            hash : % -> SingleInteger
--R index : PositiveInteger -> %          latex : % -> String
--R lookup : % -> PositiveInteger         random : () -> %
--R size : () -> NonNegativeInteger       ?~=? : (%,%) -> Boolean
--R delta : (%,PositiveInteger,PositiveInteger) -> NonNegativeInteger
--R elements : % -> List PositiveInteger
--R incrementKthElement : (%,PositiveInteger) -> Union(%,"failed")
--R member? : (PositiveInteger,%) -> Boolean
--R replaceKthElement : (%,PositiveInteger,PositiveInteger) -> Union(%,"failed")
--R setOfMinN : List PositiveInteger -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

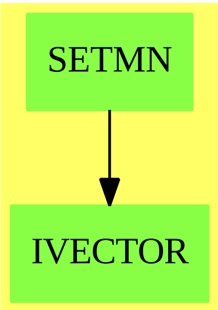
— **SetOfMIntegersInOneToN.help** —

```
====================================================================
SetOfMIntegersInOneToN examples
====================================================================

See Also:
o )show SetOfMIntegersInOneToN
```

———-

20.5.1 SetOfMIntegersInOneToN (SETMN)



Exports:

coerce	delta	elements	enumerate	hash
incrementKthElement	index	latex	lookup	member?
random	replaceKthElement	setOfMinN	size	?=?
?~=?				

— domain SETMN SetOfMIntegersInOneToN —

```
)abbrev domain SETMN SetOfMIntegersInOneToN
++ Author: Manuel Bronstein
++ Date Created: 10 January 1994
++ Date Last Updated: 10 January 1994
++ Description:
++ \spadtype{SetOfMIntegersInOneToN} implements the subsets of M integers
++ in the interval \spad{[1..n]}

SetOfMIntegersInOneToN(m, n): Exports == Implementation where
  PI ==> PositiveInteger
  N  ==> NonNegativeInteger
  U  ==> Union(%, "failed")
  n,m: PI

  Exports ==> Finite with
    incrementKthElement: (%, PI) -> U
      ++ incrementKthElement(S,k) increments the k^{th} element of S,
      ++ and returns "failed" if the result is not a set of M integers
      ++ in \spad{1..n} any more.
    replaceKthElement:   (%, PI, PI) -> U
      ++ replaceKthElement(S,k,p) replaces the k^{th} element of S by p,
      ++ and returns "failed" if the result is not a set of M integers
      ++ in \spad{1..n} any more.
    elements: % -> List PI
      ++ elements(S) returns the list of the elements of S in increasing order.
    setOfMinN: List PI -> %
      ++ setOfMinN([a_1,...,a_m]) returns the set {a_1,...,a_m}.
      ++ Error if {a_1,...,a_m} is not a set of M integers in \spad{1..n}.
    enumerate: () -> Vector %
      ++ enumerate() returns a vector of all the sets of M integers in
      ++ \spad{1..n}.
    member?:   (PI, %) -> Boolean
      ++ member?(p, s) returns true is p is in s, false otherwise.
    delta: (%, PI, PI) -> N
      ++ delta(S,k,p) returns the number of elements of S which are strictly
      ++ between p and the k^{th} element of S.

  Implementation ==> add
    Rep := Record(bits:Bits, pos:N)

    reallyEnumerate: () -> Vector %
    enum: (N, N, PI) -> List Bits
```

```
    all:Reference Vector % := ref empty()
    sz:Reference N := ref 0

    s1 = s2                 == s1.bits =$Bits s2.bits
    coerce(s:%):OutputForm == brace [i::OutputForm for i in elements s]
    random()                == index((1 + (random()$Integer rem size()))::PI)
    reallyEnumerate()       == [[b, i] for b in enum(m, n, n) for i in 1..]
    member?(p, s)           == s.bits.p

    enumerate() ==
      if empty? all() then all() := reallyEnumerate()
      all()

-- enumerates the sets of p integers in 1..q, returns them as sets in 1..n
-- must have p <= q
    enum(p, q, n) ==
      zero? p or zero? q => empty()
      p = q =>
        b := new(n, false)$Bits
        for i in 1..p repeat b.i := true
        [b]
      q1 := (q - 1)::N
      l := enum((p - 1)::N, q1, n)
      if empty? l then l := [new(n, false)$Bits]
      for s in l repeat s.q := true
      concat_!(enum(p, q1, n), l)

    size() ==
      if zero? sz() then
         sz() := binomial(n, m)$IntegerCombinatoricFunctions(Integer) :: N
      sz()

    lookup s ==
      if empty? all() then all() := reallyEnumerate()
      if zero?(s.pos) then s.pos := position(s, all()) :: N
      s.pos :: PI

    index p ==
      p > size() => error "index: argument too large"
      if empty? all() then all() := reallyEnumerate()
      all().p

    setOfMinN l ==
      s := new(n, false)$Bits
      count:N := 0
      for i in l repeat
        count := count + 1
        count > m or zero? i or i > n or s.i =>
          error "setOfMinN: improper set of integers"
```

```
      s.i := true
    count < m => error "setOfMinN: improper set of integers"
    [s, 0]

  elements s ==
    b := s.bits
    l:List PI := empty()
    found:N := 0
    i:PI := 1
    while found < m repeat
        if b.i then
            l := concat(i, l)
            found := found + 1
        i := i + 1
    reverse_! l

  incrementKthElement(s, k) ==
    b := s.bits
    found:N := 0
    i:N := 1
    while found < k repeat
        if b.i then found := found + 1
        i := i + 1
    i > n or b.i => "failed"
    newb := copy b
    newb.i := true
    newb.((i-1)::N) := false
    [newb, 0]

  delta(s, k, p) ==
    b := s.bits
    count:N := found:N := 0
    i:PI := 1
    while found < k repeat
        if b.i then
           found := found + 1
           if i > p and found < k then count := count + 1
        i := i + 1
    count

  replaceKthElement(s, k, p) ==
    b := s.bits
    found:N := 0
    i:PI := 1
    while found < k repeat
        if b.i then found := found + 1
        if found < k then i := i + 1
    b.p and i ^= p => "failed"
    newb := copy b
    newb.p := true
```

```
    newb.i := false
    [newb, (i = p => s.pos; 0)]
```

———-

— SETMN.dotabb —

```
"SETMN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SETMN"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"SETMN" -> "IVECTOR"
```

———-

20.6 domain SDPOL SequentialDifferentialPolynomial

— SequentialDifferentialPolynomial.input —

```
)set break resume
)sys rm -f SequentialDifferentialPolynomial.output
)spool SequentialDifferentialPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SequentialDifferentialPolynomial
--R SequentialDifferentialPolynomial R: Ring  is a domain constructor
--R Abbreviation for SequentialDifferentialPolynomial is SDPOL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SDPOL
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                 D : % -> % if R has DIFRING
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficients : % -> List R
--R coerce : Symbol -> %                  coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
```

```
--R ground : % -> R                     ground? : % -> Boolean
--R hash : % -> SingleInteger           initial : % -> %
--R isobaric? : % -> Boolean            latex : % -> String
--R leadingCoefficient : % -> R         leadingMonomial : % -> %
--R map : ((R -> R),%) -> %             monomial? : % -> Boolean
--R monomials : % -> List %             one? : % -> Boolean
--R order : % -> NonNegativeInteger     primitiveMonomials : % -> List %
--R recip : % -> Union(%,"failed")      reductum : % -> %
--R retract : % -> Symbol               retract : % -> R
--R sample : () -> %                    separant : % -> %
--R weight : % -> NonNegativeInteger    zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,List SequentialDifferentialVariable Symbol,List NonNegativeInteger) -> %
--R D : (%,SequentialDifferentialVariable Symbol,NonNegativeInteger) -> %
--R D : (%,List SequentialDifferentialVariable Symbol) -> %
--R D : (%,SequentialDifferentialVariable Symbol) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List SequentialDifferentialVariable Symbol,List NonNegativeInteger) -> %
--R coefficient : (%,SequentialDifferentialVariable Symbol,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents SequentialDifferentialVariable Symbol) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseMultivariatePolynomial(R,Symbol) -> %
--R coerce : SequentialDifferentialVariable Symbol -> %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SequentialDifferentialVariable Symbol) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SequentialDifferentialVariable Symbol has KONVERT INFORM and
--R convert : % -> Pattern Integer if SequentialDifferentialVariable Symbol has KONVERT PATT
--R convert : % -> Pattern Float if SequentialDifferentialVariable Symbol has KONVERT PATTER
--R degree : (%,Symbol) -> NonNegativeInteger
```

```
--R degree : (%,List SequentialDifferentialVariable Symbol) -> List NonNegativeInteger
--R degree : (%,SequentialDifferentialVariable Symbol) -> NonNegativeInteger
--R degree : % -> IndexedExponents SequentialDifferentialVariable Symbol
--R differentialVariables : % -> List Symbol
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,List SequentialDifferentialVariable Symbol,List NonNegativeInteger) -> %
--R differentiate : (%,SequentialDifferentialVariable Symbol,NonNegativeInteger) -> %
--R differentiate : (%,List SequentialDifferentialVariable Symbol) -> %
--R differentiate : (%,SequentialDifferentialVariable Symbol) -> %
--R discriminant : (%,SequentialDifferentialVariable Symbol) -> % if R has COMRING
--R eval : (%,List Symbol,List R) -> % if R has DIFRING
--R eval : (%,Symbol,R) -> % if R has DIFRING
--R eval : (%,List Symbol,List %) -> % if R has DIFRING
--R eval : (%,Symbol,%) -> % if R has DIFRING
--R eval : (%,List SequentialDifferentialVariable Symbol,List %) -> %
--R eval : (%,SequentialDifferentialVariable Symbol,%) -> %
--R eval : (%,List SequentialDifferentialVariable Symbol,List R) -> %
--R eval : (%,SequentialDifferentialVariable Symbol,R) -> %
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R isExpt : % -> Union(Record(var: SequentialDifferentialVariable Symbol,exponent: NonNegativeInteger),
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R leader : % -> SequentialDifferentialVariable Symbol
--R mainVariable : % -> Union(SequentialDifferentialVariable Symbol,"failed")
--R makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
--R makeVariable : Symbol -> (NonNegativeInteger -> %)
--R mapExponents : ((IndexedExponents SequentialDifferentialVariable Symbol -> IndexedExponents Sequenti
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SequentialDifferentialVariable Symbol) -> List NonNegativeInteger
--R minimumDegree : (%,SequentialDifferentialVariable Symbol) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents SequentialDifferentialVariable Symbol
--R monicDivide : (%,%,SequentialDifferentialVariable Symbol) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SequentialDifferentialVariable Symbol,List NonNegativeInteger) -> %
```

```
--R monomial : (%,SequentialDifferentialVariable Symbol,NonNegativeInteger) -> %
--R monomial : (R,IndexedExponents SequentialDifferentialVariable Symbol) -> %
--R multivariate : (SparseUnivariatePolynomial %,SequentialDifferentialVariable Symbol) -> %
--R multivariate : (SparseUnivariatePolynomial R,SequentialDifferentialVariable Symbol) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,Symbol) -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pomopo! : (%,R,IndexedExponents SequentialDifferentialVariable Symbol,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SequentialDifferentialVariable Symbol) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%,SequentialDifferentialVariable Symbol) -> % if R has COMRING
--R retract : % -> SparseMultivariatePolynomial(R,Symbol)
--R retract : % -> SequentialDifferentialVariable Symbol
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseMultivariatePolynomial(R,Symbol),"failed")
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(SequentialDifferentialVariable Symbol,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SequentialDifferentialVariable Symbol) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SequentialDifferentialVariable Symbol) -> SparseUnivariatePolynomial %
--R variables : % -> List SequentialDifferentialVariable Symbol
--R weight : (%,Symbol) -> NonNegativeInteger
--R weights : (%,Symbol) -> List NonNegativeInteger
--R weights : % -> List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)
```

———–

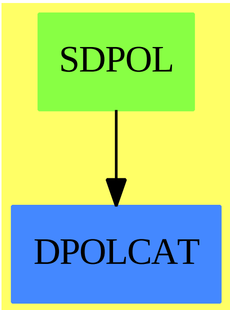
— **SequentialDifferentialPolynomial.help** —

```
====================================================================
SequentialDifferentialPolynomial examples
====================================================================

See Also:
o )show SequentialDifferentialPolynomial
```

———-

20.6.1 SequentialDifferentialPolynomial (SDPOL)



See

⇒ "OrderlyDifferentialVariable" (ODVAR) 16.17.1 on page 1816
⇒ "SequentialDifferentialVariable" (SDVAR) 20.7.1 on page 2348
⇒ "DifferentialSparseMultivariatePolynomial" (DSMP) 5.8.1 on page 526
⇒ "OrderlyDifferentialPolynomial" (ODPOL) 16.16.1 on page 1813

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentialVariables
differentiate	discriminant	eval
exquo	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	hash
initial	isExpt	isobaric?
isPlus	isTimes	latex
lcm	leader	leadingCoefficient
leadingMonomial	mainVariable	makeVariable
map	mapExponents	max
min	minimumDegree	monicDivide
monomial	monomial?	monomials
multivariate	numberOfMonomials	one?
order	patternMatch	pomopo!
prime?	primitiveMonomials	primitivePart
recip	reducedSystem	reductum
resultant	retract	retractIfCan
sample	separant	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	totalDegree	unit?
unitCanonical	unitNormal	univariate
variables	weight	weights
zero?	?^?	?*?
?**?	?+?	?-?
-?	?=?	?~=?
?/?	?<?	?<=?
?>?	?>=?	

— domain SDPOL SequentialDifferentialPolynomial —

```
)abbrev domain SDPOL SequentialDifferentialPolynomial
++ Author:  William Sit
++ Date Created: 24 September, 1991
++ Date Last Updated: 7 February, 1992
++ Basic Operations:DifferentialPolynomialCategory
++ Related Constructors: DifferentialSparseMultivariatePolynomial
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++           order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
```

```
++ \spadtype{SequentialDifferentialPolynomial} implements
++ an ordinary differential polynomial ring in arbitrary number
++ of differential indeterminates, with coefficients in a
++ ring.  The ranking on the differential indeterminate is sequential.

SequentialDifferentialPolynomial(R):
     Exports == Implementation where
  R: Ring
  S  ==> Symbol
  V  ==> SequentialDifferentialVariable S
  E   ==> IndexedExponents(V)
  SMP ==> SparseMultivariatePolynomial(R, S)
  Exports ==> Join(DifferentialPolynomialCategory(R,S,V,E),
                   RetractableTo SMP)

  Implementation ==> DifferentialSparseMultivariatePolynomial(R,S,V)
```

— **SDPOL.dotabb** —

```
"SDPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SDPOL"]
"DPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DPOLCAT"]
"SDPOL" -> "DPOLCAT"
```

20.7 domain SDVAR SequentialDifferentialVariable

— **SequentialDifferentialVariable.input** —

```
)set break resume
)sys rm -f SequentialDifferentialVariable.output
)spool SequentialDifferentialVariable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SequentialDifferentialVariable
--R SequentialDifferentialVariable S: OrderedSet  is a domain constructor
--R Abbreviation for SequentialDifferentialVariable is SDVAR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SDVAR
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : S -> %
--R coerce : % -> OutputForm              differentiate : % -> %
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R order : % -> NonNegativeInteger       retract : % -> S
--R variable : % -> S                     weight : % -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R differentiate : (%,NonNegativeInteger) -> %
--R makeVariable : (S,NonNegativeInteger) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

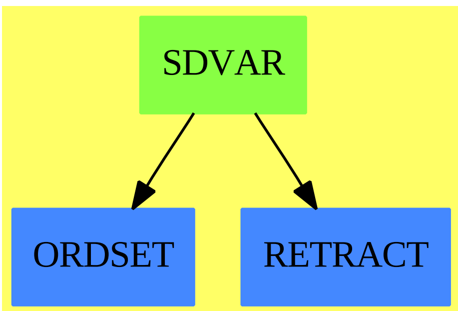
)spool
)lisp (bye)
```

— **SequentialDifferentialVariable.help** —

```
====================================================================
SequentialDifferentialVariable examples
====================================================================

See Also:
o )show SequentialDifferentialVariable
```

20.7.1 SequentialDifferentialVariable (SDVAR)



See

⇒ "OrderlyDifferentialVariable" (ODVAR) 16.17.1 on page 1816
⇒ "DifferentialSparseMultivariatePolynomial" (DSMP) 5.8.1 on page 526
⇒ "OrderlyDifferentialPolynomial" (ODPOL) 16.16.1 on page 1813
⇒ "SequentialDifferentialPolynomial" (SDPOL) 20.6.1 on page 2345

Exports:

coerce	differentiate	hash	latex	makeVariable
max	min	order	retract	retractIfCan
variable	weight	?~=?	?<?	?<=?
?=?	?>?	?>=?		

— domain SDVAR SequentialDifferentialVariable —

```
)abbrev domain SDVAR SequentialDifferentialVariable
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:differentiate, order, variable, <
++ Related Domains: OrderedVariableList,
++                  OrderlyDifferentialVariable.
++ See Also:DifferentialVariableCategory
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, sequential ranking.
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
++ \spadtype{OrderlyDifferentialVariable} adds a commonly used sequential
++ ranking to the set of derivatives of an ordered list of differential
++ indeterminates.  A sequential ranking is a ranking \spadfun{<} of the
++ derivatives with the property that for any derivative v,
++ there are only a finite number of derivatives u with u \spadfun{<} v.
++ This domain belongs to \spadtype{DifferentialVariableCategory}.  It
++ defines \spadfun{weight} to be just \spadfun{order}, and it
++ defines a sequential ranking \spadfun{<} on derivatives u by the
++ lexicographic order on the pair
++ (\spadfun{variable}(u), \spadfun{order}(u)).

SequentialDifferentialVariable(S:OrderedSet):DifferentialVariableCategory(S)
  == add
    Rep := Record(var:S, ord:NonNegativeInteger)
    makeVariable(s,n) == [s, n]
    variable v     == v.var
    order v        == v.ord
    v < u ==
      variable v = variable u => order v < order u
      variable v < variable u
```

———

— SDVAR.dotabb —

```
"SDVAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SDVAR"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"RETRACT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RETRACT"]
"SDVAR" -> "ORDSET"
"SDVAR" -> "RETRACT"
```

———

20.8 domain SEX SExpression

— SExpression.input —

```
)set break resume
)sys rm -f SExpression.output
)spool SExpression.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SExpression
--R SExpression  is a domain constructor
--R Abbreviation for SExpression is SEX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SEX
--R
--R------------------------------- Operations --------------------------------
--R #? : % -> Integer                      ?=? : (%,%) -> Boolean
--R atom? : % -> Boolean                   car : % -> %
--R cdr : % -> %                           coerce : % -> OutputForm
--R convert : OutputForm -> %              convert : DoubleFloat -> %
--R convert : Integer -> %                 convert : Symbol -> %
--R convert : String -> %                  convert : List % -> %
--R destruct : % -> List %                 ?.? : (%,List Integer) -> %
--R ?.? : (%,Integer) -> %                 eq : (%,%) -> Boolean
--R expr : % -> OutputForm                 float : % -> DoubleFloat
--R float? : % -> Boolean                  hash : % -> SingleInteger
--R integer : % -> Integer                 integer? : % -> Boolean
--R latex : % -> String                    list? : % -> Boolean
--R null? : % -> Boolean                   pair? : % -> Boolean
--R string : % -> String                   string? : % -> Boolean
--R symbol : % -> Symbol                   symbol? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
```

```
--R
--E 1

)spool
)lisp (bye)
```

———-

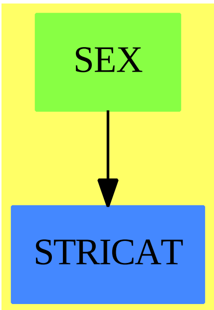
— SExpression.help —

```
====================================================================
SExpression examples
====================================================================

See Also:
o )show SExpression
```

———-

20.8.1 SExpression (SEX)



See

⇒ "SExpressionOf" (SEXOF) 20.9.1 on page 2353

Exports:

atom?	car	cdr	coerce	convert
destruct	eq	expr	float	float?
hash	integer	integer?	latex	list?
null?	pair?	string	string?	symbol
symbol?	#?	?.?	?=?	?~=?

— domain SEX SExpression —

```
)abbrev domain SEX SExpression
++ Author: S.M.Watt
```

```
++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++ This domain allows the manipulation of the usual Lisp values;

SExpression()
       == SExpressionOf(String, Symbol, Integer, DoubleFloat, OutputForm)
```

———-

— SEX.dotabb —

```
"SEX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEX"]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"SEX" -> "STRICAT"
```

———-

20.9 domain SEXOF SExpressionOf

— SExpressionOf.input —

```
)set break resume
)sys rm -f SExpressionOf.output
)spool SExpressionOf.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SExpressionOf
--R SExpressionOf(Str: SetCategory,Sym: SetCategory,Int: SetCategory,Flt: SetCategory,Expr: S
--R Abbreviation for SExpressionOf is SEXOF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SEXOF
--R
--R------------------------------- Operations --------------------------------
--R #? : % -> Integer                     ?=? : (%,%) -> Boolean
--R atom? : % -> Boolean                  car : % -> %
--R cdr : % -> %                          coerce : % -> OutputForm
--R convert : Expr -> %                   convert : Flt -> %
--R convert : Int -> %                    convert : Sym -> %
--R convert : Str -> %                    convert : List % -> %
--R destruct : % -> List %                ?.? : (%,List Integer) -> %
```

```
--R ?.? : (%,Integer) -> %              eq : (%,%) -> Boolean
--R expr : % -> Expr                    float : % -> Flt
--R float? : % -> Boolean               hash : % -> SingleInteger
--R integer : % -> Int                  integer? : % -> Boolean
--R latex : % -> String                 list? : % -> Boolean
--R null? : % -> Boolean                pair? : % -> Boolean
--R string : % -> Str                   string? : % -> Boolean
--R symbol : % -> Sym                   symbol? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R
--E 1

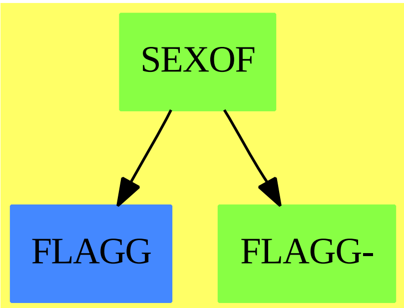
)spool
)lisp (bye)
```

— **SExpressionOf.help** —

```
====================================================================
SExpressionOf examples
====================================================================

See Also:
o )show SExpressionOf
```

20.9.1 SExpressionOf (SEXOF)



See

⇒ "SExpression" (SEX) 20.8.1 on page 2351

Exports:

atom?	car	cdr	coerce	convert
destruct	eq	expr	float	float?
hash	integer	integer?	latex	list?
null?	pair?	string	string?	symbol
symbol?	#?	?.?	?=?	?~=?

— domain SEXOF SExpressionOf —

```
)abbrev domain SEXOF SExpressionOf
++ Author: S.M.Watt
++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++ This domain allows the manipulation of Lisp values over
++ arbitrary atomic types.
-- Allows the names of the atomic types to be chosen.
-- *** Warning *** Although the parameters are declared only to be Sets,
-- *** Warning *** they must have the appropriate representations.

SExpressionOf(Str, Sym, Int, Flt, Expr): Decl == Body where
    Str, Sym, Int, Flt, Expr: SetCategory

    Decl ==> SExpressionCategory(Str, Sym, Int, Flt, Expr)

    Body ==> add
        Rep := Expr

        dotex:OutputForm := INTERN(".")$Lisp

        coerce(b:%):OutputForm ==
            null? b => paren empty()
            atom? b => coerce(b)$Rep
            r := b
            while not atom? r repeat r := cdr r
            l1 := [b1::OutputForm for b1 in (l := destruct b)]
            not null? r =>
              paren blankSeparate concat_!(l1, [dotex, r::OutputForm])
            #l = 2 and (first(l1) = QUOTE)@Boolean => quote first rest l1
            paren blankSeparate l1

        b1 = b2        == EQUAL(b1,b2)$Lisp
        eq(b1, b2)     == EQ(b1,b2)$Lisp

        null? b      == NULL(b)$Lisp
        atom? b      == ATOM(b)$Lisp
        pair? b      == CONSP(b)$Lisp

        list?    b   == CONSP(b)$Lisp or NULL(b)$Lisp
        string?  b   == STRINGP(b)$Lisp
        symbol?  b   == IDENTP(b)$Lisp
```

```
        integer? b   == INTEGERP(b)$Lisp
        float?   b   == RNUMP(b)$Lisp

        destruct b == (list? b    => b pretend List %; error "Non-list")
        string b == (STRINGP(b)$Lisp=> b pretend Str;error "Non-string")
        symbol b == (IDENTP(b)$Lisp => b pretend Sym;error "Non-symbol")
        float   b == (RNUMP(b)$Lisp  => b pretend Flt;error "Non-float")
        integer b == (INTEGERP(b)$Lisp => b pretend Int;error "Non-integer")
        expr    b == b pretend Expr

        convert(l:  List %) == l  pretend %
        convert(st: Str)    == st pretend %
        convert(sy: Sym)    == sy pretend %
        convert(n:  Int)    == n  pretend %
        convert(f:  Flt)    == f  pretend %
        convert(e:  Expr)   == e  pretend %

        car b        == CAR(b)$Lisp
        cdr b        == CDR(b)$Lisp
        #   b        == LENGTH(b)$Lisp
        elt(b:%, i:Integer)       == destruct(b).i
        elt(b:%, li:List Integer) ==
          for i in li repeat b := destruct(b).i
          b
```

———-

— **SEXOF.dotabb** —

```
"SEXOF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEXOF"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"SEXOF" -> "FLAGG"
"SEXOF" -> "FLAGG-"
```

———-

20.10 domain SAE SimpleAlgebraicExtension

— **SimpleAlgebraicExtension.input** —

```
)set break resume
)sys rm -f SimpleAlgebraicExtension.output
)spool SimpleAlgebraicExtension.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SimpleAlgebraicExtension
--R SimpleAlgebraicExtension(R: CommutativeRing,UP: UnivariatePolynomialCategory R,M: UP)  i
--R Abbreviation for SimpleAlgebraicExtension is SAE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SAE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       basis : () -> Vector %
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             convert : UP -> %
--R convert : % -> UP                    convert : Vector R -> %
--R convert : % -> Vector R              coordinates : % -> Vector R
--R definingPolynomial : () -> UP        discriminant : () -> R
--R discriminant : Vector % -> R         generator : () -> %
--R hash : % -> SingleInteger            inv : % -> % if R has FIELD
--R latex : % -> String                  lift : % -> UP
--R norm : % -> R                        one? : % -> Boolean
--R rank : () -> PositiveInteger         recip : % -> Union(%,"failed")
--R reduce : UP -> %                     represents : Vector R -> %
--R retract : % -> R                     sample : () -> %
--R trace : % -> R                       traceMatrix : () -> Matrix R
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has FIELD
--R ?*? : (Fraction Integer,%) -> % if R has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if R has FIELD
--R D : (%,(R -> R)) -> % if R has FIELD
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R D : % -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R ?^? : (%,Integer) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has FIELD
```

```
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UP
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R charthRoot : % -> % if R has FFIELDC
--R coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
--R coerce : % -> % if R has FIELD
--R conditionP : Matrix % -> Union(Vector %,"failed") if R has FFIELDC
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (%,Vector %) -> Vector R
--R createPrimitiveElement : () -> % if R has FFIELDC
--R derivationCoordinates : (Vector %,(R -> R)) -> Matrix R if R has FIELD
--R differentiate : (%,(R -> R)) -> % if R has FIELD
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R differentiate : % -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
--R discreteLog : % -> NonNegativeInteger if R has FFIELDC
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if R has FFIELDC
--R gcd : (%,%) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R index : PositiveInteger -> % if R has FINITE
--R init : () -> % if R has FFIELDC
--R lcm : (%,%) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R lookup : % -> PositiveInteger if R has FINITE
--R minimalPolynomial : % -> UP if R has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R nextItem : % -> Union(%,"failed") if R has FFIELDC
--R order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
--R order : % -> PositiveInteger if R has FFIELDC
--R prime? : % -> Boolean if R has FIELD
--R primeFrobenius : % -> % if R has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
--R primitive? : % -> Boolean if R has FFIELDC
--R primitiveElement : () -> % if R has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R ?quo? : (%,%) -> % if R has FIELD
```

```
--R random : () -> % if R has FINITE
--R reduce : Fraction UP -> Union(%,"failed") if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R regularRepresentation : % -> Matrix R
--R regularRepresentation : (%,Vector %) -> Matrix R
--R ?rem? : (%,%) -> % if R has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
--R represents : (Vector R,Vector %) -> %
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R squareFree : % -> Factored % if R has FIELD
--R squareFreePart : % -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R has
--R traceMatrix : Vector % -> Matrix R
--R unit? : % -> Boolean if R has FIELD
--R unitCanonical : % -> % if R has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has FIELD
--R
--E 1

)spool
)lisp (bye)
```

———

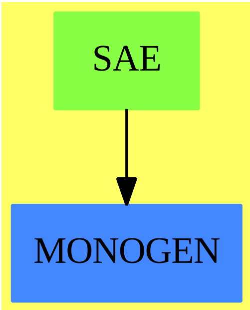
— SimpleAlgebraicExtension.help —

```
====================================================================
SimpleAlgebraicExtension examples
====================================================================

See Also:
o )show SimpleAlgebraicExtension
```

———

20.10.1 SimpleAlgebraicExtension (SAE)



Exports:

0	1	associates?
basis	characteristic	characteristicPolynomial
charthRoot	coerce	conditionP
convert	coordinates	createPrimitiveElement
D	definingPolynomial	derivationCoordinates
differentiate	discreteLog	discriminant
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor
factorsOfCyclicGroupSize	gcd	gcdPolynomial
generator	hash	index
init	inv	latex
lcm	lift	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
rank	recip	reduce
reducedSystem	regularRepresentation	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	traceMatrix
unit?	unitCanonical	unitNormal
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?quo?	?rem?

— domain SAE SimpleAlgebraicExtension —

```
)abbrev domain SAE SimpleAlgebraicExtension
++ Author: Barry Trager, Manuel Bronstein, Clifton Williamson
```

```
++ Date Created: 1986
++ Date Last Updated: 9 May 1994
++ Keywords: ring, algebraic, extension
++ Description:
++ Algebraic extension of a ring by a single polynomial.
++ Domain which represents simple algebraic extensions of arbitrary
++ rings. The first argument to the domain, R, is the underlying ring,
++ the second argument is a domain of univariate polynomials over K,
++ while the last argument specifies the defining minimal polynomial.
++ The elements of the domain are canonically represented as polynomials
++ of degree less than that of the minimal polynomial with coefficients
++ in R. The second argument is both the type of the third argument and
++ the underlying representation used by \spadtype{SAE} itself.

SimpleAlgebraicExtension(R:CommutativeRing,
 UP:UnivariatePolynomialCategory R, M:UP): MonogenicAlgebra(R, UP) == add
    --sqFr(pb): FactorS(Poly) from UnivPolySquareFree(Poly)

    --degree(M) > 0 and M must be monic if R is not a field.
    if (r := recip leadingCoefficient M) case "failed" then
                                  error "Modulus cannot be made monic"
    Rep := UP
    x,y :$
    c: R

    mkDisc   : Boolean -> Void
    mkDiscMat: Boolean -> Void

    M   := r::R * M
    d   := degree M
    d1  := subtractIfCan(d,1)::NonNegativeInteger
    discmat:Matrix(R) := zero(d, d)
    nodiscmat?:Reference(Boolean) := ref true
    disc:Reference(R) := ref 0
    nodisc?:Reference(Boolean) := ref true
    bsis := [monomial(1, i)$Rep for i in 0..d1]$Vector(Rep)

    if R has Finite then
         size == size$R ** d
         random == represents([random()$R for i in 0..d1])
    0 == 0$Rep
    1 == 1$Rep
    c * x == c *$Rep x
    n:Integer * x == n *$Rep x
    coerce(n:Integer):$   == coerce(n)$Rep
    coerce(c) == monomial(c,0)$Rep
    coerce(x):OutputForm == coerce(x)$Rep
    lift(x) == x pretend Rep
    reduce(p:UP):$ == (monicDivide(p,M)$Rep).remainder
    x = y == x =$Rep y
```

```
x + y == x +$Rep y
- x == -$Rep x
x * y == reduce((x *$Rep y) pretend UP)
coordinates(x) == [coefficient(lift(x),i) for i in 0..d1]
represents(vect) == +/[monomial(vect.(i+1),i) for i in 0..d1]
definingPolynomial()  == M
characteristic()      == characteristic()$R
rank()                == d::PositiveInteger
basis()               == copy(bsis@Vector(Rep) pretend Vector($))
--!! I inserted 'copy' in the definition of 'basis'  -- cjw 7/19/91

if R has Field then
  minimalPolynomial x == squareFreePart characteristicPolynomial x

if R has Field then
  coordinates(x:$,bas: Vector $) ==
    (m := inverse transpose coordinates bas) case "failed" =>
      error "coordinates: second argument must be a basis"
    (m :: Matrix R) * coordinates(x)

else if R has IntegralDomain then
  coordinates(x:$,bas: Vector $) ==
    -- we work over the quotient field of R to invert a matrix
    qf := Fraction R
    imatqf := InnerMatrixQuotientFieldFunctions(R,Vector R,Vector R,_
              Matrix R,qf,Vector qf,Vector qf,Matrix qf)
    mat := transpose coordinates bas
    (m := inverse(mat)$imatqf) case "failed" =>
      error "coordinates: second argument must be a basis"
    coordsQF: Vector qf :=
      map(y +-> y::qf,coordinates x)$VectorFunctions2(R,qf)
    -- here are the coordinates as elements of the quotient field:
    vecQF := (m :: Matrix qf) * coordsQF
    vec : Vector R := new(d,0)
    for i in 1..d repeat
      xi := qelt(vecQF,i)
      denom(xi) = 1 => qsetelt_!(vec,i,numer xi)
      error "coordinates: coordinates are not integral over ground ring"
    vec

reducedSystem(m:Matrix $):Matrix(R) ==
  reducedSystem(map(lift, m)$MatrixCategoryFunctions2($, Vector $,
           Vector $, Matrix $, UP, Vector UP, Vector UP, Matrix UP))

reducedSystem(m:Matrix $, v:Vector $):Record(mat:Matrix R,vec:Vector R) ==
  reducedSystem(map(lift, m)$MatrixCategoryFunctions2($, Vector $,
           Vector $, Matrix $, UP, Vector UP, Vector UP, Matrix UP),
                             map(lift, v)$VectorFunctions2($, UP))

discriminant() ==
```

```
    if nodisc?() then mkDisc false
    disc()

  mkDisc b ==
    nodisc?() := b
    disc() := discriminant M
    void

  traceMatrix() ==
    if nodiscmat?() then mkDiscMat false
    discmat

  mkDiscMat b ==
    nodiscmat?() := b
    mr := minRowIndex discmat; mc := minColIndex discmat
    for i in 0..d1 repeat
      for j in 0..d1 repeat
        qsetelt_!(discmat,mr + i,mc + j,trace reduce monomial(1,i + j))
    void

  trace x ==          --this could be coded perhaps more efficiently
    xn := x;  ans := coefficient(lift xn, 0)
    for n in 1..d1 repeat
      (xn := generator() * xn;  ans := coefficient(lift xn, n) + ans)
    ans

  if R has Finite then
     index k ==
       i:Integer := k rem size()
       p:Integer := size()$R
       ans:$ := 0
       for j in 0.. while i > 0 repeat
         h := i rem p
         -- index(p) = 0$R
         if h ^= 0 then
           -- here was a bug: "index" instead of
           -- "coerce", otherwise it wouldn't work for
           -- Rings R where "coerce: I-> R" is not surjective
           a := index(h :: PositiveInteger)$R
           ans := ans + reduce monomial(a, j)
         i := i quo p
       ans
     lookup(z : $) : PositiveInteger ==
       -- z = index lookup z, n = lookup index n
       -- the answer is merely the Horner evaluation of the
       -- representation with the size of R (as integers).
       zero?(z) => size()$$ pretend PositiveInteger
       p  :            Integer := size()$R
       co :            Integer := lookup(leadingCoefficient z)$R
       n  : NonNegativeInteger := degree(z)
```

```
         while not zero?(z := reductum z) repeat
          co := co * p ** ((n - (n := degree z)) pretend
            NonNegativeInteger) + lookup(leadingCoefficient z)$R
         n = 0 => co pretend PositiveInteger
         (co * p ** n) pretend PositiveInteger

--
--   KA:=BasicPolynomialFunctions(Poly)
--   minPoly(x) ==
--      ffe:= SqFr(resultant(M::KA, KA.var - lift(x)::KA)).fs.first
--      ffe.flag = "SQFR" => ffe.f
--      mdeg:= (degree(ffe.f) // K.characteristic)::Integer
--      mat:= Zero()::Matrix<mdeg+1,deg+mdeg+1>(K)
--      xi:=L.1;  setelt(mat,1,1,K.1);  setelt(mat,1,(deg+1),K.1)
--      for i in 1..mdeg repeat
--         xi:= x * xi;  xp:= lift(xi)
--         while xp ^= KA.0 repeat
--            setelt(mat,(mdeg+1),(degree(xp)+1),LeadingCoef(xp))
--            xp:=reductum(xp)
--         setelt(mat,(mdeg+1),(deg+i+1),K.1)
--         EchelonLastRow(mat)
--         if and/(elt(mat,(i+1),j) = K.0 for j in 1..deg)
--           then return unitNormal(+/(elt(mat,(i+1),(deg+j+1))*(B::KA)**j
--                                        for j in 0..i)).a
--      ffe.f
```

———

— **SAE.dotabb** —

```
"SAE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SAE"]
"MONOGEN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=MONOGEN"]
"SAE" -> "MONOGEN"
```

———

20.11 domain SFORT SimpleFortranProgram

— **SimpleFortranProgram.input** —

```
)set break resume
)sys rm -f SimpleFortranProgram.output
)spool SimpleFortranProgram.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show SimpleFortranProgram
--R SimpleFortranProgram(R: OrderedSet,FS: FunctionSpace R)  is a domain constructor
--R Abbreviation for SimpleFortranProgram is SFORT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SFORT
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R fortran : (Symbol,FortranScalarType,FS) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

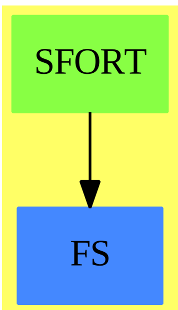
— **SimpleFortranProgram.help** —

```
====================================================================
SimpleFortranProgram examples
====================================================================

See Also:
o )show SimpleFortranProgram
```

———-

20.11.1 SimpleFortranProgram (SFORT)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranCode" (FC) 7.16.1 on page 898
⇒ "FortranProgram" (FORTRAN) 7.18.1 on page 923
⇒ "ThreeDimensionalMatrix" (M3D) 21.7.1 on page 2661
⇒ "Switch" (SWITCH) 20.36.1 on page 2588
⇒ "FortranTemplate" (FTEM) 7.20.1 on page 934
⇒ "FortranExpression" (FEXPR) 7.17.1 on page 914

Exports:
coerce fortran outputAsFortran

— domain SFORT SimpleFortranProgram —

```
)abbrev domain SFORT SimpleFortranProgram
-- Because of a bug in the compiler:
)bo $noSubsumption:=true

++ Author: Mike Dewar
++ Date Created: November 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FortranType, FortranCode, Switch
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \axiomType{SimpleFortranProgram(f,type)} provides a simple model of some
++ FORTRAN subprograms, making it possible to coerce objects of various
++ domains into a FORTRAN subprogram called \axiom{f}.
++ These can then be translated into legal FORTRAN code.

SimpleFortranProgram(R,FS): Exports == Implementation where
  R  : OrderedSet
  FS : FunctionSpace(R)

  FST ==> FortranScalarType

  Exports ==> FortranProgramCategory with
    fortran : (Symbol,FST,FS) -> $
    ++fortran(fname,ftype,body) builds an object of type
    ++\axiomType{FortranProgramCategory}. The three arguments specify
    ++the name, the type and the body of the program.

  Implementation ==> add

    Rep := Record(name : Symbol, type : FST, body : FS )

    fortran(fname, ftype, res) ==
```

```
      construct(fname,ftype,res)$Rep

    nameOf(u:$):Symbol == u . name

    typeOf(u:$):Union(FST,"void") == u . type

    bodyOf(u:$):FS == u . body

    argumentsOf(u:$):List Symbol == variables(bodyOf u)$FS

    coerce(u:$):OutputForm ==
      coerce(nameOf u)$Symbol

    outputAsFortran(u:$):Void ==
      ftype := (checkType(typeOf(u)::OutputForm)$Lisp)::OutputForm
      fname := nameOf(u)::OutputForm
      args := argumentsOf(u)
      nargs:=args::OutputForm
      val  := bodyOf(u)::OutputForm
      fortFormatHead(ftype,fname,nargs)$Lisp
      fortFormatTypes(ftype,args)$Lisp
      dispfortexp1$Lisp ["="::OutputForm, fname, val]@List(OutputForm)
      dispfortexp1$Lisp "RETURN"::OutputForm
      dispfortexp1$Lisp "END"::OutputForm
      void()$Void
```

———-

— **SFORT.dotabb** —

```
"SFORT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SFORT"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"SFORT" -> "FS"
```

———-

20.12 domain SINT SingleInteger

The definition of **one?** has been rewritten as it relies on calling **ONEP** which is a function specific to Codemist Common Lisp but is not defined in Common Lisp.

— **SingleInteger.input** —

```
)set break resume
```

```
)sys rm -f SingleInteger.output
)spool SingleInteger.output
)set message test on
)set message auto off
)clear all
--S 1 of 11
min()$SingleInteger
--R
--R
--R   (1)  - 2147483648
--R                                                    Type: SingleInteger
--E 1

--S 2 of 11
max()$SingleInteger
--R
--R
--R   (2)  2147483647
--R                                                    Type: SingleInteger
--E 2

--S 3 of 11
a := 1234 :: SingleInteger
--R
--R
--R   (3)  1234
--R                                                    Type: SingleInteger
--E 3

--S 4 of 11
b := 124$SingleInteger
--R
--R
--R   (4)  124
--R                                                    Type: SingleInteger
--E 4

--S 5 of 11
gcd(a,b)
--R
--R
--R   (5)  2
--R                                                    Type: SingleInteger
--E 5

--S 6 of 11
lcm(a,b)
--R
--R
--R   (6)  76508
```

```
--R                                                            Type: SingleInteger
--E 6

--S 7 of 11
mulmod(5,6,13)$SingleInteger
--R
--R
--R   (7)  4
--R                                                            Type: SingleInteger
--E 7

--S 8 of 11
positiveRemainder(37,13)$SingleInteger
--R
--R
--R   (8)  11
--R                                                            Type: SingleInteger
--E 8

--S 9 of 11
And(3,4)$SingleInteger
--R
--R
--R   (9)  0
--R                                                            Type: SingleInteger
--E 9

--S 10 of 11
shift(1,4)$SingleInteger
--R
--R
--R   (10)  16
--R                                                            Type: SingleInteger
--E 10

--S 11 of 11
shift(31,-1)$SingleInteger
--R
--R
--R   (11)  15
--R                                                            Type: SingleInteger
--E 11
)spool
)lisp (bye)
```

— **SingleInteger.help** —

```
====================================================================
SingleInteger examples
====================================================================

The SingleInteger domain is intended to provide support in Axiom
for machine integer arithmetic.  It is generally much faster than
(bignum) Integer arithmetic but suffers from a limited range of
values.  Since Axiom can be implemented on top of various dialects of
Lisp, the actual representation of small integers may not correspond
exactly to the host machines integer representation.

You can discover the minimum and maximum values in your implementation
by using min and max.

  min()$SingleInteger
    - 2147483648
                              Type: SingleInteger

  max()$SingleInteger
    2147483647
                              Type: SingleInteger

To avoid confusion with Integer, which is the default type for
integers, you usually need to work with declared variables.

  a := 1234 :: SingleInteger
    1234
                              Type: SingleInteger

or use package calling

  b := 124$SingleInteger
    124
                              Type: SingleInteger

You can add, multiply and subtract SingleInteger objects, and ask for
the greatest common divisor (gcd).

  gcd(a,b)
    2
                              Type: SingleInteger

The least common multiple (lcm) is also available.

  lcm(a,b)
    76508
                              Type: SingleInteger

Operations mulmod, addmod, submod, and invmod are similar - they provide
arithmetic modulo a given small integer.
```

```
Here is 5 * 6 mod 13.

  mulmod(5,6,13)$SingleInteger
    4
                              Type: SingleInteger

To reduce a small integer modulo a prime, use positiveRemainder.

  positiveRemainder(37,13)$SingleInteger
    11
                              Type: SingleInteger

Operations And, Or, xor, and Not provide bit level operations on small
integers.

  And(3,4)$SingleInteger
    0
                              Type: SingleInteger

Use shift(int,numToShift) to shift bits, where i is shifted left if
numToShift is positive, right if negative.

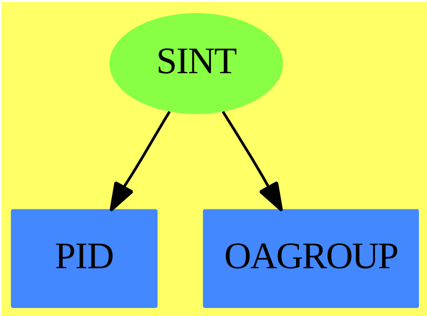
  shift(1,4)$SingleInteger
    16
                              Type: SingleInteger

  shift(31,-1)$SingleInteger
    15
                              Type: SingleInteger

Many other operations are available for small integers, including many
of those provided for Integer.

See Also:
o )help Integer
o )show SingleInteger
```

20.12.1 SingleInteger (SINT)



Exports:

0	1	abs	addmod
And	associates?	base	binomial
bit?	characteristic	coerce	convert
copy	D	dec	differentiate
divide	euclideanSize	even?	expressIdealMember
exquo	extendedEuclidean	factor	factorial
gcd	gcdPolynomial	hash	inc
init	invmod	latex	lcm
length	mask	max	min
mulmod	multiEuclidean	negative?	nextItem
Not	not?	odd?	OMwrite
one?	Or	patternMatch	permutation
principalIdeal	positive?	positiveRemainder	powmod
prime?	random	rational	rationalIfCan
rational?	recip	reducedSystem	retract
retractIfCan	sample	shift	sign
sizeLess?	squareFree	squareFreePart	subtractIfCan
submod	symmetricRemainder	unit?	unitCanonical
unitNormal	xor	zero?	?*?
?**?	?+?	?-?	-?
?/\?	?<?	?<=?	?=?
?>?	?>=?	?\/?	?^?
?	?~=?	?quo?	?rem?

— domain SINT SingleInteger —

```
)abbrev domain SINT SingleInteger

-- following patch needed to deal with *:(I,%) -> %
-- affects behavior of SourceLevelSubset
--)bo $noSubsets := true
-- No longer - JHD !! still needed 5/3/91 BMT
```

```
++ Author:  Michael Monagan
++ Date Created:
++    January 1988
++ Change History:
++ Basic Operations: max, min,
++    not, and, or, xor, Not, And, Or
++ Related Constructors:
++ Keywords: single integer
++ Description:
++ SingleInteger is intended to support machine integer arithmetic.

-- MAXINT, BASE (machine integer constants)
-- MODULUS, MULTIPLIER (random number generator constants)


-- Lisp dependencies
-- EQ, ABSVAL, TIMES, INTEGER-LENGTH, HASHEQ, REMAINDER
-- QSLESSP, QSGREATERP, QSADD1, QSSUB1, QSMINUS, QSPLUS, QSDIFFERENCE
-- QSTIMES, QSREMAINDER, QSODDP, QSZEROP, QSMAX, QSMIN, QSNOT, QSAND
-- QSOR, QSXOR, QSLEFTSHIFT, QSADDMOD, QSDIFMOD, QSMULTMOD


SingleInteger(): Join(IntegerNumberSystem,Logic,OpenMath) with
   canonical
      ++ \spad{canonical} means that mathematical equality is
      ++ implied by data structure equality.
   canonicalsClosed
      ++ \spad{canonicalClosed} means two positives multiply to
      ++ give positive.
   noetherian
      ++ \spad{noetherian} all ideals are finitely generated
      ++ (in fact principal).

   max      : () -> %
      ++ max() returns the largest single integer.
   min      : () -> %
      ++ min() returns the smallest single integer.

   -- bit operations
   "not":   % -> %
      ++ not(n) returns the bit-by-bit logical not of the single integer n.
   "~"  :   % -> %
      ++  ~ n returns the bit-by-bit logical not of the single integer n.
   "/\": (%, %) -> %
      ++ n /\ m  returns the bit-by-bit logical and of
      ++ the single integers n and m.
   "\/" : (%, %) -> %
      ++ n \/ m  returns the bit-by-bit logical or of
      ++ the single integers n and m.
```

```
  "xor": (%, %) -> %
     ++ xor(n,m)  returns the bit-by-bit logical xor of
     ++ the single integers n and m.
  Not  : % -> %
     ++ Not(n) returns the bit-by-bit logical not of the single integer n.
  And  : (%,%) -> %
     ++ And(n,m)  returns the bit-by-bit logical and of
     ++ the single integers n and m.
  Or   : (%,%) -> %
     ++ Or(n,m)  returns the bit-by-bit logical or of
     ++ the single integers n and m.

== add

  seed : % := 1$Lisp               -- for random()
  MAXINT ==> MOST_-POSITIVE_-FIXNUM$Lisp
  MININT ==> MOST_-NEGATIVE_-FIXNUM$Lisp
  BASE ==> 67108864$Lisp           -- 2**26
  MULTIPLIER ==> 314159269$Lisp    -- from Knuth's table
  MODULUS ==> 2147483647$Lisp      -- 2**31-1

  writeOMSingleInt(dev: OpenMathDevice, x: %): Void ==
   if x < 0 then
     OMputApp(dev)
     OMputSymbol(dev, "arith1", "unary_minus")
     OMputInteger(dev, convert(-x))
     OMputEndApp(dev)
   else
     OMputInteger(dev, convert(x))

  OMwrite(x: %): String ==
   s: String := ""
   sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
   dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
   OMputObject(dev)
   writeOMSingleInt(dev, x)
   OMputEndObject(dev)
   OMclose(dev)
   s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
   s

  OMwrite(x: %, wholeObj: Boolean): String ==
   s: String := ""
   sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
   dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
   if wholeObj then
     OMputObject(dev)
   writeOMSingleInt(dev, x)
   if wholeObj then
     OMputEndObject(dev)
```

```
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

   OMwrite(dev: OpenMathDevice, x: %): Void ==
    OMputObject(dev)
    writeOMSingleInt(dev, x)
    OMputEndObject(dev)

   OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
    if wholeObj then
      OMputObject(dev)
    writeOMSingleInt(dev, x)
    if wholeObj then
      OMputEndObject(dev)

   reducedSystem m      == m pretend Matrix(Integer)
   coerce(x):OutputForm == (convert(x)@Integer)::OutputForm
   convert(x:%):Integer == x pretend Integer
   i:Integer * y:%      == i::% * y
   0          == 0$Lisp
   1          == 1$Lisp
   base()     == 2$Lisp
   max()      == MAXINT
   min()      == MININT
   x = y      == EQL(x,y)$Lisp
   _~ x       == LOGNOT(x)$Lisp
   not(x)     == LOGNOT(x)$Lisp
   _/_\(x,y) == LOGAND(x,y)$Lisp
   _\_/(x,y) == LOGIOR(x,y)$Lisp
   Not(x)     == LOGNOT(x)$Lisp
   And(x,y)   == LOGAND(x,y)$Lisp
   Or(x,y)    == LOGIOR(x,y)$Lisp
   xor(x,y)   == LOGXOR(x,y)$Lisp
   x < y      == QSLESSP(x,y)$Lisp
   inc x      == QSADD1(x)$Lisp
   dec x      == QSSUB1(x)$Lisp
   - x        == QSMINUS(x)$Lisp
   x + y      == QSPLUS(x,y)$Lisp
   x:% - y:% == QSDIFFERENCE(x,y)$Lisp
   x:% * y:% == QSTIMES(x,y)$Lisp
   x:% ** n:NonNegativeInteger == ((EXPT(x, n)$Lisp) pretend Integer)::%
   x quo y    == QSQUOTIENT(x,y)$Lisp
   x rem y    == QSREMAINDER(x,y)$Lisp
   divide(x, y)   == CONS(QSQUOTIENT(x,y)$Lisp,QSREMAINDER(x,y)$Lisp)$Lisp
   gcd(x,y)   == GCD(x,y)$Lisp
   abs(x)     == QSABSVAL(x)$Lisp
   odd?(x)    == QSODDP(x)$Lisp
   zero?(x)   == QSZEROP(x)$Lisp
--   one?(x)    == ONEP(x)$Lisp
```

```
   one?(x)   == x = 1
   max(x,y)  == QSMAX(x,y)$Lisp
   min(x,y)  == QSMIN(x,y)$Lisp
   hash(x)   == SXHASH(x)$Lisp
   length(x) == INTEGER_-LENGTH(x)$Lisp
   shift(x,n)    == QSLEFTSHIFT(x,n)$Lisp
   mulmod(a,b,p) == QSMULTMOD(a,b,p)$Lisp
   addmod(a,b,p) == QSADDMOD(a,b,p)$Lisp
   submod(a,b,p) == QSDIFMOD(a,b,p)$Lisp
   negative?(x)  == QSMINUSP$Lisp x


   reducedSystem(m, v) ==
        [m pretend Matrix(Integer), v pretend Vector(Integer)]

   positiveRemainder(x,n) ==
      r := QSREMAINDER(x,n)$Lisp
      QSMINUSP(r)$Lisp =>
          QSMINUSP(n)$Lisp => QSDIFFERENCE(x, n)$Lisp
          QSPLUS(r, n)$Lisp
      r

   coerce(x:Integer):% ==
      (x <= max pretend Integer) and (x >= min pretend Integer) =>
        x pretend %
      error "integer too large to represent in a machine word"

   random() ==
      seed := REMAINDER(TIMES(MULTIPLIER,seed)$Lisp,MODULUS)$Lisp
      REMAINDER(seed,BASE)$Lisp

   random(n) == RANDOM(n)$Lisp

   UCA ==> Record(unit:%,canonical:%,associate:%)
   unitNormal x ==
      x < 0 => [-1,-x,-1]$UCA
      [1,x,1]$UCA

)bo $noSubsets := false
```

— **SINT.dotabb** —

```
"SINT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SINT",shape=ellipse]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"SINT" -> "PID"
```

```
"SINT" -> "OAGROUP"
```

20.13 domain SAOS SingletonAsOrderedSet

— SingletonAsOrderedSet.input —

```
)set break resume
)sys rm -f SingletonAsOrderedSet.output
)spool SingletonAsOrderedSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SingletonAsOrderedSet
--R SingletonAsOrderedSet  is a domain constructor
--R Abbreviation for SingletonAsOrderedSet is SAOS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SAOS
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R convert : % -> Symbol                 create : () -> %
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— SingletonAsOrderedSet.help —

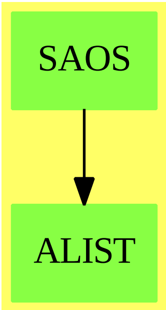
```
====================================================================
SingletonAsOrderedSet examples
====================================================================

See Also:
```

```
o )show SingletonAsOrderedSet
```

———-

20.13.1 SingletonAsOrderedSet (SAOS)



Exports:

coerce	convert	create	hash	latex
max	min	?~=?	?<?	?<=?
?=?	?>?	?>=?		

— domain SAOS SingletonAsOrderedSet —

```
)abbrev domain SAOS SingletonAsOrderedSet
++ Author: Mark Botch
++ Description:
++ This trivial domain lets us build Univariate Polynomials
++ in an anonymous variable

SingletonAsOrderedSet(): OrderedSet with
              create:() -> %
              convert:% -> Symbol
  ==  add
   create() == "?" pretend %
   a<b == false -- only one element
   coerce(a) == outputForm "?"  -- CJW doesn't like this: change ?
   a=b == true  -- only one element
   min(a,b) == a  -- only one element
   max(a,b) == a  -- only one element
   convert a == coerce("?")
```

———-

— SAOS.dotabb —

```
"SAOS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SAOS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SAOS" -> "ALIST"
```

———-

20.14 domain SMP SparseMultivariatePolynomial

— SparseMultivariatePolynomial.input —

```
)set break resume
)sys rm -f SparseMultivariatePolynomial.output
)spool SparseMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseMultivariatePolynomial
--R SparseMultivariatePolynomial(R: Ring,VarSet: OrderedSet)  is a domain constructor
--R Abbreviation for SparseMultivariatePolynomial is SMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SMP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                     ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R D : (%,List VarSet) -> %             D : (%,VarSet) -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       coefficients : % -> List R
--R coerce : VarSet -> %                 coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R differentiate : (%,VarSet) -> %      eval : (%,VarSet,%) -> %
--R eval : (%,VarSet,R) -> %             eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %                  eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %      ground : % -> R
--R ground? : % -> Boolean               hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> R
--R leadingMonomial : % -> %             map : ((R -> R),%) -> %
--R monomial? : % -> Boolean             monomials : % -> List %
--R one? : % -> Boolean                  primitiveMonomials : % -> List %
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R retract : % -> VarSet                retract : % -> R
```

```
--R sample : () -> %                      variables : % -> List VarSet
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List VarSet,List NonNegativeInteger) -> %
--R D : (%,VarSet,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List VarSet,List NonNegativeInteger) -> %
--R coefficient : (%,VarSet,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents VarSet) -> R
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : % -> % if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,VarSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if R has KONVERT INFORM and VarSet has KONVERT INFORM
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT and VarSet has KONVERT PATTERN INT
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT and VarSet has KONVERT PATTERN FLOAT
--R degree : (%,List VarSet) -> List NonNegativeInteger
--R degree : (%,VarSet) -> NonNegativeInteger
--R degree : % -> IndexedExponents VarSet
--R differentiate : (%,List VarSet,List NonNegativeInteger) -> %
--R differentiate : (%,VarSet,NonNegativeInteger) -> %
--R differentiate : (%,List VarSet) -> %
--R discriminant : (%,VarSet) -> % if R has COMRING
--R eval : (%,List VarSet,List %) -> %
--R eval : (%,List VarSet,List R) -> %
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R isExpt : % -> Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R lcm : (%,%) -> % if R has GCDDOM
```

```
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(VarSet,"failed")
--R mapExponents : ((IndexedExponents VarSet -> IndexedExponents VarSet),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List VarSet) -> List NonNegativeInteger
--R minimumDegree : (%,VarSet) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents VarSet
--R monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List VarSet,List NonNegativeInteger) -> %
--R monomial : (%,VarSet,NonNegativeInteger) -> %
--R monomial : (R,IndexedExponents VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pomopo! : (%,R,IndexedExponents VarSet,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,VarSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%,VarSet) -> % if R has COMRING
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(VarSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List VarSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,VarSet) -> SparseUnivariatePolynomial %
--R
--E 1

)spool
)lisp (bye)
```

———-

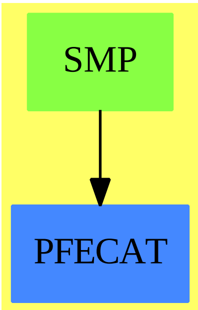
— **SparseMultivariatePolynomial.help** —

```
====================================================================
SparseMultivariatePolynomial examples
====================================================================

See Also:
o )show SparseMultivariatePolynomial
```

———-

20.14.1 SparseMultivariatePolynomial (SMP)



See

⇒ "Polynomial" (POLY) 17.25.1 on page 2037
⇒ "MultivariatePolynomial" (MPOLY) 14.16.1 on page 1645
⇒ "IndexedExponents" (INDE) 10.9.1 on page 1183

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	latex
isExpt	isPlus	isTimes
lcm	leadingCoefficient	leadingMonomial
mainVariable	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multivariate	numberOfMonomials
one?	patternMatch	pomopo!
prime?	primitivePart	primitiveMonomials
recip	reducedSystem	reductum
resultant	retract	retractIfCan
sample	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?<?	?<=?
?>?	?>=?	

— domain SMP SparseMultivariatePolynomial —

```
)abbrev domain SMP SparseMultivariatePolynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated: 30 November 1994
++ Fix History:
++ 30 Nov 94: added gcdPolynomial for float-type coefficients
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd
++ Related Constructors: Polynomial, MultivariatePolynomial
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate
++ References:
++ Description:
++ This type is the basic representation of sparse recursive multivariate
```

```
++ polynomials. It is parameterized by the coefficient ring and the
++ variable set which may be infinite. The variable ordering is determined
++ by the variable set parameter. The coefficient ring may be non-commutative,
++ but the variables are assumed to commute.

SparseMultivariatePolynomial(R: Ring,VarSet: OrderedSet): C == T where
  pgcd ==> PolynomialGcdPackage(IndexedExponents VarSet,VarSet,R,%)
  C == PolynomialCategory(R,IndexedExponents(VarSet),VarSet)
  SUP ==> SparseUnivariatePolynomial
  T == add
    --constants
    --D := F(%) replaced by next line until compiler support completed

    --representations
      D := SparseUnivariatePolynomial(%)
      VPoly:=  Record(v:VarSet,ts:D)
      Rep:=  Union(R,VPoly)

    --local function


    --declarations
      fn: R -> R
      n: Integer
      k: NonNegativeInteger
      kp:PositiveInteger
      k1:NonNegativeInteger
      c: R
      mvar: VarSet
      val : R
      var:VarSet
      up: D
      p,p1,p2,pval: %
      Lval : List(R)
      Lpval : List(%)
      Lvar : List(VarSet)

    --define
      0  == 0$R::%
      1  == 1$R::%


      zero? p == p case R and zero?(p)$R
--      one? p == p case R and one?(p)$R
      one? p == p case R and ((p) = 1)$R
    -- a local function
      red(p:%):% ==
         p case R => 0
         if ground?(reductum p.ts) then
           leadingCoefficient(reductum p.ts) else [p.v,reductum p.ts]$VPoly
```

```
      numberOfMonomials(p): NonNegativeInteger ==
        p case R =>
          zero?(p)$R => 0
          1
        +/[numberOfMonomials q for q in coefficients(p.ts)]

      coerce(mvar):% == [mvar,monomial(1,1)$D]$VPoly

      monomial? p ==
        p case R => true
        sup : D := p.ts
        1 ^= numberOfMonomials(sup) => false
        monomial? leadingCoefficient(sup)$D

--    local
      moreThanOneVariable?: % -> Boolean

      moreThanOneVariable? p ==
         p case R => false
         q:=p.ts
         any?(x1+->not ground? x1 ,coefficients q) => true
         false

      -- if we already know we use this (slighlty) faster function
      univariateKnown: % -> SparseUnivariatePolynomial R

      univariateKnown p ==
        p case R => (leadingCoefficient p) :: SparseUnivariatePolynomial(R)
        monomial( leadingCoefficient p,degree p.ts)+ univariateKnown(red p)

      univariate p ==
        p case R =>(leadingCoefficient p) :: SparseUnivariatePolynomial(R)
        moreThanOneVariable?  p => error "not univariate"
        monomial( leadingCoefficient p,degree p.ts)+ univariate(red p)

      multivariate (u:SparseUnivariatePolynomial(R),var:VarSet) ==
        ground? u => (leadingCoefficient u) ::%
        [var,monomial(leadingCoefficient u,degree u)$D]$VPoly +
           multivariate(reductum u,var)

      univariate(p:%,mvar:VarSet):SparseUnivariatePolynomial(%) ==
        p case R or mvar>p.v  => monomial(p,0)$D
        pt:=p.ts
        mvar=p.v => pt
        monomial(1,p.v,degree pt)*univariate(leadingCoefficient pt,mvar)+
          univariate(red p,mvar)

--  a local functions, used in next definition
      unlikeUnivReconstruct(u:SparseUnivariatePolynomial(%),mvar:VarSet):% ==
```

```
        zero? (d:=degree u) => coefficient(u,0)
        monomial(leadingCoefficient u,mvar,d)+
            unlikeUnivReconstruct(reductum u,mvar)

      multivariate(u:SparseUnivariatePolynomial(%),mvar:VarSet):% ==
        ground? u => coefficient(u,0)
        uu:=u
        while not zero? uu repeat
          cc:=leadingCoefficient uu
          cc case R or mvar > cc.v => uu:=reductum uu
          return unlikeUnivReconstruct(u,mvar)
        [mvar,u]$VPoly

      ground?(p:%):Boolean ==
        p case R => true
        false

--      const p ==
--        p case R => p
--        error "the polynomial is not a constant"

      monomial(p,mvar,k1) ==
        zero? k1 or zero? p => p
        p case R or mvar>p.v => [mvar,monomial(p,k1)$D]$VPoly
        p*[mvar,monomial(1,k1)$D]$VPoly

      monomial(c:R,e:IndexedExponents(VarSet)):% ==
        zero? e => (c::%)
        monomial(1,leadingSupport e, leadingCoefficient e) *
            monomial(c,reductum e)

      coefficient(p:%, e:IndexedExponents(VarSet)) : R ==
        zero? e =>
          p case R  => p::R
          coefficient(coefficient(p.ts,0),e)
        p case R => 0
        ve := leadingSupport e
        vp := p.v
        ve < vp =>
          coefficient(coefficient(p.ts,0),e)
        ve > vp => 0
        coefficient(coefficient(p.ts,leadingCoefficient e),reductum e)

--    coerce(e:IndexedExponents(VarSet)) : % ==
--      e = 0 => 1
--      monomial(1,leadingSupport e, leadingCoefficient e) *
--          (reductum e)::%

--    retract(p:%):IndexedExponents(VarSet) ==
--      q:Union(IndexedExponents(VarSet),"failed"):=retractIfCan p
```

```
--      q :: IndexedExponents(VarSet)

--    retractIfCan(p:%):Union(IndexedExponents(VarSet),"failed") ==
--      p = 0 => degree p
--      reductum(p)=0 and leadingCoefficient(p)=1 => degree p
--      "failed"

      coerce(n) == n::R::%
      coerce(c) == c::%
      characteristic == characteristic$R

      recip(p) ==
        p case R => (uu:=recip(p::R);uu case "failed" => "failed"; uu::%)
        "failed"

      - p ==
          p case R => -$R p
          [p.v, - p.ts]$VPoly
      n * p  ==
          p case R => n * p::R
          mvar:=p.v
          up:=n*p.ts
          if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
      c * p  ==
          c = 1 => p
          p case R => c * p::R
          mvar:=p.v
          up:=c*p.ts
          if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
      p1 + p2  ==
         p1 case R and p2 case R => p1 +$R p2
         p1 case R => [p2.v, p1::D + p2.ts]$VPoly
         p2 case R => [p1.v,  p1.ts + p2::D]$VPoly
         p1.v = p2.v =>
              mvar:=p1.v
              up:=p1.ts+p2.ts
              if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
         p1.v < p2.v =>
              [p2.v, p1::D + p2.ts]$VPoly
         [p1.v, p1.ts + p2::D]$VPoly

      p1 - p2  ==
         p1 case R and p2 case R => p1 -$R p2
         p1 case R => [p2.v, p1::D - p2.ts]$VPoly
         p2 case R => [p1.v,  p1.ts - p2::D]$VPoly
         p1.v = p2.v =>
              mvar:=p1.v
              up:=p1.ts-p2.ts
              if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
         p1.v < p2.v =>
```

```
          [p2.v, p1::D - p2.ts]$VPoly
       [p1.v, p1.ts - p2::D]$VPoly

    p1 = p2  ==
       p1 case R =>
           p2 case R => p1 =$R p2
           false
       p2 case R => false
       p1.v = p2.v => p1.ts = p2.ts
       false

    p1 * p2  ==
       p1 case R => p1::R * p2
       p2 case R =>
          mvar:=p1.v
          up:=p1.ts*p2
          if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
       p1.v = p2.v =>
          mvar:=p1.v
          up:=p1.ts*p2.ts
          if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
       p1.v > p2.v =>
          mvar:=p1.v
          up:=p1.ts*p2
          if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
          --- p1.v < p2.v
       mvar:=p2.v
       up:=p1*p2.ts
       if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

    p ^ kp == p ** (kp pretend NonNegativeInteger)
    p ** kp == p ** (kp pretend NonNegativeInteger )
    p ^ k == p ** k
    p ** k  ==
       p case R => p::R ** k
       -- univariate special case
       not moreThanOneVariable? p =>
           multivariate( (univariateKnown p) ** k , p.v)
       mvar:=p.v
       up:=p.ts ** k
       if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

    if R has IntegralDomain then
       UnitCorrAssoc ==> Record(unit:%,canonical:%,associate:%)
       unitNormal(p) ==
          u,c,a:R
          p case R =>
            (u,c,a):= unitNormal(p::R)$R
            [u::%,c::%,a::%]$UnitCorrAssoc
          (u,c,a):= unitNormal(leadingCoefficient(p))$R
```

```
           [u::%,(a*p)::%,a::%]$UnitCorrAssoc
        unitCanonical(p) ==
           p case R => unitCanonical(p::R)$R
           (u,c,a):= unitNormal(leadingCoefficient(p))$R
           a*p
        unit? p ==
           p case R => unit?(p::R)$R
           false
        associates?(p1,p2) ==
           p1 case R => p2 case R and associates?(p1,p2)$R
           p2 case VPoly and p1.v = p2.v and associates?(p1.ts,p2.ts)

        if R has approximate then
          p1  exquo  p2  ==
             p1 case R and p2 case R =>
               a:= (p1::R  exquo  p2::R)
               if a case "failed" then "failed" else a::%
             zero? p1 => p1
--             one? p2 => p1
             (p2 = 1) => p1
             p1 case R or p2 case VPoly and p1.v < p2.v => "failed"
             p2 case R or p1.v > p2.v =>
                a:= (p1.ts  exquo  p2::D)
                a case "failed" => "failed"
                [p1.v,a]$VPoly::%
             -- The next test is useful in the case that R has inexact
             -- arithmetic (in particular when it is Interval(...)).
             -- In the case where the test succeeds, empirical evidence
             -- suggests that it can speed up the computation several times,
             -- but in other cases where there are a lot of variables
             -- and p1 and p2 differ only in the low order terms (e.g. p1=p2+1)
             -- it slows exquo down by about 15-20%.
             p1 = p2 => 1
             a:= p1.ts  exquo  p2.ts
             a case "failed" => "failed"
             mvar:=p1.v
             up:SUP %:=a
             if ground? (up) then
               leadingCoefficient(up) else [mvar,up]$VPoly::%
        else
          p1  exquo  p2  ==
             p1 case R and p2 case R =>
               a:= (p1::R  exquo  p2::R)
               if a case "failed" then "failed" else a::%
             zero? p1 => p1
--             one? p2 => p1
             (p2 = 1) => p1
             p1 case R or p2 case VPoly and p1.v < p2.v => "failed"
             p2 case R or p1.v > p2.v =>
                a:= (p1.ts  exquo  p2::D)
```

```
              a case "failed" => "failed"
              [p1.v,a]$VPoly::%
           a:= p1.ts  exquo  p2.ts
           a case "failed" => "failed"
           mvar:=p1.v
           up:SUP %:=a
           if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly::%

    map(fn,p) ==
       p case R => fn(p)
       mvar:=p.v
       up:=map(x1+->map(fn,x1),p.ts)
       if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

    if R has Field then
      (p : %) / (r : R) == inv(r) * p

    if R has GcdDomain then
      content(p) ==
         p case R => p
         c :R :=0
         up:=p.ts
--         while not(zero? up) and not(one? c) repeat
         while not(zero? up) and not(c = 1) repeat
             c:=gcd(c,content leadingCoefficient(up))
             up := reductum up
         c

    if R has EuclideanDomain and
        R has CharacteristicZero and
         not(R has FloatingPointSystem)  then

      content(p,mvar) ==
        p case R => p
        gcd(coefficients univariate(p,mvar))$pgcd

      gcd(p1,p2) ==  gcd(p1,p2)$pgcd

      gcd(lp:List %) ==  gcd(lp)$pgcd

      gcdPolynomial(a:SUP $,b:SUP $):SUP $ == gcd(a,b)$pgcd

    else if R has GcdDomain then
      content(p,mvar) ==
        p case R => p
        content univariate(p,mvar)

      gcd(p1,p2) ==
         p1 case R =>
            p2 case R => gcd(p1,p2)$R::%
```

```
        zero? p1 => p2
        gcd(p1, content(p2.ts))
     p2 case R =>
        zero? p2 => p1
        gcd(p2, content(p1.ts))
     p1.v < p2.v => gcd(p1, content(p2.ts))
     p1.v > p2.v => gcd(content(p1.ts), p2)
     mvar:=p1.v
     up:=gcd(p1.ts, p2.ts)
     if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

  if R has FloatingPointSystem then
     -- eventually need a better notion of gcd's over floats
     -- this essentially computes the gcds of the monomial contents
     gcdPolynomial(a:SUP $,b:SUP $):SUP $ ==
        ground? (a) =>
            zero? a => b
            gcd(leadingCoefficient a, content b)::SUP $
        ground?(b) =>
            zero? b => b
            gcd(leadingCoefficient b, content a)::SUP $
        conta := content a
        mona:SUP $ := monomial(conta, minimumDegree a)
        if mona ^= 1 then
             a := (a exquo mona)::SUP $
        contb := content b
        monb:SUP $ := monomial(contb, minimumDegree b)
        if monb ^= 1 then
             b := (b exquo monb)::SUP $
        mong:SUP $  := monomial(gcd(conta, contb),
                                min(degree mona, degree monb))
        degree(a) >= degree b =>
             not((a exquo b) case "failed") =>
                  mong * b
             mong
        not((b exquo a) case "failed") => mong * a
        mong

coerce(p):OutputForm ==
  p case R => (p::R)::OutputForm
  outputForm(p.ts,p.v::OutputForm)

coefficients p ==
  p case R => list(p :: R)$List(R)
  "append"/[coefficients(p1)$% for p1 in coefficients(p.ts)]

retract(p:%):R ==
  p case R => p :: R
  error "cannot retract nonconstant polynomial"
```

```
      retractIfCan(p:%):Union(R, "failed") ==
        p case R => p::R
        "failed"

--      leadingCoefficientRecursive(p:%):% ==
--         p case R => p
--         leadingCoefficient p.ts

      mymerge:(List VarSet,List VarSet) ->List VarSet
      mymerge(l:List VarSet,m:List VarSet):List VarSet  ==
         empty? l => m
         empty? m => l
         first l = first m =>
            empty? rest l =>
                 setrest!(l,rest m)
                 l
            empty? rest m => l
            setrest!(l, mymerge(rest l, rest m))
            l
         first l > first m =>
            empty? rest l =>
                setrest!(l,m)
                l
            setrest!(l, mymerge(rest l, m))
            l
         empty? rest m =>
             setrest!(m,l)
             m
         setrest!(m,mymerge(l,rest m))
         m

      variables p ==
         p case R => empty()
         lv:List VarSet:=empty()
         q := p.ts
         while not zero? q repeat
           lv:=mymerge(lv,variables leadingCoefficient q)
           q := reductum q
         cons(p.v,lv)

      mainVariable p ==
         p case R => "failed"
         p.v

      eval(p,mvar,pval) == univariate(p,mvar)(pval)
      eval(p,mvar,val) ==  univariate(p,mvar)(val)

      evalSortedVarlist(p,Lvar,Lpval):% ==
        p case R => p
        empty? Lvar or empty? Lpval => p
```

```
  mvar := Lvar.first
  mvar > p.v => evalSortedVarlist(p,Lvar.rest,Lpval.rest)
  pval := Lpval.first
  pts := map(x1+->evalSortedVarlist(x1,Lvar,Lpval),p.ts)
  mvar=p.v =>
       pval case R => pts (pval::R)
       pts pval
  multivariate(pts,p.v)

eval(p,Lvar,Lpval) ==
  empty? rest Lvar => evalSortedVarlist(p,Lvar,Lpval)
  sorted?((x1,x2) +-> x1 > x2, Lvar) => evalSortedVarlist(p,Lvar,Lpval)
  nlvar := sort((x1,x2) +-> x1 > x2,Lvar)
  nlpval :=
     Lvar = nlvar => Lpval
     nlpval := [Lpval.position(mvar,Lvar) for mvar in nlvar]
  evalSortedVarlist(p,nlvar,nlpval)

eval(p,Lvar,Lval) ==
  eval(p,Lvar,[val::% for val in Lval]$(List %)) -- kill?

degree(p,mvar) ==
  p case R => 0
  mvar= p.v => degree p.ts
  mvar > p.v => 0    -- might as well take advantage of the order
  max(degree(leadingCoefficient p.ts,mvar),degree(red p,mvar))

degree(p,Lvar)  == [degree(p,mvar)  for mvar in Lvar]

degree p ==
  p case R => 0
  degree(leadingCoefficient(p.ts)) + monomial(degree(p.ts), p.v)

minimumDegree p ==
  p case R => 0
  md := minimumDegree p.ts
  minimumDegree(coefficient(p.ts,md)) + monomial(md, p.v)

minimumDegree(p,mvar) ==
  p case R => 0
  mvar = p.v => minimumDegree p.ts
  md:=minimumDegree(leadingCoefficient p.ts,mvar)
  zero? (p1:=red p) => md
  min(md,minimumDegree(p1,mvar))

minimumDegree(p,Lvar) ==
  [minimumDegree(p,mvar) for mvar in Lvar]

totalDegree(p, Lvar) ==
  ground? p => 0
```

```
        null setIntersection(Lvar, variables p) => 0
        u := univariate(p, mv := mainVariable(p)::VarSet)
        weight:NonNegativeInteger := (member?(mv,Lvar) => 1; 0)
        tdeg:NonNegativeInteger := 0
        while u ^= 0 repeat
            termdeg:NonNegativeInteger := weight*degree u +
                           totalDegree(leadingCoefficient u, Lvar)
            tdeg := max(tdeg, termdeg)
            u := reductum u
        tdeg

      if R has CommutativeRing then
        differentiate(p,mvar) ==
          p case R => 0
          mvar=p.v =>
             up:=differentiate p.ts
             if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
          up:=map(x1 +-> differentiate(x1,mvar),p.ts)
          if ground? up then leadingCoefficient(up) else [p.v,up]$VPoly

      leadingCoefficient(p) ==
         p case R => p
         leadingCoefficient(leadingCoefficient(p.ts))

--      trailingCoef(p) ==
--        p case R => p
--        coef(p.ts,0) case R => coef(p.ts,0)
--        trailingCoef(red p)
--      TrailingCoef(p) == trailingCoef(p)

      leadingMonomial p ==
          p case R => p
          monomial(leadingMonomial leadingCoefficient(p.ts),
                   p.v, degree(p.ts))

      reductum(p) ==
          p case R => 0
          p - leadingMonomial p


--        if R is Integer then
--           pgcd := PolynomialGcdPackage(%,VarSet)
--           gcd(p1,p2) ==
--               gcd(p1,p2)$pgcd
--
--        else if R is RationalNumber then
--           gcd(p1,p2) ==
--               mrat:= MRationalFactorize(VarSet,%)
--               gcd(p1,p2)$mrat
--
```

```
--        else gcd(p1,p2) ==
--           p1 case R =>
--              p2 case R => gcd(p1,p2)$R::%
--              p1 = 0 => p2
--              gcd(p1, content(p2.ts))
--           p2 case R =>
--              p2 = 0 => p1
--              gcd(p2, content(p1.ts))
--           p1.v < p2.v => gcd(p1, content(p2.ts))
--           p1.v > p2.v => gcd(content(p1.ts), p2)
--           PSimp(p1.v, gcd(p1.ts, p2.ts))
```

———-

— SMP.dotabb —

```
"SMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SMP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SMP" -> "PFECAT"
```

———-

20.15 domain SMTS SparseMultivariateTaylorSeries

— SparseMultivariateTaylorSeries.input —

```
)set break resume
)sys rm -f SparseMultivariateTaylorSeries.output
)spool SparseMultivariateTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 10
xts:=x::TaylorSeries Fraction Integer
--R
--R
--R   (1)  x
--R                                            Type: TaylorSeries Fraction Integer
--E 1

--S 2 of 10
yts:=y::TaylorSeries Fraction Integer
--R
```

```
--R
--R   (2)  y
--R                                          Type: TaylorSeries Fraction Integer
--E 2

--S 3 of 10
zts:=z::TaylorSeries Fraction Integer
--R
--R
--R   (3)  z
--R                                          Type: TaylorSeries Fraction Integer
--E 3

--S 4 of 10
t1:=sin(xts)
--R
--R
--R           1  3    1   5     1   7      1    9
--R   (4)  x - - x  + --- x  - ---- x  + ------ x  + O(11)
--R           6      120      5040      362880
--R                                          Type: TaylorSeries Fraction Integer
--E 4

--S 5 of 10
coefficient(t1,3)
--R
--R
--R         1  3
--R   (5)  - - x
--R         6
--R                                            Type: Polynomial Fraction Integer
--E 5

--S 6 of 10
coefficient(t1,monomial(3,x)$IndexedExponents Symbol)
--R
--R
--R         1
--R   (6)  - -
--R         6
--R                                                    Type: Fraction Integer
--E 6

--S 7 of 10
t2:=sin(xts + yts)
--R
--R
--R   (7)
--R                  1  3   1    2   1  2    1  3
--R     (y + x) + (- - y  - - x y  - - x y - - x )
```

```
--R                  6       2        2       6
--R   +
--R       1   5    1    4    1  2 3    1  3 2    1  4     1   5
--R     (--- y  + -- x y  + -- x y  + -- x y  + -- x y + --- x )
--R      120      24        12        12        24       120
--R   +
--R     PAREN
--R             1   7    1     6    1   2 5    1   3 4    1   4 3    1   5 2
--R           - ---- y  - --- x y  - --- x y  - --- x y  - --- x y  - --- x y
--R             5040      720        240        144        144        240
--R        +
--R            1   6     1   7
--R          - --- x y - ---- x
--R            720       5040
--R   +
--R     PAREN
--R            1    9    1      8     1    2 7     1   3 6     1   4 5
--R          ------ y  + ----- x y  + ----- x y  + ---- x y  + ---- x y
--R          362880      40320        10080        4320        2880
--R        +
--R            1   5 4     1   6 3     1    7 2     1    8      1    9
--R          ---- x y  + ---- x y  + ----- x y  + ----- x y + ------ x
--R          2880        4320        10080        40320       362880
--R   +
--R     O(11)
--R                                        Type: TaylorSeries Fraction Integer
--E 7

--S 8 of 10
coefficient(t2,3)
--R
--R
--R          1  3   1    2   1  2    1  3
--R   (8)  - - y  - - x y  - - x y - - x
--R          6      2        2       6
--R                                          Type: Polynomial Fraction Integer
--E 8

--S 9 of 10
coefficient(t2,monomial(3,x)$IndexedExponents Symbol)
--R
--R
--R          1
--R   (9)  - -
--R          6
--R                                                     Type: Fraction Integer
--E 9

--S 10 of 10
polynomial(t2,5)
```

```
--R
--R
--R   (10)
--R      1   5    1    4     1  2   1  3     1  3   1    2     1  4   1  2
--R     --- y  + -- x y  + (-- x  - -)y  + (-- x  - - x)y  + (-- x  - - x  + 1)y
--R     120      24         12      6       12      2          24      2
--R   +
--R      1   5   1  3
--R     --- x  - - x  + x
--R     120      6
--R                                             Type: Polynomial Fraction Integer
--E 10

)spool
)lisp (bye)
```

— **SparseMultivariateTaylorSeries.help** —

```
====================================================================
SparseMultivariateTaylorSeries examples
====================================================================

Assume we have three variables which get expressed as sparse
multivariate taylor series.

  xts:=x::TaylorSeries Fraction Integer
  yts:=y::TaylorSeries Fraction Integer
  zts:=z::TaylorSeries Fraction Integer

These will cause traditional routines to expand in series form:

  t1:=sin(xts)

         1  3    1   5     1   7      1    9
     x - - x  + --- x  - ---- x  + ------ x  + O(11)
         6      120      5040      362880


We can ask for a specific coefficient, in this case, the coefficient
of the third power.

  coefficient(t1,3)

       1  3
     - - x
       6
```

And we can get that coefficient, expressed as a monomial.

```
  coefficient(t1,monomial(3,x)$IndexedExponents Symbol)

       1
     - -
       6
```

In a multivariate version we get a polynomial in x and y

```
  t2:=sin(xts + yts)

                  1  3   1    2   1  2    1  3
     (y + x) + (- - y  - - x y  - - x y - - x )
                  6      2        2       6
   +
       1   5    1    4    1  2 3    1  3 2    1  4     1   5
     (--- y  + -- x y  + -- x y  + -- x y  + -- x y + --- x )
      120      24        12        12        24       120
   +
     PAREN
           1   7    1     6    1   2 5    1   3 4    1   4 3    1   5 2
        - ---- y  - --- x y  - --- x y  - --- x y  - --- x y  - --- x y
          5040      720        240        144        144        240
       +
          1   6     1   7
        - --- x y - ---- x
          720       5040
   +
     PAREN
          1     9     1      8     1     2 7     1   3 6     1   4 5
        ------ y  + ----- x y  + ----- x y  + ---- x y  + ---- x y
        362880      40320        10080        4320        2880
       +
          1   5 4     1   6 3     1    7 2     1    8       1    9
        ---- x y  + ---- x y  + ----- x y  + ----- x y + ------ x
        2880        4320        10080        40320       362880
   +
     O(11)
```

We can ask for the third coefficient which is

```
  coefficient(t2,3)

       1  3   1    2   1  2    1  3
     - - y  - - x y  - - x y - - x
       6      2        2       6
```

```
And we can ask for the third coefficient of that coefficient in x

  coefficient(t2,monomial(3,x)$IndexedExponents Symbol)

       1
     - -
       6

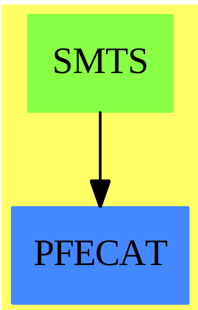
And we can convert that result to a polynomial

  polynomial(t2,5)

      1   5    1    4     1  2   1  3     1  3   1    2     1  4   1  2
     --- y  + -- x y  + (-- x  - -)y  + (-- x  - - x)y  + (-- x  - - x  + 1)y
     120      24         12      6       12      2         24      2
   +
      1   5   1  3
     --- x  - - x  + x
     120      6

See Also:
o )show SparseMultivariateTaylorSeries
o )display op coefficient
```

20.15.1 SparseMultivariateTaylorSeries (SMTS)



See

⇒ "TaylorSeries" (TS) 21.3.1 on page 2628

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	charthRoot	coefficient	coerce	complete
cos	cosh	cot	coth	csc
csch	csubst	D	degree	differentiate
eval	exp	exquo	extend	fintegrate
hash	integrate	latex	leadingCoefficient	leadingMonomial
log	map	monomial	monomial?	nthRoot
one?	order	pi	pole?	polynomial
recip	reductum	sample	sec	sech
sin	sinh	sqrt	subtractIfCan	tan
tanh	unit?	unitCanonical	unitNormal	variables
zero?	?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?	

— domain SMTS SparseMultivariateTaylorSeries —

```
)abbrev domain SMTS SparseMultivariateTaylorSeries
++ Authors: William Burge, Stephen Watt, Clifton Williamson
++ Date Created: 15 August 1988
++ Date Last Updated: 18 May 1991
++ Basic Operations:
++ Related Domains:
++ Also See: UnivariateTaylorSeries
++ AMS Classifications:
++ Keywords: multivariate, Taylor, series
++ Examples:
++ References:
++ Description:
++ This domain provides multivariate Taylor series with variables
++ from an arbitrary ordered set.  A Taylor series is represented
++ by a stream of polynomials from the polynomial domain SMP.
++ The nth element of the stream is a form of degree n.  SMTS is an
++ internal domain.

SparseMultivariateTaylorSeries(Coef,Var,SMP):_
 Exports == Implementation where
  Coef : Ring
  Var  : OrderedSet
  SMP  : PolynomialCategory(Coef,IndexedExponents Var,Var)
  I   ==> Integer
  L   ==> List
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  PS  ==> InnerTaylorSeries SMP
  RN  ==> Fraction Integer
  ST  ==> Stream
```

```
StS ==> Stream SMP
STT ==> StreamTaylorSeriesOperations SMP
STF ==> StreamTranscendentalFunctions SMP
ST2 ==> StreamFunctions2
ST3 ==> StreamFunctions3

Exports ==> MultivariateTaylorSeriesCategory(Coef,Var) with
  coefficient: (%,NNI) -> SMP
    ++ \spad{coefficient(s, n)} gives the terms of total degree n.
    ++
    ++X xts:=x::TaylorSeries Fraction Integer
    ++X t1:=sin(xts)
    ++X coefficient(t1,3)

  coerce: Var -> %
    ++ \spad{coerce(var)} converts a variable to a Taylor series
  coerce: SMP -> %
    ++ \spad{coerce(poly)} regroups the terms by total degree and forms
    ++ a series.
  "*":(SMP,%)->%
    ++\spad{smp*ts} multiplies a TaylorSeries by a monomial SMP.
  csubst:(L Var,L StS) -> (SMP -> StS)
    ++\spad{csubst(a,b)} is for internal use only

  if Coef has Algebra Fraction Integer then
    integrate: (%,Var,Coef) -> %
      ++\spad{integrate(s,v,c)} is the integral of s with respect
      ++ to v and having c as the constant of integration.
    fintegrate: (() -> %,Var,Coef) -> %
      ++\spad{fintegrate(f,v,c)} is the integral of \spad{f()} with respect
      ++ to v and having c as the constant of integration.
      ++ The evaluation of \spad{f()} is delayed.

Implementation ==> PS add

  Rep := StS -- Below we use the fact that Rep of PS is Stream SMP.
  extend(x,n) == extend(x,n + 1)$Rep
  complete x == complete(x)$Rep

  evalstream:(%,L Var,L SMP) -> StS
  evalstream(s:%,lv:(L Var),lsmp:(L SMP)):(ST SMP) ==
    scan(0,_+$SMP,
      map((z1:SMP):SMP+->eval(z1,lv,lsmp),s pretend StS))$ST2(SMP,SMP)

  addvariable:(Var,InnerTaylorSeries Coef) -> %
  addvariable(v,s) ==
    ints := integers(0)$STT pretend ST NNI
    map((n1:NNI,c2:Coef):SMP+->monomial(c2 :: SMP,v,n1)$SMP,
           ints,s pretend ST Coef)$ST3(NNI,Coef,SMP)
```

```
    -- We can extract a polynomial giving the terms of given total degree
    coefficient(s,n) == elt(s,n + 1)$Rep  -- 1-based indexing for streams

    -- Here we have to take into account that we reduce the degree of each
    -- term of the stream by a constant
    coefficient(s:%,lv:List Var,ln:List NNI):% ==
      map ((z1:SMP):SMP +-> coefficient(z1,lv,ln),rest(s,reduce(_+,ln)))

    -- the coefficient of a particular monomial:
    coefficient(s:%,m:IndexedExponents Var):Coef ==
      n:=leadingCoefficient(mon:=m)
      while not zero?(mon:=reductum mon) repeat
        n:=n+leadingCoefficient mon
      coefficient(coefficient(s,n),m)

--% creation of series

    coerce(r:Coef) == monom(r::SMP,0)$STT
    smp:SMP * p:% == (((smp) *  (p pretend Rep))$STT)pretend %
    r:Coef * p:% == (((r::SMP) *  (p pretend Rep))$STT)pretend %
    p:% * r:Coef == (((r::SMP) * ( p pretend Rep))$STT)pretend %
    mts(p:SMP):% ==
      (uv := mainVariable p) case "failed" => monom(p,0)$STT
      v := uv :: Var
      s : % := 0
      up := univariate(p,v)
      while not zero? up repeat
        s := s + monomial(1,v,degree up) * mts(leadingCoefficient up)
        up := reductum up
      s

    coerce(p:SMP) == mts p
    coerce(v:Var) == v :: SMP :: %

    monomial(r:%,v:Var,n:NNI) ==
      r * monom(monomial(1,v,n)$SMP,n)$STT

--% evaluation

    substvar: (SMP,L Var,L %) -> %
    substvar(p,vl,q) ==
      null vl => monom(p,0)$STT
      (uv := mainVariable p) case "failed" => monom(p,0)$STT
      v := uv :: Var
      v = first vl =>
        s : % := 0
        up := univariate(p,v)
        while not zero? up repeat
          c := leadingCoefficient up
          s := s + first q ** degree up * substvar(c,rest vl,rest q)
```

```
        up := reductum up
      s
    substvar(p,rest vl,rest q)

  sortmfirst:(SMP,L Var,L %) -> %
  sortmfirst(p,vl,q) ==
    nlv : L Var := sort((v1:Var,v2:Var):Boolean +-> v1 > v2,vl)
    nq : L % := [q position$(L Var) (i,vl) for i in nlv]
    substvar(p,nlv,nq)

  csubst(vl,q) == (p1:SMP):StS+->sortmfirst(p1,vl,q pretend L(%)) pretend StS

  restCheck(s:StS):StS ==
    -- checks that stream is null or first element is 0
    -- returns empty() or rest of stream
    empty? s => s
    not zero? frst s =>
      error "eval: constant coefficient should be 0"
    rst s

  eval(s:%,v:L Var,q:L %) ==
    #v ^= #q =>
      error "eval: number of variables should equal number of values"
    nq : L StS := [restCheck(i pretend StS) for i in q]
    addiag(map(csubst(v,nq),s pretend StS)$ST2(SMP,StS))$STT pretend %

  substmts(v:Var,p:SMP,q:%):% ==
    up := univariate(p,v)
    ss : % := 0
    while not zero? up repeat
      d:=degree up
      c:SMP:=leadingCoefficient up
      ss := ss + c* q ** d
      up := reductum up
    ss

  substream(v:Var,p:SMP,q:StS):StS==
    substmts(v,p,q pretend %) pretend StS

  comp1:(Var,StS,StS) -> StS
  comp1(v,r,t)==
    addiag(map((p1:SMP):StS +-> substream(v,p1,t),r)$ST2(SMP,StS))$STT

  comp(v:Var,s:StS,t:StS):StS == delay
    empty? s => s
    f := frst s; r : StS := rst s;
    empty? r => s
    empty? t => concat(f,comp1(v,r,empty()$StS))
    not zero? frst t =>
      error "eval: constant coefficient should be zero"
```

```
      concat(f,comp1(v,r,rst t))

    eval(s:%,v:Var,t:%) == comp(v,s pretend StS,t pretend StS)

--% differentiation and integration

    differentiate(s:%,v:Var):% ==
      empty? s => 0
      map((z1:SMP):SMP +-> differentiate(z1,v),rst s)

    if Coef has Algebra Fraction Integer then

      stream(x:%):Rep == x pretend Rep

      (x:%) ** (r:RN) == powern(r,stream x)$STT
      (r:RN) * (x:%)  ==
        map((z1:SMP):SMP +-> r*z1,stream x)$ST2(SMP,SMP) pretend %
      (x:%) * (r:RN)  ==
        map((z1:SMP):SMP +-> z1*r,stream x)$ST2(SMP,SMP) pretend %

      exp x == exp(stream x)$STF
      log x == log(stream x)$STF

      sin x == sin(stream x)$STF
      cos x == cos(stream x)$STF
      tan x == tan(stream x)$STF
      cot x == cot(stream x)$STF
      sec x == sec(stream x)$STF
      csc x == csc(stream x)$STF

      asin x == asin(stream x)$STF
      acos x == acos(stream x)$STF
      atan x == atan(stream x)$STF
      acot x == acot(stream x)$STF
      asec x == asec(stream x)$STF
      acsc x == acsc(stream x)$STF

      sinh x == sinh(stream x)$STF
      cosh x == cosh(stream x)$STF
      tanh x == tanh(stream x)$STF
      coth x == coth(stream x)$STF
      sech x == sech(stream x)$STF
      csch x == csch(stream x)$STF

      asinh x == asinh(stream x)$STF
      acosh x == acosh(stream x)$STF
      atanh x == atanh(stream x)$STF
      acoth x == acoth(stream x)$STF
      asech x == asech(stream x)$STF
      acsch x == acsch(stream x)$STF
```

```
    intsmp(v:Var,p: SMP): SMP ==
      up := univariate(p,v)
      ss : SMP := 0
      while not zero? up repeat
        d := degree up
        c := leadingCoefficient up
        ss := ss + inv((d+1) :: RN) * monomial(c,v,d+1)$SMP
        up := reductum up
      ss

    fintegrate(f,v,r) ==
      concat(r::SMP,delay map((z1:SMP):SMP +-> intsmp(v,z1),f() pretend StS))
    integrate(s,v,r) ==
      concat(r::SMP,map((z1:SMP):SMP +-> intsmp(v,z1),s pretend StS))

  -- If there is more than one term of the same order, group them.
  tout(p:SMP):OUT ==
    pe := p :: OUT
    monomial? p => pe
    paren pe

  showAll?: () -> Boolean
  -- check a global Lisp variable
  showAll?() == true

  coerce(s:%):OUT ==
    uu := s pretend Stream(SMP)
    empty? uu => (0$SMP) :: OUT
    n : NNI; count : NNI := _$streamCount$Lisp
    l : List OUT := empty()
    for n in 0..count while not empty? uu repeat
      if frst(uu) ^= 0 then l := concat(tout frst uu,l)
      uu := rst uu
    if showAll?() then
      for n in n.. while explicitEntries? uu and _
             not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then l := concat(tout frst uu,l)
        uu := rst uu
    l :=
      explicitlyEmpty? uu => l
      eq?(uu,rst uu) and frst uu = 0 => l
      concat(prefix("O" :: OUT,[n :: OUT]),l)
    empty? l => (0$SMP) :: OUT
    reduce("+",reverse_! l)
  if Coef has Field then
       stream(x:%):Rep == x pretend Rep
       SF2==> StreamFunctions2
       p:% / r:Coef ==
         (map((z1:SMP):SMP +-> z1/$SMP r,stream p)$SF2(SMP,SMP)) pretend %
```

———-

— SMTS.dotabb —

```
"SMTS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SMTS"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SMTS" -> "PFECAT"
```

———-

20.16 domain STBL SparseTable

— SparseTable.input —

```
)set break resume
)sys rm -f SparseTable.output
)spool SparseTable.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
t: SparseTable(Integer, String, "Try again!") := table()
--R
--R
--R   (1)  table()
--R                                    Type: SparseTable(Integer,String,Try again!)
--E 1

--S 2 of 7
t.3 := "Number three"
--R
--R
--R   (2)  "Number three"
--R                                                                 Type: String
--E 2

--S 3 of 7
t.4 := "Number four"
--R
--R
--R   (3)  "Number four"
--R                                                                 Type: String
--E 3
```

```
--S 4 of 7
t.3
--R
--R
--R   (4)  "Number three"
--R                                                              Type: String
--E 4

--S 5 of 7
t.2
--R
--R
--R   (5)  "Try again!"
--R                                                              Type: String
--E 5

--S 6 of 7
keys t
--R
--R
--R   (6)  [4,3]
--R                                                        Type: List Integer
--E 6

--S 7 of 7
entries t
--R
--R
--R   (7)  ["Number four","Number three"]
--R                                                         Type: List String
--E 7
)spool
)lisp (bye)
```

———-

— SparseTable.help —

```
====================================================================
SparseTable examples
====================================================================

The SparseTable domain provides a general purpose table type with
default entries.

Here we create a table to save strings under integer keys.  The value
"Try again!" is returned if no other value has been stored for a key.
```

```
  t: SparseTable(Integer, String, "Try again!") := table()
    table()
                              Type: SparseTable(Integer,String,Try again!)
```

Entries can be stored in the table.

```
  t.3 := "Number three"
    "Number three"
                              Type: String

  t.4 := "Number four"
    "Number four"
                              Type: String
```

These values can be retrieved as usual, but if a look up fails the default entry will be returned.

```
  t.3
    "Number three"
                              Type: String

  t.2
    "Try again!"
                              Type: String
```

To see which values are explicitly stored, the keys and entries functions can be used.

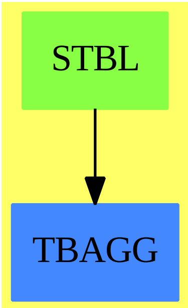
```
  keys t
    [4,3]
                              Type: List Integer

  entries t
    ["Number four","Number three"]
                              Type: List String
```

If a specific table representation is required, the GeneralSparseTable constructor should be used. The domain SparseTable(K, E, dflt)} is equivalent to GeneralSparseTable(K,E,Table(K,E), dflt).

See Also:
o)help Table
o)help GeneralSparseTable
o)show SparseTable

20.16.1 SparseTable (STBL)



See

⇒ "HashTable" (HASHTBL) 9.1.1 on page 1085
⇒ "InnerTable" (INTABL) 10.27.1 on page 1299
⇒ "Table" (TABLE) 21.1.1 on page 2621
⇒ "EqTable" (EQTBL) 6.2.1 on page 667
⇒ "StringTable" (STRTBL) 20.32.1 on page 2569
⇒ "GeneralSparseTable" (GSTBL) 8.5.1 on page 1044

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
setelt	select	select!	size?	swap!
table	#?	?=?	?~=?	?.?

— domain STBL SparseTable —

```
)abbrev domain STBL SparseTable
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
```

```
++ A sparse table has a default entry, which is returned if no other
++ value has been explicitly stored for a key.

SparseTable(Key:SetCategory, Ent:SetCategory, dent:Ent) ==
        GeneralSparseTable(Key, Ent, Table(Key, Ent), dent)
```

———-

— **STBL.dotabb** —

```
"STBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"STBL" -> "TBAGG"
```

———-

20.17 domain SULS SparseUnivariateLaurentSeries

— **SparseUnivariateLaurentSeries.input** —

```
)set break resume
)sys rm -f SparseUnivariateLaurentSeries.output
)spool SparseUnivariateLaurentSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariateLaurentSeries
--R SparseUnivariateLaurentSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for SparseUnivariateLaurentSeries is SULS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SULS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficient : (%,Integer) -> Coef     coerce : Variable var -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
```

```
--R complete : % -> %                    degree : % -> Integer
--R ?.? : (%,Integer) -> Coef            extend : (%,Integer) -> %
--R hash : % -> SingleInteger            latex : % -> String
--R leadingCoefficient : % -> Coef       leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %        monomial : (Coef,Integer) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : (%,Integer) -> Integer       order : % -> Integer
--R pole? : % -> Boolean                 recip : % -> Union(%,"failed")
--R reductum : % -> %                    removeZeroes : (Integer,%) -> %
--R removeZeroes : % -> %                sample : () -> %
--R truncate : (%,Integer) -> %          variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (SparseUnivariateTaylorSeries(Coef,var,cen),%) -> % if Coef has FIELD
--R ?*? : (%,SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if Coef has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (SparseUnivariateTaylorSeries(Coef,var,cen),SparseUnivariateTaylorSeries(Coef,var,cen)) -> % i
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?<? : (%,%) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R ?<=? : (%,%) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R ?>? : (%,%) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R ?>=? : (%,%) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R D : (%,Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIE
--R D : (%,List Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef ha
--R D : (%,Symbol,NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMB
--R D : (%,List Symbol,List NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has P
--R D : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIELD or Coef has
--R D : (%,NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef h
--R D : (%,(SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylorSeries(Coef,var,cen)),No
--R D : (%,(SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylorSeries(Coef,var,cen))) -
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R associates? : (%,%) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if SparseUnivariateTaylorSeries
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and SparseUnivariateTaylorSeries(Coe
--R coerce : Fraction Integer -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT
--R coerce : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has F
--R coerce : Symbol -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBOL an
--R coerce : SparseUnivariateTaylorSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and SparseUnivariateTa
--R convert : % -> Pattern Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has KONVERT
--R convert : % -> Pattern Float if SparseUnivariateTaylorSeries(Coef,var,cen) has KONVERT P
--R convert : % -> DoubleFloat if SparseUnivariateTaylorSeries(Coef,var,cen) has REAL and Co
--R convert : % -> Float if SparseUnivariateTaylorSeries(Coef,var,cen) has REAL and Coef has
--R convert : % -> InputForm if SparseUnivariateTaylorSeries(Coef,var,cen) has KONVERT INFOR
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING
--R differentiate : (%,List Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has P
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if SparseUnivariateTaylorSe
--R differentiate : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coe
--R differentiate : (%,NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen
--R differentiate : (%,Variable var) -> %
--R differentiate : (%,(SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylor
--R differentiate : (%,(SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylor
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivariateTaylorSerie
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,List SparseUnivariateTaylorSeries(Coef,var,cen),List SparseUnivariateTaylorSer
--R eval : (%,SparseUnivariateTaylorSeries(Coef,var,cen),SparseUnivariateTaylorSeries(Coef,v
--R eval : (%,Equation SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivariateT
--R eval : (%,List Equation SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivar
--R eval : (%,List Symbol,List SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUni
--R eval : (%,Symbol,SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivariateTay
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen) has OIN
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIE
```

```
--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if SparseUn
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R floor : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if SparseUnivariateTaylorSeries(Coef,var,cen
--R fractionPart : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has EUCDOM and Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R init : () -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has STEP and Coef has FIELD
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : (Integer,SparseUnivariateTaylorSeries(Coef,var,cen)) -> %
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylorSeries(Coef,var,cen)),%)
--R max : (%,%) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Spa
--R min : (%,%) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Spa
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R negative? : % -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIEL
--R nextItem : % -> Union(%,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen) has STEP and Coef ha
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Spars
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIEL
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R random : () -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has INS and Coef has FIELD
--R rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has LINEXP
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if SparseUniv
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix SparseUnivariateTaylorSeries(Coef,var,cen)
--R reducedSystem : Matrix % -> Matrix SparseUnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Coef has FI
--R retract : % -> Fraction Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Co
--R retract : % -> Symbol if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBOL and Coef has
--R retract : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R retractIfCan : % -> Union(Integer,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRAC
```

```
--R retractIfCan : % -> Union(Fraction Integer,"failed") if SparseUnivariateTaylorSeries(Coe
--R retractIfCan : % -> Union(Symbol,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen)
--R retractIfCan : % -> Union(SparseUnivariateTaylorSeries(Coef,var,cen),"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sign : % -> Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef ha
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylor : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R taylorIfCan : % -> Union(SparseUnivariateTaylorSeries(Coef,var,cen),"failed")
--R taylorRep : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef h
--R unitCanonical : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coe
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if SparseUnivariateTaylorSeri
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if SparseUnivariateTaylorSeri
--R
--R
--E 1

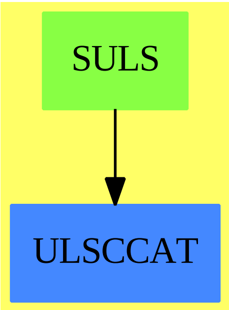
)spool
)lisp (bye)
```

— SparseUnivariateLaurentSeries.help —

```
====================================================================
SparseUnivariateLaurentSeries examples
====================================================================

See Also:
o )show SparseUnivariateLaurentSeries
```

20.17.1 SparseUnivariateLaurentSeries (SULS)



Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
approximate	asec	asech
asin	asinh	associates?
atan	atanh	ceiling
characteristic	charthRoot	center
coefficient	coerce	complete
conditionP	convert	cos
cosh	cot	coth
csc	csch	D
degree	denom	denominator
differentiate	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extend	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractionPart	gcd
gcdPolynomial	hash	init
integrate	inv	latex
laurent	lcm	leadingCoefficient
leadingMonomial	log	map
max	min	monomial
monomial?	multiEuclidean	multiplyCoefficients
multiplyExponents	negative?	nextItem
nthRoot	numer	numerator
one?	order	patternMatch
pi	pole?	positive?
prime?	principalIdeal	random
rationalFunction	recip	reducedSystem
reductum	removeZeroes	retract
retractIfCan	sample	sec
sech	series	sign
sin	sinh	sizeLess?
solveLinearPolynomialEquation	sqrt	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
tan	tanh	taylor
taylorIfCan	taylorRep	terms
truncate	unit?	unitCanonical
unitNormal	variable	variables
wholePart	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?.?	?~=?	?/?
?<?	?<=?	?>?
?>=?	?^?	?quo?
?rem?		

— domain SULS SparseUnivariateLaurentSeries —

```
)abbrev domain SULS SparseUnivariateLaurentSeries
++ Author: Clifton J. Williamson
++ Date Created: 11 November 1994
++ Date Last Updated: 10 March 1995
++ Basic Operations:
++ Related Domains: InnerSparseUnivariatePowerSeries,
++   SparseUnivariateTaylorSeries, SparseUnivariatePuiseuxSeries
++ Also See:
++ AMS Classifications:
++ Keywords: sparse, series
++ Examples:
++ References:
++ Description:
++ Sparse Laurent series in one variable
++ \spadtype{SparseUnivariateLaurentSeries} is a domain representing Laurent
++ series in one variable with coefficients in an arbitrary ring.  The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion.  For example,
++ \spad{SparseUnivariateLaurentSeries(Integer,x,3)} represents Laurent
++ series in \spad{(x - 3)} with integer coefficients.

SparseUnivariateLaurentSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I     ==> Integer
  NNI   ==> NonNegativeInteger
  OUT   ==> OutputForm
  P     ==> Polynomial Coef
  RF    ==> Fraction Polynomial Coef
  RN    ==> Fraction Integer
  S     ==> String
  SUTS  ==> SparseUnivariateTaylorSeries(Coef,var,cen)
  EFULS ==> ElementaryFunctionsUnivariateLaurentSeries(Coef,SUTS,%)

  Exports ==> UnivariateLaurentSeriesConstructorCategory(Coef,SUTS) with
    coerce: Variable(var) -> %
      ++ \spad{coerce(var)} converts the series variable \spad{var} into a
      ++ Laurent series.
    differentiate: (%,Variable(var)) -> %
      ++ \spad{differentiate(f(x),x)} returns the derivative of
      ++ \spad{f(x)} with respect to \spad{x}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
```

```
      ++ by integers.

  Implementation ==> InnerSparseUnivariatePowerSeries(Coef) add

    Rep := InnerSparseUnivariatePowerSeries(Coef)

    variable x == var
    center   x == cen

    coerce(v: Variable(var)) ==
      zero? cen => monomial(1,1)
      monomial(1,1) + monomial(cen,0)

    pole? x == negative? order(x,0)

--% operations with Taylor series

    coerce(uts:SUTS) == uts pretend %

    taylorIfCan uls ==
      pole? uls => "failed"
      uls pretend SUTS

    taylor uls ==
      (uts := taylorIfCan uls) case "failed" =>
        error "taylor: Laurent series has a pole"
      uts :: SUTS

    retractIfCan(x:%):Union(SUTS,"failed") == taylorIfCan x

    laurent(n,uts) == monomial(1,n) * (uts :: %)

    removeZeroes uls    == uls
    removeZeroes(n,uls) == uls

    taylorRep uls == taylor(monomial(1,-order(uls,0)) * uls)
    degree uls    == order(uls,0)

    numer uls == taylorRep uls
    denom uls == monomial(1,(-order(uls,0)) :: NNI)$SUTS

    (uts:SUTS) * (uls:%) == (uts :: %) * uls
    (uls:%) * (uts:SUTS) == uls * (uts :: %)

    if Coef has Field then
      (uts1:SUTS) / (uts2:SUTS) == (uts1 :: %) / (uts2 :: %)

    recip(uls) == iExquo(1,uls,false)

    if Coef has IntegralDomain then
```

```
    uls1 exquo uls2 == iExquo(uls1,uls2,false)

  if Coef has Field then
    uls1:% / uls2:% ==
      (q := uls1 exquo uls2) case "failed" =>
        error "quotient cannot be computed"
      q :: %

  differentiate(uls:%,v:Variable(var)) == differentiate uls

  elt(uls1:%,uls2:%) ==
    order(uls2,1) < 1 =>
      error "elt: second argument must have positive order"
    negative?(ord := order(uls1,0)) =>
      (recipr := recip uls2) case "failed" =>
        error "elt: second argument not invertible"
      uls3 := uls1 * monomial(1,-ord)
      iCompose(uls3,uls2) * (recipr :: %) ** ((-ord) :: NNI)
    iCompose(uls1,uls2)

  if Coef has IntegralDomain then
    rationalFunction(uls,n) ==
      zero?(e := order(uls,0)) =>
        negative? n => 0
        polynomial(taylor uls,n :: NNI) :: RF
      negative?(m := n - e) => 0
      poly := polynomial(taylor(monomial(1,-e) * uls),m :: NNI) :: RF
      v := variable(uls) :: RF; c := center(uls) :: P :: RF
      poly / (v - c) ** ((-e) :: NNI)

    rationalFunction(uls,n1,n2) == rationalFunction(truncate(uls,n1,n2),n2)

  if Coef has Algebra Fraction Integer then

    integrate uls ==
      zero? coefficient(uls,-1) =>
        error "integrate: series has term of order -1"
      integrate(uls)$Rep

    integrate(uls:%,v:Variable(var)) == integrate uls

    (uls1:%) ** (uls2:%) == exp(log(uls1) * uls2)

    exp uls   == exp(uls)$EFULS
    log uls   == log(uls)$EFULS
    sin uls   == sin(uls)$EFULS
    cos uls   == cos(uls)$EFULS
    tan uls   == tan(uls)$EFULS
    cot uls   == cot(uls)$EFULS
    sec uls   == sec(uls)$EFULS
```

```
    csc uls   == csc(uls)$EFULS
    asin uls  == asin(uls)$EFULS
    acos uls  == acos(uls)$EFULS
    atan uls  == atan(uls)$EFULS
    acot uls  == acot(uls)$EFULS
    asec uls  == asec(uls)$EFULS
    acsc uls  == acsc(uls)$EFULS
    sinh uls  == sinh(uls)$EFULS
    cosh uls  == cosh(uls)$EFULS
    tanh uls  == tanh(uls)$EFULS
    coth uls  == coth(uls)$EFULS
    sech uls  == sech(uls)$EFULS
    csch uls  == csch(uls)$EFULS
    asinh uls == asinh(uls)$EFULS
    acosh uls == acosh(uls)$EFULS
    atanh uls == atanh(uls)$EFULS
    acoth uls == acoth(uls)$EFULS
    asech uls == asech(uls)$EFULS
    acsch uls == acsch(uls)$EFULS

    if Coef has CommutativeRing then

      (uls:%) ** (r:RN) == cRationalPower(uls,r)

    else

      (uls:%) ** (r:RN) ==
        negative?(ord0 := order(uls,0)) =>
          order := ord0 :: I
          (n := order exquo denom(r)) case "failed" =>
            error "**: rational power does not exist"
          uts := retract(uls * monomial(1,-order))@SUTS
          utsPow := (uts ** r) :: %
          monomial(1,(n :: I) * numer(r)) * utsPow
        uts := retract(uls)@SUTS
        (uts ** r) :: %

--% OutputForms

  coerce(uls:%): OUT ==
    st := getStream uls
    if not(explicitlyEmpty? st or explicitEntries? st) _
      and (nx := retractIfCan(elt getRef uls))@Union(I,"failed") case I then
      count : NNI := _$streamCount$Lisp
      degr := min(count,(nx :: I) + count + 1)
      extend(uls,degr)
    seriesToOutputForm(st,getRef uls,variable uls,center uls,1)
```

———–

— SULS.dotabb —

```
"SULS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SULS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"SULS" -> "ULSCCAT"
```

20.18 domain SUP SparseUnivariatePolynomial

— SparseUnivariatePolynomial.input —

```
)set break resume
)sys rm -f SparseUnivariatePolynomial.output
)spool SparseUnivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariatePolynomial
--R SparseUnivariatePolynomial R: Ring  is a domain constructor
--R Abbreviation for SparseUnivariatePolynomial is SUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                 D : % -> %
--R D : (%,NonNegativeInteger) -> %       1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> List R            coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger      differentiate : % -> %
--R ?.? : (%,%) -> %                      ?.? : (%,R) -> R
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             init : () -> % if R has STEP
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
```

```
--R monomial? : % -> Boolean            monomials : % -> List %
--R one? : % -> Boolean                 primitiveMonomials : % -> List %
--R pseudoRemainder : (%,%) -> %        recip : % -> Union(%,"failed")
--R reductum : % -> %                   retract : % -> R
--R sample : () -> %                    zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
```

```
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R fmecg : (%,NonNegativeInteger,R,%) -> %
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
```

```
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R outputForm : (%,OutputForm) -> OutputForm
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
```

```
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)
```

———-

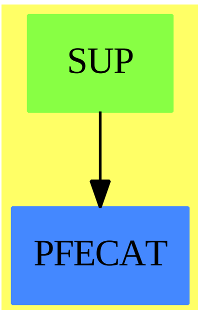
— SparseUnivariatePolynomial.help —

```
====================================================================
SparseUnivariatePolynomial examples
====================================================================

See Also:
o )show SparseUnivariatePolynomial
```

———-

20.18.1 SparseUnivariatePolynomial (SUP)



See

⇒ "FreeModule" (FM) 7.30.1 on page 980
⇒ "PolynomialRing" (PR) 17.27.1 on page 2052
⇒ "UnivariatePolynomial" (UP) 22.4.1 on page 2784

Exports:

0	1
associates?	binomThmExpt
characteristic	charthRoot
coefficient	coefficients
coerce	composite
conditionP	content
convert	D
degree	differentiate
discriminant	divide
divideExponents	elt
euclideanSize	eval
expressIdealMember	exquo
extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial
fmecg	gcd
gcdPolynomial	ground
ground?	hash
init	integrate
isExpt	isPlus
isTimes	karatsubaDivide
latex	lcm
leadingCoefficient	leadingMonomial
mainVariable	makeSUP
map	mapExponents
max	min
minimumDegree	monicDivide
monomial	monomial?
monomials	multiEuclidean
multiplyExponents	multivariate
nextItem	numberOfMonomials
one?	order
outputForm	patternMatch
pomopo!	prime?
primitiveMonomials	primitivePart
principalIdeal	pseudoDivide
pseudoQuotient	pseudoRemainder
recip	reducedSystem
reductum	resultant
retract	retractIfCan
sample	separate
shiftLeft	shiftRight
sizeLess?	solveLinearPolynomialEquation
squareFree	squareFreePart
squareFreePolynomial	subResultantGcd
subtractIfCan	totalDegree
totalDegree	unit?
unitCanonical	unitNormal
univariate	univariate
unmakeSUP	variables
vectorise	zero?
?*?	?**?
?+?	?-?
-?	?=?
?^?	?.?
?~=?	?/?
?<?	?<=?
?>?	?>=?
?quo?	?rem?

— domain SUP SparseUnivariatePolynomial —

```
)abbrev domain SUP SparseUnivariatePolynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, reductum, differentiate,
++ elt, map, resultant, discriminant
++ Related Constructors: UnivariatePolynomial, Polynomial
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents univariate polynomials over arbitrary
++ (not necessarily commutative) coefficient rings. The variable is
++ unspecified  so that the variable displays as \spad{?} on output.
++ If it is necessary to specify the variable name, use type \spadtype{UnivariatePolynomial}.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.
++ Note that if the coefficient ring is a field, this domain forms a euclidean domain.

SparseUnivariatePolynomial(R:Ring): UnivariatePolynomialCategory(R) with
     outputForm : (%,OutputForm) -> OutputForm
        ++ outputForm(p,var) converts the SparseUnivariatePolynomial p to
        ++ an output form (see \spadtype{OutputForm}) printed as a polynomial in the
        ++ output form variable.
     fmecg: (%,NonNegativeInteger,R,%) -> %
        ++ fmecg(p1,e,r,p2) finds x : p1 - r * x**e * p2
    == PolynomialRing(R,NonNegativeInteger)
  add
   --representations
   Term := Record(k:NonNegativeInteger,c:R)
   Rep  := List Term
   p:%
   n:NonNegativeInteger
   np: PositiveInteger
   FP ==> SparseUnivariatePolynomial %
   pp,qq: FP
   lpp:List FP

   -- for karatsuba
   kBound: NonNegativeInteger := 63
   upmp := UnivariatePolynomialMultiplicationPackage(R,%)


   if R has FieldOfPrimeCharacteristic  then
         p ** np == p ** (np pretend NonNegativeInteger)
         p ^ np  == p ** (np pretend NonNegativeInteger)
```

```
        p ^ n  == p ** n
        p ** n  ==
           null p => 0
           zero? n => 1
--           one? n => p
           (n = 1) => p
           empty? p.rest =>
             zero?(cc:=p.first.c ** n) => 0
             [[n * p.first.k, cc]]
           -- not worth doing special trick if characteristic is too small
           if characteristic()$R < 3 then return expt(p,n pretend PositiveInteger)$Repeated
           y:%:=1
           -- break up exponent in qn * characteristic + rn
           -- exponentiating by the characteristic is fast
           rec := divide(n, characteristic()$R)
           qn:= rec.quotient
           rn:= rec.remainder
           repeat
               if rn = 1 then y := y * p
               if rn > 1 then y:= y * binomThmExpt([p.first], p.rest, rn)
               zero? qn => return y
               -- raise to the characteristic power
               p:= [[t.k * characteristic()$R , primeFrobenius(t.c)$R ]$Term for t in p]
               rec := divide(qn, characteristic()$R)
               qn:= rec.quotient
               rn:= rec.remainder
           y



   zero?(p): Boolean == empty?(p)
--   one?(p):Boolean == not empty? p and (empty? rest p and zero? first(p).k and one? first(
   one?(p):Boolean == not empty? p and (empty? rest p and zero? first(p).k and (first(p).c =
   ground?(p): Boolean == empty? p or (empty? rest p and zero? first(p).k)
   multiplyExponents(p,n) == [ [u.k*n,u.c] for u in p]
   divideExponents(p,n) ==
     null p => p
     m:= (p.first.k :: Integer exquo n::Integer)
     m case "failed" => "failed"
     u:= divideExponents(p.rest,n)
     u case "failed" => "failed"
     [[m::Integer::NonNegativeInteger,p.first.c],:u]
   karatsubaDivide(p, n)  ==
     zero? n => [p, 0]
     lowp: Rep := p
     highp: Rep := []
     repeat
       if empty? lowp then break
       t := first lowp
       if t.k < n then break
```

```
      lowp := rest lowp
      highp := cons([subtractIfCan(t.k,n)::NonNegativeInteger,t.c]$Term,highp)
    [ reverse highp,  lowp]
  shiftRight(p, n)  ==
     [[subtractIfCan(t.k,n)::NonNegativeInteger,t.c]$Term for t in p]
  shiftLeft(p, n)  ==
     [[t.k + n,t.c]$Term for t in p]
  pomopo!(p1,r,e,p2) ==
        rout:%:= []
        for tm in p2 repeat
           e2:= e + tm.k
           c2:= r * tm.c
           c2 = 0 => "next term"
           while not null p1 and p1.first.k > e2 repeat
             (rout:=[p1.first,:rout]; p1:=p1.rest)  --use PUSH and POP?
           null p1 or p1.first.k < e2 => rout:=[[e2,c2],:rout]
           if (u:=p1.first.c + c2) ^= 0 then rout:=[[e2, u],:rout]
           p1:=p1.rest
        NRECONC(rout,p1)$Lisp

-- implementation using karatsuba algorithm conditionally
--
--   p1 * p2 ==
--     xx := p1::Rep
--     empty? xx => p1
--     yy := p2::Rep
--     empty? yy => p2
--     zero? first(xx).k => first(xx).c * p2
--     zero? first(yy).k => p1 * first(yy).c
--     (first(xx).k > kBound) and (first(yy).k > kBound) and (#xx > kBound) and (#yy > kBound) =>
--       karatsubaOnce(p1,p2)$upmp
--     xx := reverse xx
--     res : Rep := empty()
--     for tx in xx repeat res:= rep pomopo!( res,tx.c,tx.k,p2)
--     res


  univariate(p:%) == p pretend SparseUnivariatePolynomial(R)
  multivariate(sup:SparseUnivariatePolynomial(R),v:SingletonAsOrderedSet) ==
     sup pretend %
  univariate(p:%,v:SingletonAsOrderedSet) ==
    zero? p => 0
    monomial(leadingCoefficient(p)::%,degree p) +
        univariate(reductum p,v)
  multivariate(supp:SparseUnivariatePolynomial(%),v:SingletonAsOrderedSet) ==
    zero? supp => 0
    lc:=leadingCoefficient supp
    degree lc > 0 => error "bad form polynomial"
    monomial(leadingCoefficient lc,degree supp) +
        multivariate(reductum supp,v)
```

```
if R has FiniteFieldCategory and R has PolynomialFactorizationExplicit then
  RXY ==> SparseUnivariatePolynomial SparseUnivariatePolynomial R
  squareFreePolynomial pp ==
     squareFree(pp)$UnivariatePolynomialSquareFree(%,FP)
  factorPolynomial pp ==
       (generalTwoFactor(pp pretend RXY)$TwoFactorize(R))
                   pretend Factored SparseUnivariatePolynomial %
  factorSquareFreePolynomial pp ==
       (generalTwoFactor(pp pretend RXY)$TwoFactorize(R))
                   pretend Factored SparseUnivariatePolynomial %
  gcdPolynomial(pp,qq) == gcd(pp,qq)$FP
  factor p == factor(p)$DistinctDegreeFactorize(R,%)
  solveLinearPolynomialEquation(lpp,pp) ==
    solveLinearPolynomialEquation(lpp, pp)$FiniteFieldSolveLinearPolynomialEquation(R,%,FP
else if R has PolynomialFactorizationExplicit then
  import PolynomialFactorizationByRecursionUnivariate(R,%)
  solveLinearPolynomialEquation(lpp,pp)==
    solveLinearPolynomialEquationByRecursion(lpp,pp)
  factorPolynomial(pp) ==
    factorByRecursion(pp)
  factorSquareFreePolynomial(pp) ==
    factorSquareFreeByRecursion(pp)

if R has IntegralDomain then
 if R has approximate then
  p1 exquo p2  ==
     null p2 => error "Division by 0"
     p2 = 1 => p1
     p1=p2 => 1
   --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
     rout:= []@List(Term)
     while not null p1 repeat
        (a:= p1.first.c exquo p2.first.c)
        a case "failed" => return "failed"
        ee:= subtractIfCan(p1.first.k, p2.first.k)
        ee case "failed" => return "failed"
        p1:= fmecg(p1.rest, ee, a, p2.rest)
        rout:= [[ee,a], :rout]
     null p1 => reverse(rout)::%    -- nreverse?
     "failed"
 else -- R not approximate
  p1 exquo p2  ==
     null p2 => error "Division by 0"
     p2 = 1 => p1
   --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
     rout:= []@List(Term)
     while not null p1 repeat
        (a:= p1.first.c exquo p2.first.c)
        a case "failed" => return "failed"
        ee:= subtractIfCan(p1.first.k, p2.first.k)
```

```
        ee case "failed" => return "failed"
        p1:= fmecg(p1.rest, ee, a, p2.rest)
        rout:= [[ee,a], :rout]
     null p1 => reverse(rout)::%    -- nreverse?
     "failed"
fmecg(p1,e,r,p2) ==       -- p1 - r * x**e * p2
       rout:%:= []
       r:= - r
       for tm in p2 repeat
          e2:= e + tm.k
          c2:= r * tm.c
          c2 = 0 => "next term"
          while not null p1 and p1.first.k > e2 repeat
            (rout:=[p1.first,:rout]; p1:=p1.rest)  --use PUSH and POP?
          null p1 or p1.first.k < e2 => rout:=[[e2,c2],:rout]
          if (u:=p1.first.c + c2) ^= 0 then rout:=[[e2, u],:rout]
          p1:=p1.rest
       NRECONC(rout,p1)$Lisp
pseudoRemainder(p1,p2) ==
  null p2 => error "PseudoDivision by Zero"
  null p1 => 0
  co:=p2.first.c;
  e:=p2.first.k;
  p2:=p2.rest;
  e1:=max(p1.first.k:Integer-e+1,0):NonNegativeInteger
  while not null p1 repeat
    if (u:=subtractIfCan(p1.first.k,e)) case "failed" then leave
    p1:=fmecg(co * p1.rest, u, p1.first.c, p2)
    e1:= (e1 - 1):NonNegativeInteger
  e1 = 0 => p1
  co ** e1 * p1
toutput(t1:Term,v:OutputForm):OutputForm ==
  t1.k = 0 => t1.c :: OutputForm
  if t1.k = 1
    then mon:= v
    else mon := v ** t1.k::OutputForm
  t1.c = 1 => mon
  t1.c = -1 and
       ((t1.c :: OutputForm) = (-1$Integer)::OutputForm)@Boolean => - mon
  t1.c::OutputForm * mon
outputForm(p:%,v:OutputForm) ==
  l: List(OutputForm)
  l:=[toutput(t,v) for t in p]
  null l => (0$Integer)::OutputForm -- else FreeModule 0 problems
  reduce("+",l)

coerce(p:%):OutputForm == outputForm(p, "?"::OutputForm)
elt(p:%,val:R) ==
   null p => 0$R
   co:=p.first.c
```

```
      n:=p.first.k
      for tm in p.rest repeat
       co:= co * val ** (n - (n:=tm.k)):NonNegativeInteger + tm.c
      n = 0 => co
      co * val ** n
   elt(p:%,val:%) ==
      null p => 0$%
      coef:% := p.first.c :: %
      n:=p.first.k
      for tm in p.rest repeat
       coef:= coef * val ** (n-(n:=tm.k)):NonNegativeInteger+(tm.c::%)
      n = 0 => coef
      coef * val ** n

   monicDivide(p1:%,p2:%) ==
      null p2 => error "monicDivide: division by 0"
      leadingCoefficient p2 ^= 1 => error "Divisor Not Monic"
      p2 = 1 => [p1,0]
      null p1 => [0,0]
      degree p1 < (n:=degree p2) => [0,p1]
      rout:Rep := []
      p2 := p2.rest
      while not null p1 repeat
         (u:=subtractIfCan(p1.first.k, n)) case "failed" => leave
         rout:=[[u, p1.first.c], :rout]
         p1:=fmecg(p1.rest, rout.first.k, rout.first.c, p2)
      [reverse_!(rout),p1]

   if R has IntegralDomain then
       discriminant(p) == discriminant(p)$PseudoRemainderSequence(R,%)
--     discriminant(p) ==
--       null p or zero?(p.first.k) => error "cannot take discriminant of constants"
--       dp:=differentiate p
--       corr:=  p.first.c ** ((degree p - 1 - degree dp)::NonNegativeInteger)
--       (-1)**((p.first.k*(p.first.k-1)) quo 2):NonNegativeInteger
--         * (corr * resultant(p,dp) exquo p.first.c)::R

       subResultantGcd(p1,p2) == subResultantGcd(p1,p2)$PseudoRemainderSequence(R,%)
--     subResultantGcd(p1,p2) ==    --args # 0, non-coef, prim, ans not prim
--       --see algorithm 1 (p. 4) of Brown's latest (unpublished) paper
--       if p1.first.k < p2.first.k then (p1,p2):=(p2,p1)
--       p:=pseudoRemainder(p1,p2)
--       co:=1$R;
--       e:= (p1.first.k - p2.first.k):NonNegativeInteger
--       while not null p and p.first.k ^= 0 repeat
--         p1:=p2; p2:=p; p:=pseudoRemainder(p1,p2)
--         null p or p.first.k = 0 => "enuf"
--         co:=(p1.first.c ** e exquo co ** max(0, (e-1))::NonNegativeInteger)::R
--         e:= (p1.first.k - p2.first.k):NonNegativeInteger;  c1:=co**e
--         p:=[[tm.k,((tm.c exquo p1.first.c)::R exquo c1)::R] for tm in p]
```

```
--       if null p then p2 else 1$%

      resultant(p1,p2) == resultant(p1,p2)$PseudoRemainderSequence(R,%)
--    resultant(p1,p2) ==      --SubResultant PRS Algorithm
--       null p1 or null p2 => 0$R
--       0 = degree(p1) => ((first p1).c)**degree(p2)
--       0 = degree(p2) => ((first p2).c)**degree(p1)
--       if p1.first.k < p2.first.k then
--         (if odd?(p1.first.k) then p1:=-p1;  (p1,p2):=(p2,p1))
--       p:=pseudoRemainder(p1,p2)
--       co:=1$R;  e:=(p1.first.k-p2.first.k):NonNegativeInteger
--       while not null p repeat
--          if not odd?(e) then p:=-p
--          p1:=p2;  p2:=p;  p:=pseudoRemainder(p1,p2)
--          co:=(p1.first.c ** e exquo co ** max(e:Integer-1,0):NonNegativeInteger)::R
--          e:= (p1.first.k - p2.first.k):NonNegativeInteger;  c1:=co**e
--          p:=(p exquo ((leadingCoefficient p1) * c1))::%
--       degree p2 > 0 => 0$R
--       (p2.first.c**e exquo co**((e-1)::NonNegativeInteger))::R
   if R has GcdDomain then
     content(p) == if null p then 0$R else "gcd"/[tm.c for tm in p]
        --make CONTENT more efficient?

     primitivePart(p) ==
        null p => p
        ct :=content(p)
        unitCanonical((p exquo ct)::%)
               -- exquo  present since % is now an IntegralDomain

     gcd(p1,p2) ==
          gcdPolynomial(p1 pretend SparseUnivariatePolynomial R,
                        p2 pretend SparseUnivariatePolynomial R) pretend %

   if R has Field then
     divide( p1, p2)  ==
       zero? p2 => error "Division by 0"
--       one? p2 => [p1,0]
       (p2 = 1) => [p1,0]
       ct:=inv(p2.first.c)
       n:=p2.first.k
       p2:=p2.rest
       rout:=empty()$List(Term)
       while p1 ^= 0 repeat
          (u:=subtractIfCan(p1.first.k, n)) case "failed" => leave
          rout:=[[u, ct * p1.first.c], :rout]
          p1:=fmecg(p1.rest, rout.first.k, rout.first.c, p2)
       [reverse_!(rout),p1]

     p / co == inv(co) * p
```

— SUP.dotabb —

```
"SUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SUP" -> "PFECAT"
```

20.19 domain SUPEXPR SparseUnivariatePolynomialExpressions

This domain is a hack, in some sense. What I'd really like to do - automatically - is to provide all operations supported by the coefficient domain, as long as the polynomials can be retracted to that domain, i.e., as long as they are just constants. I don't see another way to do this, unfortunately.

— SparseUnivariatePolynomialExpressions.input —

```
)set break resume
)sys rm -f SparseUnivariatePolynomialExpressions.output
)spool SparseUnivariatePolynomialExpressions.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariatePolynomialExpressions
--R SparseUnivariatePolynomialExpressions R: Ring  is a domain constructor
--R Abbreviation for SparseUnivariatePolynomialExpressions is SUPEXPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUPEXPR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                 D : % -> %
--R D : (%,NonNegativeInteger) -> %       1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> List R            coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
```

```
--R degree : % -> NonNegativeInteger      differentiate : % -> %
--R ?.? : (%,%) -> %                      ?.? : (%,R) -> R
--R eval : (%,List %,List %) -> %         eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %            eval : (%,List Equation %) -> %
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             init : () -> % if R has STEP
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R monomial? : % -> Boolean              monomials : % -> List %
--R one? : % -> Boolean                   pi : () -> % if R has TRANFUN
--R primitiveMonomials : % -> List %      pseudoRemainder : (%,%) -> %
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,%) -> % if R has TRANFUN
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if R has TRANFUN
--R acosh : % -> % if R has TRANFUN
--R acot : % -> % if R has TRANFUN
--R acoth : % -> % if R has TRANFUN
--R acsc : % -> % if R has TRANFUN
--R acsch : % -> % if R has TRANFUN
--R asec : % -> % if R has TRANFUN
--R asech : % -> % if R has TRANFUN
--R asin : % -> % if R has TRANFUN
--R asinh : % -> % if R has TRANFUN
--R associates? : (%,%) -> Boolean if R has INTDOM
--R atan : % -> % if R has TRANFUN
--R atanh : % -> % if R has TRANFUN
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
```

```
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R ha
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R ha
--R cos : % -> % if R has TRANFUN
--R cosh : % -> % if R has TRANFUN
--R cot : % -> % if R has TRANFUN
--R coth : % -> % if R has TRANFUN
--R csc : % -> % if R has TRANFUN
--R csch : % -> % if R has TRANFUN
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R exp : % -> % if R has TRANFUN
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
```

```
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R log : % -> % if R has TRANFUN
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Singl
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

```
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R sec : % -> % if R has TRANFUN
--R sech : % -> % if R has TRANFUN
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sin : % -> % if R has TRANFUN
--R sinh : % -> % if R has TRANFUN
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyno
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomia
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if R has TRANFUN
--R tanh : % -> % if R has TRANFUN
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)
```

———-

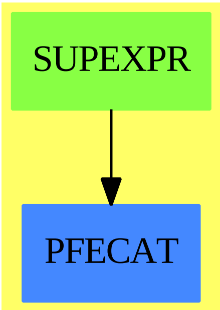
— SparseUnivariatePolynomialExpressions.help —

```
====================================================================
SparseUnivariatePolynomialExpressions examples
====================================================================
```

```
See Also:
o )show SparseUnivariatePolynomialExpressions
```

———–

20.19.1 SparseUnivariatePolynomialExpressions (SUPEXPR)



Exports:

0	1	acos
acosh	acot	acoth
acsc	acsch	asec
asech	asin	asinh
associates?	atan	atanh
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
composite	conditionP	content
convert	cos	cosh
cot	coth	csc
csch	D	degree
differentiate	discriminant	divide
divideExponents	elt	euclideanSize
eval	exp	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	gcd
gcdPolynomial	ground	ground?
hash	init	integrate
isExpt	isPlus	isTimes
karatsubaDivide	latex	lcm
leadingCoefficient	leadingMonomial	log
mainVariable	makeSUP	map
mapExponents	max	min
minimumDegree	monicDivide	monomial
monomial?	monomials	multiEuclidean
multiplyExponents	multivariate	nextItem
numberOfMonomials	one?	order
patternMatch	pi	pomopo!
prime?	primitiveMonomials	primitivePart
principalIdeal	pseudoDivide	pseudoQuotient
pseudoRemainder	recip	reducedSystem
reductum	resultant	retract
retractIfCan	sample	sec
sech	separate	shiftLeft
shiftRight	sin	sinh
sizeLess?	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subResultantGcd
subtractIfCan	tan	tanh
totalDegree	unit?	unitCanonical
unitNormal	univariate	unmakeSUP
variables	vectorise	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?.?	?~=?
?/?	?<?	?<=?
?>?	?>=?	?quo?
?rem?		

— domain SUPEXPR SparseUnivariatePolynomialExpressions —

```
)abbrev domain SUPEXPR SparseUnivariatePolynomialExpressions
++ Author: Mark Botch
++ Description:
++ This domain has no description

SparseUnivariatePolynomialExpressions(R: Ring): Exports == Implementation where

    Exports == UnivariatePolynomialCategory R with

        if R has TranscendentalFunctionCategory
        then TranscendentalFunctionCategory

    Implementation == SparseUnivariatePolynomial R add

        if R has TranscendentalFunctionCategory then
            log(p: %): % ==
                ground? p => coerce log ground p
                output(hconcat("log p for p= ", p::OutputForm))$OutputPackage
                error "SUPTRAFUN: log only defined for elements of the coefficient ring"

            exp(p: %): % ==
                ground? p => coerce exp ground p
                output(hconcat("exp p for p= ", p::OutputForm))$OutputPackage
                error "SUPTRAFUN: exp only defined for elements of the coefficient ring"
            sin(p: %): % ==
                ground? p => coerce sin ground p
                output(hconcat("sin p for p= ", p::OutputForm))$OutputPackage
                error "SUPTRAFUN: sin only defined for elements of the coefficient ring"
            asin(p: %): % ==
                ground? p => coerce asin ground p
                output(hconcat("asin p for p= ", p::OutputForm))$OutputPackage
                error "SUPTRAFUN: asin only defined for elements of the coefficient ring"
            cos(p: %): % ==
                ground? p => coerce cos ground p
                output(hconcat("cos p for p= ", p::OutputForm))$OutputPackage
                error "SUPTRAFUN: cos only defined for elements of the coefficient ring"
            acos(p: %): % ==
                ground? p => coerce acos ground p
                output(hconcat("acos p for p= ", p::OutputForm))$OutputPackage
                error "SUPTRAFUN: acos only defined for elements of the coefficient ring"
```

— SUPEXPR.dotabb —

```
"SUPEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUPEXPR"]
```

```
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SUPEXPR" -> "PFECAT"
```

20.20 domain SUPXS SparseUnivariatePuiseuxSeries

— SparseUnivariatePuiseuxSeries.input —

```
)set break resume
)sys rm -f SparseUnivariatePuiseuxSeries.output
)spool SparseUnivariatePuiseuxSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariatePuiseuxSeries
--R SparseUnivariatePuiseuxSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for SparseUnivariatePuiseuxSeries is SUPXS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUPXS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coerce : Variable var -> %            coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> Fraction Integer        hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R order : % -> Fraction Integer         pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

```
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has P
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Coef and Coef
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : SparseUnivariateTaylorSeries(Coef,var,cen) -> %
--R coerce : SparseUnivariateLaurentSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING S
--R differentiate : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDR
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef an
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef)
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
```

```
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIEL
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has var
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : % -> SparseUnivariateLaurentSeries(Coef,var,cen)
--R laurentIfCan : % -> Union(SparseUnivariateLaurentSeries(Coef,var,cen),"failed")
--R laurentRep : % -> SparseUnivariateLaurentSeries(Coef,var,cen)
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R puiseux : (Fraction Integer,SparseUnivariateLaurentSeries(Coef,var,cen)) -> %
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R retract : % -> SparseUnivariateLaurentSeries(Coef,var,cen)
--R retractIfCan : % -> Union(SparseUnivariateTaylorSeries(Coef,var,cen),"failed")
--R retractIfCan : % -> Union(SparseUnivariateLaurentSeries(Coef,var,cen),"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———

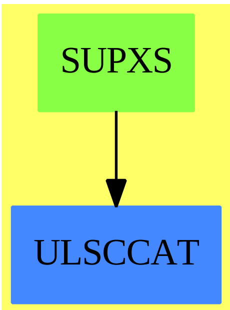
— SparseUnivariatePuiseuxSeries.help —

```
====================================================================
SparseUnivariatePuiseuxSeries examples
====================================================================

See Also:
o )show SparseUnivariatePuiseuxSeries
```

———

20.20.1 SparseUnivariatePuiseuxSeries (SUPXS)



Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
characteristic	charthRoot	center	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	laurent	laurentIfCan
laurentRep	lcm	leadingCoefficient	leadingMonomial
log	map	monomial	monomial?
multiEuclidean	multiplyExponents	nthRoot	one?
order	pi	pole?	prime?
principalIdeal	puiseux	rationalPower	recip
reductum	retract	retractIfCan	sample
sec	sech	series	sin
sinh	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	tan	tanh
terms	truncate	unit?	unitCanonical
unitNormal	variable	variables	zero?
?.?	?**?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?	?quo?	?rem?

— domain SUPXS SparseUnivariatePuiseuxSeries —

```
)abbrev domain SUPXS SparseUnivariatePuiseuxSeries
++ Author: Clifton J. Williamson
++ Date Created: 11 November 1994
++ Date Last Updated: 28 February 1995
++ Basic Operations:
++ Related Domains: InnerSparseUnivariatePowerSeries,
++   SparseUnivariateTaylorSeries, SparseUnivariateLaurentSeries
++ Also See:
++ AMS Classifications:
++ Keywords: sparse, series
++ Examples:
++ References:
++ Description:
++ Sparse Puiseux series in one variable
++ \spadtype{SparseUnivariatePuiseuxSeries} is a domain representing Puiseux
++ series in one variable with coefficients in an arbitrary ring.  The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion.  For example,
```

```
++ \spad{SparseUnivariatePuiseuxSeries(Integer,x,3)} represents Puiseux
++ series in \spad{(x - 3)} with \spadtype{Integer} coefficients.

SparseUnivariatePuiseuxSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  RN   ==> Fraction Integer
  SUTS ==> SparseUnivariateTaylorSeries(Coef,var,cen)
  SULS ==> SparseUnivariateLaurentSeries(Coef,var,cen)
  SUPS ==> InnerSparseUnivariatePowerSeries(Coef)

  Exports ==> Join(UnivariatePuiseuxSeriesConstructorCategory(Coef,SULS),_
                 RetractableTo SUTS) with
    coerce: Variable(var) -> %
      ++ coerce(var) converts the series variable \spad{var} into a
      ++ Puiseux series.
    differentiate: (%,Variable(var)) -> %
      ++ \spad{differentiate(f(x),x)} returns the derivative of
      ++ \spad{f(x)} with respect to \spad{x}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
        ++ by integers.

  Implementation ==> UnivariatePuiseuxSeriesConstructor(Coef,SULS) add

    Rep := Record(expon:RN,lSeries:SULS)

    getExpon: % -> RN
    getExpon pxs == pxs.expon

    variable x == var
    center   x == cen

    coerce(v: Variable(var)) ==
      zero? cen => monomial(1,1)
      monomial(1,1) + monomial(cen,0)

    coerce(uts:SUTS) == uts :: SULS :: %

    retractIfCan(upxs:%):Union(SUTS,"failed") ==
      (uls := retractIfCan(upxs)@Union(SULS,"failed")) case "failed" =>
        "failed"
      retractIfCan(uls :: SULS)@Union(SUTS,"failed")
```

```
    if Coef has "*": (Fraction Integer, Coef) -> Coef then
      differentiate(upxs:%,v:Variable(var)) == differentiate upxs

    if Coef has Algebra Fraction Integer then
      integrate(upxs:%,v:Variable(var)) == integrate upxs

--% OutputForms

    coerce(x:%): OUT ==
      sups : SUPS := laurentRep(x) pretend SUPS
      st := getStream sups; refer := getRef sups
      if not(explicitlyEmpty? st or explicitEntries? st) _
        and (nx := retractIfCan(elt refer)@Union(I,"failed")) case I then
        count : NNI := _$streamCount$Lisp
        degr := min(count,(nx :: I) + count + 1)
        extend(sups,degr)
      seriesToOutputForm(st,refer,variable x,center x,rationalPower x)
```

———–

— SUPXS.dotabb —

```
"SUPXS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUPXS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"SUPXS" -> "ULSCCAT"
```

———–

20.21 domain ORESUP SparseUnivariateSkewPolynomial

— SparseUnivariateSkewPolynomial.input —

```
)set break resume
)sys rm -f SparseUnivariateSkewPolynomial.output
)spool SparseUnivariateSkewPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariateSkewPolynomial
```

```
--R SparseUnivariateSkewPolynomial(R: Ring,sigma: Automorphism R,delta: (R -> R))  is a domain construct
--R Abbreviation for SparseUnivariateSkewPolynomial is ORESUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ORESUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        apply : (%,R,R) -> R
--R coefficients : % -> List R            coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger      hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R reductum : % -> %                     retract : % -> R
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R leftGcd : (%,%) -> % if R has FIELD
--R leftLcm : (%,%) -> % if R has FIELD
--R leftQuotient : (%,%) -> % if R has FIELD
--R leftRemainder : (%,%) -> % if R has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monomial : (R,NonNegativeInteger) -> %
--R outputForm : (%,OutputForm) -> OutputForm
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
```

```
--R rightGcd : (%,%) -> % if R has FIELD
--R rightLcm : (%,%) -> % if R has FIELD
--R rightQuotient : (%,%) -> % if R has FIELD
--R rightRemainder : (%,%) -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

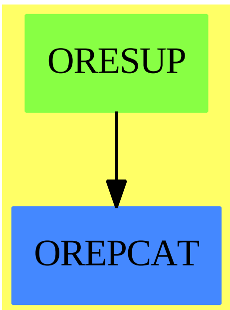
— **SparseUnivariateSkewPolynomial.help** —

```
====================================================================
SparseUnivariateSkewPolynomial examples
====================================================================

See Also:
o )show SparseUnivariateSkewPolynomial
```

———-

20.21.1 SparseUnivariateSkewPolynomial (ORESUP)



See

⇒ "Automorphism" (AUTOMOR) 2.44.1 on page 228
⇒ "UnivariateSkewPolynomial" (OREUP) 22.8.1 on page 2829

Exports:

0	1	apply	characteristic	coefficient
coefficients	coerce	content	degree	exquo
hash	latex	leadingCoefficient	leftDivide	leftExactQuotient
leftExtendedGcd	leftGcd	leftLcm	leftQuotient	leftRemainder
minimumDegree	monicLeftDivide	monicRightDivide	monomial	one?
outputForm	primitivePart	recip	reductum	retract
retractIfCan	rightDivide	rightExactQuotient	rightExtendedGcd	rightGcd
rightLcm	rightQuotient	rightRemainder	sample	subtractIfCan
zero?	?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?	

— domain ORESUP SparseUnivariateSkewPolynomial —

```
)abbrev domain ORESUP SparseUnivariateSkewPolynomial
++ Author: Manuel Bronstein
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ Description:
++ This is the domain of sparse univariate skew polynomials over an Ore
++ coefficient field.
++ The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.

SparseUnivariateSkewPolynomial(R:Ring, sigma:Automorphism R, delta: R -> R):
 UnivariateSkewPolynomialCategory R with
      outputForm: (%, OutputForm) -> OutputForm
        ++ outputForm(p, x) returns the output form of p using x for the
        ++ otherwise anonymous variable.
   == SparseUnivariatePolynomial R add
      import UnivariateSkewPolynomialCategoryOps(R, %)

      x:% * y:%      == times(x, y, sigma, delta)
      apply(p, c, r) == apply(p, c, r, sigma, delta)

      if R has IntegralDomain then
          monicLeftDivide(a, b)  == monicLeftDivide(a, b, sigma)
          monicRightDivide(a, b) == monicRightDivide(a, b, sigma)

      if R has Field then
          leftDivide(a, b)  == leftDivide(a, b, sigma)
          rightDivide(a, b) == rightDivide(a, b, sigma)
```

— ORESUP.dotabb —

```
"ORESUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ORESUP"]
```

```
"OREPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OREPCAT"]
"ORESUP" -> "OREPCAT"
```

————-

20.22 domain SUTS SparseUnivariateTaylorSeries

— SparseUnivariateTaylorSeries.input —

```
)set break resume
)sys rm -f SparseUnivariateTaylorSeries.output
)spool SparseUnivariateTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariateTaylorSeries
--R SparseUnivariateTaylorSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for SparseUnivariateTaylorSeries is SUTS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUTS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficients : % -> Stream Coef       coerce : Variable var -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial? : % -> Boolean
--R one? : % -> Boolean                   order : % -> NonNegativeInteger
--R pole? : % -> Boolean                  quoByVar : % -> %
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      series : Stream Coef -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
```

```
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Coef) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef an
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef and C
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(var,Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has P
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has va
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R univariatePolynomial : (%,NonNegativeInteger) -> UnivariatePolynomial(var,Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

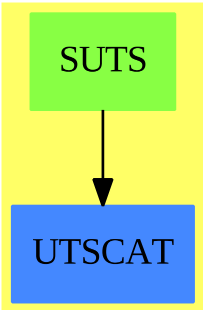
— SparseUnivariateTaylorSeries.help —

```
====================================================================
SparseUnivariateTaylorSeries examples
====================================================================

See Also:
o )show SparseUnivariateTaylorSeries
```

———-

20.22.1 SparseUnivariateTaylorSeries (SUTS)



Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	approximate	asec
asech	asin	asinh	associates?	atan
atanh	center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos	cosh
cot	coth	csc	csch	D
degree	differentiate	eval	exp	exquo
extend	hash	integrate	latex	leadingCoefficient
leadingMonomial	log	map	monomial	monomial?
multiplyCoefficients	multiplyExponents	nthRoot	one?	order
pole?	pi	polynomial	polynomial	quoByVar
recip	reductum	sample	sec	sech
series	sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate	truncate
unit?	unitCanonical	unitNormal	univariatePolynomial	variable
variables	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?
?/?	?.?			

— domain SUTS SparseUnivariateTaylorSeries —

```
)abbrev domain SUTS SparseUnivariateTaylorSeries
++ Author: Clifton J. Williamson
++ Date Created: 16 February 1990
++ Date Last Updated: 10 March 1995
++ Basic Operations:
++ Related Domains: InnerSparseUnivariatePowerSeries,
```

```
++   SparseUnivariateLaurentSeries, SparseUnivariatePuiseuxSeries
++ Also See:
++ AMS Classifications:
++ Keywords: Taylor series, sparse power series
++ Examples:
++ References:
++ Description:
++ Sparse Taylor series in one variable
++ \spadtype{SparseUnivariateTaylorSeries} is a domain representing Taylor
++ series in one variable with coefficients in an arbitrary ring.  The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion.  For example,
++ \spadtype{SparseUnivariateTaylorSeries}(Integer,x,3) represents Taylor
++ series in \spad{(x - 3)} with \spadtype{Integer} coefficients.

SparseUnivariateTaylorSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  COM  ==> OrderedCompletion Integer
  I    ==> Integer
  L    ==> List
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  P    ==> Polynomial Coef
  REF  ==> Reference OrderedCompletion Integer
  RN   ==> Fraction Integer
  Term ==> Record(k:Integer,c:Coef)
  SG   ==> String
  ST   ==> Stream Term
  UP   ==> UnivariatePolynomial(var,Coef)

  Exports ==> UnivariateTaylorSeriesCategory(Coef) with
    coerce: UP -> %
      ++\spad{coerce(p)} converts a univariate polynomial p in the variable
      ++\spad{var} to a univariate Taylor series in \spad{var}.
    univariatePolynomial: (%,NNI) -> UP
      ++\spad{univariatePolynomial(f,k)} returns a univariate polynomial
      ++ consisting of the sum of all terms of f of degree \spad{<= k}.
    coerce: Variable(var) -> %
      ++\spad{coerce(var)} converts the series variable \spad{var} into a
      ++ Taylor series.
    differentiate: (%,Variable(var)) -> %
      ++ \spad{differentiate(f(x),x)} computes the derivative of
      ++ \spad{f(x)} with respect to \spad{x}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x),x)} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
```

```
      ++ by integers.

  Implementation ==> InnerSparseUnivariatePowerSeries(Coef) add
    import REF

    Rep := InnerSparseUnivariatePowerSeries(Coef)

    makeTerm: (Integer,Coef) -> Term
    makeTerm(exp,coef) == [exp,coef]
    getCoef: Term -> Coef
    getCoef term == term.c
    getExpon: Term -> Integer
    getExpon term == term.k

    monomial(coef,expon) == monomial(coef,expon)$Rep
    extend(x,n) == extend(x,n)$Rep

    0 == monomial(0,0)$Rep
    1 == monomial(1,0)$Rep

    recip uts == iExquo(1,uts,true)

    if Coef has IntegralDomain then
      uts1 exquo uts2 == iExquo(uts1,uts2,true)

    quoByVar uts == taylorQuoByVar(uts)$Rep

    differentiate(x:%,v:Variable(var)) == differentiate x

--% Creation and destruction of series

    coerce(v: Variable(var)) ==
      zero? cen => monomial(1,1)
      monomial(1,1) + monomial(cen,0)

    coerce(p:UP) ==
      zero? p => 0
      if not zero? cen then p := p(monomial(1,1)$UP + monomial(cen,0)$UP)
      st : ST := empty()
      while not zero? p repeat
        st := concat(makeTerm(degree p,leadingCoefficient p),st)
        p := reductum p
      makeSeries(ref plusInfinity(),st)

    univariatePolynomial(x,n) ==
      extend(x,n); st := getStream x
      ans : UP := 0; oldDeg : I := 0;
      mon := monomial(1,1)$UP - monomial(center x,0)$UP; monPow : UP := 1
      while explicitEntries? st repeat
        (xExpon := getExpon(xTerm := frst st)) > n => return ans
```

```
    pow := (xExpon - oldDeg) :: NNI; oldDeg := xExpon
    monPow := monPow * mon ** pow
    ans := ans + getCoef(xTerm) * monPow
    st := rst st
  ans

polynomial(x,n) ==
  extend(x,n); st := getStream x
  ans : P := 0; oldDeg : I := 0;
  mon := (var :: P) - (center(x) :: P); monPow : P := 1
  while explicitEntries? st repeat
    (xExpon := getExpon(xTerm := frst st)) > n => return ans
    pow := (xExpon - oldDeg) :: NNI; oldDeg := xExpon
    monPow := monPow * mon ** pow
    ans := ans + getCoef(xTerm) * monPow
    st := rst st
  ans

polynomial(x,n1,n2) == polynomial(truncate(x,n1,n2),n2)

truncate(x,n)     == truncate(x,n)$Rep
truncate(x,n1,n2) == truncate(x,n1,n2)$Rep

iCoefficients: (ST,REF,I) -> Stream Coef
iCoefficients(x,refer,n) == delay
  -- when this function is called, we are computing the nth order
  -- coefficient of the series
  explicitlyEmpty? x => empty()
  -- if terms up to order n have not been computed,
  -- apply lazy evaluation
  nn := n :: COM
  while (nx := elt refer) < nn repeat lazyEvaluate x
  -- must have nx >= n
  explicitEntries? x =>
    xCoef := getCoef(xTerm := frst x); xExpon := getExpon xTerm
    xExpon = n => concat(xCoef,iCoefficients(rst x,refer,n + 1))
    -- must have nx > n
    concat(0,iCoefficients(x,refer,n + 1))
  concat(0,iCoefficients(x,refer,n + 1))

coefficients uts ==
  refer := getRef uts; x := getStream uts
  iCoefficients(x,refer,0)

terms uts == terms(uts)$Rep pretend Stream Record(k:NNI,c:Coef)

iSeries: (Stream Coef,I,REF) -> ST
iSeries(st,n,refer) == delay
  -- when this function is called, we are creating the nth order
  -- term of a series
```

```
      empty? st => (setelt(refer,plusInfinity()); empty())
      setelt(refer,n :: COM)
      zero? (coef := frst st) => iSeries(rst st,n + 1,refer)
      concat(makeTerm(n,coef),iSeries(rst st,n + 1,refer))

    series(st:Stream Coef) ==
      refer := ref(-1)
      makeSeries(refer,iSeries(st,0,refer))

    nniToI: Stream Record(k:NNI,c:Coef) -> ST
    nniToI st ==
      empty? st => empty()
      term : Term := [(frst st).k,(frst st).c]
      concat(term,nniToI rst st)

    series(st:Stream Record(k:NNI,c:Coef)) == series(nniToI st)$Rep

--% Values

    variable x == var
    center   x == cen

    coefficient(x,n) == coefficient(x,n)$Rep
    elt(x:%,n:NonNegativeInteger) == coefficient(x,n)

    pole? x == false

    order x    == (order(x)$Rep) :: NNI
    order(x,n) == (order(x,n)$Rep) :: NNI

--% Composition

    elt(uts1:%,uts2:%) ==
      zero? uts2 => coefficient(uts1,0) :: %
      not zero? coefficient(uts2,0) =>
        error "elt: second argument must have positive order"
      iCompose(uts1,uts2)

--% Integration

    if Coef has Algebra Fraction Integer then

      integrate(x:%,v:Variable(var)) == integrate x

--% Transcendental functions

      (uts1:%) ** (uts2:%) == exp(log(uts1) * uts2)

      if Coef has CommutativeRing then
```

```
    (uts:%) ** (r:RN) == cRationalPower(uts,r)

    exp uts == cExp uts
    log uts == cLog uts

    sin uts == cSin uts
    cos uts == cCos uts
    tan uts == cTan uts
    cot uts == cCot uts
    sec uts == cSec uts
    csc uts == cCsc uts

    asin uts == cAsin uts
    acos uts == cAcos uts
    atan uts == cAtan uts
    acot uts == cAcot uts
    asec uts == cAsec uts
    acsc uts == cAcsc uts

    sinh uts == cSinh uts
    cosh uts == cCosh uts
    tanh uts == cTanh uts
    coth uts == cCoth uts
    sech uts == cSech uts
    csch uts == cCsch uts

    asinh uts == cAsinh uts
    acosh uts == cAcosh uts
    atanh uts == cAtanh uts
    acoth uts == cAcoth uts
    asech uts == cAsech uts
    acsch uts == cAcsch uts

  else

    ZERO    : SG := "series must have constant coefficient zero"
    ONE     : SG := "series must have constant coefficient one"
    NPOWERS : SG := "series expansion has terms of negative degree"

    (uts:%) ** (r:RN) ==
--        not one? coefficient(uts,0) =>
      not (coefficient(uts,0) = 1) =>
        error "**: constant coefficient must be one"
      onePlusX : % := monomial(1,0) + monomial(1,1)
      ratPow := cPower(uts,r :: Coef)
      iCompose(ratPow,uts - 1)

    exp uts ==
      zero? coefficient(uts,0) =>
        expx := cExp monomial(1,1)
```

```
          iCompose(expx,uts)
        error concat("exp: ",ZERO)

      log uts ==
--        one? coefficient(uts,0) =>
        (coefficient(uts,0) = 1) =>
          log1PlusX := cLog(monomial(1,0) + monomial(1,1))
          iCompose(log1PlusX,uts - 1)
        error concat("log: ",ONE)

      sin uts ==
        zero? coefficient(uts,0) =>
          sinx := cSin monomial(1,1)
          iCompose(sinx,uts)
        error concat("sin: ",ZERO)

      cos uts ==
        zero? coefficient(uts,0) =>
          cosx := cCos monomial(1,1)
          iCompose(cosx,uts)
        error concat("cos: ",ZERO)

      tan uts ==
        zero? coefficient(uts,0) =>
          tanx := cTan monomial(1,1)
          iCompose(tanx,uts)
        error concat("tan: ",ZERO)

      cot uts ==
        zero? uts => error "cot: cot(0) is undefined"
        zero? coefficient(uts,0) => error concat("cot: ",NPOWERS)
        error concat("cot: ",ZERO)

      sec uts ==
        zero? coefficient(uts,0) =>
          secx := cSec monomial(1,1)
          iCompose(secx,uts)
        error concat("sec: ",ZERO)

      csc uts ==
        zero? uts => error "csc: csc(0) is undefined"
        zero? coefficient(uts,0) => error concat("csc: ",NPOWERS)
        error concat("csc: ",ZERO)

      asin uts ==
        zero? coefficient(uts,0) =>
          asinx := cAsin monomial(1,1)
          iCompose(asinx,uts)
        error concat("asin: ",ZERO)
```

```
atan uts ==
  zero? coefficient(uts,0) =>
    atanx := cAtan monomial(1,1)
    iCompose(atanx,uts)
  error concat("atan: ",ZERO)

acos z == error "acos: acos undefined on this coefficient domain"
acot z == error "acot: acot undefined on this coefficient domain"
asec z == error "asec: asec undefined on this coefficient domain"
acsc z == error "acsc: acsc undefined on this coefficient domain"

sinh uts ==
  zero? coefficient(uts,0) =>
    sinhx := cSinh monomial(1,1)
    iCompose(sinhx,uts)
  error concat("sinh: ",ZERO)

cosh uts ==
  zero? coefficient(uts,0) =>
    coshx := cCosh monomial(1,1)
    iCompose(coshx,uts)
  error concat("cosh: ",ZERO)

tanh uts ==
  zero? coefficient(uts,0) =>
    tanhx := cTanh monomial(1,1)
    iCompose(tanhx,uts)
  error concat("tanh: ",ZERO)

coth uts ==
  zero? uts => error "coth: coth(0) is undefined"
  zero? coefficient(uts,0) => error concat("coth: ",NPOWERS)
  error concat("coth: ",ZERO)

sech uts ==
  zero? coefficient(uts,0) =>
    sechx := cSech monomial(1,1)
    iCompose(sechx,uts)
  error concat("sech: ",ZERO)

csch uts ==
  zero? uts => error "csch: csch(0) is undefined"
  zero? coefficient(uts,0) => error concat("csch: ",NPOWERS)
  error concat("csch: ",ZERO)

asinh uts ==
  zero? coefficient(uts,0) =>
    asinhx := cAsinh monomial(1,1)
    iCompose(asinhx,uts)
  error concat("asinh: ",ZERO)
```

```
        atanh uts ==
          zero? coefficient(uts,0) =>
            atanhx := cAtanh monomial(1,1)
            iCompose(atanhx,uts)
          error concat("atanh: ",ZERO)

        acosh uts == error "acosh: acosh undefined on this coefficient domain"
        acoth uts == error "acoth: acoth undefined on this coefficient domain"
        asech uts == error "asech: asech undefined on this coefficient domain"
        acsch uts == error "acsch: acsch undefined on this coefficient domain"

    if Coef has Field then
      if Coef has Algebra Fraction Integer then

        (uts:%) ** (r:Coef) ==
--          not one? coefficient(uts,1) =>
          not (coefficient(uts,1) = 1) =>
            error "**: constant coefficient should be 1"
          cPower(uts,r)

--% OutputForms

    coerce(x:%): OUT ==
      count : NNI := _$streamCount$Lisp
      extend(x,count)
      seriesToOutputForm(getStream x,getRef x,variable x,center x,1)
```

— **SUTS.dotabb** —

```
"SUTS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUTS"]
"UTSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=UTSCAT"]
"SUTS" -> "UTSCAT"
```

20.23 domain SHDP SplitHomogeneousDirectProduct

— **SplitHomogeneousDirectProduct.input** —

```
)set break resume
)sys rm -f SplitHomogeneousDirectProduct.output
```

```
)spool SplitHomogeneousDirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SplitHomogeneousDirectProduct
--R SplitHomogeneousDirectProduct(dimtot: NonNegativeInteger,dim1: NonNegativeInteger,S: Orde
--R Abbreviation for SplitHomogeneousDirectProduct is SHDP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SHDP
--R
--R------------------------------- Operations --------------------------------
--R -? : % -> % if S has RING            1 : () -> % if S has MONOID
--R 0 : () -> % if S has CABMON          coerce : % -> Vector S
--R copy : % -> %                        directProduct : Vector S -> %
--R ?.? : (%,Integer) -> S               elt : (%,Integer,S) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List S                eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean      indices : % -> List Integer
--R map : ((S -> S),%) -> %              qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (PositiveInteger,%) -> % if S has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if S has CABMON
--R ?*? : (S,%) -> % if S has RING
--R ?*? : (%,S) -> % if S has RING
--R ?*? : (%,%) -> % if S has MONOID
--R ?*? : (Integer,%) -> % if S has RING
--R ?**? : (%,PositiveInteger) -> % if S has MONOID
--R ?**? : (%,NonNegativeInteger) -> % if S has MONOID
--R ?+? : (%,%) -> % if S has ABELSG
--R ?-? : (%,%) -> % if S has RING
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : % -> % if S has DIFRING and S has RING
--R ?^? : (%,PositiveInteger) -> % if S has MONOID
--R ?^? : (%,NonNegativeInteger) -> % if S has MONOID
--R abs : % -> % if S has ORDRING
```

```
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : S -> % if S has SETCAT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : % -> % if S has DIFRING and S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R index : PositiveInteger -> % if S has FINITE
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has MONOID
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(%,"failed") if S has MONOID
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
```

```
--R retract : % -> S if S has SETCAT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S,"failed") if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC INT and S has
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT and S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed") if S has CABMON
--R sup : (%,%) -> % if S has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R zero? : % -> Boolean if S has CABMON
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

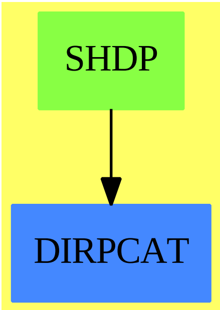
— SplitHomogeneousDirectProduct.help —

```
====================================================================
SplitHomogeneousDirectProduct examples
====================================================================

See Also:
o )show SplitHomogeneousDirectProduct
```

———-

20.23.1 SplitHomogeneousDirectProduct (SHDP)



See

⇒ "OrderedDirectProduct" (ODP) 16.13.1 on page 1778
⇒ "HomogeneousDirectProduct" (HDP) 9.9.1 on page 1138

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
#?	?*?	?**?	?+?	?-?
?/?	?<?	?<=?	?=?	?>?
?>=?	?^?	?~=?	-?	?.?

— **domain SHDP SplitHomogeneousDirectProduct** —

```
)abbrev domain SHDP SplitHomogeneousDirectProduct
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See: OrderedDirectProduct, HomogeneousDirectProduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
```

```
++ underlying ordered component type. The vectors are ordered as if
++ they were split into two blocks. The dim1 parameter specifies the
++ length of the first block. The ordering is lexicographic between
++ the blocks but acts like \spadtype{HomogeneousDirectProduct}
++ within each block. This type is a suitable third argument for
++ \spadtype{GeneralDistributedMultivariatePolynomial}.

SplitHomogeneousDirectProduct(dimtot,dim1,S) : T == C where
   NNI ==> NonNegativeInteger
   dim1,dimtot : NNI
   S         : OrderedAbelianMonoidSup

   T == DirectProductCategory(dimtot,S)
   C == DirectProduct(dimtot,S) add
        Rep:=Vector(S)
        lessThanRlex(v1:%,v2:%,low:NNI,high:NNI):Boolean ==
 -- reverse lexicographical ordering
          n1:S:=0
          n2:S:=0
          for i in low..high repeat
            n1:= n1+qelt(v1,i)
            n2:=n2+qelt(v2,i)
          n1<n2 => true
          n2<n1 => false
          for i in reverse(low..high) repeat
            if qelt(v2,i) < qelt(v1,i) then return true
            if qelt(v1,i) < qelt(v2,i) then return false
          false

        (v1:% < v2:%):Boolean ==
          lessThanRlex(v1,v2,1,dim1) => true
          for i in 1..dim1 repeat
                if qelt(v1,i) ^= qelt(v2,i) then return false
          lessThanRlex(v1,v2,dim1+1,dimtot)
```

— **SHDP.dotabb** —

```
"SHDP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SHDP"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"SHDP" -> "DIRPCAT"
```

20.24 domain SPLNODE SplittingNode

— SplittingNode.input —

```
)set break resume
)sys rm -f SplittingNode.output
)spool SplittingNode.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SplittingNode
--R SplittingNode(V: Join(SetCategory,Aggregate),C: Join(SetCategory,Aggregate))  is a domain constructo
--R Abbreviation for SplittingNode is SPLNODE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SPLNODE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R condition : % -> C                    construct : (V,List C) -> List %
--R construct : (V,C) -> %                construct : (V,C,Boolean) -> %
--R copy : % -> %                         empty : () -> %
--R empty? : % -> Boolean                 hash : % -> SingleInteger
--R latex : % -> String                   setCondition! : (%,C) -> %
--R setEmpty! : % -> %                    setStatus! : (%,Boolean) -> %
--R setValue! : (%,V) -> %                status : % -> Boolean
--R value : % -> V                        ?~=? : (%,%) -> Boolean
--R construct : List Record(val: V,tower: C) -> List %
--R construct : Record(val: V,tower: C) -> %
--R infLex? : (%,%,((V,V) -> Boolean),((C,C) -> Boolean)) -> Boolean
--R subNode? : (%,%,((C,C) -> Boolean)) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

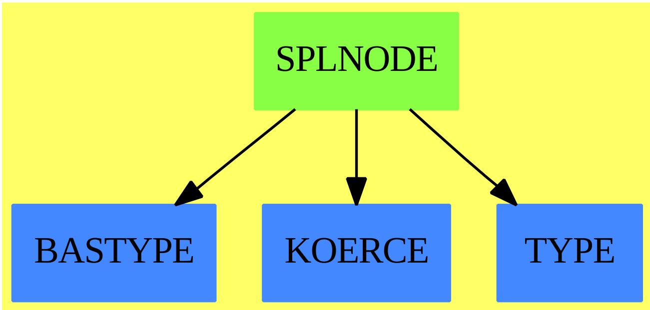
— SplittingNode.help —

```
====================================================================
SplittingNode examples
====================================================================

See Also:
o )show SplittingNode
```

———-

20.24.1 SplittingNode (SPLNODE)



See

⇒ "SplittingNode" (SPLNODE) 20.24.1 on page 2470

Exports:

coerce	condition	construct	copy	empty
empty?	hash	infLex?	latex	setCondition!
setEmpty!	setStatus!	setValue!	status	subNode?
value	?=?	?~=?		

— **domain SPLNODE SplittingNode** —

```
)abbrev domain SPLNODE SplittingNode
++ Author: Marc Moereno Maza
++ Date Created: 07/05/1996
++ Date Last Updated:  07/19/1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ References:
++ Description:
++ This domain exports a modest implementation for the
++ vertices of splitting trees. These vertices are called
++ here splitting nodes. Every of these nodes store 3 informations.
++ The first one is its value, that is the current expression
++ to evaluate. The second one is its condition, that is the
++ hypothesis under which the value has to be evaluated.
++ The last one is its status, that is a boolean flag
++ which is true iff the value is the result of its
```

```
++ evaluation under its condition. Two splitting vertices
++ are equal iff they have the sane values and the same
++ conditions (so their status do not matter).

SplittingNode(V,C) : Exports == Implementation where

  V:Join(SetCategory,Aggregate)
  C:Join(SetCategory,Aggregate)
  Z ==> Integer
  B ==> Boolean
  O ==> OutputForm
  VT ==> Record(val:V, tower:C)
  VTB ==> Record(val:V, tower:C, flag:B)

  Exports ==  SetCategory with

     empty : () -> %
       ++ \axiom{empty()} returns the same as
       ++ \axiom{[empty()$V,empty()$C,false]$%}
     empty? : % -> B
       ++ \axiom{empty?(n)} returns true iff the node n is \axiom{empty()$%}.
     value : % -> V
       ++ \axiom{value(n)} returns the value of the node n.
     condition : % -> C
       ++ \axiom{condition(n)} returns the condition of the node n.
     status : % -> B
       ++ \axiom{status(n)} returns the status of the node n.
     construct : (V,C,B) -> %
       ++ \axiom{construct(v,t,b)} returns the non-empty node with
       ++ value v, condition t and flag b
     construct : (V,C) -> %
       ++ \axiom{construct(v,t)} returns the same as
       ++ \axiom{construct(v,t,false)}
     construct : VT ->  %
       ++ \axiom{construct(vt)} returns the same as
       ++ \axiom{construct(vt.val,vt.tower)}
     construct : List VT ->  List %
       ++ \axiom{construct(lvt)} returns the same as
       ++ \axiom{[construct(vt.val,vt.tower) for vt in lvt]}
     construct : (V, List C) -> List %
       ++ \axiom{construct(v,lt)} returns the same as
       ++ \axiom{[construct(v,t) for t in lt]}
     copy : % -> %
       ++ \axiom{copy(n)} returns a copy of n.
     setValue! : (%,V) -> %
       ++ \axiom{setValue!(n,v)} returns n whose value
       ++ has been replaced by v if it is not
       ++ empty, else an error is produced.
     setCondition! : (%,C) -> %
       ++ \axiom{setCondition!(n,t)} returns n whose condition
```

```
      ++ has been replaced by t if it is not
      ++ empty, else an error is produced.
    setStatus!: (%,B) -> %
      ++ \axiom{setStatus!(n,b)} returns n whose status
      ++ has been replaced by b if it is not
      ++ empty, else an error is produced.
    setEmpty! : % -> %
      ++ \axiom{setEmpty!(n)} replaces n by \axiom{empty()$%}.
    infLex? : (%,%,(V,V) -> B,(C,C) -> B) -> B
      ++ \axiom{infLex?(n1,n2,o1,o2)} returns true iff
      ++ \axiom{o1(value(n1),value(n2))} or
      ++ \axiom{value(n1) = value(n2)} and
      ++ \axiom{o2(condition(n1),condition(n2))}.
    subNode? : (%,%,(C,C) -> B) -> B
      ++ \axiom{subNode?(n1,n2,o2)} returns true iff
      ++ \axiom{value(n1) = value(n2)} and
      ++ \axiom{o2(condition(n1),condition(n2))}

  Implementation == add

    Rep ==> VTB

    rep(n:%):Rep == n pretend Rep
    per(r:Rep):% == r pretend %

    empty() == per [empty()$V,empty()$C,false]$Rep
    empty?(n:%) == empty?((rep n).val)$V and  empty?((rep n).tower)$C
    value(n:%) == (rep n).val
    condition(n:%) == (rep n).tower
    status(n:%) == (rep n).flag
    construct(v:V,t:C,b:B) ==  per [v,t,b]$Rep
    construct(v:V,t:C) == [v,t,false]$%
    construct(vt:VT) == [vt.val,vt.tower]$%
    construct(lvt:List VT) == [[vt]$% for vt in lvt]
    construct(v:V,lt: List C) == [[v,t]$% for t in lt]
    copy(n:%) == per copy rep n
    setValue!(n:%,v:V) ==
       (rep n).val := v
       n
    setCondition!(n:%,t:C) ==
       (rep n).tower := t
       n
    setStatus!(n:%,b:B) ==
       (rep n).flag := b
       n
    setEmpty!(n:%) ==
       (rep n).val := empty()$V
       (rep n).tower := empty()$C
       n
    infLex?(n1,n2,o1,o2) ==
```

```
        o1((rep n1).val,(rep n2).val) => true
        (rep n1).val = (rep n2).val =>
           o2((rep n1).tower,(rep n2).tower)
        false
     subNode?(n1,n2,o2) ==
        (rep n1).val = (rep n2).val =>
           o2((rep n1).tower,(rep n2).tower)
        false
     -- sample() == empty()
     n1:% = n2:% ==
        (rep n1).val ~= (rep n2).val => false
        (rep n1).tower = (rep n2).tower
     n1:% ~= n2:% ==
        (rep n1).val = (rep n2).val => false
        (rep n1).tower ~= (rep n2).tower
     coerce(n:%):O ==
        l1,l2,l3,l : List O
        l1 := [message("value == "), ((rep n).val)::O]
        o1 : O := blankSeparate l1
        l2 := [message(" tower == "), ((rep n).tower)::O]
        o2 : O := blankSeparate l2
        if ((rep n).flag)
          then
            o3 := message(" closed == true")
          else
            o3 := message(" closed == false")
        l := [o1,o2,o3]
        bracket commaSeparate l
```

— **SPLNODE.dotabb** —

```
"SPLNODE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SPLNODE"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"SPLNODE" -> "BASTYPE"
"SPLNODE" -> "KOERCE"
"SPLNODE" -> "TYPE"
```

20.25 domain SPLTREE SplittingTree

— SplittingTree.input —

```
)set break resume
)sys rm -f SplittingTree.output
)spool SplittingTree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SplittingTree
--R SplittingTree(V: Join(SetCategory,Aggregate),C: Join(SetCategory,Aggregate))  is a domain
--R Abbreviation for SplittingTree is SPLTREE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SPLTREE
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %                conditions : % -> List C
--R construct : (V,C,V,List C) -> %       construct : (V,C,List %) -> %
--R copy : % -> %                         cyclic? : % -> Boolean
--R distance : (%,%) -> Integer           empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                  nodes : % -> List %
--R sample : () -> %                      updateStatus! : % -> %
--R value : % -> SplittingNode(V,C)
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if SplittingNode(V,C) has SETCAT
--R any? : ((SplittingNode(V,C) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if SplittingNode(V,C) has SETCAT
--R coerce : % -> OutputForm if SplittingNode(V,C) has SETCAT
--R construct : (V,C,List SplittingNode(V,C)) -> %
--R construct : SplittingNode(V,C) -> %
--R count : (SplittingNode(V,C),%) -> NonNegativeInteger if $ has finiteAggregate and Splitti
--R count : ((SplittingNode(V,C) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregat
--R ?.value : (%,value) -> SplittingNode(V,C)
--R eval : (%,List SplittingNode(V,C),List SplittingNode(V,C)) -> % if SplittingNode(V,C) has
--R eval : (%,SplittingNode(V,C),SplittingNode(V,C)) -> % if SplittingNode(V,C) has EVALAB SP
--R eval : (%,Equation SplittingNode(V,C)) -> % if SplittingNode(V,C) has EVALAB SPLNODE(V,C)
--R eval : (%,List Equation SplittingNode(V,C)) -> % if SplittingNode(V,C) has EVALAB SPLNODE
--R every? : ((SplittingNode(V,C) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extractSplittingLeaf : % -> Union(%,"failed")
--R hash : % -> SingleInteger if SplittingNode(V,C) has SETCAT
--R latex : % -> String if SplittingNode(V,C) has SETCAT
--R leaves : % -> List SplittingNode(V,C)
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((SplittingNode(V,C) -> SplittingNode(V,C)),%) -> %
```

```
--R map! : ((SplittingNode(V,C) -> SplittingNode(V,C)),%) -> % if $ has shallowlyMutable
--R member? : (SplittingNode(V,C),%) -> Boolean if $ has finiteAggregate and SplittingNode(V,C) has SETC
--R members : % -> List SplittingNode(V,C) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if SplittingNode(V,C) has SETCAT
--R nodeOf? : (SplittingNode(V,C),%) -> Boolean
--R parts : % -> List SplittingNode(V,C) if $ has finiteAggregate
--R remove : (SplittingNode(V,C),%) -> %
--R remove! : (SplittingNode(V,C),%) -> %
--R result : % -> List Record(val: V,tower: C)
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,SplittingNode(V,C)) -> SplittingNode(V,C) if $ has shallowlyMutable
--R setvalue! : (%,SplittingNode(V,C)) -> SplittingNode(V,C) if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R splitNodeOf! : (%,%,List SplittingNode(V,C),((C,C) -> Boolean)) -> %
--R splitNodeOf! : (%,%,List SplittingNode(V,C)) -> %
--R subNodeOf? : (SplittingNode(V,C),%,((C,C) -> Boolean)) -> Boolean
--R ?~=? : (%,%) -> Boolean if SplittingNode(V,C) has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

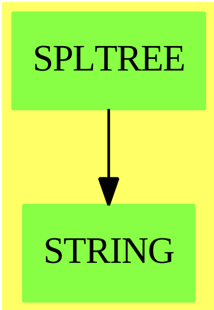
— SplittingTree.help —

```
====================================================================
SplittingTree examples
====================================================================

See Also:
o )show SplittingTree
```

———-

20.25.1 SplittingTree (SPLTREE)



See

⇒ "SplittingTree" (SPLTREE) 20.25.1 on page 2476

Exports:

any?	child?	children	coerce
conditions	construct	copy	count
cyclic?	distance	empty	empty?
eq?	eval	every?	extractSplittingLeaf
hash	latex	leaf?	leaves
less?	map	map!	member?
members	more?	node?	nodeOf?
nodes	parts	remove	remove!
result	sample	setchildren!	setelt
setvalue!	size?	splitNodeOf!	splitNodeOf!
subNodeOf?	updateStatus!	value	#?
?=?	?.value	?~=?	

— domain SPLTREE SplittingTree —

```
)abbrev domain SPLTREE SplittingTree
++ Author: Marc Moereno Maza
++ Date Created: 07/05/1996
++ Date Last Updated:  07/19/1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++      M. MORENO MAZA "Calculs de pgcd au-dessus des tours
++      d'extensions simples et resolution des systemes d'equations
++      algebriques" These, Universite P.etM. Curie, Paris, 1997.
++ Description:
++ This domain exports a modest implementation of splitting
++ trees. Spliiting trees are needed when the
```

```
++ evaluation of some quantity under some hypothesis
++ requires to split the hypothesis into sub-cases.
++ For instance by adding some new hypothesis on one
++ hand and its negation on another hand. The computations
++ are terminated is a splitting tree \axiom{a} when
++ \axiom{status(value(a))} is \axiom{true}. Thus,
++ if for the splitting tree \axiom{a} the flag
++ \axiom{status(value(a))} is \axiom{true}, then
++ \axiom{status(value(d))} is \axiom{true} for any
++ subtree \axiom{d} of \axiom{a}. This property
++ of splitting trees is called the termination
++ condition. If no vertex in a splitting tree \axiom{a}
++ is equal to another, \axiom{a} is said to satisfy
++ the no-duplicates condition. The splitting
++ tree \axiom{a} will satisfy this condition
++ if nodes are added to \axiom{a} by mean of
++ \axiom{splitNodeOf!} and if \axiom{construct}
++ is only used to create the root of \axiom{a}
++ with no children.

SplittingTree(V,C) : Exports == Implementation where

  V:Join(SetCategory,Aggregate)
  C:Join(SetCategory,Aggregate)
  B ==> Boolean
  O ==> OutputForm
  NNI ==> NonNegativeInteger
  VT ==> Record(val:V, tower:C)
  VTB ==> Record(val:V, tower:C, flag:B)
  S ==> SplittingNode(V,C)
  A ==> Record(root:S,subTrees:List(%))

  Exports ==  RecursiveAggregate(S) with
     shallowlyMutable
     finiteAggregate
     extractSplittingLeaf : % -> Union(%,"failed")
       ++ \axiom{extractSplittingLeaf(a)} returns the left
       ++ most leaf (as a tree) whose status is false
       ++ if any, else "failed" is returned.
     updateStatus! : % -> %
       ++ \axiom{updateStatus!(a)} returns a where the status
       ++ of the vertices are updated to satisfy
       ++ the "termination condition".
     construct : S -> %
       ++ \axiom{construct(s)} creates a splitting tree
       ++ with value (i.e. root vertex) given by
       ++ \axiom{s} and no children. Thus, if the
       ++ status of \axiom{s} is false, \axiom{[s]}
       ++ represents the starting point of the
       ++ evaluation \axiom{value(s)} under the
```

```
  ++ hypothesis \axiom{condition(s)}.
construct : (V,C, List %) -> %
  ++ \axiom{construct(v,t,la)} creates a splitting tree
  ++ with value (i.e. root vertex) given by
  ++ \axiom{[v,t]$S} and with \axiom{la} as
  ++ children list.
construct : (V,C,List S) -> %
  ++ \axiom{construct(v,t,ls)} creates a splitting tree
  ++ with value (i.e. root vertex) given by
  ++ \axiom{[v,t]$S} and with children list given by
  ++ \axiom{[[s]$% for s in ls]}.
construct : (V,C,V,List C) -> %
  ++ \axiom{construct(v1,t,v2,lt)} creates a splitting tree
  ++ with value (i.e. root vertex) given by
  ++ \axiom{[v,t]$S} and with children list given by
  ++ \axiom{[[[v,t]$S]$% for s in ls]}.
conditions : % -> List C
  ++ \axiom{conditions(a)} returns the list of the conditions
  ++ of the leaves of a
result : % -> List VT
  ++ \axiom{result(a)} where \axiom{ls} is the leaves list of \axiom{a}
  ++ returns \axiom{[[value(s),condition(s)]$VT for s in ls]}
  ++ if the computations are terminated in \axiom{a} else
  ++ an error is produced.
nodeOf? : (S,%) -> B
  ++ \axiom{nodeOf?(s,a)} returns true iff some node of \axiom{a}
  ++ is equal to \axiom{s}
subNodeOf? : (S,%,(C,C) -> B) -> B
  ++ \axiom{subNodeOf?(s,a,sub?)} returns true iff for some node
  ++ \axiom{n} in \axiom{a} we have \axiom{s = n} or
  ++ \axiom{status(n)} and \axiom{subNode?(s,n,sub?)}.
remove : (S,%) -> %
  ++ \axiom{remove(s,a)} returns the splitting tree obtained
  ++ from a by removing every sub-tree \axiom{b} such
  ++ that \axiom{value(b)} and \axiom{s} have the same
  ++ value, condition and status.
remove! : (S,%) -> %
  ++ \axiom{remove!(s,a)} replaces a by remove(s,a)
splitNodeOf! : (%,%,List(S)) -> %
  ++ \axiom{splitNodeOf!(l,a,ls)} returns \axiom{a} where the children
  ++ list of \axiom{l} has been set to
  ++ \axiom{[[s]$% for s in ls | not nodeOf?(s,a)]}.
  ++ Thus, if \axiom{l} is not a node of \axiom{a}, this
  ++ latter splitting tree is unchanged.
splitNodeOf! : (%,%,List(S),(C,C) -> B) -> %
  ++ \axiom{splitNodeOf!(l,a,ls,sub?)} returns \axiom{a} where the children
  ++ list of \axiom{l} has been set to
  ++ \axiom{[[s]$% for s in ls | not subNodeOf?(s,a,sub?)]}.
  ++ Thus, if \axiom{l} is not a node of \axiom{a}, this
  ++ latter splitting tree is unchanged.
```

```
Implementation == add

   Rep ==> A

   rep(n:%):Rep == n pretend Rep
   per(r:Rep):% == r pretend %

   construct(s:S) ==
      per [s,[]]$A
   construct(v:V,t:C,la:List(%)) ==
      per [[v,t]$S,la]$A
   construct(v:V,t:C,ls:List(S)) ==
      per [[v,t]$S,[[s]$% for s in ls]]$A
   construct(v1:V,t:C,v2:V,lt:List(C)) ==
      [v1,t,([v2,lt]$S)@(List S)]$%

   empty?(a:%) == empty?((rep a).root) and empty?((rep a).subTrees)
   empty() == [empty()$S]$%

   remove(s:S,a:%) ==
     empty? a => a
     (s = value(a)) and (status(s) = status(value(a))) => empty()$%
     la := children(a)
     lb : List % := []
     while (not empty? la) repeat
        lb := cons(remove(s,first la), lb)
        la := rest la
     lb := reverse remove(empty?,lb)
     [value(value(a)),condition(value(a)),lb]$%

   remove!(s:S,a:%) ==
     empty? a => a
     (s = value(a)) and (status(s) = status(value(a))) =>
       (rep a).root := empty()$S
       (rep a).subTrees := []
       a
     la := children(a)
     lb : List % := []
     while (not empty? la) repeat
        lb := cons(remove!(s,first la), lb)
        la := rest la
     lb := reverse remove(empty()$%,lb)
     setchildren!(a,lb)

   value(a:%) ==
      (rep a).root
   children(a:%) ==
      (rep a).subTrees
```

```
leaf?(a:%) ==
   empty? a => false
   empty? (rep a).subTrees
setchildren!(a:%,la:List(%)) ==
   (rep a).subTrees := la
   a
setvalue!(a:%,s:S) ==
   (rep a).root := s
   s
cyclic?(a:%) == false
map(foo:(S -> S),a:%) ==
  empty? a => a
  b : % := [foo(value(a))]$%
  leaf? a => b
  setchildren!(b,[map(foo,c) for c in children(a)])
map!(foo:(S -> S),a:%) ==
  empty? a => a
  setvalue!(a,foo(value(a)))
  leaf? a => a
  setchildren!(a,[map!(foo,c) for c in children(a)])
copy(a:%) ==
  map(copy,a)
eq?(a1:%,a2:%) ==
  error"in eq? from SPLTREE : la vache qui rit est-elle folle?"
nodes(a:%) ==
  empty? a => []
  leaf? a => [a]
  cons(a,concat([nodes(c) for c in children(a)]))
leaves(a:%) ==
  empty? a => []
  leaf? a => [value(a)]
  concat([leaves(c) for c in children(a)])
members(a:%) ==
  empty? a => []
  leaf? a => [value(a)]
  cons(value(a),concat([members(c) for c in children(a)]))
#(a:%) ==
  empty? a => 0$NNI
  leaf? a => 1$NNI
  reduce("+",[#c for c in children(a)],1$NNI)$(List NNI)
a1:% = a2:% ==
  empty? a1 => empty? a2
  empty? a2 => false
  leaf? a1 =>
    not leaf? a2 => false
    value(a1) =$S value(a2)
  leaf? a2 => false
  value(a1) ~=$S value(a2) => false
  children(a1) = children(a2)
-- sample() == [sample()$S]$%
```

```
localCoerce(a:%,k:NNI):O ==
  s : String
  if k = 1 then  s := "* " else s := "-> "
  for i in 2..k repeat s := concat("-+",s)$String
  ro : O := left(hconcat(message(s)$O,value(a)::O)$O)$O
  leaf? a => ro
  lo : List O := [localCoerce(c,k+1) for c in children(a)]
  lo := cons(ro,lo)
  vconcat(lo)$O
coerce(a:%):O ==
  empty? a => vconcat(message(" ")$O,message("* []")$O)
  vconcat(message(" ")$O,localCoerce(a,1))

extractSplittingLeaf(a:%) ==
  empty? a => "failed"::Union(%,"failed")
  status(value(a))$S => "failed"::Union(%,"failed")
  la := children(a)
  empty? la => a
  while (not empty? la) repeat
     esl := extractSplittingLeaf(first la)
     (esl case %) => return(esl)
     la := rest la
  "failed"::Union(%,"failed")

updateStatus!(a:%) ==
  la := children(a)
  (empty? la) or (status(value(a))$S) => a
  done := true
  while (not empty? la) and done repeat
     done := done and status(value(updateStatus! first la))
     la := rest la
  setStatus!(value(a),done)$S
  a

result(a:%) ==
  empty? a => []
  not status(value(a))$S =>
     error"in result from SLPTREE : mad cow!"
  ls : List S := leaves(a)
  [[value(s),condition(s)]$VT for s in ls]

conditions(a:%) ==
  empty? a => []
  ls : List S := leaves(a)
  [condition(s) for s in ls]

nodeOf?(s:S,a:%) ==
  empty? a => false
  s =$S value(a) => true
  la := children(a)
```

```
    while (not empty? la) and (not nodeOf?(s,first la)) repeat
       la := rest la
    not empty? la

  subNodeOf?(s:S,a:%,sub?:((C,C) -> B)) ==
    empty? a => false
    -- s =$S value(a) => true
    status(value(a)$%)$S and subNode?(s,value(a),sub?)$S => true
    la := children(a)
    while (not empty? la) and (not subNodeOf?(s,first la,sub?)) repeat
       la := rest la
    not empty? la

  splitNodeOf!(l:%,a:%,ls:List(S)) ==
    ln := removeDuplicates ls
    la : List % := []
    while not empty? ln repeat
       if not nodeOf?(first ln,a)
         then
           la := cons([first ln]$%, la)
       ln := rest ln
    la := reverse la
    setchildren!(l,la)$%
    if empty? la then (rep l).root := [empty()$V,empty()$C,true]$S
    updateStatus!(a)

  splitNodeOf!(l:%,a:%,ls:List(S),sub?:((C,C) -> B)) ==
    ln := removeDuplicates ls
    la : List % := []
    while not empty? ln repeat
       if not subNodeOf?(first ln,a,sub?)
         then
           la := cons([first ln]$%, la)
       ln := rest ln
    la := reverse la
    setchildren!(l,la)$%
    if empty? la then (rep l).root := [empty()$V,empty()$C,true]$S
    updateStatus!(a)
```

———-

— SPLTREE.dotabb —

```
"SPLTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SPLTREE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"SPLTREE" -> "STRING"
```

———-

20.26 domain SREGSET SquareFreeRegularTriangularSet

— SquareFreeRegularTriangularSet.input —

```
)set break resume
)sys rm -f SquareFreeRegularTriangularSet.output
)spool SquareFreeRegularTriangularSet.output
)set message test on
)set message auto off
)clear all
--S 1 of 23
R := Integer
--R
--R
--R   (1)  Integer
--R                                                                  Type: Domain
--E 1

--S 2 of 23
ls : List Symbol := [x,y,z,t]
--R
--R
--R   (2)  [x,y,z,t]
--R                                                             Type: List Symbol
--E 2

--S 3 of 23
V := OVAR(ls)
--R
--R
--R   (3)  OrderedVariableList [x,y,z,t]
--R                                                                  Type: Domain
--E 3

--S 4 of 23
E := IndexedExponents V
--R
--R
--R   (4)  IndexedExponents OrderedVariableList [x,y,z,t]
--R                                                                  Type: Domain
--E 4

--S 5 of 23
P := NSMP(R, V)
--R
--R
--R   (5)  NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--R                                                                  Type: Domain
```

```
--E 5

--S 6 of 23
x: P := 'x
--R
--R
--R   (6)  x
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 6

--S 7 of 23
y: P := 'y
--R
--R
--R   (7)  y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 7

--S 8 of 23
z: P := 'z
--R
--R
--R   (8)  z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 8

--S 9 of 23
t: P := 't
--R
--R
--R   (9)  t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 9

--S 10 of 23
ST := SREGSET(R,E,V,P)
--R
--R
--R   (10)
--R  SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [
--R  x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege
--R  r,OrderedVariableList [x,y,z,t]))
--R                                                                  Type: Domain
--E 10

--S 11 of 23
p1 := x ** 31 - x ** 6 - x - y
--R
--R
--R          31    6
```

```
--R   (11)  x   - x  - x - y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 11

--S 12 of 23
p2 := x ** 8  - z
--R
--R
--R          8
--R   (12)  x  - z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 12

--S 13 of 23
p3 := x ** 10 - t
--R
--R
--R          10
--R   (13)  x   - t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 13

--S 14 of 23
lp := [p1, p2, p3]
--R
--R
--R           31    6          8      10
--R   (14)  [x   - x  - x - y,x  - z,x   - t]
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 14

--S 15 of 23
zeroSetSplit(lp)$ST
--R
--R
--R            5    4      2     3     8     5    3    2   4               2
--R   (15)  [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z }]
--RType: List SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],Orde
--E 15

--S 16 of 23
zeroSetSplit(lp,false)$ST
--R
--R
--R   (16)
--R      5    4      2     3     8     5    3    2   4               2
--R   [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z },
--R      3      5            2    2
--R    {t  - 1,z  - t,t y + z ,z x  - t}, {t,z,y,x}]
--RType: List SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],Orde
```

```
--E 16

--S 17 of 23
T := REGSET(R,E,V,P)
--R
--R
--R   (17)
--R  RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],O
--R  rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV
--R  ariableList [x,y,z,t]))
--R                                                                  Type: Domain
--E 17

--S 18 of 23
lts := zeroSetSplit(lp,false)$T
--R
--R
--R   (18)
--R      5    4      2     3     8     5    3    2   4                2
--R   [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z },
--R      3     5          2     3         2
--R    {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}, {t,z,y,x}]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],Ord
--E 18

--S 19 of 23
ts := lts.2
--R
--R
--R           3      5          2     3         2
--R   (19)  {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}
--RType: RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedV
--E 19

--S 20 of 23
pol := select(ts,'y)$T
--R
--R
--R              2     3
--R   (20)  t z y  + 2z y + 1
--RType: Union(NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t]),...)
--E 20

--S 21 of 23
tower := collectUnder(ts,'y)$T
--R
--R
--R           3      5
--R   (21)  {t  - 1,z  - t}
--RType: RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedV
```

```
--E 21

--S 22 of 23
pack := RegularTriangularSetGcdPackage(R,E,V,P,T)
--R
--R
--R   (22)
--R  RegularTriangularSetGcdPackage(Integer,IndexedExponents OrderedVariableList [
--R  x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege
--R  r,OrderedVariableList [x,y,z,t]),RegularTriangularSet(Integer,IndexedExponent
--R  s OrderedVariableList [x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultiv
--R  ariatePolynomial(Integer,OrderedVariableList [x,y,z,t])))
--R                                                                 Type: Domain
--E 22

--S 23 of 23
toseSquareFreePart(pol,tower)$pack
--R
--R
--R                         2          3      5
--R   (23)  [[val= t y + z ,tower= {t  - 1,z  - t}]]
--RType: List Record(val: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t]),tower:
--E 23
)spool
)lisp (bye)
```

— SquareFreeRegularTriangularSet.help —

```
====================================================================
SquareFreeRegularTriangularSet examples
====================================================================

The SquareFreeRegularTriangularSet domain constructor implements
square-free regular triangular sets.  See the RegularTriangularSet
domain constructor for general regular triangular sets.  Let T be a
regular triangular set consisting of polynomials t1, ..., tm ordered
by increasing main variables.  The regular triangular set T is
square-free if T is empty or if t1, ..., tm-1 is square-free and if the
polynomial tm is square-free as a univariate polynomial with coefficients
in the tower of simple extensions associated with t1,...,tm-1.

The main interest of square-free regular triangular sets is that their
associated towers of simple extensions are product of fields.
Consequently, the saturated ideal of a square-free regular triangular
set is radical.  This property simplifies some of the operations
related to regular triangular sets.  However, building square-free
regular triangular sets is generally more expensive than building
```

```
general regular triangular sets.

As the RegularTriangularSet domain constructor, the
SquareFreeRegularTriangularSet domain constructor also implements a
method for solving polynomial systems by means of regular triangular
sets.  This is in fact the same method with some adaptations to take
into account the fact that the computed regular chains are
square-free.  Note that it is also possible to pass from a
decomposition into general regular triangular sets to a decomposition
into square-free regular triangular sets.  This conversion is used
internally by the LazardSetSolvingPackage package constructor.

N.B. When solving polynomial systems with the
SquareFreeRegularTriangularSet domain constructor or the
LazardSetSolvingPackage package constructor, decompositions have no
redundant components.  See also LexTriangularPackage and
ZeroDimensionalSolvePackage for the case of algebraic systems with a
finite number of (complex) solutions.

We shall explain now how to use the constructor SquareFreeRegularTriangularSet.

This constructor takes four arguments.  The first one, R, is the
coefficient ring of the polynomials; it must belong to the category
GcdDomain.  The second one, E, is the exponent monoid of the
polynomials; it must belong to the category OrderedAbelianMonoidSup.
the third one, V, is the ordered set of variables; it must belong to
the category OrderedSet.  The last one is the polynomial ring; it must
belong to the category RecursivePolynomialCategory(R,E,V).  The
abbreviation for SquareFreeRegularTriangularSet} is SREGSET.

Note that the way of understanding triangular decompositions
is detailed in the example of the RegularTriangularSet constructor.

Let us illustrate the use of this constructor with one example
(Donati-Traverso).  Define the coefficient ring.

  R := Integer
    Integer
                                 Type: Domain

Define the list of variables,

  ls : List Symbol := [x,y,z,t]
    [x,y,z,t]
                                 Type: List Symbol

and make it an ordered set;

  V := OVAR(ls)
    OrderedVariableList [x,y,z,t]
```

```
                                Type: Domain
```

then define the exponent monoid.

```
  E := IndexedExponents V
    IndexedExponents OrderedVariableList [x,y,z,t]
                                Type: Domain
```

Define the polynomial ring.

```
  P := NSMP(R, V)
    NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
                                Type: Domain
```

Let the variables be polynomial.

```
  x: P := 'x
    x
           Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])

  y: P := 'y
    y
           Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])

  z: P := 'z
    z
           Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])

  t: P := 't
    t
           Type: NewSparseMultivariatePolynomial(Integer,
                                  OrderedVariableList [x,y,z,t])
```

Now call the SquareFreeRegularTriangularSet domain constructor.

```
  ST := SREGSET(R,E,V,P)
  SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [
  x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege
  r,OrderedVariableList [x,y,z,t]))
                                 Type: Domain
```

Define a polynomial system.

```
  p1 := x ** 31 - x ** 6 - x - y
      31    6
     x   - x  - x - y
           Type: NewSparseMultivariatePolynomial(Integer,
```

```
                                   OrderedVariableList [x,y,z,t])

  p2 := x ** 8  - z
      8
     x  - z
           Type: NewSparseMultivariatePolynomial(Integer,
                                   OrderedVariableList [x,y,z,t])

  p3 := x ** 10 - t
      10
     x   - t
           Type: NewSparseMultivariatePolynomial(Integer,
                                   OrderedVariableList [x,y,z,t])

  lp := [p1, p2, p3]
       31    6          8      10
     [x   - x  - x - y,x  - z,x   - t]
           Type: List NewSparseMultivariatePolynomial(Integer,
                                        OrderedVariableList [x,y,z,t])
```

First of all, let us solve this system in the sense of Kalkbrener.

```
  zeroSetSplit(lp)$ST
        5    4      2     3     8     5    3    2   4               2
     [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z }]
        Type: List SquareFreeRegularTriangularSet(Integer,
                   IndexedExponents OrderedVariableList [x,y,z,t],
                   OrderedVariableList [x,y,z,t],
                   NewSparseMultivariatePolynomial(Integer,
                      OrderedVariableList [x,y,z,t]))
```

And now in the sense of Lazard (or Wu and other authors).

```
  zeroSetSplit(lp,false)$ST
      5    4      2     3     8     5    3    2   4               2
   [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z },
     3     5           2    2
    {t  - 1,z  - t,t y + z ,z x  - t}, {t,z,y,x}]
        Type: List SquareFreeRegularTriangularSet(Integer,
                   IndexedExponents OrderedVariableList [x,y,z,t],
                   OrderedVariableList [x,y,z,t],
                   NewSparseMultivariatePolynomial(Integer,
                      OrderedVariableList [x,y,z,t]))
```

Now to see the difference with the RegularTriangularSet domain constructor, we define:

```
  T := REGSET(R,E,V,P)
  RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],O
  rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV
```

```
  ariableList [x,y,z,t]))
                               Type: Domain
```

and compute:

```
  lts := zeroSetSplit(lp,false)$T
      5    4      2     3     8     5    3    2   4                2
   [{z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z },
      3      5          2     3         2
    {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}, {t,z,y,x}]
          Type: List RegularTriangularSet(Integer,
                       IndexedExponents OrderedVariableList [x,y,z,t],
                       OrderedVariableList [x,y,z,t],
                       NewSparseMultivariatePolynomial(Integer,
                          OrderedVariableList [x,y,z,t]))
```

If you look at the second set in both decompositions in the sense of Lazard, you will see that the polynomial with main variable y is not the same.

Let us understand what has happened.

We define:

```
  ts := lts.2
           3      5          2     3         2
   (19)  {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}
        Type: RegularTriangularSet(Integer,
                IndexedExponents OrderedVariableList [x,y,z,t],
                OrderedVariableList [x,y,z,t],
                NewSparseMultivariatePolynomial(Integer,
                   OrderedVariableList [x,y,z,t]))

  pol := select(ts,'y)$T
          2     3
     t z y  + 2z y + 1
      Type: Union(NewSparseMultivariatePolynomial(Integer,
                             OrderedVariableList [x,y,z,t]),...)

  tower := collectUnder(ts,'y)$T
       3      5
     {t  - 1,z  - t}
      Type: RegularTriangularSet(Integer,
              IndexedExponents OrderedVariableList [x,y,z,t],
              OrderedVariableList [x,y,z,t],
              NewSparseMultivariatePolynomial(Integer,
                 OrderedVariableList [x,y,z,t]))

  pack := RegularTriangularSetGcdPackage(R,E,V,P,T)
  RegularTriangularSetGcdPackage(Integer,IndexedExponents OrderedVariableList [
  x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege
```

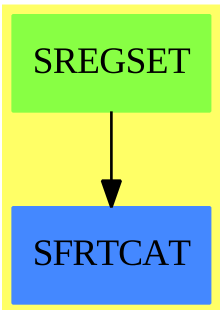
```
  r,OrderedVariableList [x,y,z,t]),RegularTriangularSet(Integer,IndexedExponent
  s OrderedVariableList [x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultiv
  ariatePolynomial(Integer,OrderedVariableList [x,y,z,t])))
                                   Type: Domain

Then we compute:

  toseSquareFreePart(pol,tower)$pack
                  2          3      5
    [[val= t y + z ,tower= {t  - 1,z  - t}]]
       Type: List Record(val: NewSparseMultivariatePolynomial(Integer,
                                 OrderedVariableList [x,y,z,t]),
                         tower: RegularTriangularSet(Integer,
                           IndexedExponents OrderedVariableList [x,y,z,t],
                           OrderedVariableList [x,y,z,t],
                           NewSparseMultivariatePolynomial(Integer,
                               OrderedVariableList [x,y,z,t])))

See Also:
o )help GcdDomain
o )help OrderedAbelianMonoidSup
o )help OrderedSet
o )help RecursivePolynomialCategory
o )help ZeroDimensionalSolvePackage
o )help LexTriangularPackage
o )help LazardSetSolvingPackage
o )help RegularTriangularSet
o )show SquareFreeRegularTriangularSet
```

20.26.1 SquareFreeRegularTriangularSet (SREGSET)



Exports:

algebraic?	algebraicCoefficients?
algebraicVariables	any?
augment	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	convert
construct	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	internalAugment
internalZeroSetSplit	intersect
invertible?	invertibleSet
invertibleElseSplit?	last
lastSubResultant	lastSubResultantElseSplit
less?	latex
mainVariable?	mainVariables
map	map!
member?	members
more?	mvar
normalized?	parts
preprocess	purelyAlgebraic?
purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce
reduceByQuasiMonic	reduced?
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
squareFreePart	stronglyReduce
stronglyReduced?	triangular?
trivialIdeal?	variables
zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?
?~=?	

— domain SREGSET SquareFreeRegularTriangularSet —

```
)abbrev domain SREGSET SquareFreeRegularTriangularSet
++ Author: Marc Moreno Maza
++ Date Created: 08/25/1998
++ Date Last Updated: 16/12/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++  [1] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
++ Description:
++ This domain provides an implementation of square-free regular chains.
++ Moreover, the operation zeroSetSplit
++ is an implementation of a new algorithm for solving polynomial systems by
++ means of regular chains.

SquareFreeRegularTriangularSet(R,E,V,P) : Exports == Implementation where

  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)
  N ==> NonNegativeInteger
  Z ==> Integer
  B ==> Boolean
  LP ==> List P
  PtoP ==> P -> P
  PS ==> GeneralPolynomialSet(R,E,V,P)
  PWT ==> Record(val : P, tower : $)
  BWT ==> Record(val : Boolean, tower : $)
  LpWT ==> Record(val : (List P), tower : $)
  Split ==> List $
  iprintpack ==> InternalPrintPackage()
  polsetpack ==> PolynomialSetUtilitiesPackage(R,E,V,P)
  quasicomppack ==> SquareFreeQuasiComponentPackage(R,E,V,P,$)
  regsetgcdpack ==> SquareFreeRegularTriangularSetGcdPackage(R,E,V,P,$)
  regsetdecomppack ==> SquareFreeRegularSetDecompositionPackage(R,E,V,P,$)

  Exports ==  SquareFreeRegularTriangularSetCategory(R,E,V,P) with

     internalAugment: (P,$,B,B,B,B,B) -> List $
       ++ \axiom{internalAugment(p,ts,b1,b2,b3,b4,b5)}
       ++ is an internal subroutine, exported only for developement.
     zeroSetSplit: (LP, B, B) -> Split
       ++ \axiom{zeroSetSplit(lp,clos?,info?)} has the same specifications as
```

```
      ++ zeroSetSplit from RegularTriangularSetCategory
      ++ from \spadtype{RegularTriangularSetCategory}
      ++ Moreover, if clos? then solves in the sense of the Zariski closure
      ++ else solves in the sense of the regular zeros. If \axiom{info?} then
      ++ do print messages during the computations.
    zeroSetSplit: (LP, B, B, B, B) -> Split
      ++ \axiom{zeroSetSplit(lp,b1,b2.b3,b4)}
      ++ is an internal subroutine, exported only for developement.
    internalZeroSetSplit: (LP, B, B, B) -> Split
      ++ \axiom{internalZeroSetSplit(lp,b1,b2,b3)}
      ++ is an internal subroutine, exported only for developement.
    pre_process: (LP, B, B) -> Record(val: LP, towers: Split)
      ++ \axiom{pre_process(lp,b1,b2)}
      ++ is an internal subroutine, exported only for developement.

  Implementation == add

    Rep ==> LP

    rep(s:$):Rep == s pretend Rep
    per(l:Rep):$ == l pretend $

    copy ts ==
      per(copy(rep(ts))$LP)
    empty() ==
      per([])
    empty?(ts:$) ==
      empty?(rep(ts))
    parts ts ==
      rep(ts)
    members ts ==
      rep(ts)
    map (f : PtoP, ts : $) : $ ==
      construct(map(f,rep(ts))$LP)$$
    map! (f : PtoP, ts : $) : $  ==
      construct(map!(f,rep(ts))$LP)$$
    member? (p,ts) ==
      member?(p,rep(ts))$LP
    unitIdealIfCan() ==
      "failed"::Union($,"failed")
    roughUnitIdeal? ts ==
      false
    coerce(ts:$) : OutputForm ==
      lp : List(P) := reverse(rep(ts))
      brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm
    mvar ts ==
      empty? ts => error "mvar$SREGSET: #1 is empty"
      mvar(first(rep(ts)))$P
    first ts ==
      empty? ts => "failed"::Union(P,"failed")
```

```
    first(rep(ts))::Union(P,"failed")
  last ts ==
    empty? ts => "failed"::Union(P,"failed")
    last(rep(ts))::Union(P,"failed")
  rest ts ==
    empty? ts => "failed"::Union($,"failed")
    per(rest(rep(ts)))::Union($,"failed")
  coerce(ts:$) : (List P) ==
    rep(ts)

  collectUpper (ts,v) ==
    empty? ts => ts
    lp := rep(ts)
    newlp : Rep := []
    while (not empty? lp) and (mvar(first(lp)) > v) repeat
      newlp := cons(first(lp),newlp)
      lp := rest lp
    per(reverse(newlp))

  collectUnder (ts,v) ==
    empty? ts => ts
    lp := rep(ts)
    while (not empty? lp) and (mvar(first(lp)) >= v) repeat
      lp := rest lp
    per(lp)

  construct(lp:List(P)) ==
    ts : $ := per([])
    empty? lp => ts
    lp := sort(infRittWu?,lp)
    while not empty? lp repeat
      eif := extendIfCan(ts,first(lp))
      not (eif case $) =>
        error"in construct : List P -> $  from SREGSET : bad #1"
      ts := eif::$
      lp := rest lp
    ts

  extendIfCan(ts:$,p:P) ==
    ground? p => "failed"::Union($,"failed")
    empty? ts =>
      p := squareFreePart primitivePart p
      (per([p]))::Union($,"failed")
    not (mvar(ts) < mvar(p)) => "failed"::Union($,"failed")
    invertible?(init(p),ts)@Boolean =>
      lts: Split := augment(p,ts)
      #lts ~= 1 => "failed"::Union($,"failed")
      (first lts)::Union($,"failed")
    "failed"::Union($,"failed")
```

```
removeZero(p:P, ts:$): P ==
  (ground? p) or (empty? ts) => p
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  if algebraic?(v,ts)
    then
      q := lazyPrem(p,select(ts,v)::P)
      zero? q => return q
      zero? removeZero(q,ts_v_-) => return 0
  empty? ts_v_- => p
  q: P := 0
  while positive? degree(p,v) repeat
     q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
     p := tail(p)
  q + removeZero(p,ts_v_-)

internalAugment(p:P,ts:$): $ ==
  -- ASSUME that adding p to ts DOES NOT require any split
  ground? p => error "in internalAugment$SREGSET: ground? #1"
  first(internalAugment(p,ts,false,false,false,false,false))

internalAugment(lp:List(P),ts:$): $ ==
  -- ASSUME that adding p to ts DOES NOT require any split
  empty? lp => ts
  internalAugment(rest lp, internalAugment(first lp, ts))

internalAugment(p:P,ts:$,rem?:B,red?:B,prim?:B,sqfr?:B,extend?:B): Split ==
  -- ASSUME p is not a constant
  -- ASSUME mvar(p) is not algebraic w.r.t. ts
  -- ASSUME init(p) invertible modulo ts
  -- if rem? then REDUCE p by remainder
  -- if prim? then REPLACE p by its main primitive part
  -- if sqfr? then FACTORIZE SQUARE FREE p over R
  -- if extend? DO NOT ASSUME every pol in ts_v_+ is invertible modulo ts
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  ts_v_+ := collectUpper(ts,v)
  if rem? then p := remainder(p,ts_v_-).polnum
  -- if rem? then p := reduceByQuasiMonic(p,ts_v_-)
  if red? then p := removeZero(p,ts_v_-)
  if prim? then p := mainPrimitivePart p
  lts: Split
  if sqfr?
    then
      lts: Split := []
      lsfp := squareFreeFactors(p)$polsetpack
      for f in lsfp repeat
        (ground? f) or (mvar(f) < v) => "leave"
        lpwt := squareFreePart(f,ts_v_-)
        for pwt in lpwt repeat
```

```
          sfp := pwt.val; us := pwt.tower
          lts := cons( per(cons(pwt.val, rep(pwt.tower))), lts)
    else
      lts: Split := [per(cons(p,rep(ts_v_-)))]
  extend? => extend(members(ts_v_+),lts)
  [per(concat(rep(ts_v_+),rep(us))) for us in lts]

augment(p:P,ts:$): List $ ==
  ground? p => error "in augment$SREGSET: ground? #1"
  algebraic?(mvar(p),ts) => error "in augment$SREGSET: bad #1"
  -- ASSUME init(p) invertible modulo ts
  -- DOES NOT ASSUME anything else.
  -- THUS reduction, mainPrimitivePart and squareFree are NEEDED
  internalAugment(p,ts,true,true,true,true,true)

extend(p:P,ts:$): List $ ==
  ground? p => error "in extend$SREGSET: ground? #1"
  v := mvar(p)
  not (mvar(ts) < mvar(p)) => error "in extend$SREGSET: bad #1"
  split: List($) := invertibleSet(init(p),ts)
  lts: List($) := []
  for us in split repeat
    lts := concat(augment(p,us),lts)
  lts

invertible?(p:P,ts:$): Boolean ==
  stoseInvertible?(p,ts)$regsetgcdpack

invertible?(p:P,ts:$): List BWT ==
  stoseInvertible?_sqfreg(p,ts)$regsetgcdpack

invertibleSet(p:P,ts:$): Split ==
  stoseInvertibleSet_sqfreg(p,ts)$regsetgcdpack

lastSubResultant(p1:P,p2:P,ts:$): List PWT ==
  stoseLastSubResultant(p1,p2,ts)$regsetgcdpack

squareFreePart(p:P, ts: $): List PWT ==
  stoseSquareFreePart(p,ts)$regsetgcdpack

intersect(p:P, ts: $): List($) == decompose([p], [ts], false, false)$regsetdecomppack

intersect(lp: LP, lts: List($)): List($) == decompose(lp, lts, false, false)$regsetdecom
   -- SOLVE in the regular zero sense
   -- and DO NOT PRINT info

decompose(p:P, ts: $): List($) == decompose([p], [ts], true, false)$regsetdecomppack

decompose(lp: LP, lts: List($)): List($) == decompose(lp, lts, true, false)$regsetdecomp
   -- SOLVE in the closure sense
```

```
       -- and DO NOT PRINT info

    zeroSetSplit(lp:List(P)) == zeroSetSplit(lp,true,false)
       -- by default SOLVE in the closure sense
       -- and DO NOT PRINT info

    zeroSetSplit(lp:List(P), clos?: B) == zeroSetSplit(lp,clos?, false)

    zeroSetSplit(lp:List(P), clos?: B, info?: B) ==
      -- if clos? then SOLVE in the closure sense
      -- if info? then PRINT info
      -- by default USE hash-tables
      -- and PREPROCESS the input system
      zeroSetSplit(lp,true,clos?,info?,true)

    zeroSetSplit(lp:List(P),hash?:B,clos?:B,info?:B,prep?:B) ==
      -- if hash? then USE hash-tables
      -- if info? then PRINT information
      -- if clos? then SOLVE in the closure sense
      -- if prep? then PREPROCESS the input system
      if hash?
        then
          s1, s2, s3, dom1, dom2, dom3: String
          e: String := empty()$String
          if info? then (s1,s2,s3) := ("w","g","i") else (s1,s2,s3) := (e,e,e)
          if info?
            then
              (dom1, dom2, dom3) := ("QCMPACK", "REGSETGCD: Gcd", "REGSETGCD: Inv Set")
            else
              (dom1, dom2, dom3) := (e,e,e)
          startTable!(s1,"W",dom1)$quasicomppack
          startTableGcd!(s2,"G",dom2)$regsetgcdpack
          startTableInvSet!(s3,"I",dom3)$regsetgcdpack
      lts := internalZeroSetSplit(lp,clos?,info?,prep?)
      if hash?
        then
          stopTable!()$quasicomppack
          stopTableGcd!()$regsetgcdpack
          stopTableInvSet!()$regsetgcdpack
      lts

    internalZeroSetSplit(lp:LP,clos?:B,info?:B,prep?:B) ==
      -- if info? then PRINT information
      -- if clos? then SOLVE in the closure sense
      -- if prep? then PREPROCESS the input system
      if prep?
        then
          pp := pre_process(lp,clos?,info?)
          lp := pp.val
          lts := pp.towers
```

```
          else
            ts: $ := [[]]
            lts := [ts]
        lp := remove(zero?, lp)
        any?(ground?, lp) => []
        empty? lp => lts
        empty? lts => lts
        lp := sort(infRittWu?,lp)
        clos? => decompose(lp,lts, clos?, info?)$regsetdecomppack
        -- IN DIM > 0 with clos? the following is not false ...
        for p in lp repeat
          lts := decompose([p],lts, clos?, info?)$regsetdecomppack
        lts

      largeSystem?(lp:LP): Boolean ==
        -- Gonnet and Gerdt and not Wu-Wang.2
        #lp > 16 => true
        #lp < 13 => false
        lts: List($) := []
        (#lp :: Z - numberOfVariables(lp,lts)$regsetdecomppack :: Z) > 3

      smallSystem?(lp:LP): Boolean ==
        -- neural, Vermeer, Liu, and not f-633 and not Hairer-2
        #lp < 5

      mediumSystem?(lp:LP): Boolean ==
        -- f-633 and not Hairer-2
        lts: List($) := []
        (numberOfVariables(lp,lts)$regsetdecomppack :: Z - #lp :: Z) < 2

--      lin?(p:P):Boolean == ground?(init(p)) and one?(mdeg(p))
      lin?(p:P):Boolean == ground?(init(p)) and (mdeg(p) = 1)

      pre_process(lp:LP,clos?:B,info?:B): Record(val: LP, towers: Split) ==
        -- if info? then PRINT information
        -- if clos? then SOLVE in the closure sense
        ts: $ := [[]];
        lts: Split := [ts]
        empty? lp => [lp,lts]
        lp1: List P := []
        lp2: List P := []
        for p in lp repeat
           ground? (tail p) => lp1 := cons(p, lp1)
           lp2 := cons(p, lp2)
        lts: Split := decompose(lp1,[ts],clos?,info?)$regsetdecomppack
        probablyZeroDim?(lp)$polsetpack =>
           largeSystem?(lp) => return [lp2,lts]
           if #lp > 7
             then
               -- Butcher (8,8) + Wu-Wang.2 (13,16)
```

```
          lp2 := crushedSet(lp2)$polsetpack
          lp2 := remove(zero?,lp2)
          any?(ground?,lp2) => return [lp2, lts]
          lp3 := [p for p in lp2 | lin?(p)]
          lp4 := [p for p in lp2 | not lin?(p)]
          if clos?
            then
              lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
            else
              lp4 := sort(infRittWu?,lp4)
              for p in lp4 repeat
                lts := decompose([p],lts, clos?, info?)$regsetdecomppack
          lp2 := lp3
        else
          lp2 := crushedSet(lp2)$polsetpack
          lp2 := remove(zero?,lp2)
          any?(ground?,lp2) => return [lp2, lts]
      if clos?
        then
          lts := decompose(lp2,lts, clos?, info?)$regsetdecomppack
        else
          lp2 := sort(infRittWu?,lp2)
          for p in lp2 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
      lp2 := []
      return [lp2,lts]
   smallSystem?(lp) => [lp2,lts]
   mediumSystem?(lp) => [crushedSet(lp2)$polsetpack,lts]
   lp3 := [p for p in lp2 | lin?(p)]
   lp4 := [p for p in lp2 | not lin?(p)]
   if clos?
     then
       lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
     else
       lp4 := sort(infRittWu?,lp4)
       for p in lp4 repeat
         lts := decompose([p],lts, clos?, info?)$regsetdecomppack
   if clos?
     then
       lts := decompose(lp3,lts, clos?, info?)$regsetdecomppack
     else
       lp3 := sort(infRittWu?,lp3)
       for p in lp3 repeat
         lts := decompose([p],lts, clos?, info?)$regsetdecomppack
   lp2 := []
   return [lp2,lts]
```

———-

— SREGSET.dotabb —

```
"SREGSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SREGSET"]
"SFRTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SFRTCAT"]
"SREGSET" -> "SFRTCAT"
```

————

20.27 domain SQMATRIX SquareMatrix

— SquareMatrix.input —

```
)set break resume
)sys rm -f SquareMatrix.output
)spool SquareMatrix.output
)set message test on
)set message auto off
)clear all
--S 1 of 6
)set expose add constructor SquareMatrix
--R
--I   SquareMatrix is now explicitly exposed in frame frame0
--E 1

--S 2 of 6
m := squareMatrix [ [1,-%i],[%i,4] ]
--R
--R
--R        +1   - %i+
--R   (1)  |        |
--R        +%i   4  +
--R                                     Type: SquareMatrix(2,Complex Integer)
--E 2

--S 3 of 6
m*m - m
--R
--R
--R        + 1   - 4%i+
--R   (2)  |          |
--R        +4%i   13  +
--R                                     Type: SquareMatrix(2,Complex Integer)
--E 3

--S 4 of 6
```

```
mm := squareMatrix [ [m, 1], [1-m, m**2] ]
--R
--R
--R        ++1   - %i+      +1  0+   +
--R        ||        |      |    |   |
--R        |+%i   4  +      +0  1+   |
--R   (3)  |                         |
--R        |+ 0    %i +  + 2   - 5%i+|
--R        ||         |  |          ||
--R        ++- %i  - 3+  +5%i   17  ++
--R                         Type: SquareMatrix(2,SquareMatrix(2,Complex Integer))
--E 4

--S 5 of 6
p := (x + m)**2
--R
--R
--R         2   + 2   - 2%i+    + 2   - 5%i+
--R   (4)  x  + |          |x + |          |
--R             +2%i    8  +    +5%i   17  +
--R                              Type: Polynomial SquareMatrix(2,Complex Integer)
--E 5

--S 6 of 6
p::SquareMatrix(2, ?)
--R
--R
--R        + 2                        +
--R        |x  + 2x + 2  - 2%i x - 5%i|
--R   (5)  |                          |
--R        |              2           |
--R        +2%i x + 5%i  x  + 8x + 17 +
--R                              Type: SquareMatrix(2,Polynomial Complex Integer)
--E 6
)spool
)lisp (bye)
```

— **SquareMatrix.help** —

```
====================================================================
SquareMatrix examples
====================================================================

The top level matrix type in Axiom is Matrix, which provides basic
arithmetic and linear algebra functions.  However, since the matrices
can be of any size it is not true that any pair can be added or
multiplied.  Thus Matrix has little algebraic structure.
```

```
Sometimes you want to use matrices as coefficients for polynomials or
in other algebraic contexts.  In this case, SquareMatrix should be
used.  The domain SquareMatrix(n,R) gives the ring of n by n square
matrices over R.

Since SquareMatrix is not normally exposed at the top level, you must
expose it before it can be used.

  )set expose add constructor SquareMatrix

Once SQMATRIX has been exposed, values can be created using the
squareMatrix function.

  m := squareMatrix [ [1,-%i],[%i,4] ]
    +1   - %i+
    |        |
    +%i   4  +
                        Type: SquareMatrix(2,Complex Integer)

The usual arithmetic operations are available.

  m*m - m
    + 1   - 4%i+
    |          |
    +4%i   13  +
                        Type: SquareMatrix(2,Complex Integer)

Square matrices can be used where ring elements are required.
For example, here is a matrix with matrix entries.

  mm := squareMatrix [ [m, 1], [1-m, m**2] ]
    ++1   - %i+      +1  0+   +
    ||        |      |    |   |
    |+%i   4  +      +0  1+   |
    |                         |
    |+ 0    %i +  + 2   - 5%i+|
    ||         |  |          ||
    ++- %i  - 3+  +5%i   17  ++
                        Type: SquareMatrix(2,SquareMatrix(2,Complex Integer))

Or you can construct a polynomial with  square matrix coefficients.

  p := (x + m)**2
     2   + 2   - 2%i+    + 2   - 5%i+
    x  + |          |x + |          |
         +2%i    8  +    +5%i   17  +
                        Type: Polynomial SquareMatrix(2,Complex Integer)

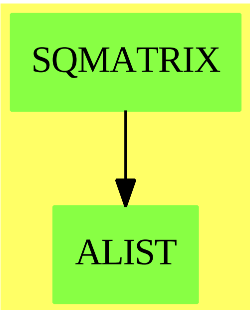
This value can be converted to a square matrix with polynomial coefficients.
```

```
  p::SquareMatrix(2, ?)
    + 2                     +
    |x  + 2x + 2  - 2%i x - 5%i|
    |                          |
    |              2           |
    +2%i x + 5%i  x  + 8x + 17 +
                        Type: SquareMatrix(2,Polynomial Complex Integer)

See Also:
o )help Matrix
o )show SquareMatrix
```

———–

20.27.1 SquareMatrix (SQMATRIX)



See

⇒ "IndexedMatrix" (IMATRIX) 10.12.1 on page 1204
⇒ "Matrix" (MATRIX) 14.7.1 on page 1586
⇒ "RectangularMatrix" (RMATRIX) 19.4.1 on page 2205

Exports:

0	1	antisymmetric?	any?	characteristic
coerce	column	convert	copy	count
D	determinant	diagonal	diagonal?	diagonalMatrix
diagonalProduct	differentiate	elt	empty	empty?
eq?	eval	every?	exquo	hash
inverse	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minordet	minRowIndex	more?
ncols	nrows	nullSpace	nullity	one?
parts	qelt	rank	recip	reducedSystem
retract	retractIfCan	row	rowEchelon	sample
scalarMatrix	size?	square?	squareMatrix	subtractIfCan
symmetric?	trace	transpose	zero?	#?
?*?	?**?	?+?	?-?	-?
?=?	?^?	?~=?	?/?	

— domain SQMATRIX SquareMatrix —

```
)abbrev domain SQMATRIX SquareMatrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: IndexedMatrix, Matrix, RectangularMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{SquareMatrix} is a matrix domain of square matrices, where the
++ number of rows (= number of columns) is a parameter of the type.

SquareMatrix(ndim,R): Exports == Implementation where
  ndim : NonNegativeInteger
  R    : Ring
  Row ==> DirectProduct(ndim,R)
  Col ==> DirectProduct(ndim,R)
  MATLIN ==> MatrixLinearAlgebraFunctions(R,Row,Col,$)

  Exports ==> Join(SquareMatrixCategory(ndim,R,Row,Col),_
                 CoercibleTo Matrix R) with

    transpose: $ -> $
      ++ \spad{transpose(m)} returns the transpose of the matrix m.
    squareMatrix: Matrix R -> $
      ++ \spad{squareMatrix(m)} converts a matrix of type \spadtype{Matrix}
      ++ to a matrix of type \spadtype{SquareMatrix}.
```

```
    coerce: $ -> Matrix R
      ++ \spad{coerce(m)} converts a matrix of type \spadtype{SquareMatrix}
      ++ to a matrix of type \spadtype{Matrix}.
--  symdecomp : $ -> Record(sym:$,antisym:$)
--    ++ \spad{symdecomp(m)} decomposes the matrix m as a sum of a symmetric
--    ++ matrix \spad{m1} and an antisymmetric matrix \spad{m2}. The object
--    ++ returned is the Record \spad{[m1,m2]}
--  if R has commutative("*") then
--    minorsVect: -> Vector(Union(R,"uncomputed")) --range: 1..2**n-1
--      ++ \spad{minorsVect(m)} returns a vector of the minors of the matrix m
    if R has commutative("*") then central
      ++ the elements of the Ring R, viewed as diagonal matrices, commute
      ++ with all matrices and, indeed, are the only matrices which commute
      ++ with all matrices.
    if R has commutative("*") and R has unitsKnown then unitsKnown
      ++ the invertible matrices are simply the matrices whose determinants
      ++ are units in the Ring R.
    if R has ConvertibleTo InputForm then ConvertibleTo InputForm

  Implementation ==> Matrix R add
    minr ==> minRowIndex
    maxr ==> maxRowIndex
    minc ==> minColIndex
    maxc ==> maxColIndex
    mini ==> minIndex
    maxi ==> maxIndex

    ZERO := scalarMatrix 0
    0    == ZERO
    ONE  := scalarMatrix 1
    1    == ONE

    characteristic() == characteristic()$R

    matrix(l: List List R) ==
      -- error check: this is a top level function
      #l ^= ndim => error "matrix: wrong number of rows"
      for ll in l repeat
        #ll ^= ndim => error "matrix: wrong number of columns"
      ans : Matrix R := new(ndim,ndim,0)
      for i in minr(ans)..maxr(ans) for ll in l repeat
        for j in minc(ans)..maxc(ans) for r in ll repeat
          qsetelt_!(ans,i,j,r)
      ans pretend $

    row(x,i)    == directProduct row(x pretend Matrix(R),i)
    column(x,j) == directProduct column(x pretend Matrix(R),j)
    coerce(x:$):OutputForm == coerce(x pretend Matrix R)$Matrix(R)

    scalarMatrix r == scalarMatrix(ndim,r)$Matrix(R) pretend $
```

```
diagonalMatrix l ==
  #l ^= ndim =>
    error "diagonalMatrix: wrong number of entries in list"
  diagonalMatrix(l)$Matrix(R) pretend $

coerce(x:$):Matrix(R) == copy(x pretend Matrix(R))

squareMatrix x ==
  (nrows(x) ^= ndim) or (ncols(x) ^= ndim) =>
    error "squareMatrix: matrix of bad dimensions"
  copy(x) pretend $

x:$ * v:Col ==
  directProduct((x pretend Matrix(R)) * (v :: Vector(R)))

v:Row * x:$ ==
  directProduct((v :: Vector(R)) * (x pretend Matrix(R)))

x:$ ** n:NonNegativeInteger ==
  ((x pretend Matrix(R)) ** n) pretend $

if R has commutative("*") then

  determinant x == determinant(x pretend Matrix(R))
  minordet x    == minordet(x pretend Matrix(R))

if R has EuclideanDomain then

  rowEchelon x == rowEchelon(x pretend Matrix(R)) pretend $

if R has IntegralDomain then

  rank x    == rank(x pretend Matrix(R))
  nullity x == nullity(x pretend Matrix(R))
  nullSpace x ==
    [directProduct c for c in nullSpace(x pretend Matrix(R))]

if R has Field then

  dimension() == (m * n) :: CardinalNumber

  inverse x ==
    (u := inverse(x pretend Matrix(R))) case "failed" => "failed"
    (u :: Matrix(R)) pretend $

  x:$ ** n:Integer ==
    ((x pretend Matrix(R)) ** n) pretend $

  recip x == inverse x
```

```
    if R has ConvertibleTo InputForm then
      convert(x:$):InputForm ==
         convert [convert("squareMatrix"::Symbol)@InputForm,
                  convert(x::Matrix(R))]$List(InputForm)
```

———-

— SQMATRIX.dotabb —

```
"SQMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SQMATRIX"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SQMATRIX" -> "ALIST"
```

———-

20.28 domain STACK Stack

— Stack.input —

```
)set break resume
)sys rm -f Stack.output
)spool Stack.output
)set message test on
)set message auto off
)clear all

--S 1 of 44
a:Stack INT:= stack [1,2,3,4,5]
--R
--R   (1)  [1,2,3,4,5]
--R                                                      Type: Stack Integer
--E 1

--S 2 of 44
pop! a
--R
--R   (2)  1
--R                                                    Type: PositiveInteger
--E 2

--S 3 of 44
a
```

```
--R
--R   (3)  [2,3,4,5]
--R                                                       Type: Stack Integer
--E 3

--S 4 of 44
extract! a
--R
--R   (4)  2
--R                                                       Type: PositiveInteger
--E 4

--S 5 of 44
a
--R
--R   (5)  [3,4,5]
--R                                                       Type: Stack Integer
--E 5

--S 6 of 44
push!(9,a)
--R
--R   (6)  9
--R                                                       Type: PositiveInteger
--E 6

--S 7 of 44
a
--R
--R   (7)  [9,3,4,5]
--R                                                       Type: Stack Integer
--E 7

--S 8 of 44
insert!(8,a)
--R
--R   (8)  [8,9,3,4,5]
--R                                                       Type: Stack Integer
--E 8

--S 9 of 44
a
--R
--R   (9)  [8,9,3,4,5]
--R                                                       Type: Stack Integer
--E 9

--S 10 of 44
inspect a
--R
```

```
--R   (10)  8
--R                                                 Type: PositiveInteger
--E 10

--S 11 of 44
empty? a
--R
--R   (11)  false
--R                                                         Type: Boolean
--E 11

--S 12 of 44
top a
--R
--R   (12)  8
--R                                                 Type: PositiveInteger
--E 12

--S 13 of 44
depth a
--R
--R   (13)  5
--R                                                 Type: PositiveInteger
--E 13

--S 14 of 44
#a
--R
--R   (14)  5
--R                                                 Type: PositiveInteger
--E 14

--S 15 of 44
less?(a,9)
--R
--R   (15)  true
--R                                                         Type: Boolean
--E 15

--S 16 of 44
more?(a,9)
--R
--R   (16)  false
--R                                                         Type: Boolean
--E 16

--S 17 of 44
size?(a,#a)
--R
--R   (17)  true
```

```
--R                                                           Type: Boolean
--E 17

--S 18 of 44
size?(a,9)
--R
--R   (18)  false
--R                                                           Type: Boolean
--E 18

--S 19 of 44
parts a
--R
--R   (19)  [8,9,3,4,5]
--R                                                      Type: List Integer
--E 19

--S 20 of 44
bag([1,2,3,4,5])$Stack(INT)
--R
--R   (20)  [5,4,3,2,1]
--R                                                     Type: Stack Integer
--E 20

--S 21 of 44
b:=empty()$(Stack INT)
--R
--R   (21)  []
--R                                                     Type: Stack Integer
--E 21

--S 22 of 44
empty? b
--R
--R   (22)  true
--R                                                           Type: Boolean
--E 22

--S 23 of 44
sample()$Stack(INT)
--R
--R   (23)  []
--R                                                     Type: Stack Integer
--E 23

--S 24 of 44
c:=copy a
--R
--R   (24)  [8,9,3,4,5]
--R                                                     Type: Stack Integer
```

```
--E 24

--S 25 of 44
eq?(a,c)
--R
--R   (25)  false
--R                                                            Type: Boolean
--E 25

--S 26 of 44
eq?(a,a)
--R
--R   (26)  true
--R                                                            Type: Boolean
--E 26

--S 27 of 44
(a=c)@Boolean
--R
--R   (27)  true
--R                                                            Type: Boolean
--E 27

--S 28 of 44
(a=a)@Boolean
--R
--R   (28)  true
--R                                                            Type: Boolean
--E 28

--S 29 of 44
a~=c
--R
--R   (29)  false
--R                                                            Type: Boolean
--E 29

--S 30 of 44
any?(x+->(x=4),a)
--R
--R   (30)  true
--R                                                            Type: Boolean
--E 30

--S 31 of 44
any?(x+->(x=11),a)
--R
--R   (31)  false
--R                                                            Type: Boolean
--E 31
```

```
--S 32 of 44
every?(x+->(x=11),a)
--R
--R   (32)  false
--R                                                            Type: Boolean
--E 32

--S 33 of 44
count(4,a)
--R
--R   (33)  1
--R                                                    Type: PositiveInteger
--E 33

--S 34 of 44
count(x+->(x>2),a)
--R
--R   (34)  5
--R                                                    Type: PositiveInteger
--E 34

--S 35 of 44
map(x+->x+10,a)
--R
--R   (35)  [18,19,13,14,15]
--R                                                      Type: Stack Integer
--E 35

--S 36 of 44
a
--R
--R   (36)  [8,9,3,4,5]
--R                                                      Type: Stack Integer
--E 36

--S 37 of 44
map!(x+->x+10,a)
--R
--R   (37)  [18,19,13,14,15]
--R                                                      Type: Stack Integer
--E 37

--S 38 of 44
a
--R
--R   (38)  [18,19,13,14,15]
--R                                                      Type: Stack Integer
--E 38
```

```
--S 39 of 44
members a
--R
--R   (39)  [18,19,13,14,15]
--R                                                          Type: List Integer
--E 39

--S 40 of 44
member?(14,a)
--R
--R   (40)  true
--R                                                               Type: Boolean
--E 40

--S 41 of 44
coerce a
--R
--R
--R   (41)  [18,19,13,14,15]
--R                                                            Type: OutputForm
--E 41

--S 42 of 44
hash a
--R
--R
--I   (42)  4999539
--R                                                         Type: SingleInteger
--E 42

--S 43 of 44
latex a
--R
--R
--R   (43)  "\mbox{\bf Unimplemented}"
--R                                                                Type: String
--E 43

--S 44 of 44
)show Stack
--R Stack S: SetCategory  is a domain constructor
--R Abbreviation for Stack is STACK
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for STACK
--R
--R------------------------------ Operations --------------------------------
--R bag : List S -> %                     copy : % -> %
--R depth : % -> NonNegativeInteger       empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
```

```
--R inspect : % -> S                    map : ((S -> S),%) -> %
--R pop! : % -> S                       push! : (S,%) -> S
--R sample : () -> %                    stack : List S -> %
--R top : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 44
)spool
)lisp (bye)
```

———-

— **Stack.help** —

```
====================================================================
Stack examples
====================================================================

A Stack object is represented as a list ordered by last-in, first-out.
It operates like a pile of books, where the "next" book is the one
on the top of the pile.

Here we create a stack of integers from a list. Notice that the
order in the list is the order in the stack.

   a:Stack INT:= stack [1,2,3,4,5]
      [1,2,3,4,5]
```

We can remove the top of the stack using pop!:

```
pop! a
   1
```

Notice that the use of pop! is destructive (destructive operations
in Axiom usually end with ! to indicate that the underylying data
structure is changed).

```
a
   [2,3,4,5]
```

The extract! operation is another name for the pop! operation and
has the same effect. This operation treats the stack as a BagAggregate:

```
extract! a
   2
```

and you can see that it also has destructively modified the stack:

```
a
   [3,4,5]
```

Next we push a new element on top of the stack:

```
push!(9,a)
   9
```

Again, the push! operation is destructive so the stack is changed:

```
a
   [9,2,3,4,5]
```

Another name for push! is insert!, which treats the stack as a BagAggregate:

```
insert!(8,a)
   [8,9,3,4,5]
```

and it modifies the stack:

```
a
   [8,9,3,4,5]
```

The inspect function returns the top of the stack without modification,
viewed as a BagAggregate:

```
inspect a
   8
```

The empty? operation returns true only if there are no element on the stack, otherwise it returns false:

```
empty? a
   false
```

The top operation returns the top of stack without modification, viewed as a Stack:

```
top a
   8
```

The depth operation returns the number of elements on the stack:

```
depth a
   5
```

which is the same as the # (length) operation:

```
#a
    5
```

The less? predicate will compare the stack length to an integer:

```
less?(a,9)
     true
```

The more? predicate will compare the stack length to an integer:

```
more?(a,9)
     false
```

The size? operation will compare the stack length to an integer:

```
size?(a,#a)
     true
```

and since the last computation must alwasy be true we try:

```
size?(a,9)
     false
```

The parts function will return the stack as a list of its elements:

```
parts a
     [8,9,3,4,5]
```

If we have a BagAggregate of elements we can use it to construct a stack. Notice that the elements are pushed in reverse order:

```
bag([1,2,3,4,5])$Stack(INT)
```

```
        [5,4,3,2,1]
```

The empty function will construct an empty stack of a given type:

```
   b:=empty()$(Stack INT)
        []
```

and the empty? predicate allows us to find out if a stack is empty:

```
   empty? b
        true
```

The sample function returns a sample, empty stack:

```
   sample()$Stack(INT)
        []
```

We can copy a stack and it does not share storage so subsequent modifications of the original stack will not affect the copy:

```
   c:=copy a
        [8,9,3,4,5]
```

The eq? function is only true if the lists are the same reference, so even though c is a copy of a, they are not the same:

```
   eq?(a,c)
        false
```

However, a clearly shares a reference with itself:

```
   eq?(a,a)
        true
```

But we can compare a and c for equality:

```
   (a=c)@Boolean
        true
```

and clearly a is equal to itself:

```
   (a=a)@Boolean
        true
```

and since a and c are equal, they are clearly NOT not-equal:

```
   a~=c
        false
```

We can use the any? function to see if a predicate is true for any element:

```
any?(x+->(x=4),a)
     true
```

or false for every element:

```
any?(x+->(x=11),a)
     false
```

We can use the every? function to check every element satisfies a predicate:

```
every?(x+->(x=11),a)
     false
```

We can count the elements that are equal to an argument of this type:

```
count(4,a)
     1
```

or we can count against a boolean function:

```
count(x+->(x>2),a)
     5
```

You can also map a function over every element, returning a new stack:

```
map(x+->x+10,a)
     [18,19,13,14,15]
```

Notice that the orignal stack is unchanged:

```
a
     [8,9,3,4,5]
```

You can use map! to map a function over every element and change
the original stack since map! is destructive:

```
map!(x+->x+10,a)
    [18,19,13,14,15]
```

Notice that the orignal stack has been changed:

```
a
    [18,19,13,14,15]
```

The member function can also get the element of the stack as a list:

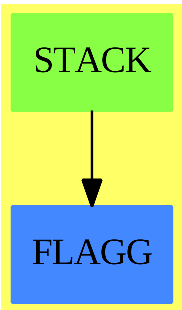
```
members a
    [18,19,13,14,15]
```

```
and using member? we can test if the stack holds a given element:

   member?(14,a)
       true

See Also:
o )show Stack
o )show ArrayStack
o )show Queue
o )show Dequeue
o )show Heap
o )show BagAggregate
```

20.28.1 Stack (STACK)



See

⇒ “ArrayStack” (ASTACK) 2.10.1 on page 65
⇒ “Queue” (QUEUE) 18.5.1 on page 2143
⇒ “Dequeue” (DEQUEUE) 5.5.1 on page 497
⇒ “Heap” (HEAP) 9.2.1 on page 1100

Exports:

any?	bag	coerce	copy	count
depth	empty	empty?	eq?	eval
every?	extract!	hash	insert!	inspect
latex	less?	map	map!	member?
members	more?	parts	pop!	push!
sample	size?	stack	top	#?
?=?	?~=?			

— domain STACK Stack —

```
)abbrev domain STACK Stack
++ Author: Michael Monagan, Stephen Watt, Timothy Daly
++ Date Created:June 86 and July 87
++ Date Last Updated:Feb 09
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Linked List implementation of a Stack
--% Dequeue and Heap data types

Stack(S:SetCategory): StackAggregate S with
    stack: List S -> %
      ++ stack([x,y,...,z]) creates a stack with first (top)
      ++ element x, second element y,...,and last element z.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]

 -- Inherited Signatures repeated for examples documentation

    pop_! : % -> S
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X pop! a
      ++X a
    extract_! : % -> S
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X extract! a
      ++X a
    push_! : (S,%) -> S
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X push!(9,a)
      ++X a
    insert_! : (S,%) -> %
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X insert!(8,a)
      ++X a
    inspect : % -> S
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X inspect a
    top : % -> S
      ++
```

```
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X top a
    depth : % -> NonNegativeInteger
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X depth a
    less? : (%,NonNegativeInteger) -> Boolean
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X less?(a,9)
    more? : (%,NonNegativeInteger) -> Boolean
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X more?(a,9)
    size? : (%,NonNegativeInteger) -> Boolean
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X size?(a,5)
    bag : List S -> %
      ++
      ++X bag([1,2,3,4,5])$Stack(INT)
    empty? : % -> Boolean
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X empty? a
    empty : () -> %
      ++
      ++X b:=empty()$(Stack INT)
    sample : () -> %
      ++
      ++X sample()$Stack(INT)
    copy : % -> %
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X copy a
    eq? : (%,%) -> Boolean
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X b:=copy a
      ++X eq?(a,b)
    map :  ((S -> S),%) -> %
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X map(x+->x+10,a)
      ++X a
    if $ has shallowlyMutable then
      map! :  ((S -> S),%) -> %
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X map!(x+->x+10,a)
```

```
      ++X a
    if S has SetCategory then
      latex : % -> String
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X latex a
      hash : % -> SingleInteger
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X hash a
      coerce : % -> OutputForm
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X coerce a
      "=": (%,%) -> Boolean
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X b:Stack INT:= stack [1,2,3,4,5]
        ++X (a=b)@Boolean
      "~=" : (%,%) -> Boolean
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X b:=copy a
        ++X (a~=b)
    if % has finiteAggregate then
      every? : ((S -> Boolean),%) -> Boolean
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X every?(x+->(x=4),a)
      any? : ((S -> Boolean),%) -> Boolean
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X any?(x+->(x=4),a)
      count :  ((S -> Boolean),%) -> NonNegativeInteger
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X count(x+->(x>2),a)
      _# : % -> NonNegativeInteger
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X #a
      parts : % -> List S
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X parts a
      members : % -> List S
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X members a
    if % has finiteAggregate and S has SetCategory then
```

```
      member? : (S,%) -> Boolean
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X member?(3,a)
      count : (S,%) -> NonNegativeInteger
        ++
        ++X a:Stack INT:= stack [1,2,3,4,5]
        ++X count(4,a)

  == add
    Rep := Reference List S
    s = t == deref s = deref t
    coerce(d:%): OutputForm == bracket [e::OutputForm for e in deref d]
    copy s == ref copy deref s
    depth s == # deref s
    # s == depth s
    pop_! (s:%):S ==
        empty? s => error "empty stack"
        e := first deref s
        setref(s,rest deref s)
        e
    extract_! (s:%):S == pop_! s
    top (s:%):S ==
        empty? s => error "empty stack"
        first deref s
    inspect s == top s
    push_!(e,s) == (setref(s,cons(e,deref s));e)
    insert_!(e:S,s:%):% == (push_!(e,s);s)
    empty() == ref nil()$List(S)
    empty? s == null deref s
    stack s == ref copy s
    parts s == copy deref s
    map(f,s) == ref map(f,deref s)
    map!(f,s) == ref map!(f,deref s)
```

— **STACK.dotabb** —

```
"STACK" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STACK"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"STACK" -> "FLAGG"
```

20.29 domain SD StochasticDifferential

— **StochasticDifferential.input** —

```
)set break resume
)sys rm -f StochasticDifferential.output
)spool StochasticDifferential.output
)set message test on
)set message auto off
)clear all

--S 1 of 12
dt := introduce!(t,dt)
--R
--R
--R   (1)  dt
--R                                  Type: Union(BasicStochasticDifferential,...)
--E 1

--S 2 of 12
dX := introduce!(X,dX)
--R
--R
--R   (2)  dX
--R                                  Type: Union(BasicStochasticDifferential,...)
--E 2

--S 3 of 12
dY := introduce!(Y,dY)
--R
--R
--R   (3)  dY
--R                                  Type: Union(BasicStochasticDifferential,...)
--E 3

--S 4 of 12
copyBSD()
--R
--R
--R   (4)  [dX,dY,dt]
--R                                        Type: List BasicStochasticDifferential
--E 4

--S 5 of 12
copyIto()
--R
--R
--R   (5)  table(t= dt,Y= dY,X= dX)
```

```
--R                             Type: Table(Symbol,BasicStochasticDifferential)
--E 5

--S 6 of 12
copyQuadVar()             -- display of multiplication table
--R
--R
--R   (6)  table()
--R   Type: Table(StochasticDifferential Integer,StochasticDifferential Integer)
--E 6

--S 7 of 12
statusIto()
--R
--R
--R        +B S D   :  dX  dY  dt+
--R        |                     |
--R        |drift   :  ?   ?   ? |
--R        |                     |
--R        |  *                  |
--R   (7)  |                     |
--R        |  dX    :  ?   ?   ? |
--R        |                     |
--R        |  dY    :  ?   ?   ? |
--R        |                     |
--R        +  dt    :  ?   ?   ? +
--R                                                         Type: OutputForm
--E 7

--S 8 of 12
copyDrift()       -- display of drift list
--R
--R
--R   (8)  table()
--R   Type: Table(StochasticDifferential Integer,StochasticDifferential Integer)
--E 8

--S 9 of 12
nbsd := #copyBSD()
--R
--R
--R   (9)  3
--R                                                    Type: PositiveInteger
--E 9

--S 10 of 12
ItoMultArray:ARRAY2(SD INT) :=new(nbsd,nbsd,0$SD(INT))
--R
--R
--R         +0  0  0+
```

```
--R          |        |
--R   (10)   |0  0  0|
--R          |        |
--R          +0  0  0+
--R                        Type: TwoDimensionalArray StochasticDifferential Integer
--E 10

--S 11 of 12
ItoMultArray
--R
--R
--R          +0  0  0+
--R          |        |
--R   (11)   |0  0  0|
--R          |        |
--R          +0  0  0+
--R                        Type: TwoDimensionalArray StochasticDifferential Integer
--E 11

--S 12 of 12
statusIto()
--R
--R
--R          +B S D   :  dX  dY  dt+
--R          |                      |
--R          |drift   :  ?   ?   ? |
--R          |                      |
--R          |  *                   |
--R   (12)   |                      |
--R          |  dX    :  ?   ?   ? |
--R          |                      |
--R          |  dY    :  ?   ?   ? |
--R          |                      |
--R          +  dt    :  ?   ?   ? +
--R                                                               Type: OutputForm
--E 12

)spool
)lisp (bye)
```

— **StochasticDifferential.help** —

```
====================================================================
StochasticDifferential examples
====================================================================
A basic implementation of StochasticDifferential(R) using the
associated domain BasicStochasticDifferential in the underlying
```

```
representation as sparse multivariate polynomials. The domain is
a module over Expression(R), and is a ring without identity
(AXIOM term is "Rng"). Note that separate instances, for example
using R=Integer and R=Float, have different hidden structure
(multiplication and drift tables).

dt := introduce!(t,dt)
   dt

dX := introduce!(X,dX)
   dX

dY := introduce!(Y,dY)
   dY

copyBSD()
   [dX,dY,dt]

copyIto()
   table(t= dt,Y= dY,X= dX)

copyQuadVar()             -- display of multiplication table
   table()

statusIto()
   +B S D   :  dX  dY  dt+
   |                     |
   |drift   :  ?   ?   ? |
   |                     |
   |  *                  |
   |                     |
   |  dX    :  ?   ?   ? |
   |                     |
   |  dY    :  ?   ?   ? |
   |                     |
   +  dt    :  ?   ?   ? +

copyDrift()       -- display of drift list
   table()

nbsd := #copyBSD()
   3

ItoMultArray:ARRAY2(SD INT) :=new(nbsd,nbsd,0$SD(INT))
   +0  0  0+
   |       |
   |0  0  0|
   |       |
   +0  0  0+
```

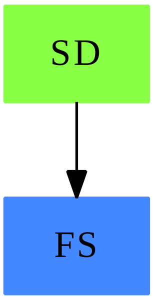
```
ItoMultArray
   +0  0  0+
   |       |
   |0  0  0|
   |       |
   +0  0  0+

statusIto()
   +B S D   :  dX  dY  dt+
   |                     |
   |drift   :  ?   ?   ? |
   |                     |
   |  *                  |
   |                     |
   |  dX    :  ?   ?   ? |
   |                     |
   |  dY    :  ?   ?   ? |
   |                     |
   +  dt    :  ?   ?   ? +


See Also:
o )help BasicStochasticDifferential
o )show StochasticDifferential
```

———–

20.29.1 StochasticDifferential (SD)



See

⇐ "BasicStochasticDifferential" (BSD) 3.6.1 on page 268

Exports:

0	coerce	copyDrift	copyQuadVar
drift	hash	latex	sample
statusIto	uncorrelated?	zero?	alterDrift!
alterQuadVar!	coefficient	coerce	equation
freeOf?	listSD	retract	retractIfCan
subtractIfCan	?*?	?**?	?+?
?-?	-?	?/?	?=?
?^?	?~=?		

— domain SD StochasticDifferential —

```
)abbrev domain SD StochasticDifferential
++ Author: Wilfrid S. Kendall
++ Last Last Updated: July 26, 1999
++ Related Domains: BasicStochasticDifferential
++ AMS Classifications:
++ Keywords: stochastic differential, semimartingale.
++ References: Ito (1975), Kendall (1991a,b; 1993a,b).
++ Description:
++ A basic implementation of StochasticDifferential(R) using the
++ associated domain BasicStochasticDifferential in the underlying
++ representation as sparse multivariate polynomials. The domain is
++ a module over Expression(R), and is a ring without identity
++ (AXIOM term is "Rng"). Note that separate instances, for example
++ using R=Integer and R=Float, have different hidden structure
++ (multiplication and drift tables).

StochasticDifferential(R:Join(OrderedSet, IntegralDomain)):
 Category == Implementation where
  ER ==> Expression(R)
  PR ==> Polynomial(R)
  FR ==> Fraction(PR)
  BSD ==> BasicStochasticDifferential
  PI ==> PositiveInteger
  NNI ==> NonNegativeInteger
  OF  ==> OutputForm
  Category ==> Join(Rng, Module(ER)) with

   RetractableTo(BSD)

   alterQuadVar!:(BSD,BSD,%) -> Union(%,"failed")
    ++ alterQuadVar! adds multiplication formula for a
    ++ pair of stochastic differentials to a private table.
    ++ Failure occurs if
    ++  (a) either of first or second arguments is not basic
    ++  (b) third argument is not exactly of first degree

   alterDrift!:(BSD,%) -> Union(%,"failed")
```

```
 ++ alterDrift! adds drift formula for a
 ++ stochastic differential to a private table.
 ++ Failure occurs if
 ++  (a) first arguments is not basic
 ++  (b) second argument is not exactly of first degree

drift:% -> %
 ++ drift(dx) returns the drift of \axiom{dx}

freeOf?:(%,BSD) -> Boolean
 ++ freeOf?(sd,dX) checks whether \axiom{dX} occurs in
 ++ \axiom{sd} as a module element

coefficient:(%,BSD) -> ER
 ++ coefficient(sd,dX) returns the coefficient of \axiom{dX}
 ++ in the stochastic differential \axiom{sd}

listSD:(%) -> List BSD
 ++ listSD(dx) returns a list of all \axiom{BSD} involved
 ++ in the generation of \axiom{dx} as a module element

equation:(%,R) -> Union(Equation %,"failed")
 ++ equation(dx,0) allows RHS of Equation % to be zero
equation:(R,%) -> Union(Equation %,"failed")
 ++ equation(0,dx) allows LHS of Equation % to be zero

copyDrift:() -> Table(%,%)
 ++ copyDrift returns private table of drifts
 ++ of basic stochastic differentials for inspection

copyQuadVar:() -> Table(%,%)
 ++ copyQuadVar returns private multiplication table
 ++ of basic stochastic differentials for inspection

"/"      : (%, ER) -> %
   ++ dx/y divides the stochastic differential dx
   ++ by the previsible function y.

"**"     : (%, PI) -> %
   ++ dx**n is dx multiplied by itself n times.

"^"     : (%, PI) -> %
   ++ dx^n is dx multiplied by itself n times.

statusIto:()  -> OF
 ++ statusIto() displays the current state of \axiom{setBSD},
 ++ \axiom{tableDrift}, and \axiom{tableQuadVar}. Question
 ++ marks are printed instead of undefined entries
 ++
 ++X dt:=introduce!(t,dt)
```

```
     ++X dX:=introduce!(X,dX)
     ++X dY:=introduce!(Y,dY)
     ++X copyBSD()
     ++X copyIto()
     ++X copyhQuadVar()
     ++X statusIto()

    uncorrelated?: (%,%) -> Boolean
     ++ uncorrelated?(dx,dy) checks whether its two arguments
     ++ have zero quadratic co-variation.
    uncorrelated?: (List %,List %) -> Boolean
     ++ uncorrelated?(l1,l2) checks whether its two arguments
     ++ are lists of stochastic differentials of zero inter-list
     ++ quadratic co-variation.
    uncorrelated?: (List List %) -> Boolean
     ++ uncorrelated?(ll) checks whether its argument is a list
     ++ of lists of stochastic differentials of zero inter-list
     ++ quadratic co-variation.

   Implementation ==> SparseMultivariatePolynomial(ER,BSD) add
    Rep:=SparseMultivariatePolynomial(ER,BSD)

    (v:% / s:ER):% == inv(s) * v

    tableQuadVar:Table(%,%)  := table()
    tableDrift:Table(%,%) := table()

    alterQuadVar!(da:BSD,db:BSD,dXdY:%):Union(%,"failed") ==
     -- next two lines for security only!
     1 < totalDegree(dXdY)                => "failed"
     0 ~= coefficient(dXdY,degree(1)$Rep) => "failed"
     not(0::% = (dXdY*dXdY)::%) => "failed"
     setelt(tableQuadVar,((da::Rep)*(db::Rep))$Rep,dXdY)$Table(%,%)
     -- We have to take care here to avoid a bad
     -- recursion on \axiom{*:(%,%)->%}

    alterDrift!(da:BSD,dx:%):Union(%,"failed") ==
     1 < totalDegree(dx)                => "failed"
     0 ~= coefficient(dx,degree(1)$Rep) => "failed"
     not(0::% = (dx*dx)::%) => "failed"
     setelt(tableDrift,da::Rep,dx)$Table(%,%)

    multSDOrError(dm:%):% ==
     c := leadingCoefficient dm
     (dmm := search(dm/c,tableQuadVar))
       case "failed" =>
        print hconcat(message("ERROR IN ")$OF,(dm/c)::OF)
        error "Above product of sd's is not defined"
     c*dmm
```

```
(dx:% * dy:%) : % ==
 1 < totalDegree(dx) =>
  print hconcat(message("ERROR IN ")$OF,dx::OF)
  error "bad sd in lhs of sd product"
 1 < totalDegree(dy) =>
  print hconcat(message("ERROR IN ")$OF,dy::OF)
  error "bad sd in rhs of sd product"
 reduce("+",map(multSDOrError,monomials((dx*dy)$Rep)),0)
 -- We have to take care here to avoid a bad
 -- recursion on \axiom{*:(%,%)->%}

(dx:% ** n:PI) : % ==
 n = 1 => dx
 n = 2 => dx*dx
 n > 2 => 0::%

(dx:% ^ n:PI) : % == dx**n

driftSDOrError(dm:%):% ==
 c := leadingCoefficient dm
 (dmm := search(dm/c,tableDrift))
   case "failed" =>
    print hconcat(message("ERROR IN ")$OF,(dm/c)::OF)
    error "drift of sd is not defined"
 c*dmm

drift(dx:%):% ==
 reduce("+",map(driftSDOrError,monomials(dx)),0)

freeOf?(sd,dX) == (0 = coefficient(sd,dX,1))

coefficient(sd:%,dX:BSD):ER ==
 retract(coefficient(sd,dX,1))@ER

listSD(sd) ==
 [retract(dX)@BSD for dX in primitiveMonomials(sd)]

equation(dx:%,zero:R):Union(Equation %,"failed") ==
 not(0 = zero) => "failed"
 equation(dx,0::%)
equation(zero:R,dx:%):Union(Equation %,"failed") ==
 not(0 = zero) => "failed"
 equation(0::%,dx)

copyDrift() == tableDrift
copyQuadVar() == tableQuadVar

xDrift(dx:BSD):OF ==
 (xdx := search(dx::Rep,tableDrift)) case "failed" => "?"::OF
 xdx::OF
```

```
   xQV(dx:BSD,dy:BSD):OF ==
    (xdxdy := search((dx::% * dy::%)$Rep,tableQuadVar))
      case "failed" => "?"::Symbol::OF
    xdxdy::OF

   statusIto():OF ==
    bsd  := copyBSD()$BSD
    bsdo := [dx::OF for dx in bsd]
    blank:= ""::Symbol::OF
    colon:= ":"::Symbol::OF
    bsdh := "B S D "::Symbol::OF
    dfth := "drift "::Symbol::OF
    qvh  := "*"::Symbol::OF
    head := append([bsdh,colon],bsdo)
    drift:= append([dfth,colon],[xDrift dx for dx in bsd])
    space:= append([qvh ,blank],[blank for dx in bsd])
    qv   := [append([dy::OF,colon],[xQV(dx,dy) for dx in bsd])
                                           for dy in bsd]
    matrix(append([head,drift,space],qv))$OF

   uncorrelated?(dx:%,dy:%): Boolean == (0::% = dx*dy)

   uncorrelated?(l1:List %,l2:List %): Boolean ==
    reduce("and", [
     reduce("and",[uncorrelated?(dx,dy) for dy in l2],true)
      for dx in l1 ],true)

   uncorrelated1?(l1:List %,ll:List List %): Boolean ==
    reduce("and",[uncorrelated?(l1,l2) for l2 in ll],true)

   uncorrelated?(ll:List List %): Boolean ==
    (0$Integer = # ll) => true
    (1 = # ll) => true
    uncorrelated1?(first ll,rest ll) and uncorrelated?(rest ll)
```

———-

— SD.dotabb —

```
"SD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SD"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"SD" -> "FS"
```

———-

20.30 domain STREAM Stream

— Stream.input —

```
)set break resume
)sys rm -f Stream.output
)spool Stream.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
ints := [i for i in 0..]
--R
--R
--R   (1)  [0,1,2,3,4,5,6,7,8,9,...]
--R                                             Type: Stream NonNegativeInteger
--E 1

--S 2 of 12
f : List INT -> List INT
--R
--R                                                                  Type: Void
--E 2

--S 3 of 12
f x == [x.1 + x.2, x.1]
--R
--R                                                                  Type: Void
--E 3

--S 4 of 12
fibs := [i.2 for i in [generate(f,[1,1])]]
--R
--R   Compiling function f with type List Integer -> List Integer
--R
--R   (4)  [1,1,2,3,5,8,13,21,34,55,...]
--R                                                        Type: Stream Integer
--E 4

--S 5 of 12
[i for i in ints | odd? i]
--R
--R
--R   (5)  [1,3,5,7,9,11,13,15,17,19,...]
--R                                             Type: Stream NonNegativeInteger
--E 5

--S 6 of 12
```

```
odds := [2*i+1 for i in ints]
--R
--R
--R   (6)  [1,3,5,7,9,11,13,15,17,19,...]
--R                                               Type: Stream NonNegativeInteger
--E 6

--S 7 of 12
scan(0,+,odds)
--R
--R
--R   (7)  [1,4,9,16,25,36,49,64,81,100,...]
--R                                               Type: Stream NonNegativeInteger
--E 7

--S 8 of 12
[i*j for i in ints for j in odds]
--R
--R
--R   (8)  [0,3,10,21,36,55,78,105,136,171,...]
--R                                               Type: Stream NonNegativeInteger
--E 8

--S 9 of 12
map(*,ints,odds)
--R
--R
--R   (9)  [0,3,10,21,36,55,78,105,136,171,...]
--R                                               Type: Stream NonNegativeInteger
--E 9

--S 10 of 12
first ints
--R
--R
--R   (10)  0
--R                                                      Type: NonNegativeInteger
--E 10

--S 11 of 12
rest ints
--R
--R
--R   (11)  [1,2,3,4,5,6,7,8,9,10,...]
--R                                               Type: Stream NonNegativeInteger
--E 11

--S 12 of 12
fibs 20
--R
```

```
--R
--R   (12)  6765
--R                                                     Type: PositiveInteger
--E 12
)spool
)lisp (bye)
```

———-

— Stream.help —

```
====================================================================
Stream examples
====================================================================

A Stream object is represented as a list whose last element contains
the wherewithal to create the next element, should it ever be required.

Let ints be the infinite stream of non-negative integers.

  ints := [i for i in 0..]
    [0,1,2,3,4,5,6,7,8,9,...]
                                 Type: Stream NonNegativeInteger

By default, ten stream elements are calculated.  This number may be
changed to something else by the system command
  )set streams calculate

More generally, you can construct a stream by specifying its initial
value and a function which, when given an element, creates the next element.

  f : List INT -> List INT
                                 Type: Void

  f x == [x.1 + x.2, x.1]
                                 Type: Void

  fibs := [i.2 for i in [generate(f,[1,1])]]
    [1,1,2,3,5,8,13,21,34,55,...]
                                 Type: Stream Integer

You can create the stream of odd non-negative integers by either filtering
them from the integers, or by evaluating an expression for each integer.

  [i for i in ints | odd? i]
    [1,3,5,7,9,11,13,15,17,19,...]
                                 Type: Stream NonNegativeInteger

  odds := [2*i+1 for i in ints]
```

```
    [1,3,5,7,9,11,13,15,17,19,...]
                                 Type: Stream NonNegativeInteger
```

You can accumulate the initial segments of a stream using the scan operation.

```
  scan(0,+,odds)
    [1,4,9,16,25,36,49,64,81,100,...]
                                 Type: Stream NonNegativeInteger
```

The corresponding elements of two or more streams can be combined in this way.

```
  [i*j for i in ints for j in odds]
    [0,3,10,21,36,55,78,105,136,171,...]
                                 Type: Stream NonNegativeInteger

  map(*,ints,odds)
    [0,3,10,21,36,55,78,105,136,171,...]
                                 Type: Stream NonNegativeInteger
```

Many operations similar to those applicable to lists are available for
streams.

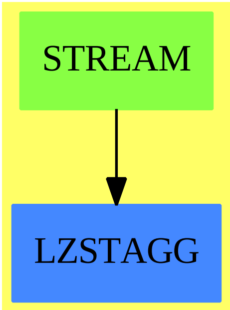
```
  first ints
    0
                                 Type: NonNegativeInteger

  rest ints
    [1,2,3,4,5,6,7,8,9,10,...]
                                 Type: Stream NonNegativeInteger

  fibs 20
    6765
                                 Type: PositiveInteger
```

See Also:
o)help StreamFunctions1
o)help StreamFunctions2
o)help StreamFunctions3
o)show Stream

20.30.1 Stream (STREAM)



Exports:

any?	child?	children
coerce	complete	concat
concat!	cons	construct
convert	copy	count
cycleEntry	cycleLength	cycleSplit!
cycleTail	cyclic?	delay
delete	distance	elt
empty	empty?	entries
entry?	eq?	eval
eval	eval	eval
every?	explicitEntries?	explicitlyEmpty?
explicitlyFinite?	extend	fill!
filterUntil	filterWhile	find
findCycle	first	frst
generate	hash	index?
indices	insert	insert
last	latex	lazy?
lazyEvaluate	leaf?	leaves
less?	map	map!
maxIndex	member?	members
minIndex	more?	new
nodes	node?	numberOfComputedEntries
output	parts	possiblyInfinite?
qelt	qsetelt!	reduce
remove	removeDuplicates	repeating
repeating?	rest	rst
sample	second	select
setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!
showAll?	showAllElements	size?
split!	swap!	tail
third	value	#?
?=?	?~=?	?.?
?.last	?.rest	?.first
?.value		

— domain STREAM Stream —

```
)abbrev domain STREAM Stream
++ Implementation of streams via lazy evaluation
++ Authors: Burge, Watt; updated by Clifton J. Williamson
++ Date Created: July 1986
++ Date Last Updated: 30 March 1990
++ Keywords: stream, infinite list, infinite sequence
++ Examples:
++ References:
```

```
++ Description:
++ A stream is an implementation of an infinite sequence using
++ a list of terms that have been computed and a function closure
++ to compute additional terms when needed.

Stream(S): Exports == Implementation where
--  problems:
--  1) dealing with functions which basically want a finite structure
--  2) 'map' doesn't deal with cycles very well

  S : Type
  B   ==> Boolean
  OUT ==> OutputForm
  I   ==> Integer
  L   ==> List
  NNI ==> NonNegativeInteger
  U   ==> UniversalSegment I

  Exports ==> LazyStreamAggregate(S) with
    shallowlyMutable
      ++ one may destructively alter a stream by assigning new
      ++ values to its entries.

    coerce: L S -> %
      ++ coerce(l) converts a list l to a stream.
      ++
      ++X m:=[1,2,3,4,5,6,7,8,9,10,11,12]
      ++X coerce(m)@Stream(Integer)
      ++X m::Stream(Integer)

    repeating: L S -> %
      ++ repeating(l) is a repeating stream whose period is the list l.
      ++
      ++X m:=repeating([-1,0,1,2,3])

    if S has SetCategory then
      repeating?: (L S,%) -> B
        ++ repeating?(l,s) returns true if a stream s is periodic
        ++ with period l, and false otherwise.
        ++
        ++X m:=[1,2,3]
        ++X n:=repeating(m)
        ++X repeating?(m,n)

    findCycle: (NNI,%) -> Record(cycle?: B, prefix: NNI, period: NNI)
      ++ findCycle(n,st) determines if st is periodic within n.
      ++
      ++X m:=[1,2,3]
      ++X n:=repeating(m)
      ++X findCycle(3,n)
```

```
    ++X findCycle(2,n)

  delay: (() -> %) -> %
    ++ delay(f) creates a stream with a lazy evaluation defined by
    ++ function f.
    ++ Caution: This function can only be called in compiled code.
  cons: (S,%) -> %
    ++ cons(a,s) returns a stream whose \spad{first} is \spad{a}
    ++ and whose \spad{rest} is s.
    ++ Note: \spad{cons(a,s) = concat(a,s)}.
    ++
    ++X m:=[1,2,3]
    ++X n:=repeating(m)
    ++X cons(4,n)

  if S has SetCategory then
    output: (I, %) -> Void
      ++ output(n,st) computes and displays the first n entries
      ++ of st.
      ++
      ++X m:=[1,2,3]
      ++X n:=repeating(m)
      ++X output(5,n)

    showAllElements: % -> OUT
      ++ showAllElements(s) creates an output form which displays all
      ++ computed elements.
      ++
      ++X m:=[1,2,3,4,5,6,7,8,9,10,11,12]
      ++X n:=m::Stream(PositiveInteger)
      ++X showAllElements n

    showAll?: () -> B
      ++ showAll?() returns true if all computed entries of streams
      ++ will be displayed.
      --!! this should be a function of one argument
  setrest_!: (%,I,%) -> %
    ++ setrest!(x,n,y) sets rest(x,n) to y. The function will expand
    ++ cycles if necessary.
    ++
    ++X p:=[i for i in 1..]
    ++X q:=[i for i in 9..]
    ++X setrest!(p,4,q)
    ++X p

  generate: (() -> S) -> %
    ++ generate(f) creates an infinite stream all of whose elements are
    ++ equal to \spad{f()}.
    ++ Note: \spad{generate(f) = [f(),f(),f(),...]}.
    ++
```

```
      ++X f():Integer == 1
      ++X generate(f)

    generate: (S -> S,S) -> %
      ++ generate(f,x) creates an infinite stream whose first element is
      ++ x and whose nth element (\spad{n > 1}) is f applied to the previous
      ++ element. Note: \spad{generate(f,x) = [x,f(x),f(f(x)),...]}.
      ++
      ++X f(x:Integer):Integer == x+10
      ++X generate(f,10)

    filterWhile: (S -> Boolean,%) -> %
      ++ filterWhile(p,s) returns \spad{[x0,x1,...,x(n-1)]} where
      ++ \spad{s = [x0,x1,x2,..]} and
      ++ n is the smallest index such that \spad{p(xn) = false}.
      ++
      ++X m:=[i for i in 1..]
      ++X f(x:PositiveInteger):Boolean ==  x < 5
      ++X filterWhile(f,m)

    filterUntil: (S -> Boolean,%) -> %
      ++ filterUntil(p,s) returns \spad{[x0,x1,...,x(n)]} where
      ++ \spad{s = [x0,x1,x2,..]} and
      ++ n is the smallest index such that \spad{p(xn) = true}.
      ++
      ++X m:=[i for i in 1..]
      ++X f(x:PositiveInteger):Boolean ==  x < 5
      ++X filterUntil(f,m)

--    if S has SetCategory then
--      map: ((S,S) -> S,%,%,S) -> %
--       ++ map(f,x,y,a) is equivalent to map(f,x,y)
--       ++ If z = map(f,x,y,a), then z = map(f,x,y) except if
--       ++ x.n = a and rest(rest(x,n)) = rest(x,n) in which case
--       ++ rest(z,n) = rest(y,n) or if y.m = a and rest(rest(y,m)) =
--       ++ rest(y,m) in which case rest(z,n) = rest(x,n).
--       ++ Think of the case where f(xi,yi) = xi + yi and a = 0.

  Implementation ==> add
    MIN ==> 1  -- minimal stream index; see also the defaults in LZSTAGG
    x:%

    import CyclicStreamTools(S,%)

--% representation

    -- This description of the rep is not quite true.
    -- The Rep is a pair of one of three forms:
    --    [value: S,             rest: %]
    --    [nullstream:    Magic, NIL    ]
```

```
    --    [nonnullstream: Magic, fun: () -> %]
    -- Could use a record of unions if we could guarantee no tags.

    NullStream:    S := _$NullStream$Lisp    pretend S
    NonNullStream: S := _$NonNullStream$Lisp pretend S

    Rep := Record(firstElt: S, restOfStream: %)

    explicitlyEmpty? x == EQ(frst x,NullStream)$Lisp
    lazy? x            == EQ(frst x,NonNullStream)$Lisp

--% signatures of local functions

    setfrst_!     : (%,S) -> S
    setrst_!      : (%,%) -> %
    setToNil_!    : % -> %
    setrestt_!    : (%,I,%) -> %
    lazyEval      : % -> %
    expand_!      : (%,I) -> %

--% functions to access or change record fields without lazy evaluation

    frst x == x.firstElt
    rst  x == x.restOfStream

    setfrst_!(x,s) == x.firstElt := s
    setrst_!(x,y)  == x.restOfStream := y

    setToNil_! x ==
    -- destructively changes x to a null stream
      setfrst_!(x,NullStream); setrst_!(x,NIL$Lisp)
      x

--% SETCAT functions

    if S has SetCategory then

      getm              : (%,L OUT,I) -> L OUT
      streamCountCoerce : % -> OUT
      listm             : (%,L OUT,I) -> L OUT

      getm(x,le,n) ==
        explicitlyEmpty? x => le
        lazy? x =>
          n > 0 =>
            empty? x => le
            getm(rst x,concat(frst(x) :: OUT,le),n - 1)
          concat(message("..."),le)
        eq?(x,rst x) => concat(overbar(frst(x) :: OUT),le)
        n > 0 => getm(rst x,concat(frst(x) :: OUT,le),n - 1)
```

```
  concat(message("..."),le)

streamCountCoerce x ==
-- this will not necessarily display all stream elements
-- which have been computed
  count := _$streamCount$Lisp
  -- compute count elements
  y := x
  for i in 1..count while not empty? y repeat y := rst y
  fc := findCycle(count,x)
  not fc.cycle? => bracket reverse_! getm(x,empty(),count)
  le : L OUT := empty()
  for i in 1..fc.prefix repeat
    le := concat(first(x) :: OUT,le)
    x := rest x
  pp : OUT :=
    fc.period = 1 => overbar(frst(x) :: OUT)
    pl : L OUT := empty()
    for i in 1..fc.period repeat
      pl := concat(frst(x) :: OUT,pl)
      x  := rest x
    overbar commaSeparate reverse_! pl
  bracket reverse_! concat(pp,le)

listm(x,le,n) ==
  explicitlyEmpty? x => le
  lazy? x =>
    n > 0 =>
      empty? x => le
      listm(rst x, concat(frst(x) :: OUT,le),n-1)
    concat(message("..."),le)
  listm(rst x,concat(frst(x) :: OUT,le),n-1)

showAllElements x ==
-- this will display all stream elements which have been computed
-- and will display at least n elements with n = streamCount$Lisp
  extend(x,_$streamCount$Lisp)
  cycElt := cycleElt x
  cycElt case "failed" =>
    le := listm(x,empty(),_$streamCount$Lisp)
    bracket reverse_! le
  cycEnt := computeCycleEntry(x,cycElt :: %)
  le : L OUT := empty()
  while not eq?(x,cycEnt) repeat
    le := concat(frst(x) :: OUT,le)
    x := rst x
  len := computeCycleLength(cycElt :: %)
  pp : OUT :=
    len = 1 => overbar(frst(x) :: OUT)
    pl : L OUT := []
```

```
          for i in 1..len repeat
            pl := concat(frst(x) :: OUT,pl)
            x := rst x
          overbar commaSeparate reverse_! pl
        bracket reverse_! concat(pp,le)

      showAll?() ==
        NULL(_$streamsShowAll$Lisp)$Lisp => false
        true

      coerce(x):OUT ==
        showAll?() => showAllElements x
        streamCountCoerce x

--% AGG functions

    lazyCopy:% -> %
    lazyCopy x == delay
      empty? x => empty()
      concat(frst x, copy rst x)

    copy x ==
      cycElt := cycleElt x
      cycElt case "failed" => lazyCopy x
      ce := cycElt :: %
      len := computeCycleLength(ce)
      e := computeCycleEntry(x,ce)
      d := distance(x,e)
      cycle := complete first(e,len)
      setrst_!(tail cycle,cycle)
      d = 0 => cycle
      head := complete first(x,d::NNI)
      setrst_!(tail head,cycle)
      head

--% CNAGG functions

    construct l ==
      -- copied from defaults to avoid loading defaults
      empty? l => empty()
      concat(first l, construct rest l)

--% ELTAGG functions

    elt(x:%,n:I) ==
      -- copied from defaults to avoid loading defaults
      n < MIN or empty? x => error "elt: no such element"
      n = MIN => frst x
      elt(rst x,n - 1)
```

```
    seteltt:(%,I,S) -> S
    seteltt(x,n,s) ==
      n = MIN => setfrst_!(x,s)
      seteltt(rst x,n - 1,s)

    setelt(x,n:I,s:S) ==
      n < MIN or empty? x => error "setelt: no such element"
      x := expand_!(x,n - MIN + 1)
      seteltt(x,n,s)

--% IXAGG functions

    removee: ((S -> Boolean),%) -> %
    removee(p,x) == delay
      empty? x => empty()
      p(frst x) => remove(p,rst x)
      concat(frst x,remove(p,rst x))

    remove(p,x) ==
      explicitlyEmpty? x => empty()
      eq?(x,rst x) =>
        p(frst x) => empty()
        x
      removee(p,x)

    selectt: ((S -> Boolean),%) -> %
    selectt(p,x) == delay
      empty? x => empty()
      not p(frst x) => select(p, rst x)
      concat(frst x,select(p,rst x))

    select(p,x) ==
      explicitlyEmpty? x => empty()
      eq?(x,rst x) =>
        p(frst x) => x
        empty()
      selectt(p,x)

    map(f,x) ==
      map(f,x pretend Stream(S))$StreamFunctions2(S,S) pretend %

    map(g,x,y) ==
      xs := x pretend Stream(S); ys := y pretend Stream(S)
      map(g,xs,ys)$StreamFunctions3(S,S,S) pretend %

    fill_!(x,s) ==
      setfrst_!(x,s)
      setrst_!(x,x)

    map_!(f,x) ==
```

```
    -- too many problems with map_! on a lazy stream, so
    -- in this case, an error message is returned
      cyclic? x =>
        tail := cycleTail x ; y := x
        until y = tail repeat
          setfrst_!(y,f frst y)
          y := rst y
        x
      explicitlyFinite? x =>
        y := x
        while not empty? y repeat
          setfrst_!(y,f frst y)
          y := rst y
        x
      error "map!: stream with lazy evaluation"

    swap_!(x,m,n) ==
      (not index?(m,x)) or (not index?(n,x)) =>
        error "swap!: no such elements"
      x := expand_!(x,max(m,n) - MIN + 1)
      xm := elt(x,m); xn := elt(x,n)
      setelt(x,m,xn); setelt(x,n,xm)
      x

--% LNAGG functions

    concat(x:%,s:S) == delay
      empty? x => concat(s,empty())
      concat(frst x,concat(rst x,s))

    concat(x:%,y:%) == delay
      empty? x => copy y
      concat(frst x,concat(rst x, y))

    concat l == delay
      empty? l => empty()
      empty?(x := first l) => concat rest l
      concat(frst x,concat(rst x,concat rest l))

    setelt(x,seg:U,s:S) ==
      low := lo seg
      hasHi seg =>
        high := hi seg
        high < low => s
        (not index?(low,x)) or (not index?(high,x)) =>
          error "setelt: index out of range"
        x := expand_!(x,high - MIN + 1)
        y := rest(x,(low - MIN) :: NNI)
        for i in 0..(high-low) repeat
          setfrst_!(y,s)
```

```
          y := rst y
        s
      not index?(low,x) => error "setelt: index out of range"
      x := rest(x,(low - MIN) :: NNI)
      setrst_!(x,x)
      setfrst_!(x,s)

--% RCAGG functions

    empty() == [NullStream, NIL$Lisp]

    lazyEval x == (rst(x):(()-> %)) ()

    lazyEvaluate x ==
      st := lazyEval x
      setfrst_!(x, frst st)
      setrst_!(x,if EQ(rst st,st)$Lisp then x else rst st)
      x

    -- empty? is the only function that explicitly causes evaluation
    -- of a stream element
    empty? x ==
      while lazy? x repeat
        st := lazyEval x
        setfrst_!(x, frst st)
        setrst_!(x,if EQ(rst st,st)$Lisp then x else rst st)
      explicitlyEmpty? x

    --setvalue(x,s) == setfirst_!(x,s)

    --setchildren(x,l) ==
      --empty? l => error "setchildren: empty list of children"
      --not(empty? rest l) => error "setchildren: wrong number of children"
      --setrest_!(x,first l)

--% URAGG functions

    first(x,n) == delay
    -- former name: take
      n = 0 or empty? x => empty()
      (concat(frst x, first(rst x,(n-1) :: NNI)))

    concat(s:S,x:%) == [s,x]
    cons(s,x) == concat(s,x)

    cycleSplit_! x ==
      cycElt := cycleElt x
      cycElt case "failed" =>
        error "cycleSplit_!: non-cyclic stream"
      y := computeCycleEntry(x,cycElt :: %)
```

```
    eq?(x,y) => (setToNil_! x; return y)
    z := rst x
    repeat
      eq?(y,z) => (setrest_!(x,empty()); return y)
      x := z ; z := rst z

  expand_!(x,n) ==
  -- expands cycles (if necessary) so that the first n
  -- elements of x will not be part of a cycle
    n < 1 => x
    y := x
    for i in 1..n while not empty? y repeat y := rst y
    cycElt := cycleElt x
    cycElt case "failed" => x
    e := computeCycleEntry(x,cycElt :: %)
    d : I := distance(x,e)
    d >= n => x
    if d = 0 then
      -- roll the cycle 1 entry
      d := 1
      t := cycleTail e
      if eq?(t,e) then
        t := concat(frst t,empty())
        e := setrst_!(t,t)
        setrst_!(x,e)
      else
        setrst_!(t,concat(frst e,rst e))
        e := rst e
    nLessD := (n-d) :: NNI
    y := complete first(e,nLessD)
    e := rest(e,nLessD)
    setrst_!(tail y,e)
    setrst_!(rest(x,(d-1) :: NNI),y)
    x

  first x ==
    empty? x => error "Can't take the first of an empty stream."
    frst x

  concat_!(x:%,y:%) ==
    empty? x => y
    setrst_!(tail x,y)

  concat_!(x:%,s:S) ==
    concat_!(x,concat(s,empty()))

  setfirst_!(x,s) == setelt(x,0,s)
  setelt(x,"first",s) == setfirst_!(x,s)
  setrest_!(x,y) ==
    empty? x => error "setrest!: empty stream"
```

```
      setrst_!(x,y)
    setelt(x,"rest",y) == setrest_!(x,y)

    setlast_!(x,s) ==
      empty? x => error "setlast!: empty stream"
      setfrst_!(tail x, s)
    setelt(x,"last",s) == setlast_!(x,s)

    split_!(x,n) ==
      n < MIN => error "split!: index out of range"
      n = MIN =>
        y : % := empty()
        setfrst_!(y,frst x)
        setrst_!(y,rst x)
        setToNil_! x
        y
      x := expand_!(x,n - MIN)
      x := rest(x,(n - MIN - 1) :: NNI)
      y := rest x
      setrst_!(x,empty())
      y

--% STREAM functions

    coerce(l: L S) == construct l

    repeating l ==
      empty? l =>
        error "Need a non-null list to make a repeating stream."
      x0 : % := x := construct l
      while not empty? rst x repeat x := rst x
      setrst_!(x,x0)

    if S has SetCategory then

      repeating?(l, x) ==
        empty? l =>
          error "Need a non-empty? list to make a repeating stream."
        empty? rest l =>
          not empty? x and frst x = first l and x = rst x
        x0 := x
        for s in l repeat
          empty? x or s ^= frst x => return false
          x := rst x
        eq?(x,x0)

    findCycle(n, x) ==
      hd := x
      -- Determine whether periodic within n.
      tl := rest(x, n)
```

```
      explicitlyEmpty? tl => [false, 0, 0]
      i := 0; while not eq?(x,tl) repeat (x := rst x; i := i + 1)
      i = n => [false, 0, 0]
      -- Find period. Now x=tl, so step over and find it again.
      x := rst x; per := 1
      while not eq?(x,tl) repeat (x := rst x; per := per + 1)
      -- Find non-periodic part.
      x := hd; xp := rest(hd, per); npp := 0
      while not eq?(x,xp) repeat (x := rst x; xp := rst xp; npp := npp+1)
      [true, npp, per]

    delay(fs:()->%) == [NonNullStream, fs pretend %]

--     explicitlyEmpty? x == markedNull? x

    explicitEntries? x ==
      not explicitlyEmpty? x and not lazy? x

    numberOfComputedEntries x ==
      explicitEntries? x => numberOfComputedEntries(rst x) + 1
      0

    if S has SetCategory then

      output(n,x) ==
        (not(n>0))or empty? x => void()
        mathPrint(frst(x)::OUT)$Lisp
        output(n-1, rst x)

    setrestt_!(x,n,y) ==
      n = 0 => setrst_!(x,y)
      setrestt_!(rst x,n-1,y)

    setrest_!(x,n,y) ==
      n < 0 or empty? x => error "setrest!: no such rest"
      x := expand_!(x,n+1)
      setrestt_!(x,n,y)

    generate f    == delay concat(f(), generate f)
    gen:(S -> S,S) -> %
    gen(f,s) == delay(ss:=f s; concat(ss, gen(f,ss)))
    generate(f,s)==concat(s,gen(f,s))

    concat(x:%,y:%) ==delay
      empty? x => y
      concat(frst x,concat(rst x,y))

    swhilee:(S -> Boolean,%) -> %
    swhilee(p,x) == delay
      empty? x      => empty()
```

```
      not p(frst x) => empty()
      concat(frst x,filterWhile(p,rst x))
    filterWhile(p,x)==
      explicitlyEmpty? x => empty()
      eq?(x,rst x) =>
        p(frst x) => x
        empty()
      swhilee(p,x)

    suntill: (S -> Boolean,%) -> %
    suntill(p,x) == delay
      empty? x  => empty()
      p(frst x) => concat(frst x,empty())
      concat(frst x, filterUntil(p, rst x))

    filterUntil(p,x)==
      explicitlyEmpty? x => empty()
      eq?(x,rst x) =>
        p(frst x) => concat(frst x,empty())
        x
      suntill(p,x)

--  if S has SetCategory then
--    mapp: ((S,S) -> S,%,%,S) -> %
--    mapp(f,x,y,a) == delay
--      empty? x or empty? y => empty()
--      concat(f(frst x,frst y), map(f,rst x,rst y,a))
--      map(f,x,y,a) ==
--      explicitlyEmpty? x => empty()
--      eq?(x,rst x) =>
--        frst x=a => y
--        map(f(frst x,#1),y)
--      explicitlyEmpty? y => empty()
--      eq?(y,rst y) =>
--        frst y=a => x
--        p(f(#1,frst y),x)
--      mapp(f,x,y,a)
```

———-

— STREAM.dotabb —

```
"STREAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STREAM"]
"LZSTAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=LZSTAGG"]
"STREAM" -> "LZSTAGG"
```

———-

20.31 domain STRING String

— String.input —

```
)set break resume
)sys rm -f String.output
)spool String.output
)set message test on
)set message auto off
)clear all
--S 1 of 35
hello := "Hello, I'm AXIOM!"
--R
--R
--R   (1)  "Hello, I'm AXIOM!"
--R                                                             Type: String
--E 1

--S 2 of 35
said := "Jane said, \_"Look!\_""
--R
--R
--R   (2)  "Jane said, \"Look!\""
--R                                                             Type: String
--E 2

--S 3 of 35
saw := "She saw exactly one underscore: \_\_."
--R
--R
--R   (3)  "She saw exactly one underscore: \\."
--R                                                             Type: String
--E 3

--S 4 of 35
gasp: String := new(32, char "x")
--R
--R
--R   (4)  "xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx"
--R                                                             Type: String
--E 4

--S 5 of 35
#gasp
--R
--R
--R   (5)  32
--R                                                    Type: PositiveInteger
```

```
--E 5

--S 6 of 35
hello.2
--R
--R
--R   (6)  e
--R                                                        Type: Character
--E 6

--S 7 of 35
hello 2
--R
--R
--R   (7)  e
--R                                                        Type: Character
--E 7

--S 8 of 35
hello(2)
--R
--R
--R   (8)  e
--R                                                        Type: Character
--E 8

--S 9 of 35
hullo := copy hello
--R
--R
--R   (9)  "Hello, I'm AXIOM!"
--R                                                           Type: String
--E 9

--S 10 of 35
hullo.2 := char "u"; [hello, hullo]
--R
--R
--R   (10)  ["Hello, I'm AXIOM!","Hullo, I'm AXIOM!"]
--R                                                      Type: List String
--E 10

--S 11 of 35
saidsaw := concat ["alpha","---","omega"]
--R
--R
--R   (11)  "alpha---omega"
--R                                                           Type: String
--E 11
```

```
--S 12 of 35
concat("hello ","goodbye")
--R
--R
--R   (12)  "hello goodbye"
--R                                                            Type: String
--E 12

--S 13 of 35
"This " "is " "several " "strings " "concatenated."
--R
--R
--R   (13)  "This is several strings concatenated."
--R                                                            Type: String
--E 13

--S 14 of 35
hello(1..5)
--R
--R
--R   (14)  "Hello"
--R                                                            Type: String
--E 14

--S 15 of 35
hello(8..)
--R
--R
--R   (15)  "I'm AXIOM!"
--R                                                            Type: String
--E 15

--S 16 of 35
split(hello, char " ")
--R
--R
--R   (16)  ["Hello,","I'm","AXIOM!"]
--R                                                       Type: List String
--E 16

--S 17 of 35
other := complement alphanumeric();
--R
--R                                                    Type: CharacterClass
--E 17

--S 18 of 35
split(saidsaw, other)
--R
--R
```

```
--R   (18)  ["alpha","omega"]
--R                                                    Type: List String
--E 18

--S 19 of 35
trim("## ++ relax ++ ##", char "#")
--R
--R
--R   (19)  " ++ relax ++ "
--R                                                         Type: String
--E 19

--S 20 of 35
trim("## ++ relax ++ ##", other)
--R
--R
--R   (20)  "relax"
--R                                                         Type: String
--E 20

--S 21 of 35
leftTrim("## ++ relax ++ ##", other)
--R
--R
--R   (21)  "relax ++ ##"
--R                                                         Type: String
--E 21

--S 22 of 35
rightTrim("## ++ relax ++ ##", other)
--R
--R
--R   (22)  "## ++ relax"
--R                                                         Type: String
--E 22

--S 23 of 35
upperCase hello
--R
--R
--R   (23)  "HELLO, I'M AXIOM!"
--R                                                         Type: String
--E 23

--S 24 of 35
lowerCase hello
--R
--R
--R   (24)  "hello, i'm axiom!"
--R                                                         Type: String
```

```
--E 24

--S 25 of 35
prefix?("He", "Hello")
--R
--R
--R   (25)  true
--R                                                            Type: Boolean
--E 25

--S 26 of 35
prefix?("Her", "Hello")
--R
--R
--R   (26)  false
--R                                                            Type: Boolean
--E 26

--S 27 of 35
suffix?("", "Hello")
--R
--R
--R   (27)  true
--R                                                            Type: Boolean
--E 27

--S 28 of 35
suffix?("LO", "Hello")
--R
--R
--R   (28)  false
--R                                                            Type: Boolean
--E 28

--S 29 of 35
substring?("ll", "Hello", 3)
--R
--R
--R   (29)  true
--R                                                            Type: Boolean
--E 29

--S 30 of 35
substring?("ll", "Hello", 4)
--R
--R
--R   (30)  false
--R                                                            Type: Boolean
--E 30
```

```
--S 31 of 35
n := position("nd", "underground", 1)
--R
--R
--R   (31)  2
--R                                                    Type: PositiveInteger
--E 31

--S 32 of 35
n := position("nd", "underground", n+1)
--R
--R
--R   (32)  10
--R                                                    Type: PositiveInteger
--E 32

--S 33 of 35
n := position("nd", "underground", n+1)
--R
--R
--R   (33)  0
--R                                                 Type: NonNegativeInteger
--E 33

--S 34 of 35
position(char "d", "underground", 1)
--R
--R
--R   (34)  3
--R                                                    Type: PositiveInteger
--E 34

--S 35 of 35
position(hexDigit(), "underground", 1)
--R
--R
--R   (35)  3
--R                                                    Type: PositiveInteger
--E 35
)spool
)lisp (bye)
```

— String.help —

```
====================================================================
String examples
====================================================================
```

```
The type String provides character strings.  Character strings
provide all the operations for a one-dimensional array of characters,
plus additional operations for manipulating text.

String values can be created using double quotes.

  hello := "Hello, I'm AXIOM!"
    "Hello, I'm AXIOM!"
                               Type: String

Note, however, that double quotes and underscores must be preceded by
an extra underscore.

  said := "Jane said, \_"Look!\_""
    "Jane said, \"Look!\""
                               Type: String

  saw := "She saw exactly one underscore: \_\_."
    "She saw exactly one underscore: \\."
                               Type: String

It is also possible to use new to create a string of any size filled
with a given character.  Since there are many new functions it is
necessary to indicate the desired type.

  gasp: String := new(32, char "x")
    "xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx"
                               Type: String

The length of a string is given by #.

  #gasp
    32
                               Type: PositiveInteger

Indexing operations allow characters to be extracted or replaced in strings.
For any string s, indices lie in the range 1..#s.

  hello.2
    e
                               Type: Character

Indexing is really just the application of a string to a subscript, so
any application syntax works.

  hello 2
    e
                               Type: Character
```

```
  hello(2)
    e
                                   Type: Character
```

If it is important not to modify a given string, it should be copied before any updating operations are used.

```
  hullo := copy hello
    "Hello, I'm AXIOM!"
                                   Type: String

  hullo.2 := char "u"; [hello, hullo]
    ["Hello, I'm AXIOM!","Hullo, I'm AXIOM!"]
                                   Type: List String
```

Operations are provided to split and join strings. The concat operation allows several strings to be joined together.

```
  saidsaw := concat ["alpha","---","omega"]
    "alpha---omega"
                                   Type: String
```

There is a version of concat that works with two strings.

```
  concat("hello ","goodbye")
    "hello goodbye"
                                   Type: String
```

Juxtaposition can also be used to concatenate strings.

```
  "This " "is " "several " "strings " "concatenated."
    "This is several strings concatenated."
                                   Type: String
```

Substrings are obtained by giving an index range.

```
  hello(1..5)
    "Hello"
                                   Type: String

  hello(8..)
    "I'm AXIOM!"
                                   Type: String
```

A string can be split into several substrings by giving a separation character or character class.

```
  split(hello, char " ")
    ["Hello,","I'm","AXIOM!"]
                                   Type: List String
```

```
  other := complement alphanumeric();
                              Type: CharacterClass

  split(saidsaw, other)
    ["alpha","omega"]
                              Type: List String
```

Unwanted characters can be trimmed from the beginning or end of a string using the operations trim, leftTrim and rightTrim.

```
  trim("## ++ relax ++ ##", char "#")
    " ++ relax ++ "
                              Type: String
```

Each of these functions takes a string and a second argument to specify the characters to be discarded.

```
  trim("## ++ relax ++ ##", other)
    "relax"
                              Type: String
```

The second argument can be given either as a single character or as a character class.

```
  leftTrim("## ++ relax ++ ##", other)
    "relax ++ ##"
                              Type: String

  rightTrim("## ++ relax ++ ##", other)
    "## ++ relax"
                              Type: String
```

Strings can be changed to upper case or lower case using the operations upperCase and lowerCase.

```
  upperCase hello
    "HELLO, I'M AXIOM!"
                              Type: String
```

The versions with the exclamation mark change the original string, while the others produce a copy.

```
  lowerCase hello
    "hello, i'm axiom!"
                              Type: String
```

Some basic string matching is provided. The function prefix? tests whether one string is an initial prefix of another.

```
prefix?("He", "Hello")
  true
                           Type: Boolean

prefix?("Her", "Hello")
  false
                           Type: Boolean
```

A similar function, suffix?, tests for suffixes.

```
suffix?("", "Hello")
  true
                           Type: Boolean

suffix?("LO", "Hello")
  false
                           Type: Boolean
```

The function substring? tests for a substring given a starting position.

```
substring?("ll", "Hello", 3)
  true
                           Type: Boolean

substring?("ll", "Hello", 4)
  false
                           Type: Boolean
```

A number of position functions locate things in strings. If the first argument to position is a string, then position(s,t,i) finds the location of s as a substring of t starting the search at position i.

```
n := position("nd", "underground", 1)
  2
                           Type: PositiveInteger

n := position("nd", "underground", n+1)
  10
                           Type: PositiveInteger
```

If s is not found, then 0 is returned (minIndex(s)-1 in IndexedString).

```
n := position("nd", "underground", n+1)
  0
                           Type: NonNegativeInteger
```

To search for a specific character or a member of a character class, a different first argument is used.

```
position(char "d", "underground", 1)
```

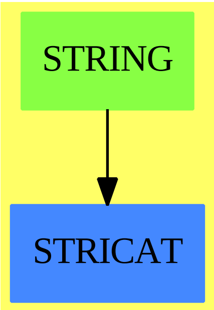
```
    3
                                Type: PositiveInteger

  position(hexDigit(), "underground", 1)
    3
                                Type: PositiveInteger

See Also:
o )help Character
o )help CharacterClass
o )show String
```

———–

20.31.1 String (STRING)



See

⇒ "Character" (CHAR) 4.3.1 on page 357
⇒ "CharacterClass" (CCLASS) 4.4.1 on page 365
⇒ "IndexedString" (ISTRING) 10.14.1 on page 1214

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	OMwrite	parts	position
prefix?	qelt	qsetelt!	reduce	removeDuplicates
replace	reverse	reverse!	rightTrim	sample
select	setelt	size?	sort	sort!
sorted?	split	string	substring?	suffix?
swap!	trim	upperCase	upperCase!	#?
?=?	?.?	?~=?	?<?	?<=?
?>?	?>=?			

— domain STRING String —

```
)abbrev domain STRING String
++ Author: Mark Botch
++ Description:
++ This is the domain of character strings. Strings are 1 based.

String(): StringCategory == IndexedString(1) add
    string n == STRINGIMAGE(n)$Lisp

    OMwrite(x: %): String ==
      s: String := ""
      sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
      dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
      OMputObject(dev)
      OMputString(dev, x pretend String)
      OMputEndObject(dev)
      OMclose(dev)
      s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
      s

    OMwrite(x: %, wholeObj: Boolean): String ==
      s: String := ""
      sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
      dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
      if wholeObj then
        OMputObject(dev)
      OMputString(dev, x pretend String)
      if wholeObj then
        OMputEndObject(dev)
      OMclose(dev)
```

```
      s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
      s

    OMwrite(dev: OpenMathDevice, x: %): Void ==
      OMputObject(dev)
      OMputString(dev, x pretend String)
      OMputEndObject(dev)

    OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
      if wholeObj then
        OMputObject(dev)
      OMputString(dev, x pretend String)
      if wholeObj then
        OMputEndObject(dev)
```

———-

— **STRING.dotabb** —

```
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"STRING" -> "STRICAT"
```

———-

20.32 domain STRTBL StringTable

— **StringTable.input** —

```
)set break resume
)sys rm -f StringTable.output
)spool StringTable.output
)set message test on
)set message auto off
)clear all
--S 1 of 3
t: StringTable(Integer) := table()
--R
--R
--R   (1)  table()
--R                                                    Type: StringTable Integer
--E 1

--S 2 of 3
```

```
for s in split("My name is Ian Watt.",char " ")
  repeat
    t.s := #s
--R
--R                                                                    Type: Void
--E 2

--S 3 of 3
for key in keys t repeat output [key, t.key]
--R
--R   ["Watt.",5]
--R   ["Ian",3]
--R   ["is",2]
--R   ["name",4]
--R   ["My",2]
--R                                                                    Type: Void
--E 3
)spool
)lisp (bye)
```

———-

— StringTable.help —

```
====================================================================
StringTable examples
====================================================================

This domain provides a table type in which the keys are known to be strings
so special techniques can be used.  Other than performance, the type
StringTable(S) should behave exactly the same way as Table(String,S).

This creates a new table whose keys are strings.

  t: StringTable(Integer) := table()
    table()
                               Type: StringTable Integer

The value associated with each string key is the number of characters
in the string.

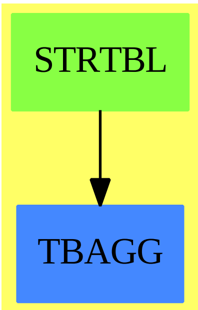
for s in split("My name is Ian Watt.",char " ")
  repeat
    t.s := #s
                               Type: Void

  for key in keys t repeat output [key, t.key]
   ["Watt.",5]
   ["Ian",3]
```

```
  ["is",2]
  ["name",4]
  ["My",2]
                                           Type: Void

See Also:
o )help Table
o )show StringTable
```

20.32.1 StringTable (STRTBL)



See

⇒ "HashTable" (HASHTBL) 9.1.1 on page 1085
⇒ "InnerTable" (INTABL) 10.27.1 on page 1299
⇒ "Table" (TABLE) 21.1.1 on page 2621
⇒ "EqTable" (EQTBL) 6.2.1 on page 667
⇒ "GeneralSparseTable" (GSTBL) 8.5.1 on page 1044
⇒ "SparseTable" (STBL) 20.16.1 on page 2409

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?~=?	?.?

— domain STRTBL StringTable —

```
)abbrev domain STRTBL StringTable
++ Author: Stephen M. Watt
++ Date Created:
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This domain provides tables where the keys are strings.
++ A specialized hash function for strings is used.

StringTable(Entry: SetCategory) ==
    HashTable(String, Entry, "CVEC")
```

———

— STRTBL.dotabb —

```
"STRTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"STRTBL" -> "TBAGG"
```

———

20.33 domain SUBSPACE SubSpace

The first argument n is the dimension of the subSpace

The SubSpace domain is implemented as a tree. The root of the tree is the only node in which the field dataList - which points to a list of points over the ring, R - is defined. The children of the root are the top level components of the SubSpace (in 2D, these would be separate curves; in 3D, these would be separate surfaces).

The pt field is only defined in the leaves.

By way of example, consider a three dimensional subspace with two components - a three by three grid and a sphere. The internal representation of this subspace is a tree with a depth of three.

The root holds a list of all the points used in the subspace (so, if the grid and the sphere share points, the shared points would not be represented redundantly but would be referenced by index).

The root, in this case, has two children - the first points to the grid component and the second to the sphere component. The grid child has four children of its own - a 3x3 grid has 4 endpoints - and each of these point to a list of four points. To see it another way, the grid (child of the root) holds a list of line components which, when placed one above the next, forms a grid. Each of these line components is a list of points.

Points could be explicitly added to subspaces at any level. A path could be given as an argument to the addPoint() function. It is a list of NonNegativeIntegers and refers, in order, to the n-th child of the current node. For example,

```
addPoint(s,[2,3],p)
```

would add the point p to the subspace s by going to the second child of the root and then the third child of that node. If the path does extend to the full depth of the tree, nodes are automatically added so that the tree is of constant depth down any path. By not specifying the full path, new components could be added - e.g. for s from SubSpace(3,Float)

```
addPoint(s,[],p)
```

would create a new child to the root (a new component in N-space) and extend a path to a leaf of depth 3 that points to the data held in p. The subspace s would now have a new component which has one child which, in turn, has one child (the leaf). The new component is then a point.

— SubSpace.input —

```
)set break resume
)sys rm -f SubSpace.output
)spool SubSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SubSpace
--R SubSpace(n: PositiveInteger,R: Ring)  is a domain constructor
--R Abbreviation for SubSpace is SUBSPACE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUBSPACE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                addPoint2 : (%,Point R) -> %
--R birth : % -> %                        children : % -> List %
--R coerce : % -> OutputForm              deepCopy : % -> %
--R extractClosed : % -> Boolean          extractPoint : % -> Point R
```

```
--R hash : % -> SingleInteger            internal? : % -> Boolean
--R latex : % -> String                  leaf? : % -> Boolean
--R level : % -> NonNegativeInteger      merge : List % -> %
--R merge : (%,%) -> %                   new : () -> %
--R parent : % -> %                      pointData : % -> List Point R
--R root? : % -> Boolean                 separate : % -> List %
--R shallowCopy : % -> %                 subspace : () -> %
--R ?~=? : (%,%) -> Boolean
--R addPoint : (%,Point R) -> NonNegativeInteger
--R addPoint : (%,List NonNegativeInteger,NonNegativeInteger) -> %
--R addPoint : (%,List NonNegativeInteger,Point R) -> %
--R addPointLast : (%,%,Point R,NonNegativeInteger) -> %
--R child : (%,NonNegativeInteger) -> %
--R closeComponent : (%,List NonNegativeInteger,Boolean) -> %
--R defineProperty : (%,List NonNegativeInteger,SubSpaceComponentProperty) -> %
--R extractIndex : % -> NonNegativeInteger
--R extractProperty : % -> SubSpaceComponentProperty
--R modifyPoint : (%,NonNegativeInteger,Point R) -> %
--R modifyPoint : (%,List NonNegativeInteger,NonNegativeInteger) -> %
--R modifyPoint : (%,List NonNegativeInteger,Point R) -> %
--R numberOfChildren : % -> NonNegativeInteger
--R traverse : (%,List NonNegativeInteger) -> %
--R
--E 1

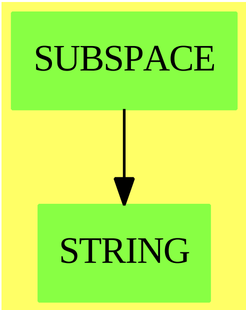
)spool
)lisp (bye)
```

— SubSpace.help —

```
====================================================================
SubSpace examples
====================================================================

See Also:
o )show SubSpace
```

20.33.1 SubSpace (SUBSPACE)



See

⇒ “Point” (POINT) 17.24.1 on page 2019
⇒ “SubSpaceComponentProperty” (COMPPROP) 20.34.1 on page 2583

Exports:

addPoint	addPointLast	addPoint2	birth	child
children	closeComponent	coerce	deepCopy	defineProperty
extractClosed	extractIndex	extractPoint	extractProperty	hash
internal?	latex	leaf?	level	merge
merge	modifyPoint	new	numberOfChildren	parent
pointData	root?	separate	shallowCopy	subspace
traverse	? =?	?=?		

— **domain SUBSPACE SubSpace** —

```
)abbrev domain SUBSPACE SubSpace
++ Author: Mark Botch
++ Description:
++ This domain is not documented

SubSpace(n:PI,R:Ring) : Exports == Implementation where
  I     ==> Integer
  PI    ==> PositiveInteger
  NNI   ==> NonNegativeInteger
  L     ==> List
  B     ==> Boolean
  POINT ==> Point(R)
  PROP ==> SubSpaceComponentProperty()
  S ==> String
  O ==> OutputForm
  empty ==> nil  -- macro to ease conversion to new aggcat.spad

  Exports ==> SetCategory with
    leaf?          : % -> B
        ++ leaf?(x) is not documented
    root?          : % -> B
```

```
    ++ root?(x) is not documented
internal?      : % -> B
    ++ internal?(x) is not documented
new            : () -> %
    ++ new() is not documented
subspace       : () -> %
    ++ subspace() is not documented
birth          : % -> %       -- returns a pointer to the baby
    ++ birth(x) is not documented
child          : (%,NNI) -> %
    ++ child(x,n) is not documented
children       : % -> List %
    ++ children(x) is not documented
numberOfChildren: % -> NNI
    ++ numberOfChildren(x) is not documented
shallowCopy    : % -> %
    ++ shallowCopy(x) is not documented
deepCopy       : % -> %
    ++ deepCopy(x) is not documented
merge          : (%,%) -> %
  ++ merge(s1,s2) the subspaces s1 and s2 into a single subspace.
merge          : List % -> %
  ++ merge(ls) a list of subspaces, ls, into one subspace.
separate       : % -> List %
  ++ separate(s) makes each of the components of the \spadtype{SubSpace},
  ++ s, into a list of separate and distinct subspaces and returns
  ++ the list.
addPoint       : (%,List NNI,POINT) -> %
  ++ addPoint(s,li,p) adds the 4 dimensional point, p, to the 3
  ++ dimensional subspace, s. The list of non negative integers, li,
  ++ dictates the path to follow, or, to look at it another way,
  ++ points to the component in which the point is to be added.  It's
  ++ length should range from 0 to \spad{n - 1} where n is the dimension
  ++ of the subspace. If the length is \spad{n - 1}, then a specific
  ++ lowest level component is being referenced.  If it is less than
  ++ \spad{n - 1}, then some higher level component (0 indicates top
  ++ level component) is being referenced and a component of that level
  ++ with the desired point is created.  The subspace s is returned
  ++ with the additional point.
addPoint2      : (%,POINT) -> %
  ++ addPoint2(s,p) adds the 4 dimensional point, p, to the 3
  ++ dimensional subspace, s.
  ++ The subspace s is returned with the additional point.
addPointLast   : (%,%,POINT, NNI) -> %
  ++ addPointLast(s,s2,li,p) adds the 4 dimensional point, p, to the 3
  ++ dimensional subspace, s. s2 point to the end of the subspace
  ++ s. n is the path in the s2 component.
  ++ The subspace s is returned with the additional point.
modifyPoint    : (%,List NNI,POINT) -> %
  ++ modifyPoint(s,li,p) replaces an existing point in the 3 dimensional
```

```
    ++ subspace, s, with the 4 dimensional point, p.  The list of non
    ++ negative integers, li, dictates the path to follow, or, to look at
    ++ it another way, points to the component in which the existing point
    ++ is to be modified.  An error message occurs if s is empty, otherwise
    ++ the subspace s is returned with the point modification.
  addPoint       : (%,List NNI,NNI) -> %
    ++ addPoint(s,li,i) adds the 4 dimensional point indicated by the
    ++ index location, i, to the 3 dimensional subspace, s. The list of
    ++ non negative integers, li, dictates the path to follow, or, to
    ++ look at it another way, points to the component in which the point
    ++ is to be added.  It's length should range from 0 to \spad{n - 1}
    ++ where n is the dimension of the subspace. If the length is
    ++ \spad{n - 1}, then a specific lowest level component is being
    ++ referenced.  If it is less than \spad{n - 1}, then some higher
    ++ level component (0 indicates top level component) is being
    ++ referenced and a component of that level with the desired point
    ++ is created.  The subspace s is returned with the additional point.
  modifyPoint    : (%,List NNI,NNI) -> %
    ++ modifyPoint(s,li,i) replaces an existing point in the 3 dimensional
    ++ subspace, s, with the 4 dimensional point indicated by the index
    ++ location, i.  The list of non negative integers, li, dictates
    ++ the path to follow, or, to look at it another way, points to the
    ++ component in which the existing point is to be modified.  An error
    ++ message occurs if s is empty, otherwise the subspace s is returned
    ++ with the point modification.
  addPoint       : (%,POINT) -> NNI
    ++ addPoint(s,p) adds the point, p, to the 3 dimensional subspace, s,
    ++ and returns the new total number of points in s.
  modifyPoint    : (%,NNI,POINT) -> %
    ++ modifyPoint(s,ind,p) modifies the point referenced by the index
    ++ location, ind, by replacing it with the point, p in the 3 dimensional
    ++ subspace, s.  An error message occurs if s is empty, otherwise the
    ++ subspace s is returned with the point modification.

  closeComponent : (%,List NNI,B)     -> %
    ++ closeComponent(s,li,b) sets the property of the component in the
    ++ 3 dimensional subspace, s, to be closed if b is true, or open if
    ++ b is false.  The list of non negative integers, li, dictates the
    ++ path to follow, or, to look at it another way, points to the
    ++ component whose closed property is to be set.  The subspace, s,
    ++ is returned with the component property modification.
  defineProperty : (%,List NNI,PROP)  -> %
    ++ defineProperty(s,li,p) defines the component property in the
    ++ 3 dimensional subspace, s, to be that of p, where p is of the
    ++ domain \spadtype{SubSpaceComponentProperty}.  The list of non
    ++ negative integers, li, dictates the path to follow, or, to look
    ++ at it another way, points to the component whose property is
    ++ being defined.  The subspace, s, is returned with the component
    ++ property definition.
  traverse       : (%,List NNI) -> %
```

```
    ++ traverse(s,li) follows the branch list of the 3 dimensional
    ++ subspace, s, along the path dictated by the list of non negative
    ++ integers, li, which points to the component which has been
    ++ traversed to.  The subspace, s, is returned, where s is now
    ++ the subspace pointed to by li.
  extractPoint    : %  -> POINT
    ++ extractPoint(s) returns the point which is given by the current
    ++ index location into the point data field of the 3 dimensional
    ++ subspace s.
  extractIndex    : % -> NNI
    ++ extractIndex(s) returns a non negative integer which is the current
    ++ index of the 3 dimensional subspace s.
  extractClosed   : %  -> B
    ++ extractClosed(s) returns the \spadtype{Boolean} value of the closed
    ++ property for the indicated 3 dimensional subspace s.  If the
    ++ property is closed, \spad{True} is returned, otherwise \spad{False}
    ++ is returned.
  extractProperty : %  -> PROP
    ++ extractProperty(s) returns the property of domain
    ++ \spadtype{SubSpaceComponentProperty} of the indicated 3 dimensional
    ++ subspace s.
  level           : % -> NNI
    ++ level(s) returns a non negative integer which is the current
    ++ level field of the indicated 3 dimensional subspace s.
  parent          : % -> %
    ++ parent(s) returns the subspace which is the parent of the indicated
    ++ 3 dimensional subspace s.  If s is the top level subspace an error
    ++ message is returned.
  pointData       : % -> L POINT
    ++ pointData(s) returns the list of points from the point data field
    ++ of the 3 dimensional subspace s.

Implementation ==> add
  import String()

  Rep := Record(pt:POINT, index:NNI, property:PROP, _
               childrenField:List %, _
               lastChild: List %, _
               levelField:NNI, _
               pointDataField:L POINT, _
               lastPoint: L POINT, _
               noPoints: NNI, _
               noChildren: NNI, _
               parentField:List %) -- needn't be list but...base case?

  TELLWATT : String := "Non-null list: Please inform Stephen Watt"

  leaf? space ==  empty? children space
  root? space == (space.levelField = 0$NNI)
  internal? space == ^(root? space and leaf? space)
```

```
new() ==
  [point(empty())$POINT,0,new()$PROP,empty(),empty(),0,_
                                 empty(),empty(),0,0,empty()]
subspace() == new()

birth momma ==
  baby := new()
  baby.levelField := momma.levelField+1
  baby.parentField := [momma]
  if not empty?(lastKid := momma.lastChild) then
    not empty? rest lastKid => error TELLWATT
  if empty? lastKid
    then
      momma.childrenField := [baby]
      momma.lastChild := momma.childrenField
      momma.noChildren := 1
    else
      setrest_!(lastKid,[baby])
      momma.lastChild := rest lastKid
      momma.noChildren := momma.noChildren + 1
  baby

child(space,num) ==
  space.childrenField.num

children space == space.childrenField
numberOfChildren space == space.noChildren

shallowCopy space ==
  node := new()
  node.pt         := space.pt
  node.index      := space.index
  node.property   := copy(space.property)
  node.levelField := space.levelField
  node.parentField := nil()
  if root? space then
    node.pointDataField := copy(space.pointDataField)
    node.lastPoint := tail(node.pointDataField)
    node.noPoints := space.noPoints
  node

deepCopy space ==
  node := shallowCopy(space)
  leaf? space => node
  for c in children space repeat
    cc := deepCopy c
    cc.parentField := [node]
    node.childrenField := cons(cc,node.childrenField)
  node.childrenField := reverse_!(node.childrenField)
```

```
    node.lastChild := tail node.childrenField
    node

  merge(s1,s2) ==
     ------------------ need to worry about reindexing s2 & parentField
    n1 : Rep := deepCopy s1
    n2 : Rep := deepCopy s2
    n1.childrenField := append(children n1,children n2)
    n1

  merge listOfSpaces ==
     ------------------ need to worry about reindexing & parentField
    empty? listOfSpaces => error "empty list passed as argument to merge"
         -- notice that the properties of the first subspace on the
         -- list are the ones that are inherited...hmmmm...
    space := deepCopy first listOfSpaces
    for s in rest listOfSpaces repeat
         -- because of the initial deepCopy, above, everything is
         -- deepCopied to be consistent...more hmmm...
      space.childrenField := append(space.childrenField,[deepCopy c for c in s.childrenFie
    space

  separate space ==
     ------------------ need to worry about reindexing & parentField
    spaceList := empty()
    for s in space.childrenField repeat
      spc:=shallowCopy space
      spc.childrenField:=[deepCopy s]
      spaceList := cons(spc,spaceList)
    spaceList

  addPoint(space:%,path:List NNI,point:POINT) ==
    if not empty?(lastPt := space.lastPoint) then
      not empty? rest lastPt => error TELLWATT
    if empty? lastPt
      then
        space.pointDataField := [point]
        space.lastPoint := space.pointDataField
      else
        setrest_!(lastPt,[point])
        space.lastPoint := rest lastPt
    space.noPoints := space.noPoints + 1
    which := space.noPoints
    node := space
    depth : NNI := 0
    for i in path repeat
      node := child(node,i)
      depth := depth + 1
    for more in depth..(n-1) repeat
      node := birth node
```

```
    node.pt := point      -- will be obsolete field
    node.index := which
    space

  addPoint2(space:%,point:POINT) ==
    if not empty?(lastPt := space.lastPoint) then
      not empty? rest lastPt => error TELLWATT
    if empty? lastPt
      then
        space.pointDataField := [point]
        space.lastPoint := space.pointDataField
      else
        setrest_!(lastPt,[point])
        space.lastPoint := rest lastPt
    space.noPoints := space.noPoints + 1
    which := space.noPoints
    node := space
    depth : NNI := 0
    node := birth node
    first := node
    for more in 1..n-1 repeat
      node := birth node
    node.pt := point      -- will be obsolete field
    node.index := which
    first

  addPointLast(space:%,node:%, point:POINT, depth:NNI) ==
    if not empty?(lastPt := space.lastPoint) then
      not empty? rest lastPt => error TELLWATT
    if empty? lastPt
      then
        space.pointDataField := [point]
        space.lastPoint := space.pointDataField
      else
        setrest_!(lastPt,[point])
        space.lastPoint := rest lastPt
    space.noPoints := space.noPoints + 1
    which := space.noPoints
    if depth = 2 then node := child(node, 2)
    for more in depth..(n-1) repeat
      node := birth node
    node.pt := point      -- will be obsolete field
    node.index := which
    node -- space

  addPoint(space:%,path:List NNI,which:NNI) ==
    node := space
    depth : NNI := 0
    for i in path repeat
      node := child(node,i)
```

```
    depth := depth + 1
  for more in depth..(n-1) repeat
    node := birth node
  node.pt := space.pointDataField.which   -- will be obsolete field
  node.index := which
  space

addPoint(space:%,point:POINT) ==
  root? space =>
    if not empty?(lastPt := space.lastPoint) then
      not empty? rest lastPt => error TELLWATT
    if empty? lastPt
      then
        space.pointDataField := [point]
        space.lastPoint := space.pointDataField
      else
        setrest_!(lastPt,[point])
        space.lastPoint := rest lastPt
    space.noPoints := space.noPoints + 1
  error "You need to pass a top level SubSpace (level should be zero)"

modifyPoint(space:%,path:List NNI,point:POINT) ==
  if not empty?(lastPt := space.lastPoint) then
    not empty? rest lastPt => error TELLWATT
  if empty? lastPt
    then
      space.pointDataField := [point]
      space.lastPoint := space.pointDataField
    else
      setrest_!(lastPt,[point])
      space.lastPoint := rest lastPt
  space.noPoints := space.noPoints + 1
  which := space.noPoints
  node := space
  for i in path repeat
    node := child(node,i)
  node.pt := point       ---------- will be obsolete field
  node.index := which
  space

modifyPoint(space:%,path:List NNI,which:NNI) ==
  node := space
  for i in path repeat
    node := child(node,i)
  node.pt := space.pointDataField.which ---------- will be obsolete field
  node.index := which
  space

modifyPoint(space:%,which:NNI,point:POINT) ==
  root? space =>
```

```
      space.pointDataField.which := point
      space
    error "You need to pass a top level SubSpace (level should be zero)"

  closeComponent(space,path,val) ==
    node := space
    for i in path repeat
      node := child(node,i)
    close(node.property,val)
    space

  defineProperty(space,path,prop) ==
    node := space
    for i in path repeat
      node := child(node,i)
    node.property := prop
    space

  traverse(space,path) ==
    for i in path repeat space := child(space,i)
    space

  extractPoint space ==
    node := space
    while ^root? node repeat node := parent node
    (node.pointDataField).(space.index)
  extractIndex space == space.index
  extractClosed space == closed? space.property
  extractProperty space == space.property

  parent space ==
    empty? space.parentField => error "This is a top level SubSpace - it does not have a parent"
    first space.parentField
  pointData space == space.pointDataField
  level space == space.levelField
  s1 = s2 ==
      ------------ extra checks for list of point data
    (leaf? s1 and leaf? s2) =>
      (s1.pt = s2.pt) and (s1.property = s2.property) and (s1.levelField = s2.levelField)
    -- note that the ordering of children is important
    #s1.childrenField ^= #s2.childrenField => false
    and/[c1 = c2 for c1 in s1.childrenField for c2 in s2.childrenField]
     and (s1.property = s2.property) and (s1.levelField = s2.levelField)
  coerce(space:%):O ==
    hconcat([n::O,"-Space with depth of "::O,                              _
            (n - space.levelField)::O," and "::O,(s:=(#space.childrenField))::O,  _
            (s=1 => " component"::O;" components"::O)])
```

———

— **SUBSPACE.dotabb** —

```
"SUBSPACE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUBSPACE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"SUBSPACE" -> "STRING"
```

————-

20.34 domain COMPPROP SubSpaceComponentProperty

— **SubSpaceComponentProperty.input** —

```
)set break resume
)sys rm -f SubSpaceComponentProperty.output
)spool SubSpaceComponentProperty.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SubSpaceComponentProperty
--R SubSpaceComponentProperty  is a domain constructor
--R Abbreviation for SubSpaceComponentProperty is COMPPROP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for COMPPROP
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean               close : (%,Boolean) -> Boolean
--R closed? : % -> Boolean               coerce : % -> OutputForm
--R copy : % -> %                        hash : % -> SingleInteger
--R latex : % -> String                  new : () -> %
--R solid : (%,Boolean) -> Boolean       solid? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

————-

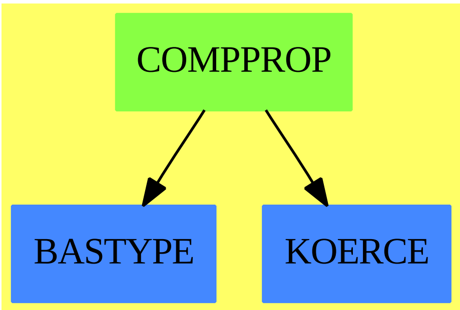
— **SubSpaceComponentProperty.help** —

```
====================================================================
SubSpaceComponentProperty examples
====================================================================

See Also:
o )show SubSpaceComponentProperty
```

———–

20.34.1 SubSpaceComponentProperty (COMPPROP)



See

⇒ "Point" (POINT) 17.24.1 on page 2019
⇒ "SubSpace" (SUBSPACE) 20.33.1 on page 2573

Exports:

close	closed?	coerce	copy	hash
latex	new	solid	solid?	?~=?
?=?				

— **domain COMPPROP SubSpaceComponentProperty** —

```
)abbrev domain COMPPROP SubSpaceComponentProperty
++ Author: Mark Botch
++ Description:
++ This domain implements some global properties of subspaces.

SubSpaceComponentProperty() : Exports == Implementation where

  O ==> OutputForm
  I    ==> Integer
  PI   ==> PositiveInteger
  NNI  ==> NonNegativeInteger
  L    ==> List
  B    ==> Boolean
```

```
  Exports ==> SetCategory with
    new     : () -> %
        ++ new() is not documented
    closed? : % -> B
        ++ closed?(x) is not documented
    solid?  : % -> B
        ++ solid?(x) is not documented
    close   : (%,B) -> B
        ++ close(x,b) is not documented
    solid   : (%,B) -> B
        ++ solid(x,b) is not documented
    copy    : % -> %
        ++ copy(x) is not documented

  Implementation ==> add
    Rep := Record(closed:B, solid:B)
    closed? p == p.closed
    solid? p == p.solid
    close(p,b) == p.closed := b
    solid(p,b) == p.solid := b
    new() == [false,false]
    copy p ==
      annuderOne := new()
      close(annuderOne,closed? p)
      solid(annuderOne,solid? p)
      annuderOne
    coerce p ==
      hconcat(["Component is "::O,
              (closed? p => ""::O; "not "::O),"closed, "::O, _
              (solid? p => ""::O; "not "::O),"solid"::O ])
```

— **COMPPROP.dotabb** —

```
"COMPPROP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COMPPROP"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"COMPPROP" -> "BASTYPE"
"COMPPROP" -> "KOERCE"
```

20.35 domain SUCH SuchThat

— SuchThat.input —

```
)set break resume
)sys rm -f SuchThat.output
)spool SuchThat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SuchThat
--R SuchThat(S1: SetCategory,S2: SetCategory)  is a domain constructor
--R Abbreviation for SuchThat is SUCH
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUCH
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R construct : (S1,S2) -> %             hash : % -> SingleInteger
--R latex : % -> String                  lhs : % -> S1
--R rhs : % -> S2                        ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

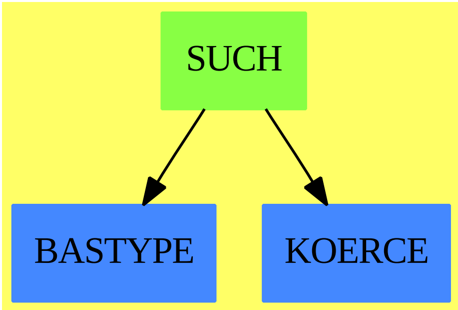
— SuchThat.help —

```
====================================================================
SuchThat examples
====================================================================

See Also:
o )show SuchThat
```

———-

20.35.1 SuchThat (SUCH)



Exports:

coerce	construct	hash	latex	lhs
rhs	?=?	?~=?		

— domain SUCH SuchThat —

```
)abbrev domain SUCH SuchThat
++ Author: Mark Botch
++ Description:
++ This domain implements "such that" forms

SuchThat(S1, S2): Cat == Capsule where
    E ==> OutputForm
    S1, S2: SetCategory

    Cat == SetCategory with
        construct: (S1, S2) -> %
                ++ construct(s,t) makes a form s:t
        lhs: % -> S1
                ++ lhs(f) returns the left side of f
        rhs: % -> S2
                ++ rhs(f) returns the right side of f

    Capsule == add
        Rep := Record(obj: S1, cond: S2)
        construct(o, c) == [o, c]$Record(obj: S1, cond: S2)
        lhs st == st.obj
        rhs st == st.cond
        coerce(w):E == infix("|"::E, w.obj::E, w.cond::E)
```

————-

— SUCH.dotabb —

```
"SUCH" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUCH"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"SUCH" -> "BASTYPE"
"SUCH" -> "KOERCE"
```

———-

20.36 domain SWITCH Switch

— Switch.input —

```
)set break resume
)sys rm -f Switch.output
)spool Switch.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Switch
--R Switch  is a domain constructor
--R Abbreviation for Switch is SWITCH
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SWITCH
--R
--R------------------------------- Operations --------------------------------
--R NOT : % -> %                          coerce : Symbol -> %
--R coerce : % -> OutputForm
--R AND : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union
--R EQ : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R GE : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R GT : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R LE : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R LT : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R NOT : Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %) -> %
--R OR : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R
--E 1

)spool
)lisp (bye)
```

———-

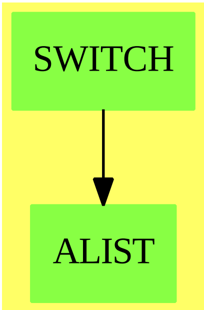
— Switch.help —

```
====================================================================
Switch examples
====================================================================

See Also:
o )show Switch
```

———-

20.36.1 Switch (SWITCH)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranCode" (FC) 7.16.1 on page 898
⇒ "FortranProgram" (FORTRAN) 7.18.1 on page 923
⇒ "ThreeDimensionalMatrix" (M3D) 21.7.1 on page 2661
⇒ "SimpleFortranProgram" (SFORT) 20.11.1 on page 2364
⇒ "FortranTemplate" (FTEM) 7.20.1 on page 934
⇒ "FortranExpression" (FEXPR) 7.17.1 on page 914

Exports:
coerce AND EQ GE GT LE LT NOT OR

— **domain SWITCH Switch** —

```
)abbrev domain SWITCH Switch
-- Because of a bug in the compiler:
)bo $noSubsumption:=false

++ Author: Mike Dewar
++ Date Created: April 1991
++ Date Last Updated: March 1994 30.6.94 Added coercion from Symbol MCD
++ Basic Operations:
++ Related Constructors: FortranProgram, FortranCode, FortranTypes
++ Also See:
```

```
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain builds representations of boolean expressions for use with
++ the \axiomType{FortranCode} domain.

Switch():public == private where
  EXPR ==> Union(I:Expression Integer,F:Expression Float,
                 CF:Expression Complex Float,switch:%)

  public ==  CoercibleTo OutputForm with
    coerce : Symbol -> $
        ++ coerce(s) is not documented
    LT : (EXPR,EXPR) -> $
      ++ LT(x,y) returns the \axiomType{Switch} expression representing \spad{x<y}.
    GT : (EXPR,EXPR) -> $
      ++ GT(x,y) returns the \axiomType{Switch} expression representing \spad{x>y}.
    LE : (EXPR,EXPR) -> $
      ++ LE(x,y) returns the \axiomType{Switch} expression representing \spad{x<=y}.
    GE : (EXPR,EXPR) -> $
      ++ GE(x,y) returns the \axiomType{Switch} expression representing \spad{x>=y}.
    OR : (EXPR,EXPR) -> $
      ++ OR(x,y) returns the \axiomType{Switch} expression representing \spad{x or y}.
    EQ : (EXPR,EXPR) -> $
      ++ EQ(x,y) returns the \axiomType{Switch} expression representing \spad{x = y}.
    AND : (EXPR,EXPR) -> $
      ++ AND(x,y) returns the \axiomType{Switch} expression representing \spad{x and y}.
    NOT : EXPR -> $
      ++ NOT(x) returns the \axiomType{Switch} expression representing \spad{\~~x}.
    NOT : $ -> $
      ++ NOT(x) returns the \axiomType{Switch} expression representing \spad{\~~x}.

  private == add
    Rep := Record(op:BasicOperator,rands:List EXPR)

    -- Public function definitions

    nullOp : BasicOperator := operator NULL

    coerce(s:%):OutputForm ==
      rat := (s . op)::OutputForm
      ran := [u::OutputForm for u in s.rands]
      (s . op) = nullOp => first ran
      #ran = 1 =>
        prefix(rat,ran)
      infix(rat,ran)

    coerce(s:Symbol):$ == [nullOp,[[s::Expression(Integer)]$EXPR]$List(EXPR)]$Rep
```

```
    NOT(r:EXPR):% ==
      [operator("~"::Symbol),[r]$List(EXPR)]$Rep

    NOT(r:%):% ==
      [operator("~"::Symbol),[[r]$EXPR]$List(EXPR)]$Rep

    LT(r1:EXPR,r2:EXPR):% ==
      [operator("<"::Symbol),[r1,r2]$List(EXPR)]$Rep

    GT(r1:EXPR,r2:EXPR):% ==
      [operator(">"::Symbol),[r1,r2]$List(EXPR)]$Rep

    LE(r1:EXPR,r2:EXPR):% ==
      [operator("<="::Symbol),[r1,r2]$List(EXPR)]$Rep

    GE(r1:EXPR,r2:EXPR):% ==
      [operator(">="::Symbol),[r1,r2]$List(EXPR)]$Rep

    AND(r1:EXPR,r2:EXPR):% ==
      [operator("and"::Symbol),[r1,r2]$List(EXPR)]$Rep

    OR(r1:EXPR,r2:EXPR):% ==
      [operator("or"::Symbol),[r1,r2]$List(EXPR)]$Rep

    EQ(r1:EXPR,r2:EXPR):% ==
      [operator("EQ"::Symbol),[r1,r2]$List(EXPR)]$Rep
```

— SWITCH.dotabb —

```
"SWITCH" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SWITCH"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SWITCH" -> "ALIST"
```

20.37 domain SYMBOL Symbol

— Symbol.input —

```
)set break resume
)sys rm -f Symbol.output
)spool Symbol.output
```

```
)set message test on
)set message auto off
)clear all
--S 1 of 24
X: Symbol := 'x
--R
--R
--R   (1)  x
--R                                                            Type: Symbol
--E 1

--S 2 of 24
XX: Symbol := x
--R
--R
--R   (2)  x
--R                                                            Type: Symbol
--E 2

--S 3 of 24
A := 'a
--R
--R
--R   (3)  a
--R                                                        Type: Variable a
--E 3

--S 4 of 24
B := b
--R
--R
--R   (4)  b
--R                                                        Type: Variable b
--E 4

--S 5 of 24
x**2 + 1
--R
--R
--R         2
--R   (5)  x  + 1
--R                                                Type: Polynomial Integer
--E 5

--S 6 of 24
"Hello"::Symbol
--R
--R
--R   (6)  Hello
--R                                                            Type: Symbol
```

```
--E 6

--S 7 of 24
new()$Symbol
--R
--R
--R   (7)  %A
--R                                                          Type: Symbol
--E 7

--S 8 of 24
new()$Symbol
--R
--R
--R   (8)  %B
--R                                                          Type: Symbol
--E 8

--S 9 of 24
new("xyz")$Symbol
--R
--R
--R   (9)  %xyz0
--R                                                          Type: Symbol
--E 9

--S 10 of 24
X[i,j]
--R
--R
--R   (10)  x
--R          i,j
--R                                                          Type: Symbol
--E 10

--S 11 of 24
U := subscript(u, [1,2,1,2])
--R
--R
--R   (11)  u
--R          1,2,1,2
--R                                                          Type: Symbol
--E 11

--S 12 of 24
V := superscript(v, [n])
--R
--R
--R          n
--R   (12)  v
```

```
--R                                                            Type: Symbol
--E 12

--S 13 of 24
P := argscript(p, [t])
--R
--R
--R   (13)  p(t)
--R                                                            Type: Symbol
--E 13

--S 14 of 24
scripted? U
--R
--R
--R   (14)  true
--R                                                           Type: Boolean
--E 14

--S 15 of 24
scripted? X
--R
--R
--R   (15)  false
--R                                                           Type: Boolean
--E 15

--S 16 of 24
string X
--R
--R
--R   (16)  "x"
--R                                                            Type: String
--E 16

--S 17 of 24
name U
--R
--R
--R   (17)  u
--R                                                            Type: Symbol
--E 17

--S 18 of 24
scripts U
--R
--R
--R   (18)  [sub= [1,2,1,2],sup= [],presup= [],presub= [],args= []]
--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: List OutputFor
--E 18
```

```
--S 19 of 24
name X
--R
--R
--R   (19)  x
--R                                                    Type: Symbol
--E 19

--S 20 of 24
scripts X
--R
--R
--R   (20)  [sub= [],sup= [],presup= [],presub= [],args= []]
--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: Lis
--E 20

--S 21 of 24
M := script(Mammoth, [ [i,j],[k,l],[0,1],[2],[u,v,w] ])
--R
--R
--R         0,1       k,l
--R   (21)     Mammoth   (u,v,w)
--R           2       i,j
--R                                                    Type: Symbol
--E 21

--S 22 of 24
scripts M
--R
--R
--R   (22)  [sub= [i,j],sup= [k,l],presup= [0,1],presub= [2],args= [u,v,w]]
--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: Lis
--E 22

--S 23 of 24
N := script(Nut, [ [i,j],[k,l],[0,1] ])
--R
--R
--R         0,1   k,l
--R   (23)     Nut
--R               i,j
--R                                                    Type: Symbol
--E 23

--S 24 of 24
scripts N
--R
--R
--R   (24)  [sub= [i,j],sup= [k,l],presup= [0,1],presub= [],args= []]
```

```
--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: List OutputFor
--E 24
)spool
)lisp (bye)
```

———-

— **Symbol.help** —

```
====================================================================
Symbol examples
====================================================================

Symbols are one of the basic types manipulated by Axiom.  The
Symbol domain provides ways to create symbols of many varieties.

The simplest way to create a symbol is to "single quote" an identifier.

  X: Symbol := 'x
    x
                                    Type: Symbol

This gives the symbol even if x has been assigned a value.  If x has
not been assigned a value, then it is possible to omit the quote.

  XX: Symbol := x
    x
                                    Type: Symbol

Declarations must be used when working with symbols, because otherwise
the interpreter tries to place values in a more specialized type Variable.

  A := 'a
    a
                                    Type: Variable a

  B := b
    b
                                    Type: Variable b

The normal way of entering polynomials uses this fact.

  x**2 + 1
     2
    x  + 1
                                    Type: Polynomial Integer

Another convenient way to create symbols is to convert a string.
This is useful when the name is to be constructed by a program.
```

```
"Hello"::Symbol
  Hello
                                   Type: Symbol
```

Sometimes it is necessary to generate new unique symbols, for example, to name constants of integration. The expression new() generates a symbol starting with %.

```
new()$Symbol
  %A
                                   Type: Symbol
```

Successive calls to new produce different symbols.

```
new()$Symbol
  %B
                                   Type: Symbol
```

The expression new("s") produces a symbol starting with %s.

```
new("xyz")$Symbol
  %xyz0
                                   Type: Symbol
```

A symbol can be adorned in various ways. The most basic thing is applying a symbol to a list of subscripts.

```
X[i,j]
   x
    i,j
                                   Type: Symbol
```

Somewhat less pretty is to attach subscripts, superscripts or arguments.

```
U := subscript(u, [1,2,1,2])
   u
    1,2,1,2
                                   Type: Symbol

V := superscript(v, [n])
    n
   v
                                   Type: Symbol

P := argscript(p, [t])
  p(t)
                                   Type: Symbol
```

It is possible to test whether a symbol has scripts using the scripted? test.

```
scripted? U
  true
                                Type: Boolean

scripted? X
  false
                                Type: Boolean
```

If a symbol is not scripted, then it may be converted to a string.

```
string X
  "x"
                                Type: String
```

The basic parts can always be extracted using the name and scripts operations.

```
name U
  u
                                Type: Symbol

scripts U
  [sub= [1,2,1,2],sup= [],presup= [],presub= [],args= []]
              Type: Record(sub: List OutputForm,
                           sup: List OutputForm,
                           presup: List OutputForm,
                           presub: List OutputForm,
                           args: List OutputForm)

name X
  x
                                Type: Symbol

scripts X
  [sub= [],sup= [],presup= [],presub= [],args= []]
              Type: Record(sub: List OutputForm,
                           sup: List OutputForm,
                           presup: List OutputForm,
                           presub: List OutputForm,
                           args: List OutputForm)
```

The most general form is obtained using the script operation. This operation takes an argument which is a list containing, in this order, lists of subscripts, superscripts, presuperscripts, presubscripts and arguments to a symbol.

```
M := script(Mammoth, [ [i,j],[k,l],[0,1],[2],[u,v,w] ])
   0,1       k,l
      Mammoth   (u,v,w)
     2       i,j
```

```
                                    Type: Symbol

  scripts M
    [sub= [i,j],sup= [k,l],presup= [0,1],presub= [2],args= [u,v,w]]
                  Type: Record(sub: List OutputForm,
                               sup: List OutputForm,
                               presup: List OutputForm,
                               presub: List OutputForm,
                               args: List OutputForm)

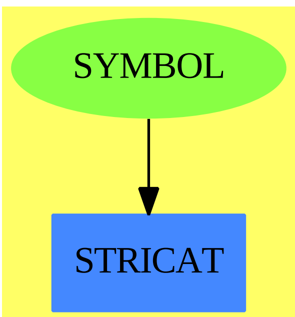
If trailing lists of scripts are omitted, they are assumed to be empty.

  N := script(Nut, [ [i,j],[k,l],[0,1] ])
     0,1   k,l
        Nut
           i,j
                                    Type: Symbol

  scripts N
    [sub= [i,j],sup= [k,l],presup= [0,1],presub= [],args= []]
                  Type: Record(sub: List OutputForm,
                               sup: List OutputForm,
                               presup: List OutputForm,
                               presub: List OutputForm,
                               args: List OutputForm)


See Also:
o )show Symbol
```

20.37.1 Symbol (SYMBOL)



Exports:

argscript	coerce	convert	hash	latex
list	max	min	name	new
OMwrite	patternMatch	resetNew	sample	script
scripts	scripted?	string	subscript	superscript
?~=?	?.?	?<?	?<=?	?=?
?>?	?>=?			

— domain SYMBOL Symbol —

```
)abbrev domain SYMBOL Symbol
++ Author: Stephen Watt
++ Date Created: 1986
++ Date Last Updated: 7 Mar 1991, 29 Apr. 1994 (FDLL)
++ Keywords: symbol.
++ Description:
++ Basic and scripted symbols.

Symbol(): Exports == Implementation where
  L ==> List OutputForm
  Scripts ==> Record(sub:L,sup:L,presup:L,presub:L,args:L)

  Exports ==> Join(OrderedSet, ConvertibleTo InputForm, OpenMath,
        ConvertibleTo Symbol,
         ConvertibleTo Pattern Integer, ConvertibleTo Pattern Float,
          PatternMatchable Integer, PatternMatchable Float) with
     new: () -> %
       ++ new() returns a new symbol whose name starts with %.
     new: % -> %
       ++ new(s) returns a new symbol whose name starts with %s.
     resetNew: () -> Void
       ++ resetNew() resets the internals counters that new() and
       ++ new(s) use to return distinct symbols every time.
     coerce: String -> %
       ++ coerce(s) converts the string s to a symbol.
     name: % -> %
       ++ name(s) returns s without its scripts.
     scripted?: % -> Boolean
       ++ scripted?(s) is true if s has been given any scripts.
     scripts: % -> Scripts
       ++ scripts(s) returns all the scripts of s.
     script: (%, List L) -> %
       ++ script(s, [a,b,c,d,e]) returns s with subscripts a,
       ++ superscripts b, pre-superscripts c, pre-subscripts d,
       ++ and argument-scripts e.  Omitted components are taken to be empty.
       ++ For example, \spad{script(s, [a,b,c])} is equivalent to
       ++ \spad{script(s,[a,b,c,[],[]])}.
     script: (%, Scripts) -> %
       ++ script(s, [a,b,c,d,e]) returns s with subscripts a,
```

```
      ++ superscripts b, pre-superscripts c, pre-subscripts d,
      ++ and argument-scripts e.
    subscript: (%, L) -> %
      ++ subscript(s, [a1,...,an]) returns s
      ++ subscripted by \spad{[a1,...,an]}.
    superscript: (%, L) -> %
      ++ superscript(s, [a1,...,an]) returns s
      ++ superscripted by \spad{[a1,...,an]}.
    argscript: (%, L) -> %
      ++ argscript(s, [a1,...,an]) returns s
      ++ arg-scripted by \spad{[a1,...,an]}.
    elt: (%, L) -> %
      ++ elt(s,[a1,...,an]) or s([a1,...,an]) returns s subscripted by \spad{[a1,...,an]}.
    string: % -> String
      ++ string(s) converts the symbol s to a string.
      ++ Error: if the symbol is subscripted.
    list: % -> List %
      ++ list(sy) takes a scripted symbol and produces a list
      ++ of the name followed by the scripts.
    sample: constant -> %
      ++ sample() returns a sample of %

  Implementation ==> add
    count: Reference(Integer) := ref 0
    xcount: AssociationList(%, Integer) := empty()
    istrings:PrimitiveArray(String) :=
                     construct ["0","1","2","3","4","5","6","7","8","9"]
    -- the following 3 strings shall be of empty intersection
    nums:String:="0123456789"
    ALPHAS:String:="ABCDEFGHIJKLMNOPQRSTUVWXYZ"
    alphas:String:="abcdefghijklmnopqrstuvwxyz"

    writeOMSym(dev: OpenMathDevice, x: %): Void ==
      scripted? x =>
        error "Cannot convert a scripted symbol to OpenMath"
      OMputVariable(dev, x pretend Symbol)

    OMwrite(x: %): String ==
      s: String := ""
      sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
      dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
      OMputObject(dev)
      writeOMSym(dev, x)
      OMputEndObject(dev)
      OMclose(dev)
      s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
      s

    OMwrite(x: %, wholeObj: Boolean): String ==
      s: String := ""
```

```
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  if wholeObj then
    OMputObject(dev)
  writeOMSym(dev, x)
  if wholeObj then
    OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  writeOMSym(dev, x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
  writeOMSym(dev, x)
  if wholeObj then
    OMputEndObject(dev)

hd:String    := "*"
lhd          := #hd
ord0         := ord char("0")$Character

istring  : Integer -> String
syprefix : Scripts -> String
syscripts: Scripts -> L

convert(s:%):InputForm == convert(s pretend Symbol)$InputForm
convert(s:%):Symbol    == s pretend Symbol
coerce(s:String):%     == VALUES(INTERN(s)$Lisp)$Lisp
x = y                  == EQUAL(x,y)$Lisp
x < y                  == GGREATERP(y, x)$Lisp
coerce(x:%):OutputForm == outputForm(x pretend Symbol)
subscript(sy, lx)      == script(sy, [lx, nil, nil(), nil(), nil()])
elt(sy,lx)             == subscript(sy,lx)
superscript(sy, lx)    == script(sy,[nil(),lx, nil(), nil(), nil()])
argscript(sy, lx)      == script(sy,[nil(),nil(), nil(), nil(), lx])

patternMatch(x:%,p:Pattern Integer,l:PatternMatchResult(Integer,%))==
  (patternMatch(x pretend Symbol, p, l pretend
   PatternMatchResult(Integer, Symbol))$PatternMatchSymbol(Integer))
     pretend PatternMatchResult(Integer, %)

patternMatch(x:%, p:Pattern Float, l:PatternMatchResult(Float, %)) ==
  (patternMatch(x pretend Symbol, p, l pretend
   PatternMatchResult(Float, Symbol))$PatternMatchSymbol(Float))
```

```
        pretend PatternMatchResult(Float, %)

    convert(x:%):Pattern(Float) ==
      coerce(x pretend Symbol)$Pattern(Float)

    convert(x:%):Pattern(Integer) ==
      coerce(x pretend Symbol)$Pattern(Integer)

    syprefix sc ==
      ns: List Integer := [#sc.presub, #sc.presup, #sc.sup, #sc.sub]
      while #ns >= 2 and zero? first ns repeat ns := rest ns
      concat concat(concat(hd, istring(#sc.args)),
                           [istring n for n in reverse_! ns])

    syscripts sc ==
      all := sc.presub
      all := concat(sc.presup, all)
      all := concat(sc.sup, all)
      all := concat(sc.sub, all)
      concat(all, sc.args)

    script(sy: %, ls: List L) ==
      sc: Scripts := [nil(), nil(), nil(), nil(), nil()]
      if not null ls then (sc.sub    := first ls; ls := rest ls)
      if not null ls then (sc.sup    := first ls; ls := rest ls)
      if not null ls then (sc.presup := first ls; ls := rest ls)
      if not null ls then (sc.presub := first ls; ls := rest ls)
      if not null ls then (sc.args   := first ls; ls := rest ls)
      script(sy, sc)

    script(sy: %, sc: Scripts) ==
      scripted? sy => error "Cannot add scripts to a scripted symbol"
      (concat(concat(syprefix sc, string name sy)::%::OutputForm,
                                          syscripts sc)) pretend %

    string e ==
      not scripted? e => PNAME(e)$Lisp
      error "Cannot form string from non-atomic symbols."

-- Scripts ==> Record(sub:L,sup:L,presup:L,presub:L,args:L)
    latex e ==
      s : String := (PNAME(name e)$Lisp) pretend String
      if #s > 1 and s.1 ^= char "\" then
        s := concat("\mbox{\it ", concat(s, "}")$String)$String
      not scripted? e => s
      ss : Scripts := scripts e
      lo : List OutputForm := ss.sub
      sc : String
      if not empty? lo then
        sc := "__{"
```

```
      while not empty? lo repeat
          sc := concat(sc, latex first lo)$String
          lo := rest lo
          if not empty? lo then sc := concat(sc, ", ")$String
      sc := concat(sc, "}")$String
      s := concat(s, sc)$String
    lo := ss.sup
    if not empty? lo then
      sc := "^{"
      while not empty? lo repeat
          sc := concat(sc, latex first lo)$String
          lo := rest lo
          if not empty? lo then sc := concat(sc, ", ")$String
      sc := concat(sc, "}")$String
      s := concat(s, sc)$String
    lo := ss.presup
    if not empty? lo then
      sc := "{}^{"
      while not empty? lo repeat
          sc := concat(sc, latex first lo)$String
          lo := rest lo
          if not empty? lo then sc := concat(sc, ", ")$String
      sc := concat(sc, "}")$String
      s := concat(sc, s)$String
    lo := ss.presub
    if not empty? lo then
      sc := "{}_{"
      while not empty? lo repeat
          sc := concat(sc, latex first lo)$String
          lo := rest lo
          if not empty? lo then sc := concat(sc, ", ")$String
      sc := concat(sc, "}")$String
      s := concat(sc, s)$String
    lo := ss.args
    if not empty? lo then
      sc := "\left( {"
      while not empty? lo repeat
          sc := concat(sc, latex first lo)$String
          lo := rest lo
          if not empty? lo then sc := concat(sc, ", ")$String
      sc := concat(sc, "} \right)")$String
      s := concat(s, sc)$String
    s

  anyRadix(n:Integer,s:String):String ==
    ns:String:=""
    repeat
      qr := divide(n,#s)
      n  := qr.quotient
      ns := concat(s.(qr.remainder+minIndex s),ns)
```

```
      if zero?(n) then return ns

    new() ==
      sym := anyRadix(count()::Integer,ALPHAS)
      count() := count() + 1
      concat("%",sym)::%

    new x ==
      n:Integer :=
        (u := search(x, xcount)) case "failed" => 0
        inc(u::Integer)
      xcount(x) := n
      xx :=
        not scripted? x => string x
        string name x
      xx := concat("%",xx)
      xx :=
        (position(xx.maxIndex(xx),nums)>=minIndex(nums)) =>
          concat(xx, anyRadix(n,alphas))
        concat(xx, anyRadix(n,nums))
      not scripted? x => xx::%
      script(xx::%,scripts x)

    resetNew() ==
      count() := 0
      for k in keys xcount repeat remove_!(k, xcount)
      void

    scripted? sy ==
      not ATOM(sy)$Lisp

    name sy ==
      not scripted? sy => sy
      str := string first list sy
      for i in lhd+1..#str repeat
        not digit?(str.i) => return((str.(i..#str))::%)
      error "Improper scripted symbol"

    scripts sy ==
      not scripted? sy => [nil(), nil(), nil(), nil(), nil()]
      nscripts: List NonNegativeInteger := [0, 0, 0, 0, 0]
      lscripts: List L := [nil(), nil(), nil(), nil(), nil()]
      str  := string first list sy
      nstr := #str
      m := minIndex nscripts
      for i in m.. for j in lhd+1..nstr while digit?(str.j) repeat
          nscripts.i := (ord(str.j) - ord0)::NonNegativeInteger
      -- Put the number of function scripts at the end.
      nscripts := concat(rest nscripts, first nscripts)
      allscripts := rest list sy
```

```
      m := minIndex lscripts
      for i in m.. for n in nscripts repeat
        #allscripts < n => error "Improper script count in symbol"
        lscripts.i := [a::OutputForm for a in first(allscripts, n)]
        allscripts := rest(allscripts, n)
      [lscripts.m, lscripts.(m+1), lscripts.(m+2),
                                   lscripts.(m+3), lscripts.(m+4)]

    istring n ==
      n > 9 => error "Can have at most 9 scripts of each kind"
      istrings.(n + minIndex istrings)

    list sy ==
      not scripted? sy =>
         error "Cannot convert a symbol to a list if it is not subscripted"
      sy pretend List(%)

    sample() == "aSymbol"::%
```

— SYMBOL.dotabb —

```
"SYMBOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMBOL",
          shape=ellipse]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"SYMBOL" -> "STRICAT"
```

20.38 domain SYMTAB SymbolTable

— SymbolTable.input —

```
)set break resume
)sys rm -f SymbolTable.output
)spool SymbolTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SymbolTable
--R SymbolTable  is a domain constructor
```

```
--R Abbreviation for SymbolTable is SYMTAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SYMTAB
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              empty : () -> %
--R externalList : % -> List Symbol       newTypeLists : % -> SExpression
--R parametersOf : % -> List Symbol       printTypes : % -> Void
--R coerce : % -> Table(Symbol,FortranType)
--R declare! : (Symbol,FortranType,%) -> FortranType
--R declare! : (List Symbol,FortranType,%) -> FortranType
--R fortranTypeOf : (Symbol,%) -> FortranType
--R symbolTable : List Record(key: Symbol,entry: FortranType) -> %
--R typeList : (FortranScalarType,%) -> List Union(name: Symbol,bounds: List Union(S: Symbol
--R typeLists : % -> List List Union(name: Symbol,bounds: List Union(S: Symbol,P: Polynomial
--R
--E 1

)spool
)lisp (bye)
```

———-

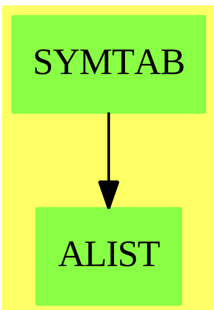
— SymbolTable.help —

```
====================================================================
SymbolTable examples
====================================================================

See Also:
o )show SymbolTable
```

———-

20.38.1 SymbolTable (SYMTAB)



See

⇒ "FortranScalarType" (FST) 7.19.1 on page 929
⇒ "FortranType" (FT) 7.21.1 on page 938
⇒ "TheSymbolTable" (SYMS) 21.6.1 on page 2655

Exports:

coerce	declare!	empty	externalList	fortranTypeOf
newTypeLists	parametersOf	printTypes	symbolTable	typeList
typeLists				

— domain SYMTAB SymbolTable —

```
)abbrev domain SYMTAB SymbolTable
++ Author: Mike Dewar
++ Date Created:  October 1992
++ Date Last Updated: 12 July 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Create and manipulate a symbol table for generated FORTRAN code

SymbolTable() : exports == implementation where

  T   ==> Union(S:Symbol,P:Polynomial Integer)
  TL1 ==> List T
  TU  ==> Union(name:Symbol,bounds:TL1)
  TL  ==> List TU
  SEX ==> SExpression
  OFORM ==> OutputForm
  L   ==> List
  FSTU ==> Union(fst:FortranScalarType,void:"void")

  exports ==> CoercibleTo OutputForm with
    coerce : $ -> Table(Symbol,FortranType)
      ++ coerce(x) returns a table view of x
    empty  : () -> $
      ++ empty() returns a new, empty symbol table
    declare! : (L Symbol,FortranType,$) -> FortranType
      ++ declare!(l,t,tab) creates new entrys in tab, declaring each of l
      ++ to be of type t
    declare! : (Symbol,FortranType,$) -> FortranType
      ++ declare!(u,t,tab) creates a new entry in tab, declaring u to be of
      ++ type t
    fortranTypeOf : (Symbol,$) -> FortranType
```

```
    ++ fortranTypeOf(u,tab) returns the type of u in tab
  parametersOf: $ -> L Symbol
    ++ parametersOf(tab) returns a list of all the symbols declared in tab
  typeList : (FortranScalarType,$) -> TL
    ++ typeList(t,tab) returns a list of all the objects of type t in tab
  externalList : $ -> L Symbol
    ++ externalList(tab) returns a list of all the external symbols in tab
  typeLists : $ -> L TL
    ++ typeLists(tab) returns a list of lists of types of objects in tab
  newTypeLists : $ -> SEX
    ++ newTypeLists(x) is not documented
  printTypes: $ -> Void
    ++ printTypes(tab) produces FORTRAN type declarations from tab, on the
    ++ current FORTRAN output stream
  symbolTable: L Record(key:Symbol,entry:FortranType) -> $
    ++ symbolTable(l) creates a symbol table from the elements of l.

implementation ==> add

  Rep := Table(Symbol,FortranType)

  coerce(t:$):OFORM ==
    coerce(t)$Rep

  coerce(t:$):Table(Symbol,FortranType) ==
    t pretend Table(Symbol,FortranType)

  symbolTable(l:L Record(key:Symbol,entry:FortranType)):$ ==
    table(l)$Rep

  empty():$ ==
    empty()$Rep

  parametersOf(tab:$):L(Symbol) ==
    keys(tab)

  declare!(name:Symbol,type:FortranType,tab:$):FortranType ==
    setelt(tab,name,type)$Rep
    type

  declare!(names:L Symbol,type:FortranType,tab:$):FortranType ==
    for name in names repeat setelt(tab,name,type)$Rep
    type

  fortranTypeOf(u:Symbol,tab:$):FortranType ==
    elt(tab,u)$Rep

  externalList(tab:$):L(Symbol) ==
   [u for u in keys(tab) | external? fortranTypeOf(u,tab)]
```

```
typeList(type:FortranScalarType,tab:$):TL ==
  scalarList := []@TL
  arrayList  := []@TL
  for u in keys(tab)$Rep repeat
    uType : FortranType := fortranTypeOf(u,tab)
    sType : FSTU := scalarTypeOf(uType)
    if (sType case fst and (sType.fst)=type) then
      uDim : TL1 := [[v]$T for v in dimensionsOf(uType)]
      if empty? uDim then
        scalarList := cons([u]$TU,scalarList)
      else
        arrayList := cons([cons([u],uDim)$TL1]$TU,arrayList)
  -- Scalars come first in case they are integers which are later
  -- used as an array dimension.
  append(scalarList,arrayList)

typeList2(type:FortranScalarType,tab:$):TL ==
  tl := []@TL
  symbolType : Symbol := coerce(type)$FortranScalarType
  for u in keys(tab)$Rep repeat
    uType : FortranType := fortranTypeOf(u,tab)
    sType : FSTU := scalarTypeOf(uType)
    if (sType case fst and (sType.fst)=type) then
      uDim : TL1 := [[v]$T for v in dimensionsOf(uType)]
      tl := if empty? uDim then cons([u]$TU,tl)
            else cons([cons([u],uDim)$TL1]$TU,tl)
  empty? tl => tl
  cons([symbolType]$TU,tl)

updateList(sType:SEX,name:SEX,lDims:SEX,tl:SEX):SEX ==
  l : SEX := ASSOC(sType,tl)$Lisp
  entry : SEX := if null?(lDims) then name else CONS(name,lDims)$Lisp
  null?(l) => CONS([sType,entry]$Lisp,tl)$Lisp
  RPLACD(l,CONS(entry,cdr l)$Lisp)$Lisp
  tl

newTypeLists(tab:$):SEX ==
  tl := []$Lisp
  for u in keys(tab)$Rep repeat
    uType : FortranType := fortranTypeOf(u,tab)
    sType : FSTU := scalarTypeOf(uType)
    dims  : L Polynomial Integer := dimensionsOf uType
    lDims : L SEX := [convert(convert(v)@InputForm)@SEX for v in dims]
    lType : SEX := if sType case void
      then convert(void::Symbol)@SEX
      else coerce(sType.fst)$FortranScalarType
    tl := updateList(lType,convert(u)@SEX,convert(lDims)@SEX,tl)
  tl

typeLists(tab:$):L(TL) ==
```

```
  fortranTypes := ["real"::FortranScalarType, _
         "double precision"::FortranScalarType, _
         "integer"::FortranScalarType, _
         "complex"::FortranScalarType, _
         "logical"::FortranScalarType, _
         "character"::FortranScalarType]@L(FortranScalarType)
  tl := []@L TL
  for u in fortranTypes repeat
    types : TL := typeList2(u,tab)
    if (not null types) then
      tl := cons(types,tl)$(L TL)
  tl

oForm2(w:T):OFORM ==
  w case S => w.S::OFORM
  w case P => w.P::OFORM

oForm(v:TU):OFORM ==
  v case name => v.name::OFORM
  v case bounds =>
    ll : L OFORM := [oForm2(uu) for uu in v.bounds]
    ll :: OFORM

outForm(t:TL):L OFORM ==
 [oForm(u) for u in t]

printTypes(tab:$):Void ==
  -- It is important that INTEGER is the first element of this
  -- list since INTEGER symbols used in type declarations must
  -- be declared in advance.
  ft := ["integer"::FortranScalarType, _
         "real"::FortranScalarType, _
         "double precision"::FortranScalarType, _
         "complex"::FortranScalarType, _
         "logical"::FortranScalarType, _
         "character"::FortranScalarType]@L(FortranScalarType)
  for ty in ft repeat
    tl : TL := typeList(ty,tab)
    otl : L OFORM := outForm(tl)
    fortFormatTypes(ty::OFORM,otl)$Lisp
  el : L OFORM := [u::OFORM for u in externalList(tab)]
  fortFormatTypes("EXTERNAL"::OFORM,el)$Lisp
  void()$Void
```

— SYMTAB.dotabb —

```
"SYMTAB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMTAB"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SYMTAB" -> "ALIST"
```

———-

20.39 domain SYMPOLY SymmetricPolynomial

— SymmetricPolynomial.input —

```
)set break resume
)sys rm -f SymmetricPolynomial.output
)spool SymmetricPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SymmetricPolynomial
--R SymmetricPolynomial R: Ring  is a domain constructor
--R Abbreviation for SymmetricPolynomial is SYMPOLY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SYMPOLY
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,Partition) -> R
--R coefficients : % -> List R            coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> Partition               ground : % -> R
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R minimumDegree : % -> Partition        monomial : (R,Partition) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
```

```
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : % -> % if R has INTDOM
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R fmecg : (%,Partition,R,%) -> % if Partition has CABMON and R has INTDOM
--R mapExponents : ((Partition -> Partition),%) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R pomopo! : (%,R,Partition,%) -> %
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

———-

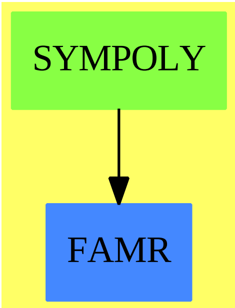
— SymmetricPolynomial.help —

```
====================================================================
SymmetricPolynomial examples
====================================================================

See Also:
o )show SymmetricPolynomial
```

———-

20.39.1 SymmetricPolynomial (SYMPOLY)



See

⇒ "Partition" (PRTITION) 17.9.1 on page 1883

Exports:

0	1	associates?	binomThmExpt
characteristic	charthRoot	coefficient	
coefficients	coerce	content	degree
exquo	exquo	fmecg	ground
ground?	hash	latex	leadingCoefficient
leadingMonomial	map	mapExponents	minimumDegree
monomial	monomial?	numberOfMonomials	one?
pomopo!	primitivePart	recip	reductum
retract	retractIfCan	sample	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?~=?	?**?	?/?	?^?
?*?	?+?	?-?	-?
?=?			

— domain SYMPOLY SymmetricPolynomial —

```
)abbrev domain SYMPOLY SymmetricPolynomial
++ Author: Mark Botch
++ Description:
++ This domain implements symmetric polynomial

SymmetricPolynomial(R:Ring) == PolynomialRing(R,Partition) add
       Term:=  Record(k:Partition,c:R)
       Rep:=  List Term

-- override PR implementation because coeff. arithmetic too expensive (??)

       if R has EntireRing then
         (p1:%) * (p2:%)  ==
            null p1 => 0
            null p2 => 0
```

```
            zero?(p1.first.k) => p1.first.c * p2
--            one? p2 => p1
            (p2 = 1) => p1
            +/[[[t1.k+t2.k,t1.c*t2.c]$Term for t2 in p2]
                   for t1 in reverse(p1)]
                   -- This 'reverse' is an efficiency improvement:
                   -- reduces both time and space [Abbott/Bradford/Davenport]
        else
         (p1:%) * (p2:%)  ==
            null p1 => 0
            null p2 => 0
            zero?(p1.first.k) => p1.first.c * p2
--            one? p2 => p1
            (p2 = 1) => p1
            +/[[[t1.k+t2.k,r]$Term for t2 in p2 | (r:=t1.c*t2.c) ^= 0]
                 for t1 in reverse(p1)]
                  -- This 'reverse' is an efficiency improvement:
                  -- reduces both time and space [Abbott/Bradford/Davenport]
```

— **SYMPOLY.dotabb** —

```
"SYMPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMPOLY"]
"FAMR" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAMR"]
"SYMPOLY" -> "FAMR"
```

Chapter 21

Chapter T

21.1 domain TABLE Table

— Table.input —

```
)set break resume
)sys rm -f Table.output
)spool Table.output
)set message test on
)set message auto off
)clear all
--S 1 of 18
t: Table(Polynomial Integer, String) := table()
--R
--R
--R   (1)  table()
--R                                      Type: Table(Polynomial Integer,String)
--E 1

--S 2 of 18
setelt(t, x**2 - 1, "Easy to factor")
--R
--R
--R   (2)  "Easy to factor"
--R                                                            Type: String
--E 2

--S 3 of 18
t(x**3 + 1) := "Harder to factor"
--R
--R
--R   (3)  "Harder to factor"
```

```
--R                                                                Type: String
--E 3

--S 4 of 18
t(x) := "The easiest to factor"
--R
--R
--R   (4)  "The easiest to factor"
--R                                                                Type: String
--E 4

--S 5 of 18
elt(t, x)
--R
--R
--R   (5)  "The easiest to factor"
--R                                                                Type: String
--E 5

--S 6 of 18
t.x
--R
--R
--R   (6)  "The easiest to factor"
--R                                                                Type: String
--E 6

--S 7 of 18
t x
--R
--R
--R   (7)  "The easiest to factor"
--R                                                                Type: String
--E 7

--S 8 of 18
t.(x**2 - 1)
--R
--R
--R   (8)  "Easy to factor"
--R                                                                Type: String
--E 8

--S 9 of 18
t (x**3 + 1)
--R
--R
--R   (9)  "Harder to factor"
--R                                                                Type: String
--E 9
```

```
--S 10 of 18
keys t
--R
--R
--R             3      2
--R   (10)  [x,x  + 1,x  - 1]
--R                                            Type: List Polynomial Integer
--E 10

--S 11 of 18
search(x, t)
--R
--R
--R   (11)  "The easiest to factor"
--R                                                  Type: Union(String,...)
--E 11

--S 12 of 18
search(x**2, t)
--R
--R
--R   (12)  "failed"
--R                                                Type: Union("failed",...)
--E 12

--S 13 of 18
search(x**2, t) case "failed"
--R
--R
--R   (13)  true
--R                                                            Type: Boolean
--E 13

--S 14 of 18
remove!(x**2-1, t)
--R
--R
--R   (14)  "Easy to factor"
--R                                                  Type: Union(String,...)
--E 14

--S 15 of 18
remove!(x-1, t)
--R
--R
--R   (15)  "failed"
--R                                                Type: Union("failed",...)
--E 15
```

```
--S 16 of 18
#t
--R
--R
--R   (16)  2
--R                                                        Type: PositiveInteger
--E 16

--S 17 of 18
members t
--R
--R
--R   (17)  ["The easiest to factor","Harder to factor"]
--R                                                           Type: List String
--E 17

--S 18 of 18
count(s: String +-> prefix?("Hard", s), t)
--R
--R
--R   (18)  1
--R                                                        Type: PositiveInteger
--E 18
)spool
)lisp (bye)
```

———-

— Table.help —

```
====================================================================
Table examples
====================================================================

The Table constructor provides a general structure for associative
storage.  This type provides hash tables in which data objects can be
saved according to keys of any type.  For a given table, specific
types must be chosen for the keys and entries.

In this example the keys to the table are polynomials with integer
coefficients.  The entries in the table are strings.

  t: Table(Polynomial Integer, String) := table()
    table()
                               Type: Table(Polynomial Integer,String)

To save an entry in the table, the setelt operation is used.  This can
be called directly, giving the table a key and an entry.
```

```
setelt(t, x**2 - 1, "Easy to factor")
  "Easy to factor"
                              Type: String
```

Alternatively, you can use assignment syntax.

```
t(x**3 + 1) := "Harder to factor"
  "Harder to factor"
                              Type: String
```

```
t(x) := "The easiest to factor"
  "The easiest to factor"
                              Type: String
```

Entries are retrieved from the table by calling the elt operation.

```
elt(t, x)
  "The easiest to factor"
                              Type: String
```

This operation is called when a table is "applied" to a key using this or the following syntax.

```
t.x
  "The easiest to factor"
                              Type: String
```

```
t x
  "The easiest to factor"
                              Type: String
```

Parentheses are used only for grouping. They are needed if the key is an infixed expression.

```
t.(x**2 - 1)
  "Easy to factor"
                              Type: String
```

Note that the elt operation is used only when the key is known to be in the table, otherwise an error is generated.

```
t (x**3 + 1)
  "Harder to factor"
                              Type: String
```

You can get a list of all the keys to a table using the keys operation.

```
keys t
      3      2
  [x,x  + 1,x  - 1]
```

```
                              Type: List Polynomial Integer
```

If you wish to test whether a key is in a table, the search operation is used. This operation returns either an entry or "failed".

```
  search(x, t)
    "The easiest to factor"
                              Type: Union(String,...)

  search(x**2, t)
    "failed"
                              Type: Union("failed",...)
```

The return type is a union so the success of the search can be tested using case.

```
  search(x**2, t) case "failed"
    true
                              Type: Boolean
```

The remove operation is used to delete values from a table.

```
  remove!(x**2-1, t)
    "Easy to factor"
                              Type: Union(String,...)
```

If an entry exists under the key, then it is returned. Otherwise remove returns "failed".

```
  remove!(x-1, t)
    "failed"
                              Type: Union("failed",...)
```

The number of key-entry pairs can be found using the # operation.

```
  #t
    2
                              Type: PositiveInteger
```

Just as keys returns a list of keys to the table, a list of all the entries can be obtained using the members operation.

```
  members t
   (17)  ["The easiest to factor","Harder to factor"]
                              Type: List String
```

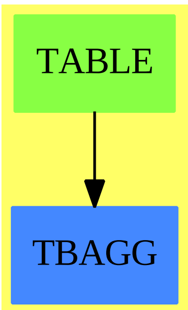
A number of useful operations take functions and map them on to the table to compute the result. Here we count the entries which have "Hard" as a prefix.

```
  count(s: String +-> prefix?("Hard", s), t)
    1
                             Type: PositiveInteger

Other table types are provided to support various needs.
  o AssociationList gives a list with a table view. This allows new
    entries to be appended onto the front of the list to cover up old
    entries. This is useful when table entries need to be stacked or when
    frequent list traversals are required.
  o EqTable gives tables in which keys are considered equal only when
    they are in fact the same instance of a structure.
  o StringTable should be used when the keys are known to be strings.
  o SparseTable provides tables with default entries, so lookup never fails.
    The GeneralSparseTable constructor can be used to make any table type
    behave this way.
  o KeyedAccessFile allows values to be saved in a file, accessed as a table.

See Also:
o )help AssociationList
o )help EqTable
o )help StringTable
o )help SparseTable
o )help GeneralSparseTable
o )help KeyedAccessFile
o )show Table
```

21.1.1 Table (TABLE)



See

⇒ "HashTable" (HASHTBL) 9.1.1 on page 1085
⇒ "InnerTable" (INTABL) 10.27.1 on page 1299
⇒ "EqTable" (EQTBL) 6.2.1 on page 667

⇒ "StringTable" (STRTBL) 20.32.1 on page 2569
⇒ "GeneralSparseTable" (GSTBL) 8.5.1 on page 1044
⇒ "SparseTable" (STBL) 20.16.1 on page 2409

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
eval	every?	extract!	fill!	find
first	hash	index?	indices	insert!
inspect	key?	keys	latex	less?
map	map	map!	maxIndex	member?
members	minIndex	more?	parts	qelt
qsetelt!	reduce	remove	remove!	removeDuplicates
sample	search	select	select!	setelt
size?	swap!	table	#?	?=?
?~=?	?.?			

— domain TABLE Table —

```
)abbrev domain TABLE Table
++ Author: Stephen M. Watt, Barry Trager
++ Date Created: 1985
++ Date Last Updated: Sept 15, 1992
++ Basic Operations:
++ Related Domains: HashTable, EqTable, StringTable, AssociationList
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This is the general purpose table type.
++ The keys are hashed to look up the entries.
++ This creates a \spadtype{HashTable} if equal for the Key
++ domain is consistent with Lisp EQUAL otherwise an
++ \spadtype{AssociationList}

Table(Key: SetCategory, Entry: SetCategory):Exports == Implementation where
    Exports ==> TableAggregate(Key, Entry) with
                     finiteAggregate

    Implementation ==> InnerTable(Key, Entry,
        if hashable(Key)$Lisp then HashTable(Key, Entry, "UEQUAL")
          else AssociationList(Key, Entry))
```

————

— TABLE.dotabb —

```
"TABLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TABLE"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"TABLE" -> "TBAGG"
```

——————

21.2 domain TABLEAU Tableau

— Tableau.input —

```
)set break resume
)sys rm -f Tableau.output
)spool Tableau.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Tableau
--R Tableau S: SetCategory  is a domain constructor
--R Abbreviation for Tableau is TABLEAU
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TABLEAU
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              listOfLists : % -> List List S
--R tableau : List List S -> %
--R
--E 1

)spool
)lisp (bye)
```

——————

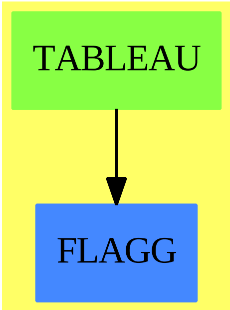
— Tableau.help —

```
====================================================================
Tableau examples
====================================================================

See Also:
o )show Tableau
```

———-

21.2.1 Tableau (TABLEAU)



Exports:
coerce listOfLists tableau

— **domain TABLEAU Tableau** —

```
)abbrev domain TABLEAU Tableau
++ Author: William H. Burge
++ Date Created: 1987
++ Date Last Updated: 23 Sept 1991
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: Young tableau
++ References:
++ Description:
++ The tableau domain is for printing Young tableaux, and
++ coercions to and from List List S where S is a set.

Tableau(S:SetCategory):Exports == Implementation where
  L   ==> List
  I   ==> Integer
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  V   ==> Vector
  fm==>formMatrix$PrintableForm()
  Exports ==>  with
    tableau : L L S -> %
      ++ tableau(ll) converts a list of lists ll to a tableau.
    listOfLists : % -> L L S
      ++ listOfLists t converts a tableau t to a list of lists.
```

```
    coerce : % -> OUT
      ++ coerce(t) converts a tableau t to an output form.
  Implementation ==> add

    Rep := L L S

    tableau(lls:(L L S)) == lls pretend %
    listOfLists(x:%):(L L S) == x pretend (L L S)
    makeupv : (NNI,L S) -> L OUT
    makeupv(n,ls)==
        v:=new(n,message " ")$(List OUT)
        for i in 1..#ls for s in  ls repeat v.i:=box(s::OUT)
        v
    maketab : L L S -> OUT
    maketab lls ==
      ll :  L OUT :=
        empty? lls => [[empty()]]
        sz:NNI:=# first lls
        [blankSeparate makeupv(sz,i) for i in lls]
      pile ll

    coerce(x:%):OUT == maketab listOfLists x
```

— **TABLEAU.dotabb** —

```
"TABLEAU" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TABLEAU"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"TABLEAU" -> "FLAGG"
```

21.3 domain TS TaylorSeries

— **TaylorSeries.input** —

```
)set break resume
)sys rm -f TaylorSeries.output
)spool TaylorSeries.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show TaylorSeries
--R TaylorSeries Coef: Ring  is a domain constructor
--R Abbreviation for TaylorSeries is TS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Coef) -> %                   ?*? : (Coef,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,List Symbol) -> %              D : (%,Symbol) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : Polynomial Coef -> %
--R coerce : Symbol -> %                  coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R differentiate : (%,Symbol) -> %       eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %       eval : (%,Symbol,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial? : % -> Boolean
--R one? : % -> Boolean                   pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      variables : % -> List Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> %
--R D : (%,Symbol,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Polynomial Coef
--R coefficient : (%,List Symbol,List NonNegativeInteger) -> %
--R coefficient : (%,Symbol,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents Symbol) -> Coef
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : % -> % if Coef has INTDOM
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R degree : % -> IndexedExponents Symbol
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> %
--R differentiate : (%,Symbol,NonNegativeInteger) -> %
--R differentiate : (%,List Symbol) -> %
--R eval : (%,List Symbol,List %) -> %
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R fintegrate : ((() -> %),Symbol,Coef) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol,Coef) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has ALGEBRA FRAC INT
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List Symbol,List NonNegativeInteger) -> %
--R monomial : (%,Symbol,NonNegativeInteger) -> %
--R monomial : (Coef,IndexedExponents Symbol) -> %
--R monomial : (%,Symbol,IndexedExponents Symbol) -> %
--R monomial : (%,List Symbol,List IndexedExponents Symbol) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Symbol,NonNegativeInteger) -> NonNegativeInteger
--R order : (%,Symbol) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
```

```
--R
--E 1

)spool
)lisp (bye)
```

———-

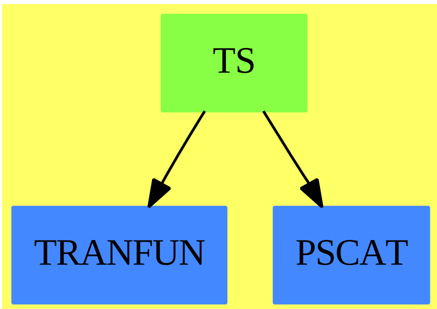
— **TaylorSeries.help** —

```
====================================================================
TaylorSeries examples
====================================================================

See Also:
o )show TaylorSeries
```

———-

21.3.1 TaylorSeries (TS)



See

⇒ "SparseMultivariateTaylorSeries" (SMTS) 20.15.1 on page 2399

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	charthRoot	coefficient	coerce	complete
cos	cosh	cot	coth	csc
csch	D	degree	differentiate	eval
exp	exquo	extend	fintegrate	hash
integrate	latex	leadingCoefficient	leadingMonomial	log
map	monomial	monomial?	nthRoot	one?
order	pi	pole?	polynomial	recip
reductum	sample	sec	sech	sin
sinh	sqrt	subtractIfCan	tan	tanh
unit?	unitCanonical	unitNormal	variables	zero?
?*?	?**?	?+?	?-?	-?
?=?	?^?	?~=?	?/?	

— domain TS TaylorSeries —

```
)abbrev domain TS TaylorSeries
++ Authors: Burge, Watt, Williamson
++ Date Created: 15 August 1988
++ Date Last Updated: 18 May 1991
++ Basic Operations:
++ Related Domains: SparseMultivariateTaylorSeries
++ Also See: UnivariateTaylorSeries
++ AMS Classifications:
++ Keywords: multivariate, Taylor, series
++ Examples:
++ References:
++ Description:
++ \spadtype{TaylorSeries} is a general multivariate Taylor series domain
++ over the ring Coef and with variables of type Symbol.

TaylorSeries(Coef): Exports == Implementation where
  Coef  : Ring
  L   ==> List
  NNI ==> NonNegativeInteger
  SMP ==> Polynomial Coef
  StS ==> Stream SMP

  Exports ==> MultivariateTaylorSeriesCategory(Coef,Symbol) with
    coefficient: (%,NNI) -> SMP
      ++\spad{coefficient(s, n)} gives the terms of total degree n.
    coerce: Symbol -> %
      ++\spad{coerce(s)} converts a variable to a Taylor series
    coerce: SMP -> %
      ++\spad{coerce(s)} regroups terms of s by total degree
      ++ and forms a series.
```

```
    if Coef has Algebra Fraction Integer then
      integrate: (%,Symbol,Coef) -> %
        ++\spad{integrate(s,v,c)} is the integral of s with respect
        ++ to v and having c as the constant of integration.
      fintegrate: (() -> %,Symbol,Coef) -> %
        ++\spad{fintegrate(f,v,c)} is the integral of \spad{f()} with respect
        ++ to v and having c as the constant of integration.
        ++ The evaluation of \spad{f()} is delayed.

  Implementation ==> SparseMultivariateTaylorSeries(Coef,Symbol,SMP) add
    Rep := StS -- Below we use the fact that Rep of PS is Stream SMP.

    polynomial(s,n) ==
      sum : SMP := 0
      for i in 0..n while not empty? s repeat
        sum := sum + frst s
        s:= rst s
      sum
```

— **TS.dotabb** —

```
"TS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TS"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"PSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PSCAT"]
"TS" -> "PSCAT"
"TS" -> "TRANFUN"
```

21.4 domain TEX TexFormat

21.4.1 product(product(i*j,i=a..b),j=c..d) fix

The expression prints properly in ascii text but the tex output is incorrect. Originally the input

```
product(product(i*j,i=a..b),j=c..d)
```

prints as

$$PI2\left(j=c,\ d,\ PI2\left(i=a,\ b,\ i\ j\right)\right) \tag{1}$$

but now says: The problem is in [[src/algebra/tex.spad.pamphlet]] in the list of constants. The code used to read

```
    plexOps      : L S := ["SIGMA","SIGMA2","PI","INTSIGN","INDEFINTEGRAL"]$(L S)
    plexPrecs    : L I := [    700, 800,      700,            700]$(L I)
```

it now reads:

— product(product(i*j,i=a..b),j=c..d) fix —

```
    plexOps      : L S := ["SIGMA","SIGMA2","PI","PI2","INTSIGN","INDEFINTEGRAL"]$(L S)
    plexPrecs    : L I := [    700, 800,     700, 800 , 700,      700]$(L I)
```

in addition we need to add a line defining [[PI2]] in [[formatPlex]]:

— define PI2 —

```
        op = "PI2"     => "\prod"
```

— TexFormat.input —

```
)set break resume
)sys rm -f TexFormat.output
)spool TexFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 11
(1/2)::TEX
--R
--R
--R   (1)  ["$$","\frac{1}{2} ","$$"]
--R                                                                 Type: TexFormat
--E 1

--S 2 of 11
(1/(x+5))::TEX
--R
--R
--R   (2)  ["$$","\frac{1}{{x+5}} ","$$"]
--R                                                                 Type: TexFormat
--E 2

--S 3 of 11
((x+3)/(y-5))::TEX
--R
--R
--R   (3)  ["$$","\frac{{x+3}}{{y -5}} ","$$"]
--R                                                                 Type: TexFormat
```

```
--E 3

--S 4 of 11
)set output fraction horizontal
--R
--E 4

--S 5 of 11
(1/2)::TEX
--R
--R
--R   (4)  ["$$","SLASH ","\left(","{1, \: 2} ","\right)","$$"]
--R                                                            Type: TexFormat
--E 5

--S 6 of 11
(1/(x+5))::TEX
--R
--R
--R   (5)
--R   ["$$","SLASH ","\left(","{1, \: {\left( x+5 ","\right)}}","\right)","$$"]
--R                                                            Type: TexFormat
--E 6

--S 7 of 11
)set output mathml on
--R
--E 7

--S 8 of 11
1/2
--R
--R
--R   (6)  1/2
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mn>1</mn><mo>/</mo><mn>2</mn></mrow>
--R</math>
--R
--R                                                     Type: Fraction Integer
--E 8

--S 9 of 11
1/(x+5)
--R
--R
--R   (7)  1/(x + 5)
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mn>1</mn><mo>/</mo><mrow><mo>(</mo><mi>x</mi><mo>+</mo><mn>5</mn><mo>)</mo></mrow><
--R</math>
--R
```

```
--R                                              Type: Fraction Polynomial Integer
--E 9

--S 10 of 11
(x+3)/(y-5)
--R
--R
--R   (8)  (x + 3)/(y - 5)
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mrow><mo>(</mo><mi>x</mi><mo>+</mo><mn>3</mn><mo>)</mo></mrow><mo>/</mo><mrow><mo>(</mo><mi>y<
--R</math>
--R
--R                                              Type: Fraction Polynomial Integer
--E 10

--S 11 of 11
)show TexFormat
--R
--R TexFormat  is a domain constructor
--R Abbreviation for TexFormat is TEX
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TEX
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : OutputForm -> %
--R coerce : % -> OutputForm              display : % -> Void
--R display : (%,Integer) -> Void         epilogue : % -> List String
--R hash : % -> SingleInteger             latex : % -> String
--R new : () -> %                         prologue : % -> List String
--R tex : % -> List String                ?~=? : (%,%) -> Boolean
--R convert : (OutputForm,Integer,OutputForm) -> %
--R convert : (OutputForm,Integer) -> %
--R setEpilogue! : (%,List String) -> List String
--R setPrologue! : (%,List String) -> List String
--R setTex! : (%,List String) -> List String
--R
--E 11

)spool
)lisp (bye)
```

———-

— TexFormat.help —

```
====================================================================
TexFormat examples
====================================================================
```

```
You can ask Axiom to show latex output. In particular, this can be
used for complex output.

(1/2)::TEX

     ["$$","\frac{1}{2} ","$$"]

(1/(x+5))::TEX

     ["$$","\frac{1}{{x+5}} ","$$"]

((x+3)/(y-5))::TEX

     ["$$","\frac{{x+3}}{{y -5}} ","$$"]


We can change the fraction display so it is horizontal:

)set output fraction horizontal

(1/2)::TEX

     ["$$","SLASH ","\left(","{1, \: 2} ","\right)","$$"]

(1/(x+5))::TEX

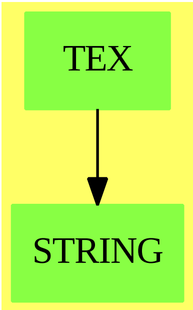
     ["$$","SLASH ","\left(","{1, \: {x+5}} ","\right)","$$"]

((x+3)/(y-5))::TEX

     ["$$","SLASH ","\left(","{{x+3}, \: {y -5}} ","\right)","$$"]


See Also:
o )show TexFormat
```

21.4.2 TexFormat (TEX)



Exports:

coerce	convert	display	epilogue	hash
latex	new	prologue	setEpilogue!	setPrologue!
setTex!	tex	convert	?=?	?~=?

— domain TEX TexFormat —

```
)abbrev domain TEX TexFormat
++ Author: Robert S. Sutor
++ Date Created: 1987 through 1992
++ Change History:
++   05/15/91 RSS Changed matrix formatting to use array environment.
++   06/27/91 RSS Fixed segments
++   08/12/91 RSS Removed some grouping for things, added newWithNum and
++                ungroup, improved line splitting
++   08/15/91 RSS Added mbox support for strings
++   10/15/91 RSS Handle \%\% at beginning of string
++   01/22/92 RSS Use \[ and \] instead of $$ and $$. Use
++                %AXIOM STEP NUMBER: instead of \leqno
++   02/27/92 RSS Escape dollar signs appearing in the input.
++   03/09/92 RSS Handle explicit blank appearing in the input.
++   11/28/93 JHD Added code for the VCONCAT and TAG operations.
++   06/27/95 RSS Change back to $$ and \leqno for Saturn
++ Basic Operations: coerce, convert, display, epilogue,
++   tex, new, prologue, setEpilogue!, setTex!, setPrologue!
++ Related Constructors: TexFormat1
++ Also See: ScriptFormulaFormat
++ AMS Classifications:
++ Keywords: TeX, LaTeX, output, format
++ References: \TeX{} is a trademark of the American Mathematical Society.
++ Description:
++ \spadtype{TexFormat} provides a coercion from \spadtype{OutputForm} to
++ \TeX{} format.  The particular dialect of \TeX{} used is \LaTeX{}.
++ The basic object consists of three parts: a prologue, a
++ tex part and an epilogue. The functions \spadfun{prologue},
```

```
++ \spadfun{tex} and \spadfun{epilogue} extract these parts,
++ respectively.  The main guts of the expression go into the tex part.
++ The other parts can be set (\spadfun{setPrologue!},
++ \spadfun{setEpilogue!}) so that contain the appropriate tags for
++ printing. For example, the prologue and epilogue might simply
++ contain ``\verb+\[+'' and ``\verb+\]+'', respectively, so that
++ the TeX section will be printed in LaTeX display math mode.

TexFormat(): public == private where
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String
  US     ==> UniversalSegment(Integer)

  public == SetCategory with
    coerce:   E -> $
      ++ coerce(o) changes o in the standard output format to TeX
      ++ format.
    convert:  (E,I) -> $
      ++ convert(o,step) changes o in standard output format to
      ++ TeX format and also adds the given step number. This is useful
      ++ if you want to create equations with given numbers or have the
      ++ equation numbers correspond to the interpreter step numbers.
    convert:  (E,I,E) -> $
      ++ convert(o,step,type) changes o in standard output format to
      ++ TeX format and also adds the given step number and type. This
      ++ is useful if you want to create equations with given numbers
      ++ or have the equation numbers correspond to the interpreter step
      ++ numbers.
    display:  ($, I) -> Void
      ++ display(t,width) outputs the TeX formatted code t so that each
      ++ line has length less than or equal to \spadvar{width}.
    display:  $ -> Void
      ++ display(t) outputs the TeX formatted code t so that each
      ++ line has length less than or equal to the value set by
      ++ the system command \spadsyscom{set output length}.
    epilogue: $ -> L S
      ++ epilogue(t) extracts the epilogue section of a TeX form t.
    tex:      $ -> L S
      ++ tex(t) extracts the TeX section of a TeX form t.
    new:      () -> $
      ++ new() create a new, empty object. Use \spadfun{setPrologue!},
      ++ \spadfun{setTex!} and \spadfun{setEpilogue!} to set the various
      ++ components of this object.
    prologue: $ -> L S
      ++ prologue(t) extracts the prologue section of a TeX form t.
    setEpilogue!: ($, L S) -> L S
      ++ setEpilogue!(t,strings) sets the epilogue section of a TeX form t
      ++ to strings.
```

```
    setTex!:  ($, L S) -> L S
      ++ setTex!(t,strings) sets the TeX section of a TeX form t to strings.
    setPrologue!: ($, L S) -> L S
      ++ setPrologue!(t,strings) sets the prologue section of a TeX form t
      ++ to strings.

  private == add
    import OutputForm
    import Character
    import Integer
    import List OutputForm
    import List String

    Rep := Record(prolog : L S, TeX : L S, epilog : L S)

    -- local variables declarations and definitions

    expr: E
    prec,opPrec: I
    str:  S
    blank         : S := " \  "

    maxPrec       : I   := 1000000
    minPrec       : I   := 0

    unaryOps      : L S := ["-","^"]$(L S)
    unaryPrecs    : L I := [700,260]$(L I)

    -- the precedence of / in the following is relatively low because
    -- the bar obviates the need for parentheses.
    binaryOps     : L S := ["+->","|","**","/","<",">","=","OVER"]$(L S)
    binaryPrecs   : L I := [0,0,900, 700,400,400,400,   700]$(L I)

    naryOps       : L S := ["-","+","*",blank,",",";"," ","ROW","",
       " \cr ","&"," \\ "]$(L S)
    naryPrecs     : L I := [700,700,800,  800,110,110,  0,    0, 0,
             0,  0,   0]$(L I)
    naryNGOps     : L S := ["ROW","&"]$(L S)

\getchunk{product(product(i*j,i=a..b),j=c..d) fix}

    specialOps    : L S := ["MATRIX","BRACKET","BRACE","CONCATB","VCONCAT",  _
                            "AGGLST","CONCAT","OVERBAR","ROOT","SUB","TAG", _
                            "SUPERSUB","ZAG","AGGSET","SC","PAREN", _
                            "SEGMENT","QUOTE","theMap" ]

    -- the next two lists provide translations for some strings for
    -- which TeX provides special macros.

    specialStrings : L S :=
```

```
      ["cos", "cot", "csc", "log", "sec", "sin", "tan",
        "cosh", "coth", "csch", "sech", "sinh", "tanh",
          "acos","asin","atan","erf","...","$","infinity"]
    specialStringsInTeX : L S :=
      ["\cos","\cot","\csc","\log","\sec","\sin","\tan",
        "\cosh","\coth","\csch","\sech","\sinh","\tanh",
          "\arccos","\arcsin","\arctan","\erf","\ldots","\$","\infty"]

    -- local function signatures

    addBraces:      S -> S
    addBrackets:    S -> S
    group:          S -> S
    formatBinary:   (S,L E, I) -> S
    formatFunction: (S,L E, I) -> S
    formatMatrix:   L E -> S
    formatNary:     (S,L E, I) -> S
    formatNaryNoGroup: (S,L E, I) -> S
    formatNullary:  S -> S
    formatPlex:     (S,L E, I) -> S
    formatSpecial:  (S,L E, I) -> S
    formatUnary:    (S,  E, I) -> S
    formatTex:      (E,I) -> S
    newWithNum:     I -> $
    parenthesize:   S -> S
    precondition:   E -> E
    postcondition:  S -> S
    splitLong:      (S,I) -> L S
    splitLong1:     (S,I) -> L S
    stringify:      E -> S
    ungroup:        S -> S

    -- public function definitions

    new() : $ ==
--    [["\["]$(L S), [""]$(L S), ["\]"]$(L S)]$Rep
      [["$$"]$(L S), [""]$(L S), ["$$"]$(L S)]$Rep

    newWithNum(stepNum: I) : $ ==
--    num : S := concat("%AXIOM STEP NUMBER: ",string(stepNum)$S)
--    [["\["]$(L S), [""]$(L S), ["\]",num]$(L S)]$Rep
      num : S := concat(concat("\leqno(",string(stepNum)$S),")")$S
      [["$$"]$(L S), [""]$(L S), [num,"$$"]$(L S)]$Rep

    coerce(expr : E): $ ==
      f : $ := new()$$
      f.TeX := [postcondition
        formatTex(precondition expr, minPrec)]$(L S)
      f
```

```
convert(expr : E, stepNum : I): $ ==
  f : $ := newWithNum(stepNum)
  f.TeX := [postcondition
    formatTex(precondition expr, minPrec)]$(L S)
  f

display(f : $, len : I) ==
  s,t : S
  for s in f.prolog repeat sayTeX$Lisp s
  for s in f.TeX repeat
    for t in splitLong(s, len) repeat sayTeX$Lisp t
  for s in f.epilog repeat sayTeX$Lisp s
  void()$Void

display(f : $) ==
  display(f, _$LINELENGTH$Lisp pretend I)

prologue(f : $) == f.prolog
tex(f : $)  == f.TeX
epilogue(f : $) == f.epilog

setPrologue!(f : $, l : L S) == f.prolog := l
setTex!(f : $, l : L S)  == f.TeX := l
setEpilogue!(f : $, l : L S) == f.epilog := l

coerce(f : $): E ==
  s,t : S
  l : L S := nil
  for s in f.prolog repeat l := concat(s,l)
  for s in f.TeX repeat
    for t in splitLong(s, (_$LINELENGTH$Lisp pretend Integer) - 4) repeat
      l := concat(t,l)
  for s in f.epilog repeat l := concat(s,l)
  (reverse l) :: E

-- local function definitions

ungroup(str: S): S ==
  len : I := #str
  len < 2 => str
  lbrace : Character := char "{"
  rbrace : Character := char "}"
  -- drop leading and trailing braces
  if (str.1 =$Character lbrace) and (str.len =$Character rbrace) then
    u : US := segment(2,len-1)$US
    str := str.u
  str

postcondition(str: S): S ==
  str := ungroup str
```

```
  len : I := #str
  plus : Character := char "+"
  minus: Character := char "-"
  len < 4 => str
  for i in 1..(len-1) repeat
    if (str.i =$Character plus) and (str.(i+1) =$Character minus)
      then setelt(str,i,char " ")$S
  str

stringify expr == (object2String$Lisp expr) pretend S

lineConcat( line : S, lines: L S ) : L S ==
  length := #line

  if ( length > 0 ) then
    -- If the last character is a backslash then split at "\ ".
    -- Reinstate the blank.

    if (line.length = char "\" ) then line := concat(line, " ")

    -- Remark: for some reason, "\%" at the beginning
    -- of a line has the "\" erased when printed

    if ( line.1 = char "%" ) then line := concat(" \", line)
    else if ( line.1 = char "\" ) and length > 1 and ( line.2 = char "%" ) then
      line := concat(" ", line)

    lines := concat(line,lines)$List(S)
  lines

splitLong(str : S, len : I): L S ==
  -- this blocks into lines
  if len < 20 then len := _$LINELENGTH$Lisp
  splitLong1(str, len)

splitLong1(str : S, len : I) ==
  -- We first build the list of lines backwards and then we
  -- reverse it.

  l : List S := nil
  s : S := ""
  ls : I := 0
  ss : S
  lss : I
  for ss in split(str,char " ") repeat
    -- have the newline macro end a line (even if it means the line
    -- is slightly too long)

    ss = "\\" =>
      l := lineConcat( concat(s,ss), l )
```

```
          s := ""
          ls := 0

        lss := #ss

        -- place certain tokens on their own lines for clarity

        ownLine : Boolean :=
          u : US := segment(1,4)$US
          (lss > 3) and ("\end" = ss.u) => true
          u      := segment(1,5)$US
          (lss > 4) and ("\left" = ss.u) => true
          u      := segment(1,6)$US
          (lss > 5) and (("\right" = ss.u) or ("\begin" = ss.u)) => true
          false

        if ownLine or (ls + lss > len) then
          if not empty? s then l := lineConcat( s, l )
          s := ""
          ls := 0

        ownLine or lss > len => l := lineConcat( ss, l )

        (lss = 1) and (ss.1 = char "\") =>
          ls := ls + lss + 2
          s := concat(s,concat(ss,"  ")$S)$S

        ls := ls + lss + 1
        s := concat(s,concat(ss," ")$S)$S

      if ls > 0 then l := lineConcat( s, l )

      reverse l

    group str ==
      concat ["{",str,"}"]

    addBraces str ==
      concat ["\left\{ ",str," \right\}"]

    addBrackets str ==
      concat ["\left[ ",str," \right]"]

    parenthesize str ==
      concat ["\left( ",str," \right)"]

    precondition expr ==
      outputTran$Lisp expr

    formatSpecial(op : S, args : L E, prec : I) : S ==
```

```
arg : E
prescript : Boolean := false
op = "theMap" => "\mbox{theMap(...)}"
op = "AGGLST" =>
  formatNary(",",args,prec)
op = "AGGSET" =>
  formatNary(";",args,prec)
op = "TAG" =>
  group concat [formatTex(first args,prec),
                "\rightarrow",
                 formatTex(second args,prec)]
op = "VCONCAT" =>
  group concat("\begin{array}{c}",
               concat(concat([concat(formatTex(u, minPrec),"\\")
                              for u in args]::L S),
                      "\end{array}"))
op = "CONCATB" =>
  formatNary(" ",args,prec)
op = "CONCAT" =>
  formatNary("",args,minPrec)
op = "QUOTE" =>
  group concat("{\tt '}",formatTex(first args, minPrec))
op = "BRACKET" =>
  group addBrackets ungroup formatTex(first args, minPrec)
op = "BRACE" =>
  group addBraces ungroup formatTex(first args, minPrec)
op = "PAREN" =>
  group parenthesize ungroup formatTex(first args, minPrec)
op = "OVERBAR" =>
  null args => ""
  group concat ["\overline ",formatTex(first args, minPrec)]
op = "ROOT" =>
  null args => ""
  tmp : S := group formatTex(first args, minPrec)
  null rest args => group concat ["\sqrt ",tmp]
  group concat
    ["\root ",group formatTex(first rest args, minPrec)," \of ",tmp]
op = "SEGMENT" =>
  tmp : S := concat [formatTex(first args, minPrec),".."]
  group
    null rest args =>  tmp
    concat [tmp,formatTex(first rest args, minPrec)]
op = "SUB" =>
  group concat [formatTex(first args, minPrec)," \sb ",
    formatSpecial("AGGLST",rest args,minPrec)]
op = "SUPERSUB" =>
  -- variable name
  form : List S := [formatTex(first args, minPrec)]
  -- subscripts
  args := rest args
```

```
        null args => concat(form)$S
        tmp : S := formatTex(first args, minPrec)
        if (tmp ^= "") and (tmp ^= "{}") and (tmp ^= " ") then
          form := append(form,[" \sb ",group tmp])$(List S)
        -- superscripts
        args := rest args
        null args => group concat(form)$S
        tmp : S := formatTex(first args, minPrec)
        if (tmp ^= "") and (tmp ^= "{}") and (tmp ^= " ") then
          form := append(form,[" \sp ",group tmp])$(List S)
        -- presuperscripts
        args := rest args
        null args => group concat(form)$S
        tmp : S := formatTex(first args, minPrec)
        if (tmp ^= "") and (tmp ^= "{}") and (tmp ^= " ") then
          form := append([" \sp ",group tmp],form)$(List S)
          prescript := true
        -- presubscripts
        args := rest args
        null args =>
          group concat
            prescript => cons("{}",form)
            form
        tmp : S := formatTex(first args, minPrec)
        if (tmp ^= "") and (tmp ^= "{}") and (tmp ^= " ") then
          form := append([" \sb ",group tmp],form)$(List S)
          prescript := true
        group concat
          prescript => cons("{}",form)
          form
      op = "SC" =>
        -- need to handle indentation someday
        null args => ""
        tmp := formatNaryNoGroup(" \\ ", args, minPrec)
        group concat ["\begin{array}{l} ",tmp," \end{array} "]
      op = "MATRIX" => formatMatrix rest args
      op = "ZAG" =>
        concat [" \zag{",formatTex(first args, minPrec),"}{",
          formatTex(first rest args,minPrec),"}"]
      concat ["not done yet for ",op]

    formatPlex(op : S, args : L E, prec : I) : S ==
      hold : S
      p : I := position(op,plexOps)
      p < 1 => error "unknown Tex unary op"
      opPrec := plexPrecs.p
      n : I := #args
      (n ^= 2) and (n ^= 3) => error "wrong number of arguments for plex"
      s : S :=
        op = "SIGMA"   => "\sum"
```

```
        op = "SIGMA2"   => "\sum"
        op = "PI"      => "\prod"
\getchunk{define PI2}
        op = "INTSIGN" => "\int"
        op = "INDEFINTEGRAL" => "\int"
        "????"
      hold := formatTex(first args,minPrec)
      args := rest args
      if op ^= "INDEFINTEGRAL" then
        if hold ^= "" then
          s := concat [s," \sb",group concat ["\displaystyle ",hold]]
        if not null rest args then
          hold := formatTex(first args,minPrec)
          if hold ^= "" then
            s := concat [s," \sp",group concat ["\displaystyle ",hold]]
          args := rest args
        s := concat [s," ",formatTex(first args,minPrec)]
      else
        hold := group concat [hold," ",formatTex(first args,minPrec)]
        s := concat [s," ",hold]
      if opPrec < prec then s := parenthesize s
      group s

    formatMatrix(args : L E) : S ==
      -- format for args is [[ROW ...],[ROW ...],[ROW ...]]
      -- generate string for formatting columns (centered)
      cols : S := "{"
      for i in 2..#(first(args) pretend L E) repeat
        cols := concat(cols,"c")
      cols := concat(cols,"} ")
      group addBrackets concat
        ["\begin{array}",cols,formatNaryNoGroup(" \\ ",args,minPrec),
          " \end{array} "]

    formatFunction(op : S, args : L E, prec : I) : S ==
      group concat [op, " ", parenthesize formatNary(",",args,minPrec)]

    formatNullary(op : S) ==
      op = "NOTHING" => ""
      group concat [op,"()"]

    formatUnary(op : S, arg : E, prec : I) ==
      p : I := position(op,unaryOps)
      p < 1 => error "unknown Tex unary op"
      opPrec := unaryPrecs.p
      s : S := concat [op,formatTex(arg,opPrec)]
      opPrec < prec => group parenthesize s
      op = "-" => s
      group s
```

```
formatBinary(op : S, args : L E, prec : I) : S ==
  p : I := position(op,binaryOps)
  p < 1 => error "unknown Tex binary op"
  op :=
    op = "|"     => " \mid "
    op = "**"    => " \sp "
    op = "/"     => " \over "
    op = "OVER"  => " \over "
    op = "+->"   => " \mapsto "
    op
  opPrec := binaryPrecs.p
  s : S := formatTex(first args, opPrec)
  if op = " \over " then
    s := concat [" \frac{",s,"}{",formatTex(first rest args, opPrec),"}"]
  else if op = " \sp " then
    s := concat [s,"^",formatTex(first rest args, opPrec)]
  else
    s := concat [s,op,formatTex(first rest args, opPrec)]
  group
    op = " \over " => s
    opPrec < prec => parenthesize s
    s

formatNary(op : S, args : L E, prec : I) : S ==
  group formatNaryNoGroup(op, args, prec)

formatNaryNoGroup(op : S, args : L E, prec : I) : S ==
  null args => ""
  p : I := position(op,naryOps)
  p < 1 => error "unknown Tex nary op"
  op :=
    op = ","     => ", \: "
    op = ";"     => "; \: "
    op = "*"     => blank
    op = " "     => " \ "
    op = "ROW"   => " & "
    op
  l : L S := nil
  opPrec := naryPrecs.p
  for a in args repeat
    l := concat(op,concat(formatTex(a,opPrec),l)$L(S))$L(S)
  s : S := concat reverse rest l
  opPrec < prec => parenthesize s
  s

formatTex(expr,prec) ==
  i,len : Integer
  intSplitLen : Integer := 20
  ATOM(expr)$Lisp pretend Boolean =>
    str := stringify expr
```

```
  len := #str
  INTEGERP$Lisp expr =>
    i := expr pretend Integer
    if (i < 0) or (i > 9)
      then
        group
           nstr : String := ""
           -- insert some blanks into the string, if too long
           while ((len := #str) > intSplitLen) repeat
             nstr := concat [nstr," ",
               elt(str,segment(1,intSplitLen)$US)]
             str := elt(str,segment(intSplitLen+1)$US)
           empty? nstr => str
           nstr :=
             empty? str => nstr
             concat [nstr," ",str]
           elt(nstr,segment(2)$US)
      else str
  str = "%pi" => "\pi"
  str = "%e"  => "e"
  str = "%i"  => "i"
  len > 1 and str.1 = char "%" and str.2 = char "%" =>
    u : US := segment(3,len)$US
    concat(" \%\%",str.u)
  len > 0 and str.1 = char "%" => concat(" \",str)
  len > 1 and digit? str.1 => group str -- should handle floats
  len > 0 and str.1 = char "_"" =>
    concat(concat(" \mbox{\tt ",str),"} ")
  len = 1 and str.1 = char " " => "{\ }"
  (i := position(str,specialStrings)) > 0 =>
    specialStringsInTeX.i
  (i := position(char " ",str)) > 0 =>
    -- We want to preserve spacing, so use a roman font.
    concat(concat(" \mbox{\rm ",str),"} ")
  str
l : L E := (expr pretend L E)
null l => blank
op : S := stringify first l
args : L E := rest l
nargs : I := #args

-- special cases
member?(op, specialOps) => formatSpecial(op,args,prec)
member?(op, plexOps)    => formatPlex(op,args,prec)

-- nullary case
0 = nargs => formatNullary op

-- unary case
(1 = nargs) and member?(op, unaryOps) =>
```

```
    formatUnary(op, first args, prec)

  -- binary case
  (2 = nargs) and member?(op, binaryOps) =>
    formatBinary(op, args, prec)

  -- nary case
  member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
  member?(op,naryOps) => formatNary(op,args, prec)
  op := formatTex(first l,minPrec)
  formatFunction(op,args,prec)
```

— **TEX.dotabb** —

```
"TEX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TEX"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"TEX" -> "STRING"
```

21.5 domain TEXTFILE TextFile

— **TextFile.input** —

```
)set break resume
)sys rm -f TextFile.output
)spool TextFile.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
f1: TextFile := open("/etc/group", "input")
--R
--R
--R   (1)  "/etc/group"
--R                                                                 Type: TextFile
--E 1

--S 2 of 10
f2: TextFile := open("MOTD", "output")
--R
--R
```

```
--R   (2)  "MOTD"
--R                                                          Type: TextFile
--E 2

--S 3 of 10
l := readLine! f1
--R
--R
--I   (3)  "ROOT:x:0:"
--R                                                            Type: String
--E 3

--S 4 of 10
writeLine!(f2, upperCase l)
--R
--R
--I   (4)  "ROOT:X:0:"
--R                                                            Type: String
--E 4

--S 5 of 10
while not endOfFile? f1 repeat
  s := readLine! f1
  writeLine!(f2, upperCase s)
--R
--R                                                              Type: Void
--E 5

--S 6 of 10
close! f1
--R
--R
--R   (6)  "/etc/group"
--R                                                          Type: TextFile
--E 6

--S 7 of 10
write!(f2, "-The-")
--R
--R
--R   (7)  "-The-"
--R                                                            Type: String
--E 7

--S 8 of 10
write!(f2, "-End-")
--R
--R
--R   (8)  "-End-"
--R                                                            Type: String
```

```
--E 8

--S 9 of 10
writeLine! f2
--R
--R
--R   (9)  ""
--R                                                               Type: String
--E 9

--S 10 of 10
close! f2
--R
--R
--R   (10)  "MOTD"
--R                                                             Type: TextFile
--E 10
)system rm -f MOTD
)spool
)lisp (bye)
```

— TextFile.help —

```
====================================================================
TextFile examples
====================================================================

The domain TextFile allows Axiom to read and write character data and
exchange text with other programs.  This type behaves in Axiom much
like a File of strings, with additional operations to cause new lines.
We give an example of how to produce an upper case copy of a file.

This is the file from which we read the text.

  f1: TextFile := open("/etc/group", "input")
    "/etc/group"
                               Type: TextFile

This is the file to which we write the text.

  f2: TextFile := open("/tmp/MOTD", "output")
    "MOTD"
                               Type: TextFile

Entire lines are handled using the readLine and writeLine operations.

  l := readLine! f1
```

```
    "root:x:0:root"
                                   Type: String

  writeLine!(f2, upperCase l)
    "ROOT:X:0:ROOT"
                                   Type: String
```

Use the endOfFile? operation to check if you have reached the end of the file.

```
  while not endOfFile? f1 repeat
    s := readLine! f1
    writeLine!(f2, upperCase s)
                                   Type: Void
```

The file f1 is exhausted and should be closed.

```
  close! f1
    "/etc/group"
                                   Type: TextFile
```

It is sometimes useful to write lines a bit at a time. The write operation allows this.

```
  write!(f2, "-The-")
    "-The-"
                                   Type: String

  write!(f2, "-End-")
    "-End-"
                                   Type: String
```

This ends the line. This is done in a machine-dependent manner.

```
  writeLine! f2
    ""
                                   Type: String

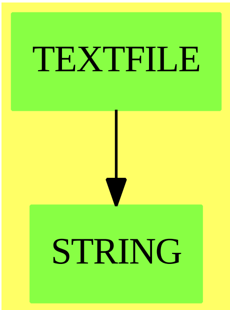
  close! f2
    "MOTD"
                                   Type: TextFile
```

Finally, clean up.

```
  )system rm /tmp/MOTD
```

See Also:
o)help File
o)help KeyedAccessFile
o)help Library
o)show TextFile

21.5.1 TextFile (TEXTFILE)



See

⇒ "File" (FILE) 7.2.1 on page 770
⇒ "BinaryFile" (BINFILE) 3.8.1 on page 277
⇒ "KeyedAccessFile" (KAFILE) 12.2.1 on page 1377
⇒ "Library" (LIB) 13.2.1 on page 1392

Exports:

close!	coerce	endOfFile?	hash	iomode
latex	name	open	read!	readIfCan!
readLine!	readLineIfCan!	reopen!	write!	writeLine!
?=?	?~=?			

— domain TEXTFILE TextFile —

```
)abbrev domain TEXTFILE TextFile
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 4, 1991
++ Basic Operations: writeLine! readLine! readLineIfCan! readIfCan! endOfFile?
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain provides an implementation of text files.  Text is stored
++ in these files using the native character set of the computer.

TextFile: Cat == Def where
```

```
StreamName ==> Union(FileName, "console")

Cat == FileCategory(FileName, String) with
    writeLine_!: (%, String) -> String
      ++ writeLine!(f,s) writes the contents of the string s
      ++ and finishes the current line in the file f.
      ++ The value of s is returned.

    writeLine_!: % -> String
      ++ writeLine!(f) finishes the current line in the file f.
      ++ An empty string is returned.  The call \spad{writeLine!(f)} is
      ++ equivalent to \spad{writeLine!(f,"")}.

    readLine_!: % -> String
      ++ readLine!(f) returns a string of the contents of a line from
      ++ the file f.

    readLineIfCan_!: % -> Union(String, "failed")
      ++ readLineIfCan!(f) returns a string of the contents of a line from
      ++ file f, if possible.  If f is not readable or if it is
      ++ positioned at the end of file, then \spad{"failed"} is returned.

    readIfCan_!: % -> Union(String, "failed")
      ++ readIfCan!(f) returns a string of the contents of a line from
      ++ file f, if possible.  If f is not readable or if it is
      ++ positioned at the end of file, then \spad{"failed"} is returned.

    endOfFile?: % -> Boolean
      ++ endOfFile?(f) tests whether the file f is positioned after the
      ++ end of all text.  If the file is open for output, then
      ++ this test is always true.

Def == File(String) add
    FileState ==> SExpression

    Rep := Record(fileName:   FileName,    _
                  fileState:  FileState,   _
                  fileIOmode: String)

    read_! f      == readLine_! f
    readIfCan_! f == readLineIfCan_! f

    readLine_! f ==
        f.fileIOmode ^= "input"  => error "File not in read state"
        s: String := read_-line(f.fileState)$Lisp
        PLACEP(s)$Lisp => error "End of file"
        s
    readLineIfCan_! f ==
        f.fileIOmode ^= "input"  => error "File not in read state"
        s: String := read_-line(f.fileState)$Lisp
```

```
            PLACEP(s)$Lisp => "failed"
            s
        write_!(f, x) ==
            f.fileIOmode ^= "output" => error "File not in write state"
            PRINTEXP(x, f.fileState)$Lisp
            x
        writeLine_! f ==
            f.fileIOmode ^= "output" => error "File not in write state"
            TERPRI(f.fileState)$Lisp
            ""
        writeLine_!(f, x) ==
            f.fileIOmode ^= "output" => error "File not in write state"
            PRINTEXP(x, f.fileState)$Lisp
            TERPRI(f.fileState)$Lisp
            x
        endOfFile? f ==
          f.fileIOmode = "output" => false
          (EOFP(f.fileState)$Lisp pretend Boolean) => true
          false
```

— **TEXTFILE.dotabb** —

```
"TEXTFILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TEXTFILE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"TEXTFILE" -> "STRING"
```

21.6 domain SYMS TheSymbolTable

— **TheSymbolTable.input** —

```
)set break resume
)sys rm -f TheSymbolTable.output
)spool TheSymbolTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TheSymbolTable
--R TheSymbolTable  is a domain constructor
```

```
--R Abbreviation for TheSymbolTable is SYMS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SYMS
--R
--R------------------------------- Operations --------------------------------
--R clearTheSymbolTable : () -> Void      coerce : % -> OutputForm
--R currentSubProgram : () -> Symbol      empty : () -> %
--R endSubProgram : () -> Symbol          newSubProgram : Symbol -> Void
--R printHeader : () -> Void              printHeader : Symbol -> Void
--R printHeader : (Symbol,%) -> Void      printTypes : Symbol -> Void
--R showTheSymbolTable : () -> %
--R argumentList! : List Symbol -> Void
--R argumentList! : (Symbol,List Symbol) -> Void
--R argumentList! : (Symbol,List Symbol,%) -> Void
--R argumentListOf : (Symbol,%) -> List Symbol
--R clearTheSymbolTable : Symbol -> Void
--R declare! : (Symbol,FortranType,Symbol) -> FortranType
--R declare! : (Symbol,FortranType) -> FortranType
--R declare! : (List Symbol,FortranType,Symbol,%) -> FortranType
--R declare! : (Symbol,FortranType,Symbol,%) -> FortranType
--R returnType! : Union(fst: FortranScalarType,void: void) -> Void
--R returnType! : (Symbol,Union(fst: FortranScalarType,void: void)) -> Void
--R returnType! : (Symbol,Union(fst: FortranScalarType,void: void),%) -> Void
--R returnTypeOf : (Symbol,%) -> Union(fst: FortranScalarType,void: void)
--R symbolTableOf : (Symbol,%) -> SymbolTable
--R
--E 1

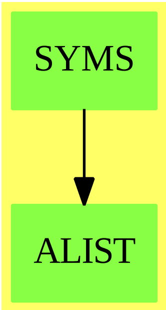
)spool
)lisp (bye)
```

— **TheSymbolTable.help** —

```
====================================================================
TheSymbolTable examples
====================================================================

See Also:
o )show TheSymbolTable
```

21.6.1 TheSymbolTable (SYMS)



See

⇒ "FortranScalarType" (FST) 7.19.1 on page 929
⇒ "FortranType" (FT) 7.21.1 on page 938
⇒ "SymbolTable" (SYMTAB) 20.38.1 on page 2606

Exports:

argumentList!	argumentListOf	clearTheSymbolTable	coerce	currentSubProgram
declare!	empty	endSubProgram	newSubProgram	printHeader
printTypes	returnType!	returnTypeOf	showTheSymbolTable	symbolTableOf

— domain SYMS TheSymbolTable —

```
)abbrev domain SYMS TheSymbolTable
++ Author: Mike Dewar
++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Creates and manipulates one global symbol table for FORTRAN
++ code generation, containing details of types, dimensions, and argument
++ lists.

TheSymbolTable() : Exports == Implementation where

  S    ==> Symbol
  FST  ==> FortranScalarType
  FSTU ==> Union(fst:FST,void:"void")

  Exports == CoercibleTo OutputForm with
    showTheSymbolTable : () -> $
```

```
  ++ showTheSymbolTable() returns the current symbol table.
clearTheSymbolTable : () -> Void
  ++ clearTheSymbolTable() clears the current symbol table.
clearTheSymbolTable : Symbol -> Void
  ++ clearTheSymbolTable(x) removes the symbol x from the table
declare! : (Symbol,FortranType,Symbol,$) -> FortranType
  ++ declare!(u,t,asp,tab) declares the parameter u of subprogram asp
  ++ to have type t in symbol table tab.
declare! : (List Symbol,FortranType,Symbol,$) -> FortranType
  ++ declare!(u,t,asp,tab) declares the parameters u of subprogram asp
  ++ to have type t in symbol table tab.
declare! : (Symbol,FortranType) -> FortranType
  ++ declare!(u,t) declares the parameter u to have type t in the
  ++ current level of the symbol table.
declare! : (Symbol,FortranType,Symbol) -> FortranType
  ++ declare!(u,t,asp) declares the parameter u to have type t in asp.
newSubProgram : Symbol -> Void
  ++ newSubProgram(f) asserts that from now on type declarations are part
  ++ of subprogram f.
currentSubProgram : () -> Symbol
  ++ currentSubProgram() returns the name of the current subprogram being
  ++ processed
endSubProgram : () -> Symbol
  ++ endSubProgram() asserts that we are no longer processing the current
  ++ subprogram.
argumentList! : (Symbol,List Symbol,$) -> Void
  ++ argumentList!(f,l,tab) declares that the argument list for subprogram f
  ++ in symbol table tab is l.
argumentList! : (Symbol,List Symbol) -> Void
  ++ argumentList!(f,l) declares that the argument list for subprogram f in
  ++ the global symbol table is l.
argumentList! : List Symbol -> Void
  ++ argumentList!(l) declares that the argument list for the current
  ++ subprogram in the global symbol table is l.
returnType! : (Symbol,FSTU,$) -> Void
  ++ returnType!(f,t,tab) declares that the return type of subprogram f in
  ++ symbol table tab is t.
returnType! : (Symbol,FSTU) -> Void
  ++ returnType!(f,t) declares that the return type of subprogram f in
  ++ the global symbol table is t.
returnType! : FSTU -> Void
  ++ returnType!(t) declares that the return type of he current subprogram
  ++ in the global symbol table is t.
printHeader : (Symbol,$) -> Void
  ++ printHeader(f,tab) produces the FORTRAN header for subprogram f in
  ++ symbol table tab on the current FORTRAN output stream.
printHeader : Symbol -> Void
  ++ printHeader(f) produces the FORTRAN header for subprogram f in
  ++ the global symbol table on the current FORTRAN output stream.
printHeader : () -> Void
```

```
    ++ printHeader() produces the FORTRAN header for the current subprogram in
    ++ the global symbol table on the current FORTRAN output stream.
  printTypes:  Symbol -> Void
    ++ printTypes(tab) produces FORTRAN type declarations from tab, on the
    ++ current FORTRAN output stream
  empty : () -> $
    ++ empty() creates a new, empty symbol table.
  returnTypeOf : (Symbol,$) -> FSTU
    ++ returnTypeOf(f,tab) returns the type of the object returned by f
  argumentListOf : (Symbol,$) -> List(Symbol)
    ++ argumentListOf(f,tab) returns the argument list of f
  symbolTableOf : (Symbol,$) -> SymbolTable
    ++ symbolTableOf(f,tab) returns the symbol table of f

 Implementation == add

  Entry : Domain  := Record(symtab:SymbolTable, _
                            returnType:FSTU, _
                            argList:List Symbol)

  Rep := Table(Symbol,Entry)

  -- These are the global variables we want to update:
  theSymbolTable : $ := empty()$Rep
  currentSubProgramName : Symbol := MAIN

  newEntry():Entry ==
    construct(empty()$SymbolTable,["void"]$FSTU,[]::List(Symbol))$Entry

  checkIfEntryExists(name:Symbol,tab:$) : Void ==
    key?(name,tab) => void()$Void
    setelt(tab,name,newEntry())$Rep
    void()$Void

  returnTypeOf(name:Symbol,tab:$):FSTU ==
    elt(elt(tab,name)$Rep,returnType)$Entry

  argumentListOf(name:Symbol,tab:$):List(Symbol) ==
    elt(elt(tab,name)$Rep,argList)$Entry

  symbolTableOf(name:Symbol,tab:$):SymbolTable ==
    elt(elt(tab,name)$Rep,symtab)$Entry

  coerce(u:$):OutputForm ==
    coerce(u)$Rep

  showTheSymbolTable():$ ==
    theSymbolTable

  clearTheSymbolTable():Void ==
```

```
    theSymbolTable := empty()$Rep
    void()$Void

  clearTheSymbolTable(u:Symbol):Void ==
    remove!(u,theSymbolTable)$Rep
    void()$Void

  empty():$ ==
    empty()$Rep

  currentSubProgram():Symbol ==
    currentSubProgramName

  endSubProgram():Symbol ==
  -- If we want to support more complex languages then we should keep
  -- a list of subprograms / blocks - but for the moment lets stick with
  -- Fortran.
    currentSubProgramName := MAIN

  newSubProgram(u:Symbol):Void ==
    setelt(theSymbolTable,u,newEntry())$Rep
    currentSubProgramName := u
    void()$Void

  argumentList!(u:Symbol,args:List Symbol,symbols:$):Void ==
    checkIfEntryExists(u,symbols)
    setelt(elt(symbols,u)$Rep,argList,args)$Entry

  argumentList!(u:Symbol,args:List Symbol):Void ==
    argumentList!(u,args,theSymbolTable)

  argumentList!(args:List Symbol):Void ==
    checkIfEntryExists(currentSubProgramName,theSymbolTable)
    setelt(elt(theSymbolTable,currentSubProgramName)$Rep, _
           argList,args)$Entry

  returnType!(u:Symbol,type:FSTU,symbols:$):Void ==
    checkIfEntryExists(u,symbols)
    setelt(elt(symbols,u)$Rep,returnType,type)$Entry

  returnType!(u:Symbol,type:FSTU):Void ==
    returnType!(u,type,theSymbolTable)

  returnType!(type:FSTU ):Void ==
    checkIfEntryExists(currentSubProgramName,theSymbolTable)
    setelt(elt(theSymbolTable,currentSubProgramName)$Rep, _
           returnType,type)$Entry

  declare!(u:Symbol,type:FortranType):FortranType ==
    declare!(u,type,currentSubProgramName,theSymbolTable)
```

```
declare!(u:Symbol,type:FortranType,asp:Symbol,symbols:$):FortranType ==
  checkIfEntryExists(asp,symbols)
  declare!(u,type, elt(elt(symbols,asp)$Rep,symtab)$Entry)$SymbolTable

declare!(u:List Symbol,type:FortranType,asp:Symbol,syms:$):FortranType ==
  checkIfEntryExists(asp,syms)
  declare!(u,type, elt(elt(syms,asp)$Rep,symtab)$Entry)$SymbolTable

declare!(u:Symbol,type:FortranType,asp:Symbol):FortranType ==
  checkIfEntryExists(asp,theSymbolTable)
  declare!(u,type,elt(elt(theSymbolTable,asp)$Rep,symtab)$Entry)$SymbolTable

printHeader(u:Symbol,symbols:$):Void ==
  entry := elt(symbols,u)$Rep
  fortFormatHead(elt(entry,returnType)$Entry::OutputForm,u::OutputForm, _
                 elt(entry,argList)$Entry::OutputForm)$Lisp
  printTypes(elt(entry,symtab)$Entry)$SymbolTable

printHeader(u:Symbol):Void ==
  printHeader(u,theSymbolTable)

printHeader():Void ==
  printHeader(currentSubProgramName,theSymbolTable)

printTypes(u:Symbol):Void ==
  printTypes(elt(elt(theSymbolTable,u)$Rep,symtab)$Entry)$SymbolTable
```

———-

— SYMS.dotabb —

```
"SYMS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SYMS" -> "ALIST"
```

———-

21.7 domain M3D ThreeDimensionalMatrix

— ThreeDimensionalMatrix.input —

```
)set break resume
)sys rm -f ThreeDimensionalMatrix.output
```

```
)spool ThreeDimensionalMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ThreeDimensionalMatrix
--R ThreeDimensionalMatrix R: SetCategory  is a domain constructor
--R Abbreviation for ThreeDimensionalMatrix is M3D
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for M3D
--R
--R------------------------------- Operations --------------------------------
--R construct : List List List R -> %     copy : % -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                map : ((R -> R),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> PrimitiveArray PrimitiveArray PrimitiveArray R
--R coerce : PrimitiveArray PrimitiveArray PrimitiveArray R -> %
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R elt : (%,NonNegativeInteger,NonNegativeInteger,NonNegativeInteger) -> R
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R identityMatrix : NonNegativeInteger -> % if R has RING
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R matrixConcat3D : (Symbol,%,%) -> %
--R matrixDimensions : % -> Vector NonNegativeInteger
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List R if $ has finiteAggregate
--R plus : (%,%) -> % if R has RING
--R setelt! : (%,NonNegativeInteger,NonNegativeInteger,NonNegativeInteger,R) -> R
--R size? : (%,NonNegativeInteger) -> Boolean
--R zeroMatrix : (NonNegativeInteger,NonNegativeInteger,NonNegativeInteger) -> % if R has RI
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

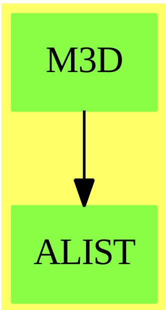
— **ThreeDimensionalMatrix.help** —

```
====================================================================
ThreeDimensionalMatrix examples
====================================================================

See Also:
o )show ThreeDimensionalMatrix
```

———-

21.7.1 ThreeDimensionalMatrix (M3D)



See

⇒ "Result" (RESULT) 19.9.1 on page 2260
⇒ "FortranCode" (FC) 7.16.1 on page 898
⇒ "FortranProgram" (FORTRAN) 7.18.1 on page 923
⇒ "SimpleFortranProgram" (SFORT) 20.11.1 on page 2364
⇒ "Switch" (SWITCH) 20.36.1 on page 2588
⇒ "FortranTemplate" (FTEM) 7.20.1 on page 934
⇒ "FortranExpression" (FEXPR) 7.17.1 on page 914

Exports:

any?	coerce	construct	copy	count
elt	empty	empty?	eq?	eval
every?	hash	identityMatrix	latex	less?
map	map!	matrixConcat3D	matrixDimensions	member?
members	more?	parts	plus	sample
setelt!	size?	zeroMatrix	#?	?=?
?~=?				

— domain M3D ThreeDimensionalMatrix —

```
)abbrev domain M3D ThreeDimensionalMatrix
++ Author: William Naylor
++ Date Created: 20 October 1993
++ Date Last Updated: 20 May 1994
++ BasicFunctions:
++ Related Constructors: Matrix
++ Also See: PrimitiveArray
++ AMS Classification:
++ Keywords:
++ References:
++ Description:
++ This domain represents three dimensional matrices over a general object type

ThreeDimensionalMatrix(R) : Exports == Implementation where

  R : SetCategory
  L ==> List
  NNI ==> NonNegativeInteger
  A1AGG ==> OneDimensionalArrayAggregate
  ARRAY1 ==> OneDimensionalArray
  PA ==> PrimitiveArray
  INT ==> Integer
  PI ==> PositiveInteger

  Exports ==> HomogeneousAggregate(R) with

    if R has Ring then
      zeroMatrix : (NNI,NNI,NNI) -> $
         ++ zeroMatrix(i,j,k) create a matrix with all zero terms
      identityMatrix : (NNI) -> $
         ++ identityMatrix(n) create an identity matrix
         ++ we note that this must be square
      plus : ($,$) -> $
         ++ plus(x,y) adds two matrices, term by term
         ++ we note that they must be the same size
    construct : (L L L R) -> $
       ++ construct(lll) creates a 3-D matrix from a List List List R lll
    elt : ($,NNI,NNI,NNI) -> R
       ++ elt(x,i,j,k) extract an element from the matrix x
    setelt! :($,NNI,NNI,NNI,R) -> R
       ++ setelt!(x,i,j,k,s) (or x.i.j.k:=s) sets a specific element of the array to some val
    coerce : (PA PA PA R) -> $
       ++ coerce(p) moves from the representation type
       ++ (PrimitiveArray  PrimitiveArray  PrimitiveArray R)
       ++ to the domain
    coerce : $ -> (PA PA PA R)
          ++ coerce(x) moves from the domain to the representation type
```

```
  matrixConcat3D : (Symbol,$,$) -> $
       ++ matrixConcat3D(s,x,y) concatenates two 3-D matrices along a specified axis
  matrixDimensions : $ -> Vector NNI
       ++ matrixDimensions(x) returns the dimensions of a matrix

 Implementation ==>  (PA PA PA R) add

  import (PA PA PA R)
  import (PA PA R)
  import (PA R)
  import R

  matrix1,matrix2,resultMatrix : $

  -- function to concatenate two matrices
  -- the first argument must be a symbol, which is either i,j or k
  -- to specify the direction in which the concatenation is to take place
  matrixConcat3D(dir : Symbol,mat1 : $,mat2 : $) : $ ==
    ^((dir = (i::Symbol)) or (dir = (j::Symbol)) or (dir = (k::Symbol)))_
     => error "the axis of concatenation must be i,j or k"
    mat1Dim := matrixDimensions(mat1)
    mat2Dim := matrixDimensions(mat2)
    iDim1 := mat1Dim.1
    jDim1 := mat1Dim.2
    kDim1 := mat1Dim.3
    iDim2 := mat2Dim.1
    jDim2 := mat2Dim.2
    kDim2 := mat2Dim.3
    matRep1 : (PA PA PA R) := copy(mat1 :: (PA PA PA R))$(PA PA PA R)
    matRep2 : (PA PA PA R) := copy(mat2 :: (PA PA PA R))$(PA PA PA R)
    retVal : $

    if (dir = (i::Symbol)) then
      -- j,k dimensions must agree
      if (^((jDim1 = jDim2) and (kDim1=kDim2)))
      then
        error "jxk do not agree"
      else
        retVal := (coerce(concat(matRep1,matRep2)$(PA PA PA R))$$)@$

    if (dir = (j::Symbol)) then
      -- i,k dimensions must agree
      if (^((iDim1 = iDim2) and (kDim1=kDim2)))
      then
        error "ixk do not agree"
      else
        for i in 0..(iDim1-1) repeat
          setelt(matRep1,i,(concat(elt(matRep1,i)$(PA PA PA R)_
            ,elt(matRep2,i)$(PA PA PA R))$(PA PA R))@(PA PA R))$(PA PA PA R)
        retVal := (coerce(matRep1)$$)@$
```

```
      if (dir = (k::Symbol)) then
        temp : (PA PA R)
        -- i,j dimensions must agree
        if (^((iDim1 = iDim2) and (jDim1=jDim2)))
        then
          error "ixj do not agree"
        else
          for i in 0..(iDim1-1) repeat
            temp := copy(elt(matRep1,i)$(PA PA PA R))$(PA PA R)
            for j in 0..(jDim1-1) repeat
              setelt(temp,j,concat(elt(elt(matRep1,i)$(PA PA PA R)_
              ,j)$(PA PA R),elt(elt(matRep2,i)$(PA PA PA R),j)$(PA PA R)_
              )$(PA R))$(PA PA R)
            setelt(matRep1,i,temp)$(PA PA PA R)
          retVal := (coerce(matRep1)$$)@$

      retVal

    matrixDimensions(mat : $) : Vector NNI ==
      matRep : (PA PA PA R) := mat :: (PA PA PA R)
      iDim : NNI := (#matRep)$(PA PA PA R)
      matRep2 : PA PA R := elt(matRep,0)$(PA PA PA R)
      jDim : NNI := (#matRep2)$(PA PA R)
      matRep3 : (PA R) := elt(matRep2,0)$(PA PA R)
      kDim : NNI := (#matRep3)$(PA R)
      retVal : Vector NNI := new(3,0)$(Vector NNI)
      retVal.1 := iDim
      retVal.2 := jDim
      retVal.3 := kDim
      retVal

    coerce(matrixRep : (PA PA PA R)) : $ == matrixRep pretend $

    coerce(mat : $) : (PA PA PA R) == mat pretend (PA PA PA R)

    -- i,j,k must be with in the bounds of the matrix
    elt(mat : $,i : NNI,j : NNI,k : NNI) : R ==
      matDims := matrixDimensions(mat)
      iLength := matDims.1
      jLength := matDims.2
      kLength := matDims.3
      ((i > iLength) or (j > jLength) or (k > kLength) or (i=0) or (j=0) or_
(k=0)) => error "coordinates must be within the bounds of the matrix"
      matrixRep : PA PA PA R := mat :: (PA PA PA R)
      elt(elt(elt(matrixRep,i-1)$(PA PA PA R),j-1)$(PA PA R),k-1)$(PA R)

    setelt!(mat : $,i : NNI,j : NNI,k : NNI,val : R)_
       : R ==
      matDims := matrixDimensions(mat)
```

```
      iLength := matDims.1
      jLength := matDims.2
      kLength := matDims.3
      ((i > iLength) or (j > jLength) or (k > kLength) or (i=0) or (j=0) or_
(k=0)) => error "coordinates must be within the bounds of the matrix"
      matrixRep : PA PA PA R := mat :: (PA PA PA R)
      row2 : PA PA R := copy(elt(matrixRep,i-1)$(PA PA PA R))$(PA PA R)
      row1 : PA R := copy(elt(row2,j-1)$(PA PA R))$(PA R)
      setelt(row1,k-1,val)$(PA R)
      setelt(row2,j-1,row1)$(PA PA R)
      setelt(matrixRep,i-1,row2)$(PA PA PA R)
      val

    if R has Ring then
      zeroMatrix(iLength:NNI,jLength:NNI,kLength:NNI) : $ ==
        (new(iLength,new(jLength,new(kLength,(0$R))$(PA R))$(PA PA R))$(PA PA PA R)) :: $

      identityMatrix(iLength:NNI) : $ ==
        retValueRep : PA PA PA R := zeroMatrix(iLength,iLength,iLength)$$ :: (PA PA PA R)
        row1 : PA R
        row2 : PA PA R
        row1empty : PA R := new(iLength,0$R)$(PA R)
        row2empty : PA PA R := new(iLength,copy(row1empty)$(PA R))$(PA PA R)
        for count in 0..(iLength-1) repeat
          row1 := copy(row1empty)$(PA R)
          setelt(row1,count,1$R)$(PA R)
          row2 := copy(row2empty)$(PA PA R)
          setelt(row2,count,copy(row1)$(PA R))$(PA PA R)
          setelt(retValueRep,count,copy(row2)$(PA PA R))$(PA PA PA R)
        retValueRep :: $


      plus(mat1 : $,mat2 :$) : $ ==

        mat1Dims := matrixDimensions(mat1)
        iLength1 := mat1Dims.1
        jLength1 := mat1Dims.2
        kLength1 := mat1Dims.3

        mat2Dims := matrixDimensions(mat2)
        iLength2 := mat2Dims.1
        jLength2 := mat2Dims.2
        kLength2 := mat2Dims.3

        -- check that the dimensions are the same
        (^(iLength1 = iLength2) or ^(jLength1 = jLength2) or ^(kLength1 = kLength2))_
         => error "error the matrices are different sizes"

        sum : R
        row1 : (PA R) := new(kLength1,0$R)$(PA R)
```

```
      row2 : (PA PA R) := new(jLength1,copy(row1)$(PA R))$(PA PA R)
      row3 : (PA PA PA R) := new(iLength1,copy(row2)$(PA PA R))$(PA PA PA R)

      for i in 1..iLength1 repeat
        for j in 1..jLength1 repeat
          for k in 1..kLength1 repeat
            sum := (elt(mat1,i,j,k)::R +$R_
                    elt(mat2,i,j,k)::R)
            setelt(row1,k-1,sum)$(PA R)
          setelt(row2,j-1,copy(row1)$(PA R))$(PA PA R)
        setelt(row3,i-1,copy(row2)$(PA PA R))$(PA PA PA R)

      resultMatrix := (row3 pretend $)

      resultMatrix

  construct(listRep : L L L R) : $ ==

    (#listRep)$(L L L R) = 0 => error "empty list"
    (#(listRep.1))$(L L R) = 0 => error "empty list"
    (#((listRep.1).1))$(L R) = 0 => error "empty list"
    iLength := (#listRep)$(L L L R)
    jLength := (#(listRep.1))$(L L R)
    kLength := (#((listRep.1).1))$(L R)

    --first check that the matrix is in the correct form
    for subList in listRep repeat
      ^((#subList)$(L L R) = jLength) => error_
"can not have an irregular shaped matrix"
      for subSubList in subList repeat
        ^((#(subSubList))$(L R) = kLength) => error_
"can not have an irregular shaped matrix"

    row1 : (PA R) := new(kLength,((listRep.1).1).1)$(PA R)
    row2 : (PA PA R) := new(jLength,copy(row1)$(PA R))$(PA PA R)
    row3 : (PA PA PA R) := new(iLength,copy(row2)$(PA PA R))$(PA PA PA R)

    for i in 1..iLength repeat
      for j in 1..jLength repeat
        for k in 1..kLength repeat

          element := elt(elt(elt(listRep,i)$(L L L R),j)$(L L R),k)$(L R)
          setelt(row1,k-1,element)$(PA R)
        setelt(row2,j-1,copy(row1)$(PA R))$(PA PA R)
      setelt(row3,i-1,copy(row2)$(PA PA R))$(PA PA PA R)

    resultMatrix := (row3 pretend $)

    resultMatrix
```

———-

— **M3D.dotabb** —

```
"M3D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=M3D"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"M3D" -> "ALIST"
```

———-

21.8 domain VIEW3D ThreeDimensionalViewport

— **ThreeDimensionalViewport.input** —

```
)set break resume
)sys rm -f ThreeDimensionalViewport.output
)spool ThreeDimensionalViewport.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ThreeDimensionalViewport
--R ThreeDimensionalViewport  is a domain constructor
--R Abbreviation for ThreeDimensionalViewport is VIEW3D
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for VIEW3D
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean              axes : (%,String) -> Void
--R clipSurface : (%,String) -> Void    close : % -> Void
--R coerce : % -> OutputForm            controlPanel : (%,String) -> Void
--R diagonals : (%,String) -> Void      drawStyle : (%,String) -> Void
--R eyeDistance : (%,Float) -> Void     hash : % -> SingleInteger
--R hitherPlane : (%,Float) -> Void     intensity : (%,Float) -> Void
--R key : % -> Integer                  latex : % -> String
--R makeViewport3D : % -> %             options : % -> List DrawOption
--R perspective : (%,String) -> Void    reset : % -> Void
--R rotate : (%,Float,Float) -> Void    showRegion : (%,String) -> Void
--R title : (%,String) -> Void          viewDeltaXDefault : Float -> Float
--R viewDeltaXDefault : () -> Float     viewDeltaYDefault : Float -> Float
--R viewDeltaYDefault : () -> Float     viewPhiDefault : Float -> Float
--R viewPhiDefault : () -> Float        viewThetaDefault : Float -> Float
--R viewThetaDefault : () -> Float      viewZoomDefault : Float -> Float
--R viewZoomDefault : () -> Float       viewport3D : () -> %
```

```
--R write : (%,String) -> String         zoom : (%,Float) -> Void
--R ?~=? : (%,%) -> Boolean
--R colorDef : (%,Color,Color) -> Void
--R dimensions : (%,NonNegativeInteger,NonNegativeInteger,PositiveInteger,PositiveInteger) ->
--R lighting : (%,Float,Float,Float) -> Void
--R makeViewport3D : (ThreeSpace DoubleFloat,List DrawOption) -> %
--R makeViewport3D : (ThreeSpace DoubleFloat,String) -> %
--R modifyPointData : (%,NonNegativeInteger,Point DoubleFloat) -> Void
--R move : (%,NonNegativeInteger,NonNegativeInteger) -> Void
--R options : (%,List DrawOption) -> %
--R outlineRender : (%,String) -> Void
--R resize : (%,PositiveInteger,PositiveInteger) -> Void
--R rotate : (%,Integer,Integer) -> Void
--R showClipRegion : (%,String) -> Void
--R subspace : (%,ThreeSpace DoubleFloat) -> %
--R subspace : % -> ThreeSpace DoubleFloat
--R translate : (%,Float,Float) -> Void
--R viewpoint : (%,Float,Float,Float) -> Void
--R viewpoint : (%,Float,Float) -> Void
--R viewpoint : (%,Integer,Integer,Float,Float,Float) -> Void
--R viewpoint : (%,Record(theta: DoubleFloat,phi: DoubleFloat,scale: DoubleFloat,scaleX: Doub
--R viewpoint : % -> Record(theta: DoubleFloat,phi: DoubleFloat,scale: DoubleFloat,scaleX: Do
--R viewpoint : (%,Float,Float,Float,Float,Float) -> Void
--R write : (%,String,List String) -> String
--R write : (%,String,String) -> String
--R zoom : (%,Float,Float,Float) -> Void
--R
--E 1

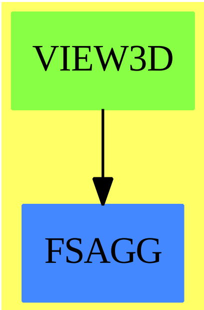
)spool
)lisp (bye)
```

— ThreeDimensionalViewport.help —

```
====================================================================
ThreeDimensionalViewport examples
====================================================================

See Also:
o )show ThreeDimensionalViewport
```

21.8.1 ThreeDimensionalViewport (VIEW3D)



Exports:

axes	clipSurface	colorDef	close	coerce
controlPanel	diagonals	dimensions	drawStyle	eyeDistance
hash	hitherPlane	intensity	key	latex
lighting	makeViewport3D	modifyPointData	move	options
outlineRender	perspective	reset	resize	rotate
showClipRegion	showRegion	subspace	title	translate
viewDeltaXDefault	viewDeltaYDefault	viewPhiDefault	viewpoint	viewThetaDefault
viewZoomDefault	viewport3D	write	zoom	?=?
?~=?				

— domain VIEW3D ThreeDimensionalViewport —

```
)abbrev domain VIEW3D ThreeDimensionalViewport
++ Author: Jim Wen
++ Date Created: 28 April 1989
++ Date Last Updated: 2 November 1991, Jim Wen
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ ThreeDimensionalViewport creates viewports to display graphs

ThreeDimensionalViewport(): Exports == Implementation where
  VIEW    ==> VIEWPORTSERVER$Lisp
  sendI   ==> SOCK_-SEND_-INT
  sendSF  ==> SOCK_-SEND_-FLOAT
  sendSTR ==> SOCK_-SEND_-STRING
  getI    ==> SOCK_-GET_-INT
  getSF   ==> SOCK_-GET_-FLOAT

  typeVIEW3D   ==> 1$I
```

```
typeVIEWTube ==> 4

makeVIEW3D ==> (-1)$SingleInteger

I   ==> Integer
PI  ==> PositiveInteger
NNI ==> NonNegativeInteger
XY  ==> Record( X:I, Y:I )
XYP ==> Record( X:PI, Y:PI )
XYNN ==> Record( X:NNI, Y:NNI )
SF  ==> DoubleFloat
F   ==> Float
L   ==> List
Pt ==> ColoredThreeDimensionalPoint
SEG ==> Segment
S   ==> String
E   ==> OutputForm
PLOT3D ==> Plot3D
TUBE   ==> TubePlot
V   ==> Record( theta:SF, phi:SF, scale:SF, scaleX:SF, scaleY:SF, scaleZ:SF, deltaX:SF, del
H   ==> Record( hueOffset:I, hueNumber:I)
FLAG   ==> Record( showCP:I, style:I, axesOn:I, diagonalsOn:I, outlineRenderOn:I, showRegi
FR  ==> Record( fn:Fn2, fc: FnU, xmin:SF, xmax:SF, ymin:SF, ymax:SF, xnum:I, ynum:I )
FParamR ==> Record( theTube:TUBE )
LR  ==> Record( lightX:SF, lightY:SF, lightZ:SF, lightTheta:SF, lightPhi:SF , translucence
UFR ==> Union(FR,FParamR,"undefined")
PR  ==> Record( perspectiveField:I, eyeDistance:SF, hitherPlane:SF)
VR  ==> Record( clipXMin:SF, clipXMax:SF, clipYMin:SF, clipYMax:SF, clipZMin:SF, clipZMax:S
C   ==> Color()
B   ==> Boolean
POINT ==> Point(SF)
SUBSPACE ==> SubSpace(3,SF)
SPACE3 ==> ThreeSpace(SF)
DROP ==> DrawOption
COORDSYS ==> CoordinateSystems(SF)

  -- the below macros correspond to the ones in include/actions.h
ROTATE       ==> 0$I  -- rotate      in actions.h
ZOOM         ==> 1$I  -- zoom        in actions.h
TRANSLATE    ==> 2    -- translate   in actions.h
rendered     ==> 3    -- render      in actions.h
hideControl  ==> 4
closeAll     ==> 5
axesOnOff    ==> 6
opaque       ==> 7    -- opaqueMesh  in action.h
contour      ==> 24
RESET        ==> 8
wireMesh     ==> 9    -- transparent in actions.h
region3D     ==> 12
smooth       ==> 22
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  diagOnOff    ==> 26
  outlineOnOff ==> 13
  zoomx        ==> 14
  zoomy        ==> 15
  zoomz        ==> 16
  perspectiveOnOff ==> 27
  clipRegionOnOff ==> 66
  clipSurfaceOnOff ==> 67

  SPADBUTTONPRESS ==> 100
  COLORDEF        ==> 101
  MOVE            ==> 102
  RESIZE          ==> 103
  TITLE           ==> 104
  lightDef        ==> 108
  translucenceDef ==> 109
  writeView       ==> 110
  eyeDistanceData ==> 111
  modifyPOINT     ==> 114
--  printViewport   ==> 115
  hitherPlaneData ==> 116
  queryVIEWPOINT  ==> 117
  changeVIEWPOINT ==> 118

  noControl ==> 0$I

  yes       ==> 1$I
  no        ==> 0$I

  EYED      ==> 500::SF  -- see draw.h, should be the same(?) as clipOffset
  HITHER    ==> (-250)::SF   -- see process.h in view3D/ (not yet passed to viewman)

  openTube  ==> 1$I
  closedTube ==> 0$I

  fun2Var3D  ==> "   Three Dimensional Viewport: Function of Two Variables"
  para1Var3D ==> "   Three Dimensional Viewport: Parametric Curve of One Variable"
  undef3D    ==> "   Three Dimensional Viewport: No function defined for this viewport yet"

  Exports ==> SetCategory with
    viewThetaDefault      : ()                                     -> F
      ++ viewThetaDefault() returns the current default longitudinal
      ++ view angle in radians.
    viewThetaDefault      : F                                      -> F
      ++ viewThetaDefault(t) sets the current default longitudinal
      ++ view angle in radians to the value t and returns t.
    viewPhiDefault        : ()                                     -> F
      ++ viewPhiDefault() returns the current default latitudinal
      ++ view angle in radians.
    viewPhiDefault        : F                                      -> F
```

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  ++ viewPhiDefault(p) sets the current default latitudinal
  ++ view angle in radians to the value p and returns p.
viewZoomDefault      : ()                                    -> F
  ++ viewZoomDefault() returns the current default graph scaling
  ++ value.
viewZoomDefault      : F                                     -> F
  ++ viewZoomDefault(s) sets the current default graph scaling
  ++ value to s and returns s.
viewDeltaXDefault    : ()                                    -> F
  ++ viewDeltaXDefault() returns the current default horizontal
  ++ offset from the center of the viewport window.
viewDeltaXDefault    : F                                     -> F
  ++ viewDeltaXDefault(dx) sets the current default horizontal
  ++ offset from the center of the viewport window to be \spad{dx}
  ++ and returns \spad{dx}.
viewDeltaYDefault    : ()                                    -> F
  ++ viewDeltaYDefault() returns the current default vertical
  ++ offset from the center of the viewport window.
viewDeltaYDefault    : F                                     -> F
  ++ viewDeltaYDefault(dy) sets the current default vertical
  ++ offset from the center of the viewport window to be \spad{dy}
  ++ and returns \spad{dy}.
viewport3D           : ()                                    -> %
  ++ viewport3D() returns an undefined three-dimensional viewport
  ++ of the domain \spadtype{ThreeDimensionalViewport} whose
  ++ contents are empty.
makeViewport3D       : %                                     -> %
  ++ makeViewport3D(v) takes the given three-dimensional viewport,
  ++ v, of the domain \spadtype{ThreeDimensionalViewport} and
  ++ displays a viewport window on the screen which contains
  ++ the contents of v.
makeViewport3D       : (SPACE3,S)                            -> %
  ++ makeViewport3D(sp,s) takes the given space, \spad{sp} which is
  ++ of the domain \spadtype{ThreeSpace} and displays a viewport
  ++ window on the screen which contains the contents of \spad{sp},
  ++ and whose title is given by s.
makeViewport3D       : (SPACE3,L DROP)                       -> %
  ++ makeViewport3D(sp,lopt) takes the given space, \spad{sp} which is
  ++ of the domain \spadtype{ThreeSpace} and displays a viewport
  ++ window on the screen which contains the contents of \spad{sp},
  ++ and whose draw options are indicated by the list \spad{lopt}, which
  ++ is a list of options from the domain \spad{DrawOption}.
subspace             : %                                     -> SPACE3
  ++ subspace(v) returns the contents of the viewport v, which is
  ++ of the domain \spadtype{ThreeDimensionalViewport}, as a subspace
  ++ of the domain \spad{ThreeSpace}.
subspace             : (%,SPACE3)                            -> %
  ++ subspace(v,sp) places the contents of the viewport v, which is
  ++ of the domain \spadtype{ThreeDimensionalViewport}, in the subspace
  ++ \spad{sp}, which is of the domain \spad{ThreeSpace}.
```

```
modifyPointData      : (%,NNI,POINT)                        -> Void
  ++ modifyPointData(v,ind,pt) takes the viewport, v, which is of the
  ++ domain \spadtype{ThreeDimensionalViewport}, and places the data
  ++ point, \spad{pt} into the list of points database of v at the index
  ++ location given by \spad{ind}.
options              : %                                    -> L DROP
  ++ options(v) takes the viewport, v, which is of the domain
  ++ \spadtype{ThreeDimensionalViewport} and returns a list of all
  ++ the draw options from the domain \spad{DrawOption} which are
  ++ being used by v.
options              : (%,L DROP)                           -> %
  ++ options(v,lopt) takes the viewport, v, which is of the domain
  ++ \spadtype{ThreeDimensionalViewport} and sets the draw options
  ++ being used by v to those indicated in the list, \spad{lopt},
  ++ which is a list of options from the domain \spad{DrawOption}.
move                 : (%,NNI,NNI)                          -> Void
  ++ move(v,x,y) displays the three-dimensional viewport, v, which
  ++ is of domain \spadtype{ThreeDimensionalViewport}, with the upper
  ++ left-hand corner of the viewport window at the screen
  ++ coordinate position x, y.
resize               : (%,PI,PI)                            -> Void
  ++ resize(v,w,h) displays the three-dimensional viewport, v, which
  ++ is of domain \spadtype{ThreeDimensionalViewport}, with a width
  ++ of w and a height of h, keeping the upper left-hand corner
  ++ position unchanged.
title                : (%,S)                                -> Void
  ++ title(v,s) changes the title which is shown in the three-dimensional
  ++ viewport window, v of domain \spadtype{ThreeDimensionalViewport}.
dimensions           : (%,NNI,NNI,PI,PI)                    -> Void
  ++ dimensions(v,x,y,width,height) sets the position of the
  ++ upper left-hand corner of the three-dimensional viewport, v,
  ++ which is of domain \spadtype{ThreeDimensionalViewport}, to
  ++ the window coordinate x, y, and sets the dimensions of the
  ++ window to that of \spad{width}, \spad{height}.  The new
  ++ dimensions are not displayed until the function
  ++ \spadfun{makeViewport3D} is executed again for v.
viewpoint            : (%,F,F,F,F,F)                        -> Void
  ++ viewpoint(v,th,phi,s,dx,dy) sets the longitudinal view angle
  ++ to \spad{th} radians, the latitudinal view angle to \spad{phi}
  ++ radians, the scale factor to \spad{s}, the horizontal viewport
  ++ offset to \spad{dx}, and the vertical viewport offset to \spad{dy}
  ++ for the viewport v, which is of the domain
  ++ \spadtype{ThreeDimensionalViewport}.  The new viewpoint position
  ++ is not displayed until the function \spadfun{makeViewport3D} is
  ++ executed again for v.
viewpoint            : (%) -> V
  ++ viewpoint(v) returns the current viewpoint setting of the given
  ++ viewport, v.  This function is useful in the situation where the
  ++ user has created a viewport, proceeded to interact with it via
  ++ the control panel and desires to save the values of the viewpoint
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    ++ as the default settings for another viewport to be created using
    ++ the system.
  viewpoint : (%,V) -> Void
    ++ viewpoint(v,viewpt) sets the viewpoint for the viewport.  The
    ++ viewport record consists of the latitudal and longitudal angles,
    ++ the zoom factor, the x,y and z scales, and the x and y displacements.
  viewpoint           : (%,I,I,F,F,F)                         -> Void
    ++ viewpoint(v,th,phi,s,dx,dy) sets the longitudinal view angle
    ++ to \spad{th} degrees, the latitudinal view angle to \spad{phi}
    ++ degrees, the scale factor to \spad{s}, the horizontal viewport
    ++ offset to \spad{dx}, and the vertical viewport offset to \spad{dy}
    ++ for the viewport v, which is of the domain
    ++ \spadtype{ThreeDimensionalViewport}.  The new viewpoint position
    ++ is not displayed until the function \spadfun{makeViewport3D} is
    ++ executed again for v.
  viewpoint           : (%,F,F)                               -> Void
    ++ viewpoint(v,th,phi) sets the longitudinal view angle to \spad{th}
    ++ radians and the latitudinal view angle to \spad{phi} radians
    ++ for the viewport v, which is of the domain
    ++ \spadtype{ThreeDimensionalViewport}.  The new viewpoint position
    ++ is not displayed until the function \spadfun{makeViewport3D} is
    ++ executed again for v.
  viewpoint           : (%,F,F,F)                             -> Void
    ++ viewpoint(v,rotx,roty,rotz) sets the rotation about the x-axis
    ++ to be \spad{rotx} radians, sets the rotation about the y-axis
    ++ to be \spad{roty} radians, and sets the rotation about the z-axis
    ++ to be \spad{rotz} radians, for the viewport v, which is of the
    ++ domain \spadtype{ThreeDimensionalViewport} and displays v with
    ++ the new view position.
  controlPanel        : (%,S)                                 -> Void
    ++ controlPanel(v,s) displays the control panel of the given
    ++ three-dimensional viewport, v, which is of domain
    ++ \spadtype{ThreeDimensionalViewport}, if s is "on", or hides
    ++ the control panel if s is "off".
  axes                : (%,S)                                 -> Void
    ++ axes(v,s) displays the axes of the given three-dimensional
    ++ viewport, v, which is of domain \spadtype{ThreeDimensionalViewport},
    ++ if s is "on", or does not display the axes if s is "off".
  diagonals           : (%,S)                                 -> Void
    ++ diagonals(v,s) displays the diagonals of the polygon outline
    ++ showing a triangularized surface instead of a quadrilateral
    ++ surface outline, for the given three-dimensional viewport v
    ++ which is of domain \spadtype{ThreeDimensionalViewport}, if s is
    ++ "on", or does not display the diagonals if s is "off".
  outlineRender       : (%,S)                                 -> Void
    ++ outlineRender(v,s) displays the polygon outline showing either
    ++ triangularized surface or a quadrilateral surface outline depending
    ++ on the whether the \spadfun{diagonals} function has been set, for
    ++ the given three-dimensional viewport v which is of domain
    ++ \spadtype{ThreeDimensionalViewport}, if s is "on", or does not
```

```
    ++ display the polygon outline if s is "off".
  drawStyle           : (%,S)                                    -> Void
    ++ drawStyle(v,s) displays the surface for the given three-dimensional
    ++ viewport v which is of domain \spadtype{ThreeDimensionalViewport}
    ++ in the style of drawing indicated by s.  If s is not a valid
    ++ drawing style the style is wireframe by default.  Possible
    ++ styles are \spad{"shade"}, \spad{"solid"} or \spad{"opaque"},
    ++ \spad{"smooth"}, and \spad{"wireMesh"}.
  rotate              : (%,F,F)                                  -> Void
    ++ rotate(v,th,phi) rotates the graph to the longitudinal view angle
    ++ \spad{th} radians and the latitudinal view angle \spad{phi} radians
    ++ for the viewport v, which is of the domain
    ++ \spadtype{ThreeDimensionalViewport}.
  rotate              : (%,I,I)                                  -> Void
    ++ rotate(v,th,phi) rotates the graph to the longitudinal view angle
    ++ \spad{th} degrees and the latitudinal view angle \spad{phi} degrees
    ++ for the viewport v, which is of the domain
    ++ \spadtype{ThreeDimensionalViewport}.  The new rotation position
    ++ is not displayed until the function \spadfun{makeViewport3D} is
    ++ executed again for v.
  zoom                : (%,F)                                    -> Void
    ++ zoom(v,s) sets the graph scaling factor to s, for the viewport v,
    ++ which is of the domain \spadtype{ThreeDimensionalViewport}.
  zoom                : (%,F,F,F)                                -> Void
    ++ zoom(v,sx,sy,sz) sets the graph scaling factors for the x-coordinate
    ++ axis to \spad{sx}, the y-coordinate axis to \spad{sy} and the
    ++ z-coordinate axis to \spad{sz} for the viewport v, which is of
    ++ the domain \spadtype{ThreeDimensionalViewport}.
  translate           : (%,F,F)                                  -> Void
    ++ translate(v,dx,dy) sets the horizontal viewport offset to \spad{dx}
    ++ and the vertical viewport offset to \spad{dy}, for the viewport v,
    ++ which is of the domain \spadtype{ThreeDimensionalViewport}.
  perspective         : (%,S)                                    -> Void
    ++ perspective(v,s) displays the graph in perspective if s is "on",
    ++ or does not display perspective if s is "off" for the given
    ++ three-dimensional viewport, v, which is of domain
    ++ \spadtype{ThreeDimensionalViewport}.
  eyeDistance         : (%,F)                                    -> Void
    ++ eyeDistance(v,d) sets the distance of the observer from the center
    ++ of the graph to d, for the viewport v, which is of the domain
    ++ \spadtype{ThreeDimensionalViewport}.
  hitherPlane         : (%,F)                                    -> Void
    ++ hitherPlane(v,h) sets the hither clipping plane of the graph to h,
    ++ for the viewport v, which is of the domain
    ++ \spadtype{ThreeDimensionalViewport}.
  showRegion          : (%,S)                                    -> Void
    ++ showRegion(v,s) displays the bounding box of the given
    ++ three-dimensional viewport, v, which is of domain
    ++ \spadtype{ThreeDimensionalViewport}, if s is "on", or does not
    ++ display the box if s is "off".
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```
showClipRegion       : (%,S)                             -> Void
  ++ showClipRegion(v,s) displays the clipping region of the given
  ++ three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}, if s is "on", or does not
  ++ display the region if s is "off".
clipSurface          : (%,S)                             -> Void
  ++ clipSurface(v,s) displays the graph with the specified
  ++ clipping region removed if s is "on", or displays the graph
  ++ without clipping implemented if s is "off", for the given
  ++ three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}.
lighting             : (%,F,F,F)                         -> Void
  ++ lighting(v,x,y,z) sets the position of the light source to
  ++ the coordinates x, y, and z and displays the graph for the given
  ++ three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}.
intensity            : (%,F)                             -> Void
  ++ intensity(v,i) sets the intensity of the light source to i, for
  ++ the given three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}.
reset                :  %                                -> Void
  ++ reset(v) sets the current state of the graph characteristics
  ++ of the given three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}, back to their initial settings.
colorDef             : (%,C,C)                           -> Void
  ++ colorDef(v,c1,c2) sets the range of colors along the colormap so
  ++ that the lower end of the colormap is defined by \spad{c1} and the
  ++ top end of the colormap is defined by \spad{c2}, for the given
  ++ three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}.
write                : (%,S)                             -> S
  ++ write(v,s) takes the given three-dimensional viewport, v, which
  ++ is of domain \spadtype{ThreeDimensionalViewport}, and creates
  ++ a directory indicated by s, which contains the graph data
  ++ file for v.
write                : (%,S,S)                           -> S
  ++ write(v,s,f) takes the given three-dimensional viewport, v, which
  ++ is of domain \spadtype{ThreeDimensionalViewport}, and creates
  ++ a directory indicated by s, which contains the graph data
  ++ file for v and an optional file type f.
write                : (%,S,L S)                         -> S
  ++ write(v,s,lf) takes the given three-dimensional viewport, v, which
  ++ is of domain \spadtype{ThreeDimensionalViewport}, and creates
  ++ a directory indicated by s, which contains the graph data
  ++ file for v and the optional file types indicated by the list lf.
close                :  %                                -> Void
  ++ close(v) closes the viewport window of the given
  ++ three-dimensional viewport, v, which is of domain
  ++ \spadtype{ThreeDimensionalViewport}, and terminates the
  ++ corresponding process ID.
```

```
    key                  :  %                                       -> I
      ++ key(v) returns the process ID number of the given three-dimensional
      ++ viewport, v, which is of domain \spadtype{ThreeDimensionalViewport}.
--    print                :  %                                       -> Void

  Implementation ==> add
    import Color()
    import ViewDefaultsPackage()
    import Plot3D()
    import TubePlot()
    import POINT
    import PointPackage(SF)
    import SubSpaceComponentProperty()
    import SPACE3
    import MeshCreationRoutinesForThreeDimensions()
    import DrawOptionFunctions0
    import COORDSYS
    import Set(PositiveInteger)

    Rep := Record (key:I, fun:I, _
               title:S, moveTo:XYNN, size:XYP, viewpoint:V, colors:H, flags:FLAG, _
               lighting:LR, perspective:PR, volume:VR, _
               space3D:SPACE3, _
               optionsField:L DROP)

    degrees := pi()$F / 180.0
    degreesSF := pi()$SF / 180
    defaultTheta  : Reference(SF) := ref(convert(pi()$F/4.0)@SF)
    defaultPhi    : Reference(SF) := ref(convert(-pi()$F/4.0)@SF)
    defaultZoom   : Reference(SF) := ref(convert(1.2)@SF)
    defaultDeltaX : Reference(SF) := ref 0
    defaultDeltaY : Reference(SF) := ref 0


--%Local Functions
    checkViewport (viewport:%):B ==
        -- checks to see if this viewport still exists
        -- by sending the key to the viewport manager and
        -- waiting for its reply after it checks it against
        -- the viewports in its list. a -1 means it doesn't
        -- exist.
      sendI(VIEW,viewport.key)$Lisp
      i := getI(VIEW)$Lisp
      (i < 0$I) =>
        viewport.key := 0$I
        error "This viewport has already been closed!"
      true

    arcsinTemp(x:SF):SF ==
      -- the asin function doesn't exist in the SF domain currently
```

```
      x >= 1  => (pi()$SF / 2)  -- to avoid floating point error from SF (ie 1.0 -> 1.00001)
      x <= -1 => 3 * pi()$SF / 2
      convert(asin(convert(x)@Float)$Float)@SF

    arctanTemp(x:SF):SF == convert(atan(convert(x)@Float)$Float)@SF

    doOptions(v:Rep):Void ==
      v.title := title(v.optionsField,"AXIOM3D")
      st:S := style(v.optionsField,"wireMesh")
      if (st = "shade" or st = "render") then
        v.flags.style := rendered
      else if (st = "solid" or st = "opaque") then
        v.flags.style := opaque
      else if (st = "contour") then
        v.flags.style := contour
      else if (st = "smooth") then
        v.flags.style := smooth
      else v.flags.style := wireMesh
      v.viewpoint := viewpoint(v.optionsField,
        [deref defaultTheta,deref defaultPhi,deref defaultZoom, _
          1$SF,1$SF,1$SF,deref defaultDeltaX, deref defaultDeltaY])
        -- etc - 3D specific stuff...

--%Exported Functions : Default Settings
    viewport3D() ==
      [0,typeVIEW3D,"AXIOM3D",[viewPosDefault().1,viewPosDefault().2], _
       [viewSizeDefault().1,viewSizeDefault().2], _
        [deref defaultTheta,deref defaultPhi,deref defaultZoom, _
          1$SF,1$SF,1$SF,deref defaultDeltaX, deref defaultDeltaY], [0,27], _
            [noControl,wireMesh,yes,no,no,no], [0$SF,0$SF,1$SF,0$SF,0$SF,1$SF], _
              [yes, EYED, HITHER], [0$SF,1$SF,0$SF,1$SF,0$SF,1$SF,no,yes], _
                create3Space()$SPACE3, [] ]

    subspace viewport ==
      viewport.space3D

    subspace(viewport,space) ==
      viewport.space3D := space
      viewport

    options viewport ==
      viewport.optionsField

    options(viewport,opts) ==
      viewport.optionsField := opts
      viewport

    makeViewport3D(space:SPACE3,Title:S):% ==
      v := viewport3D()
      v.space3D := space
```

```
    v.optionsField := [title(Title)]
    makeViewport3D v

  makeViewport3D(space:SPACE3,opts:L DROP):% ==
    v := viewport3D()
    v.space3D := space
    v.optionsField := opts
    makeViewport3D v

  makeViewport3D viewport ==
    doOptions viewport --local function to extract and assign optional arguments for 3D viewports
    sayBrightly(["   Transmitting data...":E]$List(E))$Lisp
    transform := coord(viewport.optionsField,cartesian$COORDSYS)$DrawOptionFunctions0
    check(viewport.space3D)
    lpts := lp(viewport.space3D)
    lllipts  := lllip(viewport.space3D)
    llprops := llprop(viewport.space3D)
    lprops  := lprop(viewport.space3D)
      -- check for dimensionality of points
      -- if they are all 4D points, then everything is okay
      -- if they are all 3D points, then pad an extra constant
      -- coordinate for color
      -- if they have varying dimensionalities, give an error
    s := brace()$Set(PI)
    for pt in lpts repeat
      insert_!(dimension pt,s)
    #s > 1 => error "All points should have the same dimension"
    (n := first parts s) < 3 => error "Dimension of points should be greater than 2"
    sendI(VIEW,viewport.fun)$Lisp
    sendI(VIEW,makeVIEW3D)$Lisp
    sendSTR(VIEW,viewport.title)$Lisp
    sendSF(VIEW,viewport.viewpoint.deltaX)$Lisp
    sendSF(VIEW,viewport.viewpoint.deltaY)$Lisp
    sendSF(VIEW,viewport.viewpoint.scale)$Lisp
    sendSF(VIEW,viewport.viewpoint.scaleX)$Lisp
    sendSF(VIEW,viewport.viewpoint.scaleY)$Lisp
    sendSF(VIEW,viewport.viewpoint.scaleZ)$Lisp
    sendSF(VIEW,viewport.viewpoint.theta)$Lisp
    sendSF(VIEW,viewport.viewpoint.phi)$Lisp
    sendI(VIEW,viewport.moveTo.X)$Lisp
    sendI(VIEW,viewport.moveTo.Y)$Lisp
    sendI(VIEW,viewport.size.X)$Lisp
    sendI(VIEW,viewport.size.Y)$Lisp
    sendI(VIEW,viewport.flags.showCP)$Lisp
    sendI(VIEW,viewport.flags.style)$Lisp
    sendI(VIEW,viewport.flags.axesOn)$Lisp
    sendI(VIEW,viewport.flags.diagonalsOn)$Lisp
    sendI(VIEW,viewport.flags.outlineRenderOn)$Lisp
    sendI(VIEW,viewport.flags.showRegionField)$Lisp  -- add to make3D.c
    sendI(VIEW,viewport.volume.clipRegionField)$Lisp  -- add to make3D.c
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```
  sendI(VIEW,viewport.volume.clipSurfaceField)$Lisp  -- add to make3D.c
  sendI(VIEW,viewport.colors.hueOffset)$Lisp
  sendI(VIEW,viewport.colors.hueNumber)$Lisp
  sendSF(VIEW,viewport.lighting.lightX)$Lisp
  sendSF(VIEW,viewport.lighting.lightY)$Lisp
  sendSF(VIEW,viewport.lighting.lightZ)$Lisp
  sendSF(VIEW,viewport.lighting.translucence)$Lisp
  sendI(VIEW,viewport.perspective.perspectiveField)$Lisp
  sendSF(VIEW,viewport.perspective.eyeDistance)$Lisp
    -- new, crazy points domain stuff
      -- first, send the point data list
  sendI(VIEW,#lpts)$Lisp
  for pt in lpts repeat
    aPoint := transform pt
    sendSF(VIEW,xCoord aPoint)$Lisp
    sendSF(VIEW,yCoord aPoint)$Lisp
    sendSF(VIEW,zCoord aPoint)$Lisp
    n = 3 => sendSF(VIEW,zCoord aPoint)$Lisp
    sendSF(VIEW,color aPoint)$Lisp  -- change to c
      -- now, send the 3d subspace structure
  sendI(VIEW,#lllipts)$Lisp
  for allipts in lllipts for oneprop in lprops for onelprops in llprops repeat
       -- the following is false for f(x,y) and user-defined for [x(t),y(t),z(t)]
       -- this is temporary - until the generalized points stuff gets put in
    sendI(VIEW,(closed? oneprop => yes; no))$Lisp
    sendI(VIEW,(solid? oneprop => yes; no))$Lisp
    sendI(VIEW,#allipts)$Lisp
    for alipts in allipts for tinyprop in onelprops repeat
       -- the following is false for f(x,y) and true for [x(t),y(t),z(t)]
       -- this is temporary -- until the generalized points stuff gets put in
      sendI(VIEW,(closed? tinyprop => yes;no))$Lisp
      sendI(VIEW,(solid? tinyprop => yes;no))$Lisp
      sendI(VIEW,#alipts)$Lisp
      for oneIndexedPoint in alipts repeat
        sendI(VIEW,oneIndexedPoint)$Lisp
  viewport.key := getI(VIEW)$Lisp
  viewport
     -- the key (now set to 0) should be what the viewport returns

viewThetaDefault    == convert(defaultTheta())@F
viewThetaDefault  t ==
  defaultTheta() := convert(t)@SF
  t
viewPhiDefault      == convert(defaultPhi())@F
viewPhiDefault    t ==
  defaultPhi() := convert(t)@SF
  t
viewZoomDefault     == convert(defaultZoom())@F
viewZoomDefault   t ==
  defaultZoom() := convert(t)@SF
```

```
      t
    viewDeltaXDefault   == convert(defaultDeltaX())@F
    viewDeltaXDefault t ==
      defaultDeltaX() := convert(t)@SF
      t
    viewDeltaYDefault   == convert(defaultDeltaY())@F
    viewDeltaYDefault t ==
      defaultDeltaY() := convert(t)@SF
      t

--Exported Functions: Available features for 3D viewports
    lighting(viewport,Xlight,Ylight,Zlight) ==
      viewport.lighting.lightX := convert(Xlight)@SF
      viewport.lighting.lightY := convert(Ylight)@SF
      viewport.lighting.lightZ := convert(Zlight)@SF
      (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,lightDef)$Lisp
        checkViewport viewport =>
          sendSF(VIEW,viewport.lighting.lightX)$Lisp
          sendSF(VIEW,viewport.lighting.lightY)$Lisp
          sendSF(VIEW,viewport.lighting.lightZ)$Lisp
          getI(VIEW)$Lisp          -- acknowledge

    axes (viewport,onOff) ==
      if onOff = "on" then viewport.flags.axesOn := yes
      else viewport.flags.axesOn := no
      (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,axesOnOff)$Lisp
        checkViewport viewport =>
          sendI(VIEW,viewport.flags.axesOn)$Lisp
          getI(VIEW)$Lisp          -- acknowledge

    diagonals (viewport,onOff) ==
      if onOff = "on" then viewport.flags.diagonalsOn := yes
      else viewport.flags.diagonalsOn := no
      (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,diagOnOff)$Lisp
        checkViewport viewport =>
          sendI(VIEW,viewport.flags.diagonalsOn)$Lisp
          getI(VIEW)$Lisp          -- acknowledge

    outlineRender (viewport,onOff) ==
      if onOff = "on" then viewport.flags.outlineRenderOn := yes
      else viewport.flags.outlineRenderOn := no
      (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,outlineOnOff)$Lisp
```

```
    checkViewport viewport =>
      sendI(VIEW,viewport.flags.outlineRenderOn)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

controlPanel (viewport,onOff) ==
  if onOff = "on" then viewport.flags.showCP := yes
  else viewport.flags.showCP := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,hideControl)$Lisp
    checkViewport viewport =>
      sendI(VIEW,viewport.flags.showCP)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

drawStyle (viewport,how) ==
  if (how = "shade") then                  -- render
    viewport.flags.style := rendered
  else if (how = "solid") then             -- opaque
    viewport.flags.style := opaque
  else if (how = "contour") then           -- contour
    viewport.flags.style := contour
  else if (how = "smooth") then            -- smooth
    viewport.flags.style := smooth
  else viewport.flags.style := wireMesh
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,viewport.flags.style)$Lisp
    checkViewport viewport =>
      getI(VIEW)$Lisp          -- acknowledge

reset viewport ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,SPADBUTTONPRESS)$Lisp
    checkViewport viewport =>
      sendI(VIEW,RESET)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

close viewport ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,closeAll)$Lisp
    checkViewport viewport =>
      getI(VIEW)$Lisp          -- acknowledge
      viewport.key := 0$I

viewpoint (viewport:%):V ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,queryVIEWPOINT)$Lisp
```

```
    checkViewport viewport =>
      deltaX_sf : SF := getSF(VIEW)$Lisp
      deltaY_sf : SF := getSF(VIEW)$Lisp
      scale_sf  : SF := getSF(VIEW)$Lisp
      scaleX_sf : SF := getSF(VIEW)$Lisp
      scaleY_sf : SF := getSF(VIEW)$Lisp
      scaleZ_sf : SF := getSF(VIEW)$Lisp
      theta_sf  : SF := getSF(VIEW)$Lisp
      phi_sf    : SF := getSF(VIEW)$Lisp
      getI(VIEW)$Lisp          -- acknowledge
      viewport.viewpoint :=
        [ theta_sf, phi_sf, scale_sf, scaleX_sf, scaleY_sf, scaleZ_sf,
          deltaX_sf, deltaY_sf ]
    viewport.viewpoint

viewpoint (viewport:%, viewpt:V):Void ==
  viewport.viewpoint := viewpt
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,changeVIEWPOINT)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.deltaX)$Lisp
      sendSF(VIEW,viewport.viewpoint.deltaY)$Lisp
      sendSF(VIEW,viewport.viewpoint.scale)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleX)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleY)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleZ)$Lisp
      sendSF(VIEW,viewport.viewpoint.theta)$Lisp
      sendSF(VIEW,viewport.viewpoint.phi)$Lisp
      getI(VIEW)$Lisp          -- acknowledge


viewpoint (viewport:%,Theta:F,Phi:F,Scale:F,DeltaX:F,DeltaY:F):Void ==
  viewport.viewpoint :=
    [convert(Theta)@SF,convert(Phi)@SF,convert(Scale)@SF,1$SF,1$SF,1$SF,convert(DeltaX)@SF,convert(D

viewpoint (viewport:%,Theta:I,Phi:I,Scale:F,DeltaX:F,DeltaY:F):Void ==
  viewport.viewpoint := [convert(Theta)@SF * degreesSF,convert(Phi)@SF * degreesSF,
    convert(Scale)@SF,1$SF,1$SF,1$SF,convert(DeltaX)@SF,convert(DeltaY)@SF]

viewpoint (viewport:%,Theta:F,Phi:F):Void ==
  viewport.viewpoint.theta := convert(Theta)@SF * degreesSF
  viewport.viewpoint.phi   := convert(Phi)@SF * degreesSF

viewpoint (viewport:%,X:F,Y:F,Z:F):Void ==
  Theta : F
  Phi : F
  if (X=0$F) and (Y=0$F) then
    Theta := 0$F
    if (Z>=0$F) then
```

```
      Phi := 0$F
    else
      Phi := 180.0
  else
    Theta := asin(Y/(R := sqrt(X*X+Y*Y)))
    if (Z=0$F) then
      Phi := 90.0
    else
      Phi := atan(Z/R)
  rotate(viewport, Theta * degrees, Phi * degrees)

title (viewport,Title) ==
  viewport.title := Title
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,TITLE)$Lisp
    checkViewport viewport =>
      sendSTR(VIEW,Title)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

colorDef (viewport,HueOffset,HueNumber) ==
  viewport.colors := [h := (hue HueOffset),(hue HueNumber) - h]
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,COLORDEF)$Lisp
    checkViewport viewport =>
      sendI(VIEW,hue HueOffset)$Lisp
      sendI(VIEW,hue HueNumber)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

dimensions (viewport,ViewX,ViewY,ViewWidth,ViewHeight) ==
  viewport.moveTo := [ViewX,ViewY]
  viewport.size   := [ViewWidth,ViewHeight]

move(viewport,xLoc,yLoc) ==
  viewport.moveTo := [xLoc,yLoc]
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,MOVE)$Lisp
    checkViewport viewport =>
      sendI(VIEW,xLoc)$Lisp
      sendI(VIEW,yLoc)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

resize(viewport,xSize,ySize) ==
  viewport.size := [xSize,ySize]
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,RESIZE)$Lisp
    checkViewport viewport =>
```

```
      sendI(VIEW,xSize)$Lisp
      sendI(VIEW,ySize)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

coerce viewport ==
  (key(viewport) = 0$I) =>
    hconcat
      ["Closed or Undefined ThreeDimensionalViewport: "::E,
       (viewport.title)::E]
  hconcat ["ThreeDimensionalViewport: "::E, (viewport.title)::E]

key viewport == viewport.key

rotate(viewport:%,Theta:I,Phi:I) ==
  rotate(viewport,Theta::F * degrees,Phi::F * degrees)

rotate(viewport:%,Theta:F,Phi:F) ==
  viewport.viewpoint.theta := convert(Theta)@SF
  viewport.viewpoint.phi   := convert(Phi)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,ROTATE)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.theta)$Lisp
      sendSF(VIEW,viewport.viewpoint.phi)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

zoom(viewport:%,Scale:F) ==
  viewport.viewpoint.scale := convert(Scale)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,ZOOM)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.scale)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

zoom(viewport:%,ScaleX:F,ScaleY:F,ScaleZ:F) ==
  viewport.viewpoint.scaleX := convert(ScaleX)@SF
  viewport.viewpoint.scaleY := convert(ScaleY)@SF
  viewport.viewpoint.scaleZ := convert(ScaleZ)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,zoomx)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.scaleX)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleY)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleZ)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

translate(viewport,DeltaX,DeltaY) ==
```

```
  viewport.viewpoint.deltaX := convert(DeltaX)@SF
  viewport.viewpoint.deltaY := convert(DeltaY)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,TRANSLATE)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.deltaX)$Lisp
      sendSF(VIEW,viewport.viewpoint.deltaY)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

intensity(viewport,Amount) ==
  if (Amount < 0$F) or (Amount > 1$F) then
    error "The intensity must be a value between 0 and 1, inclusively."
  viewport.lighting.translucence := convert(Amount)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,translucenceDef)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.lighting.translucence)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

write(viewport:%,Filename:S,aThingToWrite:S) ==
  write(viewport,Filename,[aThingToWrite])

write(viewport,Filename) ==
  write(viewport,Filename,viewWriteDefault())

write(viewport:%,Filename:S,thingsToWrite:L S) ==
  stringToSend : S := ""
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,writeView)$Lisp
    checkViewport viewport =>
      sendSTR(VIEW,Filename)$Lisp
      m := minIndex(avail := viewWriteAvailable())
      for aTypeOfFile in thingsToWrite repeat
        if (writeTypeInt:= position(upperCase aTypeOfFile,avail)-m) < 0 then
          sayBrightly(["  > "::E,(concat(aTypeOfFile, _
            " is not a valid file type for writing a 3D viewport"))::E]$List(E))$Lisp
        else
          sendI(VIEW,writeTypeInt+(1$I))$Lisp
      sendI(VIEW,0$I)$Lisp     -- no more types of things to write
      getI(VIEW)$Lisp          -- acknowledge
      Filename

perspective (viewport,onOff) ==
  if onOff = "on" then viewport.perspective.perspectiveField := yes
  else viewport.perspective.perspectiveField := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
```

```
      sendI(VIEW,perspectiveOnOff)$Lisp
      checkViewport viewport =>
        sendI(VIEW,viewport.perspective.perspectiveField)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  showRegion (viewport,onOff) ==
    if onOff = "on" then viewport.flags.showRegionField := yes
    else viewport.flags.showRegionField := no
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW3D)$Lisp
      sendI(VIEW,region3D)$Lisp
      checkViewport viewport =>
        sendI(VIEW,viewport.flags.showRegionField)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  showClipRegion (viewport,onOff) ==
    if onOff = "on" then viewport.volume.clipRegionField := yes
    else viewport.volume.clipRegionField := no
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW3D)$Lisp
      sendI(VIEW,clipRegionOnOff)$Lisp
      checkViewport viewport =>
        sendI(VIEW,viewport.volume.clipRegionField)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  clipSurface (viewport,onOff) ==
    if onOff = "on" then viewport.volume.clipSurfaceField := yes
    else viewport.volume.clipSurfaceField := no
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW3D)$Lisp
      sendI(VIEW,clipSurfaceOnOff)$Lisp
      checkViewport viewport =>
        sendI(VIEW,viewport.volume.clipSurfaceField)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  eyeDistance(viewport:%,EyeDistance:F) ==
    viewport.perspective.eyeDistance := convert(EyeDistance)@SF
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW3D)$Lisp
      sendI(VIEW,eyeDistanceData)$Lisp
      checkViewport viewport =>
        sendSF(VIEW,viewport.perspective.eyeDistance)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  hitherPlane(viewport:%,HitherPlane:F) ==
    viewport.perspective.hitherPlane := convert(HitherPlane)@SF
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW3D)$Lisp
      sendI(VIEW,hitherPlaneData)$Lisp
      checkViewport viewport =>
```

```
        sendSF(VIEW,viewport.perspective.hitherPlane)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

    modifyPointData(viewport,anIndex,aPoint) ==
      (n := dimension aPoint) < 3 => error "The point should have dimension of at least 3"
      viewport.space3D := modifyPointData(viewport.space3D,anIndex,aPoint)
      (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,modifyPOINT)$Lisp
        checkViewport viewport =>
          sendI(VIEW,anIndex)$Lisp
          sendSF(VIEW,xCoord aPoint)$Lisp
          sendSF(VIEW,yCoord aPoint)$Lisp
          sendSF(VIEW,zCoord aPoint)$Lisp
          if (n = 3) then sendSF(VIEW,convert(0.5)@SF)$Lisp
          else sendSF(VIEW,color aPoint)$Lisp
          getI(VIEW)$Lisp          -- acknowledge

--    print viewport ==
--      (key(viewport) ^= 0$I) =>
--        sendI(VIEW,typeVIEW3D)$Lisp
--        sendI(VIEW,printViewport)$Lisp
--        checkViewport viewport =>
--          getI(VIEW)$Lisp          -- acknowledge
```

———-

— **VIEW3D.dotabb** —

```
"VIEW3D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VIEW3D"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"VIEW3D" -> "FSAGG"
```

———-

21.9 domain SPACE3 ThreeSpace

— **ThreeSpace.input** —

```
)set break resume
)sys rm -f ThreeSpace.output
)spool ThreeSpace.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show ThreeSpace
--R ThreeSpace R: Ring  is a domain constructor
--R Abbreviation for ThreeSpace is SPACE3
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SPACE3
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                check : % -> %
--R closedCurve : % -> List Point R       closedCurve : List Point R -> %
--R closedCurve? : % -> Boolean           coerce : % -> OutputForm
--R components : % -> List %              composite : List % -> %
--R composites : % -> List %              copy : % -> %
--R create3Space : SubSpace(3,R) -> %     create3Space : () -> %
--R curve : % -> List Point R             curve : List Point R -> %
--R curve : (%,List List R) -> %          curve : (%,List Point R) -> %
--R curve? : % -> Boolean                 hash : % -> SingleInteger
--R latex : % -> String                   lp : % -> List Point R
--R merge : (%,%) -> %                    merge : List % -> %
--R mesh : % -> List List Point R         mesh : List List Point R -> %
--R mesh? : % -> Boolean                  point : % -> Point R
--R point : Point R -> %                  point : (%,List R) -> %
--R point : (%,Point R) -> %              point? : % -> Boolean
--R polygon : % -> List Point R           polygon : List Point R -> %
--R polygon : (%,List List R) -> %        polygon : (%,List Point R) -> %
--R polygon? : % -> Boolean               subspace : % -> SubSpace(3,R)
--R ?~=? : (%,%) -> Boolean
--R closedCurve : (%,List List R) -> %
--R closedCurve : (%,List Point R) -> %
--R enterPointData : (%,List Point R) -> NonNegativeInteger
--R lllip : % -> List List List NonNegativeInteger
--R lllp : % -> List List List Point R
--R llprop : % -> List List SubSpaceComponentProperty
--R lprop : % -> List SubSpaceComponentProperty
--R mesh : (List List Point R,Boolean,Boolean) -> %
--R mesh : (%,List List List R,Boolean,Boolean) -> %
--R mesh : (%,List List Point R,Boolean,Boolean) -> %
--R mesh : (%,List List List R,List SubSpaceComponentProperty,SubSpaceComponentProperty) -> %
--R mesh : (%,List List Point R,List SubSpaceComponentProperty,SubSpaceComponentProperty) -> %
--R modifyPointData : (%,NonNegativeInteger,Point R) -> %
--R numberOfComponents : % -> NonNegativeInteger
--R numberOfComposites : % -> NonNegativeInteger
--R objects : % -> Record(points: NonNegativeInteger,curves: NonNegativeInteger,polygons: NonNegativeInt
--R point : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
```

```
)lisp (bye)
```

———-

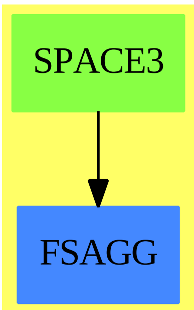
— **ThreeSpace.help** —

```
====================================================================
ThreeSpace examples
====================================================================

See Also:
o )show ThreeSpace
```

———-

21.9.1 ThreeSpace (SPACE3)



Exports:

check	closedCurve	closedCurve?	coerce	components
composite	composites	copy	create3Space	curve
curve?	enterPointData	hash	latex	lllip
lllp	llprop	lprop	lp	merge
mesh	mesh?	modifyPointData	numberOfComponents	objects
point	point?	polygon	polygon?	subspace
?=?	?~=?			

— **domain SPACE3 ThreeSpace** —

```
)abbrev domain SPACE3 ThreeSpace
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Related Constructors:
++ Also See:
```

```
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The domain ThreeSpace is used for creating three dimensional
++ objects using functions for defining points, curves, polygons, constructs
++ and the subspaces containing them.

ThreeSpace(R:Ring):Exports == Implementation where
  -- m is the dimension of the point

  I    ==> Integer
  PI   ==> PositiveInteger
  NNI  ==> NonNegativeInteger
  L    ==> List
  B    ==> Boolean
  O    ==> OutputForm
  SUBSPACE ==> SubSpace(3,R)
  POINT  ==> Point(R)
  PROP   ==> SubSpaceComponentProperty()
  REP3D  ==> Record(lp:L POINT,llliPt:L L L NNI, llProp:L L PROP, lProp:L PROP)
  OBJ3D  ==> Record(points:NNI, curves:NNI, polygons:NNI, constructs:NNI)

  Exports ==> ThreeSpaceCategory(R)
  Implementation ==> add
    import COMPPROP
    import POINT
    import SUBSPACE
    import ListFunctions2(List(R),POINT)
    import Set(NNI)

    Rep := Record( subspaceField:SUBSPACE, compositesField:L SUBSPACE, _
                   rep3DField:REP3D, objectsField:OBJ3D, _
                   converted:B)

--% Local Functions
    convertSpace : % -> %
    convertSpace space ==
      space.converted => space
      space.converted := true
      lllipt : L L L NNI := []
      llprop : L L PROP := []
      lprop : L PROP := []
      for component in children space.subspaceField repeat
        lprop := cons(extractProperty component,lprop)
        tmpllipt : L L NNI := []
        tmplprop : L PROP := []
        for curve in children component repeat
          tmplprop := cons(extractProperty curve,tmplprop)
          tmplipt : L NNI := []
```

```
        for point in children curve repeat
          tmplipt := cons(extractIndex point,tmplipt)
        tmpllipt := cons(reverse_! tmplipt,tmpllipt)
      llprop := cons(reverse_! tmplprop, llprop)
      lllipt := cons(reverse_! tmpllipt, lllipt)
    space.rep3DField := [pointData space.subspaceField,
                         reverse_! lllipt,reverse_! llprop,reverse_! lprop]
    space


--% Exported Functions
    polygon(space:%,points:L POINT) ==
      #points < 3 =>
        error "You need at least 3 points to define a polygon"
      pt := addPoint2(space.subspaceField,first points)
      points := rest points
      addPointLast(space.subspaceField, pt, first points, 1)
      for p in rest points repeat
        addPointLast(space.subspaceField, pt, p, 2)
      space.converted := false
      space
    create3Space() == [ new()$SUBSPACE, [], [ [], [], [], [] ], [0,0,0,0], false ]
    create3Space(s) == [ s, [], [ [], [], [], [] ], [0,0,0,0], false ]
    numberOfComponents(space) == #(children((space::Rep).subspaceField))
    numberOfComposites(space) == #((space::Rep).compositesField)
    merge(listOfThreeSpaces) ==
          -- * -- we may want to remove duplicate components when that functionality exists i
      newspace := create3Space(merge([ts.subspaceField for ts in listOfThreeSpaces]))
--      newspace.compositesField := [for cs in ts.compositesField for ts in listOfThreeSpaces
      for ts in listOfThreeSpaces repeat
        newspace.compositesField := append(ts.compositesField,newspace.compositesField)
      newspace
    merge(s1,s2) == merge([s1,s2])
    composite(listOfThreeSpaces) ==
      space := create3Space()
      space.subspaceField := merge [s.subspaceField for s in listOfThreeSpaces]
      space.compositesField := [deepCopy space.subspaceField]
--      for aSpace in listOfThreeSpaces repeat
          -- create a composite (which are supercomponents that group
          -- separate components together) out of all possible components
--        space.compositesField := append(children aSpace.subspaceField,space.compositesFiel
      space
    components(space) == [create3Space(s) for s in separate space.subspaceField]
    composites(space) == [create3Space(s) for s in space.compositesField]
    copy(space) ==
      spc := create3Space(deepCopy(space.subspaceField))
      spc.compositesField := [deepCopy s for s in space.compositesField]
      spc

    enterPointData(space,listOfPoints) ==
```

```
    for p in listOfPoints repeat
      addPoint(space.subspaceField,p)
    #(pointData space.subspaceField)
  modifyPointData(space,i,p) ==
    modifyPoint(space.subspaceField,i,p)
    space

    -- 3D primitives, each grouped in the following order
    --     xxx?(s)  : query whether the threespace, s, holds an xxx
    --     xxx(s)   : extract xxx from threespace, s
    --     xxx(p)   : create a new three space with xxx, p
    --     xxx(s,p) : add xxx, p, to a three space, s
    --     xxx(s,q) : add an xxx, convertable from q, to a three space, s
    --     xxx(s,i) : add an xxx, the data for xxx being indexed by reference  *** complete this
  point?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
      error "This ThreeSpace has more than one component"
      -- our 3-space has one component, a list of list of points
    #(kid:=children first c) = 1$NNI => -- the component has one subcomponent (a list of points)
      #(children first kid) = 1$NNI  -- this list of points only has one entry, so it's a point
    false
  point(space:%) ==
    point? space => extractPoint(traverse(space.subspaceField,[1,1,1]::L NNI))
    error "This ThreeSpace holds something other than a single point - try the objects() command"
  point(aPoint:POINT) == point(create3Space(),aPoint)
  point(space:%,aPoint:POINT) ==
    addPoint(space.subspaceField,[],aPoint)
    space.converted := false
    space
  point(space:%,l:L R) ==
    pt := point(l)
    point(space,pt)
  point(space:%,i:NNI) ==
    addPoint(space.subspaceField,[],i)
    space.converted := false
    space

  curve?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
      error "This ThreeSpace has more than one component"
      -- our 3-space has one component, a list of list of points
    #(children first c) = 1$NNI -- there is only one subcomponent, so it's a list of points
  curve(space:%) ==
    curve? space =>
      spc := first children first children space.subspaceField
      [extractPoint(s) for s in children spc]
    error "This ThreeSpace holds something other than a curve - try the objects() command"
  curve(points:L POINT) == curve(create3Space(),points)
  curve(space:%,points:L POINT) ==
    addPoint(space.subspaceField,[],first points)
```

```
    path : L NNI := [#(children space.subspaceField),1]
    for p in rest points repeat
      addPoint(space.subspaceField,path,p)
    space.converted := false
    space
  curve(space:%,points:L L R) ==
    pts := map(point,points)
    curve(space,pts)

  closedCurve?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
      error "This ThreeSpace has more than one component"
      -- our 3-space has one component, a list of list of points
    #(kid := children first c) = 1$NNI => -- there is one subcomponent => it's a list of po
      extractClosed first kid   -- is it a closed curve?
    false
  closedCurve(space:%) ==
    closedCurve? space =>
      spc := first children first children space.subspaceField
        -- get the list of points
      [extractPoint(s) for s in children spc]
        -- for now, we are not repeating points...
    error "This ThreeSpace holds something other than a curve - try the objects() command"
  closedCurve(points:L POINT) == closedCurve(create3Space(),points)
  closedCurve(space:%,points:L POINT) ==
    addPoint(space.subspaceField,[],first points)
    path : L NNI := [#(children space.subspaceField),1]
    closeComponent(space.subspaceField,path,true)
    for p in rest points repeat
      addPoint(space.subspaceField,path,p)
    space.converted := false
    space
  closedCurve(space:%,points:L L R) ==
    pts := map(point,points)
    closedCurve(space,pts)

  polygon?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
      error "This ThreeSpace has more than one component"
      -- our 3-space has one component, a list of list of points
    #(kid:=children first c) = 2::NNI =>
        -- there are two subcomponents
        -- the convention is to have one point in the first child and to put
        -- the remaining points (2 or more) in the second, and last, child
      #(children first kid) = 1$NNI and #(children second kid) > 2::NNI
    false  -- => returns Void...?
  polygon(space:%) ==
    polygon? space =>
      listOfPoints : L POINT :=
        [extractPoint(first children first (cs := children first children space.subspaceFie
```

```
      [extractPoint(s) for s in children second cs]
    error "This ThreeSpace holds something other than a polygon - try the objects() command"
  polygon(points:L POINT) == polygon(create3Space(),points)
  polygon(space:%,points:L L R) ==
    pts := map(point,points)
    polygon(space,pts)

  mesh?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
      error "This ThreeSpace has more than one component"
      -- our 3-space has one component, a list of list of points
    #(kid:=children first c) > 1$NNI =>
        -- there are two or more subcomponents (list of points)
        -- so this may be a definition of a mesh; if the size
        -- of each list of points is the same and they are all
        -- greater than 1(?) then we have an acceptable mesh
        -- use a set to hold the curve size info: if heterogenous
        -- curve sizes exist, then the set would hold all the sizes;
        -- otherwise it would just have the one element indicating
        -- the sizes for all the curves
        whatSizes := brace()$Set(NNI)
        for eachCurve in kid repeat
          insert_!(#children eachCurve,whatSizes)
        #whatSizes > 1 => error "Mesh defined with curves of different sizes"
        first parts whatSizes < 2 =>
          error "Mesh defined with single point curves (use curve())"
        true
    false
  mesh(space:%) ==
    mesh? space =>
      llp : L L POINT := []
      for lpSpace in children first children space.subspaceField repeat
        llp := cons([extractPoint(s) for s in children lpSpace],llp)
      llp
    error "This ThreeSpace holds something other than a mesh - try the objects() command"
  mesh(points:L L POINT) == mesh(create3Space(),points,false,false)
  mesh(points:L L POINT,prop1:B,prop2:B) == mesh(create3Space(),points,prop1,prop2)
--+ old ones \/
  mesh(space:%,llpoints:L L L R,lprops:L PROP,prop:PROP) ==
    pts := [map(point,points) for points in llpoints]
    mesh(space,pts,lprops,prop)
  mesh(space:%,llp:L L POINT,lprops:L PROP,prop:PROP) ==
    addPoint(space.subspaceField,[],first first llp)
    defineProperty(space.subspaceField,path:L NNI:=[#children space.subspaceField],prop)
    path := append(path,[1])
    defineProperty(space.subspaceField,path,first lprops)
    for p in rest (first llp) repeat
      addPoint(space.subspaceField,path,p)
    for lp in rest llp for aProp in rest lprops for count in 2.. repeat
      addPoint(space.subspaceField,path := [first path],first lp)
```

```
        path := append(path,[count])
        defineProperty(space.subspaceField,path,aProp)
        for p in rest lp repeat
          addPoint(space.subspaceField,path,p)
      space.converted := false
      space
--+ old ones /\
    mesh(space:%,llpoints:L L L R,prop1:B,prop2:B) ==
      pts := [map(point,points) for points in llpoints]
      mesh(space,pts,prop1,prop2)
    mesh(space:%,llp:L L POINT,prop1:B,prop2:B) ==
        -- prop2 refers to property of the ends of a surface (list of lists of points)
        -- while prop1 refers to the individual curves (list of points)
        -- ** note we currently use Booleans for closed (rather than a pair
        -- ** of booleans for closed and solid)
      propA : PROP := new()
      close(propA,prop1)
      propB : PROP := new()
      close(propB,prop2)
      addPoint(space.subspaceField,[],first first llp)
      defineProperty(space.subspaceField,path:L NNI:=[#children space.subspaceField],propB)
      path := append(path,[1])
      defineProperty(space.subspaceField,path,propA)
      for p in rest (first llp) repeat
        addPoint(space.subspaceField,path,p)
      for lp in rest llp for count in 2.. repeat
        addPoint(space.subspaceField,path := [first path],first lp)
        path := append(path,[count])
        defineProperty(space.subspaceField,path,propA)
        for p in rest lp repeat
          addPoint(space.subspaceField,path,p)
      space.converted := false
      space

    lp space ==
      if ^space.converted then space := convertSpace space
      space.rep3DField.lp
    lllip space   ==
      if ^space.converted then space := convertSpace space
      space.rep3DField.llliPt
--    lllp space   ==
--      if ^space.converted then space := convertSpace space
--      space.rep3DField.lllPt
    llprop space ==
      if ^space.converted then space := convertSpace space
      space.rep3DField.llProp
    lprop space  ==
      if ^space.converted then space := convertSpace space
      space.rep3DField.lProp
```

```
      -- this function is just to see how this representation really
      -- does work
    objects space ==
      if ^space.converted then space := convertSpace space
      numPts        := 0$NNI
      numCurves     := 0$NNI
      numPolys      := 0$NNI
      numConstructs := 0$NNI
      for component in children space.subspaceField repeat
        #(kid:=children component) = 1 =>
          #(children first kid) = 1 => numPts := numPts + 1
          numCurves := numCurves + 1
        (#kid = 2) and _
          (#children first kid = 1) and _
          (#children first rest kid ^= 1) =>
             numPolys := numPolys + 1
        numConstructs := numConstructs + 1
        -- otherwise, a mathematical surface is assumed
        -- there could also be garbage representation
        -- since there are always more permutations that
        -- we could ever want, so the user should not
        -- fumble around too much with the structure
        -- as other applications need to interpret it
      [numPts,numCurves,numPolys,numConstructs]

    check(s) ==
      ^s.converted => convertSpace s
      s

    subspace(s) == s.subspaceField

    coerce(s) ==
      if ^s.converted then s := convertSpace s
      hconcat(["3-Space with "::O, _
               (sizo:=#(s.rep3DField.llliPt))::O, _
               (sizo=1=>" component"::O;" components"::O)])
```

———

— SPACE3.dotabb —

```
"SPACE3" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SPACE3"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"SPACE3" -> "FSAGG"
```

———

21.10 domain TREE Tree

— Tree.input —

```
)set break resume
)sys rm -f Tree.output
)spool Tree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Tree
--R Tree S: SetCategory  is a domain constructor
--R Abbreviation for Tree is TREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TREE
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List %               copy : % -> %
--R cyclic? : % -> Boolean                cyclicCopy : % -> %
--R cyclicEntries : % -> List %           cyclicEqual? : (%,%) -> Boolean
--R cyclicParents : % -> List %           distance : (%,%) -> Integer
--R ?.value : (%,value) -> S              empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean                  leaves : % -> List S
--R map : ((S -> S),%) -> %               nodes : % -> List %
--R sample : () -> %                      tree : S -> %
--R tree : List S -> %                    tree : (S,List %) -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
```

```
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

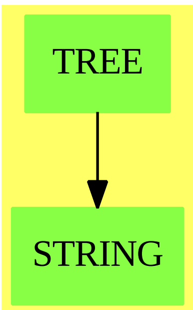
— **Tree.help** —

```
====================================================================
Tree examples
====================================================================

See Also:
o )show Tree
```

———-

21.10.1 Tree (TREE)



See

⇒ "BinaryTree" (BTREE) 3.11.1 on page 292
⇒ "BinarySearchTree" (BSTREE) 3.9.1 on page 285
⇒ "BinaryTournament" (BTOURN) 3.10.1 on page 289
⇒ "BalancedBinaryTree" (BBTREE) 3.1.1 on page 234
⇒ "PendantTree" (PENDTREE) 17.13.1 on page 1904

Exports:

any?	child?	children	coerce	copy
count	cyclic?	cyclicCopy	cyclicEntries	cyclicEqual?
cyclicParents	distance	empty	empty?	eq?
eval	every?	hash	latex	leaf?
leaves	less?	map	map!	member?
members	more?	node?	nodes	parts
sample	setchildren!	setelt	setvalue!	size?
tree	value	#?	?=?	?~=?
?.value				

— domain TREE Tree —

```
)abbrev domain TREE Tree
++ Author:W. H. Burge
++ Date Created:17 Feb 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ \spadtype{Tree(S)} is a basic domains of tree structures.
++ Each tree is either empty or else is a node consisting of a value and
++ a list of (sub)trees.

Tree(S: SetCategory): T==C where
 T== RecursiveAggregate(S) with
     finiteAggregate
     shallowlyMutable
     tree: (S,List %) -> %
       ++ tree(nd,ls) creates a tree with value nd, and children ls.
       ++
       ++X t1:=tree [1,2,3,4]
       ++X tree(5,[t1])

     tree: List S -> %
       ++ tree(ls) creates a tree from a list of elements of s.
       ++
       ++X tree [1,2,3,4]

     tree: S -> %
       ++ tree(nd) creates a tree with value nd, and no children
       ++
       ++X tree 6
```

```
    cyclic?: % -> Boolean
      ++ cyclic?(t) tests if t is a cyclic tree.
      ++
      ++X t1:=tree [1,2,3,4]
      ++X cyclic? t1

    cyclicCopy: % -> %
      ++ cyclicCopy(l) makes a copy of a (possibly) cyclic tree l.
      ++
      ++X t1:=tree [1,2,3,4]
      ++X cyclicCopy t1

    cyclicEntries:    % -> List %
      ++ cyclicEntries(t) returns a list of top-level cycles in tree t.
      ++
      ++X t1:=tree [1,2,3,4]
      ++X cyclicEntries t1

    cyclicEqual?: (%, %) -> Boolean
      ++ cyclicEqual?(t1, t2) tests of two cyclic trees have
      ++ the same structure.
      ++
      ++X t1:=tree [1,2,3,4]
      ++X t2:=tree [1,2,3,4]
      ++X cyclicEqual?(t1,t2)

    cyclicParents: % -> List %
      ++ cyclicParents(t) returns a list of cycles that are parents of t.
      ++
      ++X t1:=tree [1,2,3,4]
      ++X cyclicParents t1

C== add
   cycleTreeMax ==> 5

   Rep := Union(node:Record(value: S, args: List %),empty:"empty")
   t:%
   br:%
   s: S
   ls: List S
   empty? t == t case empty
   empty()  == ["empty"]
   children t ==
     t case empty => error "cannot take the children of an empty tree"
     (t.node.args)@List(%)
   setchildren_!(t,lt) ==
     t case empty => error "cannot set children of an empty tree"
     (t.node.args:=lt;t pretend %)
   setvalue_!(t,s) ==
     t case empty => error "cannot set value of an empty tree"
```

```
    (t.node.value:=s;s)
  count(n, t) ==
    t case empty => 0
    i := +/[count(n, c) for c in children t]
    value t = n => i + 1
    i
  count(fn: S -> Boolean, t: %): NonNegativeInteger ==
    t case empty => 0
    i := +/[count(fn, c) for c in children t]
    fn value t => i + 1
    i
  map(fn, t) ==
    t case empty => t
    tree(fn value t,[map(fn, c) for c in children t])
  map_!(fn, t) ==
    t case empty => t
    setvalue_!(t, fn value t)
    for c in children t repeat map_!(fn, c)
  tree(s,lt) == [[s,lt]]
  tree(s) == [[s,[]]]
  tree(ls) ==
    empty? ls => empty()
    tree(first ls, [tree s for s in rest ls])
  value t ==
    t case empty => error "cannot take the value of an empty tree"
    t.node.value
  child?(t1,t2) ==
    empty? t2 => false
    "or"/[t1 = t for t in children t2]
  distance1(t1: %, t2: %): Integer ==
    t1 = t2 => 0
    t2 case empty => -1
    u := [n for t in children t2 | (n := distance1(t1,t)) >= 0]
    #u > 0 => 1 + "min"/u
    -1
  distance(t1,t2) ==
    n := distance1(t1, t2)
    n >= 0 => n
    distance1(t2, t1)
  node?(t1, t2) ==
    t1 = t2 => true
    t case empty => false
    "or"/[node?(t1, t) for t in children t2]
  leaf? t ==
    t case empty => false
    empty? children t
  leaves t ==
    t case empty => empty()
    leaf? t => [value t]
    "append"/[leaves c for c in children t]
```

```
less? (t, n) == # t < n
more?(t, n) == # t > n
nodes t ==       ---buggy
  t case empty => empty()
  nl := [nodes c for c in children t]
  nl = empty() => [t]
  cons(t,"append"/nl)
size? (t, n) == # t = n
any?(fn, t) ==  ---bug fixed
  t case empty => false
  fn value t or "or"/[any?(fn, c) for c in children t]
every?(fn, t) ==
  t case empty => true
  fn value t and "and"/[every?(fn, c) for c in children t]
member?(n, t) ==
  t case empty => false
  n = value t or "or"/[member?(n, c) for c in children t]
members t == parts t
parts t == --buggy?
  t case empty => empty()
  u := [parts c for c in children t]
  u = empty() => [value t]
  cons(value t,"append"/u)

---Functions that guard against cycles: =, #, copy-------------

-----> =
equal?: (%, %, %, %, Integer) -> Boolean

t1 = t2 == equal?(t1, t2, t1, t2, 0)

equal?(t1, t2, ot1, ot2, k) ==
  k = cycleTreeMax and (cyclic? ot1 or cyclic? ot2) =>
    error "use cyclicEqual? to test equality on cyclic trees"
  t1 case empty => t2 case empty
  t2 case empty => false
  value t1 = value t2 and (c1 := children t1) = (c2 := children t2) and
    "and"/[equal?(x,y,ot1, ot2,k + 1) for x in c1 for y in c2]

-----> #
treeCount: (%, %, NonNegativeInteger) -> NonNegativeInteger
# t == treeCount(t, t, 0)
treeCount(t, origTree, k) ==
  k = cycleTreeMax and cyclic? origTree =>
    error "# is not defined on cyclic trees"
  t case empty => 0
  1 + +/[treeCount(c, origTree, k + 1) for c in children t]

-----> copy
copy1: (%, %, Integer) -> %
```

```
copy t == copy1(t, t, 0)
copy1(t, origTree, k) ==
  k = cycleTreeMax and cyclic? origTree =>
    error "use cyclicCopy to copy a cyclic tree"
  t case empty  => t
  empty? children t => tree value t
  tree(value t, [copy1(x, origTree, k + 1) for x in children t])

-----------Functions that allow cycles---------------
--local utility functions:
eqUnion: (List %, List %) -> List %
eqMember?: (%, List %) -> Boolean
eqMemberIndex: (%, List %, Integer) -> Integer
lastNode: List % -> List %
insert: (%, List %) -> List %

-----> coerce to OutputForm
if S has SetCategory then
  multipleOverbar: (OutputForm, Integer, List %) -> OutputForm
  coerce1: (%, List %, List %) -> OutputForm

  coerce(t:%): OutputForm == coerce1(t, empty()$(List %), cyclicParents t)

  coerce1(t,parents, pl) ==
    t case empty => empty()@List(S)::OutputForm
    eqMember?(t, parents) =>
      multipleOverbar(("."))::OutputForm,eqMemberIndex(t, pl,0),pl)
    empty? children t => value t::OutputForm
    nodeForm := (value t)::OutputForm
    if (k := eqMemberIndex(t, pl, 0)) > 0 then
       nodeForm := multipleOverbar(nodeForm, k, pl)
    prefix(nodeForm,
      [coerce1(br,cons(t,parents),pl) for br in children t])

  multipleOverbar(x, k, pl) ==
    k < 1 => x
    #pl = 1 => overbar x
    s : String := "abcdefghijklmnopqrstuvwxyz"
    c := s.(1 + ((k - 1) rem 26))
    overlabel(c::OutputForm, x)

-----> cyclic?
cyclic2?: (%, List %) -> Boolean

cyclic? t == cyclic2?(t, empty()$(List %))

cyclic2?(x,parents) ==
  empty? x => false
  eqMember?(x, parents) => true
  for y in children x repeat
```

```
      cyclic2?(y,cons(x, parents)) => return true
    false

  -----> cyclicCopy
  cyclicCopy2: (%, List %) -> %
  copyCycle2: (%, List %) -> %
  copyCycle4: (%, %, %, List %) -> %

  cyclicCopy(t) == cyclicCopy2(t, cyclicEntries t)

  cyclicCopy2(t, cycles) ==
    eqMember?(t, cycles) => return copyCycle2(t, cycles)
    tree(value t, [cyclicCopy2(c, cycles) for c in children t])

  copyCycle2(cycle, cycleList) ==
    newCycle := tree(value cycle, nil)
    setchildren!(newCycle,
      [copyCycle4(c,cycle,newCycle, cycleList) for c in children cycle])
    newCycle

  copyCycle4(t, cycle, newCycle, cycleList) ==
    empty? cycle => empty()
    eq?(t, cycle) => newCycle
    eqMember?(t, cycleList) => copyCycle2(t, cycleList)
    tree(value t,
         [copyCycle4(c, cycle, newCycle, cycleList) for c in children t])

  -----> cyclicEntries
  cyclicEntries3: (%, List %, List %) -> List %

  cyclicEntries(t) == cyclicEntries3(t, empty()$(List %), empty()$(List %))

  cyclicEntries3(t, parents, cl) ==
    empty? t => cl
    eqMember?(t, parents) => insert(t, cl)
    parents := cons(t, parents)
    for y in children t repeat
      cl := cyclicEntries3(t, parents, cl)
    cl

  -----> cyclicEqual?
  cyclicEqual4?: (%, %, List %, List %) -> Boolean

  cyclicEqual?(t1, t2) ==
    cp1 := cyclicParents t1
    cp2 := cyclicParents t2
    #cp1 ^= #cp2 or null cp1 => t1 = t2
    cyclicEqual4?(t1, t2, cp1, cp2)

  cyclicEqual4?(t1, t2, cp1, cp2) ==
```

```
  t1 case empty => t2 case empty
  t2 case empty => false
  0 ^= (k := eqMemberIndex(t1, cp1, 0)) => eq?(t2, cp2 . k)
  value t1 = value t2 and
    "and"/[cyclicEqual4?(x,y,cp1,cp2)
             for x in children t1 for y in children t2]

-----> cyclicParents t
cyclicParents3: (%, List %, List %) -> List %

cyclicParents t == cyclicParents3(t, empty()$(List %), empty()$(List %))

cyclicParents3(x, parents, pl) ==
  empty? x => pl
  eqMember?(x, parents) =>
    cycleMembers := [y for y in parents while not eq?(x,y)]
    eqUnion(cons(x, cycleMembers), pl)
  parents := cons(x, parents)
  for y in children x repeat
    pl := cyclicParents3(y, parents, pl)
  pl

insert(x, l) ==
  eqMember?(x, l) => l
  cons(x, l)

lastNode l ==
  empty? l => error "empty tree has no last node"
  while not empty? rest l repeat l := rest l
  l

eqMember?(y,l) ==
  for x in l repeat eq?(x,y) => return true
  false

eqMemberIndex(x, l, k) ==
  null l => k
  k := k + 1
  eq?(x, first l) => k
  eqMemberIndex(x, rest l, k)

eqUnion(u, v) ==
  null u => v
  x := first u
  newV :=
    eqMember?(x, v) => v
    cons(x, v)
  eqUnion(rest u, newV)
```

———-

— **TREE.dotabb** —

```
"TREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TREE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"TREE" -> "STRING"
```

———-

21.11 domain TUBE TubePlot

— **TubePlot.input** —

```
)set break resume
)sys rm -f TubePlot.output
)spool TubePlot.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TubePlot
--R TubePlot Curve: PlottableSpaceCurveCategory  is a domain constructor
--R Abbreviation for TubePlot is TUBE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TUBE
--R
--R------------------------------- Operations --------------------------------
--R closed? : % -> Boolean                getCurve : % -> Curve
--R open? : % -> Boolean
--R listLoops : % -> List List Point DoubleFloat
--R setClosed : (%,Boolean) -> Boolean
--R tube : (Curve,List List Point DoubleFloat,Boolean) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

— **TubePlot.help** —

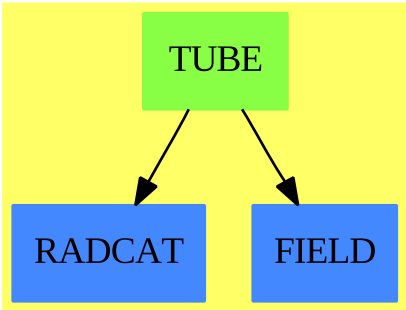
```
====================================================================
```

```
TubePlot examples
====================================================================

See Also:
o )show TubePlot
```

———-

21.11.1 TubePlot (TUBE)



Exports:
closed? getCurve listLoops open? setClosed tube

— **domain TUBE TubePlot** —

```
)abbrev domain TUBE TubePlot
++ Author: Clifton J. Williamson
++ Date Created: Bastille Day 1989
++ Date Last Updated: 5 June 1990
++ Keywords:
++ Examples:
++ Description:
++ Package for constructing tubes around 3-dimensional parametric curves.
++ Domain of tubes around 3-dimensional parametric curves.

TubePlot(Curve): Exports == Implementation where
  Curve : PlottableSpaceCurveCategory
  B   ==> Boolean
  L   ==> List
  Pt  ==> Point DoubleFloat

  Exports ==> with
    getCurve: % -> Curve
      ++ getCurve(t) returns the \spadtype{PlottableSpaceCurveCategory}
```

```
      ++ representing the parametric curve of the given tube plot t.
    listLoops: % -> L L Pt
      ++ listLoops(t) returns the list of lists of points, or the 'loops',
      ++ of the given tube plot t.
    closed?: % -> B
      ++ closed?(t) tests whether the given tube plot t is closed.
    open?: % -> B
      ++ open?(t) tests whether the given tube plot t is open.
    setClosed: (%,B) -> B
      ++ setClosed(t,b) declares the given tube plot t to be closed if
      ++ b is true, or if b is false, t is set to be open.
    tube: (Curve,L L Pt,B) -> %
      ++ tube(c,ll,b) creates a tube of the domain \spadtype{TubePlot} from a
      ++ space curve c of the category \spadtype{PlottableSpaceCurveCategory},
      ++ a list of lists of points (loops) ll and a boolean b which if
      ++ true indicates a closed tube, or if false an open tube.

  Implementation ==> add

--% representation

    Rep := Record(parCurve:Curve,loops:L L Pt,closedTube?:B)

    getCurve plot == plot.parCurve

    listLoops plot == plot.loops

    closed? plot == plot.closedTube?
    open? plot   == not plot.closedTube?

    setClosed(plot,flag) == plot.closedTube? := flag

    tube(curve,ll,b) == [curve,ll,b]
```

———-

— TUBE.dotabb —

```
"TUBE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TUBE"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"TUBE" -> "FIELD"
"TUBE" -> "RADCAT"
```

———-

21.12 domain TUPLE Tuple

— Tuple.input —

```
)set break resume
)sys rm -f Tuple.output
)spool Tuple.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Tuple
--R Tuple S: Type  is a domain constructor
--R Abbreviation for Tuple is TUPLE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TUPLE
--R
--R------------------------------- Operations --------------------------------
--R coerce : PrimitiveArray S -> %        coerce : % -> PrimitiveArray S
--R length : % -> NonNegativeInteger
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R select : (%,NonNegativeInteger) -> S
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

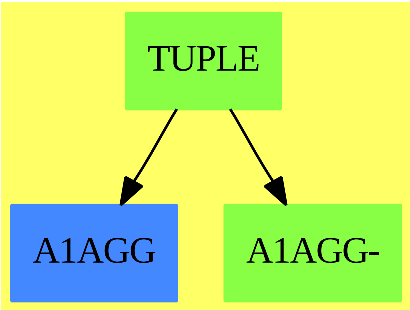
— Tuple.help —

```
====================================================================
Tuple examples
====================================================================

See Also:
o )show Tuple
```

———-

21.12.1 Tuple (TUPLE)



See

⇒ "PrimitiveArray" (PRIMARR) 17.30.1 on page 2069
⇒ "IndexedFlexibleArray" (IFARRAY) 10.10.1 on page 1187
⇒ "FlexibleArray" (FARRAY) 7.14.1 on page 853
⇒ "IndexedOneDimensionalArray" (IARRAY1) 10.13.1 on page 1208
⇒ "OneDimensionalArray" (ARRAY1) 16.3.1 on page 1736

Exports:
coerce hash latex length select ?=? ?~=?

— domain TUPLE Tuple —

```
)abbrev domain TUPLE Tuple
++ Author: Mark Botch
++ Description:
++ This domain is used to interface with the interpreter's notion
++ of comma-delimited sequences of values.

Tuple(S:Type): CoercibleTo(PrimitiveArray S) with
  coerce: PrimitiveArray S -> %
   ++ coerce(a) makes a tuple from primitive array a
   ++
   ++X t1:PrimitiveArray(Integer):= [i for i in 1..10]
   ++X t2:=coerce(t1)$Tuple(Integer)

  select: (%, NonNegativeInteger) -> S
   ++ select(x,n) returns the n-th element of tuple x.
   ++ tuples are 0-based
   ++
   ++X t1:PrimitiveArray(Integer):= [i for i in 1..10]
   ++X t2:=coerce(t1)$Tuple(Integer)
   ++X select(t2,3)

  length: % -> NonNegativeInteger
   ++ length(x) returns the number of elements in tuple x
```

```
   ++
   ++X t1:PrimitiveArray(Integer):= [i for i in 1..10]
   ++X t2:=coerce(t1)$Tuple(Integer)
   ++X length(t2)

  if S has SetCategory then SetCategory
 == add
  Rep := Record(len : NonNegativeInteger, elts : PrimitiveArray S)

  coerce(x: PrimitiveArray S): %  == [#x, x]
  coerce(x:%): PrimitiveArray(S) == x.elts
  length x == x.len

  select(x, n) ==
    n >= x.len => error "Index out of bounds"
    x.elts.n

  if S has SetCategory then
    x = y == (x.len = y.len) and (x.elts =$PrimitiveArray(S) y.elts)
    coerce(x : %): OutputForm ==
      paren [(x.elts.i)::OutputForm
             for i in minIndex x.elts .. maxIndex x.elts]$List(OutputForm)
```

— **TUPLE.dotabb** —

```
"TUPLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TUPLE"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"A1AGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=A1AGG"]
"TUPLE" -> "A1AGG"
"TUPLE" -> "A1AGG-"
```

21.13 domain ARRAY2 TwoDimensionalArray

— **TwoDimensionalArray.input** —

```
)set break resume
)sys rm -f TwoDimensionalArray.output
)spool TwoDimensionalArray.output
)set message test on
)set message auto off
```

```
)clear all
--S 1 of 20
arr : ARRAY2 INT := new(5,4,0)
--R
--R
--R        +0  0  0  0+
--R        |          |
--R        |0  0  0  0|
--R        |          |
--R   (1)  |0  0  0  0|
--R        |          |
--R        |0  0  0  0|
--R        |          |
--R        +0  0  0  0+
--R                                         Type: TwoDimensionalArray Integer
--E 1

--S 2 of 20
setelt(arr,1,1,17)
--R
--R
--R   (2)  17
--R                                                     Type: PositiveInteger
--E 2

--S 3 of 20
arr
--R
--R
--R        +17  0  0  0+
--R        |           |
--R        |0   0  0  0|
--R        |           |
--R   (3)  |0   0  0  0|
--R        |           |
--R        |0   0  0  0|
--R        |           |
--R        +0   0  0  0+
--R                                         Type: TwoDimensionalArray Integer
--E 3

--S 4 of 20
elt(arr,1,1)
--R
--R
--R   (4)  17
--R                                                     Type: PositiveInteger
--E 4

--S 5 of 20
```

```
arr(3,2) := 15
--R
--R
--R   (5)  15
--R                                                    Type: PositiveInteger
--E 5

--S 6 of 20
arr(3,2)
--R
--R
--R   (6)  15
--R                                                    Type: PositiveInteger
--E 6

--S 7 of 20
row(arr,1)
--R
--R
--R   (7)  [17,0,0,0]
--R                                        Type: OneDimensionalArray Integer
--E 7

--S 8 of 20
column(arr,1)
--R
--R
--R   (8)  [17,0,0,0,0]
--R                                        Type: OneDimensionalArray Integer
--E 8

--S 9 of 20
nrows(arr)
--R
--R
--R   (9)  5
--R                                                    Type: PositiveInteger
--E 9

--S 10 of 20
ncols(arr)
--R
--R
--R   (10)  4
--R                                                    Type: PositiveInteger
--E 10

--S 11 of 20
map(-,arr)
--R
```

```
--R
--R         +- 17   0    0  0+
--R         |               |
--R         | 0     0    0  0|
--R         |               |
--R   (11)  | 0    - 15  0  0|
--R         |               |
--R         | 0     0    0  0|
--R         |               |
--R         + 0     0    0  0+
--R                                              Type: TwoDimensionalArray Integer
--E 11

--S 12 of 20
map((x +-> x + x),arr)
--R
--R
--R         +34  0   0  0+
--R         |           |
--R         |0   0   0  0|
--R         |           |
--R   (12)  |0   30  0  0|
--R         |           |
--R         |0   0   0  0|
--R         |           |
--R         +0   0   0  0+
--R                                              Type: TwoDimensionalArray Integer
--E 12

--S 13 of 20
arrc := copy(arr)
--R
--R
--R         +17  0   0  0+
--R         |           |
--R         |0   0   0  0|
--R         |           |
--R   (13)  |0   15  0  0|
--R         |           |
--R         |0   0   0  0|
--R         |           |
--R         +0   0   0  0+
--R                                              Type: TwoDimensionalArray Integer
--E 13

--S 14 of 20
map!(-,arrc)
--R
--R
--R         +- 17   0    0  0+
```

```
--R         |                |
--R         | 0     0    0  0|
--R         |                |
--R   (14)  | 0    - 15  0  0|
--R         |                |
--R         | 0     0    0  0|
--R         |                |
--R         + 0     0    0  0+
--R                                Type: TwoDimensionalArray Integer
--E 14

--S 15 of 20
arrc
--R
--R
--R         +- 17   0    0  0+
--R         |                |
--R         | 0     0    0  0|
--R         |                |
--R   (15)  | 0    - 15  0  0|
--R         |                |
--R         | 0     0    0  0|
--R         |                |
--R         + 0     0    0  0+
--R                                Type: TwoDimensionalArray Integer
--E 15

--S 16 of 20
arr
--R
--R
--R         +17  0   0  0+
--R         |            |
--R         |0   0   0  0|
--R         |            |
--R   (16)  |0   15  0  0|
--R         |            |
--R         |0   0   0  0|
--R         |            |
--R         +0   0   0  0+
--R                                Type: TwoDimensionalArray Integer
--E 16

--S 17 of 20
member?(17,arr)
--R
--R
--R   (17)  true
--R                                                    Type: Boolean
--E 17
```

```
--S 18 of 20
member?(10317,arr)
--R
--R
--R   (18)  false
--R                                                          Type: Boolean
--E 18

--S 19 of 20
count(17,arr)
--R
--R
--R   (19)  1
--R                                                  Type: PositiveInteger
--E 19

--S 20 of 20
count(0,arr)
--R
--R
--R   (20)  18
--R                                                  Type: PositiveInteger
--E 20
)spool
)lisp (bye)
```

— **TwoDimensionalArray.help** —

```
====================================================================
TwoDimensionalArray examples
====================================================================

The TwoDimensionalArray domain is used for storing data in a two
dimensional data structure indexed by row and by column.  Such an
array is a homogeneous data structure in that all the entries of the
array must belong to the same Axiom domain..  Each array has a fixed
number of rows and columns specified by the user and arrays are not
extensible.  In Axiom, the indexing of two-dimensional arrays is
one-based.  This means that both the "first" row of an array and the
"first" column of an array are given the index 1.  Thus, the entry
in the upper left corner of an array is in position (1,1).

The operation new creates an array with a specified number of rows and
columns and fills the components of that array with a specified entry.
The arguments of this operation specify the number of rows, the number
of columns, and the entry.
```

This creates a five-by-four array of integers, all of whose entries are zero.

```
  arr : ARRAY2 INT := new(5,4,0)
        +0  0  0  0+
        |          |
        |0  0  0  0|
        |          |
        |0  0  0  0|
        |          |
        |0  0  0  0|
        |          |
        +0  0  0  0+
                         Type: TwoDimensionalArray Integer
```

The entries of this array can be set to other integers using setelt.

Issue this to set the element in the upper left corner of this array to 17.

```
  setelt(arr,1,1,17)
    17
                         Type: PositiveInteger
```

Now the first element of the array is 17.

```
  arr
        +17  0  0  0+
        |           |
        |0   0  0  0|
        |           |
        |0   0  0  0|
        |           |
        |0   0  0  0|
        |           |
        +0   0  0  0+
                         Type: TwoDimensionalArray Integer
```

Likewise, elements of an array are extracted using the operation elt.

```
  elt(arr,1,1)
    17
                         Type: PositiveInteger
```

Another way to use these two operations is as follows. This sets the element in position (3,2) of the array to 15.

```
  arr(3,2) := 15
    15
                         Type: PositiveInteger
```

This extracts the element in position (3,2) of the array.

```
  arr(3,2)
    15
                         Type: PositiveInteger
```

The operations elt and setelt come equipped with an error check which verifies that the indices are in the proper ranges. For example, the above array has five rows and four columns, so if you ask for the entry in position (6,2) with arr(6,2) Axiom displays an error message. If there is no need for an error check, you can call the operations qelt and qsetelt which provide the same functionality but without the error check. Typically, these operations are called in well-tested programs.

The operations row and column extract rows and columns, respectively, and return objects of OneDimensionalArray with the same underlying element type.

```
  row(arr,1)
    [17,0,0,0]
                         Type: OneDimensionalArray Integer

  column(arr,1)
    [17,0,0,0,0]
                         Type: OneDimensionalArray Integer
```

You can determine the dimensions of an array by calling the operations nrows and ncols, which return the number of rows and columns, respectively.

```
  nrows(arr)
    5
                         Type: PositiveInteger

  ncols(arr)
    4
                         Type: PositiveInteger
```

To apply an operation to every element of an array, use map. This creates a new array. This expression negates every element.

```
  map(-,arr)
         +- 17   0    0  0+
         |               |
         | 0     0    0  0|
         |               |
         | 0    - 15  0  0|
         |               |
         | 0     0    0  0|
         |               |
```

```
        + 0     0    0  0+
                         Type: TwoDimensionalArray Integer
```

This creates an array where all the elements are doubled.

```
  map((x +-> x + x),arr)
        +34  0   0  0+
        |           |
        |0   0   0  0|
        |           |
        |0   30  0  0|
        |           |
        |0   0   0  0|
        |           |
        +0   0   0  0+
                         Type: TwoDimensionalArray Integer
```

To change the array destructively, use map instead of map. If you need to make a copy of any array, use copy.

```
  arrc := copy(arr)
        +17  0   0  0+
        |           |
        |0   0   0  0|
        |           |
        |0   15  0  0|
        |           |
        |0   0   0  0|
        |           |
        +0   0   0  0+
                         Type: TwoDimensionalArray Integer

  map!(-,arrc)
        +- 17   0    0  0+
        |               |
        | 0     0    0  0|
        |               |
        | 0    - 15  0  0|
        |               |
        | 0     0    0  0|
        |               |
        + 0     0    0  0+
                         Type: TwoDimensionalArray Integer

  arrc
        +- 17   0    0  0+
        |               |
        | 0     0    0  0|
        |               |
        | 0    - 15  0  0|
```

```
        |               |
        | 0     0    0  0|
        |               |
        + 0     0    0  0+
                        Type: TwoDimensionalArray Integer

  arr
        +17  0   0  0+
        |            |
        |0   0   0  0|
        |            |
        |0   15  0  0|
        |            |
        |0   0   0  0|
        |            |
        +0   0   0  0+
                        Type: TwoDimensionalArray Integer

Use member? to see if a given element is in an array.

  member?(17,arr)
    true
                        Type: Boolean

  member?(10317,arr)
    false
                        Type: Boolean

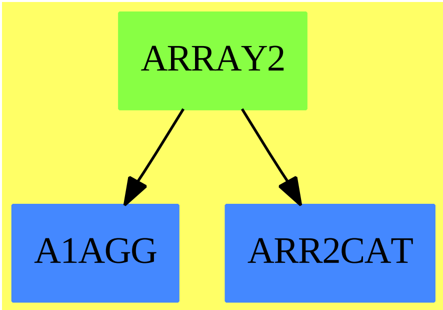
To see how many times an element appears in an array, use count.

  count(17,arr)
    1
                        Type: PositiveInteger

  count(0,arr)
    18
                        Type: PositiveInteger

See Also:
o )help Matrix
o )help OneDimensionalArray
o )show TwoDimensionalArray
```

21.13.1 TwoDimensionalArray (ARRAY2)



See

⇒ "InnerIndexedTwoDimensionalArray" (IIARRAY2) 10.23.1 on page 1254
⇒ "IndexedTwoDimensionalArray" (IARRAY2) 10.15.1 on page 1221

Exports:

any?	coerce	column	copy	count
elt	empty	empty?	eq?	eval
every?	fill!	hash	latex	less?
map	map!	maxColIndex	maxRowIndex	member?
members	more?	minColIndex	minRowIndex	ncols
new	nrows	parts	qelt	qsetelt!
row	sample	setColumn!	setRow!	setelt
size?	#?	?=?	?~=?	

— domain ARRAY2 TwoDimensionalArray —

```
)abbrev domain ARRAY2 TwoDimensionalArray
++ Author: Mark Botch
++ Description:
++ A TwoDimensionalArray is a two dimensional array with
++ 1-based indexing for both rows and columns.

TwoDimensionalArray(R):Exports == Implementation where
  R : Type
  Row ==> OneDimensionalArray R
  Col ==> OneDimensionalArray R

  Exports ==> TwoDimensionalArrayCategory(R,Row,Col) with
    shallowlyMutable
      ++ One may destructively alter TwoDimensionalArray's.

  Implementation ==> InnerIndexedTwoDimensionalArray(R,1,1,Row,Col)
```

————-

— **ARRAY2.dotabb** —

```
"ARRAY2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ARRAY2"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ARR2CAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ARR2CAT"]
"ARRAY2" -> "ARR2CAT"
"ARRAY2" -> "A1AGG"
```

21.14 domain VIEW2D TwoDimensionalViewport

— **TwoDimensionalViewport.help** —

```
====================================================================
TwoDimensionalViewport examples
====================================================================

We want to graph x^3 * (a+b*x) on the interval x=-1..1
so we clear out the workspace

We assign values to the constants
  a:=0.5
    0.5
                              Type: Float
  b:=0.5
    0.5
                              Type: Float

We draw the first case of the graph

  y1:=draw(x^3*(a+b*x),x=-1..1,title=="2.2.10 explicit")
    TwoDimensionalViewport: "2.2.10 explicit"
                              Type: TwoDimensionalViewport

We fetch the graph of the first object

  g1:=getGraph(y1,1)
    Graph with 1 point list
                              Type: GraphImage

We extract its points

  pointLists g1
   [
```

```
[[-1.,0.,1.,3.], [-0.95833333333333337,-1.8336166570216028E-2,1.,3.],
 [-0.91666666666666674,-3.2093942901234518E-2,1.,3.],
 [-0.87500000000000011,-4.18701171875E-2,1.,3.],
 [-0.83333333333333348,-4.8225308641975301E-2,1.,3.],
 [-0.79166666666666685,-5.1683967496141986E-2,1.,3.],
 [-0.75000000000000022,-5.2734375E-2,1.,3.],
 [-0.70833333333333359,-5.1828643422067916E-2,1.,3.],
 [-0.66666666666666696,-4.9382716049382741E-2,1.,3.],
 [-0.62500000000000033,-4.5776367187500042E-2,1.,3.],
 [-0.5833333333333337,-4.1353202160493867E-2,1.,3.],
 [-0.54166666666666707,-3.6420657310956832E-2,1.,3.],
 [-0.50000000000000044,-3.1250000000000056E-2,1.,3.],
 [-0.45833333333333376,-2.6076328607253136E-2,1.,3.],
 [-0.41666666666666707,-2.1098572530864244E-2,1.,3.],
 [-0.37500000000000039,-1.6479492187500042E-2,1.,3.],
 [-0.3333333333333337,-1.2345679012345713E-2,1.,3.],
 [-0.29166666666666702,-8.7875554591049648E-3,1.,3.],
 [-0.25000000000000033,-5.8593750000000208E-3,1.,3.],
 [-0.20833333333333368,-3.5792221257716214E-3,1.,3.],
 [-0.16666666666666702,-1.9290123456790237E-3,1.,3.],
 [-0.12500000000000036,-8.5449218750000705E-4,1.,3.],
 [-8.3333333333333703E-2,-2.6523919753086765E-4,1.,3.],
 [-4.1666666666667039E-2,-3.4661940586420673E-5,1.,3.],
 [-3.7470027081099033E-16,-2.6304013894372334E-47,1.,3.],
 [4.166666666666629E-2,3.7676022376542178E-5,1.,3.],
 [8.3333333333332954E-2,3.1346450617283515E-4,1.,3.],
 [0.12499999999999961,1.0986328124999894E-3,1.,3.],
 [0.16666666666666627,2.7006172839505972E-3,1.,3.],
 [0.20833333333333293,5.463023244598731E-3,1.,3.],
 [0.24999999999999958,9.765624999999948E-3,1.,3.],
 [0.29166666666666624,1.6024365837191284E-2,1.,3.],
 [0.33333333333333293,2.469135802469126E-2,1.,3.],
 [0.37499999999999961,3.6254882812499882E-2,1.,3.],
 [0.4166666666666663,5.1239390432098617E-2,1.,3.],
 [0.45833333333333298,7.0205500096450435E-2,1.,3.],
 [0.49999999999999967,9.3749999999999792E-2,1.,3.],
 [0.5416666666666663,0.12250584731867258,1.,3.],
 [0.58333333333333293,0.15714216820987617,1.,3.],
 [0.62499999999999956,0.1983642578124995,1.,3.],
 [0.66666666666666619,0.24691358024691298,1.,3.],
 [0.70833333333333282,0.30356776861496837,1.,3.],
 [0.74999999999999944,0.369140624999999,1.,3.],
 [0.79166666666666607,0.44448212046681984,1.,3.],
 [0.8333333333333327,0.530478395061727,1.,3.],
 [0.87499999999999933,0.62805175781249845,1.,3.],
 [0.91666666666666596,0.73816068672839308,1.,3.],
 [0.95833333333333259,0.86179982880015205,1.,3.], [1.,1.,1.,3.]]
]
                    Type: List List Point DoubleFloat
```

Now we create a second graph with a changed parameter

```
  b:=1.0
    1.0
                        Type: Float
```

We draw it

```
  y2:=draw(x^3*(a+b*x),x=-1..1)
    TwoDimensionalViewport: "AXIOM2D"
                        Type: TwoDimensionalViewport
```

We fetch this new graph

```
  g2:=getGraph(y2,1)
    Graph with 1 point list
                       Type: GraphImage
```

We get the points from this graph

```
  pointLists g2
   [
     [[-1.,0.5,1.,3.], [-0.95833333333333337,0.40339566454475323,1.,3.],
      [-0.91666666666666674,0.32093942901234584,1.,3.],
      [-0.87500000000000011,0.25122070312500017,1.,3.],
      [-0.83333333333333348,0.19290123456790137,1.,3.],
      [-0.79166666666666685,0.14471510898919768,1.,3.],
      [-0.75000000000000022,0.10546875000000019,1.,3.],
      [-0.70833333333333359,7.404091917438288E-2,1.,3.],
      [-0.66666666666666696,4.938271604938288E-2,1.,3.],
      [-0.62500000000000033,3.0517578125000125E-2,1.,3.],
      [-0.5833333333333337,1.6541280864197649E-2,1.,3.],
      [-0.54166666666666707,6.6219376929013279E-3,1.,3.],
      [-0.50000000000000044,5.5511151231257827E-17,1.,3.],
      [-0.45833333333333376,-4.011742862654287E-3,1.,3.],
      [-0.41666666666666707,-6.0281635802469057E-3,1.,3.],
      [-0.37500000000000039,-6.5917968750000035E-3,1.,3.],
      [-0.3333333333333337,-6.1728395061728461E-3,1.,3.],
      [-0.29166666666666702,-5.1691502700617377E-3,1.,3.],
      [-0.25000000000000033,-3.9062500000000104E-3,1.,3.],
      [-0.20833333333333368,-2.6373215663580349E-3,1.,3.],
      [-0.16666666666666702,-1.543209876543218E-3,1.,3.],
      [-0.12500000000000036,-7.3242187500000564E-4,1.,3.],
      [-8.3333333333333703E-2,-2.4112654320987957E-4,1.,3.],
      [-4.1666666666667039E-2,-3.315489969135889E-5,1.,3.],
      [-3.7470027081099033E-16,-2.6304013894372324E-47,1.,3.],
      [4.166666666666629E-2,3.9183063271603852E-5,1.,3.],
      [8.3333333333332954E-2,3.3757716049382237E-4,1.,3.],
      [0.12499999999999961,1.2207031249999879E-3,1.,3.],
      [0.16666666666666627,3.0864197530863957E-3,1.,3.],
```

```
      [0.20833333333333293,6.4049238040123045E-3,1.,3.],
      [0.24999999999999958,1.1718749999999934E-2,1.,3.],
      [0.29166666666666624,1.9642771026234473E-2,1.,3.],
      [0.33333333333333293,3.0864197530864071E-2,1.,3.],
      [0.37499999999999961,4.6142578124999847E-2,1.,3.],
      [0.4166666666666663,6.6309799382715848E-2,1.,3.],
      [0.45833333333333298,9.2270085841049135E-2,1.,3.],
      [0.49999999999999967,0.12499999999999971,1.,3.],
      [0.5416666666666663,0.16554844232253049,1.,3.],
      [0.58333333333333293,0.21503665123456736,1.,3.],
      [0.62499999999999956,0.27465820312499928,1.,3.],
      [0.66666666666666619,0.3456790123456781,1.,3.],
      [0.70833333333333282,0.42943733121141858,1.,3.],
      [0.74999999999999944,0.52734374999999845,1.,3.],
      [0.79166666666666607,0.64088119695215873,1.,3.],
      [0.8333333333333327,0.77160493827160281,1.,3.],
      [0.87499999999999933,0.92114257812499756,1.,3.],
      [0.91666666666666596,1.0911940586419722,1.,3.],
      [0.95833333333333259,1.2835316599151199,1.,3.], [1.,1.5,1.,3.]]
     ]
                         Type: List List Point DoubleFloat
```

and we put these points, g2 onto the first graph y1 as graph 2

```
  putGraph(y1,g2,2)
                         Type: Void
```

And now we do the whole sequence again

```
  b:=2.0
    2.0
                         Type: Float

  y3:=draw(x^3*(a+b*x),x=-1..1)
    TwoDimensionalViewport: "AXIOM2D"
                        Type: TwoDimensionalViewport

  g3:=getGraph(y3,1)
    Graph with 1 point list
                        Type: GraphImage

  pointLists g3
   [
     [[-1.,1.5,1.,3.], [-0.95833333333333337,1.2468593267746917,1.,3.],
      [-0.91666666666666674,1.0270061728395066,1.,3.],
      [-0.87500000000000011,0.83740234375000044,1.,3.],
      [-0.83333333333333348,0.67515432098765471,1.,3.],
      [-0.79166666666666685,0.53751326195987703,1.,3.],
      [-0.75000000000000022,0.42187500000000056,1.,3.],
      [-0.70833333333333359,0.32578004436728447,1.,3.],
```

```
      [-0.66666666666666696,0.24691358024691412,1.,3.],
      [-0.62500000000000033,0.18310546875000044,1.,3.],
      [-0.5833333333333337,0.1323302469135807,1.,3.],
      [-0.54166666666666707,9.2707127700617648E-2,1.,3.],
      [-0.50000000000000044,6.2500000000000278E-2,1.,3.],
      [-0.45833333333333376,4.0117428626543411E-2,1.,3.],
      [-0.41666666666666707,2.4112654320987775E-2,1.,3.],
      [-0.37500000000000039,1.3183593750000073E-2,1.,3.],
      [-0.3333333333333337,6.1728395061728877E-3,1.,3.],
      [-0.29166666666666702,2.0676601080247183E-3,1.,3.],
      [-0.25000000000000033,1.0408340855860843E-17,1.,3.],
      [-0.20833333333333368,-7.5352044753086191E-4,1.,3.],
      [-0.16666666666666702,-7.7160493827160663E-4,1.,3.],
      [-0.12500000000000036,-4.8828125000000282E-4,1.,3.],
      [-8.3333333333333703E-2,-1.9290123456790339E-4,1.,3.],
      [-4.1666666666667039E-2,-3.0140817901235325E-5,1.,3.],
      [-3.7470027081099033E-16,-2.6304013894372305E-47,1.,3.],
      [4.166666666666629E-2,4.21971450617272E-5,1.,3.],
      [8.3333333333332954E-2,3.8580246913579681E-4,1.,3.],
      [0.12499999999999961,1.4648437499999848E-3,1.,3.],
      [0.16666666666666627,3.8580246913579933E-3,1.,3.],
      [0.20833333333333293,8.2887249228394497E-3,1.,3.],
      [0.24999999999999958,1.562499999999991E-2,1.,3.],
      [0.29166666666666624,2.6879581404320851E-2,1.,3.],
      [0.33333333333333293,4.3209876543209694E-2,1.,3.],
      [0.37499999999999961,6.5917968749999764E-2,1.,3.],
      [0.4166666666666663,9.6450617283950296E-2,1.,3.],
      [0.45833333333333298,0.13639925733024652,1.,3.],
      [0.49999999999999967,0.18749999999999956,1.,3.],
      [0.5416666666666663,0.25163363233024633,1.,3.],
      [0.58333333333333293,0.33082561728394977,1.,3.],
      [0.62499999999999956,0.42724609374999883,1.,3.],
      [0.66666666666666619,0.5432098765432084,1.,3.],
      [0.70833333333333282,0.68117645640431912,1.,3.],
      [0.74999999999999944,0.84374999999999756,1.,3.],
      [0.79166666666666607,1.0336793499228365,1.,3.],
      [0.8333333333333327,1.2538580246913544,1.,3.],
      [0.87499999999999933,1.507324218749996,1.,3.],
      [0.91666666666666596,1.7972608024691306,1.,3.],
      [0.95833333333333259,2.1269953221450555,1.,3.], [1.,2.5,1.,3.]]
     ]
                        Type: List List Point DoubleFloat
```

and put the third graphs points g3 onto the first graph y1 as graph 3

```
  putGraph(y1,g3,3)
                        Type: Void
```

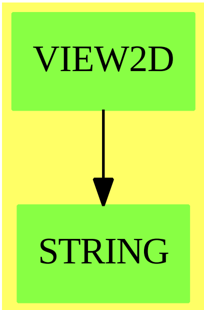
Finally we show the combined result

```
  vp:=makeViewport2D(y1)
    TwoDimensionalViewport: "2.2.10 explicit"
                        Type: TwoDimensionalViewport

See Also:
o )show TwoDimensionalViewport
```

————-

21.14.1 TwoDimensionalViewport (VIEW2D)



Exports:

axes	close	coerce	connect	controlPanel
dimensions	getGraph	getPickedPoints	graphState	graphStates
graphs	hash	key	latex	makeViewport2D
move	options	points	putGraph	region
reset	resize	scale	show	title
translate	units	update	viewport2D	write
?=?	?~=?			

— **domain VIEW2D TwoDimensionalViewport** —

```
)abbrev domain VIEW2D TwoDimensionalViewport
++ Author: Jim Wen
++ Date Created: 28 April 1989
++ Date Last Updated: 29 October 1991, Jon Steinbach
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
```

```
++ TwoDimensionalViewport creates viewports to display graphs.

TwoDimensionalViewport ():Exports == Implementation where

  VIEW    ==> VIEWPORTSERVER$Lisp
  sendI   ==> SOCK_-SEND_-INT
  sendSF  ==> SOCK_-SEND_-FLOAT
  sendSTR ==> SOCK_-SEND_-STRING
  getI    ==> SOCK_-GET_-INT
  getSF   ==> SOCK_-GET_-FLOAT

  typeGRAPH  ==> 2
  typeVIEW2D ==> 3

  makeGRAPH  ==> (-1)$SingleInteger
  makeVIEW2D ==> (-1)$SingleInteger

  I    ==> Integer
  PI   ==> PositiveInteger
  NNI  ==> NonNegativeInteger
  XY   ==> Record( X:I, Y:I )
  XYP  ==> Record( X:PI, Y:PI )
  XYNN ==> Record( X:NNI, Y:NNI )
  F    ==> Float
  SF   ==> DoubleFloat
  STR  ==> String
  L    ==> List
  V    ==> Vector
  E    ==> OutputForm
  FLAG ==> Record( showCP:I )
  PAL  ==> Palette()
  B    ==> Boolean
  G    ==> GraphImage
  GS   ==> Record( scaleX:SF, scaleY:SF, deltaX:SF, deltaY:SF, _
                  points:I, connect:I, spline:I, _
                  axes:I, axesColor:PAL, units:I, unitsColor:PAL, _
                  showing:I)
  GU   ==> Union(G,"undefined")
  DROP ==> DrawOption
  POINT ==> Point(SF)

  TRANSLATE2D     ==> 0$I
  SCALE2D         ==> 1$I
  pointsOnOff     ==> 2
  connectOnOff    ==> 3
  spline2D        ==> 4   -- used for controlling regions, now
  reset2D         ==> 5
  hideControl2D   ==> 6
  closeAll2D      ==> 7
  axesOnOff2D     ==> 8
```

```
unitsOnOff2D    ==> 9

SPADBUTTONPRESS ==> 100
MOVE            ==> 102
RESIZE          ==> 103
TITLE           ==> 104
showing2D       ==> 105  -- as defined in include/actions.h
putGraph2D      ==> 106
writeView       ==> 110
axesColor2D     ==> 112
unitsColor2D    ==> 113
getPickedPTS    ==> 119

graphStart      ==> 13   -- as defined in include/actions.h

noControl ==> 0$I

yes       ==> 1$I
no        ==> 0$I

maxGRAPHS ==> 9::I   -- should be the same as maxGraphs in include/view2d.h

fileTypeDefs ==> ["PIXMAP"]    -- see include/write.h for things to include

Exports ==> SetCategory with
  getPickedPoints : $ -> L POINT
    ++ getPickedPoints(x)
    ++ returns a list of small floats for the points the
    ++ user interactively picked on the viewport
    ++ for full integration into the system, some design
    ++ issues need to be addressed: e.g. how to go through
    ++ the GraphImage interface, how to default to graphs, etc.
  viewport2D     : ()                                  -> $
    ++ viewport2D() returns an undefined two-dimensional viewport
    ++ of the domain \spadtype{TwoDimensionalViewport} whose
    ++ contents are empty.
  makeViewport2D : $                                   -> $
    ++ makeViewport2D(v) takes the given two-dimensional viewport,
    ++ v, of the domain \spadtype{TwoDimensionalViewport} and
    ++ displays a viewport window on the screen which contains
    ++ the contents of v.
  options        : $                                   -> L DROP
    ++ options(v) takes the given two-dimensional viewport, v, of the
    ++ domain \spadtype{TwoDimensionalViewport} and returns a list
    ++ containing the draw options from the domain \spadtype{DrawOption}
    ++ for v.
  options        : ($,L DROP)                          -> $
    ++ options(v,lopt) takes the given two-dimensional viewport, v,
    ++ of the domain \spadtype{TwoDimensionalViewport} and returns
    ++ v with it's draw options modified to be those which are indicated
```

```
  ++ in the given list, \spad{lopt} of domain \spadtype{DrawOption}.
makeViewport2D : (G,L DROP)                           -> $
  ++ makeViewport2D(gi,lopt) creates and displays a viewport window
  ++ of the domain \spadtype{TwoDimensionalViewport} whose graph
  ++ field is assigned to be the given graph, \spad{gi}, of domain
  ++ \spadtype{GraphImage}, and whose options field is set to be
  ++ the list of options, \spad{lopt} of domain \spadtype{DrawOption}.
graphState     : ($,PI,SF,SF,SF,SF,I,I,I,I,PAL,I,PAL,I) -> Void
  ++ graphState(v,num,sX,sY,dX,dY,pts,lns,box,axes,axesC,un,unC,cP)
  ++ sets the state of the characteristics for the graph indicated
  ++ by \spad{num} in the given two-dimensional viewport v, of domain
  ++ \spadtype{TwoDimensionalViewport}, to the values given as
  ++ parameters.  The scaling of the graph in the x and y component
  ++ directions is set to be \spad{sX} and \spad{sY}; the window
  ++ translation in the x and y component directions is set to be
  ++ \spad{dX} and \spad{dY}; The graph points, lines, bounding box,
  ++ axes, or units will be shown in the viewport if their given
  ++ parameters \spad{pts}, \spad{lns}, \spad{box}, \spad{axes} or
  ++ \spad{un} are set to be \spad{1}, but will not be shown if they
  ++ are set to \spad{0}.  The color of the axes and the color of the
  ++ units are indicated by the palette colors \spad{axesC} and
  ++ \spad{unC} respectively.  To display the control panel when
  ++ the viewport window is displayed, set \spad{cP} to \spad{1},
  ++ otherwise set it to \spad{0}.
graphStates    : $                                    -> V GS
  ++ graphStates(v) returns and shows a listing of a record containing
  ++ the current state of the characteristics of each of the ten graph
  ++ records in the given two-dimensional viewport, v, which is of
  ++ domain \spadtype{TwoDimensionalViewport}.
graphs         : $                                    -> V GU
  ++ graphs(v) returns a vector, or list, which is a union of all
  ++ the graphs, of the domain \spadtype{GraphImage}, which are
  ++ allocated for the two-dimensional viewport, v, of domain
  ++ \spadtype{TwoDimensionalViewport}.  Those graphs which have
  ++ no data are labeled "undefined", otherwise their contents
  ++ are shown.
title          : ($,STR)                              -> Void
  ++ title(v,s) changes the title which is shown in the two-dimensional
  ++ viewport window, v of domain \spadtype{TwoDimensionalViewport}.
putGraph       : ($,G,PI)                             -> Void
  ++ putGraph(v,gi,n) sets the graph field indicated by n, of the
  ++ indicated two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, to be the graph, \spad{gi}
  ++ of domain \spadtype{GraphImage}.  The contents of viewport, v,
  ++ will contain \spad{gi} when the function \spadfun{makeViewport2D}
  ++ is called to create the an updated viewport v.
getGraph       : ($,PI)                               -> G
  ++ getGraph(v,n) returns the graph which is of the domain
  ++ \spadtype{GraphImage} which is located in graph field n
  ++ of the given two-dimensional viewport, v, which is of the
```

```
  ++ domain \spadtype{TwoDimensionalViewport}.
axes          : ($,PI,STR)                            -> Void
  ++ axes(v,n,s) displays the axes of the graph in field n of
  ++ the given two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, if s is "on", or does
  ++ not display the axes if s is "off".
axes          : ($,PI,PAL)                            -> Void
  ++ axes(v,n,c) displays the axes of the graph in field n of
  ++ the given two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, with the axes color set to
  ++ the given palette color c.
units         : ($,PI,STR)                            -> Void
  ++ units(v,n,s) displays the units of the graph in field n of
  ++ the given two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, if s is "on", or does
  ++ not display the units if s is "off".
units         : ($,PI,PAL)                            -> Void
  ++ units(v,n,c) displays the units of the graph in field n of
  ++ the given two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, with the units color set to
  ++ the given palette color c.
points        : ($,PI,STR)                            -> Void
  ++ points(v,n,s) displays the points of the graph in field n of
  ++ the given two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, if s is "on", or does
  ++ not display the points if s is "off".
region        : ($,PI,STR)                            -> Void
  ++ region(v,n,s) displays the bounding box of the graph in
  ++ field n of the given two-dimensional viewport, v, which is
  ++ of domain \spadtype{TwoDimensionalViewport}, if s is "on",
  ++ or does not display the bounding box if s is "off".
connect       : ($,PI,STR)                            -> Void
  ++ connect(v,n,s) displays the lines connecting the graph
  ++ points in field n of the given two-dimensional viewport, v,
  ++ which is of domain \spadtype{TwoDimensionalViewport}, if s
  ++ is "on", or does not display the lines if s is "off".
controlPanel  : ($,STR)                               -> Void
  ++ controlPanel(v,s) displays the control panel of the given
  ++ two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, if s is "on", or hides
  ++ the control panel if s is "off".
close         : $                                     -> Void
  ++ close(v) closes the viewport window of the given
  ++ two-dimensional viewport, v, which is of domain
  ++ \spadtype{TwoDimensionalViewport}, and terminates the
  ++ corresponding process ID.
dimensions    : ($,NNI,NNI,PI,PI)                     -> Void
  ++ dimensions(v,x,y,width,height) sets the position of the
  ++ upper left-hand corner of the two-dimensional viewport, v,
  ++ which is of domain \spadtype{TwoDimensionalViewport}, to
```

```
    ++ the window coordinate x, y, and sets the dimensions of the
    ++ window to that of \spad{width}, \spad{height}.  The new
    ++ dimensions are not displayed until the function
    ++ \spadfun{makeViewport2D} is executed again for v.
  scale          : ($,PI,F,F)                       -> Void
    ++ scale(v,n,sx,sy) displays the graph in field n of the given
    ++ two-dimensional viewport, v, which is of domain
    ++ \spadtype{TwoDimensionalViewport}, scaled by the factor \spad{sx}
    ++ in the x-coordinate direction and by the factor \spad{sy} in
    ++ the y-coordinate direction.
  translate      : ($,PI,F,F)                       -> Void
    ++ translate(v,n,dx,dy) displays the graph in field n of the given
    ++ two-dimensional viewport, v, which is of domain
    ++ \spadtype{TwoDimensionalViewport}, translated by \spad{dx} in
    ++ the x-coordinate direction from the center of the viewport, and
    ++ by \spad{dy} in the y-coordinate direction from the center.
    ++ Setting \spad{dx} and \spad{dy} to \spad{0} places the center
    ++ of the graph at the center of the viewport.
  show           : ($,PI,STR)                       -> Void
    ++ show(v,n,s) displays the graph in field n of the given
    ++ two-dimensional viewport, v, which is of domain
    ++ \spadtype{TwoDimensionalViewport}, if s is "on", or does not
    ++ display the graph if s is "off".
  move           : ($,NNI,NNI)                      -> Void
    ++ move(v,x,y) displays the two-dimensional viewport, v, which
    ++ is of domain \spadtype{TwoDimensionalViewport}, with the upper
    ++ left-hand corner of the viewport window at the screen
    ++ coordinate position x, y.
  update         :($,G,PI)                         -> Void
    ++ update(v,gr,n) drops the graph \spad{gr} in slot \spad{n}
    ++ of viewport \spad{v}. The graph gr must have been
    ++ transmitted already and acquired an integer key.
  resize         : ($,PI,PI)                        -> Void
    ++ resize(v,w,h) displays the two-dimensional viewport, v, which
    ++ is of domain \spadtype{TwoDimensionalViewport}, with a width
    ++ of w and a height of h, keeping the upper left-hand corner
    ++ position unchanged.
  write          : ($,STR)                          -> STR
    ++ write(v,s) takes the given two-dimensional viewport, v, which
    ++ is of domain \spadtype{TwoDimensionalViewport}, and creates
    ++ a directory indicated by s, which contains the graph data
    ++ files for v.
  write          : ($,STR,STR)                      -> STR
    ++ write(v,s,f) takes the given two-dimensional viewport, v, which
    ++ is of domain \spadtype{TwoDimensionalViewport}, and creates
    ++ a directory indicated by s, which contains the graph data
    ++ files for v and an optional file type f.
  write          : ($,STR,L STR)                    -> STR
    ++ write(v,s,lf) takes the given two-dimensional viewport, v, which
    ++ is of domain \spadtype{TwoDimensionalViewport}, and creates
```

```
    ++ a directory indicated by s, which contains the graph data
    ++ files for v and the optional file types indicated by the list lf.
  reset         :  $                                     -> Void
    ++ reset(v) sets the current state of the graph characteristics
    ++ of the given two-dimensional viewport, v, which is of domain
    ++ \spadtype{TwoDimensionalViewport}, back to their initial settings.
  key           :  $                                     -> I
    ++ key(v) returns the process ID number of the given two-dimensional
    ++ viewport, v, which is of domain \spadtype{TwoDimensionalViewport}.
  coerce        :  $                                     -> E
    ++ coerce(v) returns the given two-dimensional viewport, v, which
    ++ is of domain \spadtype{TwoDimensionalViewport} as output of
    ++ the domain \spadtype{OutputForm}.

Implementation ==> add

  import GraphImage()
  import Color()
  import Palette()
  import ViewDefaultsPackage()
  import DrawOptionFunctions0
  import POINT

  Rep := Record (key:I, graphsField:V GU, graphStatesField:V GS, _
               title:STR, moveTo:XYNN, size:XYP, flags:FLAG, optionsField:L DROP)

  defaultGS : GS := [convert(0.9)@SF, convert(0.9)@SF, 0$SF, 0$SF, _
                  yes, yes, no, _
                  yes, axesColorDefault(), no, unitsColorDefault(), _
                  yes]


   --% Local Functions
  checkViewport (viewport:$):B ==
      -- checks to see if this viewport still exists
      -- by sending the key to the viewport manager and
      -- waiting for its reply after it checks it against
      -- the viewports in its list. a -1 means it doesn't
      -- exist.
    sendI(VIEW,viewport.key)$Lisp
    i := getI(VIEW)$Lisp
    (i < 0$I) =>
      viewport.key := 0$I
      error "This viewport has already been closed!"
    true

  doOptions(v:Rep):Void ==
    v.title := title(v.optionsField,"AXIOM2D")
    -- etc - 2D specific stuff...
```

```
 --% Exported Functions

options viewport ==
  viewport.optionsField

options(viewport,opts) ==
  viewport.optionsField := opts
  viewport

putGraph (viewport,aGraph,which) ==
  if ((which > maxGRAPHS) or (which < 1)) then
    error "Trying to put a graph with a negative index or too big an index"
  viewport.graphsField.which := aGraph

getGraph (viewport,which) ==
  if ((which > maxGRAPHS) or (which < 1)) then
    error "Trying to get a graph with a negative index or too big an index"
  viewport.graphsField.which case "undefined" =>
    error "Graph is undefined!"
  viewport.graphsField.which::GraphImage


graphStates viewport  == viewport.graphStatesField
graphs viewport       == viewport.graphsField
key viewport          == viewport.key

dimensions(viewport,ViewX,ViewY,ViewWidth,ViewHeight) ==
  viewport.moveTo := [ViewX,ViewY]
  viewport.size   := [ViewWidth,ViewHeight]

move(viewport,xLoc,yLoc) ==
  viewport.moveTo := [xLoc,yLoc]
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,MOVE)$Lisp
    checkViewport viewport =>
      sendI(VIEW,xLoc)$Lisp
      sendI(VIEW,yLoc)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

update(viewport,graph,slot) ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,putGraph2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,key graph)$Lisp
      sendI(VIEW,slot)$Lisp
      getI(VIEW)$Lisp -- acknowledge

resize(viewport,xSize,ySize) ==
```

```
    viewport.size := [xSize,ySize]
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,RESIZE)$Lisp
      checkViewport viewport =>
        sendI(VIEW,xSize)$Lisp
        sendI(VIEW,ySize)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  translate(viewport,graphIndex,xTranslateF,yTranslateF) ==
    xTranslate := convert(xTranslateF)@SF
    yTranslate := convert(yTranslateF)@SF
    if (graphIndex > maxGRAPHS) then
      error "Referring to a graph with too big an index"
    viewport.graphStatesField.graphIndex.deltaX := xTranslate
    viewport.graphStatesField.graphIndex.deltaY := yTranslate
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,TRANSLATE2D)$Lisp
      checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        sendSF(VIEW,xTranslate)$Lisp
        sendSF(VIEW,yTranslate)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  scale(viewport,graphIndex,xScaleF,yScaleF) ==
    xScale := convert(xScaleF)@SF
    yScale := convert(yScaleF)@SF
    if (graphIndex > maxGRAPHS) then
      error "Referring to a graph with too big an index"
    viewport.graphStatesField.graphIndex.scaleX := xScale  -- check union (undefined?)
    viewport.graphStatesField.graphIndex.scaleY := yScale  -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,SCALE2D)$Lisp
      checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        sendSF(VIEW,xScale)$Lisp
        sendSF(VIEW,yScale)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  viewport2D ==
    [0,new(maxGRAPHS,"undefined"), _
     new(maxGRAPHS,copy defaultGS),"AXIOM2D", _
      [viewPosDefault().1,viewPosDefault().2],[viewSizeDefault().1,viewSizeDefault().2], _
       [noControl], [] ]

  makeViewport2D(g:G,opts:L DROP) ==
    viewport               := viewport2D()
    viewport.graphsField.1 := g
```

```
    viewport.optionsField := opts
    makeViewport2D viewport

  makeViewport2D viewportDollar ==
    viewport := viewportDollar::Rep
    doOptions viewport --local function to extract and assign optional arguments for 2D viewports
    sayBrightly(["   AXIOM2D data being transmitted to the viewport manager...":E]$List(E))$Lisp
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,makeVIEW2D)$Lisp
    sendSTR(VIEW,viewport.title)$Lisp
    sendI(VIEW,viewport.moveTo.X)$Lisp
    sendI(VIEW,viewport.moveTo.Y)$Lisp
    sendI(VIEW,viewport.size.X)$Lisp
    sendI(VIEW,viewport.size.Y)$Lisp
    sendI(VIEW,viewport.flags.showCP)$Lisp
    for i in 1..maxGRAPHS repeat
      g := (graphs viewport).i
      if g case "undefined" then
        sendI(VIEW,0$I)$Lisp
      else
        sendI(VIEW,key(g::G))$Lisp
        gs := (graphStates viewport).i
        sendSF(VIEW,gs.scaleX)$Lisp
        sendSF(VIEW,gs.scaleY)$Lisp
        sendSF(VIEW,gs.deltaX)$Lisp
        sendSF(VIEW,gs.deltaY)$Lisp
        sendI(VIEW,gs.points)$Lisp
        sendI(VIEW,gs.connect)$Lisp
        sendI(VIEW,gs.spline)$Lisp
        sendI(VIEW,gs.axes)$Lisp
        hueShade := hue hue gs.axesColor + shade gs.axesColor * numberOfHues()
        sendI(VIEW,hueShade)$Lisp
        sendI(VIEW,gs.units)$Lisp
        hueShade := hue hue gs.unitsColor + shade gs.unitsColor * numberOfHues()
        sendI(VIEW,hueShade)$Lisp
        sendI(VIEW,gs.showing)$Lisp
    viewport.key := getI(VIEW)$Lisp
    viewport

  graphState(viewport,num,sX,sY,dX,dY,Points,Lines,Spline, _
             Axes,AxesColor,Units,UnitsColor,Showing) ==
    viewport.graphStatesField.num := [sX,sY,dX,dY,Points,Lines,Spline, _
                                      Axes,AxesColor,Units,UnitsColor,Showing]

  title(viewport,Title) ==
    viewport.title := Title
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,TITLE)$Lisp
      checkViewport viewport =>
```

```
      sendSTR(VIEW,Title)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

reset viewport ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,SPADBUTTONPRESS)$Lisp
    checkViewport viewport =>
      sendI(VIEW,reset2D)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

axes (viewport:$,graphIndex:PI,onOff:STR) : Void ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  if onOff = "on" then
    status := yes
  else
    status := no
  viewport.graphStatesField.graphIndex.axes := status  -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,axesOnOff2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendI(VIEW,status)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

axes (viewport:$,graphIndex:PI,color:PAL) : Void ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  viewport.graphStatesField.graphIndex.axesColor := color
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,axesColor2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      hueShade := hue hue color + shade color * numberOfHues()
      sendI(VIEW,hueShade)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

units (viewport:$,graphIndex:PI,onOff:STR) : Void ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  if onOff = "on" then
    status := yes
  else
    status := no
  viewport.graphStatesField.graphIndex.units := status  -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
```

```
      sendI(VIEW,unitsOnOff2D)$Lisp
      checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        sendI(VIEW,status)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  units (viewport:$,graphIndex:PI,color:PAL) : Void ==
    if (graphIndex > maxGRAPHS) then
      error "Referring to a graph with too big an index"
    viewport.graphStatesField.graphIndex.unitsColor := color
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,unitsColor2D)$Lisp
      checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        hueShade := hue hue color + shade color * numberOfHues()
        sendI(VIEW,hueShade)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  connect (viewport:$,graphIndex:PI,onOff:STR) : Void ==
    if (graphIndex > maxGRAPHS) then
      error "Referring to a graph with too big an index"
    if onOff = "on" then
      status := 1$I
    else
      status := 0$I
    viewport.graphStatesField.graphIndex.connect := status  -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,connectOnOff)$Lisp
      checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        sendI(VIEW,status)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

  points (viewport:$,graphIndex:PI,onOff:STR) : Void ==
    if (graphIndex > maxGRAPHS) then
      error "Referring to a graph with too big an index"
    if onOff = "on" then
      status := 1$I
    else
      status := 0$I
    viewport.graphStatesField.graphIndex.points := status  -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
      sendI(VIEW,typeVIEW2D)$Lisp
      sendI(VIEW,pointsOnOff)$Lisp
      checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        sendI(VIEW,status)$Lisp
        getI(VIEW)$Lisp          -- acknowledge
```

```
region (viewport:$,graphIndex:PI,onOff:STR) : Void ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  if onOff = "on" then
    status := 1$I
  else
    status := 0$I
  viewport.graphStatesField.graphIndex.spline := status  -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,spline2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendI(VIEW,status)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

show (viewport,graphIndex,onOff) ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  if onOff = "on" then
    status := 1$I
  else
    status := 0$I
  viewport.graphStatesField.graphIndex.showing := status  -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,showing2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendI(VIEW,status)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

controlPanel (viewport,onOff) ==
  if onOff = "on" then viewport.flags.showCP := yes
  else viewport.flags.showCP := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,hideControl2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,viewport.flags.showCP)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

close viewport ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,closeAll2D)$Lisp
    checkViewport viewport =>
      getI(VIEW)$Lisp          -- acknowledge
      viewport.key := 0$I
```

```
coerce viewport ==
  (key(viewport) = 0$I) =>
    hconcat ["Closed or Undefined TwoDimensionalViewport: "::E,
              (viewport.title)::E]
  hconcat ["TwoDimensionalViewport: "::E, (viewport.title)::E]

write(viewport:$,Filename:STR,aThingToWrite:STR) ==
  write(viewport,Filename,[aThingToWrite])

write(viewport,Filename) ==
  write(viewport,Filename,viewWriteDefault())

write(viewport:$,Filename:STR,thingsToWrite:L STR) ==
  stringToSend : STR := ""
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,writeView)$Lisp
    checkViewport viewport =>
      sendSTR(VIEW,Filename)$Lisp
      m := minIndex(avail := viewWriteAvailable())
      for aTypeOfFile in thingsToWrite repeat
        if (writeTypeInt:= position(upperCase aTypeOfFile,avail)-m) < 0 then
          sayBrightly(["  > "::E,(concat(aTypeOfFile, _
            " is not a valid file type for writing a 2D viewport"))::E]$List(E))$Lisp
        else
          sendI(VIEW,writeTypeInt+(1$I))$Lisp
              --              stringToSend := concat [stringToSend,"%",aTypeOfFile]
              --              sendSTR(VIEW,stringToSend)$Lisp
      sendI(VIEW,0$I)$Lisp     -- no more types of things to write
      getI(VIEW)$Lisp          -- acknowledge
      Filename
```

— **VIEW2D.dotabb** —

```
"VIEW2D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VIEW2D"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"VIEW2D" -> "STRING"
```

Chapter 22

Chapter U

22.1 domain UFPS UnivariateFormalPowerSeries

— UnivariateFormalPowerSeries.input —

```
)set break resume
)sys rm -f UnivariateFormalPowerSeries.output
)spool UnivariateFormalPowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateFormalPowerSeries
--R UnivariateFormalPowerSeries Coef: Ring  is a domain constructor
--R Abbreviation for UnivariateFormalPowerSeries is UFPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UFPS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficients : % -> Stream Coef       coerce : Variable QUOTE x -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> NonNegativeInteger
--R evenlambert : % -> %                  hash : % -> SingleInteger
--R lagrange : % -> %                     lambert : % -> %
```

```
--R latex : % -> String                      leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %                 map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean                 oddlambert : % -> %
--R one? : % -> Boolean                      order : % -> NonNegativeInteger
--R pole? : % -> Boolean                     quoByVar : % -> %
--R recip : % -> Union(%,"failed")           reductum : % -> %
--R revert : % -> %                          sample : () -> %
--R series : Stream Coef -> %                variable : % -> Symbol
--R zero? : % -> Boolean                     ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Coef) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING
--R D : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PI
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef a
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef)
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) ->
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(QUOTE x,Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable QUOTE x) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has P
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R generalLambert : (%,Integer,Integer) -> %
--R integrate : (%,Variable QUOTE x) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R invmultisect : (Integer,Integer,%) -> %
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R multisect : (Integer,Integer,%) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R univariatePolynomial : (%,NonNegativeInteger) -> UnivariatePolynomial(QUOTE x,Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

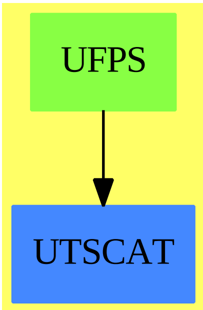
— **UnivariateFormalPowerSeries.help** —

```
====================================================================
UnivariateFormalPowerSeries examples
====================================================================

See Also:
o )show UnivariateFormalPowerSeries
```

———-

22.1.1 UnivariateFormalPowerSeries (UFPS)



Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos
cosh	cot	coth	csc
csch	D	degree	differentiate
eval	evenlambert	exp	exquo
extend	generalLambert	hash	integrate
invmultisect	lagrange	lambert	latex
leadingCoefficient	leadingMonomial	log	map
monomial	monomial?	multiplyCoefficients	multiplyExponents
multisect	nthRoot	oddlambert	one?
order	pi	pole?	polynomial
quoByVar	recip	reductum	revert
sample	sec	sech	series
sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate
unit?	unitCanonical	unitNormal	univariatePolynomial
variable	variables	zero?	?*?
?**?	?+?	?-?	-?
?=?	?^?	?~=?	?/?
?.?			

— domain UFPS UnivariateFormalPowerSeries —

```
)abbrev domain UFPS UnivariateFormalPowerSeries
++ Author: Mark Botch
++ Description:
++ This domain has no description

UnivariateFormalPowerSeries(Coef: Ring) ==
    UnivariateTaylorSeries(Coef, 'x, 0$Coef)
```

————

— UFPS.dotabb —

```
"UFPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UFPS"]
"UTSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=UTSCAT"]
"UFPS" -> "UTSCAT"
```

————

22.2 domain ULS UnivariateLaurentSeries

— UnivariateLaurentSeries.input —

```
)set break resume
)sys rm -f UnivariateLaurentSeries.output
)spool UnivariateLaurentSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeries
--R UnivariateLaurentSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for UnivariateLaurentSeries is ULS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ULS
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        center : % -> Coef
--R coefficient : (%,Integer) -> Coef     coerce : Variable var -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Integer
--R ?.? : (%,Integer) -> Coef             extend : (%,Integer) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial : (Coef,Integer) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R order : (%,Integer) -> Integer        order : % -> Integer
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     removeZeroes : (Integer,%) -> %
--R removeZeroes : % -> %                 sample : () -> %
--R truncate : (%,Integer) -> %           variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (UnivariateTaylorSeries(Coef,var,cen),%) -> % if Coef has FIELD
--R ?*? : (%,UnivariateTaylorSeries(Coef,var,cen)) -> % if Coef has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
```

```
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (UnivariateTaylorSeries(Coef,var,cen),UnivariateTaylorSeries(Coef,var,cen)) -> % if Coef has F
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?<? : (%,%) -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Uni
--R ?<=? : (%,%) -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Un
--R ?>? : (%,%) -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Uni
--R ?>=? : (%,%) -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Un
--R D : (%,Symbol) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIELD or
--R D : (%,List Symbol) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIEL
--R D : (%,Symbol,NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and
--R D : (%,List Symbol,List NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING
--R D : % -> % if UnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIELD or Coef has *: (In
--R D : (%,NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIE
--R D : (%,(UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,var,cen)),NonNegativeInt
--R D : (%,(UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,var,cen))) -> % if Coef
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIEL
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> UnivariateTaylorSeries(Coef,var,cen) if UnivariateTaylorSeries(Coef,var,cen) has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and UnivariateTaylorSeries(Coef,var,cen) has PFE
--R coerce : Fraction Integer -> % if UnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Coef has
--R coerce : % -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Univariate
--R coerce : Symbol -> % if UnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBOL and Coef has FIELD
--R coerce : UnivariateTaylorSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and UnivariateTaylorSeries(Coef,va
--R convert : % -> Pattern Integer if UnivariateTaylorSeries(Coef,var,cen) has KONVERT PATTERN INT and C
--R convert : % -> Pattern Float if UnivariateTaylorSeries(Coef,var,cen) has KONVERT PATTERN FLOAT and C
--R convert : % -> DoubleFloat if UnivariateTaylorSeries(Coef,var,cen) has REAL and Coef has FIELD
--R convert : % -> Float if UnivariateTaylorSeries(Coef,var,cen) has REAL and Coef has FIELD
--R convert : % -> InputForm if UnivariateTaylorSeries(Coef,var,cen) has KONVERT INFORM and Coef has FIE
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> UnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING S
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if UnivariateTaylorSeries(Co
--R differentiate : % -> % if UnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has F
--R differentiate : (%,NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen) has D
--R differentiate : (%,Variable var) -> %
--R differentiate : (%,(UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,v
--R differentiate : (%,(UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,v
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coef,var,ce
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,List UnivariateTaylorSeries(Coef,var,cen),List UnivariateTaylorSeries(Coef,var
--R eval : (%,UnivariateTaylorSeries(Coef,var,cen),UnivariateTaylorSeries(Coef,var,cen)) -> %
--R eval : (%,Equation UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(C
--R eval : (%,List Equation UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSer
--R eval : (%,List Symbol,List UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylor
--R eval : (%,Symbol,UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coe
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM an
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIEL
--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R floor : % -> UnivariateTaylorSeries(Coef,var,cen) if UnivariateTaylorSeries(Coef,var,cen)
--R fractionPart : % -> % if UnivariateTaylorSeries(Coef,var,cen) has EUCDOM and Coef has FIE
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R init : () -> % if UnivariateTaylorSeries(Coef,var,cen) has STEP and Coef has FIELD
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has var
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : (Integer,UnivariateTaylorSeries(Coef,var,cen)) -> %
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,var,cen)),%)
--R max : (%,%) -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD o
--R min : (%,%) -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD o
```

```
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R negative? : % -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R nextItem : % -> Union(%,"failed") if UnivariateTaylorSeries(Coef,var,cen) has STEP and Coef has FIEL
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> UnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Univa
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R random : () -> % if UnivariateTaylorSeries(Coef,var,cen) has INS and Coef has FIELD
--R rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if UnivariateTaylorSeries(Coef,var,cen) has LINEXP INT an
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if Univariate
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix UnivariateTaylorSeries(Coef,var,cen),vec:
--R reducedSystem : Matrix % -> Matrix UnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> Integer if UnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Coef has FIELD
--R retract : % -> Fraction Integer if UnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Coef has
--R retract : % -> Symbol if UnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBOL and Coef has FIELD
--R retract : % -> UnivariateTaylorSeries(Coef,var,cen)
--R retractIfCan : % -> Union(Integer,"failed") if UnivariateTaylorSeries(Coef,var,cen) has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if UnivariateTaylorSeries(Coef,var,cen) has RET
--R retractIfCan : % -> Union(Symbol,"failed") if UnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBO
--R retractIfCan : % -> Union(UnivariateTaylorSeries(Coef,var,cen),"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sign : % -> Integer if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Univ
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylor : % -> UnivariateTaylorSeries(Coef,var,cen)
--R taylorIfCan : % -> Union(UnivariateTaylorSeries(Coef,var,cen),"failed")
```

```
--R taylorRep : % -> UnivariateTaylorSeries(Coef,var,cen)
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIE
--R unitCanonical : % -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has F
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if UnivariateTaylorSeries(Coe
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> UnivariateTaylorSeries(Coef,var,cen) if UnivariateTaylorSeries(Coef,var
--R
--E 1

)spool
)lisp (bye)
```

———-

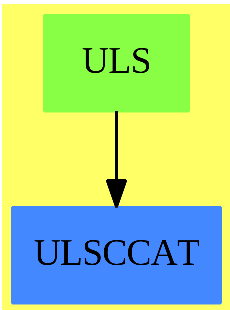
— **UnivariateLaurentSeries.help** —

```
====================================================================
UnivariateLaurentSeries examples
====================================================================

See Also:
o )show UnivariateLaurentSeries
```

———-

22.2.1 UnivariateLaurentSeries (ULS)



See

⇒ "UnivariateLaurentSeriesConstructor" (ULSCONS) 22.3.1 on page 2760

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
approximate	asec	asech
asin	asinh	associates?
atan	atanh	ceiling
center	characteristic	charthRoot
coerce	coefficient	coerce
complete	conditionP	convert
cos	cosh	cot
coth	csc	csch
D	degree	denom
denominator	differentiate	divide
euclideanSize	eval	exp
expressIdealMember	exquo	extend
extendedEuclidean	factor	factorPolynomial
factorSquareFreePolynomial	floor	fractionPart
gcd	gcdPolynomial	hash
init	integrate	inv
latex	laurent	leadingCoefficient
leadingMonomial	lcm	log
map	max	min
monomial	monomial?	multiEuclidean
multiplyCoefficients	multiplyExponents	negative?
nextItem	nthRoot	numer
numerator	one?	order
patternMatch	pi	pole?
positive?	prime?	principalIdeal
random	rationalFunction	recip
reducedSystem	reductum	removeZeroes
retract	retractIfCan	sample
sec	sech	series
sign	sin	sinh
sizeLess?	solveLinearPolynomialEquation	sqrt
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	tan	tanh
taylor	taylorIfCan	taylorRep
terms	truncate	unit?
unitCanonical	unitNormal	variable
variables	wholePart	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?.?	?~=?
?/?	?<?	?<=?
?>?	?>=?	?^?
?.?	?quo?	?rem?

— domain ULS UnivariateLaurentSeries —

```
)abbrev domain ULS UnivariateLaurentSeries
++ Author: Clifton J. Williamson
++ Date Created: 18 January 1990
++ Date Last Updated: 21 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent
++ Examples:
++ References:
++ Description:
++ Dense Laurent series in one variable
++ \spadtype{UnivariateLaurentSeries} is a domain representing Laurent
++ series in one variable with coefficients in an arbitrary ring.  The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion.  For example,
++ \spad{UnivariateLaurentSeries(Integer,x,3)} represents Laurent series in
++ \spad{(x - 3)} with integer coefficients.

UnivariateLaurentSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I   ==> Integer
  UTS ==> UnivariateTaylorSeries(Coef,var,cen)

  Exports ==> UnivariateLaurentSeriesConstructorCategory(Coef,UTS) with
    coerce: Variable(var) -> %
      ++ \spad{coerce(var)} converts the series variable \spad{var} into a
      ++ Laurent series.
    differentiate: (%,Variable(var)) -> %
      ++ \spad{differentiate(f(x),x)} returns the derivative of
      ++ \spad{f(x)} with respect to \spad{x}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
        ++ by integers.

  Implementation ==> UnivariateLaurentSeriesConstructor(Coef,UTS) add

    variable x == var
    center   x == cen

    coerce(v:Variable(var)) ==
```

```
      zero? cen => monomial(1,1)
      monomial(1,1) + monomial(cen,0)

    differentiate(x:%,v:Variable(var)) == differentiate x

    if Coef has Algebra Fraction Integer then
      integrate(x:%,v:Variable(var)) == integrate x
```

— **ULS.dotabb** —

```
"ULS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ULS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"ULS" -> "ULSCCAT"
```

22.3 domain ULSCONS UnivariateLaurentSeriesConstructor

— **UnivariateLaurentSeriesConstructor.input** —

```
)set break resume
)sys rm -f UnivariateLaurentSeriesConstructor.output
)spool UnivariateLaurentSeriesConstructor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeriesConstructor
--R UnivariateLaurentSeriesConstructor(Coef: Ring,UTS: UnivariateTaylorSeriesCategory Coef)  is a domain
--R Abbreviation for UnivariateLaurentSeriesConstructor is ULSCONS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ULSCONS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                         0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       center : % -> Coef
--R coefficient : (%,Integer) -> Coef    coerce : UTS -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R complete : % -> %                    degree : % -> Integer
--R ?.? : (%,Integer) -> Coef            extend : (%,Integer) -> %
--R hash : % -> SingleInteger            latex : % -> String
--R laurent : (Integer,UTS) -> %         leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial : (Coef,Integer) -> %       monomial? : % -> Boolean
--R one? : % -> Boolean                  order : (%,Integer) -> Integer
--R order : % -> Integer                 pole? : % -> Boolean
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R removeZeroes : (Integer,%) -> %      removeZeroes : % -> %
--R retract : % -> UTS                   sample : () -> %
--R taylor : % -> UTS                    taylorRep : % -> UTS
--R truncate : (%,Integer) -> %          variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (UTS,%) -> % if Coef has FIELD
--R ?*? : (%,UTS) -> % if Coef has FIELD
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (UTS,UTS) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?<? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R ?<=? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R ?>? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R ?>=? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R D : (%,Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,C
--R D : (%,List Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Inte
--R D : (%,Symbol,NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coe
--R D : (%,List Symbol,List NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has FI
--R D : % -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (Int
--R D : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R D : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R abs : % -> % if UTS has OINTDOM and Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> UTS if UTS has INS and Coef has FIELD
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and UTS has PFECAT and Coef has FIELD or UTS has
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Symbol -> % if UTS has RETRACT SYMBOL and Coef has FIELD
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and UTS has PFECAT and Coef has FI
--R convert : % -> Pattern Integer if UTS has KONVERT PATTERN INT and Coef has FIELD
--R convert : % -> Pattern Float if UTS has KONVERT PATTERN FLOAT and Coef has FIELD
--R convert : % -> InputForm if UTS has KONVERT INFORM and Coef has FIELD
--R convert : % -> Float if UTS has REAL and Coef has FIELD
--R convert : % -> DoubleFloat if UTS has REAL and Coef has FIELD
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> UTS if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,
--R differentiate : (%,List Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Int
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Co
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has F
--R differentiate : % -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (In
--R differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,UTS) -> % if UTS has ELTAB(UTS,UTS) and Coef has FIELD
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,List UTS,List UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,UTS,UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,Equation UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,List Equation UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,List Symbol,List UTS) -> % if UTS has IEVALAB(SYMBOL,UTS) and Coef has FIELD
--R eval : (%,Symbol,UTS) -> % if UTS has IEVALAB(SYMBOL,UTS) and Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
```

```
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIEL
--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePol
--R floor : % -> UTS if UTS has INS and Coef has FIELD
--R fractionPart : % -> % if UTS has EUCDOM and Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUniv
--R init : () -> % if UTS has STEP and Coef has FIELD
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has var
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((UTS -> UTS),%) -> % if Coef has FIELD
--R max : (%,%) -> % if UTS has ORDSET and Coef has FIELD
--R min : (%,%) -> % if UTS has ORDSET and Coef has FIELD
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R negative? : % -> Boolean if UTS has OINTDOM and Coef has FIELD
--R nextItem : % -> Union(%,"failed") if UTS has STEP and Coef has FIELD
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> UTS if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(In
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if UTS has OINTDOM and Coef has FIELD
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R random : () -> % if UTS has INS and Coef has FIELD
--R rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if UTS has LINEXP INT and Coef has FIELD
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix UTS,vec: Vector UTS) if Coef ha
--R reducedSystem : Matrix % -> Matrix UTS if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> Symbol if UTS has RETRACT SYMBOL and Coef has FIELD
--R retract : % -> Fraction Integer if UTS has RETRACT INT and Coef has FIELD
--R retract : % -> Integer if UTS has RETRACT INT and Coef has FIELD
```

```
--R retractIfCan : % -> Union(Symbol,"failed") if UTS has RETRACT SYMBOL and Coef has FIELD
--R retractIfCan : % -> Union(Fraction Integer,"failed") if UTS has RETRACT INT and Coef has FIELD
--R retractIfCan : % -> Union(Integer,"failed") if UTS has RETRACT INT and Coef has FIELD
--R retractIfCan : % -> Union(UTS,"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sign : % -> Integer if UTS has OINTDOM and Coef has FIELD
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if UTS
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylorIfCan : % -> Union(UTS,"failed")
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> UTS if UTS has EUCDOM and Coef has FIELD
--R
--E 1

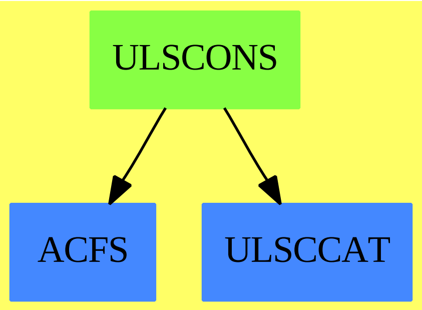
)spool
)lisp (bye)
```

— UnivariateLaurentSeriesConstructor.help —

```
====================================================================
UnivariateLaurentSeriesConstructor examples
====================================================================

See Also:
o )show UnivariateLaurentSeriesConstructor
```

22.3.1 UnivariateLaurentSeriesConstructor (ULSCONS)



See

⇒ "UnivariateLaurentSeries" (ULS) 22.2.1 on page 2752

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
approximate	asec	asech
asin	asinh	associates?
atan	atanh	ceiling
center	characteristic	charthRoot
coefficient	coerce	complete
conditionP	convert	cos
cosh	cot	coth
csc	csch	D
degree	denom	denominator
differentiate	divide	extend
euclideanSize	eval	exp
expressIdealMember	exquo	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractionPart	gcd
gcdPolynomial	hash	init
integrate	inv	latex
laurent	lcm	leadingCoefficient
leadingMonomial	log	map
max	min	monomial
monomial?	multiEuclidean	multiplyCoefficients
multiplyExponents	negative?	nextItem
nthRoot	numer	numerator
one?	order	patternMatch
pi	pole?	positive?
prime?	principalIdeal	random
rationalFunction	recip	reducedSystem
reductum	removeZeroes	retract
retractIfCan	sample	sec
sech	series	sign
sin	sinh	sizeLess?
solveLinearPolynomialEquation	sqrt	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
tan	tanh	taylor
taylorIfCan	taylorRep	terms
truncate	unit?	unitCanonical
unitNormal	variable	variables
wholePart	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?.?	?~=?	?/?
?<?	?<=?	?>?
?>=?	?quo?	?rem?

— domain ULSCONS UnivariateLaurentSeriesConstructor —

```
)abbrev domain ULSCONS UnivariateLaurentSeriesConstructor
++ Authors: Bill Burge, Clifton J. Williamson
++ Date Created: August 1988
++ Date Last Updated: 17 June 1996
++ Fix History:
++ 14 June 1996: provided missing exquo: (%,%) -> % (Frederic Lehobey)
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent, Taylor
++ Examples:
++ References:
++ Description:
++ This package enables one to construct a univariate Laurent series
++ domain from a univariate Taylor series domain. Univariate
++ Laurent series are represented by a pair \spad{[n,f(x)]}, where n is
++ an arbitrary integer and \spad{f(x)} is a Taylor series.  This pair
++ represents the Laurent series \spad{x**n * f(x)}.

UnivariateLaurentSeriesConstructor(Coef,UTS):_
 Exports == Implementation where
  Coef    : Ring
  UTS     : UnivariateTaylorSeriesCategory Coef
  I     ==> Integer
  L     ==> List
  NNI   ==> NonNegativeInteger
  OUT   ==> OutputForm
  P     ==> Polynomial Coef
  RF    ==> Fraction Polynomial Coef
  RN    ==> Fraction Integer
  ST    ==> Stream Coef
  TERM  ==> Record(k:I,c:Coef)
  monom ==> monomial$UTS
  EFULS ==> ElementaryFunctionsUnivariateLaurentSeries(Coef,UTS,%)
  STTAYLOR ==> StreamTaylorSeriesOperations Coef

  Exports ==> UnivariateLaurentSeriesConstructorCategory(Coef,UTS)

  Implementation ==> add

--% representation

    Rep := Record(expon:I,ps:UTS)

    getExpon : % -> I
    getUTS   : % -> UTS
```

```
    getExpon x == x.expon
    getUTS   x == x.ps

--% creation and destruction

    laurent(n,psr) == [n,psr]
    taylorRep x    == getUTS x
    degree x       == getExpon x

    0 == laurent(0,0)
    1 == laurent(0,1)

    monomial(s,e) == laurent(e,s::UTS)

    coerce(uts:UTS):% == laurent(0,uts)
    coerce(r:Coef):%  == r :: UTS  :: %
    coerce(i:I):%     == i :: Coef :: %

    taylorIfCan uls ==
      n := getExpon uls
      n < 0 =>
        uls := removeZeroes(-n,uls)
        getExpon(uls) < 0 => "failed"
        getUTS uls
      n = 0 => getUTS uls
      getUTS(uls) * monom(1,n :: NNI)

    taylor uls ==
      (uts := taylorIfCan uls) case "failed" =>
        error "taylor: Laurent series has a pole"
      uts :: UTS

    termExpon: TERM -> I
    termExpon term == term.k
    termCoef: TERM -> Coef
    termCoef term == term.c
    rec: (I,Coef) -> TERM
    rec(exponent,coef) == [exponent,coef]

    recs: (ST,I) -> Stream TERM
    recs(st,n) == delay
      empty? st => empty()
      zero? (coef := frst st) => recs(rst st,n + 1)
      concat(rec(n,coef),recs(rst st,n + 1))

    terms x == recs(coefficients getUTS x,getExpon x)

    recsToCoefs: (Stream TERM,I) -> ST
    recsToCoefs(st,n) == delay
```

```
      empty? st => empty()
      term := frst st; ex := termExpon term
      n = ex => concat(termCoef term,recsToCoefs(rst st,n + 1))
      concat(0,recsToCoefs(rst st,n + 1))

    series st ==
      empty? st => 0
      ex := termExpon frst st
      laurent(ex,series recsToCoefs(st,ex))

--% normalizations

    removeZeroes x ==
      empty? coefficients(xUTS := getUTS x) => 0
      coefficient(xUTS,0) = 0 =>
        removeZeroes laurent(getExpon(x) + 1,quoByVar xUTS)
      x

    removeZeroes(n,x) ==
      n <= 0 => x
      empty? coefficients(xUTS := getUTS x) => 0
      coefficient(xUTS,0) = 0 =>
        removeZeroes(n - 1,laurent(getExpon(x) + 1,quoByVar xUTS))
      x

--% predicates

    x = y ==
      EQ(x,y)$Lisp => true
      (expDiff := getExpon(x) - getExpon(y)) = 0 =>
        getUTS(x) = getUTS(y)
      abs(expDiff) > _$streamCount$Lisp => false
      expDiff > 0 =>
        getUTS(x) * monom(1,expDiff :: NNI) = getUTS(y)
      getUTS(y) * monom(1,(- expDiff) :: NNI) = getUTS(x)

    pole? x ==
      (n := degree x) >= 0 => false
      x := removeZeroes(-n,x)
      degree x < 0

--% arithmetic

    x + y  ==
      n := getExpon(x) - getExpon(y)
      n >= 0 =>
        laurent(getExpon y,getUTS(y) + getUTS(x) * monom(1,n::NNI))
      laurent(getExpon x,getUTS(x) + getUTS(y) * monom(1,(-n)::NNI))

    x - y  ==
```

```
      n := getExpon(x) - getExpon(y)
      n >= 0 =>
        laurent(getExpon y,getUTS(x) * monom(1,n::NNI) - getUTS(y))
      laurent(getExpon x,getUTS(x) - getUTS(y) * monom(1,(-n)::NNI))

    x:% * y:% == laurent(getExpon x + getExpon y,getUTS x * getUTS y)

    x:% ** n:NNI ==
      zero? n =>
        zero? x => error "0 ** 0 is undefined"
        1
      laurent(n * getExpon(x),getUTS(x) ** n)

    recip x ==
      x := removeZeroes(1000,x)
      zero? coefficient(x,d := degree x) => "failed"
      (uts := recip getUTS x) case "failed" => "failed"
      laurent(-d,uts :: UTS)

    elt(uls1:%,uls2:%) ==
      (uts := taylorIfCan uls2) case "failed" =>
        error "elt: second argument must have positive order"
      uts2 := uts :: UTS
      not zero? coefficient(uts2,0) =>
        error "elt: second argument must have positive order"
      if (deg := getExpon uls1) < 0 then uls1 := removeZeroes(-deg,uls1)
      (deg := getExpon uls1) < 0 =>
        (recipr := recip(uts2 :: %)) case "failed" =>
          error "elt: second argument not invertible"
        uts1 := taylor(uls1 * monomial(1,-deg))
        (elt(uts1,uts2) :: %) * (recipr :: %) ** ((-deg) :: NNI)
      elt(taylor uls1,uts2) :: %

    eval(uls:%,r:Coef) ==
      if (n := getExpon uls) < 0 then uls := removeZeroes(-n,uls)
      uts := getUTS uls
      (n := getExpon uls) < 0 =>
        zero? r => error "eval: 0 raised to negative power"
        (recipr := recip r) case "failed" =>
          error "eval: non-unit raised to negative power"
        (recipr :: Coef) ** ((-n) :: NNI) *$STTAYLOR eval(uts,r)
      zero? n => eval(uts,r)
      r ** (n :: NNI) *$STTAYLOR eval(uts,r)

--% values

    variable x == variable getUTS x
    center   x == center   getUTS x

    coefficient(x,n) ==
```

```
      a := n - getExpon(x)
      a >= 0 => coefficient(getUTS x,a :: NNI)
      0

    elt(x:%,n:I) == coefficient(x,n)

--% other functions

    order x == getExpon x + order getUTS x
    order(x,n) ==
      (m := n - (e := getExpon x)) < 0 => n
      e + order(getUTS x,m :: NNI)

    truncate(x,n) ==
      (m := n - (e := getExpon x)) < 0 => 0
      laurent(e,truncate(getUTS x,m :: NNI))

    truncate(x,n1,n2) ==
      if n2 < n1 then (n1,n2) := (n2,n1)
      (m1 := n1 - (e := getExpon x)) < 0 => truncate(x,n2)
      laurent(e,truncate(getUTS x,m1 :: NNI,(n2 - e) :: NNI))

    if Coef has IntegralDomain then
      rationalFunction(x,n) ==
        (m := n - (e := getExpon x)) < 0 => 0
        poly := polynomial(getUTS x,m :: NNI) :: RF
        zero? e => poly
        v := variable(x) :: RF; c := center(x) :: P :: RF
        positive? e => poly * (v - c) ** (e :: NNI)
        poly / (v - c) ** ((-e) :: NNI)

      rationalFunction(x,n1,n2) ==
        if n2 < n1 then (n1,n2) := (n2,n1)
        (m1 := n1 - (e := getExpon x)) < 0 => rationalFunction(x,n2)
        poly := polynomial(getUTS x,m1 :: NNI,(n2 - e) :: NNI) :: RF
        zero? e => poly
        v := variable(x) :: RF; c := center(x) :: P :: RF
        positive? e => poly * (v - c) ** (e :: NNI)
        poly / (v - c) ** ((-e) :: NNI)

      --  La fonction < exquo > manque dans laurent.spad,
      --les lignes suivantes le mettent en evidence :
      --
      --ls := laurent(0,series [i for i in 1..])$ULS(INT,x,0)
      ---- missing function in laurent.spad of Axiom 2.0a version of
      ---- Friday March 10, 1995 at 04:15:22 on 615:
      --exquo(ls,ls)
      --
      --  Je l'ai ajoutee a laurent.spad.
      --
```

```
  --Frederic Lehobey
  x exquo y ==
    x := removeZeroes(1000,x)
    y := removeZeroes(1000,y)
    zero? coefficient(y, d := degree y) => "failed"
    (uts := (getUTS x) exquo (getUTS y)) case "failed" => "failed"
    laurent(degree x-d,uts :: UTS)

if Coef has coerce: Symbol -> Coef then
  if Coef has "**": (Coef,I) -> Coef then

    approximate(x,n) ==
      (m := n - (e := getExpon x)) < 0 => 0
      app := approximate(getUTS x,m :: NNI)
      zero? e => app
      app * ((variable(x) :: Coef) - center(x)) ** e

complete x == laurent(getExpon x,complete getUTS x)
extend(x,n) ==
  e := getExpon x
  (m := n - e) < 0 => x
  laurent(e,extend(getUTS x,m :: NNI))

map(f:Coef -> Coef,x:%) == laurent(getExpon x,map(f,getUTS x))

multiplyCoefficients(f,x) ==
  e := getExpon x
  laurent(e,multiplyCoefficients((z1:I):Coef +-> f(e + z1),getUTS x))

multiplyExponents(x,n) ==
  laurent(n * getExpon x,multiplyExponents(getUTS x,n))

differentiate x ==
  e := getExpon x
  laurent(e - 1,
          multiplyCoefficients((z1:I):Coef +-> (e + z1)::Coef,getUTS x))

if Coef has PartialDifferentialRing(Symbol) then
  differentiate(x:%,s:Symbol) ==
    (s = variable(x)) => differentiate x
    map((z1:Coef):Coef +-> differentiate(z1,s),x)
                             - differentiate(center x,s)*differentiate(x)

characteristic() == characteristic()$Coef

if Coef has Field then

  retract(x:%):UTS                      == taylor x
  retractIfCan(x:%):Union(UTS,"failed") == taylorIfCan x
```

```
      (x:%) ** (n:I) ==
        zero? n =>
          zero? x => error "0 ** 0 is undefined"
          1
        n > 0 => laurent(n * getExpon(x),getUTS(x) ** (n :: NNI))
        xInv := inv x; minusN := (-n) :: NNI
        laurent(minusN * getExpon(xInv),getUTS(xInv) ** minusN)

      (x:UTS) * (y:%) == (x :: %) * y
      (x:%) * (y:UTS) == x * (y :: %)

      inv x ==
        (xInv := recip x) case "failed" =>
          error "multiplicative inverse does not exist"
        xInv :: %

      (x:%) / (y:%) ==
        (yInv := recip y) case "failed" =>
          error "inv: multiplicative inverse does not exist"
        x * (yInv :: %)

      (x:UTS) / (y:UTS) == (x :: %) / (y :: %)

      numer x ==
        (n := degree x) >= 0 => taylor x
        x := removeZeroes(-n,x)
        (n := degree x) = 0 => taylor x
        getUTS x

      denom x ==
        (n := degree x) >= 0 => 1
        x := removeZeroes(-n,x)
        (n := degree x) = 0 => 1
        monom(1,(-n) :: NNI)

--% algebraic and transcendental functions

    if Coef has Algebra Fraction Integer then

      coerce(r:RN) == r :: Coef :: %

      if Coef has Field then
         (x:%) ** (r:RN) == x **$EFULS r

      exp x   == exp(x)$EFULS
      log x   == log(x)$EFULS
      sin x   == sin(x)$EFULS
      cos x   == cos(x)$EFULS
      tan x   == tan(x)$EFULS
      cot x   == cot(x)$EFULS
```

```
sec x   == sec(x)$EFULS
csc x   == csc(x)$EFULS
asin x  == asin(x)$EFULS
acos x  == acos(x)$EFULS
atan x  == atan(x)$EFULS
acot x  == acot(x)$EFULS
asec x  == asec(x)$EFULS
acsc x  == acsc(x)$EFULS
sinh x  == sinh(x)$EFULS
cosh x  == cosh(x)$EFULS
tanh x  == tanh(x)$EFULS
coth x  == coth(x)$EFULS
sech x  == sech(x)$EFULS
csch x  == csch(x)$EFULS
asinh x == asinh(x)$EFULS
acosh x == acosh(x)$EFULS
atanh x == atanh(x)$EFULS
acoth x == acoth(x)$EFULS
asech x == asech(x)$EFULS
acsch x == acsch(x)$EFULS

ratInv: I -> Coef
ratInv n ==
  zero? n => 1
  inv(n :: RN) :: Coef

integrate x ==
  not zero? coefficient(x,-1) =>
    error "integrate: series has term of order -1"
  e := getExpon x
  laurent(e+1,multiplyCoefficients((z:I):Coef+->ratInv(e+1+z),getUTS x))

if Coef has integrate: (Coef,Symbol) -> Coef and _
   Coef has variables: Coef -> List Symbol then
  integrate(x:%,s:Symbol) ==
    (s = variable(x)) => integrate x
    not entry?(s,variables center x)
       => map((z1:Coef):Coef+->integrate(z1,s),x)
    error "integrate: center is a function of variable of integration"

if Coef has TranscendentalFunctionCategory and _
   Coef has PrimitiveFunctionCategory and _
   Coef has AlgebraicallyClosedFunctionSpace Integer then

  integrateWithOneAnswer: (Coef,Symbol) -> Coef
  integrateWithOneAnswer(f,s) ==
    res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
    res case Coef => res :: Coef
    first(res :: List Coef)
```

```
    integrate(x:%,s:Symbol) ==
      (s = variable(x)) => integrate x
      not entry?(s,variables center x) =>
        map((z1:Coef):Coef +-> integrateWithOneAnswer(z1,s),x)
      error "integrate: center is a function of variable of integration"

termOutput:(I,Coef,OUT) -> OUT
termOutput(k,c,vv) ==
-- creates a term c * vv ** k
  k = 0 => c :: OUT
  mon :=
    k = 1 => vv
    vv ** (k :: OUT)
  c = 1 => mon
  c = -1 => -mon
  (c :: OUT) * mon

showAll?:() -> Boolean
-- check a global Lisp variable
showAll?() == true

termsToOutputForm:(I,ST,OUT) -> OUT
termsToOutputForm(m,uu,xxx) ==
  l : L OUT := empty()
  empty? uu => (0$Coef) :: OUT
  n : NNI ; count : NNI := _$streamCount$Lisp
  for n in 0..count while not empty? uu repeat
    if frst(uu) ^= 0 then
      l := concat(termOutput((n :: I) + m,frst(uu),xxx),l)
    uu := rst uu
  if showAll?() then
    for n in (count + 1).. while explicitEntries? uu and _
           not eq?(uu,rst uu) repeat
      if frst(uu) ^= 0 then
        l := concat(termOutput((n::I) + m,frst(uu),xxx),l)
      uu := rst uu
  l :=
    explicitlyEmpty? uu => l
    eq?(uu,rst uu) and frst uu = 0 => l
    concat(prefix("O" :: OUT,[xxx ** ((n :: I) + m) :: OUT]),l)
  empty? l => (0$Coef) :: OUT
  reduce("+",reverse_! l)

coerce(x:%):OUT ==
  x := removeZeroes(_$streamCount$Lisp,x)
  m := degree x
  uts := getUTS x
  p := coefficients uts
  var := variable uts; cen := center uts
  xxx :=
```

```
        zero? cen => var :: OUT
        paren(var :: OUT - cen :: OUT)
      termsToOutputForm(m,p,xxx)
```

———-

— ULSCONS.dotabb —

```
"ULSCONS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ULSCONS"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"ULSCONS" -> "ULSCCAT"
"ULSCONS" -> "ACFS"
```

———-

22.4 domain UP UnivariatePolynomial

— UnivariatePolynomial.input —

```
)set break resume
)sys rm -f UnivariatePolynomial.output
)spool UnivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 35
(p,q) : UP(x,INT)
--R
--R                                                                   Type: Void
--E 1

--S 2 of 35
p := (3*x-1)**2 * (2*x + 8)
--R
--R
--R            3      2
--R   (2)  18x  + 60x  - 46x + 8
--R                                           Type: UnivariatePolynomial(x,Integer)
--E 2

--S 3 of 35
q := (1 - 6*x + 9*x**2)**2
--R
```

```
--R
--R           4       3      2
--R   (3)  81x  - 108x  + 54x  - 12x + 1
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 3

--S 4 of 35
p**2 + p*q
--R
--R
--R             7        6        5         4        3        2
--R   (4)  1458x  + 3240x  - 7074x  + 10584x  - 9282x  + 4120x  - 878x + 72
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 4

--S 5 of 35
leadingCoefficient p
--R
--R
--R   (5)  18
--R                                                          Type: PositiveInteger
--E 5

--S 6 of 35
degree p
--R
--R
--R   (6)  3
--R                                                          Type: PositiveInteger
--E 6

--S 7 of 35
reductum p
--R
--R
--R           2
--R   (7)  60x  - 46x + 8
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 7

--S 8 of 35
gcd(p,q)
--R
--R
--R          2
--R   (8)  9x  - 6x + 1
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 8

--S 9 of 35
```

```
lcm(p,q)
--R
--R
--R             5       4       3       2
--R   (9)  162x  + 432x  - 756x  + 408x  - 94x + 8
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 9

--S 10 of 35
resultant(p,q)
--R
--R
--R   (10)  0
--R                                                       Type: NonNegativeInteger
--E 10

--S 11 of 35
D p
--R
--R
--R            2
--R   (11)  54x  + 120x - 46
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 11

--S 12 of 35
p(2)
--R
--R
--R   (12)  300
--R                                                          Type: PositiveInteger
--E 12

--S 13 of 35
p(q)
--R
--R
--R   (13)
--R             12            11            10            9            8
--R     9565938x   - 38263752x   + 70150212x   - 77944680x  + 58852170x
--R   +
--R                7            6           5           4          3         2
--R     - 32227632x  + 13349448x  - 4280688x  + 1058184x  - 192672x  + 23328x
--R   +
--R     - 1536x + 40
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 13

--S 14 of 35
q(p)
```

```
--R
--R
--R   (14)
--R              12              11              10              9              8
--R     8503056x   + 113374080x   + 479950272x   + 404997408x  - 1369516896x
--R   +
--R                 7               6               5               4             3
--R     - 626146848x  + 2939858712x  - 2780728704x  + 1364312160x  - 396838872x
--R   +
--R              2
--R     69205896x  - 6716184x + 279841
--R                                    Type: UnivariatePolynomial(x,Integer)
--E 14

--S 15 of 35
l := coefficients p
--R
--R
--R   (15)  [18,60,- 46,8]
--R                                                       Type: List Integer
--E 15

--S 16 of 35
reduce(gcd,l)
--R
--R
--R   (16)  2
--R                                                    Type: PositiveInteger
--E 16

--S 17 of 35
content p
--R
--R
--R   (17)  2
--R                                                    Type: PositiveInteger
--E 17

--S 18 of 35
ux := (x**4+2*x+3)::UP(x,INT)
--R
--R
--R           4
--R   (18)  x  + 2x + 3
--R                                    Type: UnivariatePolynomial(x,Integer)
--E 18

--S 19 of 35
vectorise(ux,5)
--R
```

```
--R
--R   (19)  [3,2,0,0,1]
--R                                                        Type: Vector Integer
--E 19

--S 20 of 35
squareTerms(p) ==   reduce(+,[t**2 for t in monomials p])
--R
--R                                                                   Type: Void
--E 20

--S 21 of 35
p
--R
--R
--R            3      2
--R   (21)  18x  + 60x  - 46x + 8
--R                                     Type: UnivariatePolynomial(x,Integer)
--E 21

--S 22 of 35
squareTerms p
--R
--R   Compiling function squareTerms with type UnivariatePolynomial(x,
--R      Integer) -> UnivariatePolynomial(x,Integer)
--R
--R             6        4        2
--R   (22)  324x  + 3600x  + 2116x  + 64
--R                                     Type: UnivariatePolynomial(x,Integer)
--E 22

--S 23 of 35
(r,s) : UP(a1,FRAC INT)
--R
--R                                                                   Type: Void
--E 23

--S 24 of 35
r := a1**2 - 2/3
--R
--R
--R           2   2
--R   (24)  a1  - -
--R               3
--R                           Type: UnivariatePolynomial(a1,Fraction Integer)
--E 24

--S 25 of 35
s := a1 + 4
--R
```

```
--R
--R   (25)  a1 + 4
--R                              Type: UnivariatePolynomial(a1,Fraction Integer)
--E 25

--S 26 of 35
r quo s
--R
--R
--R   (26)  a1 - 4
--R                              Type: UnivariatePolynomial(a1,Fraction Integer)
--E 26

--S 27 of 35
r rem s
--R
--R
--R         46
--R   (27)  --
--R          3
--R                              Type: UnivariatePolynomial(a1,Fraction Integer)
--E 27

--S 28 of 35
d := divide(r, s)
--R
--R
--R                                            46
--R   (28)  [quotient= a1 - 4,remainder= --]
--R                                             3
--RType: Record(quotient: UnivariatePolynomial(a1,Fraction Integer),remainder: UnivariatePoly
--E 28

--S 29 of 35
r - (d.quotient * s + d.remainder)
--R
--R
--R   (29)  0
--R                              Type: UnivariatePolynomial(a1,Fraction Integer)
--E 29

--S 30 of 35
integrate r
--R
--R
--R         1   3   2
--R   (30)  - a1  - - a1
--R         3       3
--R                              Type: UnivariatePolynomial(a1,Fraction Integer)
--E 30
```

```
--S 31 of 35
integrate s
--R
--R
--R         1   2
--R   (31)  - a1  + 4a1
--R         2
--R                              Type: UnivariatePolynomial(a1,Fraction Integer)
--E 31

--S 32 of 35
t : UP(a1,FRAC POLY INT)
--R
--R                                                                   Type: Void
--E 32

--S 33 of 35
t := a1**2 - a1/b2 + (b1**2-b1)/(b2+3)
--R
--R
--R                          2
--R           2    1      b1  - b1
--R   (33)  a1  - -- a1 + --------
--R                b2      b2 + 3
--R                   Type: UnivariatePolynomial(a1,Fraction Polynomial Integer)
--E 33

--S 34 of 35
u : FRAC POLY INT := t
--R
--R
--R           2  2      2           2
--R         a1 b2  + (b1  - b1 + 3a1  - a1)b2 - 3a1
--R   (34)  ---------------------------------------
--R                           2
--R                         b2  + 3b2
--R                                           Type: Fraction Polynomial Integer
--E 34

--S 35 of 35
u :: UP(b1,?)
--R
--R
--R                                    2
--R            1     2      1        a1 b2 - a1
--R   (35)  ------ b1  - ------ b1 + ----------
--R         b2 + 3       b2 + 3          b2
--R                   Type: UnivariatePolynomial(b1,Fraction Polynomial Integer)
--E 35
```

```
)spool
)lisp (bye)
```

———

— UnivariatePolynomial.help —

```
====================================================================
UnivariatePolynomial examples
====================================================================

The domain constructor UnivariatePolynomial (abbreviated UP) creates
domains of univariate polynomials in a specified variable.  For
example, the domain UP(a1,POLY FRAC INT) provides polynomials in the
single variable a1 whose coefficients are general polynomials with
rational number coefficients.

Restriction: Axiom does not allow you to create types where
UnivariatePolynomial is contained in the coefficient type of
Polynomial. Therefore, UP(x,POLY INT) is legal but POLY UP(x,INT)
is not.

UP(x,INT) is the domain of polynomials in the single variable x with
integer coefficients.

  (p,q) : UP(x,INT)
                         Type: Void

  p := (3*x-1)**2 * (2*x + 8)
       3      2
    18x  + 60x  - 46x + 8
                         Type: UnivariatePolynomial(x,Integer)

  q := (1 - 6*x + 9*x**2)**2
       4       3      2
    81x  - 108x  + 54x  - 12x + 1
                         Type: UnivariatePolynomial(x,Integer)

The usual arithmetic operations are available for univariate polynomials.

  p**2 + p*q
         7        6        5         4        3        2
    1458x  + 3240x  - 7074x  + 10584x  - 9282x  + 4120x  - 878x + 72
                         Type: UnivariatePolynomial(x,Integer)

The operation leadingCoefficient extracts the coefficient of the term
of highest degree.

  leadingCoefficient p
```

```
    18
                              Type: PositiveInteger
```

The operation degree returns the degree of the polynomial. Since the polynomial has only one variable, the variable is not supplied to operations like degree.

```
  degree p
    3
                              Type: PositiveInteger
```

The reductum of the polynomial, the polynomial obtained by subtracting the term of highest order, is returned by reductum.

```
  reductum p
       2
    60x  - 46x + 8
                              Type: UnivariatePolynomial(x,Integer)
```

The operation gcd computes the greatest common divisor of two polynomials.

```
  gcd(p,q)
      2
    9x  - 6x + 1
                              Type: UnivariatePolynomial(x,Integer)
```

The operation lcm computes the least common multiple.

```
  lcm(p,q)
        5       4       3       2
    162x  + 432x  - 756x  + 408x  - 94x + 8
                              Type: UnivariatePolynomial(x,Integer)
```

The operation resultant computes the resultant of two univariate polynomials. In the case of p and q, the resultant is 0 because they share a common root.

```
  resultant(p,q)
    0
                              Type: NonNegativeInteger
```

To compute the derivative of a univariate polynomial with respect to its variable, use the function D.

```
  D p
       2
    54x  + 120x - 46
                              Type: UnivariatePolynomial(x,Integer)
```

Univariate polynomials can also be used as if they were functions. To

evaluate a univariate polynomial at some point, apply the polynomial
to the point.

```
  p(2)
    300
                            Type: PositiveInteger
```

The same syntax is used for composing two univariate polynomials, i.e.
substituting one polynomial for the variable in another. This substitutes
q for the variable in p.

```
  p(q)
             12            11            10            9            8
     9565938x   - 38263752x   + 70150212x   - 77944680x  + 58852170x
   +
                7            6           5           4          3         2
     - 32227632x  + 13349448x  - 4280688x  + 1058184x  - 192672x  + 23328x
   +
     - 1536x + 40
                            Type: UnivariatePolynomial(x,Integer)
```

This substitutes p for the variable in q.

```
  q(p)
             12             11             10             9              8
     8503056x   + 113374080x   + 479950272x   + 404997408x  - 1369516896x
   +
                 7              6              5              4             3
     - 626146848x  + 2939858712x  - 2780728704x  + 1364312160x  - 396838872x
   +
              2
     69205896x  - 6716184x + 279841
                            Type: UnivariatePolynomial(x,Integer)
```

To obtain a list of coefficients of the polynomial, use coefficients.

```
  l := coefficients p
    [18,60,- 46,8]
                            Type: List Integer
```

From this you can use gcd and reduce to compute the content of the polynomial.

```
  reduce(gcd,l)
    2
                            Type: PositiveInteger
```

Alternatively (and more easily), you can just call content.

```
  content p
    2
```

```
                          Type: PositiveInteger
```

Note that the operation coefficients omits the zero coefficients from the list. Sometimes it is useful to convert a univariate polynomial to a vector whose i-th position contains the degree i-1 coefficient of the polynomial.

```
  ux := (x**4+2*x+3)::UP(x,INT)
     4
    x  + 2x + 3
                          Type: UnivariatePolynomial(x,Integer)
```

To get a complete vector of coefficients, use the operation vectorise, which takes a univariate polynomial and an integer denoting the length of the desired vector.

```
  vectorise(ux,5)
    [3,2,0,0,1]
                          Type: Vector Integer
```

It is common to want to do something to every term of a polynomial, creating a new polynomial in the process.

This is a function for iterating across the terms of a polynomial, squaring each term.

```
  squareTerms(p) ==   reduce(+,[t**2 for t in monomials p])
                          Type: Void
```

Recall what p looked like.

```
  p
       3      2
    18x  + 60x  - 46x + 8
                          Type: UnivariatePolynomial(x,Integer)
```

We can demonstrate squareTerms on p.

```
  squareTerms p
        6        4        2
    324x  + 3600x  + 2116x  + 64
                          Type: UnivariatePolynomial(x,Integer)
```

When the coefficients of the univariate polynomial belong to a field, it is possible to compute quotients and remainders. For example, when the coefficients are rational numbers, as opposed to integers. The important property of a field is that non-zero elements can be divided and produce another element. The quotient of the integers 2 and 3 is not another integer.

```
  (r,s) : UP(a1,FRAC INT)
                            Type: Void

  r := a1**2 - 2/3
      2   2
    a1  - -
          3
                            Type: UnivariatePolynomial(a1,Fraction Integer)

  s := a1 + 4
    a1 + 4
                            Type: UnivariatePolynomial(a1,Fraction Integer)
```

When the coefficients are rational numbers or rational expressions, the operation quo computes the quotient of two polynomials.

```
  r quo s
    a1 - 4
                            Type: UnivariatePolynomial(a1,Fraction Integer)
```

The operation rem computes the remainder.

```
  r rem s
    46
    --
     3
                            Type: UnivariatePolynomial(a1,Fraction Integer)
```

The operation divide can be used to return a record of both components.

```
  d := divide(r, s)
                                   46
    [quotient= a1 - 4,remainder= --]
                                    3
       Type: Record(quotient: UnivariatePolynomial(a1,Fraction Integer),
                    remainder: UnivariatePolynomial(a1,Fraction Integer))
```

Now we check the arithmetic!

```
  r - (d.quotient * s + d.remainder)
    0
                            Type: UnivariatePolynomial(a1,Fraction Integer)
```

It is also possible to integrate univariate polynomials when the coefficients belong to a field.

```
  integrate r
    1   3   2
    - a1  - - a1
    3       3
```

```
                          Type: UnivariatePolynomial(a1,Fraction Integer)

  integrate s
    1   2
    - a1  + 4a1
    2
                          Type: UnivariatePolynomial(a1,Fraction Integer)
```

One application of univariate polynomials is to see expressions in terms of a specific variable.

We start with a polynomial in a1 whose coefficients are quotients of polynomials in b1 and b2.

```
  t : UP(a1,FRAC POLY INT)
                          Type: Void
```

Since in this case we are not talking about using multivariate polynomials in only two variables, we use Polynomial. We also use Fraction because we want fractions.

```
  t := a1**2 - a1/b2 + (b1**2-b1)/(b2+3)
                    2
     2    1      b1  - b1
   a1  - -- a1 + --------
         b2       b2 + 3
                   Type: UnivariatePolynomial(a1,Fraction Polynomial Integer)
```

We push all the variables into a single quotient of polynomials.

```
  u : FRAC POLY INT := t
      2  2      2           2
    a1 b2  + (b1  - b1 + 3a1  - a1)b2 - 3a1
    ---------------------------------------
                      2
                    b2  + 3b2
                          Type: Fraction Polynomial Integer
```

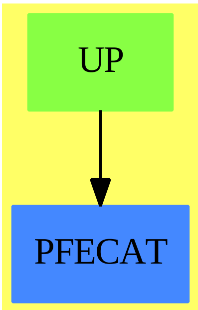
Alternatively, we can view this as a polynomial in the variable This is a mode-directed conversion: you indicate as much of the structure as you care about and let Axiom decide on the full type and how to do the transformation.

```
  u :: UP(b1,?)
                                2
       1     2      1        a1 b2 - a1
    ------ b1  - ------ b1 + ----------
    b2 + 3       b2 + 3          b2
                   Type: UnivariatePolynomial(b1,Fraction Polynomial Integer)
```

```
See Also:
o )help MultivariatePolynomial
o )help DistributedMultivariatePolynomial
o )show UnivariatePolynomial
```

———-

22.4.1 UnivariatePolynomial (UP)



See

⇒ "FreeModule" (FM) 7.30.1 on page 980
⇒ "PolynomialRing" (PR) 17.27.1 on page 2052
⇒ "SparseUnivariatePolynomial" (SUP) 20.18.1 on page 2425

Exports:

0	1
associates?	binomThmExpt
characteristic	charthRoot
coefficient	coefficients
coerce	composite
conditionP	content
convert	D
degree	differentiate
discriminant	divide
divideExponents	elt
euclideanSize	eval
expressIdealMember	exquo
extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial
fmecg	gcd
gcdPolynomial	ground
ground?	hash
init	integrate
isExpt	isPlus
isTimes	karatsubaDivide
latex	lcm
leadingCoefficient	leadingMonomial
mainVariable	makeSUP
mapExponents	map
max	min
minimumDegree	monicDivide
monomial	monomial?
monomials	multiEuclidean
multiplyExponents	multivariate
nextItem	numberOfMonomials
one?	order
patternMatch	pomopo!
prime?	primitivePart
primitiveMonomials	principalIdeal
pseudoDivide	pseudoQuotient
pseudoRemainder	recip
reducedSystem	reductum
resultant	retract
retractIfCan	sample
separate	shiftLeft
shiftRight	sizeLess?
solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial
subResultantGcd	subtractIfCan
totalDegree	totalDegree
unit?	unitCanonical
unitNormal	univariate
unmakeSUP	variables
vectorise	zero?
?*?	?**?
?+?	?-?
-?	?=?
?^?	?.?
?~=?	?/?
?<?	?<=?
?>?	?>=?
?quo?	?rem?

— domain UP UnivariatePolynomial —

```
)abbrev domain UP UnivariatePolynomial
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, reductum, differentiate,
++ elt, map, resultant, discriminant
++ Related Constructors: SparseUnivariatePolynomial, MultivariatePolynomial
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents univariate polynomials in some symbol
++ over arbitrary (not necessarily commutative) coefficient rings.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.
++ Note that if the coefficient ring is a field, then this domain
++ forms a euclidean domain.

UnivariatePolynomial(x:Symbol, R:Ring):
  UnivariatePolynomialCategory(R) with
    coerce: Variable(x) -> %
      ++ coerce(x) converts the variable x to a univariate polynomial.
    fmecg: (%,NonNegativeInteger,R,%) -> %
        ++ fmecg(p1,e,r,p2) finds x : p1 - r * x**e * p2
   == SparseUnivariatePolynomial(R)   add
    Rep:=SparseUnivariatePolynomial(R)
    coerce(p:%):OutputForm  == outputForm(p, outputForm x)
    coerce(v:Variable(x)):% == monomial(1, 1)
```

———-

— UP.dotabb —

```
"UP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"UP" -> "PFECAT"
```

———-

22.5 domain UPXS UnivariatePuiseuxSeries

— UnivariatePuiseuxSeries.input —

```
)set break resume
)sys rm -f UnivariatePuiseuxSeries.output
)spool UnivariatePuiseuxSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeries
--R UnivariatePuiseuxSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for UnivariatePuiseuxSeries is UPXS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UPXS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       center : % -> Coef
--R coerce : Variable var -> %           coerce : Integer -> %
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> Fraction Integer       hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : % -> Fraction Integer        pole? : % -> Boolean
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R sample : () -> %                     variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING SYMBOL
```

```
--R D : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRI
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Coe
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : UnivariateTaylorSeries(Coef,var,cen) -> %
--R coerce : UnivariateLaurentSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coe
--R differentiate : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef h
--R differentiate : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and C
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef)
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Int
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
```

```
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : % -> UnivariateLaurentSeries(Coef,var,cen)
--R laurentIfCan : % -> Union(UnivariateLaurentSeries(Coef,var,cen),"failed")
--R laurentRep : % -> UnivariateLaurentSeries(Coef,var,cen)
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R puiseux : (Fraction Integer,UnivariateLaurentSeries(Coef,var,cen)) -> %
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> UnivariateTaylorSeries(Coef,var,cen)
--R retract : % -> UnivariateLaurentSeries(Coef,var,cen)
--R retractIfCan : % -> Union(UnivariateTaylorSeries(Coef,var,cen),"failed")
--R retractIfCan : % -> Union(UnivariateLaurentSeries(Coef,var,cen),"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
```

```
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———-

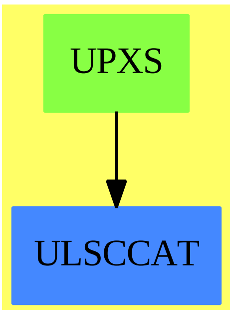
— UnivariatePuiseuxSeries.help —

```
====================================================================
UnivariatePuiseuxSeries examples
====================================================================

See Also:
o )show UnivariatePuiseuxSeries
```

———-

22.5.1 UnivariatePuiseuxSeries (UPXS)



See

⇒ "UnivariatePuiseuxSeriesConstructor" (UPXSCONS) 22.6.1 on page 2798

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	laurent	laurentIfCan
laurentRep	lcm	leadingCoefficient	leadingMonomial
log	map	monomial	monomial?
multiEuclidean	multiplyExponents	nthRoot	one?
order	pi	pole?	prime?
principalIdeal	puiseux	rationalPower	recip
reductum	retract	retractIfCan	sample
sec	sech	series	sin
sinh	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	tan	tanh
terms	truncate	unit?	unitCanonical
unitNormal	variable	variables	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?
?/?	?.?	?quo?	?rem?

— domain UPXS UnivariatePuiseuxSeries —

```
)abbrev domain UPXS UnivariatePuiseuxSeries
++ Author: Clifton J. Williamson
++ Date Created: 28 January 1990
++ Date Last Updated: 21 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux
++ Examples:
++ References:
++ Description:
++ Dense Puiseux series in one variable
++ \spadtype{UnivariatePuiseuxSeries} is a domain representing Puiseux
++ series in one variable with coefficients in an arbitrary ring.  The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion.  For example,
++ \spad{UnivariatePuiseuxSeries(Integer,x,3)} represents Puiseux series in
```

```
++ \spad{(x - 3)} with \spadtype{Integer} coefficients.

UnivariatePuiseuxSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I   ==> Integer
  L   ==> List
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  RN  ==> Fraction Integer
  ST  ==> Stream Coef
  UTS ==> UnivariateTaylorSeries(Coef,var,cen)
  ULS ==> UnivariateLaurentSeries(Coef,var,cen)

  Exports ==> Join(UnivariatePuiseuxSeriesConstructorCategory(Coef,ULS),_
                   RetractableTo UTS) with
    coerce: Variable(var) -> %
      ++ coerce(var) converts the series variable \spad{var} into a
      ++ Puiseux series.
    differentiate: (%,Variable(var)) -> %
      ++ \spad{differentiate(f(x),x)} returns the derivative of
      ++ \spad{f(x)} with respect to \spad{x}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
        ++ by integers.

  Implementation ==> UnivariatePuiseuxSeriesConstructor(Coef,ULS) add

    Rep := Record(expon:RN,lSeries:ULS)

    getExpon: % -> RN
    getExpon pxs == pxs.expon

    variable upxs == var
    center   upxs == cen

    coerce(uts:UTS) == uts :: ULS :: %

    retractIfCan(upxs:%):Union(UTS,"failed") ==
      (ulsIfCan := retractIfCan(upxs)@Union(ULS,"failed")) case "failed" =>
        "failed"
      retractIfCan(ulsIfCan :: ULS)

    --retract(upxs:%):UTS ==
      --(ulsIfCan := retractIfCan(upxs)@Union(ULS,"failed")) case "failed" =>
        --error "retractIfCan: series has fractional exponents"
```

```
  --utsIfCan := retractIfCan(ulsIfCan :: ULS)@Union(UTS,"failed")
  --utsIfCan case "failed" =>
    --error "retractIfCan: series has negative exponents"
  --utsIfCan :: UTS

coerce(v:Variable(var)) ==
  zero? cen => monomial(1,1)
  monomial(1,1) + monomial(cen,0)

if Coef has "*": (Fraction Integer, Coef) -> Coef then
  differentiate(upxs:%,v:Variable(var)) == differentiate upxs

if Coef has Algebra Fraction Integer then
  integrate(upxs:%,v:Variable(var)) == integrate upxs

if Coef has coerce: Symbol -> Coef then
  if Coef has "**": (Coef,RN) -> Coef then

    roundDown: RN -> I
    roundDown rn ==
      -- returns the largest integer <= rn
      (den := denom rn) = 1 => numer rn
      n := (num := numer rn) quo den
      positive?(num) => n
      n - 1

    stToCoef: (ST,Coef,NNI,NNI) -> Coef
    stToCoef(st,term,n,n0) ==
      (n > n0) or (empty? st) => 0
      frst(st) * term ** n + stToCoef(rst st,term,n + 1,n0)

    approximateLaurent: (ULS,Coef,I) -> Coef
    approximateLaurent(x,term,n) ==
      (m := n - (e := degree x)) < 0 => 0
      app := stToCoef(coefficients taylorRep x,term,0,m :: NNI)
      zero? e => app
      app * term ** (e :: RN)

    approximate(x,r) ==
      e := rationalPower(x)
      term := ((variable(x) :: Coef) - center(x)) ** e
      approximateLaurent(laurentRep x,term,roundDown(r / e))

termOutput:(RN,Coef,OUT) -> OUT
termOutput(k,c,vv) ==
-- creates a term c * vv ** k
  k = 0 => c :: OUT
  mon :=
    k = 1 => vv
    vv ** (k :: OUT)
```

```
      c = 1 => mon
      c = -1 => -mon
      (c :: OUT) * mon

    showAll?:() -> Boolean
    -- check a global Lisp variable
    showAll?() == true

    termsToOutputForm:(RN,RN,ST,OUT) -> OUT
    termsToOutputForm(m,rat,uu,xxx) ==
      l : L OUT := empty()
      empty? uu => 0 :: OUT
      n : NNI; count : NNI := _$streamCount$Lisp
      for n in 0..count while not empty? uu repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput((n :: I) * rat + m,frst uu,xxx),l)
        uu := rst uu
      if showAll?() then
        for n in (count + 1).. while explicitEntries? uu and _
               not eq?(uu,rst uu) repeat
          if frst(uu) ^= 0 then
            l := concat(termOutput((n :: I) * rat + m,frst uu,xxx),l)
          uu := rst uu
      l :=
        explicitlyEmpty? uu => l
        eq?(uu,rst uu) and frst uu = 0 => l
        concat(prefix("O" :: OUT,[xxx ** (((n::I) * rat + m) :: OUT)]),l)
      empty? l => 0 :: OUT
      reduce("+",reverse_! l)

    coerce(upxs:%):OUT ==
      rat := getExpon upxs; uls := laurentRep upxs
      count : I := _$streamCount$Lisp
      uls := removeZeroes(_$streamCount$Lisp,uls)
      m : RN := (degree uls) * rat
      p := coefficients taylorRep uls
      xxx :=
        zero? cen => var :: OUT
        paren(var :: OUT - cen :: OUT)
      termsToOutputForm(m,rat,p,xxx)
```

— **UPXS.dotabb** —

```
"UPXS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UPXS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"UPXS" -> "ULSCCAT"
```

———-

22.6 domain UPXSCONS UnivariatePuiseuxSeriesConstructor

— UnivariatePuiseuxSeriesConstructor.input —

```
)set break resume
)sys rm -f UnivariatePuiseuxSeriesConstructor.output
)spool UnivariatePuiseuxSeriesConstructor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeriesConstructor
--R UnivariatePuiseuxSeriesConstructor(Coef: Ring,ULS: UnivariateLaurentSeriesCategory Coef)  is a domai
--R Abbreviation for UnivariatePuiseuxSeriesConstructor is UPXSCONS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UPXSCONS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       center : % -> Coef
--R coerce : ULS -> %                    coerce : Integer -> %
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> Fraction Integer       hash : % -> SingleInteger
--R latex : % -> String                  laurent : % -> ULS
--R laurentRep : % -> ULS                leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : % -> Fraction Integer        pole? : % -> Boolean
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R retract : % -> ULS                   sample : () -> %
--R variable : % -> Symbol               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
```

```
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING S
--R D : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDR
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef an
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) 
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Co
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Co
--R differentiate : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef 
--R differentiate : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and 
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction In
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
```

```
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction Integer) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurentIfCan : % -> Union(ULS,"failed")
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R puiseux : (Fraction Integer,ULS) -> %
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retractIfCan : % -> Union(ULS,"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
```

```
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———-

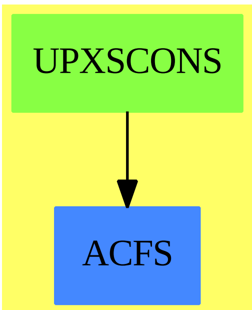
— UnivariatePuiseuxSeriesConstructor.help —

```
====================================================================
UnivariatePuiseuxSeriesConstructor examples
====================================================================

See Also:
o )show UnivariatePuiseuxSeriesConstructor
```

———-

22.6.1 UnivariatePuiseuxSeriesConstructor (UPXSCONS)



See

⇒ "UnivariatePuiseuxSeries" (UPXS) 22.5.1 on page 2790

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	laurent	laurentIfCan
laurentRep	lcm	leadingCoefficient	leadingMonomial
log	map	monomial	monomial?
multiEuclidean	multiplyExponents	nthRoot	one?
order	pi	pole?	prime?
principalIdeal	puiseux	rationalPower	recip
reductum	retract	retractIfCan	sample
sec	sech	series	sin
sinh	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	tan	tanh
terms	truncate	unit?	unitCanonical
unitNormal	variable	variables	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?
?/?	?.?	?quo?	?rem?

— domain UPXSCONS UnivariatePuiseuxSeriesConstructor —

```
)abbrev domain UPXSCONS UnivariatePuiseuxSeriesConstructor
++ Author: Clifton J. Williamson
++ Date Created: 9 May 1989
++ Date Last Updated: 30 November 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux, Laurent
++ Examples:
++ References:
++ Description:
++ This package enables one to construct a univariate Puiseux series
++ domain from a univariate Laurent series domain. Univariate
++ Puiseux series are represented by a pair \spad{[r,f(x)]}, where r is
++ a positive rational number and \spad{f(x)} is a Laurent series.
++ This pair represents the Puiseux series \spad{f(x^r)}.
```

```
UnivariatePuiseuxSeriesConstructor(Coef,ULS):_
 Exports == Implementation where
  Coef  : Ring
  ULS   : UnivariateLaurentSeriesCategory Coef
  I     ==> Integer
  L     ==> List
  NNI   ==> NonNegativeInteger
  OUT   ==> OutputForm
  PI    ==> PositiveInteger
  RN    ==> Fraction Integer
  ST    ==> Stream Coef
  LTerm ==> Record(k:I,c:Coef)
  PTerm ==> Record(k:RN,c:Coef)
  ST2LP ==> StreamFunctions2(LTerm,PTerm)
  ST2PL ==> StreamFunctions2(PTerm,LTerm)

  Exports ==> UnivariatePuiseuxSeriesConstructorCategory(Coef,ULS)

  Implementation ==> add

--% representation

    Rep := Record(expon:RN,lSeries:ULS)

    getExpon: % -> RN
    getULS  : % -> ULS

    getExpon pxs == pxs.expon
    getULS   pxs == pxs.lSeries

--% creation and destruction

    puiseux(n,ls)   == [n,ls]
    laurentRep x    == getULS x
    rationalPower x == getExpon x
    degree x        == getExpon(x) * degree(getULS(x))

    0 == puiseux(1,0)
    1 == puiseux(1,1)

    monomial(c,k) ==
      k = 0 => c :: %
      k < 0 => puiseux(-k,monomial(c,-1))
      puiseux(k,monomial(c,1))

    coerce(ls:ULS) == puiseux(1,ls)
    coerce(r:Coef) == r :: ULS  :: %
    coerce(i:I)    == i :: Coef :: %

    laurentIfCan upxs ==
```

```
      r := getExpon upxs
--      one? denom r =>
      (denom r) = 1 =>
        multiplyExponents(getULS upxs,numer(r) :: PI)
      "failed"

    laurent upxs ==
      (uls := laurentIfCan upxs) case "failed" =>
        error "laurent: Puiseux series has fractional powers"
      uls :: ULS

    multExp: (RN,LTerm) -> PTerm
    multExp(r,lTerm) == [r * lTerm.k,lTerm.c]

    terms upxs ==
      map((t1:LTerm):PTerm+->multExp(getExpon upxs,t1),terms getULS upxs)$ST2LP

    clearDen: (I,PTerm) -> LTerm
    clearDen(n,lTerm) ==
      (int := retractIfCan(n * lTerm.k)@Union(I,"failed")) case "failed" =>
        error "series: inappropriate denominator"
      [int :: I,lTerm.c]

    series(n,stream) ==
      str := map((t1:PTerm):LTerm +-> clearDen(n,t1),stream)$ST2PL
      puiseux(1/n,series str)

--% normalizations

    rewrite:(%,PI) -> %
    rewrite(upxs,m) ==
      -- rewrites a series in x**r as a series in x**(r/m)
      puiseux((getExpon upxs)*(1/m),multiplyExponents(getULS upxs,m))

    ratGcd: (RN,RN) -> RN
    ratGcd(r1,r2) ==
      -- if r1 = prod(p prime,p ** ep(r1)) and
      -- if r2 = prod(p prime,p ** ep(r2)), then
      -- ratGcd(r1,r2) = prod(p prime,p ** min(ep(r1),ep(r2)))
      gcd(numer r1,numer r2) / lcm(denom r1,denom r2)

    withNewExpon:(%,RN) -> %
    withNewExpon(upxs,r) ==
      rewrite(upxs,numer(getExpon(upxs)/r) pretend PI)

--% predicates

    upxs1 = upxs2 ==
      r1 := getExpon upxs1; r2 := getExpon upxs2
      ls1 := getULS upxs1; ls2 := getULS upxs2
```

```
      (r1 = r2) => (ls1 = ls2)
      r := ratGcd(r1,r2)
      m1 := numer(getExpon(upxs1)/r) pretend PI
      m2 := numer(getExpon(upxs2)/r) pretend PI
      multiplyExponents(ls1,m1) = multiplyExponents(ls2,m2)

    pole? upxs == pole? getULS upxs

--% arithmetic

    applyFcn:((ULS,ULS) -> ULS,%,%) -> %
    applyFcn(op,pxs1,pxs2) ==
      r1 := getExpon pxs1; r2 := getExpon pxs2
      ls1 := getULS pxs1; ls2 := getULS pxs2
      (r1 = r2) => puiseux(r1,op(ls1,ls2))
      r := ratGcd(r1,r2)
      m1 := numer(getExpon(pxs1)/r) pretend PI
      m2 := numer(getExpon(pxs2)/r) pretend PI
      puiseux(r,op(multiplyExponents(ls1,m1),multiplyExponents(ls2,m2)))

    pxs1 + pxs2     == applyFcn((z1:ULS,z2:ULS):ULS+->z1 +$ULS z2,pxs1,pxs2)
    pxs1 - pxs2     == applyFcn((z1:ULS,z2:ULS):ULS+->z1 -$ULS z2,pxs1,pxs2)
    pxs1:% * pxs2:% == applyFcn((z1:ULS,z2:ULS):ULS+->z1 *$ULS z2,pxs1,pxs2)

    pxs:% ** n:NNI == puiseux(getExpon pxs,getULS(pxs)**n)

    recip pxs ==
      rec := recip getULS pxs
      rec case "failed" => "failed"
      puiseux(getExpon pxs,rec :: ULS)

    RATALG : Boolean := Coef has Algebra(Fraction Integer)

    elt(upxs1:%,upxs2:%) ==
      uls1 := laurentRep upxs1; uls2 := laurentRep upxs2
      r1 := rationalPower upxs1; r2 := rationalPower upxs2
      (n := retractIfCan(r1)@Union(Integer,"failed")) case Integer =>
        puiseux(r2,uls1(uls2 ** r1))
      RATALG =>
        if zero? (coef := coefficient(uls2,deg := degree uls2)) then
          deg := order(uls2,deg + 1000)
          zero? (coef := coefficient(uls2,deg)) =>
            error "elt: series with many leading zero coefficients"
        -- a fractional power of a Laurent series may not be defined:
        -- if f(x) = c * x**n + ..., then f(x) ** (p/q) will be defined
        -- only if q divides n
        b := lcm(denom r1,deg); c := b quo deg
        mon : ULS := monomial(1,c)
        uls2 := elt(uls2,mon) ** r1
        puiseux(r2*(1/c),elt(uls1,uls2))
```

```
      error "elt: rational powers not available for this coefficient domain"

    if Coef has "**": (Coef,Integer) -> Coef and
       Coef has "**": (Coef, RN) -> Coef then
         eval(upxs:%,a:Coef) == eval(getULS upxs,a ** getExpon(upxs))

    if Coef has Field then

      pxs1:% / pxs2:% == applyFcn((z1:ULS,z2:ULS):ULS+->z1 /$ULS z2,pxs1,pxs2)

      inv upxs ==
        (invUpxs := recip upxs) case "failed" =>
          error "inv: multiplicative inverse does not exist"
        invUpxs :: %

--% values

    variable upxs == variable getULS upxs
    center   upxs == center   getULS upxs

    coefficient(upxs,rn) ==
--      one? denom(n := rn / getExpon upxs) =>
      (denom(n := rn / getExpon upxs)) = 1 =>
        coefficient(getULS upxs,numer n)
      0

    elt(upxs:%,rn:RN) == coefficient(upxs,rn)

--% other functions

    roundDown: RN -> I
    roundDown rn ==
      -- returns the largest integer <= rn
      (den := denom rn) = 1 => numer rn
      n := (num := numer rn) quo den
      positive?(num) => n
      n - 1

    roundUp: RN -> I
    roundUp rn ==
      -- returns the smallest integer >= rn
      (den := denom rn) = 1 => numer rn
      n := (num := numer rn) quo den
      positive?(num) => n + 1
      n

    order upxs == getExpon upxs * order getULS upxs
    order(upxs,r) ==
      e := getExpon upxs
      ord := order(getULS upxs, n := roundDown(r / e))
```

```
    ord = n => r
    ord * e

  truncate(upxs,r) ==
    e := getExpon upxs
    puiseux(e,truncate(getULS upxs,roundDown(r / e)))

  truncate(upxs,r1,r2) ==
    e := getExpon upxs
    puiseux(e,truncate(getULS upxs,roundUp(r1 / e),roundDown(r2 / e)))

  complete upxs == puiseux(getExpon upxs,complete getULS upxs)
  extend(upxs,r) ==
    e := getExpon upxs
    puiseux(e,extend(getULS upxs,roundDown(r / e)))

  map(fcn,upxs) == puiseux(getExpon upxs,map(fcn,getULS upxs))

  characteristic() == characteristic()$Coef

  -- multiplyCoefficients(f,upxs) ==
    -- r := getExpon upxs
    -- puiseux(r,multiplyCoefficients(f(#1 * r),getULS upxs))

  multiplyExponents(upxs:%,n:RN) ==
    puiseux(n * getExpon(upxs),getULS upxs)
  multiplyExponents(upxs:%,n:PI) ==
    puiseux(n * getExpon(upxs),getULS upxs)

  if Coef has "*": (Fraction Integer, Coef) -> Coef then

    differentiate upxs ==
      r := getExpon upxs
      puiseux(r,differentiate getULS upxs) * monomial(r :: Coef,r-1)

    if Coef has PartialDifferentialRing(Symbol) then

      differentiate(upxs:%,s:Symbol) ==
        (s = variable(upxs)) => differentiate upxs
        dcds := differentiate(center upxs,s)
        map((z1:Coef):Coef+->differentiate(z1,s),upxs)
                                          - dcds*differentiate(upxs)

  if Coef has Algebra Fraction Integer then

    coerce(r:RN) == r :: Coef :: %

    ratInv: RN -> Coef
    ratInv r ==
      zero? r => 1
```

```
      inv(r) :: Coef

    integrate upxs ==
      not zero? coefficient(upxs,-1) =>
        error "integrate: series has term of order -1"
      r := getExpon upxs
      uls := getULS upxs
      uls := multiplyCoefficients((z1:Integer):Coef+->ratInv(z1*r+1),uls)
      monomial(1,1) * puiseux(r,uls)

    if Coef has integrate: (Coef,Symbol) -> Coef and _
       Coef has variables: Coef -> List Symbol then

      integrate(upxs:%,s:Symbol) ==
        (s = variable(upxs)) => integrate upxs
        not entry?(s,variables center upxs)
          => map((z1:Coef):Coef +-> integrate(z1,s),upxs)
        error "integrate: center is a function of variable of integration"

    if Coef has TranscendentalFunctionCategory and _
       Coef has PrimitiveFunctionCategory and _
       Coef has AlgebraicallyClosedFunctionSpace Integer then

      integrateWithOneAnswer: (Coef,Symbol) -> Coef
      integrateWithOneAnswer(f,s) ==
        res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
        res case Coef => res :: Coef
        first(res :: List Coef)

      integrate(upxs:%,s:Symbol) ==
        (s = variable(upxs)) => integrate upxs
        not entry?(s,variables center upxs) =>
          map((z1:Coef):Coef +-> integrateWithOneAnswer(z1,s),upxs)
        error "integrate: center is a function of variable of integration"

    if Coef has Field then
       (upxs:%) ** (q:RN) ==
         num := numer q; den := denom q
--         one? den => upxs ** num
         den = 1 => upxs ** num
         r := rationalPower upxs; uls := laurentRep upxs
         deg := degree uls
         if zero?(coef := coefficient(uls,deg)) then
           deg := order(uls,deg + 1000)
           zero?(coef := coefficient(uls,deg)) =>
             error "power of series with many leading zero coefficients"
         ulsPow := (uls * monomial(1,-deg)$ULS) ** q
         puiseux(r,ulsPow) * monomial(1,deg*q*r)

    applyUnary: (ULS -> ULS,%) -> %
```

```
applyUnary(fcn,upxs) ==
  puiseux(rationalPower upxs,fcn laurentRep upxs)

exp upxs   == applyUnary(exp,upxs)
log upxs   == applyUnary(log,upxs)
sin upxs   == applyUnary(sin,upxs)
cos upxs   == applyUnary(cos,upxs)
tan upxs   == applyUnary(tan,upxs)
cot upxs   == applyUnary(cot,upxs)
sec upxs   == applyUnary(sec,upxs)
csc upxs   == applyUnary(csc,upxs)
asin upxs  == applyUnary(asin,upxs)
acos upxs  == applyUnary(acos,upxs)
atan upxs  == applyUnary(atan,upxs)
acot upxs  == applyUnary(acot,upxs)
asec upxs  == applyUnary(asec,upxs)
acsc upxs  == applyUnary(acsc,upxs)
sinh upxs  == applyUnary(sinh,upxs)
cosh upxs  == applyUnary(cosh,upxs)
tanh upxs  == applyUnary(tanh,upxs)
coth upxs  == applyUnary(coth,upxs)
sech upxs  == applyUnary(sech,upxs)
csch upxs  == applyUnary(csch,upxs)
asinh upxs == applyUnary(asinh,upxs)
acosh upxs == applyUnary(acosh,upxs)
atanh upxs == applyUnary(atanh,upxs)
acoth upxs == applyUnary(acoth,upxs)
asech upxs == applyUnary(asech,upxs)
acsch upxs == applyUnary(acsch,upxs)
```

———-

— UPXSCONS.dotabb —

```
"UPXSCONS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UPXSCONS"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UPXSCONS" -> "ACFS"
```

———-

22.7 domain UPXSSING UnivariatePuiseuxSeriesWithExponentialSingularity

— UnivariatePuiseuxSeriesWithExponentialSingularity.input —

```
)set break resume
)sys rm -f UnivariatePuiseuxSeriesWithExponentialSingularity.output
)spool UnivariatePuiseuxSeriesWithExponentialSingularity.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeriesWithExponentialSingularity
--R UnivariatePuiseuxSeriesWithExponentialSingularity(R: Join(OrderedSet,RetractableTo Integer,LinearlyE
--R Abbreviation for UnivariatePuiseuxSeriesWithExponentialSingularity is UPXSSING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UPXSSING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              ground? : % -> Boolean
--R hash : % -> SingleInteger             latex : % -> String
--R leadingMonomial : % -> %              monomial? : % -> Boolean
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if UnivariatePuiseuxSeries(FE,var,cen) has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if UnivariatePuiseuxSeries(FE,var,cen) has ALGEBRA FRAC INT
--R ?*? : (UnivariatePuiseuxSeries(FE,var,cen),%) -> %
--R ?*? : (%,UnivariatePuiseuxSeries(FE,var,cen)) -> %
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,UnivariatePuiseuxSeries(FE,var,cen)) -> % if UnivariatePuiseuxSeries(FE,var,cen) has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if UnivariatePuiseuxSeries(FE,var,cen) has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if UnivariatePuiseuxSeries(FE,var,cen) has CHARNZ
--R coefficient : (%,ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)) -> UnivariatePuiseuxSeries(FE,var
--R coefficients : % -> List UnivariatePuiseuxSeries(FE,var,cen)
--R coerce : Fraction Integer -> % if UnivariatePuiseuxSeries(FE,var,cen) has ALGEBRA FRAC INT or Univar
--R coerce : UnivariatePuiseuxSeries(FE,var,cen) -> %
--R content : % -> UnivariatePuiseuxSeries(FE,var,cen) if UnivariatePuiseuxSeries(FE,var,cen) has GCDDOM
--R degree : % -> ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)
--R dominantTerm : % -> Union(Record(%term: Record(%coef: UnivariatePuiseuxSeries(FE,var,cen),%expon: Ex
--R exquo : (%,%) -> Union(%,"failed")
--R exquo : (%,UnivariatePuiseuxSeries(FE,var,cen)) -> Union(%,"failed") if UnivariatePuiseuxSeries(FE,v
--R ground : % -> UnivariatePuiseuxSeries(FE,var,cen)
```

```
--R leadingCoefficient : % -> UnivariatePuiseuxSeries(FE,var,cen)
--R limitPlus : % -> Union(OrderedCompletion FE,"failed")
--R map : ((UnivariatePuiseuxSeries(FE,var,cen) -> UnivariatePuiseuxSeries(FE,var,cen)),%) ->
--R mapExponents : ((ExponentialOfUnivariatePuiseuxSeries(FE,var,cen) -> ExponentialOfUnivari
--R minimumDegree : % -> ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)
--R monomial : (UnivariatePuiseuxSeries(FE,var,cen),ExponentialOfUnivariatePuiseuxSeries(FE,v
--R numberOfMonomials : % -> NonNegativeInteger
--R pomopo! : (%,UnivariatePuiseuxSeries(FE,var,cen),ExponentialOfUnivariatePuiseuxSeries(FE
--R primitivePart : % -> % if UnivariatePuiseuxSeries(FE,var,cen) has GCDDOM
--R retract : % -> UnivariatePuiseuxSeries(FE,var,cen)
--R retract : % -> Fraction Integer if UnivariatePuiseuxSeries(FE,var,cen) has RETRACT FRAC I
--R retract : % -> Integer if UnivariatePuiseuxSeries(FE,var,cen) has RETRACT INT
--R retractIfCan : % -> Union(UnivariatePuiseuxSeries(FE,var,cen),"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if UnivariatePuiseuxSeries(FE,var,ce
--R retractIfCan : % -> Union(Integer,"failed") if UnivariatePuiseuxSeries(FE,var,cen) has RE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— UnivariatePuiseuxSeriesWithExponentialSingularity.help —

```
====================================================================
UnivariatePuiseuxSeriesWithExponentialSingularity examples
====================================================================

See Also:
o )show UnivariatePuiseuxSeriesWithExponentialSingularity
```

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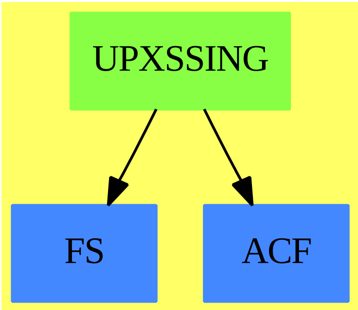
22.7.1 UnivariatePuiseuxSeriesWithExponentialSingularity (UPXSSING)



See

⇒ "ExponentialOfUnivariatePuiseuxSeries" (EXPUPXS) 6.7.1 on page 707
⇒ "ExponentialExpansion" (EXPEXPAN) 6.5.1 on page 679

Exports:

0	1	associates?	binomThmExpt	characteristic
charthRoot	coefficient	coefficients	coerce	content
degree	dominantTerm	exquo	ground	ground?
hash	latex	leadingCoefficient	leadingMonomial	limitPlus
map	mapExponents	minimumDegree	monomial	monomial?
numberOfMonomials	one?	pomopo!	primitivePart	recip
reductum	retract	retractIfCan	sample	subtractIfCan
unit?	unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?/?		

— domain UPXSSING UnivariatePuiseuxSeriesWithExponentialSingularity —

```
)abbrev domain UPXSSING UnivariatePuiseuxSeriesWithExponentialSingularity
++ Author: Clifton J. Williamson
++ Date Created: 4 August 1992
++ Date Last Updated: 27 August 1992
++ Basic Operations:
++ Related Domains: UnivariatePuiseuxSeries(FE,var,cen),
++                  ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)
++                  ExponentialExpansion(R,FE,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: limit, functional expression, power series
++ Examples:
++ References:
++ Description:
++ UnivariatePuiseuxSeriesWithExponentialSingularity is a domain used to
```

```
++ represent functions with essential singularities.  Objects in this
++ domain are sums, where each term in the sum is a univariate Puiseux
++ series times the exponential of a univariate Puiseux series.  Thus,
++ the elements of this domain are sums of expressions of the form
++ \spad{g(x) * exp(f(x))}, where g(x) is a univariate Puiseux series
++ and f(x) is a univariate Puiseux series with no terms of non-negative
++ degree.

UnivariatePuiseuxSeriesWithExponentialSingularity(R,FE,var,cen):_
  Exports == Implementation where
  R   : Join(OrderedSet,RetractableTo Integer,_
             LinearlyExplicitRingOver Integer,GcdDomain)
  FE  : Join(AlgebraicallyClosedField,TranscendentalFunctionCategory,_
             FunctionSpace R)
  var : Symbol
  cen : FE
  B       ==> Boolean
  I       ==> Integer
  L       ==> List
  RN      ==> Fraction Integer
  UPXS    ==> UnivariatePuiseuxSeries(FE,var,cen)
  EXPUPXS ==> ExponentialOfUnivariatePuiseuxSeries(FE,var,cen)
  OFE     ==> OrderedCompletion FE
  Result  ==> Union(OFE,"failed")
  PxRec   ==> Record(k: Fraction Integer,c:FE)
  Term    ==> Record(%coef:UPXS,%expon:EXPUPXS,%expTerms:List PxRec)
    -- the %expTerms field is used to record the list of the terms (a 'term'
    -- records an exponent and a coefficient) in the exponent %expon
  TypedTerm ==> Record(%term:Term,%type:String)
    -- a term together with a String which tells whether it has an infinite,
    -- zero, or unknown limit as var -> cen+
  TRec    ==> Record(%zeroTerms: List Term,_
                     %infiniteTerms: List Term,_
                     %failedTerms: List Term,_
                     %puiseuxSeries: UPXS)
  SIGNEF  ==> ElementaryFunctionSign(R,FE)

  Exports ==> Join(FiniteAbelianMonoidRing(UPXS,EXPUPXS),IntegralDomain) with
    limitPlus : % -> Union(OFE,"failed")
      ++ limitPlus(f(var)) returns \spad{limit(var -> cen+,f(var))}.
    dominantTerm : % -> Union(TypedTerm,"failed")
      ++ dominantTerm(f(var)) returns the term that dominates the limiting
      ++ behavior of \spad{f(var)} as \spad{var -> cen+} together with a
      ++ \spadtype{String} which briefly describes that behavior.  The
      ++ value of the \spadtype{String} will be \spad{"zero"} (resp.
      ++ \spad{"infinity"}) if the term tends to zero (resp. infinity)
      ++ exponentially and will \spad{"series"} if the term is a
      ++ Puiseux series.

  Implementation ==> PolynomialRing(UPXS,EXPUPXS) add
```

```
makeTerm : (UPXS,EXPUPXS) -> Term
coeff : Term -> UPXS
exponent : Term -> EXPUPXS
exponentTerms : Term -> List PxRec
setExponentTerms_! : (Term,List PxRec) -> List PxRec
computeExponentTerms_! : Term -> List PxRec
terms : % -> List Term
sortAndDiscardTerms: List Term -> TRec
termsWithExtremeLeadingCoef : (L Term,RN,I) -> Union(L Term,"failed")
filterByOrder: (L Term,(RN,RN) -> B) -> Record(%list:L Term,%order:RN)
dominantTermOnList : (L Term,RN,I) -> Union(Term,"failed")
iDominantTerm : L Term -> Union(Record(%term:Term,%type:String),"failed")

retractIfCan f ==
  (numberOfMonomials f = 1) and (zero? degree f) => leadingCoefficient f
  "failed"

recip f ==
  numberOfMonomials f = 1 =>
    monomial(inv leadingCoefficient f,- degree f)
  "failed"

makeTerm(coef,expon) == [coef,expon,empty()]
coeff term == term.%coef
exponent term == term.%expon
exponentTerms term == term.%expTerms
setExponentTerms_!(term,list) == term.%expTerms := list
computeExponentTerms_! term ==
  setExponentTerms_!(term,entries complete terms exponent term)

terms f ==
  -- terms with a higher order singularity will appear closer to the
  -- beginning of the list because of the ordering in EXPPUPXS;
  -- no "expnonent terms" are computed by this function
  zero? f => empty()
  concat(makeTerm(leadingCoefficient f,degree f),terms reductum f)

sortAndDiscardTerms termList ==
  -- 'termList' is the list of terms of some function f(var), ordered
  -- so that terms with a higher order singularity occur at the
  -- beginning of the list.
  -- This function returns lists of candidates for the "dominant
  -- term" in 'termList', i.e. the term which describes the
  -- asymptotic behavior of f(var) as var -> cen+.
  -- 'zeroTerms' will contain terms which tend to zero exponentially
  -- and contains only those terms with the lowest order singularity.
  -- 'zeroTerms' will be non-empty only when there are no terms of
  -- infinite or series type.
  -- 'infiniteTerms' will contain terms which tend to infinity
  -- exponentially and contains only those terms with the highest
```

```
-- order singularity.
-- 'failedTerms' will contain terms which have an exponential
-- singularity, where we cannot say whether the limiting value
-- is zero or infinity. Only terms with a higher order sigularity
-- than the terms on 'infiniteList' are included.
-- 'pSeries' will be a Puiseux series representing a term without an
-- exponential singularity.  'pSeries' will be non-zero only when no
-- other terms are known to tend to infinity exponentially
zeroTerms : List Term := empty()
infiniteTerms : List Term := empty()
failedTerms : List Term := empty()
-- we keep track of whether or not we've found an infinite term
-- if so, 'infTermOrd' will be set to a negative value
infTermOrd : RN := 0
-- we keep track of whether or not we've found a zero term
-- if so, 'zeroTermOrd' will be set to a negative value
zeroTermOrd : RN := 0
ord : RN := 0; pSeries : UPXS := 0  -- dummy values
while not empty? termList repeat
  -- 'expon' is a Puiseux series
  expon := exponent(term := first termList)
  -- quit if there is an infinite term with a higher order singularity
  (ord := order(expon,0)) > infTermOrd => leave "infinite term dominates"
  -- if ord = 0, we've hit the end of the list
  (ord = 0) =>
    -- since we have a series term, don't bother with zero terms
    leave(pSeries := coeff(term); zeroTerms := empty())
  coef := coefficient(expon,ord)
  -- if we can't tell if the lowest order coefficient is positive or
  -- negative, we have a "failed term"
  (signum := sign(coef)$SIGNEF) case "failed" =>
    failedTerms := concat(term,failedTerms)
    termList := rest termList
  -- if the lowest order coefficient is positive, we have an
  -- "infinite term"
  (sig := signum :: Integer) = 1 =>
    infTermOrd := ord
    infiniteTerms := concat(term,infiniteTerms)
    -- since we have an infinite term, don't bother with zero terms
    zeroTerms := empty()
    termList := rest termList
  -- if the lowest order coefficient is negative, we have a
  -- "zero term" if there are no infinite terms and no failed
  -- terms, add the term to 'zeroTerms'
  if empty? infiniteTerms then
    zeroTerms :=
      ord = zeroTermOrd => concat(term,zeroTerms)
      zeroTermOrd := ord
      list term
  termList := rest termList
```

```
    -- reverse "failed terms" so that higher order singularities
    -- appear at the beginning of the list
    [zeroTerms,infiniteTerms,reverse_! failedTerms,pSeries]

  termsWithExtremeLeadingCoef(termList,ord,signum) ==
    -- 'termList' consists of terms of the form [g(x),exp(f(x)),...];
    -- when 'signum' is +1 (resp. -1), this function filters 'termList'
    -- leaving only those terms such that coefficient(f(x),ord) is
    -- maximal (resp. minimal)
    while (coefficient(exponent first termList,ord) = 0) repeat
      termList := rest termList
    empty? termList => error "UPXSSING: can't happen"
    coefExtreme := coefficient(exponent first termList,ord)
    outList := list first termList; termList := rest termList
    for term in termList repeat
      (coefDiff := coefficient(exponent term,ord) - coefExtreme) = 0 =>
        outList := concat(term,outList)
      (sig := sign(coefDiff)$SIGNEF) case "failed" => return "failed"
      (sig :: Integer) = signum => outList := list term
    outList

  filterByOrder(termList,predicate) ==
    -- 'termList' consists of terms of the form [g(x),exp(f(x)),expTerms],
    -- where 'expTerms' is a list containing some of the terms in the
    -- series f(x).
    -- The function filters 'termList' and, when 'predicate' is < (resp. >),
    -- leaves only those terms with the lowest (resp. highest) order term
    -- in 'expTerms'
    while empty? exponentTerms first termList repeat
      termList := rest termList
      empty? termList => error "UPXSING: can't happen"
    ordExtreme := (first exponentTerms first termList).k
    outList := list first termList
    for term in rest termList repeat
      not empty? exponentTerms term =>
        (ord := (first exponentTerms term).k) = ordExtreme =>
          outList := concat(term,outList)
        predicate(ord,ordExtreme) =>
          ordExtreme := ord
          outList := list term
    -- advance pointers on "exponent terms" on terms on 'outList'
    for term in outList repeat
      setExponentTerms_!(term,rest exponentTerms term)
    [outList,ordExtreme]

  dominantTermOnList(termList,ord0,signum) ==
    -- finds dominant term on 'termList'
    -- it is known that "exponent terms" of order < 'ord0' are
    -- the same for all terms on 'termList'
    newList := termsWithExtremeLeadingCoef(termList,ord0,signum)
```

```
    newList case "failed" => "failed"
    termList := newList :: List Term
    empty? rest termList => first termList
    filtered :=
      signum = 1 => filterByOrder(termList,(x,y) +-> x < y)
      filterByOrder(termList,(x,y) +-> x > y)
    termList := filtered.%list
    empty? rest termList => first termList
    dominantTermOnList(termList,filtered.%order,signum)

  iDominantTerm termList ==
    termRecord := sortAndDiscardTerms termList
    zeroTerms := termRecord.%zeroTerms
    infiniteTerms := termRecord.%infiniteTerms
    failedTerms := termRecord.%failedTerms
    pSeries := termRecord.%puiseuxSeries
    -- in future versions, we will deal with "failed terms"
    -- at present, if any occur, we cannot determine the limit
    not empty? failedTerms => "failed"
    not zero? pSeries => [makeTerm(pSeries,0),"series"]
    not empty? infiniteTerms =>
      empty? rest infiniteTerms => [first infiniteTerms,"infinity"]
      for term in infiniteTerms repeat computeExponentTerms_! term
      ord0 := order exponent first infiniteTerms
      (dTerm := dominantTermOnList(infiniteTerms,ord0,1)) case "failed" =>
        return "failed"
      [dTerm :: Term,"infinity"]
    empty? rest zeroTerms => [first zeroTerms,"zero"]
    for term in zeroTerms repeat computeExponentTerms_! term
    ord0 := order exponent first zeroTerms
    (dTerm := dominantTermOnList(zeroTerms,ord0,-1)) case "failed" =>
      return "failed"
    [dTerm :: Term,"zero"]

  dominantTerm f == iDominantTerm terms f

  limitPlus f ==
    -- list the terms occurring in 'f'; if there are none, then f = 0
    empty?(termList := terms f) => 0
    -- compute dominant term
    (tInfo := iDominantTerm termList) case "failed" => "failed"
    termInfo := tInfo :: Record(%term:Term,%type:String)
    domTerm := termInfo.%term
    (type := termInfo.%type) = "series" =>
      -- find limit of series term
      (ord := order(pSeries := coeff domTerm,1)) > 0 => 0
      coef := coefficient(pSeries,ord)
      member?(var,variables coef) => "failed"
      ord = 0 => coef :: OFE
      -- in the case of an infinite limit, we need to know the sign
```

```
        -- of the first non-zero coefficient
        (signum := sign(coef)$SIGNEF) case "failed" => "failed"
        (signum :: Integer) = 1 => plusInfinity()
        minusInfinity()
      type = "zero" => 0
      -- examine lowest order coefficient in series part of 'domTerm'
      ord := order(pSeries := coeff domTerm)
      coef := coefficient(pSeries,ord)
      member?(var,variables coef) => "failed"
      (signum := sign(coef)$SIGNEF) case "failed" => "failed"
      (signum :: Integer) = 1 => plusInfinity()
      minusInfinity()
```

— UPXSSING.dotabb —

```
"UPXSSING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UPXSSING"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"UPXSSING" -> "ACF"
"UPXSSING" -> "FS"
```

22.8 domain OREUP UnivariateSkewPolynomial

— UnivariateSkewPolynomial.input —

```
)set break resume
)sys rm -f UnivariateSkewPolynomial.output
)spool UnivariateSkewPolynomial.output
)set message test on
)set message auto off
)set message type off
)clear all

--S 1 of 33
F:=EXPR(FRAC(INT))
--R
--R
--R   (1)  Expression Fraction Integer
--E 1
```

```
--S 2 of 33
Dx:F->F:=f+->D(f,['x])
--R
--R
--R   (2)  theMap(Closure)
--E 2

--S 3 of 33
D0:=OREUP('d,F,1,Dx)
--R
--R
--R   (3)
--I  UnivariateSkewPolynomial(d,Expression Fraction Integer,R -> R,theMap LAMBDA-C
--I  LOSURE(NIL,NIL,NIL,G9057 envArg,SPADCALL(G9057,coerceOrCroak(CONS(QUOTE List
--I  Variable x,wrap LIST QUOTE x),QUOTE List Symbol,QUOTE *1;anonymousFunction;2;
--I  frame0;internal),ELT(*1;anonymousFunction;2;frame0;internal;MV,0))))
--E 3

--S 4 of 33
u:D0:=(operator 'u)(x)
--R
--R
--R   (4)  u(x)
--E 4

--S 5 of 33
d:D0:='d
--R
--R
--R   (5)  d
--E 5

--S 6 of 33
a:D0:=u^3*d^3+u^2*d^2+u*d+1
--R
--R
--R            3 3       2 2
--R   (6)  u(x) d  + u(x) d  + u(x)d + 1
--E 6

--S 7 of 33
b:D0:=(u+1)*d^2+2*d
--R
--R
--R                     2
--R   (7)  (u(x) + 1)d  + 2d
--E 7

--S 8 of 33
r:=rightDivide(a,b)
```

```
--R
--R
--R   (8)
--R                                    3 ,           3       2
--R                     3      - u(x) u (x) - u(x)  + u(x)
--R                 u(x)
--R   [quotient= -------- d + ---------------------------,
--R               u(x) + 1               2
--R                                  u(x)  + 2u(x) + 1
--R                     3 ,          3
--R                2u(x) u (x) + 3u(x)  + u(x)
--R
--R    remainder= --------------------------- d + 1]
--R                         2
--R                     u(x)  + 2u(x) + 1
--E 8

--S 9 of 33
r.quotient
--R
--R
--R                              3 ,           3       2
--R               3      - u(x) u (x) - u(x)  + u(x)
--R           u(x)
--R   (9)  -------- d + ---------------------------
--R         u(x) + 1               2
--R                            u(x)  + 2u(x) + 1
--E 9

--S 10 of 33
r.remainder
--R
--R
--R               3 ,          3
--R          2u(x) u (x) + 3u(x)  + u(x)
--R
--R   (10)  --------------------------- d + 1
--R                   2
--R               u(x)  + 2u(x) + 1
--E 10

)clear all

--S 11 of 33
R:=UP('t,INT)
--R
--R
--R   (1)  UnivariatePolynomial(t,Integer)
--E 11
```

```
--S 12 of 33
W:=OREUP('x,R,1,D)
--R
--R
--R   (2)
--R  UnivariateSkewPolynomial(x,UnivariatePolynomial(t,Integer),R -> R,theMap(DIFR
--I  ING-;D;2S;1,411))
--E 12

--S 13 of 33
t:W:='t
--R
--R
--R   (3)  t
--E 13

--S 14 of 33
x:W:='x
--R
--R
--R   (4)  x
--E 14

--S 15 of 33
a:W:=(t-1)*x^4+(t^3+3*t+1)*x^2+2*t*x+t^3
--R
--R
--R                   4     3           2           3
--R   (5)  (t - 1)x  + (t  + 3t + 1)x  + 2t x + t
--E 15

--S 16 of 33
b:W:=(6*t^4+2*t^2)*x^3+3*t^2*x^2
--R
--R
--R            4     2  3     2 2
--R   (6)  (6t  + 2t )x  + 3t x
--E 16

--S 17 of 33
a*b
--R
--R
--R   (7)
--R        5     4     3     2  7       4      3      2        6
--R     (6t  - 6t  + 2t  - 2t )x  + (96t  - 93t  + 13t  - 16t)x
--R   +
--R        7      5     4       3       2       5
--R     (6t  + 20t  + 6t  + 438t  - 406t  - 24)x
--R   +
```

```
--R         6      5       4      3       2              4
--R     (48t  + 15t  + 152t  + 61t  + 603t  - 532t - 36)x
--R   +
--R        7      5      4       3       2               3
--R     (6t  + 74t  + 60t  + 226t  + 116t  + 168t - 140)x
--R   +
--R        5     3      2           2
--R     (3t  + 6t  + 12t  + 18t + 6)x
--E 17

--S 18 of 33
a^3
--R
--R
--R   (8)
--R       3     2           12      5     4      3      2           10
--R     (t  - 3t  + 3t - 1)x   + (3t  - 6t  + 12t  - 15t  + 3t + 3)x
--R   +
--R        3      2       9      7     6      5      4      3     2            8
--R     (6t  - 12t  + 6t)x  + (3t  - 3t  + 21t  - 18t  + 24t  - 9t  - 15t - 3)x
--R   +
--R         5      4      3      2       7
--R     (12t  - 12t  + 36t  - 24t  - 12t)x
--R   +
--R       9      7     6      5     4      3      2           6
--R     (t  + 15t  - 3t  + 45t  + 6t  + 36t  + 15t  + 9t + 1)x
--R   +
--R        7      5      3      2       5
--R     (6t  + 48t  + 54t  + 36t  + 6t)x
--R   +
--R        9      7     6      5      4      3      2  4
--R     (3t  + 21t  + 3t  + 39t  + 18t  + 39t  + 12t )x
--R   +
--R         7      5      4     3  3      9     7     6      5  2     7     9
--R     (12t  + 36t  + 12t  + 8t )x  + (3t  + 9t  + 3t  + 12t )x  + 6t x + t
--E 18

)clear all

--S 19 of 33
S:EXPR(INT)->EXPR(INT):=e+->eval(e,[n],[n+1])
--R
--R
--R   (1)  theMap(Closure)
--E 19

--S 20 of 33
DF:EXPR(INT)->EXPR(INT):=e+->eval(e,[n],[n+1])-e
--R
--R
```

```
--R   (2)  theMap(Closure)
--E 20

--S 21 of 33
DO:=OREUP('D,EXPR(INT),morphism S,DF)
--R
--R
--R   (3)
--I  UnivariateSkewPolynomial(D,Expression Integer,R -> R,theMap LAMBDA-CLOSURE(NI
--I  L,NIL,NIL,G9384 envArg,SPADCALL(SPADCALL(G9384,coerceOrCroak(CONS(QUOTE List
--I  Variable n,wrap LIST QUOTE n),QUOTE List Expression Integer,QUOTE *1;anonymou
--I  sFunction;9;frame0;internal),coerceOrCroak(CONS(QUOTE List Polynomial Integer
--I  ,wrap LIST SPADCALL(QUOTE 1(n,1 0),QUOTE 0,ELT(*1;anonymousFunction;9;frame0;
--I  internal;MV,0))),QUOTE List Expression Integer,QUOTE *1;anonymousFunction;9;f
--I  rame0;internal),ELT(*1;anonymousFunction;9;frame0;internal;MV,1)),G9384,ELT(*
--I  1;anonymousFunction;9;frame0;internal;MV,2))))
--E 21

--S 22 of 33
u:=(operator 'u)[n]
--R
--R
--R   (4)  u(n)
--E 22

--S 23 of 33
L:DO:='D+u
--R
--R
--R   (5)  D + u(n)
--E 23

--S 24 of 33
L^2
--R
--R
--R         2                2
--R   (6)  D  + 2u(n)D + u(n)
--E 24

)clear all

--S 25 of 33
)set expose add constructor SquareMatrix
--R
--I   SquareMatrix is now explicitly exposed in frame frame0
--E 25

--S 26 of 33
R:=SQMATRIX(2,INT)
```

```
--R
--R
--R   (1)  SquareMatrix(2,Integer)
--E 26

--S 27 of 33
y:R:=matrix [[1,1],[0,1]]
--R
--R
--R        +1  1+
--R   (2)  |    |
--R        +0  1+
--E 27

--S 28 of 33
delta:R->R:=r+->y*r-r*y
--R
--R
--R   (3)  theMap(Closure)
--E 28

--S 29 of 33
S:=OREUP('x,R,1,delta)
--R
--R
--R   (4)
--I  UnivariateSkewPolynomial(x,SquareMatrix(2,Integer),R -> R,theMap LAMBDA-CLOSU
--I  RE(NIL,NIL,NIL,G9459 envArg,SPADCALL(SPADCALL(getValueFromEnvironment(QUOTE y
--I  ,QUOTE SquareMatrix(2,Integer)),G9459,ELT(*1;anonymousFunction;13;frame0;inte
--I  rnal;MV,0)),SPADCALL(G9459,getValueFromEnvironment(QUOTE y,QUOTE SquareMatrix
--I  (2,Integer)),ELT(*1;anonymousFunction;13;frame0;internal;MV,0)),ELT(*1;anonym
--I  ousFunction;13;frame0;internal;MV,1))))
--E 29

--S 30 of 33
x:S:='x
--R
--R
--R   (5)  x
--E 30

--S 31 of 33
a:S:=matrix [[2,3],[1,1]]
--R
--R
--R        +2  3+
--R   (6)  |    |
--R        +1  1+
--E 31
```

```
--S 32 of 33
x^2*a
--R
--R
--R        +2  3+ 2   +2  - 2+    +0  - 2+
--R   (7)  |    |x  + |      |x + |      |
--R        +1  1+     +0  - 2+    +0   0 +
--E 32

--S 33 of 33
)show UnivariateSkewPolynomial
--R
--R UnivariateSkewPolynomial(x: Symbol,R: Ring,sigma: Automorphism R,delta: (R -> R))  is a c
--R Abbreviation for UnivariateSkewPolynomial is OREUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OREUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        apply : (%,R,R) -> R
--R coefficients : % -> List R            coerce : Variable x -> %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> R           one? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R leftGcd : (%,%) -> % if R has FIELD
--R leftLcm : (%,%) -> % if R has FIELD
--R leftQuotient : (%,%) -> % if R has FIELD
--R leftRemainder : (%,%) -> % if R has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
```

```
--R monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monomial : (R,NonNegativeInteger) -> %
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R rightGcd : (%,%) -> % if R has FIELD
--R rightLcm : (%,%) -> % if R has FIELD
--R rightQuotient : (%,%) -> % if R has FIELD
--R rightRemainder : (%,%) -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 33
)set expose drop constructor SquareMatrix

)spool
)lisp (bye)
```

———-

— UnivariateSkewPolynomial.help —

```
====================================================================
UnivariateSkewPolynomial examples
====================================================================

Skew or Ore polynomial rings provide a unified framework to
compute with differential and difference equations.

In the following, let A be an integral domain, equipped with two
endomorphisms \sigma and \delta where:

 \sigma: A -> A is an injective ring endomorphism
 \delta: A -> A, the pseudo-derivation with respect to \sigma,
        is an additive endomorphism with

    \delta(ab) = \sigma(a)\delta(b) + \delta(a)b

        for all a,b in A

Note that in the domains and categories below, these properties are
not checked.
```

```
The skew polynomial ring [\Delta;\sigma,\delta] is the ring of
polynomials in \Delta with coefficients in A, with the usual addition,
while the product is given by

 \Delta a = \sigma(a)\Delta + \delta(a) for a in A

The two most important examples of skew polynomial rings are:

  K(x)[D,1,\delta], where 1 is the identity on K and \delta is
                    the usual derviative, is the ring of differential
                    polynomials

  K [E,n,\mapsto n+1,0] is the ring of linear recurrence operators
   n                    with polynomial coefficients


---------------------------------------------------------------------

For example,

The UnivariateSkewPolynomialCategory (OREPCAT) provides a unified
framework for polynomial rings in a non-central indeterminate over
some coefficient ring R. The commutation relations between the
indeterminate x and the coefficient t is given by

   x r = \sigma(r) x + \delta(r)

where \sigma is a ring endomorphism of R
and   \delta is a \sigma-derivation of R
which is an additive map from R to R such that

   \delta(rs) = \sigma(r) \delta(s) + \delta(r) s

In case \sigma is the identity map on R, a \sigma-derivation of R
is just called a derivation. Here are some examples

We start with a linear ordinary differential operator. First, we
define the coefficient ring to be expressions in one variable x
with fractional coefficients:

   F:=EXPR(FRAC(INT))

Define Dx to be a derivative d/dx:

   Dx:F->F:=f+->D(f,['x])

Define a skew polynomial ring over F with identity endomorphism as
\sigma and derivation d/dx as \delta:

   D0:=OREUP('d,F,1,Dx)
```

```
u:DO:=(operator 'u)(x)

d:DO:='d

a:DO:=u^3*d^3+u^2*d^2+u*d+1

        3 3       2 2
    u(x) d  + u(x) d  + u(x)d + 1

b:DO:=(u+1)*d^2+2*d

              2
    (u(x) + 1)d  + 2d

r:=rightDivide(a,b)

                             3 ,          3       2
                3      - u(x) u (x) - u(x)  + u(x)
            u(x)
[quotient= -------- d + ---------------------------,
           u(x) + 1                 2
                             u(x)  + 2u(x) + 1
                3 ,          3
           2u(x) u (x) + 3u(x)  + u(x)

 remainder= --------------------------- d + 1]
                     2
                 u(x)  + 2u(x) + 1


r.quotient

                       3 ,          3       2
          3      - u(x) u (x) - u(x)  + u(x)
      u(x)
    -------- d + ----------------------------
    u(x) + 1                 2
                       u(x)  + 2u(x) + 1


r.remainder

          3 ,          3
     2u(x) u (x) + 3u(x)  + u(x)

     --------------------------- d + 1
              2
          u(x)  + 2u(x) + 1
```

```
--------------------------------------------------------------------
   )clear all

As a second example, we consider the so-called Weyl algebra.

Define the coefficient ring to be an ordinary polynomial over integers
in one variable t

   R:=UP('t,INT)

Define a skew polynomial ring over R with identity map as \sigma
and derivation d/dt as \delta. The resulting algebra is then called
a Weyl algebra. This is a simple ring over a division ring that is
non-commutative, similar to the ring of matrices.

   W:=OREUP('x,R,1,D)

   t:W:='t

   x:W:='x

Let

   a:W:=(t-1)*x^4+(t^3+3*t+1)*x^2+2*t*x+t^3

               4     3           2           3
        (t - 1)x  + (t  + 3t + 1)x  + 2t x + t

   b:W:=(6*t^4+2*t^2)*x^3+3*t^2*x^2

           4     2  3     2 2
        (6t  + 2t )x  + 3t x

Then

   a*b
           5     4     3     2  7       4      3      2        6
        (6t  - 6t  + 2t  - 2t )x  + (96t  - 93t  + 13t  - 16t)x
      +
           7      5     4       3       2       5
        (6t  + 20t  + 6t  + 438t  - 406t  - 24)x
      +
            6      5       4      3       2              4
        (48t  + 15t  + 152t  + 61t  + 603t  - 532t - 36)x
      +
           7      5      4       3       2              3
        (6t  + 74t  + 60t  + 226t  + 116t  + 168t - 140)x
      +
```

```
      5     3      2            2
   (3t  + 6t  + 12t  + 18t + 6)x

a^3
     3     2            12      5     4      3      2           10
    (t  - 3t  + 3t - 1)x   + (3t  - 6t  + 12t  - 15t  + 3t + 3)x
  +
      3      2       9      7     6      5      4      3     2            8
    (6t  - 12t  + 6t)x  + (3t  - 3t  + 21t  - 18t  + 24t  - 9t  - 15t - 3)x
  +
       5      4      3      2        7
    (12t  - 12t  + 36t  - 24t  - 12t)x
  +
     9      7     6      5     4      3      2           6
    (t  + 15t  - 3t  + 45t  + 6t  + 36t  + 15t  + 9t + 1)x
  +
      7      5      3      2       5
    (6t  + 48t  + 54t  + 36t  + 6t)x
  +
      9      7     6      5      4      3      2  4
    (3t  + 21t  + 3t  + 39t  + 18t  + 39t  + 12t )x
  +
       7      5      4     3  3      9     7     6      5  2     7     9
    (12t  + 36t  + 12t  + 8t )x  + (3t  + 9t  + 3t  + 12t )x  + 6t x + t
```

```
)clear all
```

As a third example, we construct a difference operator algebra over the ring of EXPR(INT) by using an automorphism S defined by a "shift" operation S:EXPR(INT) -> EXPR(INT)

```
s(e)(n) = e(n+1)
```

and an S-derivation defined by DF:EXPR(INT) -> EXPR(INT) as

```
DF(e)(n) = e(n+1)-e(n)
```

Define S to be a "shift" operator, which acts on expressions with the discrete variable n:

```
S:EXPR(INT)->EXPR(INT):=e+->eval(e,[n],[n+1])
```

Define DF to be a "difference" operator, which acts on expressions with a discrete variable n:

```
DF:EXPR(INT)->EXPR(INT):=e+->eval(e,[n],[n+1])-e
```

```
Then define the difference operator algebra D0:

   D0:=OREUP('D,EXPR(INT),morphism S,DF)

   u:=(operator 'u)[n]

   L:D0:='D+u

        D + u(n)

   L^2

         2                2
        D  + 2u(n)D + u(n)

--------------------------------------------------------------------

   )clear all

As a fourth example, we construct a skew polynomial ring by using an
inner derivation \delta induced by a fixed y in R:

   \delta(r) = yr - ry

First we should expose the constructor SquareMatrix so it is visible
in the interpreter:

   )set expose add constructor SquareMatrix

Define R to be the square matrix with integer entries:

   R:=SQMATRIX(2,INT)

   y:R:=matrix [[1,1],[0,1]]
        +1  1+
        |    |
        +0  1+

Define the inner derivative \delta:

   delta:R->R:=r+->y*r-r*y

Define S to be a skew polynomial determined by \sigma = 1
and \delta as an inner derivative:

   S:=OREUP('x,R,1,delta)

   x:S:='x

   a:S:=matrix [[2,3],[1,1]]
```

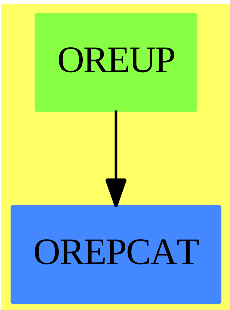
```
        +2  3+
        |    |
        +1  1+

   x^2*a
        +2  3+ 2   +2  - 2+    +0  - 2+
        |    |x  + |       |x + |      |
        +1  1+     +0  - 2+    +0   0 +


See Also:
o )show UnivariateSkewPolynomial
o )show UnivariateSkewPolynomialCategory
o )show SquareMatrix
```

—————-

22.8.1 UnivariateSkewPolynomial (OREUP)



See

⇒ "Automorphism" (AUTOMOR) 2.44.1 on page 228
⇒ "SparseUnivariateSkewPolynomial" (ORESUP) 20.21.1 on page 2450

Exports:

0	1	apply
characteristic	coefficient	coefficients
coerce	content	degree
exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient
leftExtendedGcd	leftGcd	leftLcm
leftQuotient	leftRemainder	minimumDegree
monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip
reductum	retract	retractIfCan
rightDivide	rightExactQuotient	rightExtendedGcd
rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?

— domain OREUP UnivariateSkewPolynomial —

```
)abbrev domain OREUP UnivariateSkewPolynomial
++ Author: Manuel Bronstein
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ Description:
++ This is the domain of univariate skew polynomials over an Ore
++ coefficient field in a named variable.
++ The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.

UnivariateSkewPolynomial(x:Symbol,R:Ring,sigma:Automorphism R,delta: R -> R):
 UnivariateSkewPolynomialCategory R with
   coerce: Variable x -> %
     ++ coerce(x) returns x as a skew-polynomial.
  == SparseUnivariateSkewPolynomial(R, sigma, delta) add
     Rep := SparseUnivariateSkewPolynomial(R, sigma, delta)
     coerce(v:Variable(x)):% == monomial(1, 1)
     coerce(p:%):OutputForm  == outputForm(p, outputForm x)$Rep
```

————

— OREUP.dotabb —

```
"OREUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OREUP"]
"OREPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OREPCAT"]
"OREUP" -> "OREPCAT"
```

————

22.9 domain UTS UnivariateTaylorSeries

— UnivariateTaylorSeries.input —

```
)set break resume
)sys rm -f UnivariateTaylorSeries.output
)spool UnivariateTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateTaylorSeries
--R UnivariateTaylorSeries(Coef: Ring,var: Symbol,cen: Coef)  is a domain constructor
--R Abbreviation for UnivariateTaylorSeries is UTS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UTS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,PositiveInteger) -> %       center : % -> Coef
--R coefficients : % -> Stream Coef      coerce : Variable var -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R complete : % -> %                    degree : % -> NonNegativeInteger
--R evenlambert : % -> %                 hash : % -> SingleInteger
--R lagrange : % -> %                    lambert : % -> %
--R latex : % -> String                  leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean             oddlambert : % -> %
--R one? : % -> Boolean                  order : % -> NonNegativeInteger
--R pole? : % -> Boolean                 quoByVar : % -> %
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R revert : % -> %                      sample : () -> %
--R series : Stream Coef -> %            variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Coef) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
```

```
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING
--R D : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PD
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef a
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef)
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) ->
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(var,Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> C
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coe
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef an
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Co
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegative
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R generalLambert : (%,Integer,Integer) -> %
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has va
```

```
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R invmultisect : (Integer,Integer,%) -> %
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R multisect : (Integer,Integer,%) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R univariatePolynomial : (%,NonNegativeInteger) -> UnivariatePolynomial(var,Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———

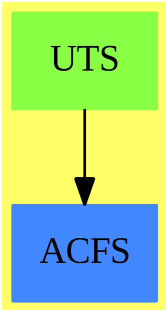
— UnivariateTaylorSeries.help —

```
====================================================================
UnivariateTaylorSeries examples
====================================================================

See Also:
o )show UnivariateTaylorSeries
```

———

22.9.1 UnivariateTaylorSeries (UTS)



See

⇒ "InnerTaylorSeries" (ITAYLOR) 10.28.1 on page 1302

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos
cosh	cot	coth	csc
csch	D	degree	differentiate
eval	evenlambert	exp	exquo
extend	generalLambert	hash	integrate
invmultisect	lagrange	lambert	latex
leadingCoefficient	leadingMonomial	log	map
monomial	monomial?	multiplyCoefficients	multiplyExponents
multisect	nthRoot	oddlambert	one?
order	pi	pole?	polynomial
quoByVar	recip	reductum	revert
sample	sec	sech	series
sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate
unit?	unitCanonical	unitNormal	univariatePolynomial
variable	variables	zero?	?*?
?**?	?+?	?-?	-?
?=?	?^?	?~=?	?.?

— domain UTS UnivariateTaylorSeries —

```
)abbrev domain UTS UnivariateTaylorSeries
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 21 September 1993
```

```
++ Basic Operations:
++ Related Domains: UnivariateLaurentSeries(Coef,var,cen),
++ UnivariatePuiseuxSeries(Coef,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: dense, Taylor series
++ Examples:
++ References:
++ Description:
++ Dense Taylor series in one variable
++ \spadtype{UnivariateTaylorSeries} is a domain representing Taylor
++ series in
++ one variable with coefficients in an arbitrary ring.  The parameters
++ of the type specify the coefficient ring, the power series variable,
++ and the center of the power series expansion.  For example,
++ \spadtype{UnivariateTaylorSeries}(Integer,x,3) represents
++ Taylor series in
++ \spad{(x - 3)} with \spadtype{Integer} coefficients.

UnivariateTaylorSeries(Coef,var,cen): Exports == Implementation where
  Coef :  Ring
  var  :  Symbol
  cen  :  Coef
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  P    ==> Polynomial Coef
  RN   ==> Fraction Integer
  ST   ==> Stream
  STT  ==> StreamTaylorSeriesOperations Coef
  TERM ==> Record(k:NNI,c:Coef)
  UP   ==> UnivariatePolynomial(var,Coef)
  Exports ==> UnivariateTaylorSeriesCategory(Coef) with
    coerce: UP -> %
      ++\spad{coerce(p)} converts a univariate polynomial p in the variable
      ++\spad{var} to a univariate Taylor series in \spad{var}.
    univariatePolynomial: (%,NNI) -> UP
      ++\spad{univariatePolynomial(f,k)} returns a univariate polynomial
      ++ consisting of the sum of all terms of f of degree \spad{<= k}.
    coerce: Variable(var) -> %
      ++\spad{coerce(var)} converts the series variable \spad{var} into a
      ++ Taylor series.
    differentiate: (%,Variable(var)) -> %
      ++ \spad{differentiate(f(x),x)} computes the derivative of
      ++ \spad{f(x)} with respect to \spad{x}.
    lagrange: % -> %
      ++\spad{lagrange(g(x))} produces the Taylor series for \spad{f(x)}
      ++ where \spad{f(x)} is implicitly defined as \spad{f(x) = x*g(f(x))}.
    lambert: % -> %
      ++\spad{lambert(f(x))} returns \spad{f(x) + f(x^2) + f(x^3) + ...}.
      ++ This function is used for computing infinite products.
```

```
      ++ \spad{f(x)} should have zero constant coefficient.
      ++ If \spad{f(x)} is a Taylor series with constant term 1, then
      ++ \spad{product(n = 1..infinity,f(x^n)) = exp(log(lambert(f(x))))}.
    oddlambert: % -> %
      ++\spad{oddlambert(f(x))} returns \spad{f(x) + f(x^3) + f(x^5) + ...}.
      ++ \spad{f(x)} should have a zero constant coefficient.
      ++ This function is used for computing infinite products.
      ++ If \spad{f(x)} is a Taylor series with constant term 1, then
      ++ \spad{product(n=1..infinity,f(x^(2*n-1)))=exp(log(oddlambert(f(x))))}.
    evenlambert: % -> %
      ++\spad{evenlambert(f(x))} returns \spad{f(x^2) + f(x^4) + f(x^6) + ...}.
      ++ \spad{f(x)} should have a zero constant coefficient.
      ++ This function is used for computing infinite products.
      ++ If \spad{f(x)} is a Taylor series with constant term 1, then
      ++ \spad{product(n=1..infinity,f(x^(2*n))) = exp(log(evenlambert(f(x))))}.
    generalLambert: (%,I,I) -> %
      ++\spad{generalLambert(f(x),a,d)} returns \spad{f(x^a) + f(x^(a + d)) +
      ++ f(x^(a + 2 d)) + ... }. \spad{f(x)} should have zero constant
      ++ coefficient and \spad{a} and d should be positive.
    revert: % -> %
      ++ \spad{revert(f(x))} returns a Taylor series \spad{g(x)} such that
      ++ \spad{f(g(x)) = g(f(x)) = x}. Series \spad{f(x)} should have constant
      ++ coefficient 0 and 1st order coefficient 1.
    multisect: (I,I,%) -> %
      ++\spad{multisect(a,b,f(x))} selects the coefficients of
      ++ \spad{x^((a+b)*n+a)}, and changes this monomial to \spad{x^n}.
    invmultisect: (I,I,%) -> %
      ++\spad{invmultisect(a,b,f(x))} substitutes \spad{x^((a+b)*n)}
      ++ for \spad{x^n} and multiples by \spad{x^b}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x),x)} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
        ++ by integers.

  Implementation ==> InnerTaylorSeries(Coef) add

    Rep := Stream Coef

--% creation and destruction of series

    stream: % -> Stream Coef
    stream x  == x pretend Stream(Coef)

    coerce(v:Variable(var)) ==
      zero? cen => monomial(1,1)
      monomial(1,1) + monomial(cen,0)

    coerce(n:I)    == n :: Coef :: %
```

```
coerce(r:Coef) == coerce(r)$STT
monomial(c,n)  == monom(c,n)$STT

getExpon: TERM -> NNI
getExpon term == term.k
getCoef: TERM -> Coef
getCoef term == term.c
rec: (NNI,Coef) -> TERM
rec(expon,coef) == [expon,coef]

recs: (ST Coef,NNI) -> ST TERM
recs(st,n) == delay$ST(TERM)
  empty? st => empty()
  zero? (coef := frst st) => recs(rst st,n + 1)
  concat(rec(n,coef),recs(rst st,n + 1))

terms x == recs(stream x,0)

recsToCoefs: (ST TERM,NNI) -> ST Coef
recsToCoefs(st,n) == delay
  empty? st => empty()
  term := frst st; expon := getExpon term
  n = expon => concat(getCoef term,recsToCoefs(rst st,n + 1))
  concat(0,recsToCoefs(st,n + 1))

series(st: ST TERM) == recsToCoefs(st,0)

stToPoly: (ST Coef,P,NNI,NNI) -> P
stToPoly(st,term,n,n0) ==
  (n > n0) or (empty? st) => 0
  frst(st) * term ** n + stToPoly(rst st,term,n + 1,n0)

polynomial(x,n) == stToPoly(stream x,(var :: P) - (cen :: P),0,n)

polynomial(x,n1,n2) ==
  if n1 > n2 then (n1,n2) := (n2,n1)
  stToPoly(rest(stream x,n1),(var :: P) - (cen :: P),n1,n2)

stToUPoly: (ST Coef,UP,NNI,NNI) -> UP
stToUPoly(st,term,n,n0) ==
  (n > n0) or (empty? st) => 0
  frst(st) * term ** n + stToUPoly(rst st,term,n + 1,n0)

univariatePolynomial(x,n) ==
  stToUPoly(stream x,monomial(1,1)$UP - monomial(cen,0)$UP,0,n)

coerce(p:UP) ==
  zero? p => 0
  if not zero? cen then
    p := p(monomial(1,1)$UP + monomial(cen,0)$UP)
```

```
      st : ST Coef := empty()
      oldDeg : NNI := degree(p) + 1
      while not zero? p repeat
        deg := degree p
        delta := (oldDeg - deg - 1) :: NNI
        for i in 1..delta repeat st := concat(0$Coef,st)
        st := concat(leadingCoefficient p,st)
        oldDeg := deg; p := reductum p
      for i in 1..oldDeg repeat st := concat(0$Coef,st)
      st

    if Coef has coerce: Symbol -> Coef then
      if Coef has "**": (Coef,NNI) -> Coef then

        stToCoef: (ST Coef,Coef,NNI,NNI) -> Coef
        stToCoef(st,term,n,n0) ==
          (n > n0) or (empty? st) => 0
          frst(st) * term ** n + stToCoef(rst st,term,n + 1,n0)

        approximate(x,n) ==
          stToCoef(stream x,(var :: Coef) - cen,0,n)

--% values

    variable x == var
    center   s == cen

    coefficient(x,n) ==
       -- Cannot use elt!  Should return 0 if stream doesn't have it.
       u := stream x
       while not empty? u and n > 0 repeat
         u := rst u
         n := (n - 1) :: NNI
       empty? u or n ^= 0 => 0
       frst u

    elt(x:%,n:NNI) == coefficient(x,n)

--% functions

    map(f,x) == map(f,x)$Rep
    eval(x:%,r:Coef) == eval(stream x,r-cen)$STT
    differentiate x == deriv(stream x)$STT
    differentiate(x:%,v:Variable(var)) == differentiate x
    if Coef has PartialDifferentialRing(Symbol) then
      differentiate(x:%,s:Symbol) ==
        (s = variable(x)) => differentiate x
        map(y +-> differentiate(y,s),x)
              - differentiate(center x,s)*differentiate(x)
    multiplyCoefficients(f,x) == gderiv(f,stream x)$STT
```

```
lagrange x == lagrange(stream x)$STT
lambert x == lambert(stream x)$STT
oddlambert x == oddlambert(stream x)$STT
evenlambert x == evenlambert(stream x)$STT
generalLambert(x:%,a:I,d:I) == generalLambert(stream x,a,d)$STT
extend(x,n) == extend(x,n+1)$Rep
complete x == complete(x)$Rep
truncate(x,n) == first(stream x,n + 1)$Rep
truncate(x,n1,n2) ==
  if n2 < n1 then (n1,n2) := (n2,n1)
  m := (n2 - n1) :: NNI
  st := first(rest(stream x,n1)$Rep,m + 1)$Rep
  for i in 1..n1 repeat st := concat(0$Coef,st)
  st
elt(x:%,y:%) == compose(stream x,stream y)$STT
revert x == revert(stream x)$STT
multisect(a,b,x) == multisect(a,b,stream x)$STT
invmultisect(a,b,x) == invmultisect(a,b,stream x)$STT
multiplyExponents(x,n) == invmultisect(n,0,x)
quoByVar x == (empty? x => 0; rst x)
if Coef has IntegralDomain then
  unit? x == unit? coefficient(x,0)
if Coef has Field then
  if Coef is RN then
    (x:%) ** (s:Coef) == powern(s,stream x)$STT
  else
    (x:%) ** (s:Coef) == power(s,stream x)$STT

if Coef has Algebra Fraction Integer then
  coerce(r:RN) == r :: Coef :: %

  integrate x == integrate(0,stream x)$STT
  integrate(x:%,v:Variable(var)) == integrate x

  if Coef has integrate: (Coef,Symbol) -> Coef and _
     Coef has variables: Coef -> List Symbol then
    integrate(x:%,s:Symbol) ==
      (s = variable(x)) => integrate x
      not entry?(s,variables center x) => map(y +-> integrate(y,s),x)
      error "integrate: center is a function of variable of integration"

  if Coef has TranscendentalFunctionCategory and _
     Coef has PrimitiveFunctionCategory and _
     Coef has AlgebraicallyClosedFunctionSpace Integer then

    integrateWithOneAnswer: (Coef,Symbol) -> Coef
    integrateWithOneAnswer(f,s) ==
      res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
      res case Coef => res :: Coef
      first(res :: List Coef)
```

```
        integrate(x:%,s:Symbol) ==
          (s = variable(x)) => integrate x
          not entry?(s,variables center x) =>
            map(y +-> integrateWithOneAnswer(y,s),x)
          error "integrate: center is a function of variable of integration"

--% OutputForms
--  We use the default coerce: % -> OutputForm in UTSCAT&
```

———-

— UTS.dotabb —

```
"UTS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UTS"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UTS" -> "ACFS"
```

———-

22.10 domain UTSZ UnivariateTaylorSeriesCZero

— UnivariateTaylorSeriesCZero.input —

```
)set break resume
)sys rm -f UnivariateTaylorSeriesCZero.output
)spool UnivariateTaylorSeriesCZero.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateTaylorSeriesCZero
--R UnivariateTaylorSeriesCZero(Coef: Ring,var: Symbol)  is a domain constructor
--R Abbreviation for UnivariateTaylorSeriesCZero is UTSZ
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UTSZ
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                    ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %         ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                              0 : () -> %
--R ?^? : (%,PositiveInteger) -> %           center : % -> Coef
--R coefficients : % -> Stream Coef          coerce : Variable var -> %
--R coerce : Integer -> %                    coerce : % -> OutputForm
--R complete : % -> %                        degree : % -> NonNegativeInteger
--R evenlambert : % -> %                     hash : % -> SingleInteger
--R lagrange : % -> %                        lambert : % -> %
--R latex : % -> String                      leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %                 map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean                 oddlambert : % -> %
--R one? : % -> Boolean                      order : % -> NonNegativeInteger
--R pole? : % -> Boolean                     quoByVar : % -> %
--R recip : % -> Union(%,"failed")           reductum : % -> %
--R revert : % -> %                          sample : () -> %
--R series : Stream Coef -> %                variable : % -> Symbol
--R zero? : % -> Boolean                     ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ?**? : (%,Coef) -> % if Coef has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef an
--R ?^? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef and C
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(var,Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
```

```
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> C
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coe
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef an
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Co
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegative
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R generalLambert : (%,Integer,Integer) -> %
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has va
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R invmultisect : (Integer,Integer,%) -> %
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R multisect : (Integer,Integer,%) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
```

```
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R univariatePolynomial : (%,NonNegativeInteger) -> UnivariatePolynomial(var,Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)
```

———-

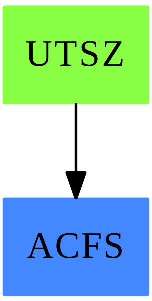
— **UnivariateTaylorSeriesCZero.help** —

```
====================================================================
UnivariateTaylorSeriesCZero examples
====================================================================

See Also:
o )show UnivariateTaylorSeriesCZero
```

———-

22.10.1 UnivariateTaylorSeriesCZero (UTSZ)



See

⇒ "InnerTaylorSeries" (ITAYLOR) 10.28.1 on page 1302

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos
cosh	cot	coth	csc
csch	D	degree	differentiate
eval	evenlambert	exp	exquo
extend	generalLambert	hash	integrate
invmultisect	lagrange	lambert	latex
leadingCoefficient	leadingMonomial	log	map
monomial	monomial?	multiplyCoefficients	multiplyExponents
multisect	nthRoot	oddlambert	one?
order	pi	pole?	polynomial
quoByVar	recip	reductum	revert
sample	sec	sech	series
sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate
unit?	unitCanonical	unitNormal	univariatePolynomial
variable	variables	zero?	?.?
?~=?	?/?	?*?	?**?
?+?	?-?	-?	?=?
?^?			

— domain UTSZ UnivariateTaylorSeriesCZero —

```
)abbrev domain UTSZ UnivariateTaylorSeriesCZero
++ Author: Gaetan Hache
++ Date Created: September 1996
++ Date Last Updated: April, 2010, by Tim Daly
++ Description:
++ Part of the Package for Algebraic Function Fields in one variable PAFF

UnivariateTaylorSeriesCZero(Coef,var): Exports == Implementation where
  Coef :  Ring
  var  :  Symbol
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  P    ==> Polynomial Coef
  RN   ==> Fraction Integer
  ST   ==> Stream
  STT  ==> StreamTaylorSeriesOperations Coef
  TERM ==> Record(k:NNI,c:Coef)
  UP   ==> UnivariatePolynomial(var,Coef)
  Exports ==> UnivariateTaylorSeriesCategory(Coef) with
    coerce: UP -> %
```

```
    ++\spad{coerce(p)} converts a univariate polynomial p in the variable
    ++\spad{var} to a univariate Taylor series in \spad{var}.
  univariatePolynomial: (%,NNI) -> UP
    ++\spad{univariatePolynomial(f,k)} returns a univariate polynomial
    ++ consisting of the sum of all terms of f of degree \spad{<= k}.
  coerce: Variable(var) -> %
    ++\spad{coerce(var)} converts the series variable \spad{var} into a
    ++ Taylor series.
  differentiate: (%,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} computes the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  lagrange: % -> %
    ++\spad{lagrange(g(x))} produces the Taylor series for \spad{f(x)}
    ++ where \spad{f(x)} is implicitly defined as \spad{f(x) = x*g(f(x))}.
  lambert: % -> %
    ++\spad{lambert(f(x))} returns \spad{f(x) + f(x^2) + f(x^3) + ...}.
    ++ This function is used for computing infinite products.
    ++ \spad{f(x)} should have zero constant coefficient.
    ++ If \spad{f(x)} is a Taylor series with constant term 1, then
    ++ \spad{product(n = 1..infinity,f(x^n)) = exp(log(lambert(f(x))))}.
  oddlambert: % -> %
    ++\spad{oddlambert(f(x))} returns \spad{f(x) + f(x^3) + f(x^5) + ...}.
    ++ \spad{f(x)} should have a zero constant coefficient.
    ++ This function is used for computing infinite products.
    ++ If \spad{f(x)} is a Taylor series with constant term 1, then
    ++ \spad{product(n=1..infinity,f(x^(2*n-1)))=exp(log(oddlambert(f(x))))}.
  evenlambert: % -> %
    ++\spad{evenlambert(f(x))} returns \spad{f(x^2) + f(x^4) + f(x^6) + ...}.
    ++ \spad{f(x)} should have a zero constant coefficient.
    ++ This function is used for computing infinite products.
    ++ If \spad{f(x)} is a Taylor series with constant term 1, then
    ++ \spad{product(n=1..infinity,f(x^(2*n)))=exp(log(evenlambert(f(x))))}.
  generalLambert: (%,I,I) -> %
    ++\spad{generalLambert(f(x),a,d)} returns \spad{f(x^a) + f(x^(a + d)) +
    ++ f(x^(a + 2 d)) + ... }. \spad{f(x)} should have zero constant
    ++ coefficient and \spad{a} and d should be positive.
  revert: % -> %
    ++ \spad{revert(f(x))} returns a Taylor series \spad{g(x)} such that
    ++ \spad{f(g(x)) = g(f(x)) = x}. Series \spad{f(x)} should have constant
    ++ coefficient 0 and 1st order coefficient 1.
  multisect: (I,I,%) -> %
    ++\spad{multisect(a,b,f(x))} selects the coefficients of
    ++ \spad{x^((a+b)*n+a)}, and changes this monomial to \spad{x^n}.
  invmultisect: (I,I,%) -> %
    ++\spad{invmultisect(a,b,f(x))} substitutes \spad{x^((a+b)*n)}
    ++ for \spad{x^n} and multiples by \spad{x^b}.
  if Coef has Algebra Fraction Integer then
    integrate: (%,Variable(var)) -> %
      ++ \spad{integrate(f(x),x)} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
```

```
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.

  Implementation ==> InnerTaylorSeries(Coef) add

    Rep := Stream Coef

    --% creation and destruction of series

    stream: % -> Stream Coef
    stream x  == x pretend Stream(Coef)

    coerce(v:Variable(var)) ==
      monomial(1,1)

    coerce(n:I)    == n :: Coef :: %
    coerce(r:Coef) == coerce(r)$STT
    monomial(c,n)  == monom(c,n)$STT

    getExpon: TERM -> NNI
    getExpon term == term.k
    getCoef: TERM -> Coef
    getCoef term == term.c
    rec: (NNI,Coef) -> TERM
    rec(expon,coef) == [expon,coef]

    recs: (ST Coef,NNI) -> ST TERM
    recs(st,n) == delay$ST(TERM)
      empty? st => empty()
      zero? (coef := frst st) => recs(rst st,n + 1)
      concat(rec(n,coef),recs(rst st,n + 1))

    terms x == recs(stream x,0)

    recsToCoefs: (ST TERM,NNI) -> ST Coef
    recsToCoefs(st,n) == delay
      empty? st => empty()
      term := frst st; expon := getExpon term
      n = expon => concat(getCoef term,recsToCoefs(rst st,n + 1))
      concat(0,recsToCoefs(st,n + 1))

    series(st: ST TERM) == recsToCoefs(st,0)

    stToPoly: (ST Coef,P,NNI,NNI) -> P
    stToPoly(st,term,n,n0) ==
      (n > n0) or (empty? st) => 0
      frst(st) * term ** n + stToPoly(rst st,term,n + 1,n0)

    polynomial(x,n) == stToPoly(stream x,(var :: P),0,n)
```

```
    polynomial(x,n1,n2) ==
      if n1 > n2 then (n1,n2) := (n2,n1)
      stToPoly(rest(stream x,n1),(var :: P),n1,n2)

    stToUPoly: (ST Coef,UP,NNI,NNI) -> UP
    stToUPoly(st,term,n,n0) ==
      (n > n0) or (empty? st) => 0
      frst(st) * term ** n + stToUPoly(rst st,term,n + 1,n0)

    univariatePolynomial(x,n) ==
      stToUPoly(stream x,monomial(1,1)$UP,0,n)

    coerce(p:UP) ==
      zero? p => 0
      st : ST Coef := empty()
      oldDeg : NNI := degree(p) + 1
      while not zero? p repeat
        deg := degree p
        delta := (oldDeg - deg - 1) :: NNI
        for i in 1..delta repeat st := concat(0$Coef,st)
        st := concat(leadingCoefficient p,st)
        oldDeg := deg; p := reductum p
      for i in 1..oldDeg repeat st := concat(0$Coef,st)
      st

    if Coef has coerce: Symbol -> Coef then
      if Coef has "**": (Coef,NNI) -> Coef then

        stToCoef: (ST Coef,Coef,NNI,NNI) -> Coef
        stToCoef(st,term,n,n0) ==
          (n > n0) or (empty? st) => 0
          frst(st) * term ** n + stToCoef(rst st,term,n + 1,n0)

        approximate(x,n) ==
          stToCoef(stream x,(var :: Coef),0,n)

--% values

    variable x == var
    center   x == 0$Coef

    coefficient(x,n) ==
       -- Cannot use elt!  Should return 0 if stream doesn't have it.
       u := stream x
       while not empty? u and n > 0 repeat
         u := rst u
         n := (n - 1) :: NNI
       empty? u or n ^= 0 => 0
       frst u
```

```
    elt(x:%,n:NNI) == coefficient(x,n)

--% functions

    map(f,x) == map(f,x)$Rep
    eval(x:%,r:Coef) == eval(stream x,r)$STT
    differentiate x == deriv(stream x)$STT
    differentiate(x:%,v:Variable(var)) == differentiate x
    if Coef has PartialDifferentialRing(Symbol) then
      differentiate(x:%,s:Symbol) ==
        (s = variable(x)) => differentiate x
        map(differentiate(#1,s),x) - differentiate(0,s)*differentiate(x)
    multiplyCoefficients(f,x) == gderiv(f,stream x)$STT
    lagrange x == lagrange(stream x)$STT
    lambert x == lambert(stream x)$STT
    oddlambert x == oddlambert(stream x)$STT
    evenlambert x == evenlambert(stream x)$STT
    generalLambert(x:%,a:I,d:I) == generalLambert(stream x,a,d)$STT
    extend(x,n) == extend(x,n+1)$Rep
    complete x == complete(x)$Rep
    truncate(x,n) == first(stream x,n + 1)$Rep
    truncate(x,n1,n2) ==
      if n2 < n1 then (n1,n2) := (n2,n1)
      m := (n2 - n1) :: NNI
      st := first(rest(stream x,n1)$Rep,m + 1)$Rep
      for i in 1..n1 repeat st := concat(0$Coef,st)
      st
    elt(x:%,y:%) == compose(stream x,stream y)$STT
    revert x == revert(stream x)$STT
    multisect(a,b,x) == multisect(a,b,stream x)$STT
    invmultisect(a,b,x) == invmultisect(a,b,stream x)$STT
    multiplyExponents(x,n) == invmultisect(n,0,x)
    quoByVar x == (empty? x => 0; rst x)
    if Coef has IntegralDomain then
      unit? x == unit? coefficient(x,0)
    if Coef has Field then
      if Coef is RN then
        (x:%) ** (s:Coef) == powern(s,stream x)$STT
      else
        (x:%) ** (s:Coef) == power(s,stream x)$STT

    if Coef has Algebra Fraction Integer then
      coerce(r:RN) == r :: Coef :: %

      integrate x == integrate(0,stream x)$STT
      integrate(x:%,v:Variable(var)) == integrate x

      if Coef has integrate: (Coef,Symbol) -> Coef and _
         Coef has variables: Coef -> List Symbol then
        integrate(x:%,s:Symbol) ==
```

```
          (s = variable(x)) => integrate x
          map(integrate(#1,s),x)

      if Coef has TranscendentalFunctionCategory and _
         Coef has PrimitiveFunctionCategory and _
         Coef has AlgebraicallyClosedFunctionSpace Integer then

        integrateWithOneAnswer: (Coef,Symbol) -> Coef
        integrateWithOneAnswer(f,s) ==
          res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
          res case Coef => res :: Coef
          first(res :: List Coef)

        integrate(x:%,s:Symbol) ==
          (s = variable(x)) => integrate x
          map(integrateWithOneAnswer(#1,s),x)
```

— **UTSZ.dotabb** —

```
"UTSZ" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UTSZ"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UTSZ" -> "ACFS"
```

22.11 domain UNISEG UniversalSegment

— **UniversalSegment.input** —

```
)set break resume
)sys rm -f UniversalSegment.output
)spool UniversalSegment.output
)set message test on
)set message auto off
)clear all
--S 1 of 9
pints := 1..
--R
--R
--R   (1)  1..
--R                                       Type: UniversalSegment PositiveInteger
--E 1
```

```
--S 2 of 9
nevens := (0..) by -2
--R
--R
--R   (2)  0.. by - 2
--R                                     Type: UniversalSegment NonNegativeInteger
--E 2

--S 3 of 9
useg: UniversalSegment(Integer) := 3..10
--R
--R
--R   (3)  3..10
--R                                                Type: UniversalSegment Integer
--E 3

--S 4 of 9
hasHi pints
--R
--R
--R   (4)  false
--R                                                                 Type: Boolean
--E 4

--S 5 of 9
hasHi nevens
--R
--R
--R   (5)  false
--R                                                                 Type: Boolean
--E 5

--S 6 of 9
hasHi useg
--R
--R
--R   (6)  true
--R                                                                 Type: Boolean
--E 6

--S 7 of 9
expand pints
--R
--R
--R   (7)  [1,2,3,4,5,6,7,8,9,10,...]
--R                                                          Type: Stream Integer
--E 7

--S 8 of 9
```

```
expand nevens
--R
--R
--R   (8)  [0,- 2,- 4,- 6,- 8,- 10,- 12,- 14,- 16,- 18,...]
--R                                                        Type: Stream Integer
--E 8

--S 9 of 9
expand [1, 3, 10..15, 100..]
--R
--R
--R   (9)  [1,3,10,11,12,13,14,15,100,101,...]
--R                                                        Type: Stream Integer
--E 9
)spool
)lisp (bye)
```

———-

— UniversalSegment.help —

```
====================================================================
UniversalSegment examples
====================================================================

The UniversalSegment domain generalizes Segment by allowing segments
without a "hi" end point.

  pints := 1..
    1..
                                Type: UniversalSegment PositiveInteger

  nevens := (0..) by -2
    0.. by - 2
                                Type: UniversalSegment NonNegativeInteger

Values of type Segment are automatically converted to type
UniversalSegment when appropriate.

  useg: UniversalSegment(Integer) := 3..10
    3..10
                                Type: UniversalSegment Integer

The operation hasHi is used to test whether a segment has a hi end point.

  hasHi pints
    false
                                Type: Boolean
```

```
  hasHi nevens
    false
                                Type: Boolean

  hasHi useg
    true
                                Type: Boolean
```

All operations available on type Segment apply to UniversalSegment, with the proviso that expansions produce streams rather than lists. This is to accommodate infinite expansions.

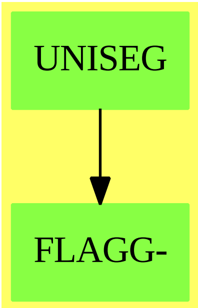
```
  expand pints
    [1,2,3,4,5,6,7,8,9,10,...]
                                Type: Stream Integer

  expand nevens
    [0,- 2,- 4,- 6,- 8,- 10,- 12,- 14,- 16,- 18,...]
                                Type: Stream Integer

  expand [1, 3, 10..15, 100..]
    [1,3,10,11,12,13,14,15,100,101,...]
                                Type: Stream Integer
```

See Also:
o)help Segment
o)help SegmentBinding
o)help List
o)help Stream
o)show UniversalSegment

22.11.1 UniversalSegment (UNISEG)



See

⇒ "Segment" (SEG) 20.2.1 on page 2319
⇒ "SegmentBinding" (SEGBIND) 20.3.1 on page 2324

Exports:

BY	coerce	convert	expand	hasHi
hash	hi	high	incr	latex
lo	low	map	segment	segment
?=?	?SEGMENT	?..?	?~=?	

— **domain UNISEG UniversalSegment** —

```
)abbrev domain UNISEG UniversalSegment
++ Author:  Robert S. Sutor
++ Date Created: 1987
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: Segment
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This domain provides segments which may be half open.
++ That is, ranges of the form \spad{a..} or \spad{a..b}.

UniversalSegment(S: Type): SegmentCategory(S) with
    SEGMENT: S -> %
        ++ \spad{l..} produces a half open segment,
        ++ that is, one with no upper bound.
    segment: S -> %
        ++ segment(l) is an alternate way to construct the segment \spad{l..}.
    coerce : Segment S -> %
        ++ coerce(x) allows \spadtype{Segment} values to be used as %.
    hasHi: % -> Boolean
```

```
        ++ hasHi(s) tests whether the segment s has an upper bound.

    if S has SetCategory then SetCategory

    if S has OrderedRing then
      SegmentExpansionCategory(S, Stream S)
--    expand : (List %, S) -> Stream S
--    expand : (%, S) -> Stream S

  == add
    Rec  ==> Record(low: S, high: S, incr: Integer)
    Rec2 ==> Record(low: S, incr: Integer)
    SEG ==> Segment S

    Rep := Union(Rec2, Rec)
    a,b : S
    s : %
    i: Integer
    ls : List %

    segment a == [a, 1]$Rec2 :: Rep
    segment(a,b) == [a,b,1]$Rec :: Rep
    BY(s,i) ==
      s case Rec => [lo s, hi s, i]$Rec ::Rep
      [lo s, i]$Rec2 :: Rep

    lo s ==
      s case Rec2 => (s :: Rec2).low
      (s :: Rec).low

    low s ==
      s case Rec2 => (s :: Rec2).low
      (s :: Rec).low

    hasHi s == s case Rec

    hi s ==
      not hasHi(s) => error "hi: segment has no upper bound"
      (s :: Rec).high

    high s ==
      not hasHi(s) => error "high: segment has no upper bound"
      (s :: Rec).high

    incr s ==
      s case Rec2 => (s :: Rec2).incr
      (s :: Rec).incr

    SEGMENT(a) == segment a
    SEGMENT(a,b) == segment(a,b)
```

```
coerce(sg : SEG): % == segment(lo sg, hi sg)

convert a == [a,a,1]

if S has SetCategory then

   (s1:%) = (s2:%) ==
      s1 case Rec2 =>
         s2 case Rec2 =>
             s1.low = s2.low and s1.incr = s2.incr
         false
      s1 case Rec =>
         s2 case Rec =>
             s2.low = s2.low and s1.high=s2.high and s1.incr=s2.incr
         false
      false

   coerce(s: %): OutputForm ==
     seg :=
       e := (lo s)::OutputForm
       hasHi s => SEGMENT(e, (hi s)::OutputForm)
       SEGMENT e
     inc := incr s
     inc = 1 => seg
     infix(" by "::OutputForm, seg, inc::OutputForm)

if S has OrderedRing then
  expand(s:%)       == expand([s])
  map(f:S->S, s:%)  == map(f, expand s)

  plusInc(t: S, a: S): S == t + a

  expand(ls: List %):Stream S ==
    st:Stream S := empty()
    null ls => st

    lb:List(Segment S) := nil()
    while not null ls and hasHi first ls repeat
        s  := first ls
        ls := rest ls
        ns := BY(SEGMENT(lo s, hi s), incr s)$Segment(S)
        lb := concat_!(lb,ns)
    if not null ls then
        s := first ls
        st: Stream S := generate(x +-> x+incr(s)::S, lo s)
    else
        st: Stream S := empty()
    concat(construct expand(lb),  st)
```

———-

— UNISEG.dotabb —

```
"UNISEG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UNISEG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"UNISEG" -> "FLAGG-"
```

———-

22.12 domain U32VEC U32Vector

— U32Vector.input —

```
)set break resume
)sys rm -f U32Vector.output
)spool U32Vector.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show U32Vector
--R U32Vector  is a domain constructor
--R Abbreviation for U32Vector is U32VEC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for U32VEC
--R
--R------------------------------- Operations --------------------------------
--R concat : List % -> %                  concat : (%,%) -> %
--R concat : (Integer,%) -> %             concat : (%,Integer) -> %
--R construct : List Integer -> %         copy : % -> %
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> Integer
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List Integer           eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List Integer
--R insert : (%,%,Integer) -> %           qelt : (%,Integer) -> Integer
--R reverse : % -> %                      sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Integer has ORDSET
--R ?<=? : (%,%) -> Boolean if Integer has ORDSET
--R ?=? : (%,%) -> Boolean if Integer has SETCAT
--R ?>? : (%,%) -> Boolean if Integer has ORDSET
--R ?>=? : (%,%) -> Boolean if Integer has ORDSET
--R any? : ((Integer -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```
--R coerce : % -> OutputForm if Integer has SETCAT
--R convert : % -> InputForm if Integer has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Integer,%) -> NonNegativeInteger if $ has finiteAggregate and Integer has SETCAT
--R count : ((Integer -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Integer) -> Integer
--R entry? : (Integer,%) -> Boolean if $ has finiteAggregate and Integer has SETCAT
--R eval : (%,List Integer,List Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R eval : (%,Integer,Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R eval : (%,Equation Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R eval : (%,List Equation Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R every? : ((Integer -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Integer) -> % if $ has shallowlyMutable
--R find : ((Integer -> Boolean),%) -> Union(Integer,"failed")
--R first : % -> Integer if Integer has ORDSET
--R hash : % -> SingleInteger if Integer has SETCAT
--R insert : (Integer,%,Integer) -> %
--R latex : % -> String if Integer has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Integer,Integer) -> Integer),%,%) -> %
--R map : ((Integer -> Integer),%) -> %
--R map! : ((Integer -> Integer),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if Integer has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Integer,%) -> Boolean if $ has finiteAggregate and Integer has SETCAT
--R members : % -> List Integer if $ has finiteAggregate
--R merge : (%,%) -> % if Integer has ORDSET
--R merge : (((Integer,Integer) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Integer has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Integer) -> %
--R parts : % -> List Integer if $ has finiteAggregate
--R position : (Integer,%,Integer) -> Integer if Integer has SETCAT
--R position : (Integer,%) -> Integer if Integer has SETCAT
--R position : ((Integer -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Integer) -> Integer if $ has shallowlyMutable
--R reduce : (((Integer,Integer) -> Integer),%) -> Integer if $ has finiteAggregate
--R reduce : (((Integer,Integer) -> Integer),%,Integer) -> Integer if $ has finiteAggregate
--R reduce : (((Integer,Integer) -> Integer),%,Integer,Integer) -> Integer if $ has finiteAggregate and
--R remove : ((Integer -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Integer,%) -> % if $ has finiteAggregate and Integer has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and Integer has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Integer -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Integer) -> Integer if $ has shallowlyMutable
--R setelt : (%,Integer,Integer) -> Integer if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
```

```
--R sort : % -> % if Integer has ORDSET
--R sort : (((Integer,Integer) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and Integer has ORDSET
--R sort! : (((Integer,Integer) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Integer has ORDSET
--R sorted? : (((Integer,Integer) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Integer has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

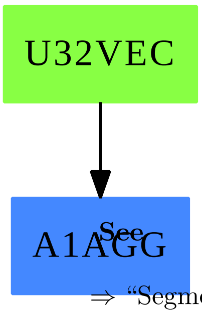
— **U32Vector.help** —

```
====================================================================
U32Vector examples
====================================================================

See Also:
o )show U32Vector
```

———-

22.12.1 U32Vector (U32VEC)



See

⇒ "Segment" (SEG) 20.2.1 on page 2319
⇒ "SegmentBinding" (SEGBIND) 20.3.1 on page 2324

Exports:

#?	?.?	?.?	?<=?	?<?
?=?	?>=?	?>?	?~=?	any?
coerce	concat	construct	convert	copy
copyInto!	count	delete	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	find	first	hash
index?	indices	insert	latex	less?
map	map!	max	maxIndex	member?
members	merge	min	minIndex	more?
new	parts	position	qelt	qsetelt!
reduce	remove	removeDuplicates	reverse	reverse!
sample	select	setelt	size?	sort
sort!	sorted?	swap!		

— domain U32VEC U32Vector —

```
)abbrev domain U32VEC U32Vector
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements vectors
++ (one dimensional arrays) of unsigned 32-bit numbers.  Indexing
++ is 0 based, there is no bound checking (unless provided by
++ lower level).
U32Vector() : OneDimensionalArrayAggregate Integer == add
   Qsize ==> QV32LEN$Lisp
   Qelt ==> ELT32$Lisp
   Qsetelt ==> SETELT32$Lisp
   Qnew ==> MAKE_-ARRAY32$Lisp

   #x                           == Qsize x
   minIndex x                   == 0
   empty()                      == Qnew(0$Lisp, 0$Lisp)
   new(n, x)                    == Qnew (n, x)
   qelt(x, i)                   == Qelt(x, i)
   elt(x:%, i:Integer)          == Qelt(x, i)
   qsetelt_!(x, i, s)           == Qsetelt(x, i, s)
   setelt(x:%, i:Integer, s:Integer) == Qsetelt(x, i, s)
   fill_!(x, s)        == (for i in 0..((Qsize x) - 1) repeat Qsetelt(x, i, s); x)
```

— U32VEC.dotabb —

```
"U32VEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=U32VEC"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"U32VEC" -> "A1AGG"
```

Chapter 23

Chapter V

23.1 domain VARIABLE Variable

— Variable.input —

```
)set break resume
)sys rm -f Variable.output
)spool Variable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Variable
--R Variable sym: Symbol  is a domain constructor
--R Abbreviation for Variable is VARIABLE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for VARIABLE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> Symbol
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   variable : () -> Symbol
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———

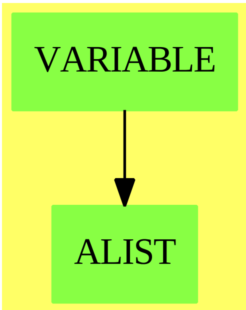
— **Variable.help** —

```
====================================================================
Variable examples
====================================================================

See Also:
o )show Variable
```

———-

23.1.1 Variable (VARIABLE)



Exports:
coerce hash latex variable ?=? ?~=?

— **domain VARIABLE Variable** —

```
)abbrev domain VARIABLE Variable
++ Author: Mark Botch
++ Description:
++ This domain implements variables

Variable(sym:Symbol): Join(SetCategory, CoercibleTo Symbol) with
        coerce  : % -> Symbol
                ++ coerce(x) returns the symbol
        variable: () -> Symbol
                ++ variable() returns the symbol
    == add
        coerce(x:%):Symbol     == sym
        coerce(x:%):OutputForm == sym::OutputForm
        variable()             == sym
        x = y                  == true
```

```
    latex(x:%):String      == latex sym
```

— VARIABLE.dotabb —

```
"VARIABLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VARIABLE"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"VARIABLE" -> "ALIST"
```

23.2 domain VECTOR Vector

— Vector.input —

```
)set break resume
)sys rm -f Vector.output
)spool Vector.output
)set message test on
)set message auto off
)clear all
--S 1 of 11
u : VECTOR INT := new(5,12)
--R
--R
--R   (1)  [12,12,12,12,12]
--R                                                      Type: Vector Integer
--E 1

--S 2 of 11
v : VECTOR INT := vector([1,2,3,4,5])
--R
--R
--R   (2)  [1,2,3,4,5]
--R                                                      Type: Vector Integer
--E 2

--S 3 of 11
#(v)
--R
--R
--R   (3)  5
--R                                                     Type: PositiveInteger
```

```
--E 3

--S 4 of 11
v.2
--R
--R
--R   (4)  2
--R                                                        Type: PositiveInteger
--E 4

--S 5 of 11
v.3 := 99
--R
--R
--R   (5)  99
--R                                                        Type: PositiveInteger
--E 5

--S 6 of 11
v
--R
--R
--R   (6)  [1,2,99,4,5]
--R                                                         Type: Vector Integer
--E 6

--S 7 of 11
5 * v
--R
--R
--R   (7)  [5,10,495,20,25]
--R                                                         Type: Vector Integer
--E 7

--S 8 of 11
v * 7
--R
--R
--R   (8)  [7,14,693,28,35]
--R                                                         Type: Vector Integer
--E 8

--S 9 of 11
w : VECTOR INT := vector([2,3,4,5,6])
--R
--R
--R   (9)  [2,3,4,5,6]
--R                                                         Type: Vector Integer
--E 9
```

```
--S 10 of 11
v + w
--R
--R
--R   (10)  [3,5,103,9,11]
--R                                                        Type: Vector Integer
--E 10

--S 11 of 11
v - w
--R
--R
--R   (11)  [- 1,- 1,95,- 1,- 1]
--R                                                        Type: Vector Integer
--E 11
)spool
)lisp (bye)
```

— Vector.help —

```
====================================================================
Vector examples
====================================================================

The Vector domain is used for storing data in a one-dimensional
indexed data structure.  A vector is a homogeneous data structure in
that all the components of the vector must belong to the same Axiom
domain.  Each vector has a fixed length specified by the user; vectors
are not extensible.  This domain is similar to the OneDimensionalArray
domain, except that when the components of a Vector belong to a Ring,
arithmetic operations are provided.

As with the OneDimensionalArray domain, a Vector can be created by
calling the operation new, its components can be accessed by calling
the operations elt and qelt, and its components can be reset by
calling the operations setelt and qsetelt.

This creates a vector of integers of length 5 all of whose components are 12.

  u : VECTOR INT := new(5,12)
    [12,12,12,12,12]
                                Type: Vector Integer

This is how you create a vector from a list of its components.

  v : VECTOR INT := vector([1,2,3,4,5])
    [1,2,3,4,5]
```

```
                                Type: Vector Integer
```

Indexing for vectors begins at 1. The last element has index equal to the length of the vector, which is computed by #.

```
 #(v)
   5
                                Type: PositiveInteger
```

This is the standard way to use elt to extract an element. Functionally, it is the same as if you had typed elt(v,2).

```
  v.2
   2
                                Type: PositiveInteger
```

This is the standard way to use setelt to change an element. It is the same as if you had typed setelt(v,3,99).

```
  v.3 := 99
   99
                                Type: PositiveInteger
```

Now look at v to see the change. You can use qelt and qsetelt (instead of elt and setelt, respectively) but only when you know that the index is within the valid range.

```
  v
   [1,2,99,4,5]
                                Type: Vector Integer
```

When the components belong to a Ring, Axiom provides arithmetic operations for Vector. These include left and right scalar multiplication.

```
  5 * v
   [5,10,495,20,25]
                                Type: Vector Integer

  v * 7
   [7,14,693,28,35]
                                Type: Vector Integer

  w : VECTOR INT := vector([2,3,4,5,6])
   [2,3,4,5,6]
                                Type: Vector Integer
```

Addition and subtraction are also available.

```
  v + w
   [3,5,103,9,11]
```

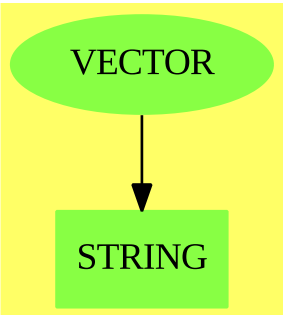
```
                                Type: Vector Integer

Of course, when adding or subtracting, the two vectors must have the same
length or an error message is displayed.

  v - w
    [- 1,- 1,95,- 1,- 1]
                                Type: Vector Integer

See Also:
o )help List
o )help Matrix
o )help OneDimensionalArray
o )help Set
o )help Table
o )help TwoDimensionalArray
o )show Vector
```

23.2.1 Vector (VECTOR)



Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	cross	delete
dot	elt	empty	empty?	entries
entry?	eq?	eval	every?	fill!
find	first	hash	index?	indices
insert	latex	length	less?	magnitude
map	map!	max	maxIndex	member?
members	merge	min	minIndex	more?
new	outerProduct	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	vector
zero	#?	?*?	?+?	?-?
?<?	?<=?	?=?	?>?	?>=?
?.?	?~=?	-?	?.?	

— domain VECTOR Vector —

```
)abbrev domain VECTOR Vector
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: IndexedVector, DirectProduct
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents vector like objects with varying lengths
++ and indexed by a finite segment of integers starting at 1.

Vector(R:Type): Exports == Implementation where
 VECTORMININDEX ==> 1       -- if you want to change this, be my guest

 Exports ==> VectorCategory R with
   vector: List R -> %
     ++ vector(l) converts the list l to a vector.
 Implementation ==>
  IndexedVector(R, VECTORMININDEX) add
     vector l == construct l
     if R has ConvertibleTo InputForm then
       convert(x:%):InputForm ==
          convert [convert("vector"::Symbol)@InputForm,
                          convert(parts x)@InputForm]
```

————

— **VECTOR.dotabb** —

```
"VECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VECTOR",
          shape=ellipse]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"VECTOR" -> "STRING"
```

————-

23.3 domain VOID Void

— **Void.input** —

```
)set break resume
)sys rm -f Void.output
)spool Void.output
)set message test on
)set message auto off
)clear all
--S 1 of 5
r := (a; b; if c then d else e; f)
--R
--R
--RDaly Bug
--R   An expression following if/when must evaluate to a Boolean and you
--R      have written one that does not.
--E 1

--S 2 of 5
a : Integer
--R
--R                                                                   Type: Void
--E 2

)set message void on

--S 3 of 5
b : Fraction Integer
--R
--R
--R   (2)  "()"
--R                                                                   Type: Void
--E 3

)set message void off
```

```
--S 4 of 5
3::Void
--R
--R                                                                   Type: Void
--E 4

--S 5 of 5
% :: PositiveInteger
--R
--R
--RDaly Bug
--R   Cannot convert from type Void to PositiveInteger for value
--R   "()"
--R
--E 5
)spool
)lisp (bye)
```

— Void.help —

```
====================================================================
Void examples
====================================================================

When an expression is not in a value context, it is given type Void.
For example, in the expression

  r := (a; b; if c then d else e; f)

values are used only from the subexpressions c and f: all others are thrown
away.  The subexpressions a, b, d and e are evaluated for side-effects only
and have type Void.  There is a unique value of type Void.

You will most often see results of type Void when you declare a variable.

  a : Integer
                                Type: Void

Usually no output is displayed for Void results.  You can force the
display of a rather ugly object by issuing

  )set message void on

  b : Fraction Integer
                                Type: Void
```

```
  )set message void off

All values can be converted to type Void.

  3::Void
                                Type: Void

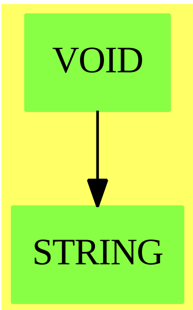
Once a value has been converted to Void, it cannot be recovered.

  % :: PositiveInteger
   Cannot convert from type Void to PositiveInteger for value "()"

See Also:
o )show Void
```

———

23.3.1 Void (VOID)



Exports:
coerce void

— domain VOID Void —

```
)abbrev domain VOID Void
-- These types act as the top and bottom of the type lattice
-- and are known to the compiler and interpreter for type resolution.
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains: ErrorFunctions, ResolveLatticeCompletion, Exit
++ Also See:
```

```
++ AMS Classifications:
++ Keywords: type, mode, coerce, no value
++ Examples:
++ References:
++ Description:
++ This type is used when no value is needed, e.g., in the \spad{then}
++ part of a one armed \spad{if}.
++ All values can be coerced to type Void.  Once a value has been coerced
++ to Void, it cannot be recovered.

Void: with
        void: () -> %
          ++ void() produces a void object.
        coerce: % -> OutputForm
          ++ coerce(v) coerces void object to outputForm.
    == add
        Rep := String
        void()      == voidValue()$Lisp
        coerce(v:%) == coerce(v)$Rep
```

— **VOID.dotabb** —

```
"VOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VOID"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"VOID" -> "STRING"
```

Chapter 24

Chapter W

24.1 domain WP WeightedPolynomials

— WeightedPolynomials.input —

```
)set break resume
)sys rm -f WeightedPolynomials.output
)spool WeightedPolynomials.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show WeightedPolynomials
--R WeightedPolynomials(R: Ring,VarSet: OrderedSet,E: OrderedAbelianMonoidSup,P: PolynomialCategory(R,E,
--R Abbreviation for WeightedPolynomials is WP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for WP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : P -> %
--R coerce : % -> P                       coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,R) -> % if R has COMRING
```

```
--R ?*? : (R,%) -> % if R has COMRING
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> Union(%,"failed") if R has FIELD
--R ?^? : (%,NonNegativeInteger) -> %
--R changeWeightLevel : NonNegativeInteger -> Void
--R characteristic : () -> NonNegativeInteger
--R coerce : R -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

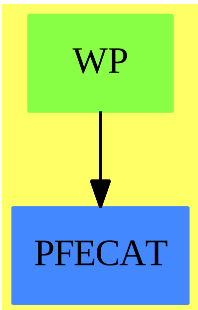
— **WeightedPolynomials.help** —

```
====================================================================
WeightedPolynomials examples
====================================================================

See Also:
o )show WeightedPolynomials
```

———-

24.1.1 WeightedPolynomials (WP)



Exports:

0	1	changeWeightLevel	characteristic
coerce	hash	latex	one?
recip	sample	subtractIfCan	zero?
?~=?	?*?	?**?	?/?
?^?	?*?	?**?	?+?
?-?	-?	?=?	

— domain WP WeightedPolynomials —

```
)abbrev domain WP WeightedPolynomials
++ Author: James Davenport
++ Date Created:  17 April 1992
++ Date Last Updated: 12 July 1992
++ Basic Functions: Ring, changeWeightLevel
++ Related Constructors: PolynomialRing
++ Also See: OrdinaryWeightedPolynomials
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents truncated weighted polynomials over a general
++ (not necessarily commutative) polynomial type. The variables must be
++ specified, as must the weights.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.

WeightedPolynomials(R:Ring,VarSet: OrderedSet, E:OrderedAbelianMonoidSup,
                           P:PolynomialCategory(R,E,VarSet),
                             vl:List VarSet, wl:List NonNegativeInteger,
                              wtlevel:NonNegativeInteger):
       Ring with
         if R has CommutativeRing then Algebra(R)
         coerce: $ -> P
                 ++ convert back into a "P", ignoring weights
         if R has Field then "/": ($,$) -> Union($,"failed")
                 ++ x/y division (only works if minimum weight
                 ++ of divisor is zero, and if R is a Field)
         coerce: P -> $
                 ++ coerce(p) coerces p into Weighted form,
                 ++ applying weights and ignoring terms
         changeWeightLevel: NonNegativeInteger -> Void
                 ++ changeWeightLevel(n) changes the weight level to
                 ++ the new value given:
                 ++ NB: previously calculated terms are not affected
    ==
  add
   --representations
   Rep  := PolynomialRing(P,NonNegativeInteger)
```

```
p:P
w,x1,x2:$
n:NonNegativeInteger
z:Integer
changeWeightLevel(n) ==
     wtlevel:=n
lookupList:List Record(var:VarSet, weight:NonNegativeInteger)
if #vl ^= #wl then error "incompatible length lists in WeightedPolynomial"
lookupList:=[[v,n] for v in vl for n in wl]
-- local operation
innercoerce:(p,z) -> $
lookup:Varset -> NonNegativeInteger
lookup v ==
   l:=lookupList
   while l ^= [] repeat
     v = l.first.var => return l.first.weight
     l:=l.rest
   0
innercoerce(p,z) ==
   z<0 => 0
   zero? p => 0
   mv:= mainVariable p
   mv case "failed" => monomial(p,0)
   n:=lookup(mv)
   up:=univariate(p,mv)
   ans:$
   ans:=0
   while not zero? up  repeat
     d:=degree up
     f:=n*d
     lcup:=leadingCoefficient up
     up:=up-leadingMonomial up
     mon:=monomial(1,mv,d)
     f<=z =>
         tmp:= innercoerce(lcup,z-f)
         while not zero? tmp repeat
           ans:=ans+ monomial(mon*leadingCoefficient(tmp),degree(tmp)+f)
           tmp:=reductum tmp
   ans
coerce(p):$ == innercoerce(p,wtlevel)
coerce(w):P ==  "+"/[c for c in coefficients w]
coerce(p:$):OutputForm ==
  zero? p => (0$Integer)::OutputForm
  degree p = 0 => leadingCoefficient(p):: OutputForm
  reduce("+",(reverse [paren(c::OutputForm) for c in coefficients p])
             ::List OutputForm)
0 == 0$Rep
1 == 1$Rep
x1 = x2 ==
   -- Note that we must strip out any terms greater than wtlevel
```

```
      while degree x1 > wtlevel repeat
            x1 := reductum x1
      while degree x2 > wtlevel repeat
            x2 := reductum x2
      x1 =$Rep x2
   x1 + x2 == x1 +$Rep x2
   -x1 == -(x1::Rep)
   x1 * x2 ==
     -- Note that this is probably an extremely inefficient definition
     w:=x1 *$Rep x2
     while degree(w) > wtlevel repeat
           w:=reductum w
     w
```

— **WP.dotabb** —

```
"WP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=WP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"WP" -> "PFECAT"
```

24.2 domain WUTSET WuWenTsunTriangularSet

— **WuWenTsunTriangularSet.input** —

```
)set break resume
)sys rm -f WuWenTsunTriangularSet.output
)spool WuWenTsunTriangularSet.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
R := Integer
--R
--R
--R   (1)  Integer
--R                                                            Type: Domain
--E 1

--S 2 of 16
ls : List Symbol := [x,y,z,t]
```

```
--R
--R
--R   (2)  [x,y,z,t]
--R                                                      Type: List Symbol
--E 2

--S 3 of 16
V := OVAR(ls)
--R
--R
--R   (3)  OrderedVariableList [x,y,z,t]
--R                                                      Type: Domain
--E 3

--S 4 of 16
E := IndexedExponents V
--R
--R
--R   (4)  IndexedExponents OrderedVariableList [x,y,z,t]
--R                                                      Type: Domain
--E 4

--S 5 of 16
P := NSMP(R, V)
--R
--R
--R   (5)  NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--R                                                      Type: Domain
--E 5

--S 6 of 16
x: P := 'x
--R
--R
--R   (6)  x
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 6

--S 7 of 16
y: P := 'y
--R
--R
--R   (7)  y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 7

--S 8 of 16
z: P := 'z
--R
--R
```

```
--R   (8)  z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 8

--S 9 of 16
t: P := 't
--R
--R
--R   (9)  t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 9

--S 10 of 16
T := WUTSET(R,E,V,P)
--R
--R
--R   (10)
--R  WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t]
--R  ,OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,Ordere
--R  dVariableList [x,y,z,t]))
--R                                                                 Type: Domain
--E 10

--S 11 of 16
p1 := x ** 31 - x ** 6 - x - y
--R
--R
--R          31    6
--R   (11)  x   - x  - x - y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 11

--S 12 of 16
p2 := x ** 8  - z
--R
--R
--R          8
--R   (12)  x  - z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 12

--S 13 of 16
p3 := x ** 10 - t
--R
--R
--R          10
--R   (13)  x   - t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 13
```

```
--S 14 of 16
lp := [p1, p2, p3]
--R
--R
--R           31    6          8      10
--R   (14)  [x   - x  - x - y,x  - z,x   - t]
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 14

--S 15 of 16
characteristicSet(lp)$T
--R
--R
--R   (15)
--R     5    4  4 2 2     3 4        7     4      6    6    3      3     3     3
--R   {z  - t ,t z y  + 2t z y + (- t  + 2t  - t)z  + t z,(t  - 1)z x - z y - t }
--RType: Union(WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t]
--E 15

--S 16 of 16
zeroSetSplit(lp)$T
--R
--R
--R   (16)
--R                  3      5    4  3     3    2
--R   [{t,z,y,x}, {t  - 1,z  - t ,z y + t ,z x  - t},
--R      5    4  4 2 2     3 4        7     4      6    6    3      3     3     3
--R    {z  - t ,t z y  + 2t z y + (- t  + 2t  - t)z  + t z,(t  - 1)z x - z y - t }]
--RType: List WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],
--E 16
)spool
)lisp (bye)
```

— **WuWenTsunTriangularSet.help** —

```
====================================================================
WuWenTsunTriangularSet examples
====================================================================

The WuWenTsunTriangularSet domain constructor implements the
characteristic set method of Wu Wen Tsun.  This algorithm computes a
list of triangular sets from a list of polynomials such that the
algebraic variety defined by the given list of polynomials decomposes
into the union of the regular-zero sets of the computed triangular
sets.  The constructor takes four arguments.  The first one, R, is the
coefficient ring of the polynomials; it must belong to the category
IntegralDomain. The second one, E, is the exponent monoid of the
```

```
polynomials; it must belong to the category OrderedAbelianMonoidSup.
The third one, V, is the ordered set of variables; it must belong to
the category OrderedSet.  The last one is the polynomial ring; it must
belong to the category RecursivePolynomialCategory(R,E,V).  The
abbreviation for WuWenTsunTriangularSet is WUTSET.

Let us illustrate the facilities by an example.

Define the coefficient ring.

  R := Integer
    Integer
                              Type: Domain

Define the list of variables,

  ls : List Symbol := [x,y,z,t]
    [x,y,z,t]
                              Type: List Symbol

and make it an ordered set;

  V := OVAR(ls)
    OrderedVariableList [x,y,z,t]
                              Type: Domain

then define the exponent monoid.

  E := IndexedExponents V
    IndexedExponents OrderedVariableList [x,y,z,t]
                              Type: Domain

Define the polynomial ring.

  P := NSMP(R, V)
    NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
                              Type: Domain

Let the variables be polynomial.

  x: P := 'x
    x
       Type: NewSparseMultivariatePolynomial(Integer,
               OrderedVariableList [x,y,z,t])

  y: P := 'y
    y
       Type: NewSparseMultivariatePolynomial(Integer,
               OrderedVariableList [x,y,z,t])
```

```
z: P := 'z
  z
     Type: NewSparseMultivariatePolynomial(Integer,
             OrderedVariableList [x,y,z,t])

t: P := 't
  t
     Type: NewSparseMultivariatePolynomial(Integer,
             OrderedVariableList [x,y,z,t])
```

Now call the WuWenTsunTriangularSet domain constructor.

```
T := WUTSET(R,E,V,P)
WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t]
,OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,Ordere
dVariableList [x,y,z,t]))
                               Type: Domain
```

Define a polynomial system.

```
p1 := x ** 31 - x ** 6 - x - y
    31    6
   x   - x  - x - y
             Type: NewSparseMultivariatePolynomial(Integer,
                     OrderedVariableList [x,y,z,t])

p2 := x ** 8  - z
    8
   x  - z
             Type: NewSparseMultivariatePolynomial(Integer,
                     OrderedVariableList [x,y,z,t])

p3 := x ** 10 - t
    10
   x   - t
             Type: NewSparseMultivariatePolynomial(Integer,
                     OrderedVariableList [x,y,z,t])

lp := [p1, p2, p3]
    31    6          8      10
  [x   - x  - x - y,x  - z,x   - t]
       Type: List NewSparseMultivariatePolynomial(Integer,
                    OrderedVariableList [x,y,z,t])
```

Compute a characteristic set of the system.

```
characteristicSet(lp)$T
   5    4  4 2 2     3 4        7     4      6    6    3      3     3     3
 {z  - t ,t z y  + 2t z y + (- t  + 2t  - t)z  + t z,(t  - 1)z x - z y - t }
          Type: Union(WuWenTsunTriangularSet(Integer,
```

```
                          IndexedExponents OrderedVariableList [x,y,z,t],
                          OrderedVariableList [x,y,z,t],
                          NewSparseMultivariatePolynomial(Integer,
                            OrderedVariableList [x,y,z,t])),...)
```

Solve the system.

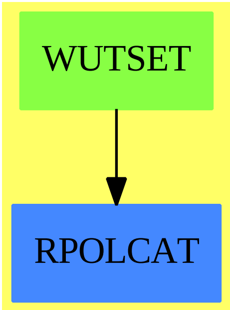
```
  zeroSetSplit(lp)$T
                3      5    4  3     3    2
  [{t,z,y,x}, {t  - 1,z  - t ,z y + t ,z x  - t},
     5    4  4 2 2     3 4        7     4      6    6    3      3     3     3
   {z  - t ,t z y  + 2t z y + (- t  + 2t  - t)z  + t z,(t  - 1)z x - z y - t }]
         Type: List WuWenTsunTriangularSet(Integer,
                      IndexedExponents OrderedVariableList [x,y,z,t],
                      OrderedVariableList [x,y,z,t],
                      NewSparseMultivariatePolynomial(Integer,
                         OrderedVariableList [x,y,z,t]))
```

The RegularTriangularSet and SquareFreeRegularTriangularSet domain constructors, the LazardSetSolvingPackage package constructors as well as, SquareFreeRegularTriangularSet and ZeroDimensionalSolvePackage package constructors also provide operations to compute triangular decompositions of algebraic varieties. These five constructor use a special kind of characteristic sets, called regular triangular sets. These special characteristic sets have better properties than the general ones. Regular triangular sets and their related concepts are presented in the paper "On the Theories of Triangular sets" By P. Aubry, D. Lazard and M. Moreno Maza (to appear in the Journal of Symbolic Computation). The decomposition algorithm (due to the third author) available in the four above constructors provide generally better timings than the characteristic set method. In fact, the WUTSET constructor remains interesting for the purpose of manipulating characteristic sets whereas the other constructors are more convenient for solving polynomial systems.

Note that the way of understanding triangular decompositions is detailed in the example of the RegularTriangularSet constructor.

See Also:
o)help RecursivePolynomialCategory
o)help RegularTriangularSet
o)help SquareFreeRegularTriangularSet
o)help LazardSetSolvingPackage
o)help ZeroDimensionalSolvePackage
o)show WuWenTsunTriangularSet

24.2.1 WuWenTsunTriangularSet (WUTSET)



See

⇒ "GeneralTriangularSet" (GTSET) 8.6.1 on page 1049

Exports:

algebraic?	algebraicVariables
any?	autoReduced?
basicSet	characteristicSerie
characteristicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	last
latex	less?
mainVariable?	mainVariables
map	map!
medialSet	member?
members	more?
mvar	normalized?
normalized?	parts
quasiComponent	reduce
reduceByQuasiMonic	reduced?
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
stronglyReduce	stronglyReduced?
triangular?	trivialIdeal?
variables	zeroSetSplit
zeroSetSplitIntoTriangularSystems	#?
?=?	?~=?

— domain WUTSET WuWenTsunTriangularSet —

```
)abbrev domain WUTSET WuWenTsunTriangularSet
```

```
++ Author: Marc Moreno Maza (marc@nag.co.uk)
++ Date Created: 11/18/1995
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++ [1] W. T. WU "A Zero Structure Theorem for polynomial equations solving"
++      MM Research Preprints, 1987.
++ [2] D. M. WANG "An implementation of the characteristic set method in Maple"
++      Proc. DISCO'92. Bath, England.
++ Description:
++ A domain constructor of the category \axiomType{GeneralTriangularSet}.
++ The only requirement for a list of polynomials to be a member of such
++ a domain is the following: no polynomial is constant and two distinct
++ polynomials have distinct main variables. Such a triangular set may
++ not be auto-reduced or consistent. The construct operation
++ does not check the previous requirement. Triangular sets are stored
++ as sorted lists w.r.t. the main variables of their members.
++ Furthermore, this domain exports operations dealing with the
++ characteristic set method of Wu Wen Tsun and some optimizations
++ mainly proposed by Dong Ming Wang.

WuWenTsunTriangularSet(R,E,V,P) : Exports == Implementation where

  R : IntegralDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)
  N ==> NonNegativeInteger
  Z ==> Integer
  B ==> Boolean
  LP ==> List P
  A ==> FiniteEdge P
  H ==> FiniteSimpleHypergraph P
  GPS ==> GeneralPolynomialSet(R,E,V,P)
  RBT ==> Record(bas:$,top:LP)
  RUL ==> Record(chs:Union($,"failed"),rfs:LP)
  pa ==> PolynomialSetUtilitiesPackage(R,E,V,P)
  NLpT ==> SplittingNode(LP,$)
  ALpT ==> SplittingTree(LP,$)
  O ==> OutputForm
  OP ==> OutputPackage

  Exports ==  TriangularSetCategory(R,E,V,P) with

     medialSet : (LP,((P,P)->B),((P,P)->P)) -> Union($,"failed")
        ++ \axiom{medialSet(ps,redOp?,redOp)} returns \axiom{bs} a basic set
```

```
      ++ (in Wu Wen Tsun sense w.r.t the reduction-test \axiom{redOp?})
      ++ of some set generating the same ideal as \axiom{ps} (with
      ++ rank not higher than  any basic set of \axiom{ps}), if no non-zero
      ++ constant polynomials appear during the computatioms, else
      ++ \axiom{"failed"} is returned. In the former case, \axiom{bs} has to be
      ++ understood as a candidate for being a characteristic set of \axiom{ps}.
      ++ In the original algorithm, \axiom{bs} is simply a basic set of \axiom{ps}.
   medialSet: LP -> Union($,"failed")
      ++ \axiom{medial(ps)} returns the same as
      ++ \axiom{medialSet(ps,initiallyReduced?,initiallyReduce)}.
   characteristicSet : (LP,((P,P)->B),((P,P)->P)) -> Union($,"failed")
      ++ \axiom{characteristicSet(ps,redOp?,redOp)} returns a non-contradictory
      ++ characteristic set of \axiom{ps} in Wu Wen Tsun sense w.r.t the
      ++ reduction-test \axiom{redOp?} (using \axiom{redOp} to reduce
      ++ polynomials w.r.t a \axiom{redOp?} basic set), if no
      ++ non-zero constant polynomial appear during those reductions,
      ++ else \axiom{"failed"} is returned.
      ++ The operations \axiom{redOp} and \axiom{redOp?} must satisfy
      ++ the following conditions: \axiom{redOp?(redOp(p,q),q)} holds
      ++ for every polynomials \axiom{p,q} and there exists an integer
      ++ \axiom{e} and a polynomial \axiom{f} such that we have
      ++ \axiom{init(q)^e*p = f*q + redOp(p,q)}.
   characteristicSet: LP -> Union($,"failed")
      ++ \axiom{characteristicSet(ps)} returns the same as
      ++ \axiom{characteristicSet(ps,initiallyReduced?,initiallyReduce)}.
   characteristicSerie  : (LP,((P,P)->B),((P,P)->P)) -> List $
      ++ \axiom{characteristicSerie(ps,redOp?,redOp)} returns a list \axiom{lts}
      ++ of triangular sets such that the zero set of \axiom{ps} is the
      ++ union of the regular zero sets of the members of \axiom{lts}.
      ++ This is made by the Ritt and Wu Wen Tsun process applying
      ++ the operation \axiom{characteristicSet(ps,redOp?,redOp)}
      ++ to compute characteristic sets in Wu Wen Tsun sense.
   characteristicSerie: LP ->  List $
      ++ \axiom{characteristicSerie(ps)} returns the same as
      ++ \axiom{characteristicSerie(ps,initiallyReduced?,initiallyReduce)}.

 Implementation == GeneralTriangularSet(R,E,V,P) add

   removeSquares: $ -> Union($,"failed")

   Rep ==> LP

   rep(s:$):Rep == s pretend Rep
   per(l:Rep):$ == l pretend $

   removeAssociates (lp:LP):LP ==
     removeDuplicates [primPartElseUnitCanonical(p) for p in lp]

   medialSetWithTrace (ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)):Union(RBT,"failed") ==
     qs := rewriteIdealWithQuasiMonicGenerators(ps,redOp?,redOp)$pa
```

```
      contradiction : B := any?(ground?,ps)
      contradiction => "failed"::Union(RBT,"failed")
      rs : LP := qs
      bs : $
      while (not empty? rs) and (not contradiction) repeat
        rec := basicSet(rs,redOp?)
        contradiction := (rec case "failed")@B
        if not contradiction
          then
            bs := (rec::RBT).bas
            rs := (rec::RBT).top
            rs :=  rewriteIdealWithRemainder(rs,bs)
--            contradiction := ((not empty? rs) and (one? first(rs)))
            contradiction := ((not empty? rs) and (first(rs) = 1))
            if (not empty? rs) and (not contradiction)
              then
                rs := rewriteSetWithReduction(rs,bs,redOp,redOp?)
--                contradiction := ((not empty? rs) and (one? first(rs)))
                contradiction := ((not empty? rs) and (first(rs) = 1))
        if (not empty? rs) and (not contradiction)
          then
            rs := removeDuplicates concat(rs,members(bs))
            rs := rewriteIdealWithQuasiMonicGenerators(rs,redOp?,redOp)$pa
--            contradiction := ((not empty? rs) and (one? first(rs)))
            contradiction := ((not empty? rs) and (first(rs) = 1))
      contradiction => "failed"::Union(RBT,"failed")
      ([bs,qs]$RBT)::Union(RBT,"failed")

    medialSet(ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)) ==
      foo: Union(RBT,"failed") := medialSetWithTrace(ps,redOp?,redOp)
      (foo case "failed") => "failed" :: Union($,"failed")
      ((foo::RBT).bas) :: Union($,"failed")

    medialSet(ps:LP) == medialSet(ps,initiallyReduced?,initiallyReduce)

    characteristicSetUsingTrace(ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)):Union($,"failed")
      ps := removeAssociates ps
      ps := remove(zero?,ps)
      contradiction : B := any?(ground?,ps)
      contradiction => "failed"::Union($,"failed")
      rs : LP := ps
      qs : LP := ps
      ms : $
      while (not empty? rs) and (not contradiction) repeat
        rec := medialSetWithTrace (qs,redOp?,redOp)
        contradiction := (rec case "failed")@B
        if not contradiction
          then
            ms := (rec::RBT).bas
            qs := (rec::RBT).top
```

```
            qs := rewriteIdealWithRemainder(qs,ms)
--            contradiction := ((not empty? qs) and (one? first(qs)))
            contradiction := ((not empty? qs) and (first(qs) = 1))
            if not contradiction
              then
                rs :=  rewriteSetWithReduction(qs,ms,lazyPrem,reduced?)
--                contradiction := ((not empty? rs) and (one? first(rs)))
                contradiction := ((not empty? rs) and (first(rs) = 1))
            if  (not contradiction) and (not empty? rs)
              then
                qs := removeDuplicates(concat(rs,concat(members(ms),qs)))
      contradiction => "failed"::Union($,"failed")
      ms::Union($,"failed")

    characteristicSet(ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)) ==
      characteristicSetUsingTrace(ps,redOp?,redOp)

    characteristicSet(ps:LP) == characteristicSet(ps,initiallyReduced?,initiallyReduce)

    characteristicSerie(ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)) ==
      a := [[ps,empty()$$]$NLpT]$ALpT
      while ((esl := extractSplittingLeaf(a)) case ALpT) repeat
         ps := value(value(esl::ALpT)$ALpT)$NLpT
         charSet? := characteristicSetUsingTrace(ps,redOp?,redOp)
         if not (charSet? case $)
            then
               setvalue!(esl::ALpT,[nil()$LP,empty()$$,true]$NLpT)
               updateStatus!(a)
            else
               cs := (charSet?)::$
               lics := initials(cs)
               lics := removeRedundantFactors(lics)$pa
               lics := sort(infRittWu?,lics)
               if empty? lics
                  then
                     setvalue!(esl::ALpT,[ps,cs,true]$NLpT)
                     updateStatus!(a)
                  else
                     ln : List NLpT := [[nil()$LP,cs,true]$NLpT]
                     while not empty? lics repeat
                        newps := cons(first(lics),concat(cs::LP,ps))
                        lics := rest lics
                        newps := removeDuplicates newps
                        newps := sort(infRittWu?,newps)
                        ln := cons([newps,empty()$$,false]$NLpT,ln)
                     splitNodeOf!(esl::ALpT,a,ln)
      remove(empty()$$,conditions(a))

    characteristicSerie(ps:LP) ==  characteristicSerie (ps,initiallyReduced?,initiallyReduce)
```

```
if R has GcdDomain
then

  removeSquares (ts:$):Union($,"failed") ==
    empty?(ts)$$ => ts::Union($,"failed")
    p := (first ts)::P
    rsts : Union($,"failed")
    rsts := removeSquares((rest ts)::$)
    not(rsts case $) => "failed"::Union($,"failed")
    newts := rsts::$
    empty? newts =>
      p := squareFreePart(p)
      (per([primitivePart(p)]$LP))::Union($,"failed")
    zero? initiallyReduce(init(p),newts) => "failed"::Union($,"failed")
    p := primitivePart(removeZero(p,newts))
    ground? p => "failed"::Union($,"failed")
    not (mvar(newts) < mvar(p)) => "failed"::Union($,"failed")
    p := squareFreePart(p)
    (per(cons(unitCanonical(p),rep(newts))))::Union($,"failed")

  zeroSetSplit lp ==
    lts : List $ := characteristicSerie(lp,initiallyReduced?,initiallyReduce)
    lts := removeDuplicates(lts)$(List $)
    newlts : List $ := []
    while not empty? lts repeat
      ts := first lts
      lts := rest lts
      iic := removeSquares(ts)
      if iic case $
        then
          newlts := cons(iic::$,newlts)
    newlts := removeDuplicates(newlts)$(List $)
    sort(infRittWu?, newlts)

else

  zeroSetSplit lp ==
    lts : List $ := characteristicSerie(lp,initiallyReduced?,initiallyReduce)
    sort(infRittWu?, removeDuplicates lts)
```

———-

— **WUTSET.dotabb** —

```
"WUTSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=WUTSET"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"WUTSET" -> "RPOLCAT"
```

———–

Chapter 25

Chapter X

25.1 domain XDPOLY XDistributedPolynomial

Polynomial arithmetic with non-commutative variables has been improved by a contribution of Michel Petitot (University of Lille I, France). The domain constructor **XDistributedPolynomial** provide a distributed representation for these polynomials. It is the non-commutative equivalent for the **DistributedMultivariatePolynomial** constructor.

— XDistributedPolynomial.input —

```
)set break resume
)sys rm -f XDistributedPolynomial.output
)spool XDistributedPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XDistributedPolynomial
--R XDistributedPolynomial(vl: OrderedSet,R: Ring)  is a domain constructor
--R Abbreviation for XDistributedPolynomial is XDPOLY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for XDPOLY
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (vl,%) -> %
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,PositiveInteger) -> %
```

```
--R coef : (%,%) -> R                    coefficients : % -> List R
--R coerce : Integer -> %                coerce : R -> %
--R coerce : vl -> %                     coerce : % -> OutputForm
--R constant : % -> R                    constant? : % -> Boolean
--R degree : % -> NonNegativeInteger     hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> R
--R lquo : (%,vl) -> %                   lquo : (%,%) -> %
--R map : ((R -> R),%) -> %              mirror : % -> %
--R monomial? : % -> Boolean             monomials : % -> List %
--R one? : % -> Boolean                  quasiRegular : % -> %
--R quasiRegular? : % -> Boolean         recip : % -> Union(%,"failed")
--R reductum : % -> %                    rquo : (%,vl) -> %
--R rquo : (%,%) -> %                    sample : () -> %
--R varList : % -> List vl               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (R,OrderedFreeMonoid vl) -> %
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid vl) -> R
--R coefficient : (%,OrderedFreeMonoid vl) -> R
--R coerce : OrderedFreeMonoid vl -> %
--R leadingMonomial : % -> OrderedFreeMonoid vl
--R leadingTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
--R listOfTerms : % -> List Record(k: OrderedFreeMonoid vl,c: R)
--R lquo : (%,OrderedFreeMonoid vl) -> %
--R maxdeg : % -> OrderedFreeMonoid vl
--R mindeg : % -> OrderedFreeMonoid vl
--R mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
--R monom : (OrderedFreeMonoid vl,R) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R retract : % -> OrderedFreeMonoid vl
--R retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
--R rquo : (%,OrderedFreeMonoid vl) -> %
--R sh : (%,%) -> % if R has COMRING
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R trunc : (%,NonNegativeInteger) -> %
--R
--E 1

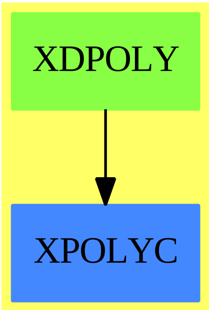
)spool
)lisp (bye)
```

— **XDistributedPolynomial.help** —

```
====================================================================
XDistributedPolynomial examples
====================================================================

See Also:
o )show XDistributedPolynomial
```

25.1.1 XDistributedPolynomial (XDPOLY)



Exports:

0	1	characteristic	coef
coefficient	coefficients	coerce	constant
constant?	degree	hash	latex
leadingCoefficient	listOfTerms	leadingMonomial	leadingTerm
lquo	map	mirror	monomial?
monomials	maxdeg	mindeg	mindegTerm
monom	numberOfMonomials	one?	quasiRegular
quasiRegular?	recip	reductum	retract
retractIfCan	rquo	sample	sh
subtractIfCan	trunc	varList	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?

— domain XDPOLY XDistributedPolynomial —

```
)abbrev domain XDPOLY XDistributedPolynomial
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
```

```
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type supports distributed multivariate polynomials
++ whose variables do not commute.
++ The coefficient ring may be non-commutative too.
++ However, coefficients and variables commute.

XDistributedPolynomial(vl:OrderedSet,R:Ring): XDPcat == XDPdef where

  WORD ==> OrderedFreeMonoid(vl)
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  TERM ==> Record(k:WORD, c:R)

  XDPcat == Join(FreeModuleCat(R, WORD), XPolynomialsCat(vl,R))

  XDPdef == XPolynomialRing(R,WORD) add

       import( WORD, TERM)

    -- Representation
       Rep  :=  List TERM

    -- local functions
       shw: (WORD , WORD) -> %    -- shuffle de 2 mots

    -- definitions

       mindegTerm p == last(p)$Rep

       if R has CommutativeRing then
         sh(p:%, n:NNI):% ==
            n=0 => 1
            n=1 => p
            n1: NNI := (n-$I 1)::NNI
            sh(p, sh(p,n1))


         sh(p1:%, p2:%) ==
           p:% := 0
           for t1 in p1 repeat
             for t2 in p2 repeat
                p := p + (t1.c * t2.c) * shw(t1.k,t2.k)
           p

       coerce(v: vl):% == coerce(v::WORD)
       v:vl * p:% ==
```

```
  [[v * t.k , t.c]$TERM for t in p]

mirror p ==
  null p => p
  monom(mirror$WORD leadingMonomial p, leadingCoefficient p) + _
        mirror reductum p

degree(p) == length(maxdeg(p))$WORD

trunc(p, n) ==
  p = 0 => p
  degree(p) > n => trunc( reductum p , n)
  p

varList p ==
  constant? p => []
  le : List vl := "setUnion"/[varList(t.k) for t in p]
  sort_!(le)

rquo(p:% , w: WORD) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= rquo(t.k,w)) case "failed" ]
lquo(p:% , w: WORD) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= lquo(t.k,w)) case "failed" ]
rquo(p:% , v: vl) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= rquo(t.k,v)) case "failed" ]
lquo(p:% , v: vl) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= lquo(t.k,v)) case "failed" ]

shw(w1,w2) ==
  w1 = 1$WORD => w2::%
  w2 = 1$WORD => w1::%
  x: vl := first w1 ; y: vl := first w2
  x * shw(rest w1,w2) + y * shw(w1,rest w2)

lquo(p:%,q:%):% ==
  +/  [r * t.c for t in q | (r := lquo(p,t.k)) ^= 0]

rquo(p:%,q:%):% ==
  +/  [r * t.c for t in q | (r := rquo(p,t.k)) ^= 0]

coef(p:%,q:%):R ==
  p = 0 => 0$R
  q = 0 => 0$R
  p.first.k > q.first.k => coef(p.rest,q)
  p.first.k < q.first.k => coef(p,q.rest)
  return p.first.c * q.first.c + coef(p.rest,q.rest)
```

———-

— XDPOLY.dotabb —

```
"XDPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XDPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XDPOLY" -> "XPOLYC"
```

———-

25.2 domain XPBWPOLY XPBWPolynomial

— XPBWPolynomial.input —

```
)set break resume
)sys rm -f XPBWPolynomial.output
)spool XPBWPolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 39
a:Symbol := 'a
--R
--R
--R   (1)  a
--R                                                                 Type: Symbol
--E 1

--S 2 of 39
b:Symbol := 'b
--R
--R
--R   (2)  b
--R                                                                 Type: Symbol
--E 2

--S 3 of 39
RN := Fraction(Integer)
--R
--R
--R   (3)  Fraction Integer
--R                                                                 Type: Domain
--E 3

--S 4 of 39
word := OrderedFreeMonoid Symbol
--R
```

```
--R
--R   (4)  OrderedFreeMonoid Symbol
--R                                                            Type: Domain
--E 4

--S 5 of 39
lword := LyndonWord(Symbol)
--R
--R
--R   (5)  LyndonWord Symbol
--R                                                            Type: Domain
--E 5

--S 6 of 39
base := PoincareBirkhoffWittLyndonBasis Symbol
--R
--R
--R   (6)  PoincareBirkhoffWittLyndonBasis Symbol
--R                                                            Type: Domain
--E 6

--S 7 of 39
dpoly := XDistributedPolynomial(Symbol, RN)
--R
--R
--R   (7)  XDistributedPolynomial(Symbol,Fraction Integer)
--R                                                            Type: Domain
--E 7

--S 8 of 39
rpoly := XRecursivePolynomial(Symbol, RN)
--R
--R
--R   (8)  XRecursivePolynomial(Symbol,Fraction Integer)
--R                                                            Type: Domain
--E 8

--S 9 of 39
lpoly := LiePolynomial(Symbol, RN)
--R
--R
--R   (9)  LiePolynomial(Symbol,Fraction Integer)
--R                                                            Type: Domain
--E 9

--S 10 of 39
poly  := XPBWPolynomial(Symbol, RN)
--R
--R
--R   (10)  XPBWPolynomial(Symbol,Fraction Integer)
```

```
--R                                                                Type: Domain
--E 10

--S 11 of 39
liste : List lword := LyndonWordsList([a,b], 6)
--R
--R
--R   (11)
--R                      2        2     3      2 2       3     4      3 2
--R   [[a], [b], [a b], [a b], [a b ], [a b], [a b ], [a b ], [a b], [a b ],
--R      2          2 3          2       4     5      4 2     3          3 3
--R    [a b a b], [a b ], [a b a b ], [a b ], [a b], [a b ], [a b a b], [a b ],
--R      2     2     2 2        2 4          3       5
--R    [a b a b ], [a b a b], [a b ], [a b a b ], [a b ]]
--R                                                 Type: List LyndonWord Symbol
--E 11

--S 12 of 39
0$poly
--R
--R
--R   (12)  0
--R                                 Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 12

--S 13 of 39
1$poly
--R
--R
--R   (13)  1
--R                                 Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 13

--S 14 of 39
p : poly := a
--R
--R
--R   (14)  [a]
--R                                 Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 14

--S 15 of 39
q : poly := b
--R
--R
--R   (15)  [b]
--R                                 Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 15

--S 16 of 39
```

```
pq: poly := p*q
--R
--R
--R   (16)  [a b] + [b][a]
--R                                   Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 16

--S 17 of 39
pq :: dpoly
--R
--R
--R   (17)  a b
--R                           Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 17

--S 18 of 39
mirror pq
--R
--R
--R   (18)  [b][a]
--R                                   Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 18

--S 19 of 39
listOfTerms pq
--R
--R
--R   (19)  [[k= [b][a],c= 1],[k= [a b],c= 1]]
--RType: List Record(k: PoincareBirkhoffWittLyndonBasis Symbol,c: Fraction Integer)
--E 19

--S 20 of 39
reductum pq
--R
--R
--R   (20)  [a b]
--R                                   Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 20

--S 21 of 39
leadingMonomial pq
--R
--R
--R   (21)  [b][a]
--R                                    Type: PoincareBirkhoffWittLyndonBasis Symbol
--E 21

--S 22 of 39
coefficients pq
--R
```

```
--R
--R   (22)  [1,1]
--R                                                  Type: List Fraction Integer
--E 22

--S 23 of 39
leadingTerm pq
--R
--R
--R   (23)  [k= [b][a],c= 1]
--R  Type: Record(k: PoincareBirkhoffWittLyndonBasis Symbol,c: Fraction Integer)
--E 23

--S 24 of 39
degree pq
--R
--R
--R   (24)  2
--R                                                      Type: PositiveInteger
--E 24

--S 25 of 39
pq4:=exp(pq,4)
--R
--R
--R   (25)
--R                            1              1     2       1      2
--R     1 + [a b] + [b][a] + - [a b][a b] + - [a b ][a] + - [b][a b]
--R                            2              2             2
--R   +
--R     3               1
--R     - [b][a b][a] + - [b][b][a][a]
--R     2               2
--R                                 Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 25

--S 26 of 39
log(pq4,4) - pq
--R
--R
--R   (26)  0
--R                                 Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 26

--S 27 of 39
lp1 :lpoly := LiePoly liste.10
--R
--R
--R           3 2
--R   (27)  [a b ]
```

```
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 27

--S 28 of 39
lp2 :lpoly := LiePoly liste.11
--R
--R
--R          2
--R   (28)  [a b a b]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 28

--S 29 of 39
lp :lpoly := [lp1, lp2]
--R
--R
--R          3 2 2
--R   (29)  [a b a b a b]
--R                                Type: LiePolynomial(Symbol,Fraction Integer)
--E 29

--S 30 of 39
lpd1: dpoly := lp1
--R
--R
--R         3 2     2         2 2                   2 2           2    2 3
--R   (30)  a b  - 2a b a b - a b a + 4a b a b a - a b a  - 2b a b a  + b a
--R                        Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 30

--S 31 of 39
lpd2: dpoly := lp2
--R
--R
--R   (31)
--R      2         2 2          2                    2 2       3       2
--R     a b a b - a b a - 3a b a b + 4a b a b a - a b a  + 2b a b - 3b a b a
--R   +
--R           2
--R     b a b a
--R                        Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 31

--S 32 of 39
lpd : dpoly := lpd1 * lpd2 - lpd2 * lpd1
--R
--R
--R   (32)
--R      3 2 2         3 2 2 2      3 2     2      3 2            3 2   2 2
--R     a b a b a b - a b a b a - 3a b a b a b + 4a b a b a b a - a b a b a
```

```
--R   +
--R       3 3 3      3 3 2       3 3     2    2       3 2     2       2 2
--R     2a b a b - 3a b a b a + a b a b a  - a b a b a b  + 3a b a b a b a
--R   +
--R       2           2       2                    2         2 2     2     2 3
--R     6a b a b a b a b - 12a b a b a b a b a + 3a b a b a b a  - 4a b a b a b
--R   +
--R       2     2 2       2     3 3    2 2 4 2     2 2 3          2 2 2   2
--R     6a b a b a b a - a b a b a  + a b a b  - 3a b a b a b + 3a b a b a b
--R   +
--R         2 2     3      2 2     2        2 2         2    2 2   2 3
--R     - 2a b a b a b + 3a b a b a b a - 3a b a b a b a  + a b a b a
--R   +
--R           2   3 2         2   2              2   2 2           2
--R     3a b a b a b  - 6a b a b a b a b - 3a b a b a b a + 12a b a b a b a b a
--R   +
--R             2     2 2         2 2     2         2 3 3             4 2
--R     - 3a b a b a b a  - 6a b a b a b a  + 3a b a b a  - 4a b a b a b
--R   +
--R                3                   2   2                  3
--R     12a b a b a b a b - 12a b a b a b a b + 8a b a b a b a b
--R   +
--R                      2                         2               2 3      2 5 2
--R     - 12a b a b a b a b a + 12a b a b a b a b a  - 4a b a b a b a  + a b a b
--R   +
--R           2 4            2 3   2        2 2   3        2 2   2
--R     - 3a b a b a b + 3a b a b a b - 2a b a b a b + 3a b a b a b a
--R   +
--R           2 2       2      2 2 2 3       3   3 2       3   2
--R     - 3a b a b a b a  + a b a b a  - 2b a b a b  + 4b a b a b a b
--R   +
--R         3   2 2        3                  3     2 2       3 2     2       3 3 3
--R     2b a b a b a - 8b a b a b a b a + 2b a b a b a  + 4b a b a b a  - 2b a b a
--R   +
--R         2   4 2       2   3            2   3 2         2   2
--R     3b a b a b  - 6b a b a b a b - 3b a b a b a + 12b a b a b a b a
--R   +
--R           2   2 2 2       2           2       2     2 3          5 2
--R     - 3b a b a b a  - 6b a b a b a b a  + 3b a b a b a  - b a b a b
--R   +
--R             4 2            3   2             3                  3 2 2
--R     3b a b a b a + 6b a b a b a b - 12b a b a b a b a + 3b a b a b a
--R   +
--R               2 3              2 2               2 2 3    2 5         2 5 2
--R     - 4b a b a b a b + 6b a b a b a b a - b a b a b a  + b a b a b - b a b a
--R   +
--R         2 4   2      2 4           2 4 2 2     2 3   2      2 3   2
--R     - 3b a b a b + 4b a b a b a - b a b a  + 2b a b a b - 3b a b a b a
--R   +
--R      2 3       2
```

```
--R     b a b a b a
--R                       Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 32

--S 33 of 39
lp :: dpoly - lpd
--R
--R
--R   (33)  0
--R                       Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 33

--S 34 of 39
p := 3 * lp
--R
--R
--R              3 2 2
--R   (34)  3[a b a b a b]
--R                               Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 34

--S 35 of 39
q := lp1
--R
--R
--R             3 2
--R   (35)  [a b ]
--R                               Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 35

--S 36 of 39
pq:= p * q
--R
--R
--R              3 2 2       3 2
--R   (36)  3[a b a b a b][a b ]
--R                               Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 36

--S 37 of 39
pr:rpoly := p :: rpoly
--R
--R
--R   (37)
--R       a
--R    *
--R           a
--R        *
--R                a b b
--R            *
```

```
--R                a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
--R              +
--R                b a(a(a b 6 + b a(- 9)) + b a a 3)
--R          +
--R              b
--R           *
--R                  a b
--R               *
--R                      a
--R                   *
--R                        a(a b b(- 3) + b b a 9)
--R                      +
--R                        b(a(a b 18 + b a(- 36)) + b a a 9)
--R                  +
--R                    b(a a(a b(- 12) + b a 18) + b a a a(- 3))
--R              +
--R                  b a
--R               *
--R                    a(a(a b b 3 + b a b(- 9)) + b a a b 9)
--R                  +
--R                    b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
--R      +
--R          b
--R       *
--R              a
--R           *
--R                  a b
--R               *
--R                      a
--R                   *
--R                        a(a b b 9 + b(a b(- 18) + b a(- 9)))
--R                      +
--R                        b(a b a 36 + b a a(- 9))
--R                  +
--R                    b(a b a a(- 18) + b a a a 9)
--R              +
--R                  b a
--R               *
--R                    a(a(a b b(- 12) + b a b 36) + b a a b(- 36))
--R                  +
--R                    b(a(a(a b 24 + b a(- 36)) + b a a 36) + b a a a(- 12))
--R          +
--R              b a a
--R           *
--R                a(a(a b b 3 + b a b(- 9)) + b a a b 9)
--R              +
--R                b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
--R  +
--R      b
--R   *
```

```
--R           a
--R        *
--R               a
--R            *
--R                   a b
--R                *
--R                       a
--R                    *
--R                            a(a b b(- 6) + b(a b 12 + b a 6))
--R                         +
--R                            b(a b a(- 24) + b a a 6)
--R                     +
--R                        b(a b a a 12 + b a a a(- 6))
--R                 +
--R                   b a
--R                *
--R                       a
--R                    *
--R                            a(a b b 9 + b(a b(- 18) + b a(- 9)))
--R                         +
--R                            b(a b a 36 + b a a(- 9))
--R                     +
--R                        b(a b a a(- 18) + b a a a 9)
--R           +
--R               b a a
--R            *
--R                 a(a(a b b(- 3) + b b a 9) + b(a(a b 18 + b a(- 36)) + b a a 9))
--R               +
--R                 b(a a(a b(- 12) + b a 18) + b a a a(- 3))
--R       +
--R           b a a a
--R        *
--R             a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
--R           +
--R             b a(a(a b 6 + b a(- 9)) + b a a 3)
--R                             Type: XRecursivePolynomial(Symbol,Fraction Integer)
--E 37

--S 38 of 39
qr:rpoly := q :: rpoly
--R
--R
--R   (38)
--R     a(a(a b b 1 + b(a b(- 2) + b a(- 1))) + b(a b a 4 + b a a(- 1)))
--R   +
--R     b(a b a a(- 2) + b a a a 1)
--R                             Type: XRecursivePolynomial(Symbol,Fraction Integer)
--E 38

--S 39 of 39
```

```
pq :: rpoly - pr*qr
--R
--R
--R   (39)  0
--R                            Type: XRecursivePolynomial(Symbol,Fraction Integer)
--E 39
)spool
)lisp (bye)
```

——————

— XPBWPolynomial.help —

```
====================================================================
XPBWPolynomial examples
====================================================================

Initialisations

  a:Symbol := 'a
    a
                                  Type: Symbol

  b:Symbol := 'b
    b
                                  Type: Symbol

  RN := Fraction(Integer)
    Fraction Integer
                                  Type: Domain

  word := OrderedFreeMonoid Symbol
    OrderedFreeMonoid Symbol
                                  Type: Domain

  lword := LyndonWord(Symbol)
    LyndonWord Symbol
                                  Type: Domain

  base := PoincareBirkhoffWittLyndonBasis Symbol
    PoincareBirkhoffWittLyndonBasis Symbol
                                  Type: Domain

  dpoly := XDistributedPolynomial(Symbol, RN)
    XDistributedPolynomial(Symbol,Fraction Integer)
                                  Type: Domain

  rpoly := XRecursivePolynomial(Symbol, RN)
    XRecursivePolynomial(Symbol,Fraction Integer)
```

```
                                  Type: Domain

  lpoly := LiePolynomial(Symbol, RN)
    LiePolynomial(Symbol,Fraction Integer)
                                  Type: Domain

  poly  := XPBWPolynomial(Symbol, RN)
    XPBWPolynomial(Symbol,Fraction Integer)
                                  Type: Domain

  liste : List lword := LyndonWordsList([a,b], 6)
                       2        2     3      2 2       3     4      3 2
   [[a], [b], [a b], [a b], [a b ], [a b], [a b ], [a b ], [a b], [a b ],
      2          2 3           2       4     5      4 2     3          3 3
    [a b a b], [a b ], [a b a b ], [a b ], [a b], [a b ], [a b a b], [a b ],
      2     2     2 2        2 4           3       5
    [a b a b ], [a b a b], [a b ], [a b a b ], [a b ]]
                                  Type: List LyndonWord Symbol
```

Let's make some polynomials

```
  0$poly
    0
                         Type: XPBWPolynomial(Symbol,Fraction Integer)

  1$poly
    1
                         Type: XPBWPolynomial(Symbol,Fraction Integer)

  p : poly := a
    [a]
                         Type: XPBWPolynomial(Symbol,Fraction Integer)

  q : poly := b
    [b]
                         Type: XPBWPolynomial(Symbol,Fraction Integer)

  pq: poly := p*q
    [a b] + [b][a]
                         Type: XPBWPolynomial(Symbol,Fraction Integer)
```

Coerce to distributed polynomial

```
  pq :: dpoly
    a b
                    Type: XDistributedPolynomial(Symbol,Fraction Integer)
```

Check some polynomial operations

```
  mirror pq
```

```
    [b][a]
                      Type: XPBWPolynomial(Symbol,Fraction Integer)

  listOfTerms pq
    [[k= [b][a],c= 1],[k= [a b],c= 1]]
      Type: List Record(k: PoincareBirkhoffWittLyndonBasis Symbol,
                        c: Fraction Integer)

  reductum pq
    [a b]
                      Type: XPBWPolynomial(Symbol,Fraction Integer)

  leadingMonomial pq
    [b][a]
                      Type: PoincareBirkhoffWittLyndonBasis Symbol

  coefficients pq
    [1,1]
                      Type: List Fraction Integer

  leadingTerm pq
    [k= [b][a],c= 1]
              Type: Record(k: PoincareBirkhoffWittLyndonBasis Symbol,
                           c: Fraction Integer)

  degree pq
    2
                      Type: PositiveInteger

  pq4:=exp(pq,4)
                          1              1     2       1      2
     1 + [a b] + [b][a] + - [a b][a b] + - [a b ][a] + - [b][a b]
                          2              2             2
   +
     3              1
     - [b][a b][a] + - [b][b][a][a]
     2              2
                      Type: XPBWPolynomial(Symbol,Fraction Integer)

  log(pq4,4) - pq
   (26)  0
                      Type: XPBWPolynomial(Symbol,Fraction Integer)
```

Calculations with verification in XDistributedPolynomial.

```
  lp1 :lpoly := LiePoly liste.10
       3 2
     [a b ]
                      Type: LiePolynomial(Symbol,Fraction Integer)
```

```
lp2 :lpoly := LiePoly liste.11
    2
  [a b a b]
                    Type: LiePolynomial(Symbol,Fraction Integer)

lp :lpoly := [lp1, lp2]
    3 2 2
  [a b a b a b]
                    Type: LiePolynomial(Symbol,Fraction Integer)

lpd1: dpoly := lp1
   3 2     2         2 2                    2 2           2    2 3
  a b  - 2a b a b - a b a + 4a b a b a - a b a  - 2b a b a  + b a
                    Type: XDistributedPolynomial(Symbol,Fraction Integer)

lpd2: dpoly := lp2
    2         2 2         2                    2 2       3        2
   a b a b - a b a - 3a b a b + 4a b a b a - a b a  + 2b a b - 3b a b a
 +
          2
   b a b a
                    Type: XDistributedPolynomial(Symbol,Fraction Integer)

lpd : dpoly := lpd1 * lpd2 - lpd2 * lpd1
    3 2 2         3 2 2 2      3 2     2      3 2             3 2   2 2
   a b a b a b - a b a b a - 3a b a b a b + 4a b a b a b a - a b a b a
 +
     3 3 3      3 3 2       3 3     2    2       3 2     2       2 2
   2a b a b - 3a b a b a + a b a b a  - a b a b a b  + 3a b a b a b a
 +
     2           2       2                    2         2 2     2     2 3
   6a b a b a b a b - 12a b a b a b a b a + 3a b a b a b a  - 4a b a b a b
 +
     2     2 2       2     3 3    2 2 4 2     2 2 3          2 2 2   2
   6a b a b a b a - a b a b a  + a b a b  - 3a b a b a b + 3a b a b a b
 +
       2 2     3      2 2     2        2 2         2    2 2   2 3
   - 2a b a b a b + 3a b a b a b a - 3a b a b a b a  + a b a b a
 +
         2   3 2         2   2              2   2 2           2
   3a b a b a b  - 6a b a b a b a b - 3a b a b a b a + 12a b a b a b a b a
 +
           2     2 2         2 2     2         2 3 3             4 2
   - 3a b a b a b a  - 6a b a b a b a  + 3a b a b a  - 4a b a b a b
 +
              3                   2 2                    3
   12a b a b a b a b - 12a b a b a b a b + 8a b a b a b a b
 +
                    2                         2               2 3      2 5 2
   - 12a b a b a b a b a + 12a b a b a b a b a  - 4a b a b a b a  + a b a b
```

```
+
        2 4            2 3   2          2 2   3          2 2   2
  - 3a b a b a b + 3a b a b a b - 2a b a b a b + 3a b a b a b a
+
        2 2        2       2 2 2 3        3   3 2        3   2
  - 3a b a b a b a  + a b a b a  - 2b a b a b  + 4b a b a b a b
+
      3   2 2         3                   3     2 2        3 2     2        3 3 3
  2b a b a b a - 8b a b a b a b a + 2b a b a b a  + 4b a b a b a  - 2b a b a
+
      2   4 2        2   3              2   3 2          2   2
  3b a b a b  - 6b a b a b a b - 3b a b a b a + 12b a b a b a b a
+
        2   2 2 2        2            2        2     2 3          5 2
  - 3b a b a b a  - 6b a b a b a b a  + 3b a b a b a  - b a b a b
+
          4 2            3   2             3                 3 2 2
  3b a b a b a + 6b a b a b a b - 12b a b a b a b a + 3b a b a b a
+
            2   3             2   2             2 2 3    2 5         2 5 2
  - 4b a b a b a b + 6b a b a b a b a - b a b a b a  + b a b a b - b a b a
+
      2 4   2       2 4            2 4 2 2     2 3   3      2 3   2
  - 3b a b a b + 4b a b a b a - b a b a  + 2b a b a b - 3b a b a b a
+
   2 3       2
  b a b a b a
                Type: XDistributedPolynomial(Symbol,Fraction Integer)

lp :: dpoly - lpd
  0
                Type: XDistributedPolynomial(Symbol,Fraction Integer)
```

Calculations with verification in XRecursivePolynomial.

```
p := 3 * lp
     3 2 2
  3[a b a b a b]
                Type: XPBWPolynomial(Symbol,Fraction Integer)

q := lp1
    3 2
   [a b ]
                Type: XPBWPolynomial(Symbol,Fraction Integer)

pq:= p * q
     3 2 2        3 2
  3[a b a b a b][a b ]
                Type: XPBWPolynomial(Symbol,Fraction Integer)
```

```
pr:rpoly := p :: rpoly
     a
  *
         a
      *
             a b b
          *
              a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
             +
               b a(a(a b 6 + b a(- 9)) + b a a 3)
         +
             b
          *
                 a b
              *
                     a
                  *
                       a(a b b(- 3) + b b a 9)
                     +
                       b(a(a b 18 + b a(- 36)) + b a a 9)
                 +
                   b(a a(a b(- 12) + b a 18) + b a a a(- 3))
             +
                 b a
              *
                   a(a(a b b 3 + b a b(- 9)) + b a a b 9)
                 +
                   b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
     +
         b
      *
             a
          *
                 a b
              *
                     a
                  *
                       a(a b b 9 + b(a b(- 18) + b a(- 9)))
                     +
                       b(a b a 36 + b a a(- 9))
                 +
                   b(a b a a(- 18) + b a a a 9)
             +
                 b a
              *
                   a(a(a b b(- 12) + b a b 36) + b a a b(- 36))
                 +
                   b(a(a(a b 24 + b a(- 36)) + b a a 36) + b a a a(- 12))
         +
             b a a
```

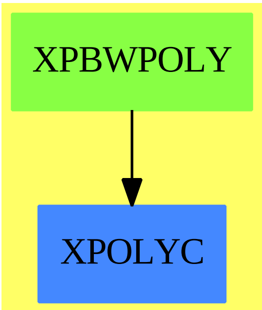
```
          *
               a(a(a b b 3 + b a b(- 9)) + b a a b 9)
             +
               b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
  +
      b
   *
          a
       *
              a
           *
                  a b
               *
                      a
                   *
                        a(a b b(- 6) + b(a b 12 + b a 6))
                      +
                        b(a b a(- 24) + b a a 6)
                  +
                    b(a b a a 12 + b a a a(- 6))
              +
                  b a
               *
                      a
                   *
                        a(a b b 9 + b(a b(- 18) + b a(- 9)))
                      +
                        b(a b a 36 + b a a(- 9))
                  +
                    b(a b a a(- 18) + b a a a 9)
          +
              b a a
           *
               a(a(a b b(- 3) + b b a 9) + b(a(a b 18 + b a(- 36)) + b a a 9))
             +
               b(a a(a b(- 12) + b a 18) + b a a a(- 3))
      +
          b a a a
       *
           a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
          +
           b a(a(a b 6 + b a(- 9)) + b a a 3)
                    Type: XRecursivePolynomial(Symbol,Fraction Integer)
```

```
qr:rpoly := q :: rpoly
   a(a(a b b 1 + b(a b(- 2) + b a(- 1))) + b(a b a 4 + b a a(- 1)))
  +
   b(a b a a(- 2) + b a a a 1)
                    Type: XRecursivePolynomial(Symbol,Fraction Integer)
```

```
  pq :: rpoly - pr*qr
    0
                      Type: XRecursivePolynomial(Symbol,Fraction Integer)

See Also:
o )show XPBWPolynomial
```

25.2.1 XPBWPolynomial (XPBWPOLY)



Exports:

0	1	characteristic	coef
coefficient	coefficients	coerce	constant
constant?	degree	exp	hash
latex	leadingCoefficient	leadingMonomial	leadingTerm
LiePolyIfCan	listOfTerms	log	lquo
map	maxdeg	mindeg	mindegTerm
mirror	monom	monomial?	monomials
numberOfMonomials	one?	product	quasiRegular
quasiRegular?	recip	reductum	retract
retractIfCan	rquo	sample	sh
subtractIfCan	trunc	varList	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?

— domain XPBWPOLY XPBWPolynomial —

```
)abbrev domain XPBWPOLY XPBWPolynomial
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
```

```
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain constructor implements polynomials in non-commutative
++ variables written in the Poincare-Birkhoff-Witt basis from the
++ Lyndon basis.
++ These polynomials can be used to compute Baker-Campbell-Hausdorff
++ relations.

XPBWPolynomial(VarSet:OrderedSet,R:CommutativeRing): XDPcat == XDPdef where

  WORD   ==> OrderedFreeMonoid(VarSet)
  LWORD  ==> LyndonWord(VarSet)
  LWORDS ==> List LWORD
  BASIS  ==> PoincareBirkhoffWittLyndonBasis(VarSet)
  TERM   ==> Record(k:BASIS, c:R)
  LTERMS ==> List(TERM)
  LPOLY  ==> LiePolynomial(VarSet,R)
  EX     ==> OutputForm
  XDPOLY ==> XDistributedPolynomial(VarSet,R)
  XRPOLY ==> XRecursivePolynomial(VarSet,R)
  TERM1  ==> Record(k:LWORD, c:R)
  NNI    ==> NonNegativeInteger
  I      ==> Integer
  RN     ==> Fraction(Integer)

  XDPcat == Join(XPolynomialsCat(VarSet,R), FreeModuleCat(R, BASIS)) with
    coerce      : LPOLY -> $
      ++ \axiom{coerce(p)} returns \axiom{p}.
    coerce      : $ -> XDPOLY
      ++ \axiom{coerce(p)} returns \axiom{p} as a distributed polynomial.
    coerce      : $ -> XRPOLY
      ++ \axiom{coerce(p)} returns \axiom{p} as a recursive polynomial.
    LiePolyIfCan: $ -> Union(LPOLY,"failed")
      ++ \axiom{LiePolyIfCan(p)} return  \axiom{p} if \axiom{p} is a Lie polynomial.
    product     : ($,$,NNI) -> $          -- produit tronque a l'ordre n
      ++ \axiom{product(a,b,n)} returns \axiom{a*b} (truncated up to order \axiom{n}).

    if R has Module(RN) then
       exp      : ($,NNI) -> $
          ++ \axiom{exp(p,n)} returns the exponential of \axiom{p}
          ++ (truncated up to order \axiom{n}).
       log      : ($,NNI) -> $
          ++ \axiom{log(p,n)} returns the logarithm of \axiom{p}
          ++ (truncated up to order \axiom{n}).
```

```
XDPdef == FreeModule1(R,BASIS) add
     import(TERM)

  -- Representation
     Rep:= LTERMS

  -- local functions
     prod1: (BASIS, $) -> $
     prod2: ($, BASIS) -> $
     prod : (BASIS, BASIS) -> $

     prod11: (BASIS, $, NNI) -> $
     prod22: ($, BASIS, NNI) -> $

     outForm : TERM -> EX
     Dexpand : BASIS -> XDPOLY
     Rexpand : BASIS -> XRPOLY
     process : (List LWORD, LWORD, List LWORD) -> $
     mirror1 : BASIS -> $

  -- functions locales
     outForm t ==
         t.c =$R 1 => t.k :: EX
         t.k =$BASIS 1 => t.c :: EX
         t.c::EX * t.k ::EX

     prod1(b:BASIS, p:$):$ ==
       +/ [t.c * prod(b, t.k) for t in p]

     prod2(p:$, b:BASIS):$ ==
       +/ [t.c * prod(t.k, b) for t in p]

     prod11(b,p,n) ==
         limit: I := n -$I length b
         +/ [t.c * prod(b, t.k) for t in p| length(t.k) :: I <= limit]

     prod22(p,b,n) ==
         limit: I := n -$I length b
         +/ [t.c * prod(t.k, b) for t in p| length(t.k) :: I <= limit]

     prod(g,d) ==
       d = 1 => monom(g,1)
       g = 1 => monom(d,1)
       process(reverse listOfTerms g, first d, rest listOfTerms d)

     Dexpand b ==
       b = 1 => 1$XDPOLY
       */ [LiePoly(l)$LPOLY :: XDPOLY for l in listOfTerms b]

     Rexpand b ==
```

```
      b = 1 => 1$XRPOLY
      */ [LiePoly(l)$LPOLY :: XRPOLY for l in listOfTerms b]

    mirror1(b:BASIS):$ ==
      b = 1 => 1
      lp: LPOLY := LiePoly first b
      lp := mirror lp
      mirror1(rest b) * lp :: $

    process(gauche, x, droite) ==    -- algo du "collect process"
      null gauche => monom( cons(x, droite) pretend BASIS, 1$R)
      r1, r2 : $
      not lexico(first gauche, x) =>     -- cas facile !!!
        monom(append(reverse gauche, cons(x, droite)) pretend BASIS , 1$R)

      p: LPOLY := [first gauche , x]      -- on crochete !!!
      null droite =>
        r1 :=  +/ [t.c * process(rest gauche, t.k, droite) for t in _
                   listOfTerms p]
        r2 :=  process( rest gauche, x, list first gauche)
        r1 + r2
      rd: List LWORD := rest droite; fd: LWORD := first droite
      r1 := +/ [t.c * process(list t.k, fd, rd) for t in  listOfTerms p]
      r1 := +/ [t.c * process(rest gauche, first t.k, rest listOfTerms(t.k))_
               for t in  r1]
      r2 := process([first gauche, x], fd, rd)
      r2 := +/ [t.c * process(rest gauche, first t.k, rest listOfTerms(t.k))_
               for t in  r2]
      r1 + r2

 -- definitions
    1 == monom(1$BASIS, 1$R)

    coerce(r:R):$ == [[1$BASIS , r]$TERM ]

    coerce(p:$):EX ==
      null p => (0$R) :: EX
      le : List EX := nil
      for rec in p repeat le := cons(outForm rec, le)
      reduce(_+, le)$List(EX)

    coerce(v: VarSet):$ == monom(v::BASIS , 1$R)
    coerce(p: LPOLY):$ ==
       [[t.k :: BASIS , t.c ]$TERM for t in listOfTerms p]

    coerce(p:$):XDPOLY ==
      +/ [t.c * Dexpand t.k for t in p]

    coerce(p:$):XRPOLY ==
      p = 0 => 0$XRPOLY
```

```
          +/ [t.c * Rexpand t.k for t in p]

        constant? p == (null p) or (leadingMonomial(p) =$BASIS 1)
        constant p ==
          null p => 0$R
          p.last.k = 1$BASIS => p.last.c
          0$R

        quasiRegular? p == (p=0) or (p.last.k ^= 1$BASIS)
        quasiRegular p ==
          p = 0 => p
          p.last.k = 1$BASIS => delete(p, maxIndex p)
          p

        x:$ * y:$ ==
          y = 0$$ => 0
          +/ [t.c * prod1(t.k, y) for t in x]

--        listOfTerms p == p pretend LTERMS

        varList p ==
           lv: List VarSet := "setUnion"/ [varList(b.k)$BASIS for b in p]
           sort(lv)

        degree(p) ==
           p=0 => error "null polynomial"
           length(leadingMonomial p)

        trunc(p, n) ==
          p = 0 => p
          degree(p) > n => trunc( reductum p , n)
          p

        product(x,y,n) ==
          x = 0 => 0
          y = 0 => 0
          +/ [t.c * prod11(t.k, y, n) for t in x]

        if R has Module(RN) then
          exp (p,n) ==
              p = 0 => 1
              not quasiRegular? p =>
                error "a proper polynomial is required"
              s : $ := 1 ; r: $ := 1                  -- resultat
              for i in 1..n repeat
                 k1 :RN := 1/i
                 k2 : R := k1 * 1$R
                 s := k2 * product(p, s, n)
                 r := r + s
              r
```

```
    log (p,n) ==
        p = 1 => 0
        p1: $ := 1 - p
        not quasiRegular? p1 =>
          error "constant term <> 1, impossible log "
        s : $ := - 1 ; r: $ := 0                 -- resultat
        for i in 1..n repeat
          k1 :RN := 1/i
          k2 : R := k1 * 1$R
          s := product(p1, s, n)
          r := k2 * s + r
        r

  LiePolyIfCan p ==
    p = 0 => 0$LPOLY
    "and"/ [retractable?(t.k)$BASIS for t in p] =>
       lt : List TERM1 := _
            [[retract(t.k)$BASIS, t.c]$TERM1 for t in p]
       lt pretend LPOLY
    "failed"

  mirror p ==
    +/ [t.c * mirror1(t.k) for t in p]
```

— **XPBWPOLY.dotabb** —

```
"XPBWPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XPBWPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XPBWPOLY" -> "XPOLYC"
```

25.3 domain XPOLY XPolynomial

— **XPolynomial.input** —

```
)set break resume
)sys rm -f XPolynomial.output
)spool XPolynomial.output
)set message test on
)set message auto off
```

```
)clear all
--S 1 of 14
poly := XPolynomial(Integer)
--R
--R
--R   (1)  XPolynomial Integer
--R                                                            Type: Domain
--E 1

--S 2 of 14
pr: poly := 2*x + 3*y-5
--R
--R
--R   (2)  - 5 + x 2 + y 3
--R                                                Type: XPolynomial Integer
--E 2

--S 3 of 14
pr2: poly := pr*pr
--R
--R
--R   (3)  25 + x(- 20 + x 4 + y 6) + y(- 30 + x 6 + y 9)
--R                                                Type: XPolynomial Integer
--E 3

--S 4 of 14
pd  := expand pr
--R
--R
--R   (4)  - 5 + 2x + 3y
--R                                 Type: XDistributedPolynomial(Symbol,Integer)
--E 4

--S 5 of 14
pd2 := pd*pd
--R
--R
--R                            2                   2
--R   (5)  25 - 20x - 30y + 4x  + 6x y + 6y x + 9y
--R                                 Type: XDistributedPolynomial(Symbol,Integer)
--E 5

--S 6 of 14
expand(pr2) - pd2
--R
--R
--R   (6)  0
--R                                 Type: XDistributedPolynomial(Symbol,Integer)
--E 6
```

```
--S 7 of 14
qr :=  pr**3
--R
--R
--R   (7)
--R     - 125 + x(150 + x(- 60 + x 8 + y 12) + y(- 90 + x 12 + y 18))
--R   +
--R     y(225 + x(- 90 + x 12 + y 18) + y(- 135 + x 18 + y 27))
--R                                                 Type: XPolynomial Integer
--E 7

--S 8 of 14
qd :=  pd**3
--R
--R
--R   (8)
--R                             2                       2     3      2
--R     - 125 + 150x + 225y - 60x  - 90x y - 90y x - 135y  + 8x  + 12x y + 12x y x
--R   +
--R          2        2                2       3
--R     18x y  + 12y x  + 18y x y + 18y x + 27y
--R                              Type: XDistributedPolynomial(Symbol,Integer)
--E 8

--S 9 of 14
trunc(qd,2)
--R
--R
--R                                2                       2
--R   (9)  - 125 + 150x + 225y - 60x  - 90x y - 90y x - 135y
--R                              Type: XDistributedPolynomial(Symbol,Integer)
--E 9

--S 10 of 14
trunc(qr,2)
--R
--R
--R   (10)  - 125 + x(150 + x(- 60) + y(- 90)) + y(225 + x(- 90) + y(- 135))
--R                                                 Type: XPolynomial Integer
--E 10

--S 11 of 14
Word := OrderedFreeMonoid Symbol
--R
--R
--R   (11)  OrderedFreeMonoid Symbol
--R                                                              Type: Domain
--E 11

--S 12 of 14
```

```
w: Word := x*y**2
--R
--R
--R            2
--R   (12)  x y
--R                                              Type: OrderedFreeMonoid Symbol
--E 12

--S 13 of 14
rquo(qr,w)
--R
--R
--R   (13)  18
--R                                                   Type: XPolynomial Integer
--E 13

--S 14 of 14
sh(pr,w::poly)
--R
--R
--R   (14)  x(x y y 4 + y(x y 2 + y(- 5 + x 2 + y 9))) + y x y y 3
--R                                                   Type: XPolynomial Integer
--E 14
)spool
)lisp (bye)
```

— XPolynomial.help —

```
====================================================================
XPolynomial examples
====================================================================

The XPolynomial domain constructor implements multivariate polynomials
whose set of variables is Symbol.  These variables do not commute.
The only parameter of this construtor is the coefficient ring which
may be non-commutative.  However, coefficients and variables commute.
The representation of the polynomials is recursive.  The abbreviation
for XPolynomial is XPOLY.

Other constructors like XPolynomialRing, XRecursivePolynomial as well
as XDistributedPolynomial, LiePolynomial and XPBWPolynomial implement
multivariate polynomials in non-commutative variables.

We illustrate now some of the facilities of the XPOLY domain constructor.

Define a polynomial ring over the integers.
```

```
poly := XPolynomial(Integer)
  XPolynomial Integer
                                 Type: Domain
```

Define a first polynomial,

```
pr: poly := 2*x + 3*y-5
  - 5 + x 2 + y 3
                                 Type: XPolynomial Integer
```

and a second one.

```
pr2: poly := pr*pr
  25 + x(- 20 + x 4 + y 6) + y(- 30 + x 6 + y 9)
                                 Type: XPolynomial Integer
```

Rewrite pr in a distributive way,

```
pd  := expand pr
  - 5 + 2x + 3y
                                 Type: XDistributedPolynomial(Symbol,Integer)
```

compute its square,

```
pd2 := pd*pd
                     2                    2
  25 - 20x - 30y + 4x  + 6x y + 6y x + 9y
                                 Type: XDistributedPolynomial(Symbol,Integer)
```

and checks that:

```
expand(pr2) - pd2
  0
                                 Type: XDistributedPolynomial(Symbol,Integer)
```

We define:

```
qr :=  pr**3
   - 125 + x(150 + x(- 60 + x 8 + y 12) + y(- 90 + x 12 + y 18))
 +
   y(225 + x(- 90 + x 12 + y 18) + y(- 135 + x 18 + y 27))
                                 Type: XPolynomial Integer
```

and:

```
qd :=  pd**3
                          2                        2     3      2
   - 125 + 150x + 225y - 60x  - 90x y - 90y x - 135y  + 8x  + 12x y + 12x y x
 +
       2        2                2       3
```

```
     18x y  + 12y x  + 18y x y + 18y x + 27y
                                 Type: XDistributedPolynomial(Symbol,Integer)
```

We truncate qd at degree 3.

```
  trunc(qd,2)
                           2                       2
    - 125 + 150x + 225y - 60x  - 90x y - 90y x - 135y
                                 Type: XDistributedPolynomial(Symbol,Integer)
```

The same for qr:

```
  trunc(qr,2)
    - 125 + x(150 + x(- 60) + y(- 90)) + y(225 + x(- 90) + y(- 135))
                                 Type: XPolynomial Integer
```

We define:

```
  Word := OrderedFreeMonoid Symbol
    OrderedFreeMonoid Symbol
                                 Type: Domain
```

and:

```
  w: Word := x*y**2
       2
    x y
                                 Type: OrderedFreeMonoid Symbol
```

We can compute the right-quotient of qr by r:

```
  rquo(qr,w)
    18
                                 Type: XPolynomial Integer
```

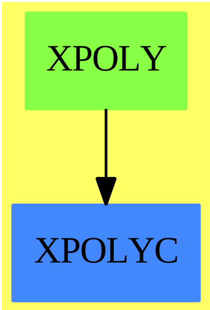
and the shuffle-product of pr by r:

```
  sh(pr,w::poly)
    x(x y y 4 + y(x y 2 + y(- 5 + x 2 + y 9))) + y x y y 3
                                 Type: XPolynomial Integer
```

See Also:
o)help XPBWPolynomial
o)help LiePolynomial
o)help XDistributedPolynomial
o)help XRecursivePolynomial
o)help XPolynomialRing
o)show XPolynomial

———-

25.3.1 XPolynomial (XPOLY)



Exports:

0	1	characteristic	coef	coerce
constant	constant?	degree	expand	hash
latex	lquo	map	maxdeg	mindeg
mindegTerm	mirror	monom	monomial?	RemainderList
one?	quasiRegular	quasiRegular?	recip	retract
retractIfCan	rquo	sample	sh	subtractIfCan
trunc	unexpand	varList	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?			

— **domain XPOLY XPolynomial** —

```
)abbrev domain XPOLY XPolynomial
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++   extend renomme en expand
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type supports multivariate polynomials whose set of variables
++ is \spadtype{Symbol}. The representation is recursive.
++ The coefficient ring may be non-commutative and the variables
++ do not commute. However, coefficients and variables commute.
```

```
XPolynomial(R:Ring) == XRecursivePolynomial(Symbol, R)
```

———-

— XPOLY.dotabb —

```
"XPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XPOLY" -> "XPOLYC"
```

———-

25.4 domain XPR XPolynomialRing

— XPolynomialRing.input —

```
)set break resume
)sys rm -f XPolynomialRing.output
)spool XPolynomialRing.output
)set message test on
)set message auto off
)clear all
--S 1 of 15
Word := OrderedFreeMonoid(Symbol)
--R
--R
--R   (1)  OrderedFreeMonoid Symbol
--R                                                              Type: Domain
--E 1

--S 2 of 15
poly:= XPR(Integer,Word)
--R
--R
--R   (2)  XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--R                                                              Type: Domain
--E 2

--S 3 of 15
p:poly := 2 * x - 3 * y + 1
--R
--R
--R   (3)  1 + 2x - 3y
```

```
--R                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 3

--S 4 of 15
q:poly := 2 * x + 1
--R
--R
--R   (4)  1 + 2x
--R                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 4

--S 5 of 15
p + q
--R
--R
--R   (5)  2 + 4x - 3y
--R                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 5

--S 6 of 15
p * q
--R
--R
--R                         2
--R   (6)  1 + 4x - 3y + 4x  - 6y x
--R                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 6

--S 7 of 15
(p+q)**2-p**2-q**2-2*p*q
--R
--R
--R   (7)  - 6x y + 6y x
--R                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 7

--S 8 of 15
M := SquareMatrix(2,Fraction Integer)
--R
--R
--R   (8)  SquareMatrix(2,Fraction Integer)
--R                                                                   Type: Domain
--E 8

--S 9 of 15
poly1:= XPR(M,Word)
--R
--R
--R   (9)
--R   XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
```

```
--R                                                                Type: Domain
--E 9

--S 10 of 15
m1:M := matrix [ [i*j**2 for i in 1..2] for j in 1..2]
--R
--R
--R         +1  2+
--R   (10)  |    |
--R         +4  8+
--R                                     Type: SquareMatrix(2,Fraction Integer)
--E 10

--S 11 of 15
m2:M := m1 - 5/4
--R
--R
--R         +  1    +
--R         |- -  2 |
--R         |  4    |
--R   (11)  |       |
--R         |     27|
--R         | 4   --|
--R         +      4+
--R                                     Type: SquareMatrix(2,Fraction Integer)
--E 11

--S 12 of 15
m3: M := m2**2
--R
--R
--R         +129     +
--R         |---  13 |
--R         | 16     |
--R   (12)  |        |
--R         |     857|
--R         |26   ---|
--R         +      16+
--R                                     Type: SquareMatrix(2,Fraction Integer)
--E 12

--S 13 of 15
pm:poly1   := m1*x + m2*y + m3*z - 2/3
--R
--R
--R         +  2     +             +  1    +    +129     +
--R         |- -   0 |             |- -  2 |    |---  13 |
--R         |  3     |   +1  2+    |  4    |    | 16     |
--R   (13)  |        | + |    |x + |       |y + |        |z
--R         |       2|   +4  8+    |     27|    |     857|
```

```
--R         | 0    - -|              | 4    --|    |26    ---|
--R         +       3+              +      4+    +      16+
--RType: XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
--E 13

--S 14 of 15
qm:poly1 := pm - m1*x
--R
--R
--R         +  2     +   +  1    +    +129     +
--R         |- -   0 |   |- -  2 |    |---  13 |
--R         |  3     |   |  4    |    | 16     |
--R   (14)  |        | + |       |y + |        |z
--R         |       2|   |     27|    |     857|
--R         | 0   - -|   | 4   --|    |26   ---|
--R         +       3+   +      4+    +      16+
--RType: XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
--E 14

--S 15 of 15
qm**3
--R
--R
--R   (15)
--R     +   8      +   +  1  8+    +43   52 +    +  129       +
--R     |- --   0  |   |- -  -|    |--   -- |    |- ---  - 26 |
--R     |  27      |   |  3  3|    | 4    3 |    |   8        | 2
--R     |          | + |      |y + |        |z + |            |y
--R     |         8|   |16    |    |104  857|    |         857|
--R     | 0    - --|   |--   9|    |---  ---|    |- 52   - ---|
--R     +        27+   + 3    +    + 3    12+    +          8 +
--R   +
--R     +  3199      831 +      +  3199     831 +      +  103169     6409 +
--R     |- ----    - --- |      |- ----   - --- |      |- ------   - ---- |
--R     |   32       4   |      |   32       4  |      |    128       4  | 2
--R     |                |y z + |               |z y + |                 |z
--R     |  831     26467|      |  831     26467|      |   6409     820977|
--R     |- ---   - -----|      |- ---   - -----|      | - ----   - ------|
--R     +   2        32 +      +   2        32 +      +     2       128 +
--R   +
--R     +3199   831 +     +103169   6409 +      +103169   6409 +
--R     |----   --- |     |------   ---- |      |------   ---- |
--R     | 64     8  | 3   |  256      8  | 2    |  256      8  |
--R     |           |y  + |              |y z + |              |y z y
--R     |831   26467|     | 6409   820977|      | 6409   820977|
--R     |---   -----|     | ----   ------|      | ----   ------|
--R     + 4      64 +     +   4      256 +      +   4      256 +
--R   +
--R     +3178239   795341 +       +103169   6409 +       +3178239   795341 +
--R     |-------   ------ |       |------   ---- |       |-------   ------ |
```

```
--R     |  1024      128  |    2   |  256      8  |    2   |  1024      128  |
--R     |                 |y z  + |              |z y  + |                 |z y z
--R     |795341   25447787|       | 6409   820977|       |795341   25447787|
--R     |------   --------|       | ----   ------|       |------   --------|
--R     +  64       1024  +       +   4      256 +       +  64       1024  +
--R   +
--R     +3178239   795341 +      +98625409  12326223 +
--R     |-------   ------ |      |--------  -------- |
--R     |  1024      128  | 2    |  4096       256   | 3
--R     |                 |z y + |                   |z
--R     |795341   25447787|      |12326223  788893897|
--R     |------   --------|      |--------  ---------|
--R     +  64       1024  +      +   128       4096  +
--RType: XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
--E 15
)spool
)lisp (bye)
```

— **XPolynomialRing.help** —

```
====================================================================
XPolynomialRing examples
====================================================================

The XPolynomialRing domain constructor implements generalized
polynomials with coefficients from an arbitrary Ring (not necessarily
commutative) and whose exponents are words from an arbitrary
OrderedMonoid (not necessarily commutative too).  Thus these
polynomials are (finite) linear combinations of words.

This constructor takes two arguments.  The first one is a Ring and the
second is an OrderedMonoid.  The abbreviation for XPolynomialRing is XPR.

Other constructors like XPolynomial, XRecursivePolynomial,
XDistributedPolynomial, LiePolynomial and XPBWPolynomial implement
multivariate polynomials in non-commutative variables.

We illustrate now some of the facilities of the XPR domain constructor.

Define the free ordered monoid generated by the symbols.

  Word := OrderedFreeMonoid(Symbol)
    OrderedFreeMonoid Symbol
                             Type: Domain

Define the linear combinations of these words with integer coefficients.
```

```
poly:= XPR(Integer,Word)
  XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
                               Type: Domain
```

Then we define a first element from poly.

```
p:poly := 2 * x - 3 * y + 1
  1 + 2x - 3y
                 Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

And a second one.

```
q:poly := 2 * x + 1
  1 + 2x
                 Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

We compute their sum,

```
p + q
  2 + 4x - 3y
                 Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

their product,

```
p * q
                  2
  1 + 4x - 3y + 4x  - 6y x
                 Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

and see that variables do not commute.

```
(p+q)**2-p**2-q**2-2*p*q
  - 6x y + 6y x
                 Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

Now we define a ring of square matrices,

```
M := SquareMatrix(2,Fraction Integer)
  SquareMatrix(2,Fraction Integer)
                               Type: Domain
```

and the linear combinations of words with these matrices as coefficients.

```
poly1:= XPR(M,Word)
 XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
                               Type: Domain
```

Define a first matrix,

```
m1:M := matrix [ [i*j**2 for i in 1..2] for j in 1..2]
```

```
        +1  2+
        |    |
        +4  8+
                                Type: SquareMatrix(2,Fraction Integer)
```

a second one,

```
  m2:M := m1 - 5/4
        +  1    +
        |- -  2 |
        |  4    |
        |       |
        |     27|
        | 4   --|
        +      4+
                               Type: SquareMatrix(2,Fraction Integer)
```

and a third one.

```
  m3: M := m2**2
        +129     +
        |---  13 |
        | 16     |
        |        |
        |     857|
        |26   ---|
        +      16+
                              Type: SquareMatrix(2,Fraction Integer)
```

Define a polynomial,

```
  pm:poly1   := m1*x + m2*y + m3*z - 2/3
        +  2     +             +  1    +    +129     +
        |- -   0 |             |- -  2 |    |---  13 |
        |  3     |   +1  2+    |  4    |    | 16     |
        |        | + |    |x + |       |y + |        |z
        |       2|   +4  8+    |     27|    |     857|
        | 0   - -|             | 4   --|    |26   ---|
        +       3+             +      4+    +      16+
     Type: XPolynomialRing(SquareMatrix(2,Fraction Integer),
                           OrderedFreeMonoid Symbol)
```

a second one,

```
  qm:poly1 := pm - m1*x
        +  2     +   +  1    +    +129     +
        |- -   0 |   |- -  2 |    |---  13 |
        |  3     |   |  4    |    | 16     |
        |        | + |       |y + |        |z
        |       2|   |     27|    |     857|
```

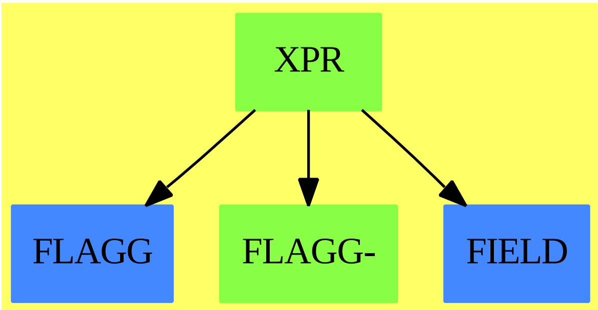
```
        | 0   - -|   | 4   --|    |26   ---|
        +       3+   +      4+    +      16+
     Type: XPolynomialRing(SquareMatrix(2,Fraction Integer),
                           OrderedFreeMonoid Symbol)
```

and the following power.

```
  qm**3
     +   8      +    +  1  8+    +43   52 +    +  129       +
     |- --   0  |    |- -  -|    |--   -- |    |- ---  - 26 |
     |  27      |    |  3  3|    | 4    3 |    |   8        | 2
     |          | +  |      |y + |        |z + |            |y
     |         8|    |16    |    |104  857|    |         857|
     | 0    - --|    |--   9|    |---  ---|    |- 52   - ---|
     +        27+    + 3    +    + 3    12+    +          8 +
   +
     +  3199     831 +      +  3199     831 +      +  103169     6409 +
     |- ----   - --- |      |- ----   - --- |      |- ------   - ---- |
     |   32       4  |      |   32       4  |      |    128        4  | 2
     |               |y z + |               |z y + |                  |z
     |  831     26467|      |  831     26467|      |  6409     820977|
     |- ---   - -----|      |- ---   - -----|      | - ----   - ------|
     +   2        32 +      +   2        32 +      +     2        128 +
   +
     +3199   831 +     +103169   6409 +      +103169   6409 +
     |----   --- |     |------   ---- |      |------   ---- |
     | 64     8  | 3   |  256      8  | 2    |  256      8  |
     |           |y  + |              |y z + |              |y z y
     |831   26467|     | 6409   820977|      | 6409   820977|
     |---   -----|     | ----   ------|      | ----   ------|
     + 4      64 +     +   4      256 +      +   4      256 +
   +
     +3178239   795341 +       +103169   6409 +       +3178239   795341 +
     |-------   ------ |       |------   ---- |       |-------   ------ |
     |  1024      128  |   2   |  256      8  |   2   |  1024      128  |
     |                 |y z  + |              |z y  + |                 |z y z
     |795341   25447787|       | 6409   820977|       |795341   25447787|
     |------   --------|       | ----   ------|       |------   --------|
     +  64       1024  +       +   4      256 +       +  64       1024  +
   +
     +3178239   795341 +      +98625409  12326223 +
     |-------   ------ |      |--------  -------- |
     |  1024      128  | 2    |  4096       256   | 3
     |                 |z y + |                   |z
     |795341   25447787|      |12326223  788893897|
     |------   --------|      |--------  ---------|
     +  64       1024  +      +   128       4096  +
        Type: XPolynomialRing(SquareMatrix(2,Fraction Integer),
                              OrderedFreeMonoid Symbol)
```

```
See Also:
o )help XPBWPolynomial
o )help LiePolynomial
o )help XDistributedPolynomial
o )help XRecursivePolynomial
o )help XPolynomial
o )show XPolynomialRing
```

25.4.1 XPolynomialRing (XPR)



Exports:

0	1	characteristic	coef
coefficient	coefficients	coerce	constant
constant?	hash	latex	leadingCoefficient
leadingMonomial	leadingTerm	listOfTerms	map
maxdeg	mindeg	monom	monomial?
monomials	numberOfMonomials	one?	quasiRegular
quasiRegular?	recip	reductum	retract
retractIfCan	sample	subtractIfCan	zero?
#?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?			

— domain XPR XPolynomialRing —

```
)abbrev domain XPR XPolynomialRing
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
```

```
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents generalized polynomials with coefficients
++ (from a not necessarily commutative ring), and words
++ belonging to an arbitrary \spadtype{OrderedMonoid}.
++ This type is used, for instance, by the \spadtype{XDistributedPolynomial}
++ domain constructor where the Monoid is free.

XPolynomialRing(R:Ring,E:OrderedMonoid): T == C where
  TERM   ==> Record(k: E, c: R)
  EX     ==> OutputForm
  NNI    ==> NonNegativeInteger

  T == Join(Ring, XAlgebra(R), FreeModuleCat(R,E)) with
    --operations
      "*": (%,R) -> %
        ++ \spad{p*r} returns the product of \spad{p} by \spad{r}.
      "#": % -> NonNegativeInteger
        ++ \spad{# p} returns the number of terms in \spad{p}.
      coerce: E -> %
        ++ \spad{coerce(e)} returns \spad{1*e}
      maxdeg: % -> E
        ++ \spad{maxdeg(p)} returns the greatest word occurring in the polynomial \spad{p}
        ++ with a non-zero coefficient. An error is produced if  \spad{p} is zero.
      mindeg: % -> E
        ++ \spad{mindeg(p)} returns the smallest word occurring in the polynomial \spad{p}
        ++ with a non-zero coefficient. An error is produced if  \spad{p} is zero.
      reductum : % -> %
        ++ \spad{reductum(p)} returns \spad{p} minus its leading term.
        ++ An error is produced if  \spad{p} is zero.
      coef  : (%,E) -> R
        ++ \spad{coef(p,e)} extracts the coefficient of the monomial \spad{e}.
        ++ Returns zero if \spad{e} is not present.
      constant?:% -> Boolean
        ++ \spad{constant?(p)} tests whether the polynomial \spad{p} belongs to the
        ++ coefficient ring.
      constant: % -> R
        ++ \spad{constant(p)} return the constant term of \spad{p}.
      quasiRegular? : % -> Boolean
        ++ \spad{quasiRegular?(x)} return true if \spad{constant(p)} is zero.
      quasiRegular : % -> %
        ++ \spad{quasiRegular(x)} return \spad{x} minus its constant term.
      map   : (R -> R, %) -> %
        ++ \spad{map(fn,x)} returns \spad{Sum(fn(r_i) w_i)} if \spad{x} writes \spad{Sum(r_i
      if R has Field then "/" : (%,R) -> %
        ++ \spad{p/r} returns \spad{p*(1/r)}.
```

```
  --assertions
    if R has noZeroDivisors then noZeroDivisors
    if R has unitsKnown then unitsKnown
    if R has canonicalUnitNormal then canonicalUnitNormal
        ++ canonicalUnitNormal guarantees that the function
        ++ unitCanonical returns the same representative for all
        ++ associates of any particular element.


 C == FreeModule1(R,E) add
  --representations
     Rep:=  List TERM
  --uses
     repeatMultExpt: (%,NonNegativeInteger) -> %
  --define
     1  == [[1$E,1$R]]

     characteristic  == characteristic$R
     #x == #$Rep x
     maxdeg p == if null p then  error " polynome nul !!"
                           else p.first.k
     mindeg p == if null p then  error " polynome nul !!"
                           else (last p).k

     coef(p,e)  ==
        for tm in p repeat
          tm.k=e => return tm.c
          tm.k < e => return 0$R
        0$R

     constant? p == (p = 0) or (maxdeg(p) = 1$E)
     constant  p == coef(p,1$E)

     quasiRegular? p == (p=0) or (last p).k ^= 1$E
     quasiRegular  p ==
        quasiRegular?(p) => p
        [t for t in p | not(t.k = 1$E)]

     recip(p) ==
         p=0 => "failed"
         p.first.k > 1$E => "failed"
         (u:=recip(p.first.c)) case "failed" => "failed"
         (u::R)::%

     coerce(r:R) == if r=0$R then 0$% else [[1$E,r]]
     coerce(n:Integer) == (n::R)::%

     if R has noZeroDivisors then
       p1:% * p2:%  ==
```

```
            null p1 => 0
            null p2 => 0
            p1.first.k = 1$E => p1.first.c * p2
            p2 = 1 => p1
--            +/[[[t1.k*t2.k,t1.c*t2.c]$TERM for t2 in p2]
--                   for t1 in reverse(p1)]
            +/[[[t1.k*t2.k,t1.c*t2.c]$TERM for t2 in p2]
                   for t1 in p1]
        else
         p1:% * p2:%  ==
            null p1 => 0
            null p2 => 0
            p1.first.k = 1$E => p1.first.c * p2
            p2 = 1 => p1
--            +/[[[t1.k*t2.k,r]$TERM for t2 in p2 | not (r:=t1.c*t2.c) =$R 0]
--                   for t1 in reverse(p1)]
            +/[[[t1.k*t2.k,r]$TERM for t2 in p2 | not (r:=t1.c*t2.c) =$R 0]
                   for t1 in p1]
       p:% ** nn:NNI  == repeatMultExpt(p,nn)
       repeatMultExpt(x,nn) ==
               nn = 0 => 1
               y:% := x
               for i in 2..nn repeat y:= x * y
               y

       outTerm(r:R, m:E):EX ==
            r=1 => m::EX
            m=1 => r::EX
            r::EX * m::EX

--       coerce(x:%) : EX ==
--         null x => (0$R) :: EX
--         le : List EX := nil
--         for rec in x repeat
--           rec.c = 1$R => le := cons(rec.k :: EX, le)
--           rec.k = 1$E => le := cons(rec.c :: EX, le)
--           le := cons(mkBinary("*"::EX,rec.c :: EX,
--             rec.k :: EX), le)
--         1 = #le => first le
--         mkNary("+" :: EX,le)

       coerce(a:%):EX ==
            empty? a => (0$R)::EX
            reduce(_+, reverse_! [outTerm(t.c, t.k) for t in a])$List(EX)


       if R has Field then
          x/r == inv(r)*x
```

———-

— **XPR.dotabb** —

```
"XPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XPR"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FIELD"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"XPR" -> "FLAGG"
"XPR" -> "FLAGG-"
"XPR" -> "FIELD"
```

———-

25.5 domain XRPOLY XRecursivePolynomial

— **XRecursivePolynomial.input** —

```
)set break resume
)sys rm -f XRecursivePolynomial.output
)spool XRecursivePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XRecursivePolynomial
--R XRecursivePolynomial(VarSet: OrderedSet,R: Ring)  is a domain constructor
--R Abbreviation for XRecursivePolynomial is XRPOLY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for XRPOLY
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (VarSet,%) -> %                ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                     ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,PositiveInteger) -> %
--R coef : (%,%) -> R                    coerce : VarSet -> %
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             constant : % -> R
--R constant? : % -> Boolean             degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger            latex : % -> String
```

```
--R lquo : (%,%) -> %                    lquo : (%,VarSet) -> %
--R map : ((R -> R),%) -> %              mirror : % -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R quasiRegular : % -> %                quasiRegular? : % -> Boolean
--R recip : % -> Union(%,"failed")       rquo : (%,%) -> %
--R rquo : (%,VarSet) -> %               sample : () -> %
--R varList : % -> List VarSet           zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R RemainderList : % -> List Record(k: VarSet,c: %)
--R ?^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid VarSet) -> R
--R coerce : OrderedFreeMonoid VarSet -> %
--R expand : % -> XDistributedPolynomial(VarSet,R)
--R lquo : (%,OrderedFreeMonoid VarSet) -> %
--R maxdeg : % -> OrderedFreeMonoid VarSet
--R mindeg : % -> OrderedFreeMonoid VarSet
--R mindegTerm : % -> Record(k: OrderedFreeMonoid VarSet,c: R)
--R monom : (OrderedFreeMonoid VarSet,R) -> %
--R retract : % -> OrderedFreeMonoid VarSet
--R retractIfCan : % -> Union(OrderedFreeMonoid VarSet,"failed")
--R rquo : (%,OrderedFreeMonoid VarSet) -> %
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R sh : (%,%) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R trunc : (%,NonNegativeInteger) -> %
--R unexpand : XDistributedPolynomial(VarSet,R) -> %
--R
--E 1

)spool
)lisp (bye)
```

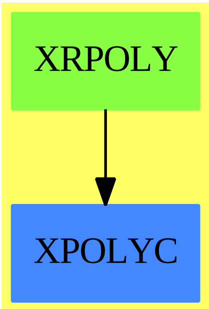
— **XRecursivePolynomial.help** —

```
====================================================================
XRecursivePolynomial examples
====================================================================

See Also:
o )show XRecursivePolynomial
```

Polynomial arithmetic with non-commutative variables has been improved by a contribution of Michel Petitot (University of Lille I, France). The domain constructors **XRecursivePolynomial** provides a recursive for these polynomials. It is the non-commutative equivalents for the **SparseMultivariatePolynomial** constructor.

25.5.1 XRecursivePolynomial (XRPOLY)



Exports:

0	1	characteristic	coef	coerce
constant	constant?	degree	expand	hash
latex	lquo	map	maxdeg	mindeg
mindegTerm	mirror	monom	monomial?	one?
quasiRegular	quasiRegular?	recip	RemainderList	retract
retractIfCan	rquo	sample	sh	subtractIfCan
trunc	unexpand	varList	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?			

— domain XRPOLY XRecursivePolynomial —

```
)abbrev domain XRPOLY XRecursivePolynomial
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++   extend renomme en expand
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type supports multivariate polynomials whose variables do not commute.
++ The representation is recursive. The coefficient ring may be
++ non-commutative. Coefficients and variables commute.
```

```
XRecursivePolynomial(VarSet:OrderedSet,R:Ring):  Xcat == Xdef where
  I      ==> Integer
  NNI    ==> NonNegativeInteger
  XDPOLY ==> XDistributedPolynomial(VarSet, R)
  EX     ==> OutputForm
  WORD   ==> OrderedFreeMonoid(VarSet)
  TERM   ==> Record(k:VarSet , c:%)
  LTERMS ==> List(TERM)
  REGPOLY==> FreeModule1(%, VarSet)
  VPOLY  ==> Record(c0:R, reg:REGPOLY)

  Xcat == XPolynomialsCat(VarSet,R) with
       expand: % -> XDPOLY
         ++ \spad{expand(p)} returns \spad{p} in distributed form.
       unexpand : XDPOLY -> %
         ++ \spad{unexpand(p)} returns \spad{p} in recursive form.
       RemainderList: % -> LTERMS
         ++ \spad{RemainderList(p)} returns the regular part of \spad{p}
         ++ as a list of terms.

  Xdef == add
       import(VPOLY)

    -- representation
       Rep     := Union(R,VPOLY)

    -- local functions
       construct: LTERMS -> REGPOLY
       simplifie: VPOLY -> %
       lquo1: (LTERMS,LTERMS) -> %        -- a ajouter
       coef1: (LTERMS,LTERMS) -> R        -- a ajouter
       outForm: REGPOLY -> EX

    --define
       construct(lt) == lt pretend REGPOLY
       p1:%  =  p2:%  ==
         p1 case R =>
             p2 case R => p1 =$R p2
             false
         p2 case R => false
         p1.c0 =$R p2.c0 and p1.reg =$REGPOLY p2.reg

       monom(w, r) ==
         r =0 => 0
         r * w::%

--       if R has Field then                  -- Bug non resolu !!!!!!!!
--         p:% / r: R == inv(r) * p

       rquo(p1:%, p2:%):% ==
```

```
  p2 case R => p1 * p2::R
  p1 case R => p1  * p2.c0
  x:REGPOLY := construct [[t.k, a]$TERM for t in listOfTerms(p1.reg) _
                 | (a:= rquo(t.c,p2)) ^= 0$% ]$LTERMS
  simplifie [coef(p1,p2) , x]$VPOLY

trunc(p,n) ==
  n = 0 or (p case R) => (constant p)::%
  n1: NNI := (n-1)::NNI
  lt: LTERMS := [[t.k, r]$TERM for t in listOfTerms p.reg _
                 | (r := trunc(t.c, n1)) ^= 0]$LTERMS
  x: REGPOLY := construct lt
  simplifie [constant p, x]$VPOLY

unexpand p ==
  constant? p => (constant p)::%
  vl: List VarSet := sort((y,z) +-> y > z, varList p)
  x : REGPOLY := _
    construct [[v, unexpand r]$TERM for v in vl| (r:=lquo(p,v)) ^= 0]
  [constant p, x]$VPOLY

if R has CommutativeRing then
  sh(p:%, n:NNI):% ==
     n = 0 => 1
     p case R => (p::R)** n
     n1: NNI := (n-1)::NNI
     p1: % := n * sh(p, n1)
     lt: LTERMS := [[t.k, sh(t.c, p1)]$TERM for t in listOfTerms p.reg]
     [p.c0 ** n, construct lt]$VPOLY

  sh(p1:%, p2:%) ==
     p1 case R => p1::R * p2
     p2 case R => p1 * p2::R
     lt1:LTERMS := listOfTerms p1.reg ; lt2:LTERMS := listOfTerms p2.reg
     x: REGPOLY := construct [[t.k,sh(t.c,p2)]$TERM for t in lt1]
     y: REGPOLY := construct [[t.k,sh(p1,t.c)]$TERM for t in lt2]
     [p1.c0*p2.c0,x + y]$VPOLY

RemainderList p ==
    p case R => []
    listOfTerms( p.reg)$REGPOLY

lquo(p1:%,p2:%):% ==
  p2 case R => p1 * p2
  p1 case R => p1  *$R p2.c0
  p1 * p2.c0 +$% lquo1(listOfTerms p1.reg, listOfTerms p2.reg)

lquo1(x:LTERMS,y:LTERMS):% ==
  null x => 0$%
  null y => 0$%
```

```
  x.first.k < y.first.k => lquo1(x,y.rest)
  x.first.k = y.first.k =>
      lquo(x.first.c,y.first.c) + lquo1(x.rest,y.rest)
  return lquo1(x.rest,y)

coef(p1:%, p2:%):R ==
  p1 case R => p1::R * constant p2
  p2 case R => p1.c0 * p2::R
  p1.c0 * p2.c0 +$R coef1(listOfTerms p1.reg, listOfTerms p2.reg)

coef1(x:LTERMS,y:LTERMS):R ==
  null x => 0$R
  null y => 0$R
  x.first.k < y.first.k => coef1(x,y.rest)
  x.first.k = y.first.k =>
      coef(x.first.c,y.first.c) + coef1(x.rest,y.rest)
  return coef1(x.rest,y)

--------------------------------------------------------------
outForm(p:REGPOLY): EX ==
   le : List EX :=  [t.k::EX * t.c::EX for t in listOfTerms p]
   reduce(_+, reverse_! le)$List(EX)

coerce(p:$): EX ==
   p case R => (p::R)::EX
   p.c0 = 0 => outForm p.reg
   p.c0::EX + outForm p.reg

0 == 0$R::%
1 == 1$R::%
constant? p ==  p case R
constant p ==
   p case R => p
   p.c0

simplifie p ==
  p.reg = 0$REGPOLY => (p.c0)::%
  p

coerce (v:VarSet):% ==
  [0$R,coerce(v)$REGPOLY]$VPOLY

coerce (r:R):% == r::%
coerce (n:Integer) == n::R::%
coerce (w:WORD) ==
  w = 1 => 1$R
  (first w) * coerce(rest w)

expand p ==
  p case R => p::R::XDPOLY
```

```
      lt:LTERMS := listOfTerms(p.reg)
      ep:XDPOLY := (p.c0)::XDPOLY
      for t in lt repeat
        ep:= ep + t.k * expand(t.c)
      ep

    - p:% ==
      p case R => -$R p
      [- p.c0, - p.reg]$VPOLY

    p1 + p2 ==
      p1 case R and p2 case R => p1 +$R p2
      p1 case R => [p1 + p2.c0 , p2.reg]$VPOLY
      p2 case R => [p2 + p1.c0 , p1.reg]$VPOLY
      simplifie [p1.c0 + p2.c0 , p1.reg +$REGPOLY p2.reg]$VPOLY

    p1 - p2 ==
      p1 case R and p2 case R => p1 -$R p2
      p1 case R => [p1 - p2.c0 , -p2.reg]$VPOLY
      p2 case R => [p1.c0 - p2 , p1.reg]$VPOLY
      simplifie [p1.c0 - p2.c0 , p1.reg -$REGPOLY p2.reg]$VPOLY

    n:Integer * p:% ==
      n=0 => 0$%
      p case R => n *$R p
      -- [ n*p.c0,n*p.reg]$VPOLY
      simplifie [ n*p.c0,n*p.reg]$VPOLY

    r:R * p:% ==
      r=0 => 0$%
      p case R => r *$R p
      -- [ r*p.c0,r*p.reg]$VPOLY
      simplifie [ r*p.c0,r*p.reg]$VPOLY

    p:% * r:R ==
      r=0 => 0$%
      p case R => p *$R r
      -- [ p.c0 * r,p.reg * r]$VPOLY
      simplifie [ r*p.c0,r*p.reg]$VPOLY

    v:VarSet * p:% ==
       p = 0 => 0$%
       [0$R, v *$REGPOLY p]$VPOLY

    p1:% * p2:% ==
      p1 case R => p1::R * p2
      p2 case R => p1 * p2::R
      x:REGPOLY := p1.reg *$REGPOLY p2
      y:REGPOLY := (p1.c0)::% *$REGPOLY p2.reg  -- maladroit:(p1.c0)::% !!
      -- [ p1.c0 * p2.c0 , x+y ]$VPOLY
```

```
  simplifie [ p1.c0 * p2.c0 , x+y ]$VPOLY

lquo(p:%, v:VarSet):% ==
  p case R => 0
  coefficient(p.reg,v)$REGPOLY

lquo(p:%, w:WORD):% ==
  w = 1$WORD => p
  lquo(lquo(p,first w),rest w)

rquo(p:%, v:VarSet):% ==
  p case R => 0
  x:REGPOLY := construct [[t.k, a]$TERM for t in listOfTerms(p.reg)
                  | (a:= rquo(t.c,v)) ^= 0 ]
  simplifie [constant(coefficient(p.reg,v)) , x]$VPOLY

rquo(p:%, w:WORD):% ==
  w = 1$WORD => p
  rquo(rquo(p,rest w),first w)

coef(p:%, w:WORD):R ==
  constant lquo(p,w)

quasiRegular? p ==
  p case R => p = 0$R
  p.c0 = 0$R

quasiRegular p ==
  p case R => 0$%
  [0$R,p.reg]$VPOLY

characteristic == characteristic()$R
recip p ==
  p case R => recip(p::R)
  "failed"

mindeg p ==
  p case R =>
    p = 0 => error "XRPOLY.mindeg: polynome nul !!"
    1$WORD
  p.c0 ^= 0 => 1$WORD
  "min"/[(t.k) *$WORD mindeg(t.c) for t in listOfTerms p.reg]

maxdeg p ==
  p case R =>
     p = 0 => error "XRPOLY.maxdeg: polynome nul !!"
     1$WORD
  "max"/[(t.k) *$WORD maxdeg(t.c) for t in listOfTerms p.reg]

degree p ==
```

```
     p = 0 => error "XRPOLY.degree: polynome nul !!"
     length(maxdeg p)

  map(fn,p) ==
    p case R => fn(p::R)
    x:REGPOLY := construct [[t.k,a]$TERM for t in listOfTerms p.reg
                    |(a := map(fn,t.c)) ^= 0$R]
    simplifie [fn(p.c0),x]$VPOLY

  varList p ==
    p case R => []
    lv: List VarSet := "setUnion"/[varList(t.c) for t in listOfTerms p.reg]
    lv:= setUnion(lv,[t.k for t in listOfTerms p.reg])
    sort_!(lv)
```

———

— **XRPOLY.dotabb** —

```
"XRPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XRPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XRPOLY" -> "XPOLYC"
```

———

Chapter 26

Chapter Y

Chapter 27

Chapter Z

Chapter 28

The bootstrap code

28.1 BOOLEAN.lsp

BOOLEAN depends on **ORDSET** which depends on **SETCAT** which depends on **BASTYPE** which depends on **BOOLEAN**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated BOOLEAN domain which we can write into the **MID** directory. We compile the lisp code and copy the **BOOLEAN.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

— BOOLEAN.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |BOOLEAN;test;2$;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|a|) |a|)))

(DEFUN |BOOLEAN;test;2$;1| (|a| |$|) |a|)

(DEFUN |BOOLEAN;nt| (|b| |$|)
  (COND (|b| (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(PUT
  (QUOTE |BOOLEAN;true;$;3|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL (QUOTE T))))

(DEFUN |BOOLEAN;true;$;3| (|$|)
  (QUOTE T))

(PUT
```

```
  (QUOTE |BOOLEAN;false;$;4|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL NIL)))

(DEFUN |BOOLEAN;false;$;4| (|$|) NIL)

(DEFUN |BOOLEAN;not;2$;5| (|b| |$|)
  (COND
    (|b| (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;^;2$;6| (|b| |$|)
  (COND
    (|b| (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;~;2$;7| (|b| |$|)
  (COND
    (|b| (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;and;3$;8| (|a| |b| |$|)
  (COND
    (|a| |b|)
    ((QUOTE T) (QUOTE NIL))))

(DEFUN |BOOLEAN;/\\;3$;9| (|a| |b| |$|)
  (COND
    (|a| |b|)
    ((QUOTE T) (QUOTE NIL))))

(DEFUN |BOOLEAN;or;3$;10| (|a| |b| |$|)
  (COND
    (|a| (QUOTE T))
    ((QUOTE T) |b|)))

(DEFUN |BOOLEAN;\\/;3$;11| (|a| |b| |$|)
  (COND
    (|a| (QUOTE T))
    ((QUOTE T) |b|)))

(DEFUN |BOOLEAN;xor;3$;12| (|a| |b| |$|)
  (COND
    (|a| (|BOOLEAN;nt| |b| |$|))
    ((QUOTE T) |b|)))

(DEFUN |BOOLEAN;nor;3$;13| (|a| |b| |$|)
  (COND
    (|a| (QUOTE NIL))
    ((QUOTE T) (|BOOLEAN;nt| |b| |$|))))
```

```
(DEFUN |BOOLEAN;nand;3$;14| (|a| |b| |$|)
  (COND
    (|a| (|BOOLEAN;nt| |b| |$|))
    ((QUOTE T) (QUOTE T))))

(PUT
  (QUOTE |BOOLEAN;=;3$;15|)
  (QUOTE |SPADreplace|)
  (QUOTE |BooleanEquality|))

(DEFUN |BOOLEAN;=;3$;15| (|a| |b| |$|)
  (|BooleanEquality| |a| |b|))

(DEFUN |BOOLEAN;implies;3$;16| (|a| |b| |$|)
  (COND
    (|a| |b|)
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;<;3$;17| (|a| |b| |$|)
  (COND
    (|b|
      (COND
        (|a| (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
    ((QUOTE T) (QUOTE NIL))))

(PUT
  (QUOTE |BOOLEAN;size;Nni;18|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 2)))

(DEFUN |BOOLEAN;size;Nni;18| (|$|) 2)

(DEFUN |BOOLEAN;index;Pi$;19| (|i| |$|)
  (COND
    ((SPADCALL |i| (QREFELT |$| 26)) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;lookup;$Pi;20| (|a| |$|)
  (COND
    (|a| 1)
    ((QUOTE T) 2)))

(DEFUN |BOOLEAN;random;$;21| (|$|)
  (COND
    ((SPADCALL (|random|) (QREFELT |$| 26)) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;convert;$If;22| (|x| |$|)
```

```
  (COND
    (|x| (SPADCALL (SPADCALL "true" (QREFELT |$| 33)) (QREFELT |$| 35)))
    ((QUOTE T)
      (SPADCALL (SPADCALL "false" (QREFELT |$| 33)) (QREFELT |$| 35)))))

(DEFUN |BOOLEAN;coerce;$Of;23| (|x| |$|)
  (COND
    (|x| (SPADCALL "true" (QREFELT |$| 38)))
    ((QUOTE T) (SPADCALL "false" (QREFELT |$| 38)))))

(DEFUN |Boolean| ()
  (PROG ()
    (RETURN
      (PROG (G82461)
        (RETURN
          (COND
            ((LETT G82461 (HGET |$ConstructorCache| '|Boolean|)
                   |Boolean|)
             (|CDRwithIncrement| (CDAR G82461)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Boolean|
                                   (LIST
                                    (CONS NIL (CONS 1 (|Boolean;|))))))
                 (LETT G82461 T |Boolean|))
               (COND
                 ((NOT G82461) (HREM |$ConstructorCache| '|Boolean|)))))))))))

(DEFUN |Boolean;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|Boolean|) |Boolean|)
        (LETT $ (make-array 41) |Boolean|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |Boolean|))
        (|haddProp| |$ConstructorCache| '|Boolean| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        $))))

(setf (get
  (QUOTE |Boolean|)
  (QUOTE |infovec|))
    (LIST
      (QUOTE
        #(NIL NIL NIL NIL NIL NIL
          (|Boolean|)
          |BOOLEAN;test;2$;1|
          (CONS IDENTITY
```

```
          (FUNCALL (|dispatchFunction| |BOOLEAN;true;$;3|) |$|))
        (CONS IDENTITY
          (FUNCALL (|dispatchFunction| |BOOLEAN;false;$;4|) |$|))
        |BOOLEAN;not;2$;5|
        |BOOLEAN;^;2$;6|
        |BOOLEAN;~;2$;7|
        |BOOLEAN;and;3$;8|
        |BOOLEAN;/\\;3$;9|
        |BOOLEAN;or;3$;10|
        |BOOLEAN;\\/;3$;11|
        |BOOLEAN;xor;3$;12|
        |BOOLEAN;nor;3$;13|
        |BOOLEAN;nand;3$;14|
        |BOOLEAN;=;3$;15|
        |BOOLEAN;implies;3$;16|
        |BOOLEAN;<;3$;17|
        (|NonNegativeInteger|)
        |BOOLEAN;size;Nni;18|
        (|Integer|)
        (0 . |even?|)
        (|PositiveInteger|)
        |BOOLEAN;index;Pi$;19|
        |BOOLEAN;lookup;$Pi;20|
        |BOOLEAN;random;$;21|
        (|String|)
        (|Symbol|)
        (5 . |coerce|)
        (|InputForm|)
        (10 . |convert|)
        |BOOLEAN;convert;$If;22|
        (|OutputForm|)
        (15 . |message|)
        |BOOLEAN;coerce;$Of;23|
        (|SingleInteger|)))
    (QUOTE
      #(|~=| 20 |~| 26 |xor| 31 |true| 37 |test| 41 |size| 46 |random| 50
        |or| 54 |not| 60 |nor| 65 |nand| 71 |min| 77 |max| 83 |lookup| 89
        |latex| 94 |index| 99 |implies| 104 |hash| 110 |false| 115
        |convert| 119 |coerce| 124 |and| 129 |^| 135 |\\/| 140 |>=| 146
        |>| 152 |=| 158 |<=| 164 |<| 170 |/\\| 176))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0 0)))
      (CONS
        (QUOTE
          #(|OrderedSet&| NIL |Logic&| |SetCategory&| NIL |BasicType&| NIL))
        (CONS
          (QUOTE
            #((|OrderedSet|)
              (|Finite|)
```

```
                (|Logic|)
                (|SetCategory|)
                (|ConvertibleTo| 34)
                (|BasicType|)
                (|CoercibleTo| 37)))
            (|makeByteWordVec2|
             40
             (QUOTE
               (1 25 6 0 26 1 32 0 31 33 1 34 0 32 35 1 37 0 31 38 2 0 6 0 0
                1 1 0 0 0 12 2 0 0 0 0 17 0 0 0 8 1 0 6 0 7 0 0 23 24 0 0 0
                30 2 0 0 0 0 15 1 0 0 0 10 2 0 0 0 0 18 2 0 0 0 0 19 2 0 0 0
                0 1 2 0 0 0 0 1 1 0 27 0 29 1 0 31 0 1 1 0 0 27 28 2 0 0 0 0
                21 1 0 40 0 1 0 0 0 9 1 0 34 0 36 1 0 37 0 39 2 0 0 0 0 13 1
                0 0 0 11 2 0 0 0 0 16 2 0 6 0 0 1 2 0 6 0 0 1 2 0 6 0 0 20 2
                0 6 0 0 1 2 0 6 0 0 22 2 0 0 0 0 14))))))
      (QUOTE |lookupComplete|)))

(setf (get (QUOTE |Boolean|) (QUOTE NILADIC)) T)
```

28.2 CHAR.lsp BOOTSTRAP

CHAR depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CHAR** category which we can write into the **MID** directory. We compile the lisp code and copy the **CHAR.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— CHAR.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |CHAR;=;2$B;1|) (QUOTE |SPADreplace|) (QUOTE EQL))

(DEFUN |CHAR;=;2$B;1| (|a| |b| |$|) (EQL |a| |b|))

(PUT (QUOTE |CHAR;<;2$B;2|) (QUOTE |SPADreplace|) (QUOTE QSLESSP))

(DEFUN |CHAR;<;2$B;2| (|a| |b| |$|) (QSLESSP |a| |b|))

(PUT (QUOTE |CHAR;size;Nni;3|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 256)))

(DEFUN |CHAR;size;Nni;3| (|$|) 256)
```

```
(DEFUN |CHAR;index;Pi$;4| (|n| |$|) (SPADCALL (|-| |n| 1) (QREFELT |$| 18)))

(DEFUN |CHAR;lookup;$Pi;5| (|c| $)
  (PROG (G90919)
    (RETURN
      (PROG1 (LETT G90919 (+ 1 (SPADCALL |c| (QREFELT $ 21)))
                   |CHAR;lookup;$Pi;5|)
        (|check-subtype| (> G90919 0) '(|PositiveInteger|) G90919)))))

(DEFUN |CHAR;char;I$;6| (|n| |$|) (SPADCALL |n| (QREFELT |$| 23)))

(PUT (QUOTE |CHAR;ord;$I;7|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|c|) |c|)))

(DEFUN |CHAR;ord;$I;7| (|c| |$|) |c|)

(DEFUN |CHAR;random;$;8| (|$|)
 (SPADCALL (REMAINDER2 (|random|) (SPADCALL (QREFELT |$| 16)))
           (QREFELT |$| 18)))

(PUT (QUOTE |CHAR;space;$;9|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (QENUM "   " 0))))

(DEFUN |CHAR;space;$;9| (|$|) (QENUM "   " 0))

(PUT (QUOTE |CHAR;quote;$;10|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (QENUM "\" " 0))))

(DEFUN |CHAR;quote;$;10| (|$|) (QENUM "\" " 0))

(PUT (QUOTE |CHAR;escape;$;11|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (QENUM "_ " 0))))

(DEFUN |CHAR;escape;$;11| (|$|) (QENUM "_ " 0))

(DEFUN |CHAR;coerce;$Of;12| (|c| |$|)
 (ELT (QREFELT |$| 10)
      (|+| (QREFELT |$| 11) (SPADCALL |c| (QREFELT |$| 21)))))

(DEFUN |CHAR;digit?;$B;13| (|c| |$|)
 (SPADCALL |c| (|spadConstant| |$| 31) (QREFELT |$| 33)))

(DEFUN |CHAR;hexDigit?;$B;14| (|c| |$|)
 (SPADCALL |c| (|spadConstant| |$| 35) (QREFELT |$| 33)))

(DEFUN |CHAR;upperCase?;$B;15| (|c| |$|)
 (SPADCALL |c| (|spadConstant| |$| 37) (QREFELT |$| 33)))

(DEFUN |CHAR;lowerCase?;$B;16| (|c| |$|)
 (SPADCALL |c| (|spadConstant| |$| 39) (QREFELT |$| 33)))
```

```
(DEFUN |CHAR;alphabetic?;$B;17| (|c| |$|)
 (SPADCALL |c| (|spadConstant| |$| 41) (QREFELT |$| 33)))

(DEFUN |CHAR;alphanumeric?;$B;18| (|c| |$|)
 (SPADCALL |c| (|spadConstant| |$| 43) (QREFELT |$| 33)))

(DEFUN |CHAR;latex;$S;19| (|c| |$|)
 (STRCONC "\\mbox{`" (STRCONC (|MAKE-FULL-CVEC| 1 |c|) "'}")))

(DEFUN |CHAR;char;S$;20| (|s| |$|)
 (COND
  ((EQL (QCSIZE |s|) 1)
    (SPADCALL |s| (SPADCALL |s| (QREFELT |$| 47)) (QREFELT |$| 48)))
  ((QUOTE T) (|error| "String is not a single character"))))

(DEFUN |CHAR;upperCase;2$;21| (|c| |$|)
 (QENUM (PNAME (UPCASE (code-char (SPADCALL |c| (QREFELT |$| 21))))) 0))

(DEFUN |CHAR;lowerCase;2$;22| (|c| |$|)
 (QENUM (PNAME (DOWNCASE (code-char (SPADCALL |c| (QREFELT |$| 21))))) 0))

(DEFUN |Character| ()
  (PROG ()
    (RETURN
      (PROG (G90941)
        (RETURN
          (COND
            ((LETT G90941 (HGET |$ConstructorCache| '|Character|)
                   |Character|)
             (|CDRwithIncrement| (CDAR G90941)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Character|
                                   (LIST
                                    (CONS NIL (CONS 1 (|Character;|))))))
                 (LETT G90941 T |Character|))
               (COND
                ((NOT G90941) (HREM |$ConstructorCache| '|Character|)))))))))))

(DEFUN |Character;| ()
  (PROG (|dv$| $ |pv$| G90939 |i|)
    (RETURN
      (SEQ (PROGN
             (LETT |dv$| '(|Character|) |Character|)
             (LETT $ (make-array 53) |Character|)
             (QSETREFV $ 0 |dv$|)
             (QSETREFV $ 3
                 (LETT |pv$| (|buildPredVector| 0 0 NIL) |Character|))
             (|haddProp| |$ConstructorCache| '|Character| NIL
                 (CONS 1 $))
```

```
            (|stuffDomainSlots| $)
            (QSETREFV $ 6 (|SingleInteger|))
            (QSETREFV $ 10
                (SPADCALL
                    (PROGN
                      (LETT G90939 NIL |Character|)
                      (SEQ (LETT |i| 0 |Character|) G190
                           (COND ((QSGREATERP |i| 255) (GO G191)))
                           (SEQ (EXIT (LETT G90939
                                            (CONS (code-char |i|) G90939)
                                            |Character|)))
                           (LETT |i| (QSADD1 |i|) |Character|)
                           (GO G190) G191 (EXIT (NREVERSE0 G90939))))
                    (QREFELT $ 9)))
            (QSETREFV $ 11 0)
            $)))))

(setf (get (QUOTE |Character|) (QUOTE |infovec|))
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (QUOTE |Rep|) (|List| 28) (|PrimitiveArray| 28)
   (0 . |construct|) (QUOTE |OutChars|) (QUOTE |minChar|) (|Boolean|)
   |CHAR;=;2$B;1| |CHAR;<;2$B;2| (|NonNegativeInteger|) |CHAR;size;Nni;3|
   (|Integer|) |CHAR;char;I$;6| (|PositiveInteger|) |CHAR;index;Pi$;4|
   |CHAR;ord;$I;7| |CHAR;lookup;$Pi;5| (5 . |coerce|) |CHAR;random;$;8|
   |CHAR;space;$;9| |CHAR;quote;$;10| |CHAR;escape;$;11| (|OutputForm|)
   |CHAR;coerce;$Of;12| (|CharacterClass|) (10 . |digit|) (|Character|)
   (14 . |member?|) |CHAR;digit?;$B;13| (20 . |hexDigit|)
   |CHAR;hexDigit?;$B;14| (24 . |upperCase|) |CHAR;upperCase?;$B;15|
   (28 . |lowerCase|) |CHAR;lowerCase?;$B;16| (32 . |alphabetic|)
   |CHAR;alphabetic?;$B;17| (36 . |alphanumeric|) |CHAR;alphanumeric?;$B;18|
   (|String|) |CHAR;latex;$S;19| (40 . |minIndex|) (45 . |elt|)
   |CHAR;char;S$;20| |CHAR;upperCase;2$;21| |CHAR;lowerCase;2$;22|
   (|SingleInteger|))) (QUOTE #(|~=| 51 |upperCase?| 57 |upperCase| 62
   |space| 67 |size| 71 |random| 75 |quote| 79 |ord| 83 |min| 88 |max| 94
   |lowerCase?| 100 |lowerCase| 105 |lookup| 110 |latex| 115 |index| 120
   |hexDigit?| 125 |hash| 130 |escape| 135 |digit?| 139 |coerce| 144 |char|
   149 |alphanumeric?| 159 |alphabetic?| 164 |>=| 169 |>| 175 |=| 181 |<=|
   187 |<| 193)) (QUOTE NIL)
   (CONS
    (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0)))
    (CONS
     (QUOTE #(NIL |OrderedSet&| NIL |SetCategory&| |BasicType&| NIL))
     (CONS
      (QUOTE #((|OrderedFinite|) (|OrderedSet|) (|Finite|) (|SetCategory|)
               (|BasicType|) (|CoercibleTo| 28)))
       (|makeByteWordVec2| 52
        (QUOTE (1 8 0 7 9 1 6 0 17 23 0 30 0 31 2 30 12 32 0 33 0 30 0 35
                0 30 0 37 0 30 0 39 0 30 0 41 0 30 0 43 1 45 17 0 47 2 45
                32 0 17 48 2 0 12 0 0 1 1 0 12 0 38 1 0 0 0 50 0 0 0 25 0
                0 15 16 0 0 0 24 0 0 0 26 1 0 17 0 21 2 0 0 0 0 1 2 0 0 0
```

```
                0 1 1 0 12 0 40 1 0 0 0 51 1 0 19 0 22 1 0 45 0 46 1 0 0 19
                20 1 0 12 0 36 1 0 52 0 1 0 0 0 27 1 0 12 0 34 1 0 28 0 29
                1 0 0 45 49 1 0 0 17 18 1 0 12 0 44 1 0 12 0 42 2 0 12 0 0
                1 2 0 12 0 0 1 2 0 12 0 0 13 2 0 12 0 0 1 2 0 12 0 0 14))))))
  (QUOTE |lookupComplete|)))

(setf (get (QUOTE |Character|) (QUOTE NILADIC)) T)
```

28.3 DFLOAT.lsp BOOTSTRAP

DFLOAT depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DFLOAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **DFLOAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— DFLOAT.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |DFLOAT;OMwrite;$S;1| (|x| |$|)
 (PROG (|sp| |dev| |s|)
  (RETURN
   (SEQ
    (LETT |s| "" |DFLOAT;OMwrite;$S;1|)
    (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |DFLOAT;OMwrite;$S;1|)
    (LETT |dev|
     (SPADCALL |sp| (SPADCALL (QREFELT |$| 7)) (QREFELT |$| 10))
     |DFLOAT;OMwrite;$S;1|)
    (SPADCALL |dev| (QREFELT |$| 12))
    (SPADCALL |dev| |x| (QREFELT |$| 14))
    (SPADCALL |dev| (QREFELT |$| 15))
    (SPADCALL |dev| (QREFELT |$| 16))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |DFLOAT;OMwrite;$S;1|)
    (EXIT |s|)))))

(DEFUN |DFLOAT;OMwrite;$BS;2| (|x| |wholeObj| |$|)
 (PROG (|sp| |dev| |s|)
  (RETURN
   (SEQ
    (LETT |s| "" |DFLOAT;OMwrite;$BS;2|)
    (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |DFLOAT;OMwrite;$BS;2|)
    (LETT |dev|
     (SPADCALL |sp| (SPADCALL (QREFELT |$| 7)) (QREFELT |$| 10))
     |DFLOAT;OMwrite;$BS;2|)
```

```
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 12))))
    (SPADCALL |dev| |x| (QREFELT |$| 14))
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 15))))
    (SPADCALL |dev| (QREFELT |$| 16))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |DFLOAT;OMwrite;$BS;2|)
    (EXIT |s|)))))

(DEFUN |DFLOAT;OMwrite;Omd$V;3| (|dev| |x| |$|)
 (SEQ
  (SPADCALL |dev| (QREFELT |$| 12))
  (SPADCALL |dev| |x| (QREFELT |$| 14))
  (EXIT (SPADCALL |dev| (QREFELT |$| 15)))))

(DEFUN |DFLOAT;OMwrite;Omd$BV;4| (|dev| |x| |wholeObj| |$|)
 (SEQ
  (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 12))))
  (SPADCALL |dev| |x| (QREFELT |$| 14))
  (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 15)))))))

(PUT (QUOTE |DFLOAT;checkComplex|) (QUOTE |SPADreplace|) (QUOTE |C-TO-R|))

(DEFUN |DFLOAT;checkComplex| (|x| |$|) (|C-TO-R| |x|))

(PUT
 (QUOTE |DFLOAT;base;Pi;6|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL (|FLOAT-RADIX| 0.0))))

(DEFUN |DFLOAT;base;Pi;6| (|$|) (|FLOAT-RADIX| 0.0))

(DEFUN |DFLOAT;mantissa;$I;7| (|x| |$|) (QCAR (|DFLOAT;manexp| |x| |$|)))

(DEFUN |DFLOAT;exponent;$I;8| (|x| |$|) (QCDR (|DFLOAT;manexp| |x| |$|)))

(PUT
 (QUOTE |DFLOAT;precision;Pi;9|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL (|FLOAT-DIGITS| 0.0))))

(DEFUN |DFLOAT;precision;Pi;9| (|$|) (|FLOAT-DIGITS| 0.0))

(DEFUN |DFLOAT;bits;Pi;10| ($)
  (PROG (G105705)
    (RETURN
      (COND
        ((EQL (FLOAT-RADIX 0.0) 2) (FLOAT-DIGITS 0.0))
        ((EQL (FLOAT-RADIX 0.0) 16) (* 4 (FLOAT-DIGITS 0.0)))
        ('T
         (PROG1 (LETT G105705
                     (truncate (SPADCALL (FLOAT-DIGITS 0.0)
```

```
                              (SPADCALL
                                  (FLOAT (FLOAT-RADIX 0.0)
                                   MOST-POSITIVE-LONG-FLOAT)
                                  (QREFELT $ 28))
                              (QREFELT $ 29)))
                     |DFLOAT;bits;Pi;10|)
           (|check-subtype| (> G105705 0) '(|PositiveInteger|) G105705)))))))

(PUT
 (QUOTE |DFLOAT;max;$;11|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL |MOST-POSITIVE-LONG-FLOAT|)))

(DEFUN |DFLOAT;max;$;11| (|$|) |MOST-POSITIVE-LONG-FLOAT|)

(PUT
 (QUOTE |DFLOAT;min;$;12|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL |MOST-NEGATIVE-LONG-FLOAT|)))

(DEFUN |DFLOAT;min;$;12| (|$|) |MOST-NEGATIVE-LONG-FLOAT|)

(DEFUN |DFLOAT;order;$I;13| (|a| |$|)
 (|-| (|+| (|FLOAT-DIGITS| 0.0) (SPADCALL |a| (QREFELT |$| 26))) 1))

(PUT
 (QUOTE |DFLOAT;Zero;$;14|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL (FLOAT 0 |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |DFLOAT;Zero;$;14| (|$|) (FLOAT 0 |MOST-POSITIVE-LONG-FLOAT|))

(PUT
 (QUOTE |DFLOAT;One;$;15|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL (FLOAT 1 |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |DFLOAT;One;$;15| (|$|) (FLOAT 1 |MOST-POSITIVE-LONG-FLOAT|))

(DEFUN |DFLOAT;exp1;$;16| (|$|)
 (|/|
  (FLOAT 534625820200 |MOST-POSITIVE-LONG-FLOAT|)
  (FLOAT 196677847971 |MOST-POSITIVE-LONG-FLOAT|)))

(PUT (QUOTE |DFLOAT;pi;$;17|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL PI)))

(DEFUN |DFLOAT;pi;$;17| (|$|) PI)

(DEFUN |DFLOAT;coerce;$Of;18| (|x| |$|) (SPADCALL |x| (QREFELT |$| 39)))
```

```
(DEFUN |DFLOAT;convert;$If;19| (|x| |$|) (SPADCALL |x| (QREFELT |$| 42)))

(PUT (QUOTE |DFLOAT;<;2$B;20|) (QUOTE |SPADreplace|) (QUOTE |<|))

(DEFUN |DFLOAT;<;2$B;20| (|x| |y| |$|) (|<| |x| |y|))

(PUT (QUOTE |DFLOAT;-;2$;21|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |DFLOAT;-;2$;21| (|x| |$|) (|-| |x|))

(PUT (QUOTE |DFLOAT;+;3$;22|) (QUOTE |SPADreplace|) (QUOTE |+|))

(DEFUN |DFLOAT;+;3$;22| (|x| |y| |$|) (|+| |x| |y|))

(PUT (QUOTE |DFLOAT;-;3$;23|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |DFLOAT;-;3$;23| (|x| |y| |$|) (|-| |x| |y|))

(PUT (QUOTE |DFLOAT;*;3$;24|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |DFLOAT;*;3$;24| (|x| |y| |$|) (|*| |x| |y|))

(PUT (QUOTE |DFLOAT;*;I2$;25|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |DFLOAT;*;I2$;25| (|i| |x| |$|) (|*| |i| |x|))

(PUT (QUOTE |DFLOAT;max;3$;26|) (QUOTE |SPADreplace|) (QUOTE MAX))

(DEFUN |DFLOAT;max;3$;26| (|x| |y| |$|) (MAX |x| |y|))

(PUT (QUOTE |DFLOAT;min;3$;27|) (QUOTE |SPADreplace|) (QUOTE MIN))

(DEFUN |DFLOAT;min;3$;27| (|x| |y| |$|) (MIN |x| |y|))

(PUT (QUOTE |DFLOAT;=;2$B;28|) (QUOTE |SPADreplace|) (QUOTE |=|))

(DEFUN |DFLOAT;=;2$B;28| (|x| |y| |$|) (|=| |x| |y|))

(PUT (QUOTE |DFLOAT;/;$I$;29|) (QUOTE |SPADreplace|) (QUOTE |/|))

(DEFUN |DFLOAT;/;$I$;29| (|x| |i| |$|) (|/| |x| |i|))

(DEFUN |DFLOAT;sqrt;2$;30| (|x| |$|) (|DFLOAT;checkComplex| (SQRT |x|) |$|))

(DEFUN |DFLOAT;log10;2$;31| (|x| |$|) (|DFLOAT;checkComplex| (|log| |x|) |$|))

(PUT (QUOTE |DFLOAT;**;$I$;32|) (QUOTE |SPADreplace|) (QUOTE EXPT))

(DEFUN |DFLOAT;**;$I$;32| (|x| |i| |$|) (EXPT |x| |i|))
```

```
(DEFUN |DFLOAT;**;3$;33| (|x| |y| |$|)
 (|DFLOAT;checkComplex| (EXPT |x| |y|) |$|))

(PUT
 (QUOTE |DFLOAT;coerce;I$;34|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|i|) (FLOAT |i| |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |DFLOAT;coerce;I$;34| (|i| |$|) (FLOAT |i| |MOST-POSITIVE-LONG-FLOAT|))

(PUT (QUOTE |DFLOAT;exp;2$;35|) (QUOTE |SPADreplace|) (QUOTE EXP))

(DEFUN |DFLOAT;exp;2$;35| (|x| |$|) (EXP |x|))

(DEFUN |DFLOAT;log;2$;36| (|x| |$|) (|DFLOAT;checkComplex| (LN |x|) |$|))

(DEFUN |DFLOAT;log2;2$;37| (|x| |$|) (|DFLOAT;checkComplex| (LOG2 |x|) |$|))

(PUT (QUOTE |DFLOAT;sin;2$;38|) (QUOTE |SPADreplace|) (QUOTE SIN))

(DEFUN |DFLOAT;sin;2$;38| (|x| |$|) (SIN |x|))

(PUT (QUOTE |DFLOAT;cos;2$;39|) (QUOTE |SPADreplace|) (QUOTE COS))

(DEFUN |DFLOAT;cos;2$;39| (|x| |$|) (COS |x|))

(PUT (QUOTE |DFLOAT;tan;2$;40|) (QUOTE |SPADreplace|) (QUOTE TAN))

(DEFUN |DFLOAT;tan;2$;40| (|x| |$|) (TAN |x|))

(PUT (QUOTE |DFLOAT;cot;2$;41|) (QUOTE |SPADreplace|) (QUOTE COT))

(DEFUN |DFLOAT;cot;2$;41| (|x| |$|) (COT |x|))

(PUT (QUOTE |DFLOAT;sec;2$;42|) (QUOTE |SPADreplace|) (QUOTE SEC))

(DEFUN |DFLOAT;sec;2$;42| (|x| |$|) (SEC |x|))

(PUT (QUOTE |DFLOAT;csc;2$;43|) (QUOTE |SPADreplace|) (QUOTE CSC))

(DEFUN |DFLOAT;csc;2$;43| (|x| |$|) (CSC |x|))

(DEFUN |DFLOAT;asin;2$;44| (|x| |$|) (|DFLOAT;checkComplex| (ASIN |x|) |$|))

(DEFUN |DFLOAT;acos;2$;45| (|x| |$|) (|DFLOAT;checkComplex| (ACOS |x|) |$|))

(PUT (QUOTE |DFLOAT;atan;2$;46|) (QUOTE |SPADreplace|) (QUOTE ATAN))

(DEFUN |DFLOAT;atan;2$;46| (|x| |$|) (ATAN |x|))
```

```
(DEFUN |DFLOAT;acsc;2$;47| (|x| |$|) (|DFLOAT;checkComplex| (ACSC |x|) |$|))

(PUT (QUOTE |DFLOAT;acot;2$;48|) (QUOTE |SPADreplace|) (QUOTE ACOT))

(DEFUN |DFLOAT;acot;2$;48| (|x| |$|) (ACOT |x|))

(DEFUN |DFLOAT;asec;2$;49| (|x| |$|) (|DFLOAT;checkComplex| (ASEC |x|) |$|))

(PUT (QUOTE |DFLOAT;sinh;2$;50|) (QUOTE |SPADreplace|) (QUOTE SINH))

(DEFUN |DFLOAT;sinh;2$;50| (|x| |$|) (SINH |x|))

(PUT (QUOTE |DFLOAT;cosh;2$;51|) (QUOTE |SPADreplace|) (QUOTE COSH))

(DEFUN |DFLOAT;cosh;2$;51| (|x| |$|) (COSH |x|))

(PUT (QUOTE |DFLOAT;tanh;2$;52|) (QUOTE |SPADreplace|) (QUOTE TANH))

(DEFUN |DFLOAT;tanh;2$;52| (|x| |$|) (TANH |x|))

(PUT (QUOTE |DFLOAT;csch;2$;53|) (QUOTE |SPADreplace|) (QUOTE CSCH))

(DEFUN |DFLOAT;csch;2$;53| (|x| |$|) (CSCH |x|))

(PUT (QUOTE |DFLOAT;coth;2$;54|) (QUOTE |SPADreplace|) (QUOTE COTH))

(DEFUN |DFLOAT;coth;2$;54| (|x| |$|) (COTH |x|))

(PUT (QUOTE |DFLOAT;sech;2$;55|) (QUOTE |SPADreplace|) (QUOTE SECH))

(DEFUN |DFLOAT;sech;2$;55| (|x| |$|) (SECH |x|))

(PUT (QUOTE |DFLOAT;asinh;2$;56|) (QUOTE |SPADreplace|) (QUOTE ASINH))

(DEFUN |DFLOAT;asinh;2$;56| (|x| |$|) (ASINH |x|))

(DEFUN |DFLOAT;acosh;2$;57| (|x| |$|) (|DFLOAT;checkComplex| (ACOSH |x|) |$|))

(DEFUN |DFLOAT;atanh;2$;58| (|x| |$|) (|DFLOAT;checkComplex| (ATANH |x|) |$|))

(PUT (QUOTE |DFLOAT;acsch;2$;59|) (QUOTE |SPADreplace|) (QUOTE ACSCH))

(DEFUN |DFLOAT;acsch;2$;59| (|x| |$|) (ACSCH |x|))

(DEFUN |DFLOAT;acoth;2$;60| (|x| |$|) (|DFLOAT;checkComplex| (ACOTH |x|) |$|))

(DEFUN |DFLOAT;asech;2$;61| (|x| |$|) (|DFLOAT;checkComplex| (ASECH |x|) |$|))

(PUT (QUOTE |DFLOAT;/;3$;62|) (QUOTE |SPADreplace|) (QUOTE |/|))
```

```
(DEFUN |DFLOAT;/;3$;62| (|x| |y| |$|) (|/| |x| |y|))

(PUT (QUOTE |DFLOAT;negative?;$B;63|) (QUOTE |SPADreplace|) (QUOTE MINUSP))

(DEFUN |DFLOAT;negative?;$B;63| (|x| |$|) (MINUSP |x|))

(PUT (QUOTE |DFLOAT;zero?;$B;64|) (QUOTE |SPADreplace|) (QUOTE ZEROP))

(DEFUN |DFLOAT;zero?;$B;64| (|x| |$|) (ZEROP |x|))

(PUT (QUOTE |DFLOAT;hash;$I;65|) (QUOTE |SPADreplace|) (QUOTE SXHASH))

(DEFUN |DFLOAT;hash;$I;65| (|x| |$|) (SXHASH |x|))

(DEFUN |DFLOAT;recip;$U;66| (|x| |$|)
 (COND
  ((ZEROP |x|) (CONS 1 "failed"))
  ((QUOTE T) (CONS 0 (|/| 1.0 |x|)))))

(PUT
 (QUOTE |DFLOAT;differentiate;2$;67|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) 0.0)))

(DEFUN |DFLOAT;differentiate;2$;67| (|x| |$|) 0.0)

(DEFUN |DFLOAT;Gamma;2$;68| (|x| |$|) (SPADCALL |x| (QREFELT |$| 93)))

(DEFUN |DFLOAT;Beta;3$;69| (|x| |y| |$|) (SPADCALL |x| |y| (QREFELT |$| 95)))

(PUT (QUOTE |DFLOAT;wholePart;$I;70|) (QUOTE |SPADreplace|) (QUOTE truncate))

(DEFUN |DFLOAT;wholePart;$I;70| (|x| |$|) (truncate |x|))

(DEFUN |DFLOAT;float;2IPi$;71| (|ma| |ex| |b| |$|)
 (|*| |ma| (EXPT (FLOAT |b| |MOST-POSITIVE-LONG-FLOAT|) |ex|)))

(PUT
 (QUOTE |DFLOAT;convert;2$;72|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) |x|)))

(DEFUN |DFLOAT;convert;2$;72| (|x| |$|) |x|)

(DEFUN |DFLOAT;convert;$F;73| (|x| |$|) (SPADCALL |x| (QREFELT |$| 101)))

(DEFUN |DFLOAT;rationalApproximation;$NniF;74| (|x| |d| |$|)
 (SPADCALL |x| |d| 10 (QREFELT |$| 105)))

(DEFUN |DFLOAT;atan;3$;75| (|x| |y| |$|)
```

```
 (PROG (|theta|)
  (RETURN
   (SEQ
    (COND
     ((|=| |x| 0.0)
      (COND
       ((|<| 0.0 |y|) (|/| PI 2))
       ((|<| |y| 0.0) (|-| (|/| PI 2)))
       ((QUOTE T) 0.0)))
     ((QUOTE T)
      (SEQ
       (LETT |theta|
        (ATAN (|FLOAT-SIGN| 1.0 (|/| |y| |x|)))
        |DFLOAT;atan;3$;75|)
       (COND
        ((|<| |x| 0.0) (LETT |theta| (|-| PI |theta|) |DFLOAT;atan;3$;75|)))
       (COND ((|<| |y| 0.0) (LETT |theta| (|-| |theta|) |DFLOAT;atan;3$;75|)))
       (EXIT |theta|))))))))

(DEFUN |DFLOAT;retract;$F;76| (|x| $)
  (PROG (G105780)
    (RETURN
      (SPADCALL |x|
          (PROG1 (LETT G105780 (- (FLOAT-DIGITS 0.0) 1)
                      |DFLOAT;retract;$F;76|)
            (|check-subtype| (>= G105780 0) '(|NonNegativeInteger|)
                G105780))
          (FLOAT-RADIX 0.0) (QREFELT $ 105)))))

(DEFUN |DFLOAT;retractIfCan;$U;77| (|x| $)
  (PROG (G105785)
    (RETURN
      (CONS 0
            (SPADCALL |x|
                (PROG1 (LETT G105785 (- (FLOAT-DIGITS 0.0) 1)
                            |DFLOAT;retractIfCan;$U;77|)
                  (|check-subtype| (>= G105785 0)
                      '(|NonNegativeInteger|) G105785))
                (FLOAT-RADIX 0.0) (QREFELT $ 105))))))

(DEFUN |DFLOAT;retract;$I;78| (|x| |$|)
 (PROG (|n|)
  (RETURN
   (SEQ
    (LETT |n| (truncate |x|) |DFLOAT;retract;$I;78|)
    (EXIT
     (COND
      ((|=| |x| (FLOAT |n| |MOST-POSITIVE-LONG-FLOAT|)) |n|)
      ((QUOTE T) (|error| "Not an integer"))))))))
```

```
(DEFUN |DFLOAT;retractIfCan;$U;79| (|x| |$|)
 (PROG (|n|)
  (RETURN
   (SEQ
    (LETT |n| (truncate |x|) |DFLOAT;retractIfCan;$U;79|)
    (EXIT
     (COND
      ((|=| |x| (FLOAT |n| |MOST-POSITIVE-LONG-FLOAT|)) (CONS 0 |n|))
      ((QUOTE T) (CONS 1 "failed"))))))))

(DEFUN |DFLOAT;sign;$I;80| (|x| |$|)
 (SPADCALL (|FLOAT-SIGN| |x| 1.0) (QREFELT |$| 111)))

(PUT
 (QUOTE |DFLOAT;abs;2$;81|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) (|FLOAT-SIGN| 1.0 |x|))))

(DEFUN |DFLOAT;abs;2$;81| (|x| |$|) (|FLOAT-SIGN| 1.0 |x|))

(DEFUN |DFLOAT;manexp| (|x| $)
  (PROG (|s| G105806 |me| |two53|)
    (RETURN
      (SEQ (EXIT (COND
                   ((ZEROP |x|) (CONS 0 0))
                   ('T
                    (SEQ (LETT |s| (SPADCALL |x| (QREFELT $ 114))
                               |DFLOAT;manexp|)
                         (LETT |x| (FLOAT-SIGN 1.0 |x|)
                               |DFLOAT;manexp|)
                         (COND
                           ((< MOST-POSITIVE-LONG-FLOAT |x|)
                            (PROGN
                              (LETT G105806
                                    (CONS
                                     (+
                                      (* |s|
                                       (SPADCALL
                                        MOST-POSITIVE-LONG-FLOAT
                                        (QREFELT $ 25)))
                                      1)
                                     (SPADCALL MOST-POSITIVE-LONG-FLOAT
                                      (QREFELT $ 26)))
                                    |DFLOAT;manexp|)
                              (GO G105806))))
                         (LETT |me| (MANEXP |x|) |DFLOAT;manexp|)
                         (LETT |two53|
                               (EXPT (FLOAT-RADIX 0.0)
                                     (FLOAT-DIGITS 0.0))
                               |DFLOAT;manexp|)
```

```
                         (EXIT (CONS (* |s|
                                      (truncate (* |two53| (QCAR |me|))))
                                     (- (QCDR |me|) (FLOAT-DIGITS 0.0)))))))
           G105806 (EXIT G105806)))))

(DEFUN |DFLOAT;rationalApproximation;$2NniF;83| (|f| |d| |b| $)
  (PROG (|#G102| |nu| |ex| BASE G105809 |de| |tol| |#G103| |q| |r| |p2|
                |q2| G105827 |#G104| |#G105| |p0| |p1| |#G106| |#G107|
                |q0| |q1| |#G108| |#G109| |s| |t| G105825)
    (RETURN
      (SEQ (EXIT (SEQ
       (PROGN
        (LETT |#G102| (|DFLOAT;manexp| |f| $)
                         |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |nu| (QCAR |#G102|)
                         |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |ex| (QCDR |#G102|)
                         |DFLOAT;rationalApproximation;$2NniF;83|)
        |#G102|)
       (LETT BASE (FLOAT-RADIX 0.0)
                       |DFLOAT;rationalApproximation;$2NniF;83|)
       (EXIT
        (COND
         ((< |ex| 0)
          (SEQ
           (LETT |de|
            (EXPT BASE
             (PROG1
              (LETT G105809 (- |ex|) |DFLOAT;rationalApproximation;$2NniF;83|)
              (|check-subtype| (>= G105809 0) '(|NonNegativeInteger|)
                G105809)))
            |DFLOAT;rationalApproximation;$2NniF;83|)
           (EXIT
            (COND
             ((< |b| 2) (|error| "base must be > 1"))
             (t
              (SEQ
               (LETT |tol| (EXPT |b| |d|)
                  |DFLOAT;rationalApproximation;$2NniF;83|)
               (LETT |s| |nu|
                  |DFLOAT;rationalApproximation;$2NniF;83|)
               (LETT |t| |de|
                  |DFLOAT;rationalApproximation;$2NniF;83|)
               (LETT |p0| 0
                  |DFLOAT;rationalApproximation;$2NniF;83|)
               (LETT |p1| 1
                  |DFLOAT;rationalApproximation;$2NniF;83|)
               (LETT |q0| 1
                  |DFLOAT;rationalApproximation;$2NniF;83|)
               (LETT |q1| 0
```

```
   |DFLOAT;rationalApproximation;$2NniF;83|)
(EXIT
 (SEQ G190 NIL
  (SEQ
   (PROGN
    (LETT |#G103| (DIVIDE2 |s| |t|)
     |DFLOAT;rationalApproximation;$2NniF;83|)
    (LETT |q| (QCAR |#G103|)
     |DFLOAT;rationalApproximation;$2NniF;83|)
    (LETT |r| (QCDR |#G103|)
     |DFLOAT;rationalApproximation;$2NniF;83|)
    |#G103|)
   (LETT |p2| (+ (* |q| |p1|) |p0|)
    |DFLOAT;rationalApproximation;$2NniF;83|)
   (LETT |q2| (+ (* |q| |q1|) |q0|)
    |DFLOAT;rationalApproximation;$2NniF;83|)
   (COND
    ((OR (EQL |r| 0)
      (< (SPADCALL |tol| (ABS (- (* |nu| |q2|) (* |de| |p2|)))
                   (QREFELT $ 118))
                   (* |de| (ABS |p2|))))
     (EXIT
      (PROGN
       (LETT G105827
        (SPADCALL |p2| |q2| (QREFELT $ 117))
        |DFLOAT;rationalApproximation;$2NniF;83|)
       (GO G105827)))))
      (PROGN
       (LETT |#G104| |p1|
        |DFLOAT;rationalApproximation;$2NniF;83|)
       (LETT |#G105| |p2|
        |DFLOAT;rationalApproximation;$2NniF;83|)
       (LETT |p0| |#G104|
        |DFLOAT;rationalApproximation;$2NniF;83|)
       (LETT |p1| |#G105|
        |DFLOAT;rationalApproximation;$2NniF;83|))
       (PROGN
        (LETT |#G106| |q1|
         |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |#G107| |q2|
         |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |q0| |#G106|
         |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |q1| |#G107|
         |DFLOAT;rationalApproximation;$2NniF;83|))
        (EXIT
         (PROGN
          (LETT |#G108| |t|
           |DFLOAT;rationalApproximation;$2NniF;83|)
          (LETT |#G109| |r|
```

```
                          |DFLOAT;rationalApproximation;$2NniF;83|)
                         (LETT |s| |#G108|
                          |DFLOAT;rationalApproximation;$2NniF;83|)
                         (LETT |t| |#G109|
                          |DFLOAT;rationalApproximation;$2NniF;83|))))
                         NIL (GO G190) G191
                         (EXIT NIL))))))))
         ('T
          (SPADCALL
           (* |nu|
            (EXPT BASE
             (PROG1
              (LETT G105825 |ex| |DFLOAT;rationalApproximation;$2NniF;83|)
              (|check-subtype| (>= G105825 0) '(|NonNegativeInteger|)
                 G105825))))
           (QREFELT $ 119)))))))
    G105827
      (EXIT G105827)))))

(DEFUN |DFLOAT;**;$F$;84| (|x| |r| $)
  (PROG (|n| |d| G105837)
    (RETURN
      (SEQ (EXIT (COND
                   ((ZEROP |x|)
                    (COND
                      ((SPADCALL |r| (QREFELT $ 120))
                       (|error| "0**0 is undefined"))
                      ((SPADCALL |r| (QREFELT $ 121))
                       (|error| "division by 0"))
                      ('T 0.0)))
                   ((OR (SPADCALL |r| (QREFELT $ 120))
                        (SPADCALL |x| (QREFELT $ 122)))
                    1.0)
                   ('T
                    (COND
                      ((SPADCALL |r| (QREFELT $ 123)) |x|)
                      ('T
                       (SEQ (LETT |n| (SPADCALL |r| (QREFELT $ 124))
                                  |DFLOAT;**;$F$;84|)
                            (LETT |d| (SPADCALL |r| (QREFELT $ 125))
                                  |DFLOAT;**;$F$;84|)
                            (EXIT (COND
                                    ((MINUSP |x|)
                                     (COND
                                       ((ODDP |d|)
                                        (COND
                                          ((ODDP |n|)
                                           (PROGN
                                             (LETT G105837
                                              (-
```

```
                                                (SPADCALL (- |x|) |r|
                                                 (QREFELT $ 126)))
                                               |DFLOAT;**;$F$;84|)
                                              (GO G105837)))
                                           ('T
                                            (PROGN
                                              (LETT G105837
                                               (SPADCALL (- |x|) |r|
                                                (QREFELT $ 126))
                                               |DFLOAT;**;$F$;84|)
                                              (GO G105837)))))
                                        ('T (|error| "negative root"))))
                                     ((EQL |d| 2)
                                      (EXPT
                                       (SPADCALL |x| (QREFELT $ 54))
                                       |n|))
                                     ('T
                                      (SPADCALL |x|
                                       (/
                                        (FLOAT |n|
                                         MOST-POSITIVE-LONG-FLOAT)
                                        (FLOAT |d|
                                         MOST-POSITIVE-LONG-FLOAT))
                                       (QREFELT $ 57)))))))))))
           G105837 (EXIT G105837)))))

(DEFUN |DoubleFloat| ()
  (PROG ()
    (RETURN
      (PROG (G105850)
        (RETURN
          (COND
            ((LETT G105850 (HGET |$ConstructorCache| '|DoubleFloat|)
                   |DoubleFloat|)
             (|CDRwithIncrement| (CDAR G105850)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|DoubleFloat|
                                   (LIST
                                    (CONS NIL
                                     (CONS 1 (|DoubleFloat;|))))))
                 (LETT G105850 T |DoubleFloat|))
               (COND
                 ((NOT G105850)
                  (HREM |$ConstructorCache| '|DoubleFloat|)))))))))))

(DEFUN |DoubleFloat;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
```

```
        (LETT |dv$| '(|DoubleFloat|) |DoubleFloat|)
        (LETT $ (make-array 140) |DoubleFloat|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |DoubleFloat|))
        (|haddProp| |$ConstructorCache| '|DoubleFloat| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        $))))

(setf (get
 (QUOTE |DoubleFloat|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|OpenMathEncoding|) (0 . |OMencodingXML|)
  (|String|) (|OpenMathDevice|) (4 . |OMopenString|) (|Void|)
  (10 . |OMputObject|) (|DoubleFloat|) (15 . |OMputFloat|)
  (21 . |OMputEndObject|) (26 . |OMclose|) |DFLOAT;OMwrite;$S;1|
  (|Boolean|) |DFLOAT;OMwrite;$BS;2| |DFLOAT;OMwrite;Omd$V;3|
  |DFLOAT;OMwrite;Omd$BV;4| (|PositiveInteger|) |DFLOAT;base;Pi;6|
  (|Integer|) |DFLOAT;mantissa;$I;7| |DFLOAT;exponent;$I;8|
  |DFLOAT;precision;Pi;9| |DFLOAT;log2;2$;37| (31 . |*|)
  |DFLOAT;bits;Pi;10| |DFLOAT;max;$;11| |DFLOAT;min;$;12|
  |DFLOAT;order;$I;13|
  (CONS IDENTITY (FUNCALL (|dispatchFunction| |DFLOAT;Zero;$;14|) |$|))
  (CONS IDENTITY (FUNCALL (|dispatchFunction| |DFLOAT;One;$;15|) |$|))
  |DFLOAT;exp1;$;16| |DFLOAT;pi;$;17| (|OutputForm|) (37 . |outputForm|)
  |DFLOAT;coerce;$Of;18| (|InputForm|) (42 . |convert|)
  |DFLOAT;convert;$If;19| |DFLOAT;<;2$B;20| |DFLOAT;-;2$;21|
  |DFLOAT;+;3$;22| |DFLOAT;-;3$;23| |DFLOAT;*;3$;24| |DFLOAT;*;I2$;25|
  |DFLOAT;max;3$;26| |DFLOAT;min;3$;27| |DFLOAT;=;2$B;28|
  |DFLOAT;/;$I$;29| |DFLOAT;sqrt;2$;30| |DFLOAT;log10;2$;31|
  |DFLOAT;**;$I$;32| |DFLOAT;**;3$;33| |DFLOAT;coerce;I$;34|
  |DFLOAT;exp;2$;35| |DFLOAT;log;2$;36| |DFLOAT;sin;2$;38|
  |DFLOAT;cos;2$;39| |DFLOAT;tan;2$;40| |DFLOAT;cot;2$;41|
  |DFLOAT;sec;2$;42| |DFLOAT;csc;2$;43| |DFLOAT;asin;2$;44|
  |DFLOAT;acos;2$;45| |DFLOAT;atan;2$;46| |DFLOAT;acsc;2$;47|
  |DFLOAT;acot;2$;48| |DFLOAT;asec;2$;49| |DFLOAT;sinh;2$;50|
  |DFLOAT;cosh;2$;51| |DFLOAT;tanh;2$;52| |DFLOAT;csch;2$;53|
  |DFLOAT;coth;2$;54| |DFLOAT;sech;2$;55| |DFLOAT;asinh;2$;56|
  |DFLOAT;acosh;2$;57| |DFLOAT;atanh;2$;58| |DFLOAT;acsch;2$;59|
  |DFLOAT;acoth;2$;60| |DFLOAT;asech;2$;61| |DFLOAT;/;3$;62|
  |DFLOAT;negative?;$B;63| |DFLOAT;zero?;$B;64| |DFLOAT;hash;$I;65|
  (|Union| |$| (QUOTE "failed")) |DFLOAT;recip;$U;66|
  |DFLOAT;differentiate;2$;67| (|DoubleFloatSpecialFunctions|)
  (47 . |Gamma|) |DFLOAT;Gamma;2$;68| (52 . |Beta|) |DFLOAT;Beta;3$;69|
  |DFLOAT;wholePart;$I;70| |DFLOAT;float;2IPi$;71| |DFLOAT;convert;2$;72|
  (|Float|) (58 . |convert|) |DFLOAT;convert;$F;73| (|Fraction| 24)
  (|NonNegativeInteger|) |DFLOAT;rationalApproximation;$2NniF;83|
  |DFLOAT;rationalApproximation;$NniF;74| |DFLOAT;atan;3$;75|
  |DFLOAT;retract;$F;76| (|Union| 103 (QUOTE "failed"))
```

```
|DFLOAT;retractIfCan;$U;77| |DFLOAT;retract;$I;78|
(|Union| 24 (QUOTE "failed")) |DFLOAT;retractIfCan;$U;79|
|DFLOAT;sign;$I;80| |DFLOAT;abs;2$;81| (63 . |Zero|) (67 . |/|)
(73 . |*|) (79 . |coerce|) (84 . |zero?|) (89 . |negative?|)
(94 . |one?|) (99 . |one?|) (104 . |numer|) (109 . |denom|)
|DFLOAT;**;$F$;84| (|Pattern| 100) (|PatternMatchResult| 100 |$|)
(|Factored| |$|) (|Union| 131 (QUOTE "failed")) (|List| |$|)
(|Record| (|:| |coef1| |$|) (|:| |coef2| |$|) (|:| |generator| |$|))
(|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
(|Union| 133 (QUOTE "failed"))
(|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
(|Record| (|:| |coef| 131) (|:| |generator| |$|))
(|SparseUnivariatePolynomial| |$|) (|Record| (|:| |unit| |$|)
(|:| |canonical| |$|) (|:| |associate| |$|)) (|SingleInteger|)))
(QUOTE #(|~=| 114 |zero?| 120 |wholePart| 125 |unitNormal| 130
 |unitCanonical| 135 |unit?| 140 |truncate| 145 |tanh| 150 |tan|
 155 |subtractIfCan| 160 |squareFreePart| 166 |squareFree| 171
 |sqrt| 176 |sizeLess?| 181 |sinh| 187 |sin| 192 |sign| 197 |sech|
 202 |sec| 207 |sample| 212 |round| 216 |retractIfCan| 221 |retract|
 231 |rem| 241 |recip| 247 |rationalApproximation| 252 |quo| 265
 |principalIdeal| 271 |prime?| 276 |precision| 281 |positive?| 285
 |pi| 290 |patternMatch| 294 |order| 301 |one?| 306 |nthRoot| 311
 |norm| 317 |negative?| 322 |multiEuclidean| 327 |min| 333 |max| 343
 |mantissa| 353 |log2| 358 |log10| 363 |log| 368 |lcm| 373 |latex|
 384 |inv| 389 |hash| 394 |gcdPolynomial| 404 |gcd| 410 |fractionPart|
 421 |floor| 426 |float| 431 |factor| 444 |extendedEuclidean| 449
 |exquo| 462 |expressIdealMember| 468 |exponent| 474 |exp1| 479 |exp|
 483 |euclideanSize| 488 |divide| 493 |digits| 499 |differentiate|
 503 |csch| 514 |csc| 519 |coth| 524 |cot| 529 |cosh| 534 |cos| 539
 |convert| 544 |coerce| 564 |characteristic| 594 |ceiling| 598 |bits|
 603 |base| 607 |atanh| 611 |atan| 616 |associates?| 627 |asinh| 633
 |asin| 638 |asech| 643 |asec| 648 |acsch| 653 |acsc| 658 |acoth| 663
 |acot| 668 |acosh| 673 |acos| 678 |abs| 683 |^| 688 |Zero| 706 |One|
 710 |OMwrite| 714 |Gamma| 738 D 743 |Beta| 754 |>=| 760 |>| 766 |=|
 772 |<=| 778 |<| 784 |/| 790 |-| 802 |+| 813 |**| 819 |*| 849))
(QUOTE ((|approximate| . 0) (|canonicalsClosed| . 0)
 (|canonicalUnitNormal| . 0) (|noZeroDivisors| . 0)
 ((|commutative| "*") . 0) (|rightUnitary| . 0) (|leftUnitary| . 0)
 (|unitsKnown| . 0)))
(CONS
 (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0)))
 (CONS
  (QUOTE #(|FloatingPointSystem&| |RealNumberSystem&| |Field&|
   |EuclideanDomain&| NIL |UniqueFactorizationDomain&| |GcdDomain&|
   |DivisionRing&| |IntegralDomain&| |Algebra&| |Algebra&|
   |DifferentialRing&| NIL |OrderedRing&| |Module&| NIL NIL |Module&|
   NIL NIL NIL |Ring&| NIL NIL NIL NIL NIL NIL NIL |AbelianGroup&| NIL
   NIL |AbelianMonoid&| |Monoid&| NIL |OrderedSet&| |AbelianSemiGroup&|
   |SemiGroup&| |TranscendentalFunctionCategory&| NIL |SetCategory&| NIL
```

```
    |ElementaryFunctionCategory&| NIL |HyperbolicFunctionCategory&|
    |ArcTrigonometricFunctionCategory&| |TrigonometricFunctionCategory&|
    NIL NIL |RadicalCategory&| |RetractableTo&| |RetractableTo&| NIL
    NIL |BasicType&| NIL)) (CONS (QUOTE #((|FloatingPointSystem|)
    (|RealNumberSystem|) (|Field|) (|EuclideanDomain|)
    (|PrincipalIdealDomain|) (|UniqueFactorizationDomain|)
    (|GcdDomain|) (|DivisionRing|) (|IntegralDomain|) (|Algebra| 103)
    (|Algebra| |$$|) (|DifferentialRing|) (|CharacteristicZero|)
    (|OrderedRing|) (|Module| 103) (|EntireRing|) (|CommutativeRing|)
    (|Module| |$$|) (|OrderedAbelianGroup|) (|BiModule| 103 103)
    (|BiModule| |$$| |$$|) (|Ring|) (|OrderedCancellationAbelianMonoid|)
    (|RightModule| 103) (|LeftModule| 103) (|LeftModule| |$$|) (|Rng|)
    (|RightModule| |$$|) (|OrderedAbelianMonoid|) (|AbelianGroup|)
    (|OrderedAbelianSemiGroup|) (|CancellationAbelianMonoid|)
    (|AbelianMonoid|) (|Monoid|) (|PatternMatchable| 100) (|OrderedSet|)
    (|AbelianSemiGroup|) (|SemiGroup|) (|TranscendentalFunctionCategory|)
    (|RealConstant|) (|SetCategory|) (|ConvertibleTo| 41)
    (|ElementaryFunctionCategory|) (|ArcHyperbolicFunctionCategory|)
    (|HyperbolicFunctionCategory|) (|ArcTrigonometricFunctionCategory|)
    (|TrigonometricFunctionCategory|) (|OpenMath|) (|ConvertibleTo| 127)
    (|RadicalCategory|) (|RetractableTo| 103) (|RetractableTo| 24)
    (|ConvertibleTo| 100) (|ConvertibleTo| 13) (|BasicType|)
    (|CoercibleTo| 38)))
 (|makeByteWordVec2| 139
  (QUOTE (0 6 0 7 2 9 0 8 6 10 1 9 11 0 12 2 9 11 0 13 14 1 9 11 0 15
   1 9 11 0 16 2 0 0 22 0 29 1 38 0 13 39 1 41 0 13 42 1 92 13 13 93 2 92
   13 13 13 95 1 100 0 13 101 0 103 0 116 2 103 0 24 24 117 2 24 0 104 0
   118 1 103 0 24 119 1 103 18 0 120 1 103 18 0 121 1 0 18 0 122 1 103 18
   0 123 1 103 24 0 124 1 103 24 0 125 2 0 18 0 0 1 1 0 18 0 87 1 0 24 0
   97 1 0 138 0 1 1 0 0 0 1 1 0 18 0 1 1 0 0 0 1 1 0 0 0 75 1 0 0 0 63 2
   0 89 0 0 1 1 0 0 0 1 1 0 129 0 1 1 0 0 0 54 2 0 18 0 0 1 1 0 0 0 73 1
   0 0 0 61 1 0 24 0 114 1 0 0 0 78 1 0 0 0 65 0 0 0 1 1 0 0 0 1 1 0 109
   0 110 1 0 112 0 113 1 0 103 0 108 1 0 24 0 111 2 0 0 0 0 1 1 0 89 0
   90 2 0 103 0 104 106 3 0 103 0 104 104 105 2 0 0 0 0 1 1 0 136 131 1
   1 0 18 0 1 0 0 22 27 1 0 18 0 1 0 0 0 37 3 0 128 0 127 128 1 1 0 24
   0 33 1 0 18 0 122 2 0 0 0 24 1 1 0 0 0 1 1 0 18 0 86 2 0 130 131 0 1
   0 0 0 32 2 0 0 0 0 51 0 0 0 31 2 0 0 0 0 50 1 0 24 0 25 1 0 0 0 28 1
   0 0 0 55 1 0 0 0 60 1 0 0 131 1 2 0 0 0 0 1 1 0 8 0 1 1 0 0 0 1 1 0
   24 0 88 1 0 139 0 1 2 0 137 137 137 1 1 0 0 131 1 2 0 0 0 0 1 1 0 0
   0 1 1 0 0 0 1 3 0 0 24 24 22 98 2 0 0 24 24 1 1 0 129 0 1 2 0 132 0
   0 1 3 0 134 0 0 0 1 2 0 89 0 0 1 2 0 130 131 0 1 1 0 24 0 26 0 0 0
   36 1 0 0 0 59 1 0 104 0 1 2 0 135 0 0 1 0 0 22 1 1 0 0 0 91 2 0 0 0
   104 1 1 0 0 0 76 1 0 0 0 66 1 0 0 0 77 1 0 0 0 64 1 0 0 0 74 1 0 0 0
   62 1 0 41 0 43 1 0 127 0 1 1 0 13 0 99 1 0 100 0 102 1 0 0 103 1 1 0
   0 24 58 1 0 0 103 1 1 0 0 24 58 1 0 0 0 1 1 0 38 0 40 0 0 104 1 1 0
   0 0 1 0 0 22 30 0 0 22 23 1 0 0 0 81 2 0 0 0 0 107 1 0 0 0 69 2 0 18
   0 0 1 1 0 0 0 79 1 0 0 0 67 1 0 0 0 84 1 0 0 0 72 1 0 0 0 82 1 0 0 0
   70 1 0 0 0 83 1 0 0 0 71 1 0 0 0 80 1 0 0 0 68 1 0 0 0 115 2 0 0 0
   24 1 2 0 0 0 104 1 2 0 0 0 22 1 0 0 0 34 0 0 0 35 2 0 11 9 0 20 3 0
   11 9 0 18 21 1 0 8 0 17 2 0 8 0 18 19 1 0 0 0 94 1 0 0 0 1 2 0 0 0
```

```
   104 1 2 0 0 0 0 96 2 0 18 0 0 1 2 0 18 0 0 1 2 0 18 0 0 52 2 0 18 0
   0 1 2 0 18 0 0 44 2 0 0 0 24 53 2 0 0 0 0 85 2 0 0 0 0 47 1 0 0 0
   45 2 0 0 0 0 46 2 0 0 0 0 57 2 0 0 0 103 126 2 0 0 0 24 56 2 0 0 0
   104 1 2 0 0 0 22 1 2 0 0 0 103 1 2 0 0 103 0 1 2 0 0 0 0 48 2 0 0
   24 0 49 2 0 0 104 0 1 2 0 0 22 0 29))))))
  (QUOTE |lookupComplete|)))

(setf (get (QUOTE |DoubleFloat|) (QUOTE NILADIC)) T)
```

28.4 ILIST.lsp BOOTSTRAP

ILIST depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ILIST** category which we can write into the **MID** directory. We compile the lisp code and copy the **ILIST.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ILIST.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |ILIST;#;$Nni;1|) (QUOTE |SPADreplace|) (QUOTE LENGTH))

(DEFUN |ILIST;#;$Nni;1| (|x| |$|) (LENGTH |x|))

(PUT (QUOTE |ILIST;concat;S2$;2|) (QUOTE |SPADreplace|) (QUOTE CONS))

(DEFUN |ILIST;concat;S2$;2| (|s| |x| |$|) (CONS |s| |x|))

(PUT (QUOTE |ILIST;eq?;2$B;3|) (QUOTE |SPADreplace|) (QUOTE EQ))

(DEFUN |ILIST;eq?;2$B;3| (|x| |y| |$|) (EQ |x| |y|))

(PUT (QUOTE |ILIST;first;$S;4|) (QUOTE |SPADreplace|) (QUOTE |SPADfirst|))

(DEFUN |ILIST;first;$S;4| (|x| |$|) (|SPADfirst| |x|))

(PUT
 (QUOTE |ILIST;elt;$firstS;5|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x| "first") (|SPADfirst| |x|))))

(DEFUN |ILIST;elt;$firstS;5| (|x| G101995 |$|) (|SPADfirst| |x|))

(PUT (QUOTE |ILIST;empty;$;6|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL NIL)))
```

```
(DEFUN |ILIST;empty;$;6| (|$|) NIL)

(PUT (QUOTE |ILIST;empty?;$B;7|) (QUOTE |SPADreplace|) (QUOTE NULL))

(DEFUN |ILIST;empty?;$B;7| (|x| |$|) (NULL |x|))

(PUT (QUOTE |ILIST;rest;2$;8|) (QUOTE |SPADreplace|) (QUOTE CDR))

(DEFUN |ILIST;rest;2$;8| (|x| |$|) (CDR |x|))

(PUT
 (QUOTE |ILIST;elt;$rest$;9|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x| "rest") (CDR |x|))))

(DEFUN |ILIST;elt;$rest$;9| (|x| G102000 |$|) (CDR |x|))

(DEFUN |ILIST;setfirst!;$2S;10| (|x| |s| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
  ((QUOTE T) (QCAR (RPLACA |x| |s|)))))

(DEFUN |ILIST;setelt;$first2S;11| (|x| G102005 |s| |$|)
 (COND
   ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
   ((QUOTE T) (QCAR (RPLACA |x| |s|)))))

(DEFUN |ILIST;setrest!;3$;12| (|x| |y| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
  ((QUOTE T) (QCDR (RPLACD |x| |y|)))))

(DEFUN |ILIST;setelt;$rest2$;13| (|x| G102010 |y| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
  ((QUOTE T) (QCDR (RPLACD |x| |y|)))))

(PUT
 (QUOTE |ILIST;construct;L$;14|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|l|) |l|)))

(DEFUN |ILIST;construct;L$;14| (|l| |$|) |l|)

(PUT
 (QUOTE |ILIST;parts;$L;15|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|s|) |s|)))
```

```
(DEFUN |ILIST;parts;$L;15| (|s| |$|) |s|)

(PUT (QUOTE |ILIST;reverse!;2$;16|) (QUOTE |SPADreplace|) (QUOTE NREVERSE))

(DEFUN |ILIST;reverse!;2$;16| (|x| |$|) (NREVERSE |x|))

(PUT (QUOTE |ILIST;reverse;2$;17|) (QUOTE |SPADreplace|) (QUOTE REVERSE))

(DEFUN |ILIST;reverse;2$;17| (|x| |$|) (REVERSE |x|))

(DEFUN |ILIST;minIndex;$I;18| (|x| |$|) (QREFELT |$| 7))

(DEFUN |ILIST;rest;$Nni$;19| (|x| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |i| 1 |ILIST;rest;$Nni$;19|)
     G190
     (COND ((QSGREATERP |i| |n|) (GO G191)))
     (SEQ
      (COND ((NULL |x|) (|error| "index out of range")))
      (EXIT (LETT |x| (QCDR |x|) |ILIST;rest;$Nni$;19|)))
     (LETT |i| (QSADD1 |i|) |ILIST;rest;$Nni$;19|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |ILIST;copy;2$;20| (|x| |$|)
 (PROG (|i| |y|)
  (RETURN
   (SEQ
    (LETT |y| (SPADCALL (QREFELT |$| 16)) |ILIST;copy;2$;20|)
    (SEQ
     (LETT |i| 0 |ILIST;copy;2$;20|)
     G190
     (COND
      ((NULL (COND ((NULL |x|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
     (SEQ
      (COND
       ((EQ |i| 1000)
        (COND ((SPADCALL |x| (QREFELT |$| 33)) (|error| "cyclic list")))))
      (LETT |y| (CONS (QCAR |x|) |y|) |ILIST;copy;2$;20|)
      (EXIT (LETT |x| (QCDR |x|) |ILIST;copy;2$;20|)))
     (LETT |i| (QSADD1 |i|) |ILIST;copy;2$;20|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (NREVERSE |y|))))))
```

```
(DEFUN |ILIST;coerce;$Of;21| (|x| |$|)
 (PROG (|s| |y| |z|)
  (RETURN
   (SEQ
    (LETT |y| NIL |ILIST;coerce;$Of;21|)
    (LETT |s| (SPADCALL |x| (QREFELT |$| 35)) |ILIST;coerce;$Of;21|)
    (SEQ
     G190
     (COND ((NULL (NEQ |x| |s|)) (GO G191)))
     (SEQ
      (LETT |y|
       (CONS (SPADCALL (SPADCALL |x| (QREFELT |$| 13)) (QREFELT |$| 37)) |y|)
       |ILIST;coerce;$Of;21|)
      (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 18)) |ILIST;coerce;$Of;21|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (LETT |y| (NREVERSE |y|) |ILIST;coerce;$Of;21|)
    (EXIT
     (COND
      ((SPADCALL |s| (QREFELT |$| 17)) (SPADCALL |y| (QREFELT |$| 39)))
      ((QUOTE T)
       (SEQ
        (LETT |z|
         (SPADCALL
          (SPADCALL (SPADCALL |x| (QREFELT |$| 13)) (QREFELT |$| 37))
          (QREFELT |$| 41))
         |ILIST;coerce;$Of;21|)
        (SEQ
         G190
         (COND ((NULL (NEQ |s| (SPADCALL |x| (QREFELT |$| 18)))) (GO G191)))
         (SEQ
          (LETT |x| (SPADCALL |x| (QREFELT |$| 18)) |ILIST;coerce;$Of;21|)
          (EXIT
           (LETT |z|
            (CONS
             (SPADCALL (SPADCALL |x| (QREFELT |$| 13)) (QREFELT |$| 37))
             |z|)
            |ILIST;coerce;$Of;21|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT
         (SPADCALL
          (SPADCALL |y|
           (SPADCALL
            (SPADCALL
```

```
             (NREVERSE |z|)
             (QREFELT |$| 42))
            (QREFELT |$| 43))
           (QREFELT |$| 44))
          (QREFELT |$| 39)))))))))))

(DEFUN |ILIST;=;2$B;22| (|x| |y| $)
  (PROG (G102042)
    (RETURN
      (SEQ (EXIT (COND
                   ((EQ |x| |y|) 'T)
                   ('T
                    (SEQ (SEQ G190
                              (COND
                                ((NULL (COND
                                         ((OR (NULL |x|) (NULL |y|))
                                          'NIL)
                                         ('T 'T)))
                                 (GO G191)))
                              (SEQ (EXIT
                                     (COND
                                       ((NULL
                                         (SPADCALL (QCAR |x|) (QCAR |y|)
                                          (QREFELT $ 46)))
                                        (PROGN
                                          (LETT G102042 'NIL
                                           |ILIST;=;2$B;22|)
                                          (GO G102042)))
                                       ('T
                                        (SEQ
                                         (LETT |x| (QCDR |x|)
                                          |ILIST;=;2$B;22|)
                                         (EXIT
                                          (LETT |y| (QCDR |y|)
                                           |ILIST;=;2$B;22|)))))))
                              NIL (GO G190) G191 (EXIT NIL))
                         (EXIT (COND
                                 ((NULL |x|) (NULL |y|))
                                 ('T 'NIL)))))))
           G102042 (EXIT G102042)))))

(DEFUN |ILIST;latex;$S;23| (|x| |$|)
 (PROG (|s|)
  (RETURN
   (SEQ
    (LETT |s| "\\left[" |ILIST;latex;$S;23|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |x|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
```

```
     (GO G191)))
    (SEQ
     (LETT |s|
       (STRCONC |s| (SPADCALL (QCAR |x|) (QREFELT |$| 49)))
       |ILIST;latex;$S;23|)
     (LETT |x| (QCDR |x|) |ILIST;latex;$S;23|)
     (EXIT
      (COND
       ((NULL (NULL |x|))
         (LETT |s| (STRCONC |s| ", ") |ILIST;latex;$S;23|)))))
    NIL
    (GO G190)
    G191
    (EXIT NIL))
   (EXIT (STRCONC |s| " \\right]"))))))

(DEFUN |ILIST;member?;S$B;24| (|s| |x| $)
  (PROG (G102052)
    (RETURN
      (SEQ (EXIT (SEQ (SEQ G190
                           (COND
                             ((NULL (COND ((NULL |x|) 'NIL) ('T 'T)))
                              (GO G191)))
                           (SEQ (EXIT (COND
                                        ((SPADCALL |s| (QCAR |x|)
                                           (QREFELT $ 46))
                                         (PROGN
                                           (LETT G102052 'T
                                            |ILIST;member?;S$B;24|)
                                           (GO G102052)))
                                        ('T
                                         (LETT |x| (QCDR |x|)
                                          |ILIST;member?;S$B;24|)))))
                           NIL (GO G190) G191 (EXIT NIL))
                      (EXIT 'NIL)))
           G102052 (EXIT G102052)))))

(DEFUN |ILIST;concat!;3$;25| (|x| |y| |$|)
 (PROG (|z|)
  (RETURN
   (SEQ
    (COND
     ((NULL |x|)
      (COND
       ((NULL |y|) |x|)
       ((QUOTE T)
        (SEQ
         (PUSH (SPADCALL |y| (QREFELT |$| 13)) |x|)
         (QRPLACD |x| (SPADCALL |y| (QREFELT |$| 18))) (EXIT |x|)))))
     ((QUOTE T)
```

```
      (SEQ
       (LETT |z| |x| |ILIST;concat!;3$;25|)
       (SEQ
        G190
        (COND
         ((NULL (COND ((NULL (QCDR |z|)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
           (GO G191)))
        (SEQ (EXIT (LETT |z| (QCDR |z|) |ILIST;concat!;3$;25|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (QRPLACD |z| |y|)
       (EXIT |x|))))))))

(DEFUN |ILIST;removeDuplicates!;2$;26| (|l| |$|)
 (PROG (|f| |p| |pr| |pp|)
  (RETURN
   (SEQ
    (LETT |p| |l| |ILIST;removeDuplicates!;2$;26|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |p|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
     (SEQ
      (LETT |pp| |p| |ILIST;removeDuplicates!;2$;26|)
      (LETT |f| (QCAR |p|) |ILIST;removeDuplicates!;2$;26|)
      (LETT |p| (QCDR |p|) |ILIST;removeDuplicates!;2$;26|)
      (EXIT
       (SEQ
        G190
        (COND
         ((NULL
          (COND
           ((NULL (LETT |pr| (QCDR |pp|) |ILIST;removeDuplicates!;2$;26|))
             (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((SPADCALL (QCAR |pr|) |f| (QREFELT |$| 46))
             (QRPLACD |pp| (QCDR |pr|)))
           ((QUOTE T) (LETT |pp| |pr| |ILIST;removeDuplicates!;2$;26|)))))
        NIL
        (GO G190)
        G191
        (EXIT NIL))))
     NIL
     (GO G190)
```

```
     G191
     (EXIT NIL))
    (EXIT |l|)))))

(DEFUN |ILIST;sort!;M2$;27| (|f| |l| |$|)
 (|ILIST;mergeSort| |f| |l| (SPADCALL |l| (QREFELT |$| 9)) |$|))

(DEFUN |ILIST;merge!;M3$;28| (|f| |p| |q| |$|)
 (PROG (|r| |t|)
  (RETURN
   (SEQ
    (COND
     ((NULL |p|) |q|)
     ((NULL |q|) |p|)
     ((EQ |p| |q|) (|error| "cannot merge a list into itself"))
     ((QUOTE T)
      (SEQ
       (COND
        ((SPADCALL (QCAR |p|) (QCAR |q|) |f|)
         (SEQ
          (LETT |r| (LETT |t| |p| |ILIST;merge!;M3$;28|) |ILIST;merge!;M3$;28|)
          (EXIT (LETT |p| (QCDR |p|) |ILIST;merge!;M3$;28|))))
        ((QUOTE T)
         (SEQ
          (LETT |r| (LETT |t| |q| |ILIST;merge!;M3$;28|) |ILIST;merge!;M3$;28|)
          (EXIT (LETT |q| (QCDR |q|) |ILIST;merge!;M3$;28|)))))
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((OR (NULL |p|) (NULL |q|)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((SPADCALL (QCAR |p|) (QCAR |q|) |f|)
            (SEQ
             (QRPLACD |t| |p|)
             (LETT |t| |p| |ILIST;merge!;M3$;28|)
             (EXIT (LETT |p| (QCDR |p|) |ILIST;merge!;M3$;28|))))
           ((QUOTE T)
            (SEQ
             (QRPLACD |t| |q|)
             (LETT |t| |q| |ILIST;merge!;M3$;28|)
             (EXIT (LETT |q| (QCDR |q|) |ILIST;merge!;M3$;28|)))))))
        NIL
        (GO G190)
        G191
```

```
        (EXIT NIL))
       (QRPLACD |t| (COND ((NULL |p|) |q|) ((QUOTE T) |p|)))
       (EXIT |r|))))))))

(DEFUN |ILIST;split!;$I$;29| (|p| |n| $)
  (PROG (G102085 |q|)
    (RETURN
      (SEQ (COND
             ((< |n| 1) (|error| "index out of range"))
             ('T
              (SEQ (LETT |p|
                         (SPADCALL |p|
                             (PROG1 (LETT G102085 (- |n| 1)
                                     |ILIST;split!;$I$;29|)
                               (|check-subtype| (>= G102085 0)
                                   '(|NonNegativeInteger|) G102085))
                             (QREFELT $ 32))
                         |ILIST;split!;$I$;29|)
                   (LETT |q| (QCDR |p|) |ILIST;split!;$I$;29|)
                   (QRPLACD |p| NIL) (EXIT |q|))))))))

(DEFUN |ILIST;mergeSort| (|f| |p| |n| $)
  (PROG (G102089 |l| |q|)
    (RETURN
      (SEQ (COND
             ((EQL |n| 2)
              (COND
                ((SPADCALL
                     (SPADCALL (SPADCALL |p| (QREFELT $ 18))
                         (QREFELT $ 13))
                     (SPADCALL |p| (QREFELT $ 13)) |f|)
                 (LETT |p| (SPADCALL |p| (QREFELT $ 28))
                       |ILIST;mergeSort|)))))
           (EXIT (COND
                   ((< |n| 3) |p|)
                   ('T
                    (SEQ (LETT |l|
                               (PROG1 (LETT G102089 (QUOTIENT2 |n| 2)
                                       |ILIST;mergeSort|)
                                 (|check-subtype| (>= G102089 0)
                                     '(|NonNegativeInteger|) G102089))
                               |ILIST;mergeSort|)
                         (LETT |q| (SPADCALL |p| |l| (QREFELT $ 57))
                               |ILIST;mergeSort|)
                         (LETT |p| (|ILIST;mergeSort| |f| |p| |l| $)
                               |ILIST;mergeSort|)
                         (LETT |q|
                               (|ILIST;mergeSort| |f| |q| (- |n| |l|)
                                   $)
                               |ILIST;mergeSort|)
```

```
                         (EXIT (SPADCALL |f| |p| |q| (QREFELT $ 56))))))))))

(DEFUN |IndexedList| (&REST G102103 &AUX G102101)
  (DSETQ G102101 G102103)
  (PROG ()
    (RETURN
      (PROG (G102102)
        (RETURN
          (COND
            ((LETT G102102
                   (|lassocShiftWithFunction| (|devaluateList| G102101)
                       (HGET |$ConstructorCache| '|IndexedList|)
                       '|domainEqualList|)
                   |IndexedList|)
             (|CDRwithIncrement| G102102))
            ('T
             (UNWIND-PROTECT
               (PROG1 (APPLY (|function| |IndexedList;|) G102101)
                 (LETT G102102 T |IndexedList|))
               (COND
                 ((NOT G102102)
                  (HREM |$ConstructorCache| '|IndexedList|)))))))))))

(DEFUN |IndexedList;| (|#1| |#2|)
  (PROG (DV$1 DV$2 |dv$| $ G102100 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |IndexedList|)
        (LETT DV$2 (|devaluate| |#2|) |IndexedList|)
        (LETT |dv$| (LIST '|IndexedList| DV$1 DV$2) |IndexedList|)
        (LETT $ (make-array 71) |IndexedList|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$|
                  (|buildPredVector| 0 0
                      (LIST (|HasCategory| |#1| '(|SetCategory|))
                            (|HasCategory| |#1|
                                '(|ConvertibleTo| (|InputForm|)))
                            (LETT G102100
                                  (|HasCategory| |#1| '(|OrderedSet|))
                                  |IndexedList|)
                            (OR G102100
                                (|HasCategory| |#1| '(|SetCategory|)))
                            (|HasCategory| (|Integer|) '(|OrderedSet|))
                            (AND (|HasCategory| |#1|
                                         (LIST '|Evalable|
                                          (|devaluate| |#1|)))
                                 (|HasCategory| |#1| '(|SetCategory|)))
                            (OR (AND (|HasCategory| |#1|
                                          (LIST '|Evalable|
```

```
                                        (|devaluate| |#1|)))
                                      G102100)
                                 (AND (|HasCategory| |#1|
                                       (LIST '|Evalable|
                                        (|devaluate| |#1|)))
                                      (|HasCategory| |#1|
                                       '(|SetCategory|))))))
                 |IndexedList|))
        (|haddProp| |$ConstructorCache| '|IndexedList| (LIST DV$1 DV$2)
            (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 |#1|)
        (QSETREFV $ 7 |#2|)
        (COND
          ((|testBitVector| |pv$| 1)
           (PROGN
             (QSETREFV $ 45
                 (CONS (|dispatchFunction| |ILIST;coerce;$Of;21|) $))
             (QSETREFV $ 47
                 (CONS (|dispatchFunction| |ILIST;=;2$B;22|) $))
             (QSETREFV $ 50
                 (CONS (|dispatchFunction| |ILIST;latex;$S;23|) $))
             (QSETREFV $ 51
                 (CONS (|dispatchFunction| |ILIST;member?;S$B;24|) $)))))
        (COND
          ((|testBitVector| |pv$| 1)
           (QSETREFV $ 53
               (CONS (|dispatchFunction|
                         |ILIST;removeDuplicates!;2$;26|)
                     $))))
        $))))

(setf (get
 (QUOTE |IndexedList|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(
   NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|NonNegativeInteger|) |ILIST;#;$Nni;1| |ILIST;concat;S2$;2| (|Boolean|)
   |ILIST;eq?;2$B;3| |ILIST;first;$S;4| (QUOTE "first") |ILIST;elt;$firstS;5|
   |ILIST;empty;$;6| |ILIST;empty?;$B;7| |ILIST;rest;2$;8| (QUOTE "rest")
   |ILIST;elt;$rest$;9| |ILIST;setfirst!;$2S;10| |ILIST;setelt;$first2S;11|
   |ILIST;setrest!;3$;12| |ILIST;setelt;$rest2$;13| (|List| 6)
   |ILIST;construct;L$;14| |ILIST;parts;$L;15| |ILIST;reverse!;2$;16|
   |ILIST;reverse;2$;17| (|Integer|) |ILIST;minIndex;$I;18|
   |ILIST;rest;$Nni$;19| (0 . |cyclic?|) |ILIST;copy;2$;20|
   (5 . |cycleEntry|) (|OutputForm|) (10 . |coerce|) (|List| |$|)
   (15 . |bracket|) (|List| 36) (20 . |list|) (25 . |commaSeparate|)
   (30 . |overbar|) (35 . |concat!|) (41 . |coerce|) (46 . |=|) (52 . |=|)
   (|String|) (58 . |latex|) (63 . |latex|) (68 . |member?|)
```

```
 |ILIST;concat!;3$;25| (74 . |removeDuplicates!|) (|Mapping| 11 6 6)
 |ILIST;sort!;M2$;27| |ILIST;merge!;M3$;28| |ILIST;split!;$I$;29|
 (|Mapping| 6 6 6) (|Equation| 6) (|List| 59) (|Mapping| 11 6) (|Void|)
 (|UniversalSegment| 30) (QUOTE "last") (QUOTE "value") (|Mapping| 6 6)
 (|InputForm|) (|SingleInteger|) (|List| 30) (|Union| 6 (QUOTE "failed"))))
(QUOTE #(
 |~=| 79 |value| 85 |third| 90 |tail| 95 |swap!| 100 |split!| 107
 |sorted?| 113 |sort!| 124 |sort| 135 |size?| 146 |setvalue!| 152
 |setrest!| 158 |setlast!| 164 |setfirst!| 170 |setelt| 176
 |setchildren!| 218 |select!| 224 |select| 230 |second| 236 |sample|
 241 |reverse!| 245 |reverse| 250 |rest| 255 |removeDuplicates!|
 266 |removeDuplicates| 271 |remove!| 276 |remove| 288 |reduce|
 300 |qsetelt!| 321 |qelt| 328 |possiblyInfinite?| 334 |position|
 339 |parts| 358 |nodes| 363 |node?| 368 |new| 374 |more?| 380
 |minIndex| 386 |min| 391 |merge!| 397 |merge| 410 |members| 423
 |member?| 428 |maxIndex| 434 |max| 439 |map!| 445 |map| 451 |list|
 464 |less?| 469 |leaves| 475 |leaf?| 480 |latex| 485 |last| 490
 |insert!| 501 |insert| 515 |indices| 529 |index?| 534 |hash| 540
 |first| 545 |find| 556 |fill!| 562 |explicitlyFinite?| 568 |every?|
 573 |eval| 579 |eq?| 605 |entry?| 611 |entries| 617 |empty?| 622
 |empty| 627 |elt| 631 |distance| 674 |delete!| 680 |delete| 692
 |cyclic?| 704 |cycleTail| 709 |cycleSplit!| 714 |cycleLength| 719
 |cycleEntry| 724 |count| 729 |copyInto!| 741 |copy| 748 |convert|
 753 |construct| 758 |concat!| 763 |concat| 775 |coerce| 798
 |children| 803 |child?| 808 |any?| 814 |>=| 820 |>| 826 |=| 832
 |<=| 838 |<| 844 |#| 850))
(QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
(CONS
 (|makeByteWordVec2| 7 (QUOTE (0 0 0 0 0 0 0 0 0 0 3 0 0 7 4 0 0 7 1 2 4)))
 (CONS
  (QUOTE #(|ListAggregate&| |StreamAggregate&| |ExtensibleLinearAggregate&|
    |FiniteLinearAggregate&| |UnaryRecursiveAggregate&| |LinearAggregate&|
    |RecursiveAggregate&| |IndexedAggregate&| |Collection&|
    |HomogeneousAggregate&| |OrderedSet&| |Aggregate&| |EltableAggregate&|
    |Evalable&| |SetCategory&| NIL NIL |InnerEvalable&| NIL NIL
    |BasicType&|))
  (CONS
   (QUOTE #(
    (|ListAggregate| 6) (|StreamAggregate| 6)
    (|ExtensibleLinearAggregate| 6) (|FiniteLinearAggregate| 6)
    (|UnaryRecursiveAggregate| 6) (|LinearAggregate| 6)
    (|RecursiveAggregate| 6) (|IndexedAggregate| 30 6)
    (|Collection| 6) (|HomogeneousAggregate| 6) (|OrderedSet|)
    (|Aggregate|) (|EltableAggregate| 30 6) (|Evalable| 6) (|SetCategory|)
    (|Type|) (|Eltable| 30 6) (|InnerEvalable| 6 6) (|CoercibleTo| 36)
    (|ConvertibleTo| 67) (|BasicType|)))
   (|makeByteWordVec2| 70
    (QUOTE (1 0 11 0 33 1 0 0 0 35 1 6 36 0 37 1 36 0 38 39 1 40 0 36
     41 1 36 0 38 42 1 36 0 0 43 2 40 0 0 36 44 1 0 36 0 45 2 6 11 0 0
     46 2 0 11 0 0 47 1 6 48 0 49 1 0 48 0 50 2 0 11 6 0 51 1 0 0 0 53
```

```
      2 1 11 0 0 1 1 0 6 0 1 1 0 6 0 1 1 0 0 0 1 3 0 62 0 30 30 1 2 0 0
      0 30 57 1 3 11 0 1 2 0 11 54 0 1 1 3 0 0 1 2 0 0 54 0 55 1 3 0 0 1
      2 0 0 54 0 1 2 0 11 0 8 1 2 0 6 0 6 1 2 0 0 0 0 23 2 0 6 0 6 1 2 0
      6 0 6 21 3 0 6 0 30 6 1 3 0 6 0 63 6 1 3 0 6 0 64 6 1 3 0 0 0 19 0
      24 3 0 6 0 14 6 22 3 0 6 0 65 6 1 2 0 0 0 38 1 2 0 0 61 0 1 2 0 0
      61 0 1 1 0 6 0 1 0 0 0 1 1 0 0 0 28 1 0 0 0 29 2 0 0 0 8 32 1 0 0
      0 18 1 1 0 0 53 1 1 0 0 1 2 1 0 6 0 1 2 0 0 61 0 1 2 1 0 6 0 1 2 0
      0 61 0 1 4 1 6 58 0 6 6 1 2 0 6 58 0 1 3 0 6 58 0 6 1 3 0 6 0 30 6
      1 2 0 6 0 30 1 1 0 11 0 1 2 1 30 6 0 1 3 1 30 6 0 30 1 2 0 30 61 0
      1 1 0 25 0 27 1 0 38 0 1 2 1 11 0 0 1 2 0 0 8 6 1 2 0 11 0 8 1 1 5
      30 0 31 2 3 0 0 0 1 2 3 0 0 0 1 3 0 0 54 0 0 56 2 3 0 0 0 1 3 0 0
      54 0 0 1 1 0 25 0 1 2 1 11 6 0 51 1 5 30 0 1 2 3 0 0 0 1 2 0 0 66
      0 1 3 0 0 58 0 0 1 2 0 0 66 0 1 1 0 0 6 1 2 0 11 0 8 1 1 0 25 0 1
      1 0 11 0 1 1 1 48 0 50 2 0 0 0 8 1 1 0 6 0 1 3 0 0 6 0 30 1 3 0 0
      0 0 30 1 3 0 0 0 0 30 1 3 0 0 6 0 30 1 1 0 69 0 1 2 0 11 30 0 1 1
      1 68 0 1 2 0 0 0 8 1 1 0 6 0 13 2 0 70 61 0 1 2 0 0 0 6 1 1 0 11
      0 1 2 0 11 61 0 1 3 6 0 0 6 6 1 3 6 0 0 25 25 1 2 6 0 0 59 1 2 6
      0 0 60 1 2 0 11 0 0 12 2 1 11 6 0 1 1 0 25 0 1 1 0 11 0 17 0 0 0
      16 2 0 6 0 30 1 3 0 6 0 30 6 1 2 0 0 0 63 1 2 0 6 0 64 1 2 0 0 0
      19 20 2 0 6 0 14 15 2 0 6 0 65 1 2 0 30 0 0 1 2 0 0 0 63 1 2 0 0 0
      30 1 2 0 0 0 63 1 2 0 0 0 30 1 1 0 11 0 33 1 0 0 0 1 1 0 0 0 1 1 0
      8 0 1 1 0 0 0 35 2 1 8 6 0 1 2 0 8 61 0 1 3 0 0 0 0 30 1 1 0 0 0
      34 1 2 67 0 1 1 0 0 25 26 2 0 0 0 0 52 2 0 0 0 6 1 1 0 0 38 1 2 0
      0 0 6 1 2 0 0 6 0 10 2 0 0 0 0 1 1 1 36 0 45 1 0 38 0 1 2 1 11 0
      0 1 2 0 11 61 0 1 2 3 11 0 0 1 2 3 11 0 0 1 2 1 11 0 0 47 2 3 11
      0 0 1 2 3 11 0 0 1 1 0 8 0 9))))))
  (QUOTE |lookupComplete|)))
```

28.5 INT.lsp BOOTSTRAP

INT depends on **OINTDOM** which depends on **ORDRING** which depends on **INT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INT** category which we can write into the **MID** directory. We compile the lisp code and copy the **INT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— INT.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |INT;writeOMInt| (|dev| |x| |$|)
 (SEQ
  (COND
```

```
   ((|<| |x| 0)
    (SEQ
     (SPADCALL |dev| (QREFELT |$| 8))
     (SPADCALL |dev| "arith1" "unary_minus" (QREFELT |$| 10))
     (SPADCALL |dev| (|-| |x|) (QREFELT |$| 12))
     (EXIT (SPADCALL |dev| (QREFELT |$| 13)))))
   ((QUOTE T) (SPADCALL |dev| |x| (QREFELT |$| 12))))))

(DEFUN |INT;OMwrite;$S;2| (|x| |$|)
 (PROG (|sp| |dev| |s|)
  (RETURN
   (SEQ
    (LETT |s| "" |INT;OMwrite;$S;2|)
    (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |INT;OMwrite;$S;2|)
    (LETT |dev|
     (SPADCALL |sp| (SPADCALL (QREFELT |$| 15)) (QREFELT |$| 16))
     |INT;OMwrite;$S;2|)
    (SPADCALL |dev| (QREFELT |$| 17))
    (|INT;writeOMInt| |dev| |x| |$|)
    (SPADCALL |dev| (QREFELT |$| 18))
    (SPADCALL |dev| (QREFELT |$| 19))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |INT;OMwrite;$S;2|)
    (EXIT |s|)))))

(DEFUN |INT;OMwrite;$BS;3| (|x| |wholeObj| |$|)
 (PROG (|sp| |dev| |s|)
  (RETURN
   (SEQ
    (LETT |s| "" |INT;OMwrite;$BS;3|)
    (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |INT;OMwrite;$BS;3|)
    (LETT |dev|
     (SPADCALL |sp| (SPADCALL (QREFELT |$| 15)) (QREFELT |$| 16))
     |INT;OMwrite;$BS;3|)
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 17))))
    (|INT;writeOMInt| |dev| |x| |$|)
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 18))))
    (SPADCALL |dev| (QREFELT |$| 19))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |INT;OMwrite;$BS;3|)
    (EXIT |s|)))))

(DEFUN |INT;OMwrite;Omd$V;4| (|dev| |x| |$|)
 (SEQ
  (SPADCALL |dev| (QREFELT |$| 17))
  (|INT;writeOMInt| |dev| |x| |$|)
  (EXIT (SPADCALL |dev| (QREFELT |$| 18)))))

(DEFUN |INT;OMwrite;Omd$BV;5| (|dev| |x| |wholeObj| |$|)
 (SEQ
  (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 17))))
  (|INT;writeOMInt| |dev| |x| |$|)
```

```
  (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 18)))))))

(PUT (QUOTE |INT;zero?;$B;6|) (QUOTE |SPADreplace|) (QUOTE ZEROP))

(DEFUN |INT;zero?;$B;6| (|x| |$|) (ZEROP |x|))

(PUT (QUOTE |INT;Zero;$;7|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 0)))

(DEFUN |INT;Zero;$;7| (|$|) 0)

(PUT (QUOTE |INT;One;$;8|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 1)))

(DEFUN |INT;One;$;8| (|$|) 1)

(PUT (QUOTE |INT;base;$;9|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 2)))

(DEFUN |INT;base;$;9| (|$|) 2)

(PUT (QUOTE |INT;copy;2$;10|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INT;copy;2$;10| (|x| |$|) |x|)

(PUT
 (QUOTE |INT;inc;2$;11|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) (|+| |x| 1))))

(DEFUN |INT;inc;2$;11| (|x| |$|) (|+| |x| 1))

(PUT
 (QUOTE |INT;dec;2$;12|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) (|-| |x| 1))))

(DEFUN |INT;dec;2$;12| (|x| |$|) (|-| |x| 1))

(PUT (QUOTE |INT;hash;2$;13|) (QUOTE |SPADreplace|) (QUOTE SXHASH))

(DEFUN |INT;hash;2$;13| (|x| |$|) (SXHASH |x|))

(PUT (QUOTE |INT;negative?;$B;14|) (QUOTE |SPADreplace|) (QUOTE MINUSP))

(DEFUN |INT;negative?;$B;14| (|x| |$|) (MINUSP |x|))

(DEFUN |INT;coerce;$Of;15| (|x| |$|) (SPADCALL |x| (QREFELT |$| 35)))

(PUT
 (QUOTE |INT;coerce;2$;16|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|m|) |m|)))
```

```
(DEFUN |INT;coerce;2$;16| (|m| |$|) |m|)

(PUT
 (QUOTE |INT;convert;2$;17|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INT;convert;2$;17| (|x| |$|) |x|)

(PUT
 (QUOTE |INT;length;2$;18|)
 (QUOTE |SPADreplace|)
 (QUOTE |INTEGER-LENGTH|))

(DEFUN |INT;length;2$;18| (|a| |$|) (|INTEGER-LENGTH| |a|))

(DEFUN |INT;addmod;4$;19| (|a| |b| |p| $)
  (PROG (|c| G86338)
    (RETURN
      (SEQ (EXIT (SEQ (SEQ (LETT |c| (+ |a| |b|) |INT;addmod;4$;19|)
                           (EXIT (COND
                                   ((NULL (< |c| |p|))
                                    (PROGN
                                      (LETT G86338 (- |c| |p|)
                                       |INT;addmod;4$;19|)
                                      (GO G86338))))))
                      (EXIT |c|)))
           G86338 (EXIT G86338)))))

(DEFUN |INT;submod;4$;20| (|a| |b| |p| |$|)
 (PROG (|c|)
  (RETURN
   (SEQ
    (LETT |c| (|-| |a| |b|) |INT;submod;4$;20|)
    (EXIT (COND ((|<| |c| 0) (|+| |c| |p|)) ((QUOTE T) |c|)))))))

(DEFUN |INT;mulmod;4$;21| (|a| |b| |p| |$|) (REMAINDER2 (|*| |a| |b|) |p|))

(DEFUN |INT;convert;$F;22| (|x| |$|) (SPADCALL |x| (QREFELT |$| 44)))

(PUT
 (QUOTE |INT;convert;$Df;23|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|x|) (FLOAT |x| |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |INT;convert;$Df;23| (|x| |$|) (FLOAT |x| |MOST-POSITIVE-LONG-FLOAT|))

(DEFUN |INT;convert;$If;24| (|x| |$|) (SPADCALL |x| (QREFELT |$| 49)))
```

```
(PUT (QUOTE |INT;convert;$S;25|) (QUOTE |SPADreplace|) (QUOTE STRINGIMAGE))

(DEFUN |INT;convert;$S;25| (|x| |$|) (STRINGIMAGE |x|))

(DEFUN |INT;latex;$S;26| (|x| |$|)
 (PROG (|s|)
  (RETURN
   (SEQ
    (LETT |s| (STRINGIMAGE |x|) |INT;latex;$S;26|)
    (COND ((|<| -1 |x|) (COND ((|<| |x| 10) (EXIT |s|)))))
    (EXIT (STRCONC "{" (STRCONC |s| "}")))))))

(DEFUN |INT;positiveRemainder;3$;27| (|a| |b| |$|)
 (PROG (|r|)
  (RETURN
   (COND
    ((MINUSP (LETT |r| (REMAINDER2 |a| |b|) |INT;positiveRemainder;3$;27|))
      (COND
       ((MINUSP |b|) (|-| |r| |b|))
       ((QUOTE T) (|+| |r| |b|))))
    ((QUOTE T) |r|)))))

(PUT
 (QUOTE |INT;reducedSystem;2M;28|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|m|) |m|)))

(DEFUN |INT;reducedSystem;2M;28| (|m| |$|) |m|)

(DEFUN |INT;reducedSystem;MVR;29| (|m| |v| |$|) (CONS |m| (QUOTE |vec|)))

(PUT (QUOTE |INT;abs;2$;30|) (QUOTE |SPADreplace|) (QUOTE ABS))

(DEFUN |INT;abs;2$;30| (|x| |$|) (ABS |x|))

(PUT (QUOTE |INT;random;$;31|) (QUOTE |SPADreplace|) (QUOTE |random|))

(DEFUN |INT;random;$;31| (|$|) (|random|))

(PUT (QUOTE |INT;random;2$;32|) (QUOTE |SPADreplace|) (QUOTE RANDOM))

(DEFUN |INT;random;2$;32| (|x| |$|) (RANDOM |x|))

(PUT (QUOTE |INT;=;2$B;33|) (QUOTE |SPADreplace|) (QUOTE EQL))

(DEFUN |INT;=;2$B;33| (|x| |y| |$|) (EQL |x| |y|))

(PUT (QUOTE |INT;<;2$B;34|) (QUOTE |SPADreplace|) (QUOTE |<|))

(DEFUN |INT;<;2$B;34| (|x| |y| |$|) (|<| |x| |y|))
```

```
(PUT (QUOTE |INT;-;2$;35|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |INT;-;2$;35| (|x| |$|) (|-| |x|))

(PUT (QUOTE |INT;+;3$;36|) (QUOTE |SPADreplace|) (QUOTE |+|))

(DEFUN |INT;+;3$;36| (|x| |y| |$|) (|+| |x| |y|))

(PUT (QUOTE |INT;-;3$;37|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |INT;-;3$;37| (|x| |y| |$|) (|-| |x| |y|))

(PUT (QUOTE |INT;*;3$;38|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |INT;*;3$;38| (|x| |y| |$|) (|*| |x| |y|))

(PUT (QUOTE |INT;*;3$;39|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |INT;*;3$;39| (|m| |y| |$|) (|*| |m| |y|))

(PUT (QUOTE |INT;**;$Nni$;40|) (QUOTE |SPADreplace|) (QUOTE EXPT))

(DEFUN |INT;**;$Nni$;40| (|x| |n| |$|) (EXPT |x| |n|))

(PUT (QUOTE |INT;odd?;$B;41|) (QUOTE |SPADreplace|) (QUOTE ODDP))

(DEFUN |INT;odd?;$B;41| (|x| |$|) (ODDP |x|))

(PUT (QUOTE |INT;max;3$;42|) (QUOTE |SPADreplace|) (QUOTE MAX))

(DEFUN |INT;max;3$;42| (|x| |y| |$|) (MAX |x| |y|))

(PUT (QUOTE |INT;min;3$;43|) (QUOTE |SPADreplace|) (QUOTE MIN))

(DEFUN |INT;min;3$;43| (|x| |y| |$|) (MIN |x| |y|))

(PUT (QUOTE |INT;divide;2$R;44|) (QUOTE |SPADreplace|) (QUOTE DIVIDE2))

(DEFUN |INT;divide;2$R;44| (|x| |y| |$|) (DIVIDE2 |x| |y|))

(PUT (QUOTE |INT;quo;3$;45|) (QUOTE |SPADreplace|) (QUOTE QUOTIENT2))

(DEFUN |INT;quo;3$;45| (|x| |y| |$|) (QUOTIENT2 |x| |y|))

(PUT (QUOTE |INT;rem;3$;46|) (QUOTE |SPADreplace|) (QUOTE REMAINDER2))

(DEFUN |INT;rem;3$;46| (|x| |y| |$|) (REMAINDER2 |x| |y|))

(PUT (QUOTE |INT;shift;3$;47|) (QUOTE |SPADreplace|) (QUOTE ASH))
```

```
(DEFUN |INT;shift;3$;47| (|x| |y| |$|) (ASH |x| |y|))

(DEFUN |INT;exquo;2$U;48| (|x| |y| |$|)
 (COND
  ((OR (ZEROP |y|) (NULL (ZEROP (REMAINDER2 |x| |y|)))) (CONS 1 "failed"))
  ((QUOTE T) (CONS 0 (QUOTIENT2 |x| |y|)))))

(DEFUN |INT;recip;$U;49| (|x| |$|)
 (COND
  ((OR (EQL |x| 1) (EQL |x| -1)) (CONS 0 |x|))
  ((QUOTE T) (CONS 1 "failed"))))

(PUT (QUOTE |INT;gcd;3$;50|) (QUOTE |SPADreplace|) (QUOTE GCD))

(DEFUN |INT;gcd;3$;50| (|x| |y| |$|) (GCD |x| |y|))

(DEFUN |INT;unitNormal;$R;51| (|x| |$|)
 (COND
  ((|<| |x| 0) (VECTOR -1 (|-| |x|) -1))
  ((QUOTE T) (VECTOR 1 |x| 1))))

(PUT (QUOTE |INT;unitCanonical;2$;52|) (QUOTE |SPADreplace|) (QUOTE ABS))

(DEFUN |INT;unitCanonical;2$;52| (|x| |$|) (ABS |x|))

(DEFUN |INT;solveLinearPolynomialEquation| (|lp| |p| |$|)
 (SPADCALL |lp| |p| (QREFELT |$| 91)))

(DEFUN |INT;squareFreePolynomial| (|p| |$|) (SPADCALL |p| (QREFELT |$| 95)))

(DEFUN |INT;factorPolynomial| (|p| $)
  (PROG (|pp| G86409)
    (RETURN
      (SEQ (LETT |pp| (SPADCALL |p| (QREFELT $ 96))
                 |INT;factorPolynomial|)
           (EXIT (COND
                   ((EQL (SPADCALL |pp| (QREFELT $ 97))
                         (SPADCALL |p| (QREFELT $ 97)))
                    (SPADCALL |p| (QREFELT $ 99)))
                   ('T
                    (SPADCALL (SPADCALL |pp| (QREFELT $ 99))
                        (SPADCALL (CONS #'|INT;factorPolynomial!0| $)
                            (SPADCALL
                                (PROG2 (LETT G86409
                                        (SPADCALL
                                         (SPADCALL |p| (QREFELT $ 97))
                                         (SPADCALL |pp| (QREFELT $ 97))
                                         (QREFELT $ 81))
                                        |INT;factorPolynomial|)
```

```
                                       (QCDR G86409)
                                  (|check-union| (QEQCAR G86409 0) $
                                      G86409))
                                (QREFELT $ 102))
                            (QREFELT $ 106))
                        (QREFELT $ 108)))))))))

(DEFUN |INT;factorPolynomial!0| (|#1| |$|) (SPADCALL |#1| (QREFELT |$| 100)))

(DEFUN |INT;factorSquareFreePolynomial| (|p| |$|)
 (SPADCALL |p| (QREFELT |$| 109)))

(DEFUN |INT;gcdPolynomial;3Sup;57| (|p| |q| |$|)
 (COND
  ((SPADCALL |p| (QREFELT |$| 110)) (SPADCALL |q| (QREFELT |$| 111)))
  ((SPADCALL |q| (QREFELT |$| 110)) (SPADCALL |p| (QREFELT |$| 111)))
  ((QUOTE T) (SPADCALL (LIST |p| |q|) (QREFELT |$| 114)))))

(DEFUN |Integer| ()
  (PROG ()
    (RETURN
      (PROG (G86434)
        (RETURN
          (COND
            ((LETT G86434 (HGET |$ConstructorCache| '|Integer|)
                   |Integer|)
             (|CDRwithIncrement| (CDAR G86434)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Integer|
                                   (LIST
                                    (CONS NIL (CONS 1 (|Integer;|))))))
                 (LETT G86434 T |Integer|))
               (COND
                 ((NOT G86434) (HREM |$ConstructorCache| '|Integer|)))))))))))

(DEFUN |Integer;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|Integer|) |Integer|)
        (LETT $ (make-array 130) |Integer|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |Integer|))
        (|haddProp| |$ConstructorCache| '|Integer| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 69
            (QSETREFV $ 68 (CONS (|dispatchFunction| |INT;*;3$;39|) $)))
        $))))
```

```
(setf (get
 (QUOTE |Integer|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE
   #(NIL NIL NIL NIL NIL NIL (|Void|) (|OpenMathDevice|) (0 . |OMputApp|)
    (|String|) (5 . |OMputSymbol|) (|Integer|) (12 . |OMputInteger|)
    (18 . |OMputEndApp|) (|OpenMathEncoding|) (23 . |OMencodingXML|)
    (27 . |OMopenString|) (33 . |OMputObject|) (38 . |OMputEndObject|)
    (43 . |OMclose|) |INT;OMwrite;$S;2| (|Boolean|) |INT;OMwrite;$BS;3|
    |INT;OMwrite;Omd$V;4| |INT;OMwrite;Omd$BV;5| |INT;zero?;$B;6|
    (CONS IDENTITY (FUNCALL (|dispatchFunction| |INT;Zero;$;7|) |$|))
    (CONS IDENTITY (FUNCALL (|dispatchFunction| |INT;One;$;8|) |$|))
    |INT;base;$;9| |INT;copy;2$;10| |INT;inc;2$;11| |INT;dec;2$;12|
    |INT;hash;2$;13| |INT;negative?;$B;14| (|OutputForm|)
    (48 . |outputForm|) |INT;coerce;$Of;15| |INT;coerce;2$;16|
    |INT;convert;2$;17| |INT;length;2$;18| |INT;addmod;4$;19|
    |INT;submod;4$;20| |INT;mulmod;4$;21| (|Float|) (53 . |coerce|)
    |INT;convert;$F;22| (|DoubleFloat|) |INT;convert;$Df;23| (|InputForm|)
    (58 . |convert|) |INT;convert;$If;24| |INT;convert;$S;25|
    |INT;latex;$S;26| |INT;positiveRemainder;3$;27| (|Matrix| 11)
    (|Matrix| |$|) |INT;reducedSystem;2M;28|
    (|Record| (|:| |mat| 54) (|:| |vec| (|Vector| 11)))
    (|Vector| |$|) |INT;reducedSystem;MVR;29| |INT;abs;2$;30|
    |INT;random;$;31| |INT;random;2$;32| |INT;=;2$B;33|
    |INT;<;2$B;34| |INT;-;2$;35| |INT;+;3$;36| |INT;-;3$;37| NIL NIL
    (|NonNegativeInteger|) |INT;**;$Nni$;40| |INT;odd?;$B;41|
    |INT;max;3$;42| |INT;min;3$;43|
    (|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
    |INT;divide;2$R;44| |INT;quo;3$;45| |INT;rem;3$;46| |INT;shift;3$;47|
    (|Union| |$| (QUOTE "failed")) |INT;exquo;2$U;48| |INT;recip;$U;49|
    |INT;gcd;3$;50|
    (|Record| (|:| |unit| |$|) (|:| |canonical| |$|) (|:| |associate| |$|))
    |INT;unitNormal;$R;51| |INT;unitCanonical;2$;52|
    (|Union| 88 (QUOTE "failed")) (|List| 89)
    (|SparseUnivariatePolynomial| 11)
    (|IntegerSolveLinearPolynomialEquation|)
    (63 . |solveLinearPolynomialEquation|) (|Factored| 93)
    (|SparseUnivariatePolynomial| |$$|)
    (|UnivariatePolynomialSquareFree| |$$| 93) (69 . |squareFree|)
    (74 . |primitivePart|) (79 . |leadingCoefficient|)
    (|GaloisGroupFactorizer| 93) (84 . |factor|) (89 . |coerce|)
    (|Factored| |$|) (94 . |factor|) (|Mapping| 93 |$$|)
    (|Factored| |$$|) (|FactoredFunctions2| |$$| 93) (99 . |map|)
    (|FactoredFunctionUtilities| 93) (105 . |mergeFactors|)
    (111 . |factorSquareFree|) (116 . |zero?|) (121 . |unitCanonical|)
    (|List| 93) (|HeuGcd| 93) (126 . |gcd|)
    (|SparseUnivariatePolynomial| |$|) |INT;gcdPolynomial;3Sup;57|
    (|Union| 118 (QUOTE "failed")) (|Fraction| 11)
```

```
  (|PatternMatchResult| 11 |$|) (|Pattern| 11)
  (|Union| 11 (QUOTE "failed")) (|Union| 123 (QUOTE "failed"))
  (|List| |$|)
  (|Record| (|:| |coef| 123) (|:| |generator| |$|))
  (|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
  (|Union| 125 (QUOTE "failed"))
  (|Record| (|:| |coef1| |$|) (|:| |coef2| |$|) (|:| |generator| |$|))
  (|PositiveInteger|) (|SingleInteger|)))
(QUOTE #(|~=| 131 |zero?| 137 |unitNormal| 142 |unitCanonical| 147
     |unit?| 152 |symmetricRemainder| 157 |subtractIfCan| 163 |submod| 169
     |squareFreePart| 176 |squareFree| 181 |sizeLess?| 186 |sign| 192
     |shift| 197 |sample| 203 |retractIfCan| 207 |retract| 212 |rem| 217
     |reducedSystem| 223 |recip| 234 |rationalIfCan| 239 |rational?| 244
     |rational| 249 |random| 254 |quo| 263 |principalIdeal| 269
     |prime?| 274 |powmod| 279 |positiveRemainder| 286 |positive?| 292
     |permutation| 297 |patternMatch| 303 |one?| 310 |odd?| 315
     |nextItem| 320 |negative?| 325 |multiEuclidean| 330 |mulmod| 336
     |min| 343 |max| 349 |mask| 355 |length| 360 |lcm| 365 |latex| 376
     |invmod| 381 |init| 387 |inc| 391 |hash| 396 |gcdPolynomial| 406
     |gcd| 412 |factorial| 423 |factor| 428 |extendedEuclidean| 433
     |exquo| 446 |expressIdealMember| 452 |even?| 458
     |euclideanSize| 463 |divide| 468 |differentiate| 474 |dec| 485
     |copy| 490 |convert| 495 |coerce| 525 |characteristic| 545
     |bit?| 549 |binomial| 555 |base| 561 |associates?| 565
     |addmod| 571 |abs| 578 |^| 583 |Zero| 595 |One| 599
     |OMwrite| 603 D 627 |>=| 638 |>| 644 |=| 650 |<=| 656 |<| 662
     |-| 668 |+| 679 |**| 685 |*| 697))
(QUOTE (
 (|infinite| . 0) (|noetherian| . 0) (|canonicalsClosed| . 0)
 (|canonical| . 0) (|canonicalUnitNormal| . 0)
 (|multiplicativeValuation| . 0) (|noZeroDivisors| . 0)
 ((|commutative| "*") . 0) (|rightUnitary| . 0) (|leftUnitary| . 0)
 (|unitsKnown| . 0)))
(CONS
  (|makeByteWordVec2| 1
   (QUOTE (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0)))
  (CONS
   (QUOTE #(
    |IntegerNumberSystem&| |EuclideanDomain&| |UniqueFactorizationDomain&|
    NIL NIL |GcdDomain&| |IntegralDomain&| |Algebra&| NIL NIL
    |DifferentialRing&| |OrderedRing&| NIL NIL |Module&| NIL NIL
    |Ring&| NIL NIL NIL NIL NIL |AbelianGroup&| NIL NIL
    |AbelianMonoid&| |Monoid&| NIL NIL |OrderedSet&|
    |AbelianSemiGroup&| |SemiGroup&| NIL |SetCategory&|
    NIL NIL NIL NIL NIL NIL NIL |RetractableTo&| NIL |BasicType&| NIL))
   (CONS
    (QUOTE #(
     (|IntegerNumberSystem|) (|EuclideanDomain|)
     (|UniqueFactorizationDomain|) (|PrincipalIdealDomain|)
```

```
        (|OrderedIntegralDomain|) (|GcdDomain|) (|IntegralDomain|)
        (|Algebra| |$$|) (|CharacteristicZero|) (|LinearlyExplicitRingOver| 11)
        (|DifferentialRing|) (|OrderedRing|) (|CommutativeRing|) (|EntireRing|)
        (|Module| |$$|) (|OrderedAbelianGroup|) (|BiModule| |$$| |$$|)
        (|Ring|) (|OrderedCancellationAbelianMonoid|) (|LeftModule| |$$|)
        (|Rng|) (|RightModule| |$$|) (|OrderedAbelianMonoid|) (|AbelianGroup|)
        (|OrderedAbelianSemiGroup|) (|CancellationAbelianMonoid|)
        (|AbelianMonoid|) (|Monoid|) (|StepThrough|) (|PatternMatchable| 11)
        (|OrderedSet|) (|AbelianSemiGroup|) (|SemiGroup|) (|RealConstant|)
        (|SetCategory|) (|OpenMath|) (|ConvertibleTo| 9)
        (|ConvertibleTo| 43) (|ConvertibleTo| 46)
        (|CombinatorialFunctionCategory|) (|ConvertibleTo| 120)
        (|ConvertibleTo| 48) (|RetractableTo| 11) (|ConvertibleTo| 11)
        (|BasicType|) (|CoercibleTo| 34)))
       (|makeByteWordVec2| 129 (QUOTE (1 7 6 0 8 3 7 6 0 9 9 10 2 7 6 0 11
        12 1 7 6 0 13 0 14 0 15 2 7 0 9 14 16 1 7 6 0 17 1 7 6 0 18 1 7 6 0
        19 1 34 0 11 35 1 43 0 11 44 1 48 0 11 49 2 90 87 88 89 91 1 94 92
        93 95 1 93 0 0 96 1 93 2 0 97 1 98 92 93 99 1 93 0 2 100 1 0 101 0
        102 2 105 92 103 104 106 2 107 92 92 92 108 1 98 92 93 109 1 93 21
        0 110 1 93 0 0 111 1 113 93 112 114 2 0 21 0 0 1 1 0 21 0 25 1 0 84
        0 85 1 0 0 0 86 1 0 21 0 1 2 0 0 0 0 1 2 0 80 0 0 1 3 0 0 0 0 0 41
        1 0 0 0 1 1 0 101 0 1 2 0 21 0 0 1 1 0 11 0 1 2 0 0 0 0 79 0 0 0 1
        1 0 121 0 1 1 0 11 0 1 2 0 0 0 0 78 2 0 57 55 58 59 1 0 54 55 56 1
        0 80 0 82 1 0 117 0 1 1 0 21 0 1 1 0 118 0 1 1 0 0 0 62 0 0 0 61 2
        0 0 0 0 77 1 0 124 123 1 1 0 21 0 1 3 0 0 0 0 0 1 2 0 0 0 0 53 1 0
        21 0 1 2 0 0 0 0 1 3 0 119 0 120 119 1 1 0 21 0 1 1 0 21 0 72 1 0
        80 0 1 1 0 21 0 33 2 0 122 123 0 1 3 0 0 0 0 0 42 2 0 0 0 0 74 2 0
        0 0 0 73 1 0 0 0 1 1 0 0 0 39 1 0 0 123 1 2 0 0 0 0 1 1 0 9 0 52 2
        0 0 0 0 1 0 0 0 1 1 0 0 0 30 1 0 0 0 32 1 0 129 0 1 2 0 115 115 115
        116 2 0 0 0 0 83 1 0 0 123 1 1 0 0 0 1 1 0 101 0 102 3 0 126 0 0 0
        1 2 0 127 0 0 1 2 0 80 0 0 81 2 0 122 123 0 1 1 0 21 0 1 1 0 70 0
        1 2 0 75 0 0 76 1 0 0 0 1 2 0 0 0 70 1 1 0 0 0 31 1 0 0 0 29 1 0 9
        0 51 1 0 46 0 47 1 0 43 0 45 1 0 48 0 50 1 0 120 0 1 1 0 11 0 38 1
        0 0 11 37 1 0 0 11 37 1 0 0 0 1 1 0 34 0 36 0 0 70 1 2 0 21 0 0 1
        2 0 0 0 0 1 0 0 0 28 2 0 21 0 0 1 3 0 0 0 0 0 40 1 0 0 0 60 2 0 0
        0 70 1 2 0 0 0 128 1 0 0 0 26 0 0 0 27 3 0 6 7 0 21 24 2 0 9 0 21
        22 2 0 6 7 0 23 1 0 9 0 20 1 0 0 0 1 2 0 0 0 70 1 2 0 21 0 0 1 2
        0 21 0 0 1 2 0 21 0 0 63 2 0 21 0 0 1 2 0 21 0 0 64 2 0 0 0 0 67
        1 0 0 0 65 2 0 0 0 0 66 2 0 0 0 70 71 2 0 0 0 128 1 2 0 0 0 0 68
        2 0 0 11 0 69 2 0 0 70 0 1 2 0 0 128 0 1))))))
   (QUOTE |lookupComplete|)))

(setf (get (QUOTE |Integer|) (QUOTE NILADIC)) T)
```

28.6 ISTRING.lsp BOOTSTRAP

ISTRING depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ISTRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **ISTRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ISTRING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT '|ISTRING;new;NniC$;1| '|SPADreplace| 'MAKE-FULL-CVEC)

(DEFUN |ISTRING;new;NniC$;1| (|n| |c| |$|) (|MAKE-FULL-CVEC| |n| |c|))

(PUT '|ISTRING;empty;$;2| '|SPADreplace|
     '(XLAM NIL (MAKE-FULL-CVEC 0)))

(DEFUN |ISTRING;empty;$;2| (|$|) (|MAKE-FULL-CVEC| 0))

(DEFUN |ISTRING;empty?;$B;3| (|s| |$|) (EQL (QCSIZE |s|) 0))

(PUT '|ISTRING;#;$Nni;4| '|SPADreplace| 'QCSIZE)

(DEFUN |ISTRING;#;$Nni;4| (|s| |$|) (QCSIZE |s|))

(PUT '|ISTRING;=;2$B;5| '|SPADreplace| 'EQUAL)

(DEFUN |ISTRING;=;2$B;5| (|s| |t| |$|) (EQUAL |s| |t|))

(PUT '|ISTRING;<;2$B;6| '|SPADreplace|
     '(XLAM (|s| |t|) (CGREATERP |t| |s|)))

(DEFUN |ISTRING;<;2$B;6| (|s| |t| |$|) (CGREATERP |t| |s|))

(PUT '|ISTRING;concat;3$;7| '|SPADreplace| 'STRCONC)

(DEFUN |ISTRING;concat;3$;7| (|s| |t| |$|) (STRCONC |s| |t|))

(PUT '|ISTRING;copy;2$;8| '|SPADreplace| 'COPY-SEQ)

(DEFUN |ISTRING;copy;2$;8| (|s| |$|) (|COPY-SEQ| |s|))

(DEFUN |ISTRING;insert;2$I$;9| (|s| |t| |i| $)
  (SPADCALL
      (SPADCALL
          (SPADCALL |s|
```

```
              (SPADCALL (QREFELT $ 6) (- |i| 1) (QREFELT $ 20))
              (QREFELT $ 21))
          |t| (QREFELT $ 16))
      (SPADCALL |s| (SPADCALL |i| (QREFELT $ 22)) (QREFELT $ 21))
      (QREFELT $ 16)))

(DEFUN |ISTRING;coerce;$Of;10| (|s| |$|) (SPADCALL |s| (QREFELT |$| 26)))

(DEFUN |ISTRING;minIndex;$I;11| (|s| |$|) (QREFELT |$| 6))

(DEFUN |ISTRING;upperCase!;2$;12| (|s| $)
  (SPADCALL (ELT $ 31) |s| (QREFELT $ 33)))

(DEFUN |ISTRING;lowerCase!;2$;13| (|s| $)
  (SPADCALL (ELT $ 36) |s| (QREFELT $ 33)))

(DEFUN |ISTRING;latex;$S;14| (|s| $)
  (STRCONC "\\mbox{``" (STRCONC |s| "''}")))

(DEFUN |ISTRING;replace;$Us2$;15| (|s| |sg| |t| $)
  (PROG (|l| |m| |n| |h| G91425 |r| G91433 G91432 |i| G91431
             |k|)
    (RETURN
      (SEQ (LETT |l| (- (SPADCALL |sg| (QREFELT $ 39)) (QREFELT $ 6))
                 |ISTRING;replace;$Us2$;15|)
           (LETT |m| (SPADCALL |s| (QREFELT $ 13))
                 |ISTRING;replace;$Us2$;15|)
           (LETT |n| (SPADCALL |t| (QREFELT $ 13))
                 |ISTRING;replace;$Us2$;15|)
           (LETT |h|
                 (COND
                   ((SPADCALL |sg| (QREFELT $ 40))
                    (- (SPADCALL |sg| (QREFELT $ 41)) (QREFELT $ 6)))
                   ('T (- (SPADCALL |s| (QREFELT $ 42)) (QREFELT $ 6))))
                 |ISTRING;replace;$Us2$;15|)
           (COND
             ((OR (OR (< |l| 0) (NULL (< |h| |m|))) (< |h| (- |l| 1)))
              (EXIT (|error| "index out of range"))))
           (LETT |r|
                 (SPADCALL
                     (PROG1 (LETT G91425
                                  (+ (- |m| (+ (- |h| |l|) 1)) |n|)
                                  |ISTRING;replace;$Us2$;15|)
                       (|check-subtype| (>= G91425 0)
                           '(|NonNegativeInteger|) G91425))
                     (SPADCALL (QREFELT $ 43)) (QREFELT $ 9))
                 |ISTRING;replace;$Us2$;15|)
           (SEQ (LETT |i| 0 |ISTRING;replace;$Us2$;15|)
                (LETT G91433 (- |l| 1) |ISTRING;replace;$Us2$;15|)
                (LETT |k| 0 |ISTRING;replace;$Us2$;15|) G190
```

```
                (COND ((QSGREATERP |i| G91433) (GO G191)))
                (SEQ (EXIT (QESET |r| |k| (QENUM |s| |i|))))
                (LETT |k|
                      (PROG1 (QSADD1 |k|)
                        (LETT |i| (QSADD1 |i|)
                              |ISTRING;replace;$Us2$;15|))
                      |ISTRING;replace;$Us2$;15|)
                (GO G190) G191 (EXIT NIL))
           (SEQ (LETT |i| 0 |ISTRING;replace;$Us2$;15|)
                (LETT G91432 (- |n| 1) |ISTRING;replace;$Us2$;15|)
                (LETT |k| |k| |ISTRING;replace;$Us2$;15|) G190
                (COND ((QSGREATERP |i| G91432) (GO G191)))
                (SEQ (EXIT (QESET |r| |k| (QENUM |t| |i|))))
                (LETT |k|
                      (PROG1 (+ |k| 1)
                        (LETT |i| (QSADD1 |i|)
                              |ISTRING;replace;$Us2$;15|))
                      |ISTRING;replace;$Us2$;15|)
                (GO G190) G191 (EXIT NIL))
           (SEQ (LETT |i| (+ |h| 1) |ISTRING;replace;$Us2$;15|)
                (LETT G91431 (- |m| 1) |ISTRING;replace;$Us2$;15|)
                (LETT |k| |k| |ISTRING;replace;$Us2$;15|) G190
                (COND ((> |i| G91431) (GO G191)))
                (SEQ (EXIT (QESET |r| |k| (QENUM |s| |i|))))
                (LETT |k|
                      (PROG1 (+ |k| 1)
                        (LETT |i| (+ |i| 1) |ISTRING;replace;$Us2$;15|))
                      |ISTRING;replace;$Us2$;15|)
                (GO G190) G191 (EXIT NIL))
           (EXIT |r|)))))

(DEFUN |ISTRING;setelt;$I2C;16| (|s| |i| |c| $)
  (SEQ (COND
         ((OR (< |i| (QREFELT $ 6))
              (< (SPADCALL |s| (QREFELT $ 42)) |i|))
          (|error| "index out of range"))
         ('T (SEQ (QESET |s| (- |i| (QREFELT $ 6)) |c|) (EXIT |c|))))))

(DEFUN |ISTRING;substring?;2$IB;17| (|part| |whole| |startpos| $)
  (PROG (|np| |nw| |iw| |ip| G91443 G91442 G91438)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |np| (QCSIZE |part|)
                            |ISTRING;substring?;2$IB;17|)
                      (LETT |nw| (QCSIZE |whole|)
                            |ISTRING;substring?;2$IB;17|)
                      (LETT |startpos| (- |startpos| (QREFELT $ 6))
                            |ISTRING;substring?;2$IB;17|)
                      (EXIT (COND
                              ((< |startpos| 0)
                               (|error| "index out of bounds"))
```

```
                              ((< (- |nw| |startpos|) |np|) 'NIL)
                              ('T
                               (SEQ (SEQ
                                     (EXIT
                                      (SEQ
                                       (LETT |iw| |startpos|
                                        |ISTRING;substring?;2$IB;17|)
                                       (LETT |ip| 0
                                        |ISTRING;substring?;2$IB;17|)
                                       (LETT G91443 (- |np| 1)
                                        |ISTRING;substring?;2$IB;17|)
                                       G190
                                       (COND
                                         ((QSGREATERP |ip| G91443)
                                          (GO G191)))
                                       (SEQ
                                        (EXIT
                                         (COND
                                           ((NULL
                                              (EQL (QENUM |part| |ip|)
                                               (QENUM |whole| |iw|)))
                                             (PROGN
                                               (LETT G91438
                                                (PROGN
                                                  (LETT G91442 'NIL
                                                   |ISTRING;substring?;2$IB;17|)
                                                  (GO G91442))
                                                |ISTRING;substring?;2$IB;17|)
                                               (GO G91438))))))
                                       (LETT |ip|
                                        (PROG1 (QSADD1 |ip|)
                                          (LETT |iw| (+ |iw| 1)
                                           |ISTRING;substring?;2$IB;17|))
                                        |ISTRING;substring?;2$IB;17|)
                                       (GO G190) G191 (EXIT NIL)))
                                     G91438 (EXIT G91438))
                                    (EXIT 'T)))))))
           G91442 (EXIT G91442)))))

(DEFUN |ISTRING;position;2$2I;18| (|s| |t| |startpos| $)
  (PROG (|r|)
    (RETURN
      (SEQ (LETT |startpos| (- |startpos| (QREFELT $ 6))
                 |ISTRING;position;2$2I;18|)
           (EXIT (COND
                   ((< |startpos| 0) (|error| "index out of bounds"))
                   ((NULL (< |startpos| (QCSIZE |t|)))
                    (- (QREFELT $ 6) 1))
                   ('T
                    (SEQ (LETT |r| (STRPOS |s| |t| |startpos| NIL)
```

```
                              |ISTRING;position;2$2I;18|)
                        (EXIT (COND
                                ((EQ |r| NIL) (- (QREFELT $ 6) 1))
                                ('T (+ |r| (QREFELT $ 6))))))))))))

(DEFUN |ISTRING;position;C$2I;19| (|c| |t| |startpos| $)
  (PROG (|r| G91454 G91453)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |startpos| (- |startpos| (QREFELT $ 6))
                            |ISTRING;position;C$2I;19|)
                      (EXIT (COND
                              ((< |startpos| 0)
                               (|error| "index out of bounds"))
                              ((NULL (< |startpos| (QCSIZE |t|)))
                               (- (QREFELT $ 6) 1))
                              ('T
                               (SEQ (SEQ
                                     (LETT |r| |startpos|
                                      |ISTRING;position;C$2I;19|)
                                     (LETT G91454
                                      (QSDIFFERENCE (QCSIZE |t|) 1)
                                      |ISTRING;position;C$2I;19|)
                                     G190
                                     (COND
                                       ((> |r| G91454) (GO G191)))
                                     (SEQ
                                      (EXIT
                                       (COND
                                         ((EQL (QENUM |t| |r|) |c|)
                                          (PROGN
                                            (LETT G91453
                                             (+ |r| (QREFELT $ 6))
                                             |ISTRING;position;C$2I;19|)
                                            (GO G91453))))))
                                     (LETT |r| (+ |r| 1)
                                      |ISTRING;position;C$2I;19|)
                                     (GO G190) G191 (EXIT NIL))
                                    (EXIT (- (QREFELT $ 6) 1))))))))
           G91453 (EXIT G91453)))))

(DEFUN |ISTRING;position;Cc$2I;20| (|cc| |t| |startpos| $)
  (PROG (|r| G91461 G91460)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |startpos| (- |startpos| (QREFELT $ 6))
                            |ISTRING;position;Cc$2I;20|)
                      (EXIT (COND
                              ((< |startpos| 0)
                               (|error| "index out of bounds"))
                              ((NULL (< |startpos| (QCSIZE |t|)))
                               (- (QREFELT $ 6) 1))
```

```
                              ('T
                               (SEQ (SEQ
                                     (LETT |r| |startpos|
                                      |ISTRING;position;Cc$2I;20|)
                                     (LETT G91461
                                      (QSDIFFERENCE (QCSIZE |t|) 1)
                                      |ISTRING;position;Cc$2I;20|)
                                     G190
                                     (COND
                                       ((> |r| G91461) (GO G191)))
                                     (SEQ
                                      (EXIT
                                       (COND
                                         ((SPADCALL (QENUM |t| |r|)
                                           |cc| (QREFELT $ 49))
                                          (PROGN
                                            (LETT G91460
                                             (+ |r| (QREFELT $ 6))
                                             |ISTRING;position;Cc$2I;20|)
                                            (GO G91460))))))
                                     (LETT |r| (+ |r| 1)
                                      |ISTRING;position;Cc$2I;20|)
                                     (GO G190) G191 (EXIT NIL))
                                    (EXIT (- (QREFELT $ 6) 1)))))))
           G91460 (EXIT G91460)))))

(DEFUN |ISTRING;suffix?;2$B;21| (|s| |t| $)
  (PROG (|n| |m|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |t| (QREFELT $ 42))
                 |ISTRING;suffix?;2$B;21|)
           (LETT |m| (SPADCALL |s| (QREFELT $ 42))
                 |ISTRING;suffix?;2$B;21|)
           (EXIT (COND
                   ((< |n| |m|) 'NIL)
                   ('T
                    (SPADCALL |s| |t| (- (+ (QREFELT $ 6) |n|) |m|)
                        (QREFELT $ 46)))))))))

(DEFUN |ISTRING;split;$CL;22| (|s| |c| $)
  (PROG (|n| |j| |i| |l|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
                 |ISTRING;split;$CL;22|)
           (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;split;$CL;22|) G190
                (COND
                  ((OR (> |i| |n|)
                       (NULL (SPADCALL
                                  (SPADCALL |s| |i| (QREFELT $ 52)) |c|
                                  (QREFELT $ 53))))
```

```
                (GO G191)))
             (SEQ (EXIT 0))
             (LETT |i| (+ |i| 1) |ISTRING;split;$CL;22|) (GO G190)
             G191 (EXIT NIL))
        (LETT |l| (SPADCALL (QREFELT $ 55)) |ISTRING;split;$CL;22|)
        (SEQ G190
             (COND
               ((NULL (COND
                        ((< |n| |i|) 'NIL)
                        ('T
                         (SEQ (LETT |j|
                                    (SPADCALL |c| |s| |i|
                                     (QREFELT $ 48))
                                    |ISTRING;split;$CL;22|)
                              (EXIT (COND
                                      ((< |j| (QREFELT $ 6)) 'NIL)
                                      ('T 'T)))))))
                (GO G191)))
             (SEQ (LETT |l|
                        (SPADCALL
                            (SPADCALL |s|
                                (SPADCALL |i| (- |j| 1)
                                 (QREFELT $ 20))
                                (QREFELT $ 21))
                            |l| (QREFELT $ 56))
                        |ISTRING;split;$CL;22|)
                  (EXIT (SEQ (LETT |i| |j| |ISTRING;split;$CL;22|)
                             G190
                             (COND
                               ((OR (> |i| |n|)
                                    (NULL
                                     (SPADCALL
                                      (SPADCALL |s| |i| (QREFELT $ 52))
                                      |c| (QREFELT $ 53))))
                                (GO G191)))
                             (SEQ (EXIT 0))
                             (LETT |i| (+ |i| 1)
                                   |ISTRING;split;$CL;22|)
                             (GO G190) G191 (EXIT NIL))))
             NIL (GO G190) G191 (EXIT NIL))
        (COND
          ((NULL (< |n| |i|))
           (LETT |l|
                 (SPADCALL
                     (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
                         (QREFELT $ 21))
                     |l| (QREFELT $ 56))
                 |ISTRING;split;$CL;22|)))
        (EXIT (SPADCALL |l| (QREFELT $ 57))))))
```

```
(DEFUN |ISTRING;split;$CcL;23| (|s| |cc| $)
  (PROG (|n| |j| |i| |l|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
                 |ISTRING;split;$CcL;23|)
           (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;split;$CcL;23|) G190
                (COND
                  ((OR (> |i| |n|)
                       (NULL (SPADCALL
                                 (SPADCALL |s| |i| (QREFELT $ 52)) |cc|
                                 (QREFELT $ 49))))
                   (GO G191)))
                (SEQ (EXIT 0))
                (LETT |i| (+ |i| 1) |ISTRING;split;$CcL;23|) (GO G190)
                G191 (EXIT NIL))
           (LETT |l| (SPADCALL (QREFELT $ 55)) |ISTRING;split;$CcL;23|)
           (SEQ G190
                (COND
                  ((NULL (COND
                           ((< |n| |i|) 'NIL)
                           ('T
                            (SEQ (LETT |j|
                                       (SPADCALL |cc| |s| |i|
                                        (QREFELT $ 50))
                                       |ISTRING;split;$CcL;23|)
                                 (EXIT (COND
                                         ((< |j| (QREFELT $ 6)) 'NIL)
                                         ('T 'T)))))))
                   (GO G191)))
                (SEQ (LETT |l|
                           (SPADCALL
                               (SPADCALL |s|
                                   (SPADCALL |i| (- |j| 1)
                                    (QREFELT $ 20))
                                   (QREFELT $ 21))
                               |l| (QREFELT $ 56))
                           |ISTRING;split;$CcL;23|)
                     (EXIT (SEQ (LETT |i| |j| |ISTRING;split;$CcL;23|)
                                G190
                                (COND
                                  ((OR (> |i| |n|)
                                    (NULL
                                     (SPADCALL
                                      (SPADCALL |s| |i| (QREFELT $ 52))
                                      |cc| (QREFELT $ 49))))
                                   (GO G191)))
                                (SEQ (EXIT 0))
                                (LETT |i| (+ |i| 1)
                                      |ISTRING;split;$CcL;23|)
                                (GO G190) G191 (EXIT NIL))))
```

```
                NIL (GO G190) G191 (EXIT NIL))
           (COND
             ((NULL (< |n| |i|))
              (LETT |l|
                    (SPADCALL
                        (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
                            (QREFELT $ 21))
                        |l| (QREFELT $ 56))
                    |ISTRING;split;$CcL;23|)))
           (EXIT (SPADCALL |l| (QREFELT $ 57)))))))

(DEFUN |ISTRING;leftTrim;$C$;24| (|s| |c| $)
  (PROG (|n| |i|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
                 |ISTRING;leftTrim;$C$;24|)
           (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;leftTrim;$C$;24|) G190
                (COND
                  ((OR (> |i| |n|)
                       (NULL (SPADCALL
                                 (SPADCALL |s| |i| (QREFELT $ 52)) |c|
                                 (QREFELT $ 53))))
                   (GO G191)))
                (SEQ (EXIT 0))
                (LETT |i| (+ |i| 1) |ISTRING;leftTrim;$C$;24|)
                (GO G190) G191 (EXIT NIL))
           (EXIT (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
                     (QREFELT $ 21)))))))

(DEFUN |ISTRING;leftTrim;$Cc$;25| (|s| |cc| $)
  (PROG (|n| |i|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
                 |ISTRING;leftTrim;$Cc$;25|)
           (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;leftTrim;$Cc$;25|)
                G190
                (COND
                  ((OR (> |i| |n|)
                       (NULL (SPADCALL
                                 (SPADCALL |s| |i| (QREFELT $ 52)) |cc|
                                 (QREFELT $ 49))))
                   (GO G191)))
                (SEQ (EXIT 0))
                (LETT |i| (+ |i| 1) |ISTRING;leftTrim;$Cc$;25|)
                (GO G190) G191 (EXIT NIL))
           (EXIT (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
                     (QREFELT $ 21)))))))

(DEFUN |ISTRING;rightTrim;$C$;26| (|s| |c| $)
  (PROG (|j| G91487)
```

```
    (RETURN
      (SEQ (SEQ (LETT |j| (SPADCALL |s| (QREFELT $ 42))
                      |ISTRING;rightTrim;$C$;26|)
                (LETT G91487 (QREFELT $ 6)
                      |ISTRING;rightTrim;$C$;26|)
                G190
                (COND
                  ((OR (< |j| G91487)
                       (NULL (SPADCALL
                                 (SPADCALL |s| |j| (QREFELT $ 52)) |c|
                                 (QREFELT $ 53))))
                   (GO G191)))
                (SEQ (EXIT 0))
                (LETT |j| (+ |j| -1) |ISTRING;rightTrim;$C$;26|)
                (GO G190) G191 (EXIT NIL))
           (EXIT (SPADCALL |s|
                     (SPADCALL (SPADCALL |s| (QREFELT $ 28)) |j|
                         (QREFELT $ 20))
                     (QREFELT $ 21)))))))

(DEFUN |ISTRING;rightTrim;$Cc$;27| (|s| |cc| $)
  (PROG (|j| G91491)
    (RETURN
      (SEQ (SEQ (LETT |j| (SPADCALL |s| (QREFELT $ 42))
                      |ISTRING;rightTrim;$Cc$;27|)
                (LETT G91491 (QREFELT $ 6)
                      |ISTRING;rightTrim;$Cc$;27|)
                G190
                (COND
                  ((OR (< |j| G91491)
                       (NULL (SPADCALL
                                 (SPADCALL |s| |j| (QREFELT $ 52)) |cc|
                                 (QREFELT $ 49))))
                   (GO G191)))
                (SEQ (EXIT 0))
                (LETT |j| (+ |j| -1) |ISTRING;rightTrim;$Cc$;27|)
                (GO G190) G191 (EXIT NIL))
           (EXIT (SPADCALL |s|
                     (SPADCALL (SPADCALL |s| (QREFELT $ 28)) |j|
                         (QREFELT $ 20))
                     (QREFELT $ 21)))))))


(DEFUN |ISTRING;concat;L$;28| (|l| $)
  (PROG (G91500 G91494 G91492 G91493 |t| |s| G91499 |i|)
    (RETURN
      (SEQ (LETT |t|
                 (SPADCALL
                     (PROGN
                       (LETT G91493 NIL |ISTRING;concat;L$;28|)
```

```
                      (SEQ (LETT |s| NIL |ISTRING;concat;L$;28|)
                           (LETT G91500 |l| |ISTRING;concat;L$;28|)
                           G190
                           (COND
                             ((OR (ATOM G91500)
                                  (PROGN
                                    (LETT |s| (CAR G91500)
                                     |ISTRING;concat;L$;28|)
                                    NIL))
                              (GO G191)))
                           (SEQ (EXIT (PROGN
                                        (LETT G91494
                                         (SPADCALL |s| (QREFELT $ 13))
                                         |ISTRING;concat;L$;28|)
                                        (COND
                                          (G91493
                                           (LETT G91492
                                            (+ G91492 G91494)
                                            |ISTRING;concat;L$;28|))
                                          ('T
                                           (PROGN
                                             (LETT G91492 G91494
                                              |ISTRING;concat;L$;28|)
                                             (LETT G91493 'T
                                              |ISTRING;concat;L$;28|)))))))
                           (LETT G91500 (CDR G91500)
                                 |ISTRING;concat;L$;28|)
                           (GO G190) G191 (EXIT NIL))
                      (COND (G91493 G91492) ('T 0)))
                    (SPADCALL (QREFELT $ 43)) (QREFELT $ 9))
                |ISTRING;concat;L$;28|)
          (LETT |i| (QREFELT $ 6) |ISTRING;concat;L$;28|)
          (SEQ (LETT |s| NIL |ISTRING;concat;L$;28|)
               (LETT G91499 |l| |ISTRING;concat;L$;28|) G190
               (COND
                 ((OR (ATOM G91499)
                      (PROGN
                        (LETT |s| (CAR G91499)
                              |ISTRING;concat;L$;28|)
                        NIL))
                  (GO G191)))
               (SEQ (SPADCALL |t| |s| |i| (QREFELT $ 65))
                    (EXIT (LETT |i|
                                (+ |i| (SPADCALL |s| (QREFELT $ 13)))
                                |ISTRING;concat;L$;28|)))
               (LETT G91499 (CDR G91499) |ISTRING;concat;L$;28|)
               (GO G190) G191 (EXIT NIL))
          (EXIT |t|)))))

(DEFUN |ISTRING;copyInto!;2$I$;29| (|y| |x| |s| $)
```

```
  (PROG (|m| |n|)
    (RETURN
      (SEQ (LETT |m| (SPADCALL |x| (QREFELT $ 13))
                 |ISTRING;copyInto!;2$I$;29|)
           (LETT |n| (SPADCALL |y| (QREFELT $ 13))
                 |ISTRING;copyInto!;2$I$;29|)
           (LETT |s| (- |s| (QREFELT $ 6)) |ISTRING;copyInto!;2$I$;29|)
           (COND
             ((OR (< |s| 0) (< |n| (+ |s| |m|)))
              (EXIT (|error| "index out of range"))))
           (RPLACSTR |y| |s| |m| |x| 0 |m|) (EXIT |y|)))))


(DEFUN |ISTRING;elt;$IC;30| (|s| |i| $)
  (COND
    ((OR (< |i| (QREFELT $ 6)) (< (SPADCALL |s| (QREFELT $ 42)) |i|))
     (|error| "index out of range"))
    ('T (QENUM |s| (- |i| (QREFELT $ 6))))))

(DEFUN |ISTRING;elt;$Us$;31| (|s| |sg| $)
  (PROG (|l| |h|)
    (RETURN
      (SEQ (LETT |l| (- (SPADCALL |sg| (QREFELT $ 39)) (QREFELT $ 6))
                 |ISTRING;elt;$Us$;31|)
           (LETT |h|
                 (COND
                   ((SPADCALL |sg| (QREFELT $ 40))
                    (- (SPADCALL |sg| (QREFELT $ 41)) (QREFELT $ 6)))
                   ('T (- (SPADCALL |s| (QREFELT $ 42)) (QREFELT $ 6))))
                 |ISTRING;elt;$Us$;31|)
           (COND
             ((OR (< |l| 0)
                  (NULL (< |h| (SPADCALL |s| (QREFELT $ 13)))))
              (EXIT (|error| "index out of bound"))))
           (EXIT (SUBSTRING |s| |l| (MAX 0 (+ (- |h| |l|) 1))))))))


(DEFUN |ISTRING;hash;$I;32| (|s| $)
  (PROG (|n|)
    (RETURN
      (SEQ (LETT |n| (QCSIZE |s|) |ISTRING;hash;$I;32|)
           (EXIT (COND
                   ((ZEROP |n|) 0)
                   ((EQL |n| 1)
                    (SPADCALL
                        (SPADCALL |s| (QREFELT $ 6) (QREFELT $ 52))
                        (QREFELT $ 67)))
                   ('T
                    (* (* (SPADCALL
                              (SPADCALL |s| (QREFELT $ 6)
```

```
                                (QREFELT $ 52))
                            (QREFELT $ 67))
                        (SPADCALL
                            (SPADCALL |s| (- (+ (QREFELT $ 6) |n|) 1)
                                (QREFELT $ 52))
                            (QREFELT $ 67)))
                     (SPADCALL
                         (SPADCALL |s|
                             (+ (QREFELT $ 6) (QUOTIENT2 |n| 2))
                             (QREFELT $ 52))
                         (QREFELT $ 67))))))))))

(PUT '|ISTRING;match;2$CNni;33| '|SPADreplace| '|stringMatch|)

(DEFUN |ISTRING;match;2$CNni;33| (|pattern| |target| |wildcard| $)
  (|stringMatch| |pattern| |target| |wildcard|))

(DEFUN |ISTRING;match?;2$CB;34| (|pattern| |target| |dontcare| $)
  (PROG (|n| |m| G91514 G91516 |s| G91518 G91526 |i| |p|
             G91519 |q|)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |n| (SPADCALL |pattern| (QREFELT $ 42))
                            |ISTRING;match?;2$CB;34|)
                      (LETT |p|
                            (PROG1 (LETT G91514
                                    (SPADCALL |dontcare| |pattern|
                                     (LETT |m|
                                      (SPADCALL |pattern|
                                       (QREFELT $ 28))
                                      |ISTRING;match?;2$CB;34|)
                                     (QREFELT $ 48))
                                    |ISTRING;match?;2$CB;34|)
                              (|check-subtype| (>= G91514 0)
                                  '(|NonNegativeInteger|) G91514))
                            |ISTRING;match?;2$CB;34|)
                      (EXIT (COND
                              ((EQL |p| (- |m| 1))
                               (SPADCALL |pattern| |target|
                                   (QREFELT $ 14)))
                              ('T
                               (SEQ (COND
                                      ((NULL (EQL |p| |m|))
                                       (COND
                                         ((NULL
                                           (SPADCALL
                                            (SPADCALL |pattern|
                                             (SPADCALL |m| (- |p| 1)
                                              (QREFELT $ 20))
                                             (QREFELT $ 21))
                                            |target| (QREFELT $ 70)))
```

```
      (EXIT 'NIL)))))
(LETT |i| |p|
 |ISTRING;match?;2$CB;34|)
(LETT |q|
 (PROG1
  (LETT G91516
   (SPADCALL |dontcare| |pattern|
    (+ |p| 1) (QREFELT $ 48))
   |ISTRING;match?;2$CB;34|)
   (|check-subtype| (>= G91516 0)
    '(|NonNegativeInteger|)
    G91516))
 |ISTRING;match?;2$CB;34|)
(SEQ G190
 (COND
   ((NULL
     (COND
       ((EQL |q| (- |m| 1)) 'NIL)
       ('T 'T)))
    (GO G191)))
 (SEQ
  (LETT |s|
   (SPADCALL |pattern|
    (SPADCALL (+ |p| 1) (- |q| 1)
     (QREFELT $ 20))
    (QREFELT $ 21))
   |ISTRING;match?;2$CB;34|)
  (LETT |i|
   (PROG1
    (LETT G91518
     (SPADCALL |s| |target| |i|
      (QREFELT $ 47))
     |ISTRING;match?;2$CB;34|)
     (|check-subtype|
      (>= G91518 0)
      '(|NonNegativeInteger|)
      G91518))
   |ISTRING;match?;2$CB;34|)
  (EXIT
   (COND
     ((EQL |i| (- |m| 1))
      (PROGN
        (LETT G91526 'NIL
         |ISTRING;match?;2$CB;34|)
        (GO G91526)))
     ('T
      (SEQ
       (LETT |i|
        (+ |i|
         (SPADCALL |s|
```

```
                                                  (QREFELT $ 13)))
                                                |ISTRING;match?;2$CB;34|)
                                               (LETT |p| |q|
                                                |ISTRING;match?;2$CB;34|)
                                               (EXIT
                                                (LETT |q|
                                                 (PROG1
                                                  (LETT G91519
                                                   (SPADCALL |dontcare|
                                                    |pattern| (+ |q| 1)
                                                    (QREFELT $ 48))
                                                   |ISTRING;match?;2$CB;34|)
                                                   (|check-subtype|
                                                    (>= G91519 0)
                                                    '(|NonNegativeInteger|)
                                                    G91519))
                                                 |ISTRING;match?;2$CB;34|)))))))
                                      NIL (GO G190) G191 (EXIT NIL))
                                     (COND
                                       ((NULL (EQL |p| |n|))
                                        (COND
                                          ((NULL
                                            (SPADCALL
                                             (SPADCALL |pattern|
                                              (SPADCALL (+ |p| 1) |n|
                                               (QREFELT $ 20))
                                              (QREFELT $ 21))
                                             |target| (QREFELT $ 51)))
                                           (EXIT 'NIL)))))
                                     (EXIT 'T)))))))
           G91526 (EXIT G91526)))))

(DEFUN |IndexedString| (G91535)
  (PROG ()
    (RETURN
      (PROG (G91536)
        (RETURN
          (COND
            ((LETT G91536
                   (|lassocShiftWithFunction|
                       (LIST (|devaluate| G91535))
                       (HGET |$ConstructorCache| '|IndexedString|)
                       '|domainEqualList|)
                   |IndexedString|)
             (|CDRwithIncrement| G91536))
            ('T
             (UNWIND-PROTECT
               (PROG1 (|IndexedString;| G91535)
                 (LETT G91536 T |IndexedString|))
               (COND
```

```
                 ((NOT G91536)
                  (HREM |$ConstructorCache| '|IndexedString|)))))))))))

(DEFUN |IndexedString;| (|#1|)
  (PROG (DV$1 |dv$| $ G91534 G91533 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |IndexedString|)
        (LETT |dv$| (LIST '|IndexedString| DV$1) |IndexedString|)
        (LETT $ (make-array 83) |IndexedString|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$|
                  (|buildPredVector| 0 0
                      (LIST (|HasCategory| (|Character|)
                                '(|SetCategory|))
                            (|HasCategory| (|Character|)
                                '(|ConvertibleTo| (|InputForm|)))
                            (LETT G91534
                                  (|HasCategory| (|Character|)
                                      '(|OrderedSet|))
                                  |IndexedString|)
                            (OR G91534
                                (|HasCategory| (|Character|)
                                    '(|SetCategory|)))
                            (|HasCategory| (|Integer|) '(|OrderedSet|))
                            (LETT G91533
                                  (AND (|HasCategory| (|Character|)
                                        '(|Evalable| (|Character|)))
                                       (|HasCategory| (|Character|)
                                        '(|SetCategory|)))
                                  |IndexedString|)
                            (OR (AND (|HasCategory| (|Character|)
                                      '(|Evalable| (|Character|)))
                                     G91534)
                                G91533)))
                  |IndexedString|))
        (|haddProp| |$ConstructorCache| '|IndexedString| (LIST DV$1)
            (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 |#1|)
        $))))

(setf (get '|IndexedString| '|infovec|)
    (LIST '#(NIL NIL NIL NIL NIL NIL (|local| |#1|)
             (|NonNegativeInteger|) (|Character|) |ISTRING;new;NniC$;1|
             |ISTRING;empty;$;2| (|Boolean|) |ISTRING;empty?;$B;3|
             |ISTRING;#;$Nni;4| |ISTRING;=;2$B;5| |ISTRING;<;2$B;6|
             |ISTRING;concat;3$;7| |ISTRING;copy;2$;8| (|Integer|)
             (|UniversalSegment| 18) (0 . SEGMENT)
```

```
    |ISTRING;elt;$Us$;31| (6 . SEGMENT)
    |ISTRING;insert;2$I$;9| (|String|) (|OutputForm|)
    (11 . |outputForm|) |ISTRING;coerce;$Of;10|
    |ISTRING;minIndex;$I;11| (|CharacterClass|)
    (16 . |upperCase|) (20 . |upperCase|) (|Mapping| 8 8)
    (25 . |map!|) |ISTRING;upperCase!;2$;12|
    (31 . |lowerCase|) (35 . |lowerCase|)
    |ISTRING;lowerCase!;2$;13| |ISTRING;latex;$S;14|
    (40 . |lo|) (45 . |hasHi|) (50 . |hi|) (55 . |maxIndex|)
    (60 . |space|) |ISTRING;replace;$Us2$;15|
    |ISTRING;setelt;$I2C;16| |ISTRING;substring?;2$IB;17|
    |ISTRING;position;2$2I;18| |ISTRING;position;C$2I;19|
    (64 . |member?|) |ISTRING;position;Cc$2I;20|
    |ISTRING;suffix?;2$B;21| |ISTRING;elt;$IC;30| (70 . =)
    (|List| $$) (76 . |empty|) (80 . |concat|)
    (86 . |reverse!|) (|List| $) |ISTRING;split;$CL;22|
    |ISTRING;split;$CcL;23| |ISTRING;leftTrim;$C$;24|
    |ISTRING;leftTrim;$Cc$;25| |ISTRING;rightTrim;$C$;26|
    |ISTRING;rightTrim;$Cc$;27| |ISTRING;copyInto!;2$I$;29|
    |ISTRING;concat;L$;28| (91 . |ord|) |ISTRING;hash;$I;32|
    |ISTRING;match;2$CNni;33| (96 . |prefix?|)
    |ISTRING;match?;2$CB;34| (|List| 8) (|List| 74)
    (|Equation| 8) (|Mapping| 8 8 8) (|InputForm|)
    (|SingleInteger|) (|Mapping| 11 8) (|Mapping| 11 8 8)
    (|Void|) (|Union| 8 '"failed") (|List| 18))
 '#(~= 102 |upperCase!| 108 |upperCase| 113 |trim| 118 |swap!|
    130 |suffix?| 137 |substring?| 143 |split| 150 |sorted?|
    162 |sort!| 173 |sort| 184 |size?| 195 |setelt| 201
    |select| 215 |sample| 221 |rightTrim| 225 |reverse!| 237
    |reverse| 242 |replace| 247 |removeDuplicates| 254
    |remove| 259 |reduce| 271 |qsetelt!| 292 |qelt| 299
    |prefix?| 305 |position| 311 |parts| 344 |new| 349 |more?|
    355 |minIndex| 361 |min| 366 |merge| 372 |members| 385
    |member?| 390 |maxIndex| 396 |max| 401 |match?| 407
    |match| 414 |map!| 421 |map| 427 |lowerCase!| 440
    |lowerCase| 445 |less?| 450 |leftTrim| 456 |latex| 468
    |insert| 473 |indices| 487 |index?| 492 |hash| 498 |first|
    508 |find| 513 |fill!| 519 |every?| 525 |eval| 531 |eq?|
    557 |entry?| 563 |entries| 569 |empty?| 574 |empty| 579
    |elt| 583 |delete| 608 |count| 620 |copyInto!| 632 |copy|
    639 |convert| 644 |construct| 649 |concat| 654 |coerce|
    677 |any?| 687 >= 693 > 699 = 705 <= 711 < 717 |#| 723)
 '((|shallowlyMutable| . 0) (|finiteAggregate| . 0))
 (CONS (|makeByteWordVec2| 7
           '(0 0 0 0 0 0 0 3 0 0 7 4 0 0 7 1 2 4))
       (CONS '#(|StringAggregate&|
                 |OneDimensionalArrayAggregate&|
                 |FiniteLinearAggregate&| |LinearAggregate&|
                 |IndexedAggregate&| |Collection&|
                 |HomogeneousAggregate&| |OrderedSet&|
```

```
   |Aggregate&| |EltableAggregate&| |Evalable&|
   |SetCategory&| NIL NIL |InnerEvalable&| NIL
   NIL |BasicType&|)
(CONS '#((|StringAggregate|)
         (|OneDimensionalArrayAggregate| 8)
         (|FiniteLinearAggregate| 8)
         (|LinearAggregate| 8)
         (|IndexedAggregate| 18 8)
         (|Collection| 8)
         (|HomogeneousAggregate| 8)
         (|OrderedSet|) (|Aggregate|)
         (|EltableAggregate| 18 8) (|Evalable| 8)
         (|SetCategory|) (|Type|)
         (|Eltable| 18 8) (|InnerEvalable| 8 8)
         (|CoercibleTo| 25) (|ConvertibleTo| 76)
         (|BasicType|))
      (|makeByteWordVec2| 82
          '(2 19 0 18 18 20 1 19 0 18 22 1 25 0
            24 26 0 29 0 30 1 8 0 0 31 2 0 0 32 0
            33 0 29 0 35 1 8 0 0 36 1 19 18 0 39
            1 19 11 0 40 1 19 18 0 41 1 0 18 0 42
            0 8 0 43 2 29 11 8 0 49 2 8 11 0 0 53
            0 54 0 55 2 54 0 2 0 56 1 54 0 0 57 1
            8 18 0 67 2 0 11 0 0 70 2 1 11 0 0 1
            1 0 0 0 34 1 0 0 0 1 2 0 0 0 8 1 2 0
            0 0 29 1 3 0 80 0 18 18 1 2 0 11 0 0
            51 3 0 11 0 0 18 46 2 0 58 0 29 60 2
            0 58 0 8 59 1 3 11 0 1 2 0 11 79 0 1
            1 3 0 0 1 2 0 0 79 0 1 1 3 0 0 1 2 0
            0 79 0 1 2 0 11 0 7 1 3 0 8 0 19 8 1
            3 0 8 0 18 8 45 2 0 0 78 0 1 0 0 0 1
            2 0 0 0 8 63 2 0 0 0 29 64 1 0 0 0 1
            1 0 0 0 1 3 0 0 0 19 0 44 1 1 0 0 1 2
            1 0 8 0 1 2 0 0 78 0 1 4 1 8 75 0 8 8
            1 3 0 8 75 0 8 1 2 0 8 75 0 1 3 0 8 0
            18 8 1 2 0 8 0 18 1 2 0 11 0 0 70 3 1
            18 8 0 18 48 2 1 18 8 0 1 3 0 18 29 0
            18 50 3 0 18 0 0 18 47 2 0 18 78 0 1
            1 0 72 0 1 2 0 0 7 8 9 2 0 11 0 7 1 1
            5 18 0 28 2 3 0 0 0 1 2 3 0 0 0 1 3 0
            0 79 0 0 1 1 0 72 0 1 2 1 11 8 0 1 1
            5 18 0 42 2 3 0 0 0 1 3 0 11 0 0 8 71
            3 0 7 0 0 8 69 2 0 0 32 0 33 3 0 0 75
            0 0 1 2 0 0 32 0 1 1 0 0 0 37 1 0 0 0
            1 2 0 11 0 7 1 2 0 0 0 8 61 2 0 0 0
            29 62 1 1 24 0 38 3 0 0 8 0 18 1 3 0
            0 0 0 18 23 1 0 82 0 1 2 0 11 18 0 1
            1 1 77 0 1 1 0 18 0 68 1 5 8 0 1 2 0
            81 78 0 1 2 0 0 0 8 1 2 0 11 78 0 1 3
            6 0 0 72 72 1 3 6 0 0 8 8 1 2 6 0 0
```

```
                              73 1 2 6 0 0 74 1 2 0 11 0 0 1 2 1 11
                              8 0 1 1 0 72 0 1 1 0 11 0 12 0 0 0 10
                              2 0 0 0 0 1 2 0 0 0 19 21 2 0 8 0 18
                              52 3 0 8 0 18 8 1 2 0 0 0 18 1 2 0 0
                              0 19 1 2 1 7 8 0 1 2 0 7 78 0 1 3 0 0
                              0 0 18 65 1 0 0 0 17 1 2 76 0 1 1 0 0
                              72 1 1 0 0 58 66 2 0 0 0 0 16 2 0 0 0
                              8 1 2 0 0 8 0 1 1 1 25 0 27 1 0 0 8 1
                              2 0 11 78 0 1 2 3 11 0 0 1 2 3 11 0 0
                              1 2 1 11 0 0 14 2 3 11 0 0 1 2 3 11 0
                              0 15 1 0 7 0 13)))))
          '|lookupComplete|))
```

28.7 LIST.lsp BOOTSTRAP

LIST depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LIST** category which we can write into the **MID** directory. We compile the lisp code and copy the **LIST.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **LIST.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |LIST;nil;$;1|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL NIL)))

(DEFUN |LIST;nil;$;1| (|$|) NIL)

(PUT (QUOTE |LIST;null;$B;2|) (QUOTE |SPADreplace|) (QUOTE NULL))

(DEFUN |LIST;null;$B;2| (|l| |$|) (NULL |l|))

(PUT (QUOTE |LIST;cons;S2$;3|) (QUOTE |SPADreplace|) (QUOTE CONS))

(DEFUN |LIST;cons;S2$;3| (|s| |l| |$|) (CONS |s| |l|))

(PUT (QUOTE |LIST;append;3$;4|) (QUOTE |SPADreplace|) (QUOTE APPEND))

(DEFUN |LIST;append;3$;4| (|l| |t| |$|) (APPEND |l| |t|))

(DEFUN |LIST;writeOMList| (|dev| |x| |$|)
 (SEQ
  (SPADCALL |dev| (QREFELT |$| 14))
```

```
  (SPADCALL |dev| "list1" "list" (QREFELT |$| 16))
  (SEQ
   G190
   (COND
    ((NULL (COND ((NULL |x|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
   (SEQ
    (SPADCALL |dev| (|SPADfirst| |x|) (QUOTE NIL) (QREFELT |$| 17))
    (EXIT (LETT |x| (CDR |x|) |LIST;writeOMList|)))
   NIL
   (GO G190)
   G191
   (EXIT NIL))
  (EXIT (SPADCALL |dev| (QREFELT |$| 18)))))

(DEFUN |LIST;OMwrite;$S;6| (|x| |$|)
 (PROG (|sp| |dev| |s|)
  (RETURN
   (SEQ
    (LETT |s| "" |LIST;OMwrite;$S;6|)
    (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |LIST;OMwrite;$S;6|)
    (LETT |dev|
     (SPADCALL |sp| (SPADCALL (QREFELT |$| 20)) (QREFELT |$| 21))
     |LIST;OMwrite;$S;6|)
    (SPADCALL |dev| (QREFELT |$| 22))
    (|LIST;writeOMList| |dev| |x| |$|)
    (SPADCALL |dev| (QREFELT |$| 23))
    (SPADCALL |dev| (QREFELT |$| 24))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |LIST;OMwrite;$S;6|)
    (EXIT |s|)))))

(DEFUN |LIST;OMwrite;$BS;7| (|x| |wholeObj| |$|)
 (PROG (|sp| |dev| |s|)
  (RETURN
   (SEQ
    (LETT |s| "" |LIST;OMwrite;$BS;7|)
    (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |LIST;OMwrite;$BS;7|)
    (LETT |dev|
     (SPADCALL |sp| (SPADCALL (QREFELT |$| 20)) (QREFELT |$| 21))
     |LIST;OMwrite;$BS;7|)
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 22))))
    (|LIST;writeOMList| |dev| |x| |$|)
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 23))))
    (SPADCALL |dev| (QREFELT |$| 24))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |LIST;OMwrite;$BS;7|)
    (EXIT |s|)))))

(DEFUN |LIST;OMwrite;Omd$V;8| (|dev| |x| |$|)
 (SEQ
  (SPADCALL |dev| (QREFELT |$| 22))
  (|LIST;writeOMList| |dev| |x| |$|)
```

```
  (EXIT (SPADCALL |dev| (QREFELT |$| 23)))))

(DEFUN |LIST;OMwrite;Omd$BV;9| (|dev| |x| |wholeObj| |$|)
 (SEQ
  (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 22))))
  (|LIST;writeOMList| |dev| |x| |$|)
  (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 23)))))))

(DEFUN |LIST;setUnion;3$;10| (|l1| |l2| |$|)
 (SPADCALL (SPADCALL |l1| |l2| (QREFELT |$| 29)) (QREFELT |$| 30)))

(DEFUN |LIST;setIntersection;3$;11| (|l1| |l2| |$|)
 (PROG (|u|)
  (RETURN
   (SEQ
    (LETT |u| NIL |LIST;setIntersection;3$;11|)
    (LETT |l1| (SPADCALL |l1| (QREFELT |$| 30)) |LIST;setIntersection;3$;11|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |l1|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (COND
       ((SPADCALL (|SPADfirst| |l1|) |l2| (QREFELT |$| 32))
        (LETT |u| (CONS (|SPADfirst| |l1|) |u|) |LIST;setIntersection;3$;11|)))
      (EXIT (LETT |l1| (CDR |l1|) |LIST;setIntersection;3$;11|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |u|)))))

(DEFUN |LIST;setDifference;3$;12| (|l1| |l2| |$|)
 (PROG (|l11| |lu|)
  (RETURN
   (SEQ
    (LETT |l1| (SPADCALL |l1| (QREFELT |$| 30)) |LIST;setDifference;3$;12|)
    (LETT |lu| NIL |LIST;setDifference;3$;12|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |l1|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |l11|
       (SPADCALL |l1| 1 (QREFELT |$| 35))
       |LIST;setDifference;3$;12|)
      (COND
       ((NULL (SPADCALL |l11| |l2| (QREFELT |$| 32)))
```

```
        (LETT |lu| (CONS |l11| |lu|) |LIST;setDifference;3$;12|)))
      (EXIT (LETT |l1| (CDR |l1|) |LIST;setDifference;3$;12|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |lu|)))))

(DEFUN |LIST;convert;$If;13| (|x| $)
  (PROG (G102544 |a| G102545)
    (RETURN
      (SEQ (SPADCALL
              (CONS (SPADCALL (SPADCALL "construct" (QREFELT $ 38))
                         (QREFELT $ 40))
                     (PROGN
                       (LETT G102544 NIL |LIST;convert;$If;13|)
                       (SEQ (LETT |a| NIL |LIST;convert;$If;13|)
                            (LETT G102545 |x| |LIST;convert;$If;13|)
                            G190
                            (COND
                              ((OR (ATOM G102545)
                                   (PROGN
                                     (LETT |a| (CAR G102545)
                                      |LIST;convert;$If;13|)
                                     NIL))
                               (GO G191)))
                            (SEQ (EXIT (LETT G102544
                                         (CONS
                                          (SPADCALL |a| (QREFELT $ 41))
                                          G102544)
                                         |LIST;convert;$If;13|)))
                            (LETT G102545 (CDR G102545)
                                  |LIST;convert;$If;13|)
                            (GO G190) G191 (EXIT (NREVERSE0 G102544)))))
              (QREFELT $ 43))))))

(DEFUN |List| (G102555)
  (PROG ()
    (RETURN
      (PROG (G102556)
        (RETURN
          (COND
            ((LETT G102556
                   (|lassocShiftWithFunction|
                       (LIST (|devaluate| G102555))
                       (HGET |$ConstructorCache| '|List|)
                       '|domainEqualList|)
                   |List|)
             (|CDRwithIncrement| G102556))
            ('T
```

```
            (UNWIND-PROTECT
              (PROG1 (|List;| G102555) (LETT G102556 T |List|))
              (COND
                ((NOT G102556) (HREM |$ConstructorCache| '|List|))))))))))

(DEFUN |List;| (|#1|)
  (PROG (DV$1 |dv$| $ G102554 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |List|)
        (LETT |dv$| (LIST '|List| DV$1) |List|)
        (LETT $ (make-array 62) |List|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$|
                  (|buildPredVector| 0 0
                      (LIST (|HasCategory| |#1| '(|SetCategory|))
                            (|HasCategory| |#1|
                                '(|ConvertibleTo| (|InputForm|)))
                            (LETT G102554
                                  (|HasCategory| |#1| '(|OrderedSet|))
                                  |List|)
                            (OR G102554
                                (|HasCategory| |#1| '(|SetCategory|)))
                            (|HasCategory| |#1| '(|OpenMath|))
                            (|HasCategory| (|Integer|) '(|OrderedSet|))
                            (AND (|HasCategory| |#1|
                                     (LIST '|Evalable|
                                      (|devaluate| |#1|)))
                                 (|HasCategory| |#1| '(|SetCategory|)))
                            (OR (AND (|HasCategory| |#1|
                                      (LIST '|Evalable|
                                       (|devaluate| |#1|)))
                                     G102554)
                                (AND (|HasCategory| |#1|
                                      (LIST '|Evalable|
                                       (|devaluate| |#1|)))
                                     (|HasCategory| |#1|
                                      '(|SetCategory|))))))
                  |List|))
        (|haddProp| |$ConstructorCache| '|List| (LIST DV$1) (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 |#1|)
        (COND
          ((|testBitVector| |pv$| 5)
           (PROGN
            (QSETREFV $ 25
                (CONS (|dispatchFunction| |LIST;OMwrite;$S;6|) $))
            (QSETREFV $ 26
                (CONS (|dispatchFunction| |LIST;OMwrite;$BS;7|) $))
```

```
            (QSETREFV $ 27
                (CONS (|dispatchFunction| |LIST;OMwrite;Omd$V;8|) $))
            (QSETREFV $ 28
                (CONS (|dispatchFunction| |LIST;OMwrite;Omd$BV;9|) $)))))
        (COND
          ((|testBitVector| |pv$| 1)
           (PROGN
            (QSETREFV $ 31
                (CONS (|dispatchFunction| |LIST;setUnion;3$;10|) $))
            (QSETREFV $ 33
                (CONS (|dispatchFunction|
                          |LIST;setIntersection;3$;11|)
                      $))
            (QSETREFV $ 36
                (CONS (|dispatchFunction| |LIST;setDifference;3$;12|)
                      $)))))
        (COND
          ((|testBitVector| |pv$| 2)
           (QSETREFV $ 44
               (CONS (|dispatchFunction| |LIST;convert;$If;13|) $))))
       $))))

(setf (get
 (QUOTE |List|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(
   NIL NIL NIL NIL NIL (|IndexedList| 6 (NRTEVAL 1)) (|local| |#1|)
   |LIST;nil;$;1| (|Boolean|) |LIST;null;$B;2| |LIST;cons;S2$;3|
   |LIST;append;3$;4| (|Void|) (|OpenMathDevice|) (0 . |OMputApp|)
   (|String|) (5 . |OMputSymbol|) (12 . |OMwrite|) (19 . |OMputEndApp|)
   (|OpenMathEncoding|) (24 . |OMencodingXML|) (28 . |OMopenString|)
   (34 . |OMputObject|) (39 . |OMputEndObject|) (44 . |OMclose|)
   (49 . |OMwrite|) (54 . |OMwrite|) (60 . |OMwrite|) (66 . |OMwrite|)
   (73 . |concat|) (79 . |removeDuplicates|) (84 . |setUnion|)
   (90 . |member?|) (96 . |setIntersection|) (|Integer|) (102 . |elt|)
   (108 . |setDifference|) (|Symbol|) (114 . |coerce|) (|InputForm|)
   (119 . |convert|) (124 . |convert|) (|List| |$|) (129 . |convert|)
   (134 . |convert|) (|Mapping| 6 6 6) (|NonNegativeInteger|)
   (|List| 6) (|List| 49) (|Equation| 6) (|Mapping| 8 6)
   (|Mapping| 8 6 6) (|UniversalSegment| 34) (QUOTE "last")
   (QUOTE "rest") (QUOTE "first") (QUOTE "value") (|Mapping| 6 6)
   (|SingleInteger|) (|OutputForm|) (|List| 34) (|Union| 6 (QUOTE "failed"))))
  (QUOTE #(|setUnion| 139 |setIntersection| 145 |setDifference| 151
   |removeDuplicates| 157 |null| 162 |nil| 167 |member?| 171 |elt| 177
   |convert| 183 |cons| 188 |concat| 194 |append| 200 |OMwrite| 206))
  (QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
  (CONS
   (|makeByteWordVec2| 8 (QUOTE (0 0 0 0 0 0 0 0 0 0 3 0 0 8 4 0 0 8 1 2 4 5)))
   (CONS (QUOTE #(
```

```
    |ListAggregate&| |StreamAggregate&| |ExtensibleLinearAggregate&|
    |FiniteLinearAggregate&| |UnaryRecursiveAggregate&| |LinearAggregate&|
    |RecursiveAggregate&| |IndexedAggregate&| |Collection&|
    |HomogeneousAggregate&| |OrderedSet&| |Aggregate&| |EltableAggregate&|
    |Evalable&| |SetCategory&| NIL NIL |InnerEvalable&| NIL NIL
    |BasicType&| NIL))
   (CONS
     (QUOTE #((|ListAggregate| 6) (|StreamAggregate| 6)
      (|ExtensibleLinearAggregate| 6) (|FiniteLinearAggregate| 6)
      (|UnaryRecursiveAggregate| 6) (|LinearAggregate| 6)
      (|RecursiveAggregate| 6) (|IndexedAggregate| 34 6) (|Collection| 6)
      (|HomogeneousAggregate| 6) (|OrderedSet|) (|Aggregate|)
      (|EltableAggregate| 34 6) (|Evalable| 6) (|SetCategory|)
      (|Type|) (|Eltable| 34 6) (|InnerEvalable| 6 6) (|CoercibleTo| 59)
      (|ConvertibleTo| 39) (|BasicType|) (|OpenMath|)))
    (|makeByteWordVec2| 44
     (QUOTE (
      1 13 12 0 14 3 13 12 0 15 15 16 3 6 12 13 0 8 17 1 13 12 0 18 0
      19 0 20 2 13 0 15 19 21 1 13 12 0 22 1 13 12 0 23 1 13 12 0 24 1 0 15
      0 25 2 0 15 0 8 26 2 0 12 13 0 27 3 0 12 13 0 8 28 2 0 0 0 0 29 1 0 0
      0 30 2 0 0 0 0 31 2 0 8 6 0 32 2 0 0 0 0 33 2 0 6 0 34 35 2 0 0 0 0 36
      1 37 0 15 38 1 39 0 37 40 1 6 39 0 41 1 39 0 42 43 1 0 39 0 44 2 1 0 0
      0 31 2 1 0 0 0 33 2 1 0 0 0 36 1 1 0 0 30 1 0 8 0 9 0 0 0 7 2 1 8 6 0
      32 2 0 6 0 34 35 1 2 39 0 44 2 0 0 6 0 10 2 0 0 0 0 29 2 0 0 0 0 11 3
      5 12 13 0 8 28 2 5 12 13 0 27 1 5 15 0 25 2 5 15 0 8 26))))))
  (QUOTE |lookupIncomplete|)))
```

———-

28.8 NNI.lsp BOOTSTRAP

NNI depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **NNI** category which we can write into the **MID** directory. We compile the lisp code and copy the **NNI.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— NNI.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |$CategoryFrame|
      (|put| '|NonNegativeInteger| '|SuperDomain| '(|Integer|)
             (|put| '(|Integer|) '|SubDomain|
                    (CONS '(|NonNegativeInteger| COND ((< |#1| 0) 'NIL)
                                   ('T 'T))
                          (DELASC '|NonNegativeInteger|
```

```
                                   (|get| '(|Integer|) '|SubDomain|
                                          |$CategoryFrame|)))
                     |$CategoryFrame|)))

(PUT '|NNI;sup;3$;1| '|SPADreplace| 'MAX)

(DEFUN |NNI;sup;3$;1| (|x| |y| |$|) (MAX |x| |y|))

(PUT '|NNI;shift;$I$;2| '|SPADreplace| 'ASH)

(DEFUN |NNI;shift;$I$;2| (|x| |n| |$|) (ASH |x| |n|))

(DEFUN |NNI;subtractIfCan;2$U;3| (|x| |y| |$|)
  (PROG (|c|)
    (RETURN
      (SEQ
        (LETT |c| (|-| |x| |y|) |NNI;subtractIfCan;2$U;3|)
        (EXIT
          (COND
            ((|<| |c| 0) (CONS 1 "failed"))
            ((QUOTE T) (CONS 0 |c|))))))))

(DEFUN |NonNegativeInteger| ()
  (PROG ()
    (RETURN
      (PROG (G96708)
        (RETURN
          (COND
            ((LETT G96708
                   (HGET |$ConstructorCache| '|NonNegativeInteger|)
                   |NonNegativeInteger|)
             (|CDRwithIncrement| (CDAR G96708)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache|
                                   '|NonNegativeInteger|
                                   (LIST
                                    (CONS NIL
                                     (CONS 1 (|NonNegativeInteger;|))))))
                 (LETT G96708 T |NonNegativeInteger|))
               (COND
                 ((NOT G96708)
                  (HREM |$ConstructorCache| '|NonNegativeInteger|)))))))))))

(DEFUN |NonNegativeInteger;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|NonNegativeInteger|) |NonNegativeInteger|)
        (LETT $ (make-array 17) |NonNegativeInteger|)
```

```
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL)
                  |NonNegativeInteger|))
        (|haddProp| |$ConstructorCache| '|NonNegativeInteger| NIL
            (CONS 1 $))
        (|stuffDomainSlots| $)
        $))))

(setf (get
  (QUOTE |NonNegativeInteger|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL
        (|Integer|)
        |NNI;sup;3$;1|
        |NNI;shift;$I$;2|
        (|Union| |$| (QUOTE "failed"))
        |NNI;subtractIfCan;2$U;3|
        (|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
        (|PositiveInteger|)
        (|Boolean|)
        (|NonNegativeInteger|)
        (|SingleInteger|)
        (|String|)
        (|OutputForm|)))
    (QUOTE
      #(|~=| 0 |zero?| 6 |sup| 11 |subtractIfCan| 17 |shift| 23 |sample| 29
        |rem| 33 |recip| 39 |random| 44 |quo| 49 |one?| 55 |min| 60 |max| 66
        |latex| 72 |hash| 77 |gcd| 82 |exquo| 88 |divide| 94 |coerce| 100
        |^| 105 |Zero| 117 |One| 121 |>=| 125 |>| 131 |=| 137 |<=| 143
        |<| 149 |+| 155 |**| 161 |*| 173))
    (QUOTE (((|commutative| "*") . 0)))
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0 0 0 0 0 0 0 0 0)))
      (CONS
        (QUOTE
          #(NIL NIL NIL NIL NIL
            |Monoid&|
            |AbelianMonoid&|
            |OrderedSet&|
            |SemiGroup&|
            |AbelianSemiGroup&|
            |SetCategory&|
            |BasicType&|
            NIL))
        (CONS
          (QUOTE
            #((|OrderedAbelianMonoidSup|)
```

```
              (|OrderedCancellationAbelianMonoid|)
              (|OrderedAbelianMonoid|)
              (|OrderedAbelianSemiGroup|)
              (|CancellationAbelianMonoid|)
              (|Monoid|)
              (|AbelianMonoid|)
              (|OrderedSet|)
              (|SemiGroup|)
              (|AbelianSemiGroup|)
              (|SetCategory|)
              (|BasicType|)
              (|CoercibleTo| 16)))
          (|makeByteWordVec2| 16
            (QUOTE
              (2 0 12 0 0 1 1 0 12 0 1 2 0 0 0 0 6 2 0 8 0 0 9 2 0 0 0 5 7 0 0
               0 1 2 0 0 0 0 1 1 0 8 0 1 1 0 0 0 1 2 0 0 0 0 1 1 0 12 0 1 2 0
               0 0 0 1 2 0 0 0 0 1 1 0 15 0 1 1 0 14 0 1 2 0 0 0 0 1 2 0 8 0 0
               1 2 0 10 0 0 1 1 0 16 0 1 2 0 0 0 11 1 2 0 0 0 13 1 0 0 0 1 0 0
               0 1 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12
               0 0 1 2 0 0 0 0 1 2 0 0 0 11 1 2 0 0 0 13 1 2 0 0 0 0 1 2 0 0
               11 0 1 2 0 0 13 0 1))))))
     (QUOTE |lookupComplete|)))

(setf (get (QUOTE |NonNegativeInteger|) (QUOTE NILADIC)) T)
```

———-

28.9 OUTFORM.lsp BOOTSTRAP

OUTFORM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **OUTFORM** category which we can write into the **MID** directory. We compile the lisp code and copy the **OUTFORM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— OUTFORM.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |OUTFORM;print;$V;1|) (QUOTE |SPADreplace|) (QUOTE |mathprint|))

(DEFUN |OUTFORM;print;$V;1| (|x| |$|) (|mathprint| |x|))

(DEFUN |OUTFORM;message;S$;2| (|s| |$|)
 (COND
  ((SPADCALL |s| (QREFELT |$| 11)) (SPADCALL (QREFELT |$| 12)))
```

```
  ((QUOTE T) |s|)))

(DEFUN |OUTFORM;messagePrint;SV;3| (|s| |$|)
 (SPADCALL (SPADCALL |s| (QREFELT |$| 13)) (QREFELT |$| 8)))

(PUT (QUOTE |OUTFORM;=;2$B;4|) (QUOTE |SPADreplace|) (QUOTE EQUAL))

(DEFUN |OUTFORM;=;2$B;4| (|a| |b| |$|) (EQUAL |a| |b|))

(DEFUN |OUTFORM;=;3$;5| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "=" |$|) |a| |b|))

(PUT
 (QUOTE |OUTFORM;coerce;2$;6|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|a|) |a|)))

(DEFUN |OUTFORM;coerce;2$;6| (|a| |$|) |a|)

(PUT
 (QUOTE |OUTFORM;outputForm;I$;7|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|n|) |n|)))

(DEFUN |OUTFORM;outputForm;I$;7| (|n| |$|) |n|)

(PUT
 (QUOTE |OUTFORM;outputForm;S$;8|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|e|) |e|)))

(DEFUN |OUTFORM;outputForm;S$;8| (|e| |$|) |e|)

(PUT
 (QUOTE |OUTFORM;outputForm;Df$;9|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM (|f|) |f|)))

(DEFUN |OUTFORM;outputForm;Df$;9| (|f| |$|) |f|)

(PUT (QUOTE |OUTFORM;sform|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|s|) |s|)))

(DEFUN |OUTFORM;sform| (|s| |$|) |s|)

(PUT (QUOTE |OUTFORM;eform|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|e|) |e|)))

(DEFUN |OUTFORM;eform| (|e| |$|) |e|)

(PUT (QUOTE |OUTFORM;iform|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|n|) |n|)))
```

```
(DEFUN |OUTFORM;iform| (|n| |$|) |n|)

(DEFUN |OUTFORM;outputForm;S$;13| (|s| |$|)
 (|OUTFORM;sform|
   (SPADCALL
    (SPADCALL (QREFELT |$| 26))
    (SPADCALL |s| (SPADCALL (QREFELT |$| 26)) (QREFELT |$| 27))
    (QREFELT |$| 28))
   |$|))

(PUT
 (QUOTE |OUTFORM;width;$I;14|)
 (QUOTE |SPADreplace|)
 (QUOTE |outformWidth|))

(DEFUN |OUTFORM;width;$I;14| (|a| |$|) (|outformWidth| |a|))

(PUT (QUOTE |OUTFORM;height;$I;15|) (QUOTE |SPADreplace|) (QUOTE |height|))

(DEFUN |OUTFORM;height;$I;15| (|a| |$|) (|height| |a|))

(PUT
 (QUOTE |OUTFORM;subHeight;$I;16|)
 (QUOTE |SPADreplace|)
 (QUOTE |subspan|))

(DEFUN |OUTFORM;subHeight;$I;16| (|a| |$|) (|subspan| |a|))

(PUT
 (QUOTE |OUTFORM;superHeight;$I;17|)
 (QUOTE |SPADreplace|)
 (QUOTE |superspan|))

(DEFUN |OUTFORM;superHeight;$I;17| (|a| |$|) (|superspan| |a|))

(PUT
 (QUOTE |OUTFORM;height;I;18|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL 20)))

(DEFUN |OUTFORM;height;I;18| (|$|) 20)

(PUT
 (QUOTE |OUTFORM;width;I;19|)
 (QUOTE |SPADreplace|)
 (QUOTE (XLAM NIL 66)))

(DEFUN |OUTFORM;width;I;19| (|$|) 66)

(DEFUN |OUTFORM;center;$I$;20| (|a| |w| |$|)
```

```
 (SPADCALL
  (SPADCALL
   (QUOTIENT2 (|-| |w| (SPADCALL |a| (QREFELT |$| 30))) 2)
   (QREFELT |$| 36))
  |a|
  (QREFELT |$| 37)))

(DEFUN |OUTFORM;left;$I$;21| (|a| |w| |$|)
 (SPADCALL
  |a|
  (SPADCALL (|-| |w| (SPADCALL |a| (QREFELT |$| 30))) (QREFELT |$| 36))
  (QREFELT |$| 37)))

(DEFUN |OUTFORM;right;$I$;22| (|a| |w| |$|)
 (SPADCALL
  (SPADCALL (|-| |w| (SPADCALL |a| (QREFELT |$| 30))) (QREFELT |$| 36))
  |a|
  (QREFELT |$| 37)))

(DEFUN |OUTFORM;center;2$;23| (|a| |$|)
 (SPADCALL |a| (SPADCALL (QREFELT |$| 35)) (QREFELT |$| 38)))

(DEFUN |OUTFORM;left;2$;24| (|a| |$|)
 (SPADCALL |a| (SPADCALL (QREFELT |$| 35)) (QREFELT |$| 39)))

(DEFUN |OUTFORM;right;2$;25| (|a| |$|)
 (SPADCALL |a| (SPADCALL (QREFELT |$| 35)) (QREFELT |$| 40)))

(DEFUN |OUTFORM;vspace;I$;26| (|n| |$|)
 (COND
  ((EQL |n| 0) (SPADCALL (QREFELT |$| 12)))
  ((QUOTE T)
   (SPADCALL
    (|OUTFORM;sform| " " |$|)
    (SPADCALL (|-| |n| 1) (QREFELT |$| 44))
    (QREFELT |$| 45)))))

(DEFUN |OUTFORM;hspace;I$;27| (|n| |$|)
 (COND
  ((EQL |n| 0) (SPADCALL (QREFELT |$| 12)))
  ((QUOTE T) (|OUTFORM;sform| (|fillerSpaces| |n|) |$|))))

(DEFUN |OUTFORM;rspace;2I$;28| (|n| |m| |$|)
 (COND
  ((OR (EQL |n| 0) (EQL |m| 0)) (SPADCALL (QREFELT |$| 12)))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL |n| (QREFELT |$| 36))
    (SPADCALL |n| (|-| |m| 1) (QREFELT |$| 46))
    (QREFELT |$| 45)))))
```

```
(DEFUN |OUTFORM;matrix;L$;29| (|ll| $)
  (PROG (G82748 |l| G82749 |lv|)
    (RETURN
      (SEQ (LETT |lv|
                 (PROGN
                   (LETT G82748 NIL |OUTFORM;matrix;L$;29|)
                   (SEQ (LETT |l| NIL |OUTFORM;matrix;L$;29|)
                        (LETT G82749 |ll| |OUTFORM;matrix;L$;29|) G190
                        (COND
                          ((OR (ATOM G82749)
                               (PROGN
                                 (LETT |l| (CAR G82749)
                                       |OUTFORM;matrix;L$;29|)
                                 NIL))
                           (GO G191)))
                        (SEQ (EXIT (LETT G82748
                                         (CONS (LIST2VEC |l|) G82748)
                                         |OUTFORM;matrix;L$;29|)))
                        (LETT G82749 (CDR G82749)
                              |OUTFORM;matrix;L$;29|)
                        (GO G190) G191 (EXIT (NREVERSE0 G82748))))
                 |OUTFORM;matrix;L$;29|)
           (EXIT (CONS (|OUTFORM;eform| 'MATRIX $) (LIST2VEC |lv|)))))))

(DEFUN |OUTFORM;pile;L$;30| (|l| |$|)
 (CONS (|OUTFORM;eform| (QUOTE SC) |$|) |l|))

(DEFUN |OUTFORM;commaSeparate;L$;31| (|l| |$|)
 (CONS (|OUTFORM;eform| (QUOTE AGGLST) |$|) |l|))

(DEFUN |OUTFORM;semicolonSeparate;L$;32| (|l| |$|)
 (CONS (|OUTFORM;eform| (QUOTE AGGSET) |$|) |l|))

(DEFUN |OUTFORM;blankSeparate;L$;33| (|l| $)
  (PROG (|c| |u| G82757 |l1|)
    (RETURN
      (SEQ (LETT |c| (|OUTFORM;eform| 'CONCATB $)
                 |OUTFORM;blankSeparate;L$;33|)
           (LETT |l1| NIL |OUTFORM;blankSeparate;L$;33|)
           (SEQ (LETT |u| NIL |OUTFORM;blankSeparate;L$;33|)
                (LETT G82757 (SPADCALL |l| (QREFELT $ 53))
                      |OUTFORM;blankSeparate;L$;33|)
                G190
                (COND
                  ((OR (ATOM G82757)
                       (PROGN
                         (LETT |u| (CAR G82757)
                               |OUTFORM;blankSeparate;L$;33|)
                         NIL))
```

```
                (GO G191)))
             (SEQ (EXIT (COND
                         ((EQCAR |u| |c|)
                          (LETT |l1|
                                (SPADCALL (CDR |u|) |l1|
                                 (QREFELT $ 54))
                                |OUTFORM;blankSeparate;L$;33|))
                         ('T
                          (LETT |l1| (CONS |u| |l1|)
                                |OUTFORM;blankSeparate;L$;33|)))))
             (LETT G82757 (CDR G82757)
                   |OUTFORM;blankSeparate;L$;33|)
             (GO G190) G191 (EXIT NIL))
         (EXIT (CONS |c| |l1|))))))

(DEFUN |OUTFORM;brace;2$;34| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE BRACE) |$|) |a|))

(DEFUN |OUTFORM;brace;L$;35| (|l| |$|)
 (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 56)))

(DEFUN |OUTFORM;bracket;2$;36| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE BRACKET) |$|) |a|))

(DEFUN |OUTFORM;bracket;L$;37| (|l| |$|)
 (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 58)))

(DEFUN |OUTFORM;paren;2$;38| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE PAREN) |$|) |a|))

(DEFUN |OUTFORM;paren;L$;39| (|l| |$|)
 (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 60)))

(DEFUN |OUTFORM;sub;3$;40| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SUB) |$|) |a| |b|))

(DEFUN |OUTFORM;super;3$;41| (|a| |b| |$|)
 (LIST
  (|OUTFORM;eform| (QUOTE SUPERSUB) |$|)
  |a|
  (|OUTFORM;sform| " " |$|) |b|))

(DEFUN |OUTFORM;presub;3$;42| (|a| |b| |$|)
 (LIST
  (|OUTFORM;eform| (QUOTE SUPERSUB) |$|)
  |a|
  (|OUTFORM;sform| " " |$|)
  (|OUTFORM;sform| " " |$|)
  (|OUTFORM;sform| " " |$|)
  |b|))
```

```
(DEFUN |OUTFORM;presuper;3$;43| (|a| |b| |$|)
 (LIST
  (|OUTFORM;eform| (QUOTE SUPERSUB) |$|)
  |a|
  (|OUTFORM;sform| " " |$|)
  (|OUTFORM;sform| " " |$|)
  |b|))

(DEFUN |OUTFORM;scripts;$L$;44| (|a| |l| |$|)
 (COND
  ((SPADCALL |l| (QREFELT |$| 66)) |a|)
  ((SPADCALL (SPADCALL |l| (QREFELT |$| 67)) (QREFELT |$| 66))
    (SPADCALL |a| (SPADCALL |l| (QREFELT |$| 68)) (QREFELT |$| 62)))
  ((QUOTE T) (CONS (|OUTFORM;eform| (QUOTE SUPERSUB) |$|) (CONS |a| |l|)))))

(DEFUN |OUTFORM;supersub;$L$;45| (|a| |l| |$|)
 (SEQ
  (COND
   ((ODDP (SPADCALL |l| (QREFELT |$| 71)))
    (LETT |l|
     (SPADCALL |l| (LIST (SPADCALL (QREFELT |$| 12))) (QREFELT |$| 73))
     |OUTFORM;supersub;$L$;45|)))
  (EXIT (CONS (|OUTFORM;eform| (QUOTE ALTSUPERSUB) |$|) (CONS |a| |l|)))))

(DEFUN |OUTFORM;hconcat;3$;46| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE CONCAT) |$|) |a| |b|))

(DEFUN |OUTFORM;hconcat;L$;47| (|l| |$|)
 (CONS (|OUTFORM;eform| (QUOTE CONCAT) |$|) |l|))

(DEFUN |OUTFORM;vconcat;3$;48| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE VCONCAT) |$|) |a| |b|))

(DEFUN |OUTFORM;vconcat;L$;49| (|l| |$|)
 (CONS (|OUTFORM;eform| (QUOTE VCONCAT) |$|) |l|))

(DEFUN |OUTFORM;^=;3$;50| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "^=" |$|) |a| |b|))

(DEFUN |OUTFORM;<;3$;51| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "<" |$|) |a| |b|))

(DEFUN |OUTFORM;>;3$;52| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| ">" |$|) |a| |b|))

(DEFUN |OUTFORM;<=;3$;53| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "<=" |$|) |a| |b|))

(DEFUN |OUTFORM;>=;3$;54| (|a| |b| |$|)
```

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 (LIST (|OUTFORM;sform| ">=" |$|) |a| |b|))

(DEFUN |OUTFORM;+;3$;55| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "+" |$|) |a| |b|))

(DEFUN |OUTFORM;-;3$;56| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "-" |$|) |a| |b|))

(DEFUN |OUTFORM;-;2$;57| (|a| |$|)
 (LIST (|OUTFORM;sform| "-" |$|) |a|))

(DEFUN |OUTFORM;*;3$;58| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "*" |$|) |a| |b|))

(DEFUN |OUTFORM;/;3$;59| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "/" |$|) |a| |b|))

(DEFUN |OUTFORM;**;3$;60| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "**" |$|) |a| |b|))

(DEFUN |OUTFORM;div;3$;61| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "div" |$|) |a| |b|))

(DEFUN |OUTFORM;rem;3$;62| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "rem" |$|) |a| |b|))

(DEFUN |OUTFORM;quo;3$;63| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "quo" |$|) |a| |b|))

(DEFUN |OUTFORM;exquo;3$;64| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "exquo" |$|) |a| |b|))

(DEFUN |OUTFORM;and;3$;65| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "and" |$|) |a| |b|))

(DEFUN |OUTFORM;or;3$;66| (|a| |b| |$|)
 (LIST (|OUTFORM;sform| "or" |$|) |a| |b|))

(DEFUN |OUTFORM;not;2$;67| (|a| |$|)
 (LIST (|OUTFORM;sform| "not" |$|) |a|))

(DEFUN |OUTFORM;SEGMENT;3$;68| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SEGMENT) |$|) |a| |b|))

(DEFUN |OUTFORM;SEGMENT;2$;69| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SEGMENT) |$|) |a|))

(DEFUN |OUTFORM;binomial;3$;70| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE BINOMIAL) |$|) |a| |b|))
```

```
(DEFUN |OUTFORM;empty;$;71| (|$|)
 (LIST (|OUTFORM;eform| (QUOTE NOTHING) |$|)))

(DEFUN |OUTFORM;infix?;$B;72| (|a| $)
  (PROG (G82802 |e|)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |e|
                            (COND
                              ((IDENTP |a|) |a|)
                              ((STRINGP |a|) (INTERN |a|))
                              ('T
                               (PROGN
                                 (LETT G82802 'NIL
                                       |OUTFORM;infix?;$B;72|)
                                 (GO G82802))))
                            |OUTFORM;infix?;$B;72|)
                      (EXIT (COND ((GET |e| 'INFIXOP) 'T) ('T 'NIL)))))
           G82802 (EXIT G82802)))))

(PUT (QUOTE |OUTFORM;elt;$L$;73|) (QUOTE |SPADreplace|) (QUOTE CONS))

(DEFUN |OUTFORM;elt;$L$;73| (|a| |l| |$|) (CONS |a| |l|))

(DEFUN |OUTFORM;prefix;$L$;74| (|a| |l| |$|)
 (COND
  ((NULL (SPADCALL |a| (QREFELT |$| 98))) (CONS |a| |l|))
  ((QUOTE T)
   (SPADCALL |a|
    (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 60))
    (QREFELT |$| 37)))))

(DEFUN |OUTFORM;infix;$L$;75| (|a| |l| |$|)
 (COND
  ((SPADCALL |l| (QREFELT |$| 66)) (SPADCALL (QREFELT |$| 12)))
  ((SPADCALL (SPADCALL |l| (QREFELT |$| 67)) (QREFELT |$| 66))
    (SPADCALL |l| (QREFELT |$| 68)))
  ((SPADCALL |a| (QREFELT |$| 98)) (CONS |a| |l|))
  ((QUOTE T)
   (SPADCALL
    (LIST
     (SPADCALL |l| (QREFELT |$| 68))
     |a|
     (SPADCALL |a| (SPADCALL |l| (QREFELT |$| 101)) (QREFELT |$| 102)))
    (QREFELT |$| 75)))))

(DEFUN |OUTFORM;infix;4$;76| (|a| |b| |c| |$|)
 (COND
  ((SPADCALL |a| (QREFELT |$| 98)) (LIST |a| |b| |c|))
  ((QUOTE T) (SPADCALL (LIST |b| |a| |c|) (QREFELT |$| 75)))))
```

```
(DEFUN |OUTFORM;postfix;3$;77| (|a| |b| |$|)
 (SPADCALL |b| |a| (QREFELT |$| 37)))

(DEFUN |OUTFORM;string;2$;78| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE STRING) |$|) |a|))

(DEFUN |OUTFORM;quote;2$;79| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE QUOTE) |$|) |a|))

(DEFUN |OUTFORM;overbar;2$;80| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE OVERBAR) |$|) |a|))

(DEFUN |OUTFORM;dot;2$;81| (|a| |$|)
 (SPADCALL |a| (|OUTFORM;sform| "." |$|) (QREFELT |$| 63)))

(DEFUN |OUTFORM;prime;2$;82| (|a| |$|)
 (SPADCALL |a| (|OUTFORM;sform| "," |$|) (QREFELT |$| 63)))

(DEFUN |OUTFORM;dot;$Nni$;83| (|a| |nn| |$|)
 (PROG (|s|)
  (RETURN
   (SEQ
    (LETT |s|
     (|MAKE-FULL-CVEC| |nn| (SPADCALL "." (QREFELT |$| 110)))
     |OUTFORM;dot;$Nni$;83|)
    (EXIT (SPADCALL |a| (|OUTFORM;sform| |s| |$|) (QREFELT |$| 63)))))))

(DEFUN |OUTFORM;prime;$Nni$;84| (|a| |nn| |$|)
 (PROG (|s|)
  (RETURN
   (SEQ
    (LETT |s|
     (|MAKE-FULL-CVEC| |nn| (SPADCALL "," (QREFELT |$| 110)))
     |OUTFORM;prime;$Nni$;84|)
    (EXIT (SPADCALL |a| (|OUTFORM;sform| |s| |$|) (QREFELT |$| 63)))))))

(DEFUN |OUTFORM;overlabel;3$;85| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE OVERLABEL) |$|) |a| |b|))

(DEFUN |OUTFORM;box;2$;86| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE BOX) |$|) |a|))

(DEFUN |OUTFORM;zag;3$;87| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE ZAG) |$|) |a| |b|))

(DEFUN |OUTFORM;root;2$;88| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE ROOT) |$|) |a|))

(DEFUN |OUTFORM;root;3$;89| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE ROOT) |$|) |a| |b|))
```

```
(DEFUN |OUTFORM;over;3$;90| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE OVER) |$|) |a| |b|))

(DEFUN |OUTFORM;slash;3$;91| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SLASH) |$|) |a| |b|))

(DEFUN |OUTFORM;assign;3$;92| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE LET) |$|) |a| |b|))

(DEFUN |OUTFORM;label;3$;93| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE EQUATNUM) |$|) |a| |b|))

(DEFUN |OUTFORM;rarrow;3$;94| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE TAG) |$|) |a| |b|))

(DEFUN |OUTFORM;differentiate;$Nni$;95| (|a| |nn| $)
  (PROG (G82832 |r| |s|)
    (RETURN
      (SEQ (COND
             ((ZEROP |nn|) |a|)
             ((< |nn| 4) (SPADCALL |a| |nn| (QREFELT $ 112)))
             ('T
              (SEQ (LETT |r|
                         (SPADCALL
                             (PROG1 (LETT G82832 |nn|
                                     |OUTFORM;differentiate;$Nni$;95|)
                               (|check-subtype| (> G82832 0)
                                   '(|PositiveInteger|) G82832))
                             (QREFELT $ 125))
                         |OUTFORM;differentiate;$Nni$;95|)
                   (LETT |s| (SPADCALL |r| (QREFELT $ 126))
                         |OUTFORM;differentiate;$Nni$;95|)
                   (EXIT (SPADCALL |a|
                             (SPADCALL (|OUTFORM;sform| |s| $)
                                 (QREFELT $ 60))
                             (QREFELT $ 63))))))))))

(DEFUN |OUTFORM;sum;2$;96| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SIGMA) |$|) (SPADCALL (QREFELT |$| 12)) |a|))

(DEFUN |OUTFORM;sum;3$;97| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SIGMA) |$|) |b| |a|))

(DEFUN |OUTFORM;sum;4$;98| (|a| |b| |c| |$|)
 (LIST (|OUTFORM;eform| (QUOTE SIGMA2) |$|) |b| |c| |a|))

(DEFUN |OUTFORM;prod;2$;99| (|a| |$|)
 (LIST (|OUTFORM;eform| (QUOTE PI) |$|) (SPADCALL (QREFELT |$| 12)) |a|))
```

```
(DEFUN |OUTFORM;prod;3$;100| (|a| |b| |$|)
 (LIST (|OUTFORM;eform| (QUOTE PI) |$|) |b| |a|))

(DEFUN |OUTFORM;prod;4$;101| (|a| |b| |c| |$|)
 (LIST (|OUTFORM;eform| (QUOTE PI2) |$|) |b| |c| |a|))

(DEFUN |OUTFORM;int;2$;102| (|a| |$|)
 (LIST
  (|OUTFORM;eform| (QUOTE INTSIGN) |$|)
  (SPADCALL (QREFELT |$| 12))
  (SPADCALL (QREFELT |$| 12))
  |a|))

(DEFUN |OUTFORM;int;3$;103| (|a| |b| |$|)
 (LIST
  (|OUTFORM;eform| (QUOTE INTSIGN) |$|)
  |b|
  (SPADCALL (QREFELT |$| 12))
  |a|))

(DEFUN |OUTFORM;int;4$;104| (|a| |b| |c| |$|)
 (LIST (|OUTFORM;eform| (QUOTE INTSIGN) |$|) |b| |c| |a|))

(DEFUN |OutputForm| ()
  (PROG ()
    (RETURN
      (PROG (G82846)
        (RETURN
          (COND
            ((LETT G82846 (HGET |$ConstructorCache| '|OutputForm|)
                   |OutputForm|)
             (|CDRwithIncrement| (CDAR G82846)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|OutputForm|
                                   (LIST
                                    (CONS NIL (CONS 1 (|OutputForm;|))))))
                 (LETT G82846 T |OutputForm|))
               (COND
                 ((NOT G82846)
                  (HREM |$ConstructorCache| '|OutputForm|)))))))))))

(DEFUN |OutputForm;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|OutputForm|) |OutputForm|)
        (LETT $ (make-array 138) |OutputForm|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
```

```
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |OutputForm|))
        (|haddProp| |$ConstructorCache| '|OutputForm| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 (|List| $))
        $))))

(setf (get
 (QUOTE |OutputForm|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(
   NIL NIL NIL NIL NIL NIL (QUOTE |Rep|) (|Void|) |OUTFORM;print;$V;1|
   (|Boolean|) (|String|) (0 . |empty?|) |OUTFORM;empty;$;71|
   |OUTFORM;message;S$;2| |OUTFORM;messagePrint;SV;3|
   |OUTFORM;=;2$B;4| |OUTFORM;=;3$;5| (|OutputForm|)
   |OUTFORM;coerce;2$;6| (|Integer|) |OUTFORM;outputForm;I$;7|
   (|Symbol|) |OUTFORM;outputForm;S$;8| (|DoubleFloat|)
   |OUTFORM;outputForm;Df$;9| (|Character|) (5 . |quote|)
   (9 . |concat|) (15 . |concat|) |OUTFORM;outputForm;S$;13|
   |OUTFORM;width;$I;14| |OUTFORM;height;$I;15|
   |OUTFORM;subHeight;$I;16| |OUTFORM;superHeight;$I;17|
   |OUTFORM;height;I;18| |OUTFORM;width;I;19| |OUTFORM;hspace;I$;27|
   |OUTFORM;hconcat;3$;46| |OUTFORM;center;$I$;20|
   |OUTFORM;left;$I$;21| |OUTFORM;right;$I$;22| |OUTFORM;center;2$;23|
   |OUTFORM;left;2$;24| |OUTFORM;right;2$;25| |OUTFORM;vspace;I$;26|
   |OUTFORM;vconcat;3$;48| |OUTFORM;rspace;2I$;28| (|List| 49)
   |OUTFORM;matrix;L$;29| (|List| |$|) |OUTFORM;pile;L$;30|
   |OUTFORM;commaSeparate;L$;31| |OUTFORM;semicolonSeparate;L$;32|
   (21 . |reverse|) (26 . |append|) |OUTFORM;blankSeparate;L$;33|
   |OUTFORM;brace;2$;34| |OUTFORM;brace;L$;35| |OUTFORM;bracket;2$;36|
   |OUTFORM;bracket;L$;37| |OUTFORM;paren;2$;38| |OUTFORM;paren;L$;39|
   |OUTFORM;sub;3$;40| |OUTFORM;super;3$;41| |OUTFORM;presub;3$;42|
   |OUTFORM;presuper;3$;43| (32 . |null|) (37 . |rest|) (42 . |first|)
   |OUTFORM;scripts;$L$;44| (|NonNegativeInteger|) (47 . |#|)
   (|List| |$$|) (52 . |append|) |OUTFORM;supersub;$L$;45|
   |OUTFORM;hconcat;L$;47| |OUTFORM;vconcat;L$;49| |OUTFORM;^=;3$;50|
   |OUTFORM;<;3$;51| |OUTFORM;>;3$;52| |OUTFORM;<=;3$;53|
   |OUTFORM;>=;3$;54| |OUTFORM;+;3$;55| |OUTFORM;-;3$;56|
   |OUTFORM;-;2$;57| |OUTFORM;*;3$;58| |OUTFORM;/;3$;59|
   |OUTFORM;**;3$;60| |OUTFORM;div;3$;61| |OUTFORM;rem;3$;62|
   |OUTFORM;quo;3$;63| |OUTFORM;exquo;3$;64| |OUTFORM;and;3$;65|
   |OUTFORM;or;3$;66| |OUTFORM;not;2$;67| |OUTFORM;SEGMENT;3$;68|
   |OUTFORM;SEGMENT;2$;69| |OUTFORM;binomial;3$;70|
   |OUTFORM;infix?;$B;72| |OUTFORM;elt;$L$;73| |OUTFORM;prefix;$L$;74|
   (58 . |rest|) |OUTFORM;infix;$L$;75| |OUTFORM;infix;4$;76|
   |OUTFORM;postfix;3$;77| |OUTFORM;string;2$;78| |OUTFORM;quote;2$;79|
   |OUTFORM;overbar;2$;80| |OUTFORM;dot;2$;81| |OUTFORM;prime;2$;82|
   (63 . |char|) |OUTFORM;dot;$Nni$;83| |OUTFORM;prime;$Nni$;84|
   |OUTFORM;overlabel;3$;85| |OUTFORM;box;2$;86| |OUTFORM;zag;3$;87|
   |OUTFORM;root;2$;88| |OUTFORM;root;3$;89| |OUTFORM;over;3$;90|
```

```
 |OUTFORM;slash;3$;91| |OUTFORM;assign;3$;92|
 |OUTFORM;label;3$;93| |OUTFORM;rarrow;3$;94| (|PositiveInteger|)
 (|NumberFormats|) (68 . |FormatRoman|) (73 . |lowerCase|)
 |OUTFORM;differentiate;$Nni$;95| |OUTFORM;sum;2$;96|
 |OUTFORM;sum;3$;97| |OUTFORM;sum;4$;98| |OUTFORM;prod;2$;99|
 |OUTFORM;prod;3$;100| |OUTFORM;prod;4$;101| |OUTFORM;int;2$;102|
 |OUTFORM;int;3$;103| |OUTFORM;int;4$;104| (|SingleInteger|)))
(QUOTE #(|~=| 78 |zag| 84 |width| 90 |vspace| 99 |vconcat| 104
 |supersub| 115 |superHeight| 121 |super| 126 |sum| 132 |subHeight|
 150 |sub| 155 |string| 161 |slash| 166 |semicolonSeparate| 172
 |scripts| 177 |rspace| 183 |root| 189 |right| 200 |rem| 211
 |rarrow| 217 |quote| 223 |quo| 228 |prod| 234 |print| 252
 |prime| 257 |presuper| 268 |presub| 274 |prefix| 280 |postfix|
 286 |pile| 292 |paren| 297 |overlabel| 307 |overbar| 313 |over|
 318 |outputForm| 324 |or| 344 |not| 350 |messagePrint| 355
 |message| 360 |matrix| 365 |left| 370 |latex| 381 |label| 386
 |int| 392 |infix?| 410 |infix| 415 |hspace| 428 |height| 433
 |hconcat| 442 |hash| 453 |exquo| 458 |empty| 464 |elt| 468 |dot|
 474 |div| 485 |differentiate| 491 |commaSeparate| 497 |coerce|
 502 |center| 507 |bracket| 518 |brace| 528 |box| 538 |blankSeparate|
 543 |binomial| 548 |assign| 554 |and| 560 |^=| 566 SEGMENT 572
 |>=| 583 |>| 589 |=| 595 |<=| 607 |<| 613 |/| 619 |-| 625 |+|
 636 |**| 642 |*| 648))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE (0 0 0)))
 (CONS
  (QUOTE #(|SetCategory&| |BasicType&| NIL))
  (CONS
   (QUOTE #((|SetCategory|) (|BasicType|) (|CoercibleTo| 17)))
   (|makeByteWordVec2| 137 (QUOTE (1 10 9 0 11 0 25 0 26 2 10 0 0 25
     27 2 10 0 25 0 28 1 6 0 0 53 2 6 0 0 0 54 1 6 9 0 66 1 6 0 0 67 1
     6 2 0 68 1 6 70 0 71 2 72 0 0 0 73 1 72 0 0 101 1 25 0 10 110 1 124
     10 123 125 1 10 0 0 126 2 0 9 0 0 1 2 0 0 0 0 115 0 0 19 35 1 0 19
     0 30 1 0 0 19 44 1 0 0 49 76 2 0 0 0 0 45 2 0 0 0 49 74 1 0 19 0
     33 2 0 0 0 0 63 2 0 0 0 0 129 3 0 0 0 0 0 130 1 0 0 0 128 1 0 19
     0 32 2 0 0 0 0 62 1 0 0 0 105 2 0 0 0 0 119 1 0 0 49 52 2 0 0 0
     49 69 2 0 0 19 19 46 1 0 0 0 116 2 0 0 0 0 117 1 0 0 0 43 2 0 0
     0 19 40 2 0 0 0 0 89 2 0 0 0 0 122 1 0 0 0 106 2 0 0 0 0 90 3 0
     0 0 0 0 133 1 0 0 0 131 2 0 0 0 0 132 1 0 7 0 8 2 0 0 0 70 112 1
     0 0 0 109 2 0 0 0 0 65 2 0 0 0 0 64 2 0 0 0 49 100 2 0 0 0 0 104
     1 0 0 49 50 1 0 0 49 61 1 0 0 0 60 2 0 0 0 0 113 1 0 0 0 107 2 0
     0 0 0 118 1 0 0 10 29 1 0 0 23 24 1 0 0 21 22 1 0 0 19 20 2 0 0
     0 0 93 1 0 0 0 94 1 0 7 10 14 1 0 0 10 13 1 0 0 47 48 1 0 0 0 42
     2 0 0 0 19 39 1 0 10 0 1 2 0 0 0 0 121 3 0 0 0 0 0 136 2 0 0 0 0
     135 1 0 0 0 134 1 0 9 0 98 2 0 0 0 49 102 3 0 0 0 0 0 103 1 0 0
     19 36 0 0 19 34 1 0 19 0 31 1 0 0 49 75 2 0 0 0 0 37 1 0 137 0 1
     2 0 0 0 0 91 0 0 0 12 2 0 0 0 49 99 2 0 0 0 70 111 1 0 0 0 108 2
     0 0 0 0 88 2 0 0 0 70 127 1 0 0 49 51 1 0 17 0 18 1 0 0 0 41 2 0
     0 0 19 38 1 0 0 0 58 1 0 0 49 59 1 0 0 49 57 1 0 0 0 56 1 0 0 0
```

```
      114 1 0 0 49 55 2 0 0 0 0 97 2 0 0 0 0 120 2 0 0 0 0 92 2 0 0 0
      0 77 1 0 0 0 96 2 0 0 0 0 95 2 0 0 0 0 81 2 0 0 0 0 79 2 0 0 0 0
      16 2 0 9 0 0 15 2 0 0 0 0 80 2 0 0 0 0 78 2 0 0 0 0 86 1 0 0 0 84
      2 0 0 0 0 83 2 0 0 0 0 82 2 0 0 0 0 87 2 0 0 0 0 85))))))
  (QUOTE |lookupComplete|)))

(setf (get (QUOTE |OutputForm|) (QUOTE NILADIC)) T)
```

———-

28.10 PI.lsp BOOTSTRAP

PI depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PI** category which we can write into the **MID** directory. We compile the lisp code and copy the **PI.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **PI.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(SETQ |$CategoryFrame|
      (|put| '|PositiveInteger| '|SuperDomain| '(|NonNegativeInteger|)
             (|put| '(|NonNegativeInteger|) '|SubDomain|
                    (CONS '(|PositiveInteger| < 0 |#1|)
                          (DELASC '|PositiveInteger|
                                  (|get| '(|NonNegativeInteger|)
                                         '|SubDomain| |$CategoryFrame|)))
                    |$CategoryFrame|)))

(DEFUN |PositiveInteger| ()
  (PROG ()
    (RETURN
      (PROG (G96739)
        (RETURN
          (COND
            ((LETT G96739 (HGET |$ConstructorCache| '|PositiveInteger|)
                   |PositiveInteger|)
             (|CDRwithIncrement| (CDAR G96739)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache|
                                   '|PositiveInteger|
                                   (LIST
                                    (CONS NIL
                                     (CONS 1 (|PositiveInteger;|))))))
```

```
                (LETT G96739 T |PositiveInteger|))
              (COND
                ((NOT G96739)
                 (HREM |$ConstructorCache| '|PositiveInteger|)))))))))))

(DEFUN |PositiveInteger;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|PositiveInteger|) |PositiveInteger|)
        (LETT $ (make-array 12) |PositiveInteger|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |PositiveInteger|))
        (|haddProp| |$ConstructorCache| '|PositiveInteger| NIL
            (CONS 1 $))
        (|stuffDomainSlots| $)
        $))))

(setf (get
  (QUOTE |PositiveInteger|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL
        (|NonNegativeInteger|)
        (|PositiveInteger|)
        (|Boolean|)
        (|Union| |$| (QUOTE "failed"))
        (|SingleInteger|)
        (|String|)
        (|OutputForm|)))
    (QUOTE #(|~=| 0 |sample| 6 |recip| 10 |one?| 15 |min| 20 |max| 26
             |latex| 32 |hash| 37 |gcd| 42 |coerce| 48 |^| 53 |One| 65
             |>=| 69 |>| 75 |=| 81 |<=| 87 |<| 93 |+| 99 |**| 105 |*| 117))
    (QUOTE (((|commutative| "*") . 0)))
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0 0)))
      (CONS
        (QUOTE #(|Monoid&| |AbelianSemiGroup&| |SemiGroup&| |OrderedSet&|
                 |SetCategory&| |BasicType&| NIL))
        (CONS
          (QUOTE #(
            (|Monoid|)
            (|AbelianSemiGroup|)
            (|SemiGroup|)
            (|OrderedSet|)
            (|SetCategory|)
            (|BasicType|)
            (|CoercibleTo| 11)))
```

```
        (|makeByteWordVec2| 11
         (QUOTE (2 0 7 0 0 1 0 0 0 1 1 0 8 0 1 1 0 7 0 1 2 0 0 0 0 1 2 0 0 0
                 0 1 1 0 10 0 1 1 0 9 0 1 2 0 0 0 0 1 1 0 11 0 1 2 0 0 0 6 1
                 2 0 0 0 5 1 0 0 0 1 2 0 7 0 0 1 2 0 7 0 0 1 2 0 7 0 0 1 2 0
                 7 0 0 1 2 0 7 0 0 1 2 0 0 0 0 1 2 0 0 0 6 1 2 0 0 0 5 1 2 0
                 0 0 0 1 2 0 0 6 0 1))))))
    (QUOTE |lookupComplete|)))

(setf (get (QUOTE |PositiveInteger|) (QUOTE NILADIC)) T)
```

———-

28.11 PRIMARR.lsp BOOTSTRAP

PRIMARR depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PRIMARR** category which we can write into the **MID** directory. We compile the lisp code and copy the **PRIMARR.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— PRIMARR.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |PRIMARR;#;$Nni;1|) (QUOTE |SPADreplace|) (QUOTE QVSIZE))

(DEFUN |PRIMARR;#;$Nni;1| (|x| |$|) (QVSIZE |x|))

(PUT (QUOTE |PRIMARR;minIndex;$I;2|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM (|x|) 0)))

(DEFUN |PRIMARR;minIndex;$I;2| (|x| |$|) 0)

(PUT (QUOTE |PRIMARR;empty;$;3|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (make-array 0))))

(DEFUN |PRIMARR;empty;$;3| (|$|) (make-array 0))

(DEFUN |PRIMARR;new;NniS$;4| (|n| |x| |$|)
 (SPADCALL (make-array |n|) |x| (QREFELT |$| 12)))

(PUT (QUOTE |PRIMARR;qelt;$IS;5|) (QUOTE |SPADreplace|) (QUOTE ELT))

(DEFUN |PRIMARR;qelt;$IS;5| (|x| |i| |$|) (ELT |x| |i|))

(PUT (QUOTE |PRIMARR;elt;$IS;6|) (QUOTE |SPADreplace|) (QUOTE ELT))
```

```
(DEFUN |PRIMARR;elt;$IS;6| (|x| |i| |$|) (ELT |x| |i|))

(PUT (QUOTE |PRIMARR;qsetelt!;$I2S;7|) (QUOTE |SPADreplace|) (QUOTE SETELT))

(DEFUN |PRIMARR;qsetelt!;$I2S;7| (|x| |i| |s| |$|) (SETELT |x| |i| |s|))

(PUT (QUOTE |PRIMARR;setelt;$I2S;8|) (QUOTE |SPADreplace|) (QUOTE SETELT))

(DEFUN |PRIMARR;setelt;$I2S;8| (|x| |i| |s| |$|) (SETELT |x| |i| |s|))

(DEFUN |PRIMARR;fill!;$S$;9| (|x| |s| $)
  (PROG (|i| G82338)
    (RETURN
      (SEQ (SEQ (LETT |i| 0 |PRIMARR;fill!;$S$;9|)
                (LETT G82338 (QVMAXINDEX |x|) |PRIMARR;fill!;$S$;9|)
                G190 (COND ((QSGREATERP |i| G82338) (GO G191)))
                (SEQ (EXIT (SETELT |x| |i| |s|)))
                (LETT |i| (QSADD1 |i|) |PRIMARR;fill!;$S$;9|) (GO G190)
                G191 (EXIT NIL))
           (EXIT |x|)))))

(DEFUN |PrimitiveArray| (G82348)
  (PROG ()
    (RETURN
      (PROG (G82349)
        (RETURN
          (COND
            ((LETT G82349
                   (|lassocShiftWithFunction|
                       (LIST (|devaluate| G82348))
                       (HGET |$ConstructorCache| '|PrimitiveArray|)
                       '|domainEqualList|)
                   |PrimitiveArray|)
             (|CDRwithIncrement| G82349))
            ('T
             (UNWIND-PROTECT
               (PROG1 (|PrimitiveArray;| G82348)
                 (LETT G82349 T |PrimitiveArray|))
               (COND
                 ((NOT G82349)
                  (HREM |$ConstructorCache| '|PrimitiveArray|)))))))))))

(DEFUN |PrimitiveArray;| (|#1|)
  (PROG (DV$1 |dv$| $ G82347 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |PrimitiveArray|)
        (LETT |dv$| (LIST '|PrimitiveArray| DV$1) |PrimitiveArray|)
        (LETT $ (make-array 35) |PrimitiveArray|)
```

```
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$|
                  (|buildPredVector| 0 0
                      (LIST (|HasCategory| |#1| '(|SetCategory|))
                            (|HasCategory| |#1|
                                '(|ConvertibleTo| (|InputForm|)))
                            (LETT G82347
                                  (|HasCategory| |#1| '(|OrderedSet|))
                                  |PrimitiveArray|)
                            (OR G82347
                                (|HasCategory| |#1| '(|SetCategory|)))
                            (|HasCategory| (|Integer|) '(|OrderedSet|))
                            (AND (|HasCategory| |#1|
                                     (LIST '|Evalable|
                                      (|devaluate| |#1|)))
                                 (|HasCategory| |#1| '(|SetCategory|)))
                            (OR (AND (|HasCategory| |#1|
                                      (LIST '|Evalable|
                                       (|devaluate| |#1|)))
                                     G82347)
                                (AND (|HasCategory| |#1|
                                      (LIST '|Evalable|
                                       (|devaluate| |#1|)))
                                     (|HasCategory| |#1|
                                      '(|SetCategory|))))))
                  |PrimitiveArray|))
        (|haddProp| |$ConstructorCache| '|PrimitiveArray| (LIST DV$1)
            (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 |#1|)
        $))))

(setf (get (QUOTE |PrimitiveArray|) (QUOTE |infovec|))
 (LIST
  (QUOTE
   #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|NonNegativeInteger|)
     |PRIMARR;#;$Nni;1| (|Integer|) |PRIMARR;minIndex;$I;2|
     |PRIMARR;empty;$;3| |PRIMARR;fill!;$S$;9| |PRIMARR;new;NniS$;4|
     |PRIMARR;qelt;$IS;5| |PRIMARR;elt;$IS;6| |PRIMARR;qsetelt!;$I2S;7|
     |PRIMARR;setelt;$I2S;8| (|Mapping| 6 6 6) (|Boolean|) (|List| 6)
     (|Equation| 6) (|List| 21) (|Mapping| 19 6) (|Mapping| 19 6 6)
     (|UniversalSegment| 9) (|Void|) (|Mapping| 6 6) (|InputForm|)
     (|OutputForm|) (|String|) (|SingleInteger|) (|List| |$|)
     (|Union| 6 (QUOTE "failed")) (|List| 9)))
  (QUOTE
   #(|~=| 0 |swap!| 6 |sorted?| 13 |sort!| 24 |sort| 35 |size?| 46 |setelt|
     52 |select| 66 |sample| 72 |reverse!| 76 |reverse| 81 |removeDuplicates|
     86 |remove| 91 |reduce| 103 |qsetelt!| 124 |qelt| 131 |position| 137
     |parts| 156 |new| 161 |more?| 167 |minIndex| 173 |min| 178 |merge| 184
```

```
    |members| 197 |member?| 202 |maxIndex| 208 |max| 213 |map!| 219 |map|
    225 |less?| 238 |latex| 244 |insert| 249 |indices| 263 |index?| 268
    |hash| 274 |first| 279 |find| 284 |fill!| 290 |every?| 296 |eval| 302
    |eq?| 328 |entry?| 334 |entries| 340 |empty?| 345 |empty| 350 |elt| 354
    |delete| 373 |count| 385 |copyInto!| 397 |copy| 404 |convert| 409
    |construct| 414 |concat| 419 |coerce| 442 |any?| 447 |>=| 453 |>| 459
    |=| 465 |<=| 471 |<| 477 |#| 483))
(QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
(CONS
  (|makeByteWordVec2| 7 (QUOTE (0 0 0 0 0 0 3 0 0 7 4 0 0 7 1 2 4)))
  (CONS
   (QUOTE #(|OneDimensionalArrayAggregate&| |FiniteLinearAggregate&|
    |LinearAggregate&| |IndexedAggregate&| |Collection&|
    |HomogeneousAggregate&| |OrderedSet&| |Aggregate&| |EltableAggregate&|
    |Evalable&| |SetCategory&| NIL NIL |InnerEvalable&| NIL NIL |BasicType&|))
   (CONS
    (QUOTE
     #((|OneDimensionalArrayAggregate| 6) (|FiniteLinearAggregate| 6)
       (|LinearAggregate| 6) (|IndexedAggregate| 9 6) (|Collection| 6)
       (|HomogeneousAggregate| 6) (|OrderedSet|) (|Aggregate|)
       (|EltableAggregate| 9 6) (|Evalable| 6) (|SetCategory|) (|Type|)
       (|Eltable| 9 6) (|InnerEvalable| 6 6) (|CoercibleTo| 29)
       (|ConvertibleTo| 28) (|BasicType|)))
    (|makeByteWordVec2| 34
     (QUOTE
      (2 1 19 0 0 1 3 0 26 0 9 9 1 1 3 19 0 1 2 0 19 24 0 1 1 3 0 0 1 2 0 0
       24 0 1 1 3 0 0 1 2 0 0 24 0 1 2 0 19 0 7 1 3 0 6 0 25 6 1 3 0 6 0 9
       6 17 2 0 0 23 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 1 1 0 0 1 2 1 0 6 0 1
       2 0 0 23 0 1 4 1 6 18 0 6 6 1 3 0 6 18 0 6 1 2 0 6 18 0 1 3 0 6 0 9
       6 16 2 0 6 0 9 14 2 1 9 6 0 1 3 1 9 6 0 9 1 2 0 9 23 0 1 1 0 20 0 1
       2 0 0 7 6 13 2 0 19 0 7 1 1 5 9 0 10 2 3 0 0 0 1 2 3 0 0 0 1 3 0 0
       24 0 0 1 1 0 20 0 1 2 1 19 6 0 1 1 5 9 0 1 2 3 0 0 0 1 2 0 0 27 0 1
       3 0 0 18 0 0 1 2 0 0 27 0 1 2 0 19 0 7 1 1 1 30 0 1 3 0 0 0 0 9 1 3
       0 0 6 0 9 1 1 0 34 0 1 2 0 19 9 0 1 1 1 31 0 1 1 5 6 0 1 2 0 33 23
       0 1 2 0 0 0 6 12 2 0 19 23 0 1 3 6 0 0 20 20 1 2 6 0 0 21 1 3 6 0 0
       6 6 1 2 6 0 0 22 1 2 0 19 0 0 1 2 1 19 6 0 1 1 0 20 0 1 1 0 19 0 1
       0 0 0 11 2 0 0 0 25 1 2 0 6 0 9 15 3 0 6 0 9 6 1 2 0 0 0 9 1 2 0 0
       0 25 1 2 1 7 6 0 1 2 0 7 23 0 1 3 0 0 0 0 9 1 1 0 0 0 1 1 2 28 0 1
       1 0 0 20 1 1 0 0 32 1 2 0 0 6 0 1 2 0 0 0 0 1 2 0 0 0 6 1 1 1 29 0
       1 2 0 19 23 0 1 2 3 19 0 0 1 2 3 19 0 0 1 2 1 19 0 0 1 2 3 19 0 0 1
       2 3 19 0 0 1 1 0 7 0 8))))))
(QUOTE |lookupComplete|)))
```

28.12 REF.lsp BOOTSTRAP

REF depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **REF** category which we can write into the **MID** directory. We compile the lisp code and copy the **REF.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— REF.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |REF;=;2$B;1|) (QUOTE |SPADreplace|) (QUOTE EQ))

(DEFUN |REF;=;2$B;1| (|p| |q| |$|) (EQ |p| |q|))

(PUT (QUOTE |REF;ref;S$;2|) (QUOTE |SPADreplace|) (QUOTE LIST))

(DEFUN |REF;ref;S$;2| (|v| |$|) (LIST |v|))

(PUT (QUOTE |REF;elt;$S;3|) (QUOTE |SPADreplace|) (QUOTE QCAR))

(DEFUN |REF;elt;$S;3| (|p| |$|) (QCAR |p|))

(DEFUN |REF;setelt;$2S;4| (|p| |v| |$|) (PROGN (RPLACA |p| |v|) (QCAR |p|)))

(PUT (QUOTE |REF;deref;$S;5|) (QUOTE |SPADreplace|) (QUOTE QCAR))

(DEFUN |REF;deref;$S;5| (|p| |$|) (QCAR |p|))

(DEFUN |REF;setref;$2S;6| (|p| |v| |$|) (PROGN (RPLACA |p| |v|) (QCAR |p|)))

(DEFUN |REF;coerce;$Of;7| (|p| |$|)
 (SPADCALL
  (SPADCALL "ref" (QREFELT |$| 17))
  (LIST (SPADCALL (QCAR |p|) (QREFELT |$| 18)))
  (QREFELT |$| 20)))

(DEFUN |Reference| (G82336)
  (PROG ()
    (RETURN
      (PROG (G82337)
        (RETURN
          (COND
            ((LETT G82337
                   (|lassocShiftWithFunction|
                       (LIST (|devaluate| G82336))
                       (HGET |$ConstructorCache| '|Reference|)
                       '|domainEqualList|)
                   |Reference|)
             (|CDRwithIncrement| G82337))
            ('T
```

```
            (UNWIND-PROTECT
              (PROG1 (|Reference;| G82336)
                (LETT G82337 T |Reference|))
              (COND
               ((NOT G82337) (HREM |$ConstructorCache| '|Reference|)))))))))))

(DEFUN |Reference;| (|#1|)
  (PROG (DV$1 |dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |Reference|)
        (LETT |dv$| (LIST '|Reference| DV$1) |Reference|)
        (LETT $ (make-array 23) |Reference|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$|
                  (|buildPredVector| 0 0
                      (LIST (|HasCategory| |#1| '(|SetCategory|))))
                  |Reference|))
        (|haddProp| |$ConstructorCache| '|Reference| (LIST DV$1)
            (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 |#1|)
        (QSETREFV $ 7 (|Record| (|:| |value| |#1|)))
        (COND
          ((|testBitVector| |pv$| 1)
           (QSETREFV $ 21
               (CONS (|dispatchFunction| |REF;coerce;$Of;7|) $))))
        $))))

(setf (get
 (QUOTE |Reference|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (QUOTE |Rep|) (|Boolean|)
    |REF;=;2$B;1| |REF;ref;S$;2| |REF;elt;$S;3| |REF;setelt;$2S;4|
    |REF;deref;$S;5| |REF;setref;$2S;6| (|String|) (|OutputForm|)
    (0 . |message|) (5 . |coerce|) (|List| |$|) (10 . |prefix|)
    (16 . |coerce|) (|SingleInteger|)))
  (QUOTE #(|~=| 21 |setref| 27 |setelt| 33 |ref| 39 |latex| 44 |hash| 49
         |elt| 54 |deref| 59 |coerce| 64 |=| 69))
   (QUOTE NIL)
   (CONS
    (|makeByteWordVec2| 1 (QUOTE (1 0 1 1)))
    (CONS
     (QUOTE #(|SetCategory&| NIL |BasicType&| NIL))
     (CONS
      (QUOTE #((|SetCategory|) (|Type|) (|BasicType|) (|CoercibleTo| 16)))
      (|makeByteWordVec2| 22
```

```
      (QUOTE (1 16 0 15 17 1 6 16 0 18 2 16 0 0 19 20 1 0 16 0 21 2 1 8 0
              0 1 2 0 6 0 6 14 2 0 6 0 6 12 1 0 0 6 10 1 1 15 0 1 1 1 22
              0 1 1 0 6 0 11 1 0 6 0 13 1 1 16 0 21 2 0 8 0 0 9))))))
   (QUOTE |lookupComplete|)))
```

———-

28.13 SINT.lsp BOOTSTRAP

— SINT.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(DEFUN |SINT;writeOMSingleInt| (|dev| |x| $)
  (SEQ
    (COND
      ((QSLESSP |x| 0)
        (SEQ
          (SPADCALL |dev| (QREFELT $ 9))
          (SPADCALL |dev| "arith1" "unaryminus" (QREFELT $ 11))
          (SPADCALL |dev| (QSMINUS |x|) (QREFELT $ 13))
          (EXIT (SPADCALL |dev| (QREFELT $ 14)))))
      ((QUOTE T) (SPADCALL |dev| |x| (QREFELT $ 13))))))

(DEFUN |SINT;OMwrite;$S;2| (|x| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |SINT;OMwrite;$S;2|)
        (LETT |sp| (OM-STRINGTOSTRINGPTR |s|) |SINT;OMwrite;$S;2|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT $ 16)) (QREFELT $ 17))
          |SINT;OMwrite;$S;2|)
        (SPADCALL |dev| (QREFELT $ 18))
        (|SINT;writeOMSingleInt| |dev| |x| $)
        (SPADCALL |dev| (QREFELT $ 19))
        (SPADCALL |dev| (QREFELT $ 20))
        (LETT |s| (OM-STRINGPTRTOSTRING |sp|) |SINT;OMwrite;$S;2|)
        (EXIT |s|)))))

(DEFUN |SINT;OMwrite;$BS;3| (|x| |wholeObj| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |SINT;OMwrite;$BS;3|)
        (LETT |sp| (OM-STRINGTOSTRINGPTR |s|) |SINT;OMwrite;$BS;3|)
```

```
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT $ 16)) (QREFELT $ 17))
          |SINT;OMwrite;$BS;3|)
        (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 18))))
        (|SINT;writeOMSingleInt| |dev| |x| $)
        (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 19))))
        (SPADCALL |dev| (QREFELT $ 20))
        (LETT |s| (OM-STRINGPTRTOSTRING |sp|) |SINT;OMwrite;$BS;3|)
        (EXIT |s|)))))

(DEFUN |SINT;OMwrite;Omd$V;4| (|dev| |x| $)
  (SEQ
    (SPADCALL |dev| (QREFELT $ 18))
    (|SINT;writeOMSingleInt| |dev| |x| $)
    (EXIT (SPADCALL |dev| (QREFELT $ 19)))))

(DEFUN |SINT;OMwrite;Omd$BV;5| (|dev| |x| |wholeObj| $)
  (SEQ
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 18))))
    (|SINT;writeOMSingleInt| |dev| |x| $)
    (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 19)))))))

(PUT
  (QUOTE |SINT;reducedSystem;MM;6|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|m|) |m|)))

(DEFUN |SINT;reducedSystem;MM;6| (|m| $) |m|)

(DEFUN |SINT;coerce;$Of;7| (|x| $)
  (SPADCALL |x| (QREFELT $ 30)))

(PUT
  (QUOTE |SINT;convert;$I;8|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |SINT;convert;$I;8| (|x| $) |x|)

(DEFUN |SINT;*;I2$;9| (|i| |y| $)
  (QSTIMES (SPADCALL |i| (QREFELT $ 33)) |y|))

(PUT
  (QUOTE |SINT;Zero;$;10|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |SINT;Zero;$;10| ($) 0)

(PUT
```

```
  (QUOTE |SINT;One;$;11|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 1)))

(DEFUN |SINT;One;$;11| ($) 1)

(PUT
  (QUOTE |SINT;base;$;12|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 2)))

(DEFUN |SINT;base;$;12| ($) 2)

(PUT
  (QUOTE |SINT;max;$;13|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL MOST-POSITIVE-FIXNUM)))

(DEFUN |SINT;max;$;13| ($) MOST-POSITIVE-FIXNUM)

(PUT
  (QUOTE |SINT;min;$;14|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL MOST-NEGATIVE-FIXNUM)))

(DEFUN |SINT;min;$;14| ($) MOST-NEGATIVE-FIXNUM)

(PUT
  (QUOTE |SINT;=;2$B;15|)
  (QUOTE |SPADreplace|)
  (QUOTE EQL))

(DEFUN |SINT;=;2$B;15| (|x| |y| $)
  (EQL |x| |y|))

(PUT
  (QUOTE |SINT;~;2$;16|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGNOT))

(DEFUN |SINT;~;2$;16| (|x| $)
  (LOGNOT |x|))

(PUT
  (QUOTE |SINT;not;2$;17|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGNOT))

(DEFUN |SINT;not;2$;17| (|x| $)
  (LOGNOT |x|))
```

```
(PUT
  (QUOTE |SINT;/\\;3$;18|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGAND))

(DEFUN |SINT;/\\;3$;18| (|x| |y| $)
  (LOGAND |x| |y|))

(PUT
  (QUOTE |SINT;\\/;3$;19|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGIOR))

(DEFUN |SINT;\\/;3$;19| (|x| |y| $)
  (LOGIOR |x| |y|))

(PUT
  (QUOTE |SINT;Not;2$;20|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGNOT))

(DEFUN |SINT;Not;2$;20| (|x| $)
  (LOGNOT |x|))

(PUT
  (QUOTE |SINT;And;3$;21|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGAND))

(DEFUN |SINT;And;3$;21| (|x| |y| $)
  (LOGAND |x| |y|))

(PUT
  (QUOTE |SINT;Or;3$;22|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGIOR))

(DEFUN |SINT;Or;3$;22| (|x| |y| $)
  (LOGIOR |x| |y|))

(PUT
  (QUOTE |SINT;xor;3$;23|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGXOR))

(DEFUN |SINT;xor;3$;23| (|x| |y| $)
  (LOGXOR |x| |y|))

(PUT
```

```
  (QUOTE |SINT;<;2$B;24|)
  (QUOTE |SPADreplace|)
  (QUOTE QSLESSP))

(DEFUN |SINT;<;2$B;24| (|x| |y| $)
  (QSLESSP |x| |y|))

(PUT
  (QUOTE |SINT;inc;2$;25|)
  (QUOTE |SPADreplace|)
  (QUOTE QSADD1))

(DEFUN |SINT;inc;2$;25| (|x| $)
  (QSADD1 |x|))

(PUT
  (QUOTE |SINT;dec;2$;26|)
  (QUOTE |SPADreplace|)
  (QUOTE QSSUB1))

(DEFUN |SINT;dec;2$;26| (|x| $)
  (QSSUB1 |x|))

(PUT
  (QUOTE |SINT;-;2$;27|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMINUS))

(DEFUN |SINT;-;2$;27| (|x| $)
  (QSMINUS |x|))

(PUT
  (QUOTE |SINT;+;3$;28|)
  (QUOTE |SPADreplace|)
  (QUOTE QSPLUS))

(DEFUN |SINT;+;3$;28| (|x| |y| $)
  (QSPLUS |x| |y|))

(PUT
  (QUOTE |SINT;-;3$;29|)
  (QUOTE |SPADreplace|)
  (QUOTE QSDIFFERENCE))

(DEFUN |SINT;-;3$;29| (|x| |y| $)
  (QSDIFFERENCE |x| |y|))

(PUT
  (QUOTE |SINT;*;3$;30|)
  (QUOTE |SPADreplace|)
```

```
  (QUOTE QSTIMES))

(DEFUN |SINT;*;3$;30| (|x| |y| $)
  (QSTIMES |x| |y|))

(DEFUN |SINT;**;$Nni$;31| (|x| |n| $)
  (SPADCALL (EXPT |x| |n|) (QREFELT $ 33)))

(PUT
  (QUOTE |SINT;quo;3$;32|)
  (QUOTE |SPADreplace|)
  (QUOTE QSQUOTIENT))

(DEFUN |SINT;quo;3$;32| (|x| |y| $)
  (QSQUOTIENT |x| |y|))

(PUT
  (QUOTE |SINT;rem;3$;33|)
  (QUOTE |SPADreplace|)
  (QUOTE QSREMAINDER))

(DEFUN |SINT;rem;3$;33| (|x| |y| $)
  (QSREMAINDER |x| |y|))

(DEFUN |SINT;divide;2$R;34| (|x| |y| $)
  (CONS (QSQUOTIENT |x| |y|) (QSREMAINDER |x| |y|)))

(PUT (QUOTE |SINT;gcd;3$;35|)
  (QUOTE |SPADreplace|) (QUOTE GCD))

(DEFUN |SINT;gcd;3$;35| (|x| |y| $)
  (GCD |x| |y|))

(PUT
  (QUOTE |SINT;abs;2$;36|)
  (QUOTE |SPADreplace|)
  (QUOTE QSABSVAL))

(DEFUN |SINT;abs;2$;36| (|x| $)
  (QSABSVAL |x|))

(PUT
  (QUOTE |SINT;odd?;$B;37|)
  (QUOTE |SPADreplace|)
  (QUOTE QSODDP))

(DEFUN |SINT;odd?;$B;37| (|x| $)
  (QSODDP |x|))

(PUT
```

```
  (QUOTE |SINT;zero?;$B;38|)
  (QUOTE |SPADreplace|)
  (QUOTE QSZEROP))

(DEFUN |SINT;zero?;$B;38| (|x| $)
  (QSZEROP |x|))

(PUT
  (QUOTE |SINT;max;3$;39|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMAX))

(DEFUN |SINT;max;3$;39| (|x| |y| $)
  (QSMAX |x| |y|))

(PUT
  (QUOTE |SINT;min;3$;40|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMIN))

(DEFUN |SINT;min;3$;40| (|x| |y| $)
  (QSMIN |x| |y|))

(PUT
  (QUOTE |SINT;hash;2$;41|)
  (QUOTE |SPADreplace|)
  (QUOTE SXHASH))

(DEFUN |SINT;hash;2$;41| (|x| $)
  (SXHASH |x|))

(PUT
  (QUOTE |SINT;length;2$;42|)
  (QUOTE |SPADreplace|)
  (QUOTE INTEGER-LENGTH))

(DEFUN |SINT;length;2$;42| (|x| $)
  (INTEGER-LENGTH |x|))

(PUT
  (QUOTE |SINT;shift;3$;43|)
  (QUOTE |SPADreplace|)
  (QUOTE QSLEFTSHIFT))

(DEFUN |SINT;shift;3$;43| (|x| |n| $)
  (QSLEFTSHIFT |x| |n|))

(PUT
  (QUOTE |SINT;mulmod;4$;44|)
  (QUOTE |SPADreplace|)
```

```
  (QUOTE QSMULTMOD))

(DEFUN |SINT;mulmod;4$;44| (|a| |b| |p| $)
  (QSMULTMOD |a| |b| |p|))

(PUT
  (QUOTE |SINT;addmod;4$;45|)
  (QUOTE |SPADreplace|)
  (QUOTE QSADDMOD))

(DEFUN |SINT;addmod;4$;45| (|a| |b| |p| $)
  (QSADDMOD |a| |b| |p|))

(PUT
  (QUOTE |SINT;submod;4$;46|)
  (QUOTE |SPADreplace|)
  (QUOTE QSDIFMOD))

(DEFUN |SINT;submod;4$;46| (|a| |b| |p| $)
  (QSDIFMOD |a| |b| |p|))

(PUT
  (QUOTE |SINT;negative?;$B;47|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMINUSP))

(DEFUN |SINT;negative?;$B;47| (|x| $)
  (QSMINUSP |x|))

(PUT
  (QUOTE |SINT;reducedSystem;MVR;48|)
  (QUOTE |SPADreplace|)
  (QUOTE CONS))

(DEFUN |SINT;reducedSystem;MVR;48| (|m| |v| $)
  (CONS |m| |v|))

(DEFUN |SINT;positiveRemainder;3$;49| (|x| |n| $)
  (PROG (|r|)
    (RETURN
      (SEQ
        (LETT |r| (QSREMAINDER |x| |n|) |SINT;positiveRemainder;3$;49|)
        (EXIT
          (COND
            ((QSMINUSP |r|)
              (COND
                ((QSMINUSP |n|) (QSDIFFERENCE |x| |n|))
                ((QUOTE T) (QSPLUS |r| |n|))))
            ((QUOTE T) |r|)))))))
```

```
(DEFUN |SINT;coerce;I$;50| (|x| $)
  (SEQ
    (COND
      ((NULL (< MOST-POSITIVE-FIXNUM |x|))
        (COND ((NULL (< |x| MOST-NEGATIVE-FIXNUM)) (EXIT |x|)))))
     (EXIT (|error| "integer too large to represent in a machine word"))))

(DEFUN |SINT;random;$;51| ($)
  (SEQ
    (SETELT $ 6 (REMAINDER (TIMES 314159269 (QREFELT $ 6)) 2147483647))
    (EXIT (REMAINDER (QREFELT $ 6) 67108864))))

(PUT
  (QUOTE |SINT;random;2$;52|)
  (QUOTE |SPADreplace|)
  (QUOTE RANDOM))

(DEFUN |SINT;random;2$;52| (|n| $)
  (RANDOM |n|))

(DEFUN |SINT;unitNormal;$R;53| (|x| $)
  (COND
    ((QSLESSP |x| 0) (VECTOR -1 (QSMINUS |x|) -1))
    ((QUOTE T) (VECTOR 1 |x| 1))))

(DEFUN |SingleInteger| ()
  (PROG ()
    (RETURN
      (PROG (G1358)
        (RETURN
          (COND
            ((LETT G1358 (HGET |$ConstructorCache| '|SingleInteger|)
                   |SingleInteger|)
             (|CDRwithIncrement| (CDAR G1358)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|SingleInteger|
                                   (LIST
                                    (CONS NIL
                                     (CONS 1 (|SingleInteger;|))))))
                 (LETT G1358 T |SingleInteger|))
               (COND
                 ((NOT G1358)
                  (HREM |$ConstructorCache| '|SingleInteger|)))))))))))

(DEFUN |SingleInteger;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|SingleInteger|) |SingleInteger|)
```

```
        (LETT $ (make-array 103) |SingleInteger|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |SingleInteger|))
        (|haddProp| |$ConstructorCache| '|SingleInteger| NIL
            (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 1)
        $))))

(setf (get '|SingleInteger| '|infovec|)
    (LIST '#(NIL NIL NIL NIL NIL NIL '|seed| (|Void|)
             (|OpenMathDevice|) (0 . |OMputApp|) (|String|)
             (5 . |OMputSymbol|) (|Integer|) (12 . |OMputInteger|)
             (18 . |OMputEndApp|) (|OpenMathEncoding|)
             (23 . |OMencodingXML|) (27 . |OMopenString|)
             (33 . |OMputObject|) (38 . |OMputEndObject|)
             (43 . |OMclose|) |SINT;OMwrite;$S;2| (|Boolean|)
             |SINT;OMwrite;$BS;3| |SINT;OMwrite;Omd$V;4|
             |SINT;OMwrite;Omd$BV;5| (|Matrix| 12) (|Matrix| $)
             |SINT;reducedSystem;MM;6| (|OutputForm|) (48 . |coerce|)
             |SINT;coerce;$Of;7| |SINT;convert;$I;8| (53 . |coerce|)
             |SINT;*;I2$;9|
             (CONS IDENTITY
                   (FUNCALL (|dispatchFunction| |SINT;Zero;$;10|) $))
             (CONS IDENTITY
                   (FUNCALL (|dispatchFunction| |SINT;One;$;11|) $))
             |SINT;base;$;12| |SINT;max;$;13| |SINT;min;$;14|
             |SINT;=;2$B;15| |SINT;~;2$;16| |SINT;not;2$;17|
             |SINT;/\\;3$;18| |SINT;\\/;3$;19| |SINT;Not;2$;20|
             |SINT;And;3$;21| |SINT;Or;3$;22| |SINT;xor;3$;23|
             |SINT;<;2$B;24| |SINT;inc;2$;25| |SINT;dec;2$;26|
             |SINT;-;2$;27| |SINT;+;3$;28| |SINT;-;3$;29|
             |SINT;*;3$;30| (|NonNegativeInteger|) |SINT;**;$Nni$;31|
             |SINT;quo;3$;32| |SINT;rem;3$;33|
             (|Record| (|:| |quotient| $) (|:| |remainder| $))
             |SINT;divide;2$R;34| |SINT;gcd;3$;35| |SINT;abs;2$;36|
             |SINT;odd?;$B;37| |SINT;zero?;$B;38| |SINT;max;3$;39|
             |SINT;min;3$;40| |SINT;hash;2$;41| |SINT;length;2$;42|
             |SINT;shift;3$;43| |SINT;mulmod;4$;44| |SINT;addmod;4$;45|
             |SINT;submod;4$;46| |SINT;negative?;$B;47|
             (|Record| (|:| |mat| 26) (|:| |vec| (|Vector| 12)))
             (|Vector| $) |SINT;reducedSystem;MVR;48|
             |SINT;positiveRemainder;3$;49| |SINT;coerce;I$;50|
             |SINT;random;$;51| |SINT;random;2$;52|
             (|Record| (|:| |unit| $) (|:| |canonical| $)
                 (|:| |associate| $))
             |SINT;unitNormal;$R;53| (|Union| 85 '"failed")
             (|Fraction| 12) (|Union| $ '"failed") (|Float|)
             (|DoubleFloat|) (|Pattern| 12) (|PatternMatchResult| 12 $)
```

```
    (|InputForm|) (|Union| 12 '"failed")
    (|Record| (|:| |coef| 94) (|:| |generator| $)) (|List| $)
    (|Union| 94 '"failed")
    (|Record| (|:| |coef1| $) (|:| |coef2| $)
       (|:| |generator| $))
    (|Record| (|:| |coef1| $) (|:| |coef2| $))
    (|Union| 97 '"failed") (|Factored| $)
    (|SparseUnivariatePolynomial| $) (|PositiveInteger|)
    (|SingleInteger|))
 '#(~= 58 ~ 64 |zero?| 69 |xor| 74 |unitNormal| 80
    |unitCanonical| 85 |unit?| 90 |symmetricRemainder| 95
    |subtractIfCan| 101 |submod| 107 |squareFreePart| 114
    |squareFree| 119 |sizeLess?| 124 |sign| 130 |shift| 135
    |sample| 141 |retractIfCan| 145 |retract| 150 |rem| 155
    |reducedSystem| 161 |recip| 172 |rationalIfCan| 177
    |rational?| 182 |rational| 187 |random| 192 |quo| 201
    |principalIdeal| 207 |prime?| 212 |powmod| 217
    |positiveRemainder| 224 |positive?| 230 |permutation| 235
    |patternMatch| 241 |one?| 248 |odd?| 253 |not| 258
    |nextItem| 263 |negative?| 268 |multiEuclidean| 273
    |mulmod| 279 |min| 286 |max| 296 |mask| 306 |length| 311
    |lcm| 316 |latex| 327 |invmod| 332 |init| 338 |inc| 342
    |hash| 347 |gcdPolynomial| 357 |gcd| 363 |factorial| 374
    |factor| 379 |extendedEuclidean| 384 |exquo| 397
    |expressIdealMember| 403 |even?| 409 |euclideanSize| 414
    |divide| 419 |differentiate| 425 |dec| 436 |copy| 441
    |convert| 446 |coerce| 471 |characteristic| 491 |bit?| 495
    |binomial| 501 |base| 507 |associates?| 511 |addmod| 517
    |abs| 524 ^ 529 |\\/| 541 |Zero| 547 |Or| 551 |One| 557
    |OMwrite| 561 |Not| 585 D 590 |And| 601 >= 607 > 613 = 619
    <= 625 < 631 |/\\| 637 - 643 + 654 ** 660 * 672)
 '((|noetherian| . 0) (|canonicalsClosed| . 0)
   (|canonical| . 0) (|canonicalUnitNormal| . 0)
   (|multiplicativeValuation| . 0) (|noZeroDivisors| . 0)
   ((|commutative| "*") . 0) (|rightUnitary| . 0)
   (|leftUnitary| . 0) (|unitsKnown| . 0))
 (CONS (|makeByteWordVec2| 1
          '(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
            0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
            0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
            0))
       (CONS '#(|IntegerNumberSystem&| |EuclideanDomain&|
                |UniqueFactorizationDomain&| NIL NIL
                |GcdDomain&| |IntegralDomain&| |Algebra&|
                |Module&| NIL |Module&| NIL NIL |Module&| NIL
                |DifferentialRing&| |OrderedRing&| NIL
                |Module&| NIL |Module&| NIL NIL NIL NIL NIL
                NIL |Ring&| NIL NIL NIL NIL NIL NIL NIL NIL
                NIL NIL NIL NIL NIL |AbelianGroup&| NIL NIL
                |AbelianMonoid&| |Monoid&| NIL NIL NIL NIL
```

```
                |OrderedSet&| |AbelianSemiGroup&| |SemiGroup&|
                |Logic&| NIL |SetCategory&| NIL NIL NIL NIL
                |RetractableTo&| NIL NIL NIL |RetractableTo&|
                NIL NIL NIL NIL NIL NIL |RetractableTo&| NIL
                |BasicType&| NIL)
             (CONS '#((|IntegerNumberSystem|)
                      (|EuclideanDomain|)
                      (|UniqueFactorizationDomain|)
                      (|PrincipalIdealDomain|)
                      (|OrderedIntegralDomain|) (|GcdDomain|)
                      (|IntegralDomain|) (|Algebra| $$)
                      (|Module| 12)
                      (|LinearlyExplicitRingOver| 12)
                      (|Module| G1062)
                      (|LinearlyExplicitRingOver| G1062)
                      (|CharacteristicZero|)
                      (|Module| G106217)
                      (|LinearlyExplicitRingOver| G106217)
                      (|DifferentialRing|) (|OrderedRing|)
                      (|CommutativeRing|) (|Module| |t#1|)
                      (|EntireRing|) (|Module| $$)
                      (|BiModule| 12 12)
                      (|BiModule| G1062 G1062)
                      (|BiModule| G106217 G106217)
                      (|OrderedAbelianGroup|)
                      (|BiModule| |t#1| |t#1|)
                      (|BiModule| $$ $$) (|Ring|)
                      (|RightModule| 12) (|LeftModule| 12)
                      (|RightModule| G1062)
                      (|LeftModule| G1062)
                      (|RightModule| G106217)
                      (|LeftModule| G106217)
                      (|OrderedCancellationAbelianMonoid|)
                      (|RightModule| |t#1|)
                      (|LeftModule| |t#1|) (|LeftModule| $$)
                      (|Rng|) (|RightModule| $$)
                      (|OrderedAbelianMonoid|)
                      (|AbelianGroup|)
                      (|OrderedAbelianSemiGroup|)
                      (|CancellationAbelianMonoid|)
                      (|AbelianMonoid|) (|Monoid|)
                      (|PatternMatchable| 12)
                      (|PatternMatchable| G1065)
                      (|StepThrough|)
                      (|PatternMatchable| G106220)
                      (|OrderedSet|) (|AbelianSemiGroup|)
                      (|SemiGroup|) (|Logic|) (|RealConstant|)
                      (|SetCategory|) (|OpenMath|)
                      (|CoercibleTo| G82356)
                      (|ConvertibleTo| 89)
```

```
    (|ConvertibleTo| 91)
    (|RetractableTo| 12)
    (|ConvertibleTo| 12)
    (|ConvertibleTo| G1064)
    (|ConvertibleTo| G1063)
    (|RetractableTo| G1061)
    (|ConvertibleTo| G1060)
    (|ConvertibleTo| 87)
    (|ConvertibleTo| 88)
    (|CombinatorialFunctionCategory|)
    (|ConvertibleTo| G106219)
    (|ConvertibleTo| G106218)
    (|RetractableTo| G106216)
    (|ConvertibleTo| G106215) (|BasicType|)
    (|CoercibleTo| 29))
 (|makeByteWordVec2| 102
     '(1 8 7 0 9 3 8 7 0 10 10 11 2 8 7 0 12
       13 1 8 7 0 14 0 15 0 16 2 8 0 10 15
       17 1 8 7 0 18 1 8 7 0 19 1 8 7 0 20 1
       12 29 0 30 1 0 0 12 33 2 0 22 0 0 1 1
       0 0 0 41 1 0 22 0 65 2 0 0 0 0 48 1 0
       82 0 83 1 0 0 0 1 1 0 22 0 1 2 0 0 0
       0 1 2 0 86 0 0 1 3 0 0 0 0 0 73 1 0 0
       0 1 1 0 99 0 1 2 0 22 0 0 1 1 0 12 0
       1 2 0 0 0 0 70 0 0 0 1 1 0 92 0 1 1 0
       12 0 1 2 0 0 0 0 59 1 0 26 27 28 2 0
       75 27 76 77 1 0 86 0 1 1 0 84 0 1 1 0
       22 0 1 1 0 85 0 1 1 0 0 0 81 0 0 0 80
       2 0 0 0 0 58 1 0 93 94 1 1 0 22 0 1 3
       0 0 0 0 0 1 2 0 0 0 0 78 1 0 22 0 1 2
       0 0 0 0 1 3 0 90 0 89 90 1 1 0 22 0 1
       1 0 22 0 64 1 0 0 0 42 1 0 86 0 1 1 0
       22 0 74 2 0 95 94 0 1 3 0 0 0 0 0 71
       0 0 0 39 2 0 0 0 0 67 0 0 0 38 2 0 0
       0 0 66 1 0 0 0 1 1 0 0 0 69 1 0 0 94
       1 2 0 0 0 0 1 1 0 10 0 1 2 0 0 0 0 1
       0 0 0 1 1 0 0 0 50 1 0 0 0 68 1 0 102
       0 1 2 0 100 100 100 1 1 0 0 94 1 2 0
       0 0 0 62 1 0 0 0 1 1 0 99 0 1 2 0 96
       0 0 1 3 0 98 0 0 0 1 2 0 86 0 0 1 2 0
       95 94 0 1 1 0 22 0 1 1 0 56 0 1 2 0
       60 0 0 61 1 0 0 0 1 2 0 0 0 56 1 1 0
       0 0 51 1 0 0 0 1 1 0 87 0 1 1 0 88 0
       1 1 0 89 0 1 1 0 91 0 1 1 0 12 0 32 1
       0 0 12 79 1 0 0 0 1 1 0 0 12 79 1 0
       29 0 31 0 0 56 1 2 0 22 0 0 1 2 0 0 0
       0 1 0 0 0 37 2 0 22 0 0 1 3 0 0 0 0 0
       72 1 0 0 0 63 2 0 0 0 56 1 2 0 0 0
       101 1 2 0 0 0 0 44 0 0 0 35 2 0 0 0 0
       47 0 0 0 36 3 0 7 8 0 22 25 2 0 10 0
```

```
                                 22 23 2 0 7 8 0 24 1 0 10 0 21 1 0 0
                                 0 45 1 0 0 0 1 2 0 0 0 56 1 2 0 0 0 0
                                 46 2 0 22 0 0 1 2 0 22 0 0 1 2 0 22 0
                                 0 40 2 0 22 0 0 1 2 0 22 0 0 49 2 0 0
                                 0 0 43 1 0 0 0 52 2 0 0 0 0 54 2 0 0
                                 0 0 53 2 0 0 0 56 57 2 0 0 0 101 1 2
                                 0 0 0 0 55 2 0 0 12 0 34 2 0 0 56 0 1
                                 2 0 0 101 0 1)))))
          '|lookupComplete|))

(setf (get (QUOTE |SingleInteger|) (QUOTE NILADIC)) T)
```

28.14 SYMBOL.lsp BOOTSTRAP

SYMBOL depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SYMBOL** category which we can write into the **MID** directory. We compile the lisp code and copy the **SYMBOL.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— SYMBOL.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |SYMBOL;writeOMSym| (|dev| |x| $)
  (COND
    ((SPADCALL |x| (QREFELT $ 21))
     (|error| "Cannot convert a scripted symbol to OpenMath"))
    ('T (SPADCALL |dev| |x| (QREFELT $ 25)))))

(DEFUN |SYMBOL;OMwrite;$S;2| (|x| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ (LETT |s| "" |SYMBOL;OMwrite;$S;2|)
           (LETT |sp| (OM-STRINGTOSTRINGPTR |s|) |SYMBOL;OMwrite;$S;2|)
           (LETT |dev|
                 (SPADCALL |sp| (SPADCALL (QREFELT $ 27))
                     (QREFELT $ 29))
                 |SYMBOL;OMwrite;$S;2|)
           (SPADCALL |dev| (QREFELT $ 30))
           (|SYMBOL;writeOMSym| |dev| |x| $)
           (SPADCALL |dev| (QREFELT $ 31))
           (SPADCALL |dev| (QREFELT $ 32))
           (LETT |s| (OM-STRINGPTRTOSTRING |sp|) |SYMBOL;OMwrite;$S;2|)
```

```
          (EXIT |s|)))))

(DEFUN |SYMBOL;OMwrite;$BS;3| (|x| |wholeObj| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ (LETT |s| "" |SYMBOL;OMwrite;$BS;3|)
           (LETT |sp| (OM-STRINGTOSTRINGPTR |s|)
                 |SYMBOL;OMwrite;$BS;3|)
           (LETT |dev|
                 (SPADCALL |sp| (SPADCALL (QREFELT $ 27))
                     (QREFELT $ 29))
                 |SYMBOL;OMwrite;$BS;3|)
           (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 30))))
           (|SYMBOL;writeOMSym| |dev| |x| $)
           (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 31))))
           (SPADCALL |dev| (QREFELT $ 32))
           (LETT |s| (OM-STRINGPTRTOSTRING |sp|)
                 |SYMBOL;OMwrite;$BS;3|)
           (EXIT |s|)))))

(DEFUN |SYMBOL;OMwrite;Omd$V;4| (|dev| |x| $)
  (SEQ (SPADCALL |dev| (QREFELT $ 30))
       (|SYMBOL;writeOMSym| |dev| |x| $)
       (EXIT (SPADCALL |dev| (QREFELT $ 31)))))

(DEFUN |SYMBOL;OMwrite;Omd$BV;5| (|dev| |x| |wholeObj| $)
  (SEQ (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 30))))
       (|SYMBOL;writeOMSym| |dev| |x| $)
       (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 31)))))))

(DEFUN |SYMBOL;convert;$If;6| (|s| |$|) (SPADCALL |s| (QREFELT |$| 44)))

(PUT '|SYMBOL;convert;2$;7| '|SPADreplace| '(XLAM (|s|) |s|))

(DEFUN |SYMBOL;convert;2$;7| (|s| |$|) |s|)

(DEFUN |SYMBOL;coerce;S$;8| (|s| |$|) (VALUES (INTERN |s|)))

(PUT '|SYMBOL;=;2$B;9| '|SPADreplace| 'EQUAL)

(DEFUN |SYMBOL;=;2$B;9| (|x| |y| |$|) (EQUAL |x| |y|))

(PUT '|SYMBOL;<;2$B;10| '|SPADreplace|
     '(XLAM (|x| |y|) (GGREATERP |y| |x|)))

(DEFUN |SYMBOL;<;2$B;10| (|x| |y| |$|) (GGREATERP |y| |x|))

(DEFUN |SYMBOL;coerce;$Of;11| (|x| |$|) (SPADCALL |x| (QREFELT |$| 51)))

(DEFUN |SYMBOL;subscript;$L$;12| (|sy| |lx| $)
```

```
  (SPADCALL |sy| (LIST |lx| NIL NIL NIL NIL) (QREFELT $ 54)))

(DEFUN |SYMBOL;elt;$L$;13| (|sy| |lx| $)
  (SPADCALL |sy| |lx| (QREFELT $ 56)))

(DEFUN |SYMBOL;superscript;$L$;14| (|sy| |lx| $)
  (SPADCALL |sy| (LIST NIL |lx| NIL NIL NIL) (QREFELT $ 54)))

(DEFUN |SYMBOL;argscript;$L$;15| (|sy| |lx| $)
  (SPADCALL |sy| (LIST NIL NIL NIL NIL |lx|) (QREFELT $ 54)))

(DEFUN |SYMBOL;patternMatch;$P2Pmr;16| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 63)))

(DEFUN |SYMBOL;patternMatch;$P2Pmr;17| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 69)))

(DEFUN |SYMBOL;convert;$P;18| (|x| |$|) (SPADCALL |x| (QREFELT |$| 72)))

(DEFUN |SYMBOL;convert;$P;19| (|x| |$|) (SPADCALL |x| (QREFELT |$| 74)))

(DEFUN |SYMBOL;syprefix| (|sc| $)
  (PROG (|ns| G108218 |n| G108219)
    (RETURN
      (SEQ (LETT |ns|
                 (LIST (LENGTH (QVELT |sc| 3)) (LENGTH (QVELT |sc| 2))
                       (LENGTH (QVELT |sc| 1)) (LENGTH (QVELT |sc| 0)))
                 |SYMBOL;syprefix|)
           (SEQ G190
                (COND
                  ((NULL (COND
                           ((< (LENGTH |ns|) 2) 'NIL)
                           ('T (ZEROP (|SPADfirst| |ns|)))))
                   (GO G191)))
                (SEQ (EXIT (LETT |ns| (CDR |ns|) |SYMBOL;syprefix|)))
                NIL (GO G190) G191 (EXIT NIL))
           (EXIT (SPADCALL
                     (CONS (STRCONC (QREFELT $ 37)
                                    (|SYMBOL;istring|
                                     (LENGTH (QVELT |sc| 4)) $))
                           (PROGN
                             (LETT G108218 NIL |SYMBOL;syprefix|)
                             (SEQ (LETT |n| NIL |SYMBOL;syprefix|)
                                  (LETT G108219 (NREVERSE |ns|)
                                        |SYMBOL;syprefix|)
                                  G190
                                  (COND
                                    ((OR (ATOM G108219)
                                      (PROGN
                                        (LETT |n| (CAR G108219)
```

```
                                        |SYMBOL;syprefix|)
                                       NIL))
                                    (GO G191)))
                                 (SEQ (EXIT
                                       (LETT G108218
                                        (CONS (|SYMBOL;istring| |n| $)
                                         G108218)
                                        |SYMBOL;syprefix|)))
                                 (LETT G108219 (CDR G108219)
                                       |SYMBOL;syprefix|)
                                 (GO G190) G191
                                 (EXIT (NREVERSE0 G108218)))))
                    (QREFELT $ 77)))))))

(DEFUN |SYMBOL;syscripts| (|sc| $)
  (PROG (|all|)
    (RETURN
      (SEQ (LETT |all| (QVELT |sc| 3) |SYMBOL;syscripts|)
           (LETT |all| (SPADCALL (QVELT |sc| 2) |all| (QREFELT $ 78))
                 |SYMBOL;syscripts|)
           (LETT |all| (SPADCALL (QVELT |sc| 1) |all| (QREFELT $ 78))
                 |SYMBOL;syscripts|)
           (LETT |all| (SPADCALL (QVELT |sc| 0) |all| (QREFELT $ 78))
                 |SYMBOL;syscripts|)
           (EXIT (SPADCALL |all| (QVELT |sc| 4) (QREFELT $ 78)))))))

(DEFUN |SYMBOL;script;$L$;22| (|sy| |ls| $)
  (PROG (|sc|)
    (RETURN
      (SEQ (LETT |sc| (VECTOR NIL NIL NIL NIL NIL)
                 |SYMBOL;script;$L$;22|)
           (COND
             ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 0 (|SPADfirst| |ls|))
                   (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|)))))
           (COND
             ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 1 (|SPADfirst| |ls|))
                   (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|)))))
           (COND
             ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 2 (|SPADfirst| |ls|))
                   (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|)))))
           (COND
             ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 3 (|SPADfirst| |ls|))
                   (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|)))))
           (COND
             ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 4 (|SPADfirst| |ls|))
```

```
                   (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|)))))
           (EXIT (SPADCALL |sy| |sc| (QREFELT $ 80)))))))

(DEFUN |SYMBOL;script;$R$;23| (|sy| |sc| $)
  (COND
    ((SPADCALL |sy| (QREFELT $ 21))
     (|error| "Cannot add scripts to a scripted symbol"))
    ('T
     (CONS (SPADCALL
               (SPADCALL
                   (STRCONC (|SYMBOL;syprefix| |sc| $)
                            (SPADCALL (SPADCALL |sy| (QREFELT $ 81))
                                (QREFELT $ 82)))
                   (QREFELT $ 47))
               (QREFELT $ 52))
           (|SYMBOL;syscripts| |sc| $)))))

(DEFUN |SYMBOL;string;$S;24| (|e| $)
  (COND
    ((NULL (SPADCALL |e| (QREFELT $ 21))) (PNAME |e|))
    ('T (|error| "Cannot form string from non-atomic symbols."))))

(DEFUN |SYMBOL;latex;$S;25| (|e| $)
  (PROG (|ss| |lo| |sc| |s|)
    (RETURN
      (SEQ (LETT |s| (PNAME (SPADCALL |e| (QREFELT $ 81)))
                 |SYMBOL;latex;$S;25|)
           (COND
             ((< 1 (QCSIZE |s|))
              (COND
                ((NULL (SPADCALL (SPADCALL |s| 1 (QREFELT $ 83))
                           (SPADCALL "\\" (QREFELT $ 40))
                           (QREFELT $ 84)))
                 (LETT |s| (STRCONC "\\mbox{\\it " (STRCONC |s| "}"))
                       |SYMBOL;latex;$S;25|)))))
           (COND ((NULL (SPADCALL |e| (QREFELT $ 21))) (EXIT |s|)))
           (LETT |ss| (SPADCALL |e| (QREFELT $ 85))
                 |SYMBOL;latex;$S;25|)
           (LETT |lo| (QVELT |ss| 0) |SYMBOL;latex;$S;25|)
           (COND
             ((NULL (NULL |lo|))
              (SEQ (LETT |sc| "_{" |SYMBOL;latex;$S;25|)
                   (SEQ G190
                        (COND
                          ((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
                           (GO G191)))
                        (SEQ (LETT |sc|
                                   (STRCONC |sc|
                                    (SPADCALL (|SPADfirst| |lo|)
                                     (QREFELT $ 86)))
```

```
                          |SYMBOL;latex;$S;25|)
                    (LETT |lo| (CDR |lo|)
                          |SYMBOL;latex;$S;25|)
                    (EXIT (COND
                            ((NULL (NULL |lo|))
                             (LETT |sc| (STRCONC |sc| ", ")
                              |SYMBOL;latex;$S;25|)))))
               NIL (GO G190) G191 (EXIT NIL))
          (LETT |sc| (STRCONC |sc| "}") |SYMBOL;latex;$S;25|)
          (EXIT (LETT |s| (STRCONC |s| |sc|)
                      |SYMBOL;latex;$S;25|)))))
  (LETT |lo| (QVELT |ss| 1) |SYMBOL;latex;$S;25|)
  (COND
    ((NULL (NULL |lo|))
     (SEQ (LETT |sc| "^{" |SYMBOL;latex;$S;25|)
          (SEQ G190
               (COND
                 ((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
                  (GO G191)))
               (SEQ (LETT |sc|
                          (STRCONC |sc|
                           (SPADCALL (|SPADfirst| |lo|)
                            (QREFELT $ 86)))
                          |SYMBOL;latex;$S;25|)
                    (LETT |lo| (CDR |lo|)
                          |SYMBOL;latex;$S;25|)
                    (EXIT (COND
                            ((NULL (NULL |lo|))
                             (LETT |sc| (STRCONC |sc| ", ")
                              |SYMBOL;latex;$S;25|)))))
               NIL (GO G190) G191 (EXIT NIL))
          (LETT |sc| (STRCONC |sc| "}") |SYMBOL;latex;$S;25|)
          (EXIT (LETT |s| (STRCONC |s| |sc|)
                      |SYMBOL;latex;$S;25|)))))
  (LETT |lo| (QVELT |ss| 2) |SYMBOL;latex;$S;25|)
  (COND
    ((NULL (NULL |lo|))
     (SEQ (LETT |sc| "{}^{" |SYMBOL;latex;$S;25|)
          (SEQ G190
               (COND
                 ((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
                  (GO G191)))
               (SEQ (LETT |sc|
                          (STRCONC |sc|
                           (SPADCALL (|SPADfirst| |lo|)
                            (QREFELT $ 86)))
                          |SYMBOL;latex;$S;25|)
                    (LETT |lo| (CDR |lo|)
                          |SYMBOL;latex;$S;25|)
                    (EXIT (COND
```

```
                            ((NULL (NULL |lo|))
                             (LETT |sc| (STRCONC |sc| ", ")
                              |SYMBOL;latex;$S;25|)))))
              NIL (GO G190) G191 (EXIT NIL))
         (LETT |sc| (STRCONC |sc| "}") |SYMBOL;latex;$S;25|)
         (EXIT (LETT |s| (STRCONC |sc| |s|)
                     |SYMBOL;latex;$S;25|)))))
(LETT |lo| (QVELT |ss| 3) |SYMBOL;latex;$S;25|)
(COND
  ((NULL (NULL |lo|))
   (SEQ (LETT |sc| "{}_{" |SYMBOL;latex;$S;25|)
        (SEQ G190
             (COND
               ((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
                (GO G191)))
             (SEQ (LETT |sc|
                        (STRCONC |sc|
                         (SPADCALL (|SPADfirst| |lo|)
                          (QREFELT $ 86)))
                        |SYMBOL;latex;$S;25|)
                  (LETT |lo| (CDR |lo|)
                        |SYMBOL;latex;$S;25|)
                  (EXIT (COND
                          ((NULL (NULL |lo|))
                           (LETT |sc| (STRCONC |sc| ", ")
                            |SYMBOL;latex;$S;25|)))))
              NIL (GO G190) G191 (EXIT NIL))
         (LETT |sc| (STRCONC |sc| "}") |SYMBOL;latex;$S;25|)
         (EXIT (LETT |s| (STRCONC |sc| |s|)
                     |SYMBOL;latex;$S;25|)))))
(LETT |lo| (QVELT |ss| 4) |SYMBOL;latex;$S;25|)
(COND
  ((NULL (NULL |lo|))
   (SEQ (LETT |sc| "\\left( {" |SYMBOL;latex;$S;25|)
        (SEQ G190
             (COND
               ((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
                (GO G191)))
             (SEQ (LETT |sc|
                        (STRCONC |sc|
                         (SPADCALL (|SPADfirst| |lo|)
                          (QREFELT $ 86)))
                        |SYMBOL;latex;$S;25|)
                  (LETT |lo| (CDR |lo|)
                        |SYMBOL;latex;$S;25|)
                  (EXIT (COND
                          ((NULL (NULL |lo|))
                           (LETT |sc| (STRCONC |sc| ", ")
                            |SYMBOL;latex;$S;25|)))))
              NIL (GO G190) G191 (EXIT NIL))
```

```
                   (LETT |sc| (STRCONC |sc| "} \\right)")
                        |SYMBOL;latex;$S;25|)
                   (EXIT (LETT |s| (STRCONC |s| |sc|)
                               |SYMBOL;latex;$S;25|)))))
           (EXIT |s|)))))

(DEFUN |SYMBOL;anyRadix| (|n| |s| $)
  (PROG (|qr| |ns| G108274)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |ns| "" |SYMBOL;anyRadix|)
                      (EXIT (SEQ G190 NIL
                                 (SEQ (LETT |qr|
                                       (DIVIDE2 |n| (QCSIZE |s|))
                                       |SYMBOL;anyRadix|)
                                      (LETT |n| (QCAR |qr|)
                                       |SYMBOL;anyRadix|)
                                      (LETT |ns|
                                       (SPADCALL
                                        (SPADCALL |s|
                                         (+ (QCDR |qr|)
                                          (SPADCALL |s| (QREFELT $ 88)))
                                         (QREFELT $ 83))
                                        |ns| (QREFELT $ 89))
                                       |SYMBOL;anyRadix|)
                                      (EXIT
                                       (COND
                                         ((ZEROP |n|)
                                          (PROGN
                                            (LETT G108274 |ns|
                                             |SYMBOL;anyRadix|)
                                            (GO G108274))))))
                                 NIL (GO G190) G191 (EXIT NIL)))))
           G108274 (EXIT G108274)))))

(DEFUN |SYMBOL;new;$;27| ($)
  (PROG (|sym|)
    (RETURN
      (SEQ (LETT |sym|
                 (|SYMBOL;anyRadix|
                     (SPADCALL (QREFELT $ 9) (QREFELT $ 90))
                     (QREFELT $ 18) $)
                 |SYMBOL;new;$;27|)
           (SPADCALL (QREFELT $ 9)
               (+ (SPADCALL (QREFELT $ 9) (QREFELT $ 90)) 1)
               (QREFELT $ 91))
           (EXIT (SPADCALL (STRCONC "%" |sym|) (QREFELT $ 47)))))))

(DEFUN |SYMBOL;new;2$;28| (|x| $)
  (PROG (|u| |n| |xx|)
    (RETURN
```

```
      (SEQ (LETT |n|
                 (SEQ (LETT |u|
                            (SPADCALL |x| (QREFELT $ 12)
                                (QREFELT $ 94))
                            |SYMBOL;new;2$;28|)
                      (EXIT (COND
                              ((QEQCAR |u| 1) 0)
                              ('T (+ (QCDR |u|) 1)))))
                 |SYMBOL;new;2$;28|)
           (SPADCALL (QREFELT $ 12) |x| |n| (QREFELT $ 95))
           (LETT |xx|
                 (COND
                   ((NULL (SPADCALL |x| (QREFELT $ 21)))
                    (SPADCALL |x| (QREFELT $ 82)))
                   ('T
                    (SPADCALL (SPADCALL |x| (QREFELT $ 81))
                        (QREFELT $ 82))))
                 |SYMBOL;new;2$;28|)
           (LETT |xx| (STRCONC "%" |xx|) |SYMBOL;new;2$;28|)
           (LETT |xx|
                 (COND
                   ((NULL (< (SPADCALL
                                 (SPADCALL |xx|
                                     (SPADCALL |xx| (QREFELT $ 96))
                                     (QREFELT $ 83))
                                 (QREFELT $ 17) (QREFELT $ 97))
                             (SPADCALL (QREFELT $ 17) (QREFELT $ 88))))
                    (STRCONC |xx|
                             (|SYMBOL;anyRadix| |n| (QREFELT $ 19) $)))
                   ('T
                    (STRCONC |xx|
                             (|SYMBOL;anyRadix| |n| (QREFELT $ 17) $))))
                 |SYMBOL;new;2$;28|)
           (COND
             ((NULL (SPADCALL |x| (QREFELT $ 21)))
              (EXIT (SPADCALL |xx| (QREFELT $ 47)))))
           (EXIT (SPADCALL (SPADCALL |xx| (QREFELT $ 47))
                     (SPADCALL |x| (QREFELT $ 85)) (QREFELT $ 80))))))

(DEFUN |SYMBOL;resetNew;V;29| ($)
  (PROG (|k| G108297)
    (RETURN
      (SEQ (SPADCALL (QREFELT $ 9) 0 (QREFELT $ 91))
           (SEQ (LETT |k| NIL |SYMBOL;resetNew;V;29|)
                (LETT G108297 (SPADCALL (QREFELT $ 12) (QREFELT $ 100))
                      |SYMBOL;resetNew;V;29|)
                G190
                (COND
                  ((OR (ATOM G108297)
                       (PROGN
```

```
                         (LETT |k| (CAR G108297)
                               |SYMBOL;resetNew;V;29|)
                         NIL))
                   (GO G191)))
                (SEQ (EXIT (SPADCALL |k| (QREFELT $ 12)
                               (QREFELT $ 101))))
                (LETT G108297 (CDR G108297) |SYMBOL;resetNew;V;29|)
                (GO G190) G191 (EXIT NIL))
           (EXIT (SPADCALL (QREFELT $ 102)))))))

(DEFUN |SYMBOL;scripted?;$B;30| (|sy| $)
  (COND ((ATOM |sy|) 'NIL) ('T 'T)))

(DEFUN |SYMBOL;name;2$;31| (|sy| $)
  (PROG (|str| |i| G108304 G108303 G108301)
    (RETURN
      (SEQ (EXIT (COND
                   ((NULL (SPADCALL |sy| (QREFELT $ 21))) |sy|)
                   ('T
                    (SEQ (LETT |str|
                               (SPADCALL
                                   (SPADCALL
                                    (SPADCALL |sy| (QREFELT $ 104))
                                    (QREFELT $ 105))
                                   (QREFELT $ 82))
                               |SYMBOL;name;2$;31|)
                         (SEQ (EXIT (SEQ
                                     (LETT |i| (+ (QREFELT $ 38) 1)
                                      |SYMBOL;name;2$;31|)
                                     (LETT G108304 (QCSIZE |str|)
                                      |SYMBOL;name;2$;31|)
                                     G190
                                     (COND
                                       ((> |i| G108304) (GO G191)))
                                     (SEQ
                                      (EXIT
                                       (COND
                                         ((NULL
                                           (SPADCALL
                                            (SPADCALL |str| |i|
                                             (QREFELT $ 83))
                                            (QREFELT $ 106)))
                                          (PROGN
                                            (LETT G108301
                                             (PROGN
                                               (LETT G108303
                                                (SPADCALL
                                                 (SPADCALL |str|
                                                  (SPADCALL |i|
                                                   (QCSIZE |str|)
```

```
                                                   (QREFELT $ 108))
                                                  (QREFELT $ 109))
                                                 (QREFELT $ 47))
                                                |SYMBOL;name;2$;31|)
                                               (GO G108303))
                                              |SYMBOL;name;2$;31|)
                                             (GO G108301))))))
                                      (LETT |i| (+ |i| 1)
                                       |SYMBOL;name;2$;31|)
                                      (GO G190) G191 (EXIT NIL)))
                               G108301 (EXIT G108301))
                          (EXIT (|error| "Improper scripted symbol"))))))
           G108303 (EXIT G108303)))))

(DEFUN |SYMBOL;scripts;$R;32| (|sy| $)
  (PROG (|lscripts| |str| |nstr| |j| G108307 |nscripts| |m| |n| G108316
            |i| G108317 |a| G108318 |allscripts|)
    (RETURN
      (SEQ (COND
             ((NULL (SPADCALL |sy| (QREFELT $ 21)))
              (VECTOR NIL NIL NIL NIL NIL))
             ('T
              (SEQ (LETT |nscripts| (LIST 0 0 0 0 0)
                         |SYMBOL;scripts;$R;32|)
                   (LETT |lscripts| (LIST NIL NIL NIL NIL NIL)
                         |SYMBOL;scripts;$R;32|)
                   (LETT |str|
                         (SPADCALL
                             (SPADCALL (SPADCALL |sy| (QREFELT $ 104))
                                 (QREFELT $ 105))
                             (QREFELT $ 82))
                         |SYMBOL;scripts;$R;32|)
                   (LETT |nstr| (QCSIZE |str|) |SYMBOL;scripts;$R;32|)
                   (LETT |m| (SPADCALL |nscripts| (QREFELT $ 111))
                         |SYMBOL;scripts;$R;32|)
                   (SEQ (LETT |j| (+ (QREFELT $ 38) 1)
                              |SYMBOL;scripts;$R;32|)
                        (LETT |i| |m| |SYMBOL;scripts;$R;32|) G190
                        (COND
                          ((OR (> |j| |nstr|)
                               (NULL (SPADCALL
                                      (SPADCALL |str| |j|
                                       (QREFELT $ 83))
                                      (QREFELT $ 106))))
                           (GO G191)))
                        (SEQ (EXIT (SPADCALL |nscripts| |i|
                                        (PROG1
                                         (LETT G108307
                                          (-
                                           (SPADCALL
```

```
                         (SPADCALL |str| |j|
                          (QREFELT $ 83))
                         (QREFELT $ 41))
                        (QREFELT $ 42))
                       |SYMBOL;scripts;$R;32|)
                       (|check-subtype| (>= G108307 0)
                        '(|NonNegativeInteger|) G108307))
                     (QREFELT $ 113))))
     (LETT |i|
           (PROG1 (+ |i| 1)
             (LETT |j| (+ |j| 1)
                   |SYMBOL;scripts;$R;32|))
           |SYMBOL;scripts;$R;32|)
     (GO G190) G191 (EXIT NIL))
(LETT |nscripts|
      (SPADCALL (CDR |nscripts|)
          (|SPADfirst| |nscripts|) (QREFELT $ 114))
      |SYMBOL;scripts;$R;32|)
(LETT |allscripts|
      (SPADCALL (SPADCALL |sy| (QREFELT $ 104))
          (QREFELT $ 115))
      |SYMBOL;scripts;$R;32|)
(LETT |m| (SPADCALL |lscripts| (QREFELT $ 116))
      |SYMBOL;scripts;$R;32|)
(SEQ (LETT |n| NIL |SYMBOL;scripts;$R;32|)
     (LETT G108316 |nscripts|
           |SYMBOL;scripts;$R;32|)
     (LETT |i| |m| |SYMBOL;scripts;$R;32|) G190
     (COND
       ((OR (ATOM G108316)
            (PROGN
              (LETT |n| (CAR G108316)
                    |SYMBOL;scripts;$R;32|)
              NIL))
        (GO G191)))
     (SEQ (EXIT (COND
                  ((<
                    (SPADCALL |allscripts|
                     (QREFELT $ 117))
                    |n|)
                   (|error|
                    "Improper script count in symbol"))
                  ('T
                   (SEQ
                    (SPADCALL |lscripts| |i|
                     (PROGN
                       (LETT G108317 NIL
                        |SYMBOL;scripts;$R;32|)
                       (SEQ
                        (LETT |a| NIL
```

```
                                         |SYMBOL;scripts;$R;32|)
                                        (LETT G108318
                                         (SPADCALL |allscripts| |n|
                                          (QREFELT $ 118))
                                         |SYMBOL;scripts;$R;32|)
                                        G190
                                        (COND
                                          ((OR (ATOM G108318)
                                            (PROGN
                                              (LETT |a|
                                               (CAR G108318)
                                               |SYMBOL;scripts;$R;32|)
                                              NIL))
                                           (GO G191)))
                                        (SEQ
                                         (EXIT
                                          (LETT G108317
                                           (CONS
                                            (SPADCALL |a|
                                             (QREFELT $ 52))
                                            G108317)
                                           |SYMBOL;scripts;$R;32|)))
                                        (LETT G108318 (CDR G108318)
                                         |SYMBOL;scripts;$R;32|)
                                        (GO G190) G191
                                        (EXIT (NREVERSEO G108317))))
                                     (QREFELT $ 119))
                                    (EXIT
                                     (LETT |allscripts|
                                      (SPADCALL |allscripts| |n|
                                       (QREFELT $ 120))
                                      |SYMBOL;scripts;$R;32|)))))))
                   (LETT |i|
                         (PROG1 (+ |i| 1)
                           (LETT G108316 (CDR G108316)
                                 |SYMBOL;scripts;$R;32|))
                         |SYMBOL;scripts;$R;32|)
                   (GO G190) G191 (EXIT NIL))
              (EXIT (VECTOR (SPADCALL |lscripts| |m|
                                (QREFELT $ 121))
                            (SPADCALL |lscripts| (+ |m| 1)
                                (QREFELT $ 121))
                            (SPADCALL |lscripts| (+ |m| 2)
                                (QREFELT $ 121))
                            (SPADCALL |lscripts| (+ |m| 3)
                                (QREFELT $ 121))
                            (SPADCALL |lscripts| (+ |m| 4)
                                (QREFELT $ 121)))))))))

(DEFUN |SYMBOL;istring| (|n| $)
```

```
  (COND
    ((< 9 |n|) (|error| "Can have at most 9 scripts of each kind"))
    ('T (ELT (QREFELT $ 16) (+ |n| 0)))))

(DEFUN |SYMBOL;list;$L;34| (|sy| $)
  (COND
    ((NULL (SPADCALL |sy| (QREFELT $ 21)))
     (|error| "Cannot convert a symbol to a list if it is not subscripted"))
    ('T |sy|)))

(DEFUN |SYMBOL;sample;$;35| (|$|) (SPADCALL "aSymbol" (QREFELT |$| 47)))

(DEFUN |Symbol| ()
  (PROG ()
    (RETURN
      (PROG (G108325)
        (RETURN
          (COND
            ((LETT G108325 (HGET |$ConstructorCache| '|Symbol|)
                   |Symbol|)
             (|CDRwithIncrement| (CDAR G108325)))
            ('T
             (UNWIND-PROTECT
               (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Symbol|
                                   (LIST
                                    (CONS NIL (CONS 1 (|Symbol;|))))))
                 (LETT G108325 T |Symbol|))
               (COND
                 ((NOT G108325) (HREM |$ConstructorCache| '|Symbol|)))))))))))

(DEFUN |Symbol;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|Symbol|) |Symbol|)
        (LETT $ (make-array 124) |Symbol|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$| (|buildPredVector| 0 0 NIL) |Symbol|))
        (|haddProp| |$ConstructorCache| '|Symbol| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 9 (SPADCALL 0 (QREFELT $ 8)))
        (QSETREFV $ 12 (SPADCALL (QREFELT $ 11)))
        (QSETREFV $ 16
            (SPADCALL (LIST "0" "1" "2" "3" "4" "5" "6" "7" "8" "9")
                (QREFELT $ 15)))
        (QSETREFV $ 17 "0123456789")
        (QSETREFV $ 18 "ABCDEFGHIJKLMNOPQRSTUVWXYZ")
        (QSETREFV $ 19 "abcdefghijklmnopqrstuvwxyz")
        (QSETREFV $ 37 "*")
```

```
        (QSETREFV $ 38 (QCSIZE (QREFELT $ 37)))
        (QSETREFV $ 42
            (SPADCALL (SPADCALL "0" (QREFELT $ 40)) (QREFELT $ 41)))
        $))))

(setf (get '|Symbol| '|infovec|)
    (LIST '#(NIL NIL NIL NIL NIL NIL (|Integer|) (|Reference| 6)
             (0 . |ref|) '|count| (|AssociationList| $$ 6)
             (5 . |empty|) '|xcount| (|List| 28) (|PrimitiveArray| 28)
             (9 . |construct|) '|istrings| '|nums| 'ALPHAS '|alphas|
             (|Boolean|) |SYMBOL;scripted?;$B;30| (|Void|) (|Symbol|)
             (|OpenMathDevice|) (14 . |OMputVariable|)
             (|OpenMathEncoding|) (20 . |OMencodingXML|) (|String|)
             (24 . |OMopenString|) (30 . |OMputObject|)
             (35 . |OMputEndObject|) (40 . |OMclose|)
             |SYMBOL;OMwrite;$S;2| |SYMBOL;OMwrite;$BS;3|
             |SYMBOL;OMwrite;Omd$V;4| |SYMBOL;OMwrite;Omd$BV;5| '|hd|
             '|lhd| (|Character|) (45 . |char|) (50 . |ord|) '|ord0|
             (|InputForm|) (55 . |convert|) |SYMBOL;convert;$If;6|
             |SYMBOL;convert;2$;7| |SYMBOL;coerce;S$;8|
             |SYMBOL;=;2$B;9| |SYMBOL;<;2$B;10| (|OutputForm|)
             (60 . |outputForm|) |SYMBOL;coerce;$Of;11| (|List| 55)
             |SYMBOL;script;$L$;22| (|List| 50)
             |SYMBOL;subscript;$L$;12| |SYMBOL;elt;$L$;13|
             |SYMBOL;superscript;$L$;14| |SYMBOL;argscript;$L$;15|
             (|PatternMatchResult| 6 23) (|Pattern| 6)
             (|PatternMatchSymbol| 6) (65 . |patternMatch|)
             (|PatternMatchResult| 6 $) |SYMBOL;patternMatch;$P2Pmr;16|
             (|PatternMatchResult| (|Float|) 23) (|Pattern| (|Float|))
             (|PatternMatchSymbol| (|Float|)) (72 . |patternMatch|)
             (|PatternMatchResult| (|Float|) $)
             |SYMBOL;patternMatch;$P2Pmr;17| (79 . |coerce|)
             |SYMBOL;convert;$P;18| (84 . |coerce|)
             |SYMBOL;convert;$P;19| (|List| $) (89 . |concat|)
             (94 . |concat|)
             (|Record| (|:| |sub| 55) (|:| |sup| 55) (|:| |presup| 55)
                 (|:| |presub| 55) (|:| |args| 55))
             |SYMBOL;script;$R$;23| |SYMBOL;name;2$;31|
             |SYMBOL;string;$S;24| (100 . |elt|) (106 . =)
             |SYMBOL;scripts;$R;32| (112 . |latex|)
             |SYMBOL;latex;$S;25| (117 . |minIndex|) (122 . |concat|)
             (128 . |elt|) (133 . |setelt|) |SYMBOL;new;$;27|
             (|Union| 6 '"failed") (139 . |search|) (145 . |setelt|)
             (152 . |maxIndex|) (157 . |position|) |SYMBOL;new;2$;28|
             (|List| $$) (163 . |keys|) (168 . |remove!|)
             (174 . |void|) |SYMBOL;resetNew;V;29| |SYMBOL;list;$L;34|
             (178 . |first|) (183 . |digit?|) (|UniversalSegment| 6)
             (188 . SEGMENT) (194 . |elt|) (|List| 112)
             (200 . |minIndex|) (|NonNegativeInteger|) (205 . |setelt|)
             (212 . |concat|) (218 . |rest|) (223 . |minIndex|)
```

```
   (228 . |#|) (233 . |first|) (239 . |setelt|)
   (246 . |rest|) (252 . |elt|)
   (CONS IDENTITY
         (FUNCALL (|dispatchFunction| |SYMBOL;sample;$;35|)
                  $))
   (|SingleInteger|))
'#(~= 258 |superscript| 264 |subscript| 270 |string| 276
   |scripts| 281 |scripted?| 286 |script| 291 |sample| 303
   |resetNew| 307 |patternMatch| 311 |new| 325 |name| 334
   |min| 339 |max| 345 |list| 351 |latex| 356 |hash| 361
   |elt| 366 |convert| 372 |coerce| 392 |argscript| 402
   |OMwrite| 408 >= 432 > 438 = 444 <= 450 < 456)
'NIL
(CONS (|makeByteWordVec2| 1 '(0 0 0 0 0 0 0 0 0 0 0 0))
      (CONS '#(|OrderedSet&| NIL NIL |SetCategory&|
               |BasicType&| NIL NIL NIL NIL NIL NIL)
            (CONS '#((|OrderedSet|)
                     (|PatternMatchable| (|Float|))
                     (|PatternMatchable| 6) (|SetCategory|)
                     (|BasicType|) (|ConvertibleTo| 67)
                     (|ConvertibleTo| 61)
                     (|ConvertibleTo| 23) (|OpenMath|)
                     (|ConvertibleTo| 43) (|CoercibleTo| 50))
                  (|makeByteWordVec2| 123
                      '(1 7 0 6 8 0 10 0 11 1 14 0 13 15 2 24
                        22 0 23 25 0 26 0 27 2 24 0 28 26 29
                        1 24 22 0 30 1 24 22 0 31 1 24 22 0
                        32 1 39 0 28 40 1 39 6 0 41 1 43 0 23
                        44 1 50 0 23 51 3 62 60 23 61 60 63 3
                        68 66 23 67 66 69 1 67 0 23 72 1 61 0
                        23 74 1 28 0 76 77 2 55 0 0 0 78 2 28
                        39 0 6 83 2 39 20 0 0 84 1 50 28 0 86
                        1 28 6 0 88 2 28 0 39 0 89 1 7 6 0 90
                        2 7 6 0 6 91 2 10 93 2 0 94 3 10 6 0
                        2 6 95 1 28 6 0 96 2 28 6 39 0 97 1
                        10 99 0 100 2 10 93 2 0 101 0 22 0
                        102 1 99 2 0 105 1 39 20 0 106 2 107
                        0 6 6 108 2 28 0 0 107 109 1 110 6 0
                        111 3 110 112 0 6 112 113 2 110 0 0
                        112 114 1 99 0 0 115 1 53 6 0 116 1
                        99 112 0 117 2 99 0 0 112 118 3 53 55
                        0 6 55 119 2 99 0 0 112 120 2 53 55 0
                        6 121 2 0 20 0 0 1 2 0 0 0 55 58 2 0
                        0 0 55 56 1 0 28 0 82 1 0 79 0 85 1 0
                        20 0 21 2 0 0 0 53 54 2 0 0 0 79 80 0
                        0 0 122 0 0 22 103 3 0 64 0 61 64 65
                        3 0 70 0 67 70 71 1 0 0 0 98 0 0 0 92
                        1 0 0 0 81 2 0 0 0 0 1 2 0 0 0 0 1 1
                        0 76 0 104 1 0 28 0 87 1 0 123 0 1 2
                        0 0 0 55 57 1 0 61 0 75 1 0 67 0 73 1
```

```
                                  0 23 0 46 1 0 43 0 45 1 0 0 28 47 1 0
                                  50 0 52 2 0 0 0 55 59 3 0 22 24 0 20
                                  36 2 0 28 0 20 34 2 0 22 24 0 35 1 0
                                  28 0 33 2 0 20 0 0 1 2 0 20 0 0 1 2 0
                                  20 0 0 48 2 0 20 0 0 1 2 0 20 0 0 49)))))
        '|lookupComplete|))

(setf (get '|Symbol| 'NILADIC) T)
```

———-

28.15 VECTOR.lsp BOOTSTRAP

VECTOR depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **VECTOR** category which we can write into the **MID** directory. We compile the lisp code and copy the **VECTOR.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— VECTOR.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |VECTOR;vector;L$;1| (|l| |$|) (SPADCALL |l| (QREFELT |$| 8)))

(DEFUN |VECTOR;convert;$If;2| (|x| |$|)
 (SPADCALL
  (LIST
   (SPADCALL (SPADCALL "vector" (QREFELT |$| 12)) (QREFELT |$| 14))
   (SPADCALL (SPADCALL |x| (QREFELT |$| 15)) (QREFELT |$| 16)))
  (QREFELT |$| 18)))

(DEFUN |Vector| (G84134)
  (PROG ()
    (RETURN
      (PROG (G84135)
        (RETURN
          (COND
            ((LETT G84135
                   (|lassocShiftWithFunction|
                       (LIST (|devaluate| G84134))
                       (HGET |$ConstructorCache| '|Vector|)
                       '|domainEqualList|)
                   |Vector|)
             (|CDRwithIncrement| G84135))
            ('T
```

```
             (UNWIND-PROTECT
               (PROG1 (|Vector;| G84134) (LETT G84135 T |Vector|))
               (COND
                 ((NOT G84135) (HREM |$ConstructorCache| '|Vector|)))))))))))

(DEFUN |Vector;| (|#1|)
  (PROG (DV$1 |dv$| $ G84133 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |Vector|)
        (LETT |dv$| (LIST '|Vector| DV$1) |Vector|)
        (LETT $ (make-array 36) |Vector|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
            (LETT |pv$|
                  (|buildPredVector| 0 0
                      (LIST (|HasCategory| |#1| '(|SetCategory|))
                            (|HasCategory| |#1|
                                '(|ConvertibleTo| (|InputForm|)))
                            (LETT G84133
                                  (|HasCategory| |#1| '(|OrderedSet|))
                                  |Vector|)
                            (OR G84133
                                (|HasCategory| |#1| '(|SetCategory|)))
                            (|HasCategory| (|Integer|) '(|OrderedSet|))
                            (|HasCategory| |#1| '(|AbelianSemiGroup|))
                            (|HasCategory| |#1| '(|AbelianMonoid|))
                            (|HasCategory| |#1| '(|AbelianGroup|))
                            (|HasCategory| |#1| '(|Monoid|))
                            (|HasCategory| |#1| '(|Ring|))
                            (AND (|HasCategory| |#1|
                                     '(|RadicalCategory|))
                                 (|HasCategory| |#1| '(|Ring|)))
                            (AND (|HasCategory| |#1|
                                     (LIST '|Evalable|
                                      (|devaluate| |#1|)))
                                 (|HasCategory| |#1| '(|SetCategory|)))
                            (OR (AND (|HasCategory| |#1|
                                      (LIST '|Evalable|
                                       (|devaluate| |#1|)))
                                     G84133)
                                (AND (|HasCategory| |#1|
                                      (LIST '|Evalable|
                                       (|devaluate| |#1|)))
                                     (|HasCategory| |#1|
                                      '(|SetCategory|))))))
                  |Vector|))
        (|haddProp| |$ConstructorCache| '|Vector| (LIST DV$1)
            (CONS 1 $))
        (|stuffDomainSlots| $)
```

```
        (QSETREFV $ 6 |#1|)
        (COND
          ((|testBitVector| |pv$| 2)
           (QSETREFV $ 19
               (CONS (|dispatchFunction| |VECTOR;convert;$If;2|) $))))
        $))))

(setf (get
 (QUOTE |Vector|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL (|IndexedVector| 6 (NRTEVAL 1)) (|local| |#1|)
    (|List| 6) (0 . |construct|) |VECTOR;vector;L$;1| (|String|) (|Symbol|)
    (5 . |coerce|) (|InputForm|) (10 . |convert|) (15 . |parts|)
    (20 . |convert|) (|List| |$|) (25 . |convert|) (30 . |convert|)
    (|Mapping| 6 6 6) (|Boolean|) (|NonNegativeInteger|) (|List| 24)
    (|Equation| 6) (|Integer|) (|Mapping| 21 6) (|Mapping| 21 6 6)
    (|UniversalSegment| 25) (|Void|) (|Mapping| 6 6) (|Matrix| 6)
    (|OutputForm|) (|SingleInteger|) (|Union| 6 (QUOTE "failed"))
    (|List| 25)))
  (QUOTE #(|vector| 35 |parts| 40 |convert| 45 |construct| 50))
  (QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
  (CONS
   (|makeByteWordVec2| 13 (QUOTE (0 0 0 0 0 0 0 3 0 0 13 4 0 0 13 1 2 4)))
   (CONS
    (QUOTE #(|VectorCategory&| |OneDimensionalArrayAggregate&|
      |FiniteLinearAggregate&| |LinearAggregate&| |IndexedAggregate&|
      |Collection&| |HomogeneousAggregate&| |OrderedSet&| |Aggregate&|
      |EltableAggregate&| |Evalable&| |SetCategory&| NIL NIL
      |InnerEvalable&| NIL NIL |BasicType&|))
    (CONS
     (QUOTE #((|VectorCategory| 6) (|OneDimensionalArrayAggregate| 6)
      (|FiniteLinearAggregate| 6) (|LinearAggregate| 6)
      (|IndexedAggregate| 25 6) (|Collection| 6)
      (|HomogeneousAggregate| 6) (|OrderedSet|) (|Aggregate|)
      (|EltableAggregate| 25 6) (|Evalable| 6) (|SetCategory|)
      (|Type|) (|Eltable| 25 6) (|InnerEvalable| 6 6)
      (|CoercibleTo| 32) (|ConvertibleTo| 13) (|BasicType|)))
     (|makeByteWordVec2| 19
      (QUOTE (1 0 0 7 8 1 11 0 10 12 1 13 0 11 14 1 0 7 0 15 1 7 13 0 16 1 13
          0 17 18 1 0 13 0 19 1 0 0 7 9 1 0 7 0 15 1 2 13 0 19 1 0 0 7 8))))))
  (QUOTE |lookupIncomplete|)))
```

———-

Chapter 29

Chunk collections

— algebra —

```
\getchunk{domain AFFPL AffinePlane}
\getchunk{domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField}
\getchunk{domain AFFSP AffineSpace}
\getchunk{domain ALGSC AlgebraGivenByStructuralConstants}
\getchunk{domain ALGFF AlgebraicFunctionField}
\getchunk{domain AN AlgebraicNumber}
\getchunk{domain ANON AnonymousFunction}
\getchunk{domain ANTISYM AntiSymm}
\getchunk{domain ANY Any}
\getchunk{domain ASTACK ArrayStack}
\getchunk{domain ASP1 Asp1}
\getchunk{domain ASP10 Asp10}
\getchunk{domain ASP12 Asp12}
\getchunk{domain ASP19 Asp19}
\getchunk{domain ASP20 Asp20}
\getchunk{domain ASP24 Asp24}
\getchunk{domain ASP27 Asp27}
\getchunk{domain ASP28 Asp28}
\getchunk{domain ASP29 Asp29}
\getchunk{domain ASP30 Asp30}
\getchunk{domain ASP31 Asp31}
\getchunk{domain ASP33 Asp33}
\getchunk{domain ASP34 Asp34}
\getchunk{domain ASP35 Asp35}
\getchunk{domain ASP4 Asp4}
\getchunk{domain ASP41 Asp41}
\getchunk{domain ASP42 Asp42}
\getchunk{domain ASP49 Asp49}
\getchunk{domain ASP50 Asp50}
```

```
\getchunk{domain ASP55 Asp55}
\getchunk{domain ASP6 Asp6}
\getchunk{domain ASP7 Asp7}
\getchunk{domain ASP73 Asp73}
\getchunk{domain ASP74 Asp74}
\getchunk{domain ASP77 Asp77}
\getchunk{domain ASP78 Asp78}
\getchunk{domain ASP8 Asp8}
\getchunk{domain ASP80 Asp80}
\getchunk{domain ASP9 Asp9}
\getchunk{domain JORDAN AssociatedJordanAlgebra}
\getchunk{domain LIE AssociatedLieAlgebra}
\getchunk{domain ALIST AssociationList}
\getchunk{domain ATTRBUT AttributeButtons}
\getchunk{domain AUTOMOR Automorphism}

\getchunk{domain BBTREE BalancedBinaryTree}
\getchunk{domain BPADIC BalancedPAdicInteger}
\getchunk{domain BPADICRT BalancedPAdicRational}
\getchunk{domain BFUNCT BasicFunctions}
\getchunk{domain BOP BasicOperator}
\getchunk{domain BINARY BinaryExpansion}
\getchunk{domain BINFILE BinaryFile}
\getchunk{domain BSTREE BinarySearchTree}
\getchunk{domain BTOURN BinaryTournament}
\getchunk{domain BTREE BinaryTree}
\getchunk{domain BITS Bits}
\getchunk{domain BLHN BlowUpWithHamburgerNoether}
\getchunk{domain BLQT BlowUpWithQuadTrans}
\getchunk{domain BOOLEAN Boolean}

\getchunk{domain CARD CardinalNumber}
\getchunk{domain CARTEN CartesianTensor}
\getchunk{domain CHAR Character}
\getchunk{domain CCLASS CharacterClass}
\getchunk{domain CLIF CliffordAlgebra}
\getchunk{domain COLOR Color}
\getchunk{domain COMM Commutator}
\getchunk{domain COMPLEX Complex}
\getchunk{domain CDFMAT ComplexDoubleFloatMatrix}
\getchunk{domain CDFVEC ComplexDoubleFloatVector}
\getchunk{domain CONTFRAC ContinuedFraction}

\getchunk{domain DHMATRIX DenavitHartenbergMatrix}
\getchunk{domain DBASE Database}
\getchunk{domain DLIST DataList}
\getchunk{domain DECIMAL DecimalExpansion}
\getchunk{domain DEQUEUE Dequeue}
\getchunk{domain DERHAM DeRhamComplex}
\getchunk{domain DSMP DifferentialSparseMultivariatePolynomial}
```

```
\getchunk{domain DIRPROD DirectProduct}
\getchunk{domain DPMM DirectProductMatrixModule}
\getchunk{domain DPMO DirectProductModule}
\getchunk{domain DIRRING DirichletRing}
\getchunk{domain DMP DistributedMultivariatePolynomial}
\getchunk{domain DIV Divisor}
\getchunk{domain DFLOAT DoubleFloat}
\getchunk{domain DFMAT DoubleFloatMatrix}
\getchunk{domain DFVEC DoubleFloatVector}
\getchunk{domain DROPT DrawOption}
\getchunk{domain D01AJFA d01ajfAnnaType}
\getchunk{domain D01AKFA d01akfAnnaType}
\getchunk{domain D01ALFA d01alfAnnaType}
\getchunk{domain D01AMFA d01amfAnnaType}
\getchunk{domain D01ANFA d01anfAnnaType}
\getchunk{domain D01APFA d01apfAnnaType}
\getchunk{domain D01AQFA d01aqfAnnaType}
\getchunk{domain D01ASFA d01asfAnnaType}
\getchunk{domain D01FCFA d01fcfAnnaType}
\getchunk{domain D01GBFA d01gbfAnnaType}
\getchunk{domain D01TRNS d01TransformFunctionType}
\getchunk{domain D02BBFA d02bbfAnnaType}
\getchunk{domain D02BHFA d02bhfAnnaType}
\getchunk{domain D02CJFA d02cjfAnnaType}
\getchunk{domain D02EJFA d02ejfAnnaType}
\getchunk{domain D03EEFA d03eefAnnaType}
\getchunk{domain D03FAFA d03fafAnnaType}

\getchunk{domain EQTBL EqTable}
\getchunk{domain EQ Equation}
\getchunk{domain EXPEXPAN ExponentialExpansion}
\getchunk{domain EXPUPXS ExponentialOfUnivariatePuiseuxSeries}
\getchunk{domain EMR EuclideanModularRing}
\getchunk{domain EXIT Exit}
\getchunk{domain EXPR Expression}
\getchunk{domain EAB ExtAlgBasis}
\getchunk{domain E04DGFA e04dgfAnnaType}
\getchunk{domain E04FDFA e04fdfAnnaType}
\getchunk{domain E04GCFA e04gcfAnnaType}
\getchunk{domain E04JAFA e04jafAnnaType}
\getchunk{domain E04MBFA e04mbfAnnaType}
\getchunk{domain E04NAFA e04nafAnnaType}
\getchunk{domain E04UCFA e04ucfAnnaType}

\getchunk{domain FR Factored}
\getchunk{domain FILE File}
\getchunk{domain FNAME FileName}
\getchunk{domain FARRAY FlexibleArray}
\getchunk{domain FDIV FiniteDivisor}
\getchunk{domain FF FiniteField}
```

```
\getchunk{domain FFCG FiniteFieldCyclicGroup}
\getchunk{domain FFCGX FiniteFieldCyclicGroupExtension}
\getchunk{domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial}
\getchunk{domain FFX FiniteFieldExtension}
\getchunk{domain FFP FiniteFieldExtensionByPolynomial}
\getchunk{domain FFNB FiniteFieldNormalBasis}
\getchunk{domain FFNBX FiniteFieldNormalBasisExtension}
\getchunk{domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial}
\getchunk{domain FLOAT Float}
\getchunk{domain FC FortranCode}
\getchunk{domain FEXPR FortranExpression}
\getchunk{domain FORTRAN FortranProgram}
\getchunk{domain FST FortranScalarType}
\getchunk{domain FTEM FortranTemplate}
\getchunk{domain FT FortranType}
\getchunk{domain FCOMP FourierComponent}
\getchunk{domain FSERIES FourierSeries}
\getchunk{domain FRAC Fraction}
\getchunk{domain FRIDEAL FractionalIdeal}
\getchunk{domain FRMOD FramedModule}
\getchunk{domain FAGROUP FreeAbelianGroup}
\getchunk{domain FAMONOID FreeAbelianMonoid}
\getchunk{domain FGROUP FreeGroup}
\getchunk{domain FM FreeModule}
\getchunk{domain FM1 FreeModule1}
\getchunk{domain FMONOID FreeMonoid}
\getchunk{domain FNLA FreeNilpotentLie}
\getchunk{domain FPARFRAC FullPartialFractionExpansion}
\getchunk{domain FUNCTION FunctionCalled}

\getchunk{domain GDMP GeneralDistributedMultivariatePolynomial}
\getchunk{domain GMODPOL GeneralModulePolynomial}
\getchunk{domain GCNAALG GenericNonAssociativeAlgebra}
\getchunk{domain GPOLSET GeneralPolynomialSet}
\getchunk{domain GSTBL GeneralSparseTable}
\getchunk{domain GTSET GeneralTriangularSet}
\getchunk{domain GSERIES GeneralUnivariatePowerSeries}
\getchunk{domain GRIMAGE GraphImage}
\getchunk{domain GOPT GuessOption}
\getchunk{domain GOPT0 GuessOptionFunctions0}

\getchunk{domain HASHTBL HashTable}
\getchunk{domain HEAP Heap}
\getchunk{domain HEXADEC HexadecimalExpansion}
\getchunk{domain HTMLFORM HTMLFormat}
\getchunk{domain HDP HomogeneousDirectProduct}
\getchunk{domain HDMP HomogeneousDistributedMultivariatePolynomial}
\getchunk{domain HELLFDIV HyperellipticFiniteDivisor}

\getchunk{domain ICP InfClsPt}
```

```
\getchunk{domain ICARD IndexCard}
\getchunk{domain IBITS IndexedBits}
\getchunk{domain IDPAG IndexedDirectProductAbelianGroup}
\getchunk{domain IDPAM IndexedDirectProductAbelianMonoid}
\getchunk{domain IDPO IndexedDirectProductObject}
\getchunk{domain IDPOAM IndexedDirectProductOrderedAbelianMonoid}
\getchunk{domain IDPOAMS IndexedDirectProductOrderedAbelianMonoidSup}
\getchunk{domain INDE IndexedExponents}
\getchunk{domain IFARRAY IndexedFlexibleArray}
\getchunk{domain ILIST IndexedList}
\getchunk{domain IMATRIX IndexedMatrix}
\getchunk{domain IARRAY1 IndexedOneDimensionalArray}
\getchunk{domain ISTRING IndexedString}
\getchunk{domain IARRAY2 IndexedTwoDimensionalArray}
\getchunk{domain IVECTOR IndexedVector}
\getchunk{domain ITUPLE InfiniteTuple}
\getchunk{domain INFCLSPT InfinitlyClosePoint}
\getchunk{domain INFCLSPS InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField}
\getchunk{domain IAN InnerAlgebraicNumber}
\getchunk{domain IFF InnerFiniteField}
\getchunk{domain IFAMON InnerFreeAbelianMonoid}
\getchunk{domain IIARRAY2 InnerIndexedTwoDimensionalArray}
\getchunk{domain IPADIC InnerPAdicInteger}
\getchunk{domain IPF InnerPrimeField}
\getchunk{domain ISUPS InnerSparseUnivariatePowerSeries}
\getchunk{domain INTABL InnerTable}
\getchunk{domain ITAYLOR InnerTaylorSeries}
\getchunk{domain INFORM InputForm}
\getchunk{domain INT Integer}
\getchunk{domain ZMOD IntegerMod}
\getchunk{domain INTFTBL IntegrationFunctionsTable}
\getchunk{domain IR IntegrationResult}
\getchunk{domain INTRVL Interval}

\getchunk{domain KERNEL Kernel}
\getchunk{domain KAFILE KeyedAccessFile}

\getchunk{domain LAUPOL LaurentPolynomial}
\getchunk{domain LIB Library}
\getchunk{domain LEXP LieExponentials}
\getchunk{domain LPOLY LiePolynomial}
\getchunk{domain LSQM LieSquareMatrix}
\getchunk{domain LODO LinearOrdinaryDifferentialOperator}
\getchunk{domain LODO1 LinearOrdinaryDifferentialOperator1}
\getchunk{domain LODO2 LinearOrdinaryDifferentialOperator2}
\getchunk{domain LIST List}
\getchunk{domain LMOPS ListMonoidOps}
\getchunk{domain LMDICT ListMultiDictionary}
\getchunk{domain LA LocalAlgebra}
\getchunk{domain LO Localize}
```

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\getchunk{domain LWORD LyndonWord}

\getchunk{domain MCMPLX MachineComplex}
\getchunk{domain MFLOAT MachineFloat}
\getchunk{domain MINT MachineInteger}
\getchunk{domain MAGMA Magma}
\getchunk{domain MKCHSET MakeCachableSet}
\getchunk{domain MMLFORM MathMLFormat}
\getchunk{domain MATRIX Matrix}
\getchunk{domain MODMON ModMonic}
\getchunk{domain MODMONOM ModuleMonomial}
\getchunk{domain MODFIELD ModularField}
\getchunk{domain MODRING ModularRing}
\getchunk{domain MODOP ModuleOperator}
\getchunk{domain MOEBIUS MoebiusTransform}
\getchunk{domain MRING MonoidRing}
\getchunk{domain MSET Multiset}
\getchunk{domain MPOLY MultivariatePolynomial}
\getchunk{domain MYEXPR MyExpression}
\getchunk{domain MYUP MyUnivariatePolynomial}

\getchunk{domain NSDPS NeitherSparseOrDensePowerSeries}
\getchunk{domain NSMP NewSparseMultivariatePolynomial}
\getchunk{domain NSUP NewSparseUnivariatePolynomial}
\getchunk{domain NONE None}
\getchunk{domain NNI NonNegativeInteger}
\getchunk{domain NOTTING NottinghamGroup}
\getchunk{domain NIPROB NumericalIntegrationProblem}
\getchunk{domain ODEPROB NumericalODEProblem}
\getchunk{domain OPTPROB NumericalOptimizationProblem}
\getchunk{domain PDEPROB NumericalPDEProblem}

\getchunk{domain OCT Octonion}
\getchunk{domain ODEIFTBL ODEIntensityFunctionsTable}
\getchunk{domain ARRAY1 OneDimensionalArray}
\getchunk{domain ONECOMP OnePointCompletion}
\getchunk{domain OMCONN OpenMathConnection}
\getchunk{domain OMDEV OpenMathDevice}
\getchunk{domain OMENC OpenMathEncoding}
\getchunk{domain OMERR OpenMathError}
\getchunk{domain OMERRK OpenMathErrorKind}
\getchunk{domain OP Operator}
\getchunk{domain OMLO OppositeMonogenicLinearOperator}
\getchunk{domain ORDCOMP OrderedCompletion}
\getchunk{domain ODP OrderedDirectProduct}
\getchunk{domain OFMONOID OrderedFreeMonoid}
\getchunk{domain OVAR OrderedVariableList}
\getchunk{domain ODPOL OrderlyDifferentialPolynomial}
\getchunk{domain ODVAR OrderlyDifferentialVariable}
\getchunk{domain ODR OrdinaryDifferentialRing}
```

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\getchunk{domain OWP OrdinaryWeightedPolynomials}
\getchunk{domain OSI OrdSetInts}
\getchunk{domain OUTFORM OutputForm}

\getchunk{domain PADIC PAdicInteger}
\getchunk{domain PADICRC PAdicRationalConstructor}
\getchunk{domain PADICRAT PAdicRational}
\getchunk{domain PALETTE Palette}
\getchunk{domain PARPCURV ParametricPlaneCurve}
\getchunk{domain PARSCURV ParametricSpaceCurve}
\getchunk{domain PARSURF ParametricSurface}
\getchunk{domain PFR PartialFraction}
\getchunk{domain PLACES Places}
\getchunk{domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField}
\getchunk{domain PRTITION Partition}
\getchunk{domain PATTERN Pattern}
\getchunk{domain PATLRES PatternMatchListResult}
\getchunk{domain PATRES PatternMatchResult}
\getchunk{domain PENDTREE PendantTree}
\getchunk{domain PERM Permutation}
\getchunk{domain PERMGRP PermutationGroup}
\getchunk{domain HACKPI Pi}
\getchunk{domain ACPLOT PlaneAlgebraicCurvePlot}
\getchunk{domain PLCS Plcs}
\getchunk{domain PLOT Plot}
\getchunk{domain PLOT3D Plot3D}
\getchunk{domain PBWLB PoincareBirkhoffWittLyndonBasis}
\getchunk{domain POINT Point}
\getchunk{domain POLY Polynomial}
\getchunk{domain IDEAL PolynomialIdeals}
\getchunk{domain PR PolynomialRing}
\getchunk{domain PI PositiveInteger}
\getchunk{domain PF PrimeField}
\getchunk{domain PRIMARR PrimitiveArray}
\getchunk{domain PRODUCT Product}
\getchunk{domain PROJPL ProjectivePlane}
\getchunk{domain PROJSP ProjectiveSpace}
\getchunk{domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber}
\getchunk{domain PACOFF PseudoAlgebraicClosureOfFiniteField}
\getchunk{domain PACRAT PseudoAlgebraicClosureOfRationalNumber}

\getchunk{domain QFORM QuadraticForm}
\getchunk{domain QALGSET QuasiAlgebraicSet}
\getchunk{domain QUAT Quaternion}
\getchunk{domain QEQUAT QueryEquation}
\getchunk{domain QUEUE Queue}

\getchunk{domain RADFF RadicalFunctionField}
\getchunk{domain RADIX RadixExpansion}
\getchunk{domain RECLOS RealClosure}
```

```
\getchunk{domain RMATRIX RectangularMatrix}
\getchunk{domain REF Reference}
\getchunk{domain RGCHAIN RegularChain}
\getchunk{domain REGSET RegularTriangularSet}
\getchunk{domain RESRING ResidueRing}
\getchunk{domain RESULT Result}
\getchunk{domain RULE RewriteRule}
\getchunk{domain ROIRC RightOpenIntervalRootCharacterization}
\getchunk{domain ROMAN RomanNumeral}
\getchunk{domain ROUTINE RoutinesTable}
\getchunk{domain RULECOLD RuleCalled}
\getchunk{domain RULESET Ruleset}

\getchunk{domain FORMULA ScriptFormulaFormat}
\getchunk{domain SEG Segment}
\getchunk{domain SEGBIND SegmentBinding}
\getchunk{domain SET Set}
\getchunk{domain SEX SExpression}
\getchunk{domain SEXOF SExpressionOf}
\getchunk{domain SAE SimpleAlgebraicExtension}
\getchunk{domain SFORT SimpleFortranProgram}
\getchunk{domain SINT SingleInteger}
\getchunk{domain SAOS SingletonAsOrderedSet}
\getchunk{domain SDPOL SequentialDifferentialPolynomial}
\getchunk{domain SDVAR SequentialDifferentialVariable}
\getchunk{domain SETMN SetOfMIntegersInOneToN}
\getchunk{domain SMP SparseMultivariatePolynomial}
\getchunk{domain SMTS SparseMultivariateTaylorSeries}
\getchunk{domain STBL SparseTable}
\getchunk{domain SULS SparseUnivariateLaurentSeries}
\getchunk{domain SUP SparseUnivariatePolynomial}
\getchunk{domain SUPEXPR SparseUnivariatePolynomialExpressions}
\getchunk{domain SUPXS SparseUnivariatePuiseuxSeries}
\getchunk{domain ORESUP SparseUnivariateSkewPolynomial}
\getchunk{domain SUTS SparseUnivariateTaylorSeries}
\getchunk{domain SHDP SplitHomogeneousDirectProduct}
\getchunk{domain SPLNODE SplittingNode}
\getchunk{domain SPLTREE SplittingTree}
\getchunk{domain SREGSET SquareFreeRegularTriangularSet}
\getchunk{domain SQMATRIX SquareMatrix}
\getchunk{domain STACK Stack}
\getchunk{domain STREAM Stream}
\getchunk{domain STRING String}
\getchunk{domain STRTBL StringTable}
\getchunk{domain SUBSPACE SubSpace}
\getchunk{domain COMPPROP SubSpaceComponentProperty}
\getchunk{domain SUCH SuchThat}
\getchunk{domain SWITCH Switch}
\getchunk{domain SYMBOL Symbol}
\getchunk{domain SYMTAB SymbolTable}
```

```
\getchunk{domain SYMPOLY SymmetricPolynomial}

\getchunk{domain TABLE Table}
\getchunk{domain TABLEAU Tableau}
\getchunk{domain TS TaylorSeries}
\getchunk{domain TEX TexFormat}
\getchunk{domain TEXTFILE TextFile}
\getchunk{domain SYMS TheSymbolTable}
\getchunk{domain M3D ThreeDimensionalMatrix}
\getchunk{domain VIEW3D ThreeDimensionalViewport}
\getchunk{domain SPACE3 ThreeSpace}
\getchunk{domain TREE Tree}
\getchunk{domain TUBE TubePlot}
\getchunk{domain TUPLE Tuple}
\getchunk{domain ARRAY2 TwoDimensionalArray}
\getchunk{domain VIEW2D TwoDimensionalViewport}

\getchunk{domain UFPS UnivariateFormalPowerSeries}
\getchunk{domain ULS UnivariateLaurentSeries}
\getchunk{domain ULSCONS UnivariateLaurentSeriesConstructor}
\getchunk{domain UP UnivariatePolynomial}
\getchunk{domain UPXS UnivariatePuiseuxSeries}
\getchunk{domain UPXSCONS UnivariatePuiseuxSeriesConstructor}
\getchunk{domain UPXSSING UnivariatePuiseuxSeriesWithExponentialSingularity}
\getchunk{domain OREUP UnivariateSkewPolynomial}
\getchunk{domain UTS UnivariateTaylorSeries}
\getchunk{domain UTSZ UnivariateTaylorSeriesCZero}
\getchunk{domain UNISEG UniversalSegment}

\getchunk{domain VARIABLE Variable}
\getchunk{domain VECTOR Vector}
\getchunk{domain VOID Void}

\getchunk{domain WP WeightedPolynomials}
\getchunk{domain WUTSET WuWenTsunTriangularSet}

\getchunk{domain XDPOLY XDistributedPolynomial}
\getchunk{domain XPBWPOLY XPBWPolynomial}
\getchunk{domain XPOLY XPolynomial}
\getchunk{domain XPR XPolynomialRing}
\getchunk{domain XRPOLY XRecursivePolynomial}
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